浙江工商大学 ZHEJIANG GONGSHANG UNIVERSITY | 金融学院 School of Finance

Portfolio Management

Portfolio Theory(extra)

Learning Objectives

- How to estimate expected returns and risk for individual securities
- What happens to risk and return when securities are combined in a portfolio
- Why diversification is so important to investors

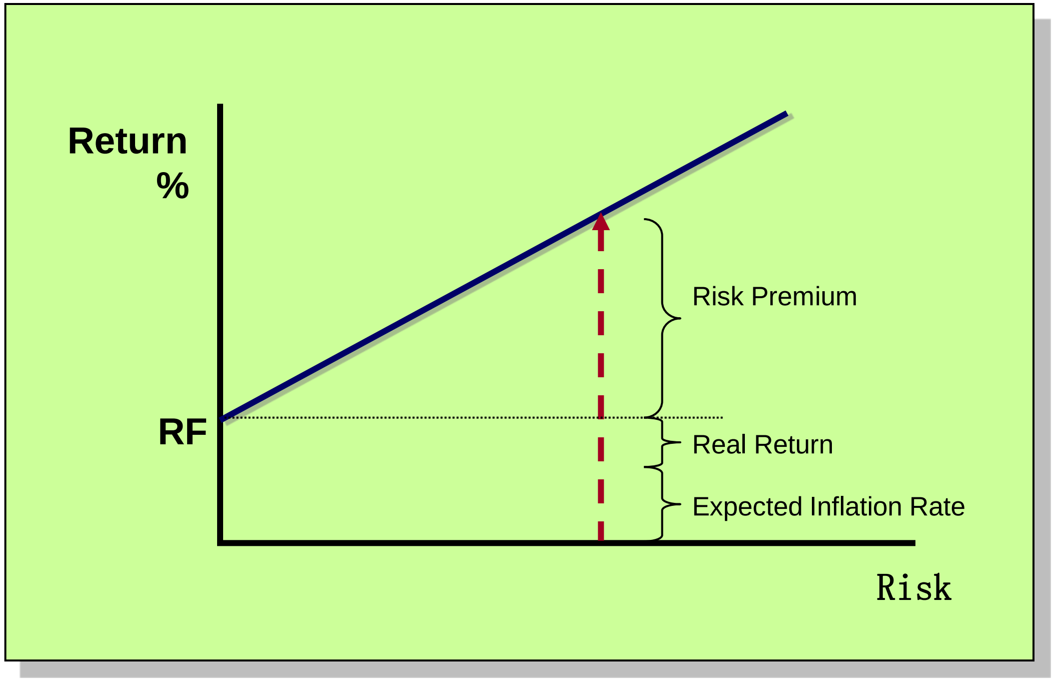
Introduction to Risk and Return

Risk and return are the two most important attributes of an investment.

Research has shown that the two are linked in the capital markets and that generally, higher returns can only be achieved by taking on greater risk.

Risk isn't just the potential loss of return, it is the potential loss of the entire investment itself (loss of both principal and interest).

Consequently, taking on additional risk in search of higher returns is a decision that should not be taking lightly.



Summary on Return and Risk

- The greater the risk of a security, the higher is expected return
- Return is the compensation that has to be paid to induce investors to accept risk
- Success in investing is about balancing risk and return to achieve an optimal combination
- The risk always remains because of unpredictable variability in the returns on assets

Measuring Returns

Ex Ante Returns

Return calculations may be done ‘before-the-fact,’ in which case, assumptions must be made about the future

Ex Post Returns

Return calculations done ‘after-the-fact,’ in order to analyze what rate of return was earned.

Measuring Returns

You have learned that the constant growth DDM can be decomposed into the two forms of income that equity investors may receive, dividends and capital gains.

$$k_c = \left[\frac{D_1}{P_0}\right] + [g] = [\text{Income / Dividend Yield}] + [\text{Capital Gain (or loss) Yield}]$$

WHEREAS

Fixed-income investors (bond investors for example) can expect to earn interest income as well as (depending on the movement of interest rates) either capital gains or capital losses.

Measuring Returns

Income yield is the return earned in the form of a periodic cash flow received by investors.

The income yield return is calculated by the periodic cash flow divided by the purchase price.

$$\text{Income yield} = \frac{CF_1}{P_0}$$

Where CF_1 = the expected cash flow to be received

P_0 = the purchase price

Measuring Returns

Investors in market-traded securities (bonds or stock) receive investment returns in two different form:

- Income yield
- Capital gain (or loss) yield

The investor will receive dollar returns, for example:

- $1.00 of dividends
- Share price rise of $2.00

To be useful, dollar returns must be converted to percentage returns as a function of the original investment. (Because a $3.00 return on a $30 investment might be good, but a $3.00 return on a $300 investment would be unsatisfactory!)

Measuring Returns

An investor receives the following dollar returns a stock investment of $25:

$1.00 of dividends

Share price rise of $2.00

The capital gain (or loss) return component of total return is calculated: ending price – minus beginning price, divided by beginning price

$$\text{Capital gain (loss) return} = \frac{P_1 - P_0}{P_0} = \frac{\$27 - \$25}{\$25} = .08 = 8\%$$

Measuring Returns

The investor's total return (holding period return) is:

$$\text{Total return} = \text{Income yield} + \text{Capital gain (or loss) yield}$$

$$= \frac{CF_1 + P_1 - P_0}{P_0}$$

$$= \left[\frac{CF_1}{P_0}\right] + \left[\frac{P_1 - P_0}{P_0}\right]$$

$$= \left[\frac{\$1.00}{\$25}\right] + \left[\frac{\$27 - \$25}{\$25}\right] = 0.04 + 0.08 = 0.12 = 12\%$$

Measuring Average Returns

Ex Post Returns

- Measurement of historical rates of return that have been earned on a security or a class of securities allows us to identify trends or tendencies that may be useful in predicting the future.
- There are two different types of ex post mean or average returns used:
  - Arithmetic average
  - Geometric mean

Measuring Average Returns

Arithmetic Average

$$\text{Arithmetic Average (AM)} = \frac{\sum_{i=1}^{n} r_i}{n}$$

Where:

r_i = the individual returns

n = the total number of observations

Most commonly used value in statistics

Sum of all returns divided by the total number of observations

Measuring Average Returns

Geometric Mean

$$\text{Geometric Mean (GM)} = [(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_n)]^{\frac{1}{n}} - 1$$

Measures the average or compound growth rate over multiple periods.

Measuring Average Returns

Geometric Mean versus Arithmetic Average

If all returns (values) are identical the geometric mean = arithmetic average.

If the return values are volatile the geometric mean < arithmetic average

The greater the volatility of returns, the greater the difference between geometric mean and arithmetic average.

