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Semester :- 5 . Subject : AAOA.
Seat No :- CS5A021
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D) i) We first show that vertex cover ∈ NP. Suppose we are given a graph $G = (V, E)$ and an integer k.
ii) The certificate we'll choose is vertex cover $V' \subseteq V$ itself. The verification algorithm affirms that $|V'| = k$ and then it checks for each edge $(u, v) \in E$ that $u \in V'$ or $v \in V'$
iii) This verification can be performed straight forwardly in polynomial time.
iv) We prove that vertex-cover problem is NP-hard by showing that CLIQUE $\leq_P$ VERTEX COVER. This reduction is based on notation of complement of graph as G(Bar).

Proof :-
Reduction algorithm takes instance (G, K) of clique problem and calculate G(Bar) in polynomial time.
Output is instance (G-Bar, |V|-k) of vertex cover problem.
To compleme a proof we have to show that a graph G has a clique of size k iff G-bar has a vertex cover of size |V|-k.
Suppose G has a clique V' such that $|V'| = k$. Then we claim that V-V' is vertex cover in G-bar.

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D) ~~Let u and v~~

- Let (u,v) be any edge in E-bar then $(u,v) \notin E$ which implies at least one of u or v does not belong to V'. Since every pair of vertices in V' is connected by an edge in E.
- Equivalently, atleast one of u or v is in $V-V'$ which means edge (u,v) is covered by $V-V'$
- Since (u,v) was chosen arbitrarily from ebar hence every edge in E-bar is covered by vertex in $V-V'$
- Hence set $V-V'$ which has size $|V|-K$ forms a vertex cover for G-bar.

Conversely, suppose G-bar has a vertex cover V' where $|V'| = |V| - K$.
then for all $u, v \in V$, if $(u,v) \in$ E-bar then $u \in V'$ or $v \in V'$ or both.
the contrapositive of this implication is that for all of $u, v \in V$ if $u \notin V'$ and $v \in V'$ then $(u,v) \in E$.
In other words, $V-V'$ is a clique of size $|V| - |V'| = k$,
So clique is polynomially reducible to vertex ~~cov~~ cover.

Hence vertex-cover is NP complete.

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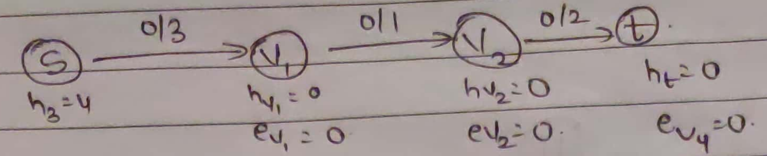
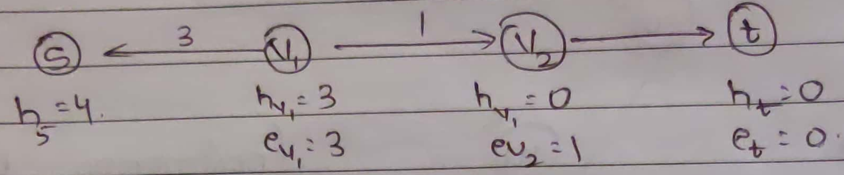
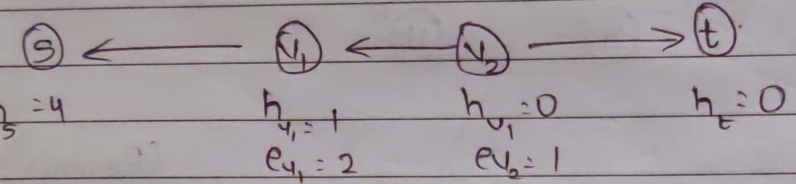
E)



L	V_1	V_2
N	S	V_1
	V_2	t

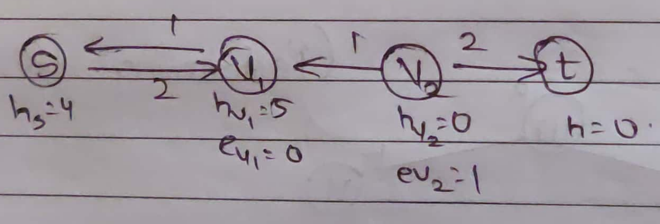
The vertex v_1 will undergo discharge operation
Vertex V_1 push 1 flow to V_2.