Measuring Average Returns

Average Investment Returns and Standard Deviations

Table 8 - 2 Average Investment Returns and Standard Deviations, 1938-2005

	Annual Arithmetic Average (%)	Annual Geometric Mean (%)	Standard Deviation of Annual Returns (%)
Government of Canada treasury bills	5.20	5.11	4.32
Government of Canada bonds	6.62	6.24	9.32
Canadian stocks	11.79	10.60	16.22
U.S. stocks	13.15	11.76	17.54

Source: Data are from the Canadian Institute of Actuaries

The greater the difference, the greater the volatility of annual returns.

Measuring Expected (Ex Ante) Returns

While past returns might be interesting, investor's are most concerned with future returns.

Sometimes, historical average returns will not be realized in the future.

Developing an independent estimate of ex ante returns usually involves use of forecasting discrete scenarios with outcomes and probabilities of occurrence.

Estimating Expected Returns

Estimating Ex Ante (Forecast) Returns

The general formula

$$\text{Expected Return (ER)} = \sum_{i=1}^{n} (r_i \times \text{Prob}_\text{i})$$

Where:

ER = the expected return on an investment

R_i = the estimated return in scenario *i*

$Prob_i$ = the probability of state *i* occurring

Estimating Expected Returns

Estimating Ex Ante (Forecast) Returns

Example:

This is type of forecast data that are required to make an ex ante estimate of expected return.

State of the Economy	Probability of Occurrence	Possible Returns on Stock A in that State
Economic Expansion	25.0%	30%
Normal Economy	50.0%	12%
Recession	25.0%	-25%

Estimating Expected Returns

Estimating Ex Ante (Forecast) Returns Using a Spreadsheet Approach

Example Solution:

Sum the products of the probabilities and possible returns in each state of the economy.

(1) **State of the Economy**	(2) **Probability of Occurrence**	(3) **Possible Returns on Stock A in that State**	(4)=(2)×(1) **Weighted Possible Returns on the Stock**
Economic Expansion	25.0%	30%	7.50%
Normal Economy	50.0%	12%	6.00%
Recession	25.0%	-25%	-6.25%
	Expected Return on the Stock =		**7.25%**

Estimating Expected Returns

Estimating Ex Ante (Forecast) Returns Using a Formula Approach

Example Solution:

Sum the products of the probabilities and possible returns in each state of the economy.

$$\begin{aligned}
\text{Expected Return (ER)} &= \sum_{i=1}^{n} (r_i \times \text{Prob}_\text{i}) \\
&= (\text{r}_1 \times \text{Prob}_1) + (\text{r}_2 \times \text{Prob}_2) + (\text{r}_3 \times \text{Prob}_3) \\
&= (30\% \times 0.25) + (12\% \times 0.5) + (\text{-}25\% \times 0.25) \\
&= 7.25\%
\end{aligned}$$

Sources of Risk

The risk inherent in holding a security is the variability, or the uncertainty, of its return

Factors that affect risk are

1. Maturity
   - Underlying factors have more chance to change over a longer horizon
   - Maturity value of the security may be eroded by inflation or currency fluctuations
   - Increased chance of the issuer defaulting the longer is the time horizon

Sources of Risk

2. Creditworthiness

The governments of the US, UK and other developed countries are all judged as safe since they have no history of default in the payment of their liabilities

Some other countries have defaulted in the recent past

Corporations vary even more in their creditworthiness. Some are so lacking in creditworthiness that an active "junk bond" market exists for high return, high risk corporate bonds that are judged very likely to default

Sources of Risk

3. Priority

Bond holders have the first claim on the assets of a liquidated firm

Bond holders are also able to put the corporation into bankruptcy if it defaults on payment

4. Liquidity

Liquidity relates to how easy it is to sell an asset

The existence of a highly developed and active secondary market raises liquidity

A security's risk is raised if it is lacking liquidity

Sources of Risk

5. Underlying Activities

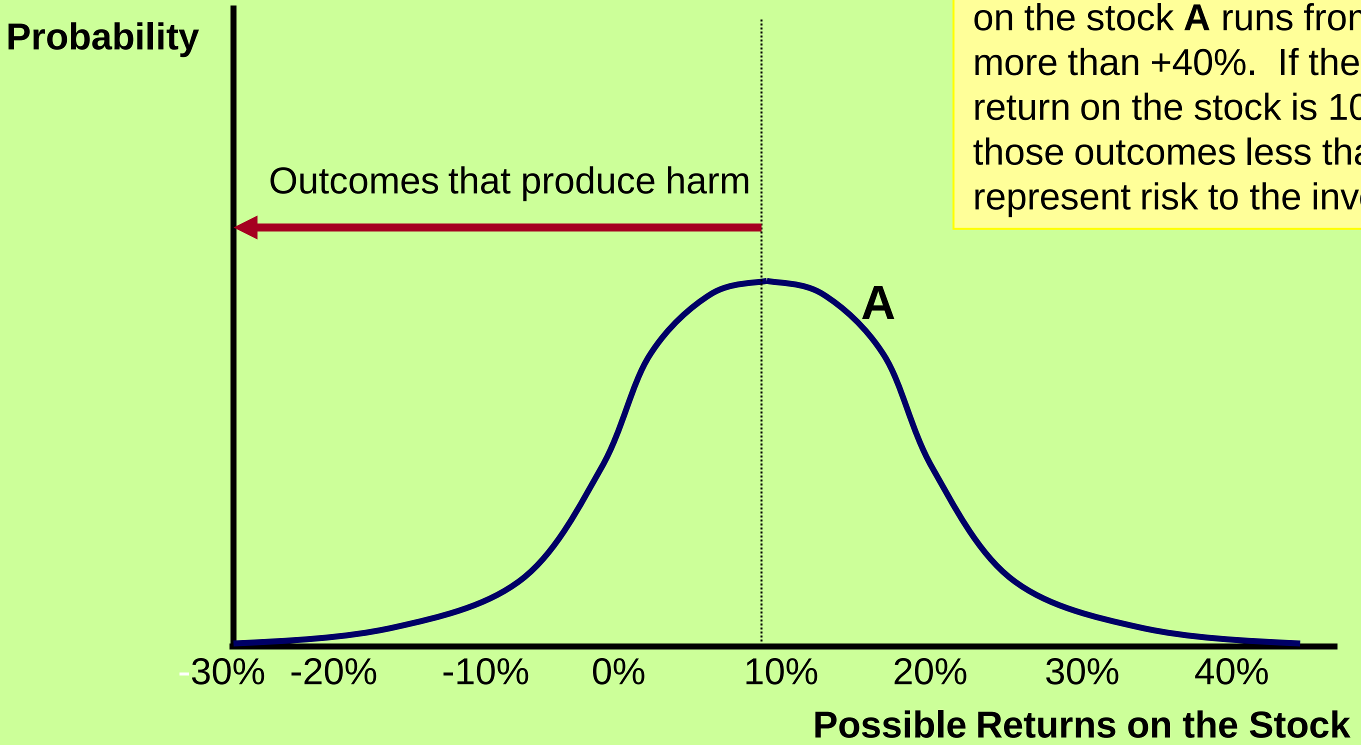
The economic activities of the issuer of the security can affect how risky it is

Stock in small firms and in firms operating in high-technology sectors are on average more risky than those of large firms in traditional sectors

Risk

Illustrated

The range of total possible returns on the stock **A** runs from -30% to more than +40%. If the required return on the stock is 10%, then those outcomes less than 10% represent risk to the investor.

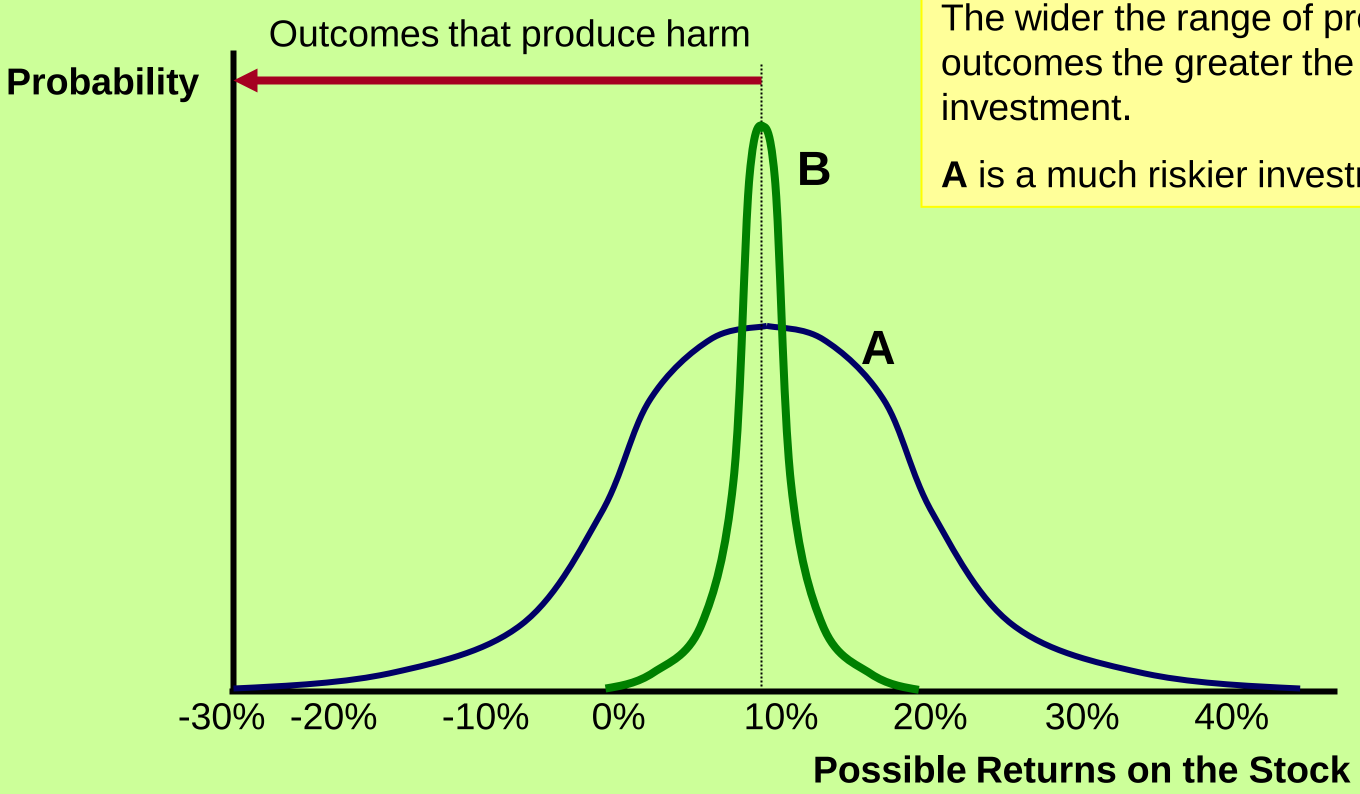


Range

- The difference between the maximum and minimum values is called the range
  - Common stocks have had a range of annual returns of 74.36 % over the 1938-2005 period
  - Treasury bills had a range of 21.07% over the same period.
- As a rough measure of risk, range tells us that common stock is more risky than treasury bills.