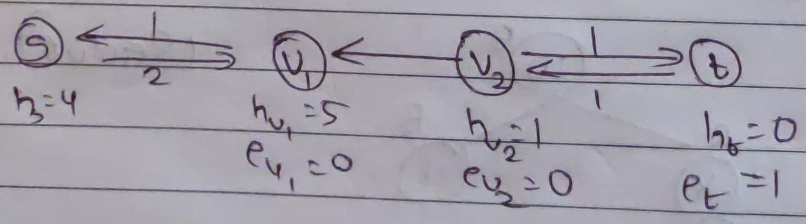
L	V_1	V_2
N	S	V_1
	V_2	t

Vertex v_1 still has $e_{v_1} = 2$ thus it will ~~relabel~~ relabel $h = 5$ and pushes how 2 to vertex S.



L	V_1	V_2
	S	T
	V_2	t

Vertex V_2 performs relabel operation and pushes 1 to t.



L	V_1	V_2
N	S	V_1
	V_2	t

Vertex V_1 now follows V_2 in 1 But V_1 has no excess flow. Thus relabel to front has reached the end of list 1
Here the maximum flow is 1.

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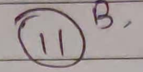
A) 11, 10, 9, 4, 6, 1.

Insert 11 :- 

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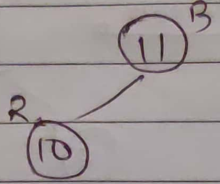
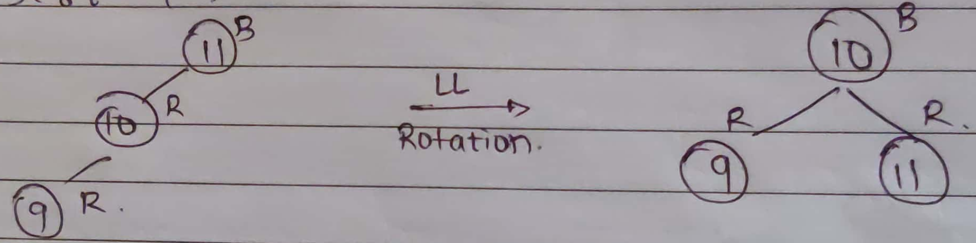
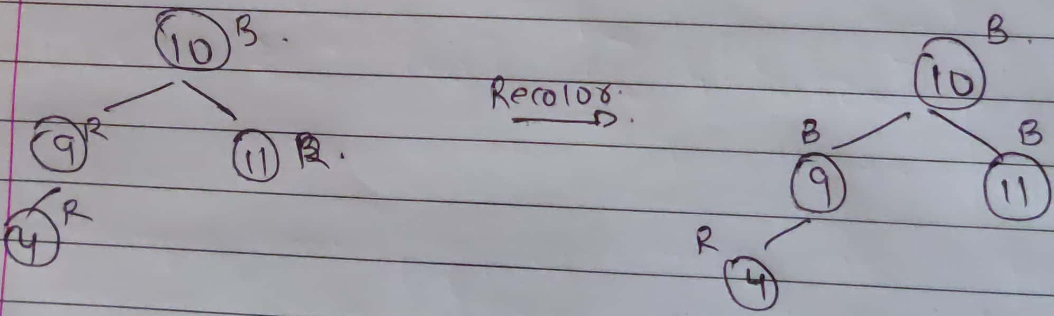


Insert 9 :-



Insert 4 :-



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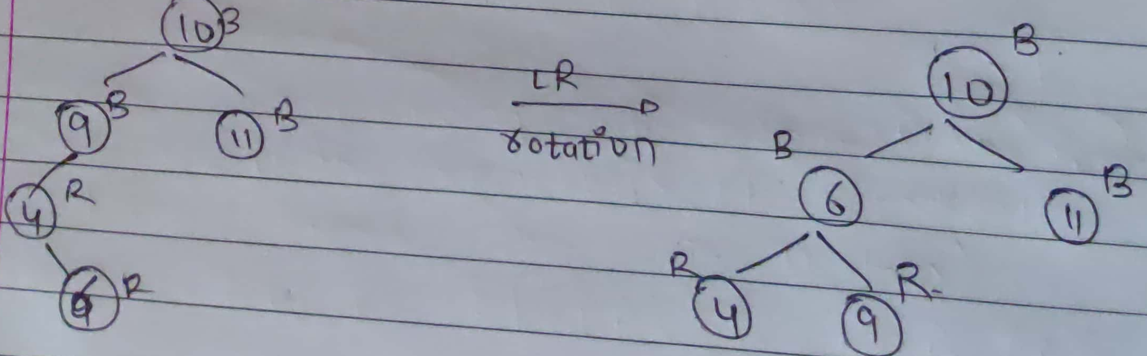