Differences in Levels of Risk

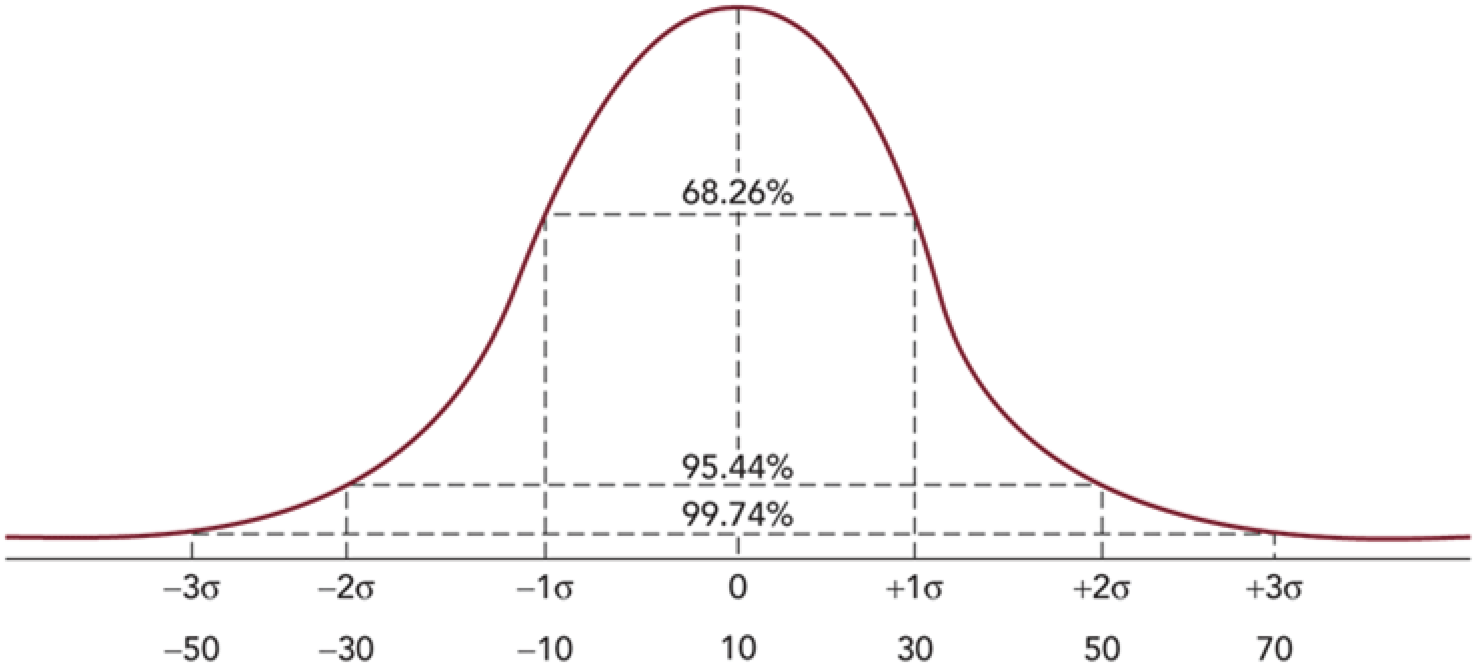
Illustrated



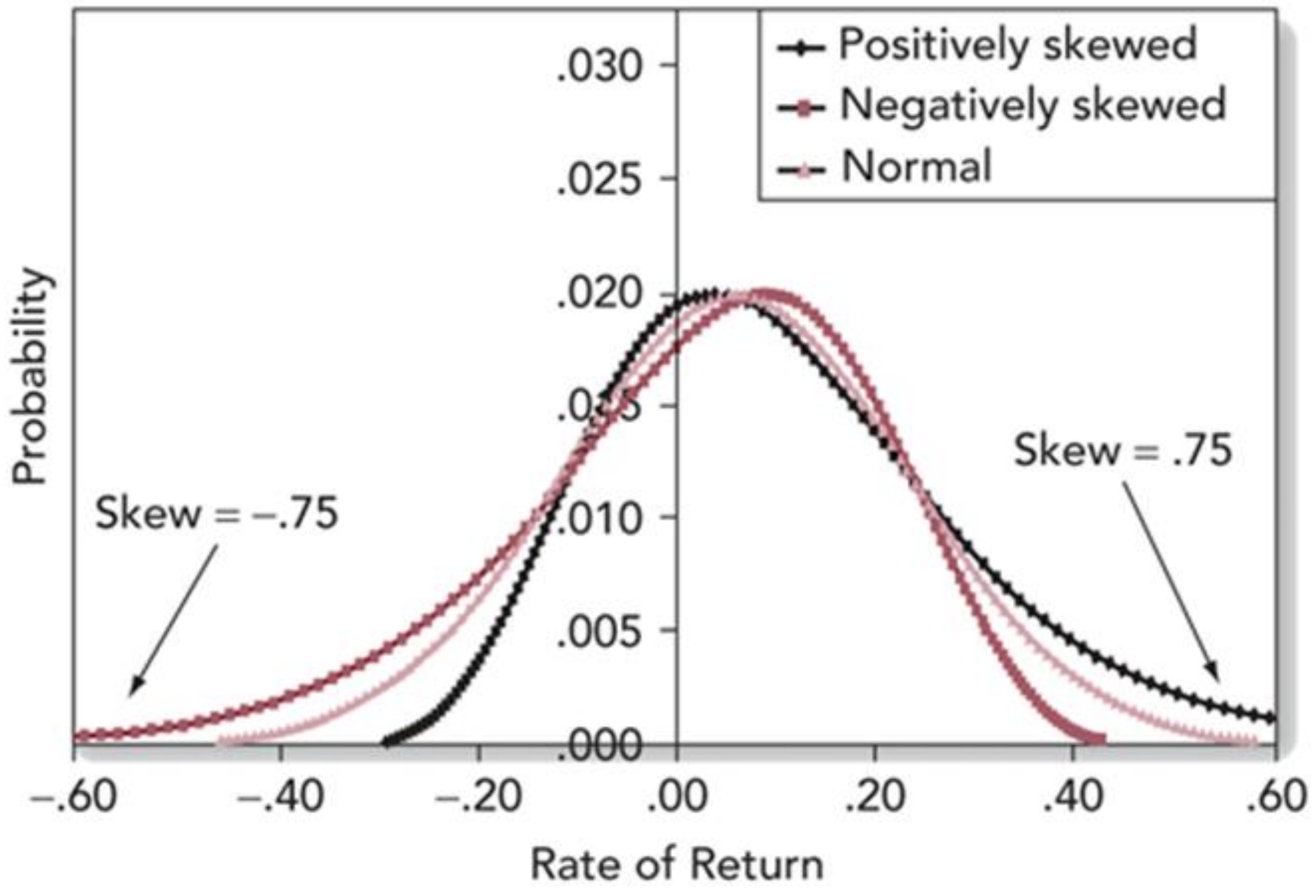
The wider the range of probable outcomes the greater the risk of the investment.

A is a much riskier investment than **B**

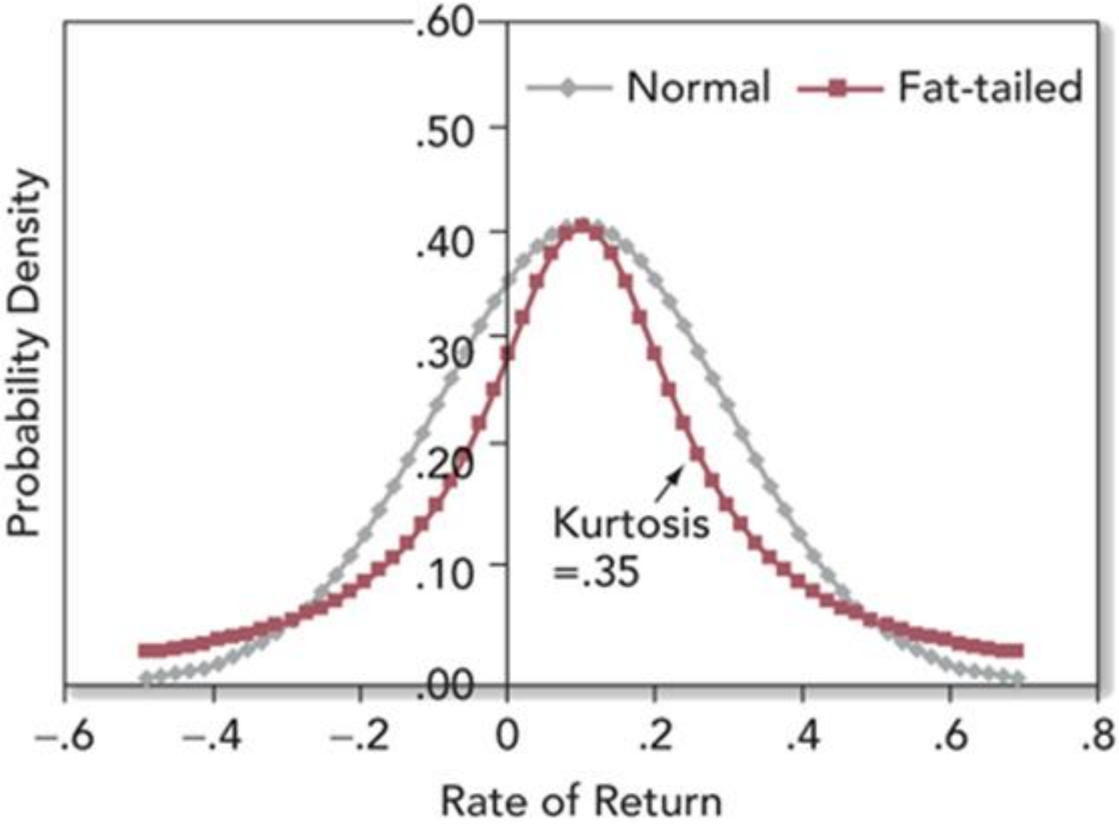
Normal distribution



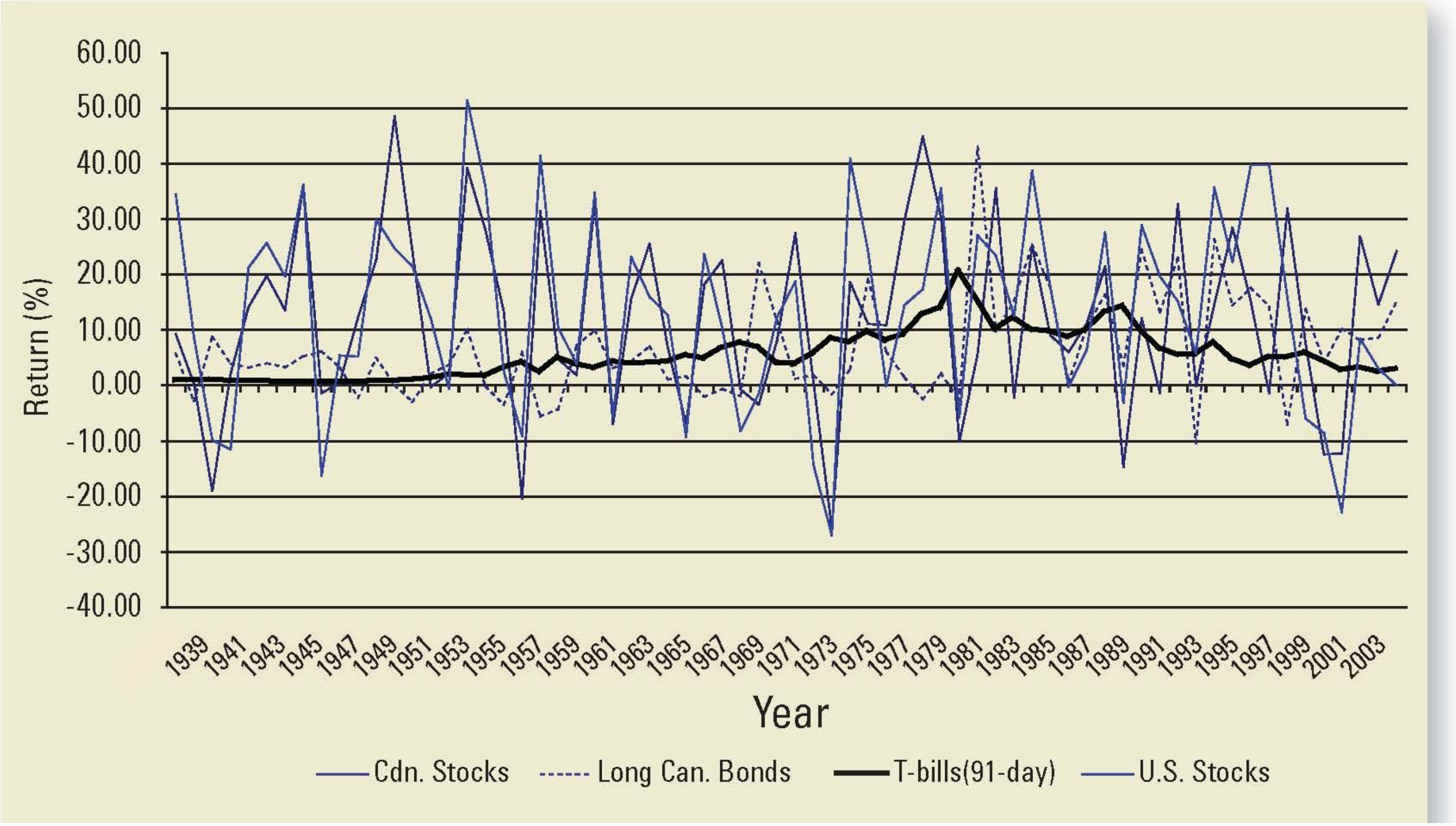
Skewed Distributions



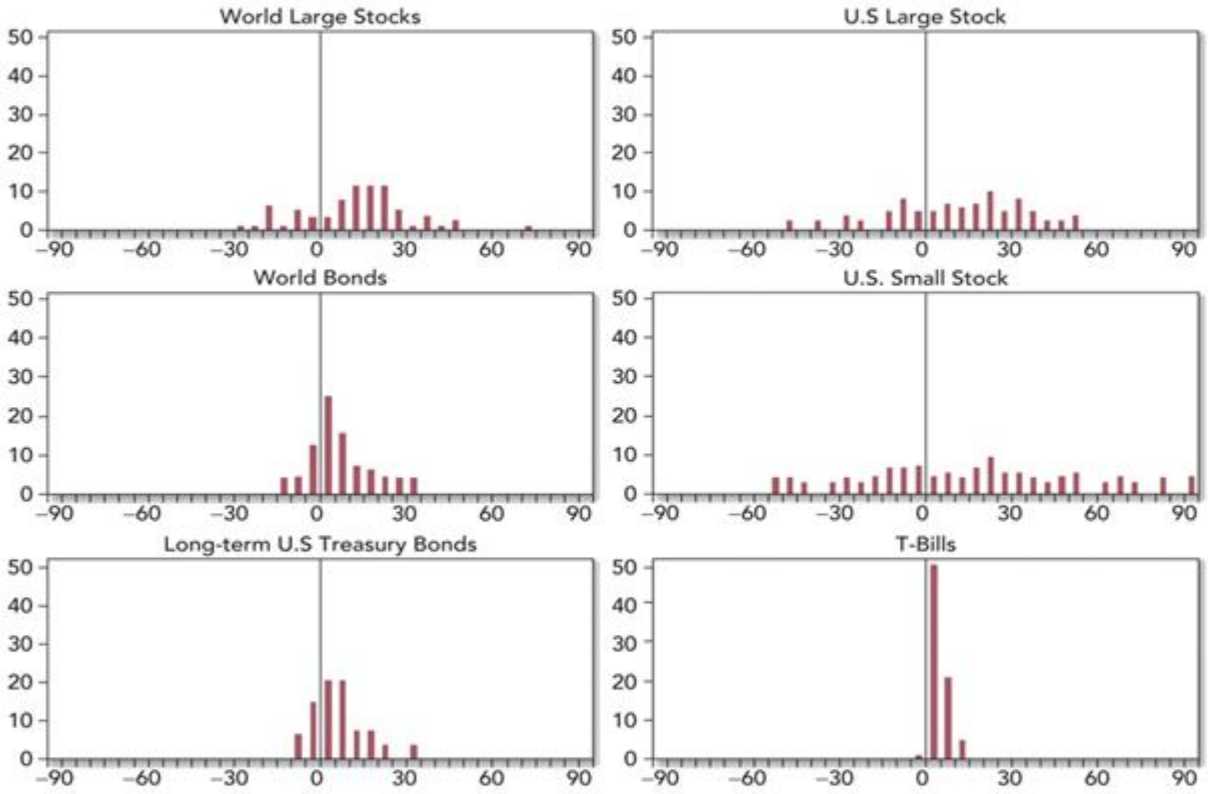
Fat-Tailed Distribution



Historical Returns on Different Asset Classes



Historical Returns on Different Asset Classes



Refining the Measurement of Risk

Standard Deviation (σ)

- Range measures risk based on only two observations (minimum and maximum value)
- Standard deviation uses all observations.
  - Standard deviation can be calculated on forecast or possible returns as well as historical or ex post returns.

Measuring Risk

Ex post Standard Deviation

$$\text{Ex post}\,\sigma = \sqrt{\frac{\sum_{i=1}^{n}(r_i - \bar{r})^2}{n-1}}$$

$Where:$

σ = the standard deviation

$\bar{r}$ = the average return

r_i = the return in year i

n = the number of observations

Measuring Risk

Example Using the Ex post Standard Deviation

<u>Problem</u>

Estimate the standard deviation of the historical returns on investment A that were: 10%, 24%, -12%, 8% and 10%.

<u>Step 1 – Calculate the Historical Average Return</u>

$$\text{Arithmetic Average (AM)} = \frac{\sum_{i=1}^{n} r_i}{n} = \frac{10+24-12+8+10}{5} = \frac{40}{5} = 8.0\%$$

<u>Step 2 – Calculate the Standard Deviation</u>

$$\text{Ex post}\,\sigma = \sqrt{\frac{\sum_{i=1}^{n}(r_i - \bar{r})^2}{n-1}} = \sqrt{\frac{(10-8)^2 + (24-8)^2 + (-12-8)^2 + (8-8)^2 + (14-8)^2}{5-1}}$$

$$= \sqrt{\frac{2^2 + 16^2 - 20^2 + 0^2 + 2^2}{4}} = \sqrt{\frac{4+256+400+0+4}{4}} = \sqrt{\frac{664}{4}} = \sqrt{166} = 12.88\%$$

Measuring Risk

Ex ante Standard Deviation

A Scenario-Based Estimate of Risk

$$\text{Ex ante}\,\sigma = \sqrt{\sum_{i=1}^{n} (\text{Prob}_\text{i}) \times (r_i - ER_i)^2}$$