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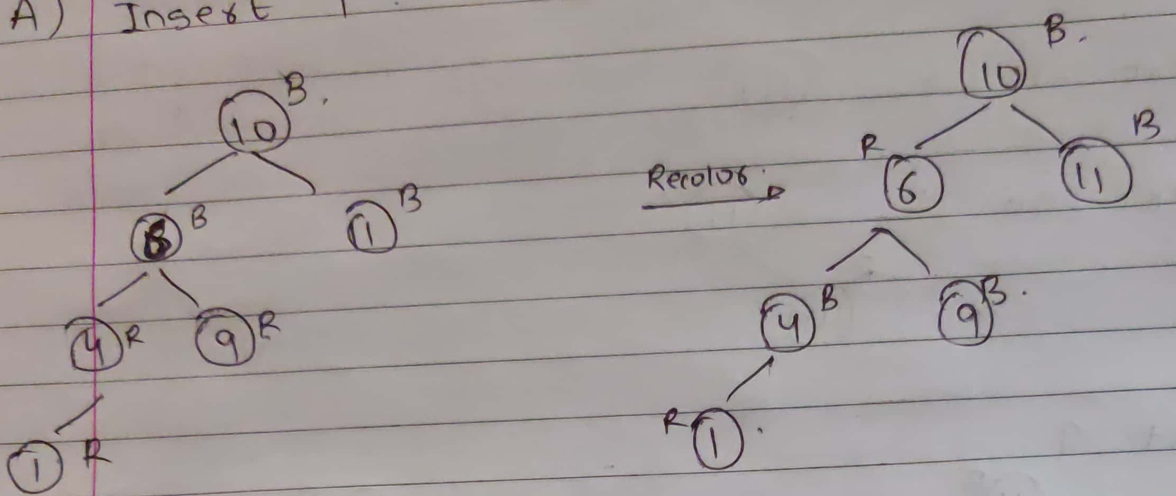
A) Insert 1 :-



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F) In order to analyze the hiring problem we need a convenient way to convert between probabilities and expectations.
we will use indicator random variable to help us do this.
Given a sample space S and an event A.
then the indicator random variable $I\{A\}$ associated with event A is define as :-

$$I\{A\} = \begin{cases} 1 & \text{if } A \text{ occurs.} \\ 0 & \text{if } A \text{ does not occur.} \end{cases}$$

Example :-
Suppose our sample space $S : \{H, T\}$ with $Pr(H) = Pr(T) = 1/2$
we can associate an indicator random variable. X_H associated with coin coming up heads.

$$X_H = I(H) = \begin{cases} 1 & \text{if } H \text{ occurs.} \\ 0 & \text{if } T \text{ occurs.} \end{cases}$$

The expected number of Heads in one coin flip is then

$$E(X_H) = E\{I(H)\}$$
$$= 1 \cdot Pr(H) + 0 \cdot Pr(T)$$
$$= 1 \cdot (1/2) + 0 \cdot (1/2)$$
$$= 1/2.$$

Given a sample space S and an event A in S let $X_A = I\{A\}$ then $E[X_A] = Pr\{A\}$.

Proof :- $E[X_A] = E[I\{A\}] = 1 * Pr\{A\} + 0 * Pr\{A\} = Pr\{A\}$.

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Analysis of Hiring problem.

Let x be the indicator random variable which is 1 if candidate i is hired and 0 otherwise.

Let $X = \sum_{i=1}^{n} X_i$

By our lemma $E[X_i] = P_r$ (candidate i is hired)

Candidate i will be hired if i is better than each of candidate 1 through $i-1$

As each candidate arrives in random order, any one of first candidate i is equally likely to be best candidate so far.

So $E[X_i] = 1/i$.

$$E[X] = E[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} 1/i = \ln n + O(1)$$

Lemma:-

Assume that the candidates are present in random order, then algorithm hire assistant has a hiring cost of $O(c_h \ln n)$

Proof:-

From being hiring cost is $O(m^* c_h)$ where m is the number of candidates hired

From lemma, this is $O(\ln n)$.