Portfolios

A portfolio is a collection of different securities such as stocks and bonds, that are combined and considered a single asset

The risk-return characteristics of the portfolio is demonstrably different than the characteristics of the assets that make up that portfolio, especially with regard to risk.

Combining different securities into portfolios is done to achieve *diversification*.

Diversification

Diversification has two faces:

1. Diversification results in an overall reduction in portfolio risk (return volatility over time) with little sacrifice in returns, and
2. Diversification helps to immunize the portfolio from potentially catastrophic events such as the outright failure of one of the constituent investments.

(If only one investment is held, and the issuing firm goes bankrupt, the entire portfolio value and returns are lost. If a portfolio is made up of many different investments, the outright failure of one is more than likely to be offset by gains on others, helping to make the portfolio immune to such events.)

Expected Return of a Portfolio

Modern Portfolio Theory

The Expected Return on a Portfolio is simply the weighted average of the returns of the individual assets that make up the portfolio:

$$ER_p = \sum_{\mathrm{i=1}}^{\mathrm{n}} (w_i \times ER_i)$$

The portfolio weight of a particular security is the percentage of the portfolio's total value that is invested in that security.

Expected Return of a Portfolio

Example

Portfolio value = \$2,000 + \$5,000 = \$7,000

$r_A = 14\%,\ r_B = 6\%,$

w_A = *weight of security A* = \$2,000 / \$7,000 = 28.6%

w_B = *weight of security B* = \$5,000 / \$7,000 = (1-28.6%)= 71.4%

$$ER_p = \sum_{i=1}^{n} (w_i \times ER_i) = (.286 \times 14\%) + (.714 \times 6\%)$$

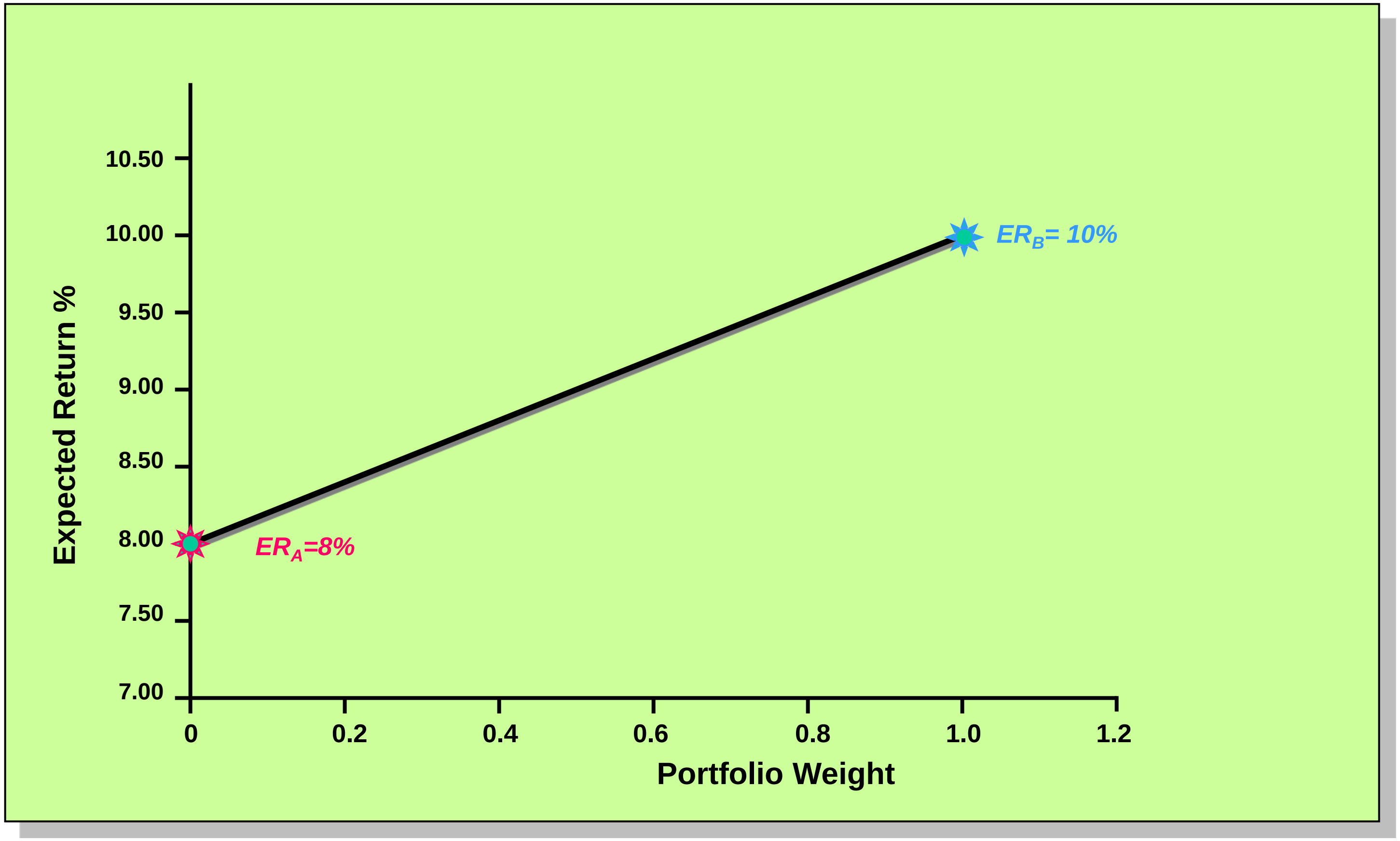
$$= 4.004\% + 4.284\% = 8.288\%$$

Range of Returns in a Two Asset Portfolio

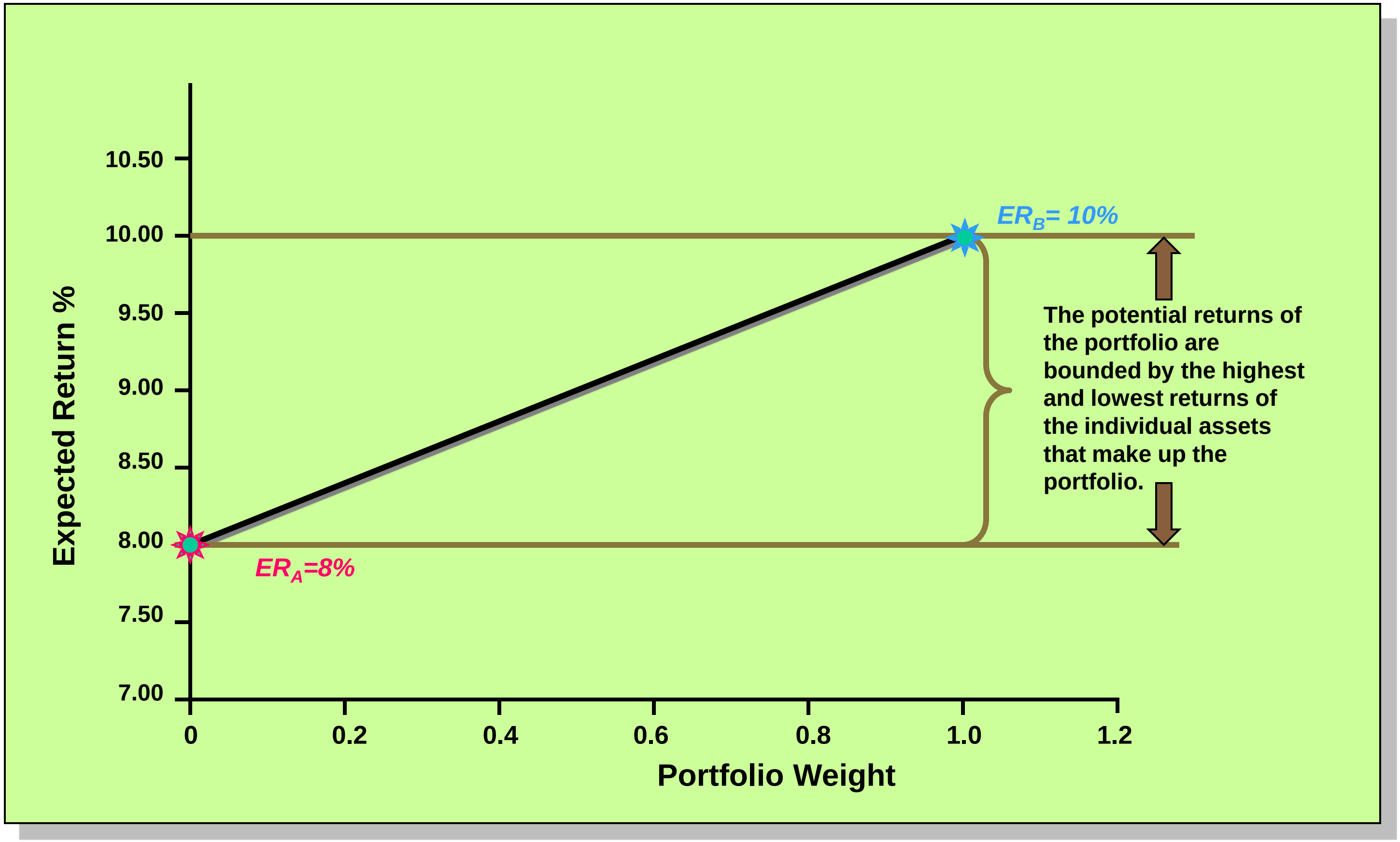
In a two asset portfolio, simply by changing the weight of the constituent assets, different portfolio returns can be achieved.

Because the expected return on the portfolio is a simple weighted average of the individual returns of the assets, you can achieve portfolio returns bounded by the highest and the lowest individual asset returns.

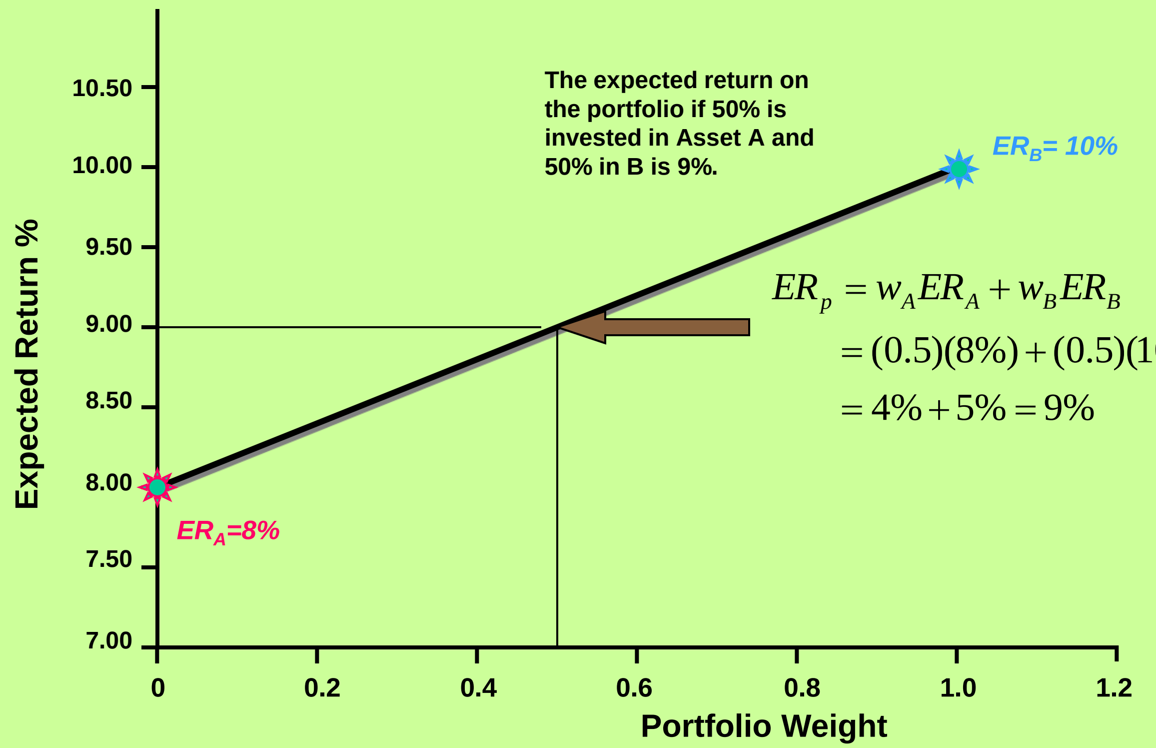
Range of Returns in a Two Asset Portfolio



Range of Returns in a Two Asset Portfolio



Range of Returns in a Two Asset Portfolio



$$ER_p = w_A ER_A + w_B ER_B$$
$$= (0.5)(8\%) + (0.5)(10\%)$$
$$= 4\% + 5\% = 9\%$$

Expected Portfolio Returns

Example of a Three Asset Portfolio

	Relative Weight	Expected Return	Weighted Return
Stock X	0.400	8.0%	0.03
Stock Y	0.350	15.0%	0.05
Stock Z	0.250	25.0%	0.06
	Expected Portfolio Return =		**14.70%**

Risk in Portfolios

Prior to the establishment of Modern Portfolio Theory (MPT), most people only focused upon investment returns…they ignored risk.

With MPT, investors had a tool that they could use to *dramatically reduce the risk* of the portfolio *without a significant reduction* in the expected return of the portfolio.

Expected Return and Risk For Portfolios

Standard Deviation of a Two-Asset Portfolio using Covariance

$$\sigma_p = \sqrt{\underbrace{(w_A)^2(\sigma_A)^2}_{\text{Risk of Asset A adjusted for weight in the portfolio}} + \underbrace{(w_B)^2(\sigma_B)^2}_{\text{Risk of Asset B adjusted for weight in the portfolio}} + \underbrace{2(w_A)(w_B)(COV_{A,B})}_{\text{Factor to take into account comovement of returns}}}$$

Risk of Asset A adjusted for weight in the portfolio

Risk of Asset B adjusted for weight in the portfolio

Factor to take into account comovement of returns. This factor can be negative.

Expected Return and Risk For Portfolios

Standard Deviation of a Two-Asset Portfolio using Correlation Coefficient

$$\sigma_p = \sqrt{(w_A)^2(\sigma_A)^2 + (w_B)^2(\sigma_B)^2 + 2\underbrace{(w_A)(w_B)(\rho_{A,B})(\sigma_A)(\sigma_B)}}$$

Factor that takes into account the degree of comovement of returns. It can have a negative value if correlation is negative.

Grouping Individual Assets into Portfolios

The riskiness of a portfolio that is made of different risky assets is a function of three different factors:

- the riskiness of the individual assets that make up the portfolio
- the relative weights of the assets in the portfolio
- the degree of comovement of returns of the assets making up the portfolio

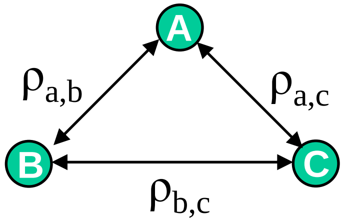
The standard deviation of a two-asset portfolio may be measured by:

$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

Risk of a Three-Asset Portfolio

The data requirements for a three-asset portfolio grows dramatically if we are using Markowitz Portfolio selection formulae.

We need 3 (three) correlation coefficients between A and B; A and C; and B and C.



$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + \sigma_C^2 w_C^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B + 2 w_B w_C \rho_{B,C} \sigma_B \sigma_C + 2 w_A w_C \rho_{A,C} \sigma_A \sigma_C}$$

Covariance

A statistical measure of the correlation of the fluctuations of the annual rates of return of different investments.

$$COV_{AB} = \sum_{i=1}^{n} \mathrm{Prob}_{\mathrm{i}} (k_{A,i} - \bar{k}_{i})(k_{B,i} - \bar{k}_{B})$$

Correlation

The degree to which the returns of two stocks co-move is measured by the correlation coefficient (ρ）.

The correlation coefficient (ρ）between the returns on two securities will lie in the range of +1 through - 1.

+1 is perfect positive correlation

-1 is perfect negative correlation

$$\rho_{AB} = \frac{COV_{AB}}{\sigma_A \sigma_B}$$

Covariance and Correlation Coefficient

Solving for covariance given the correlation coefficient and standard deviation of the two assets:

$$COV_{AB} = \rho_{AB}\sigma_A\sigma_B$$

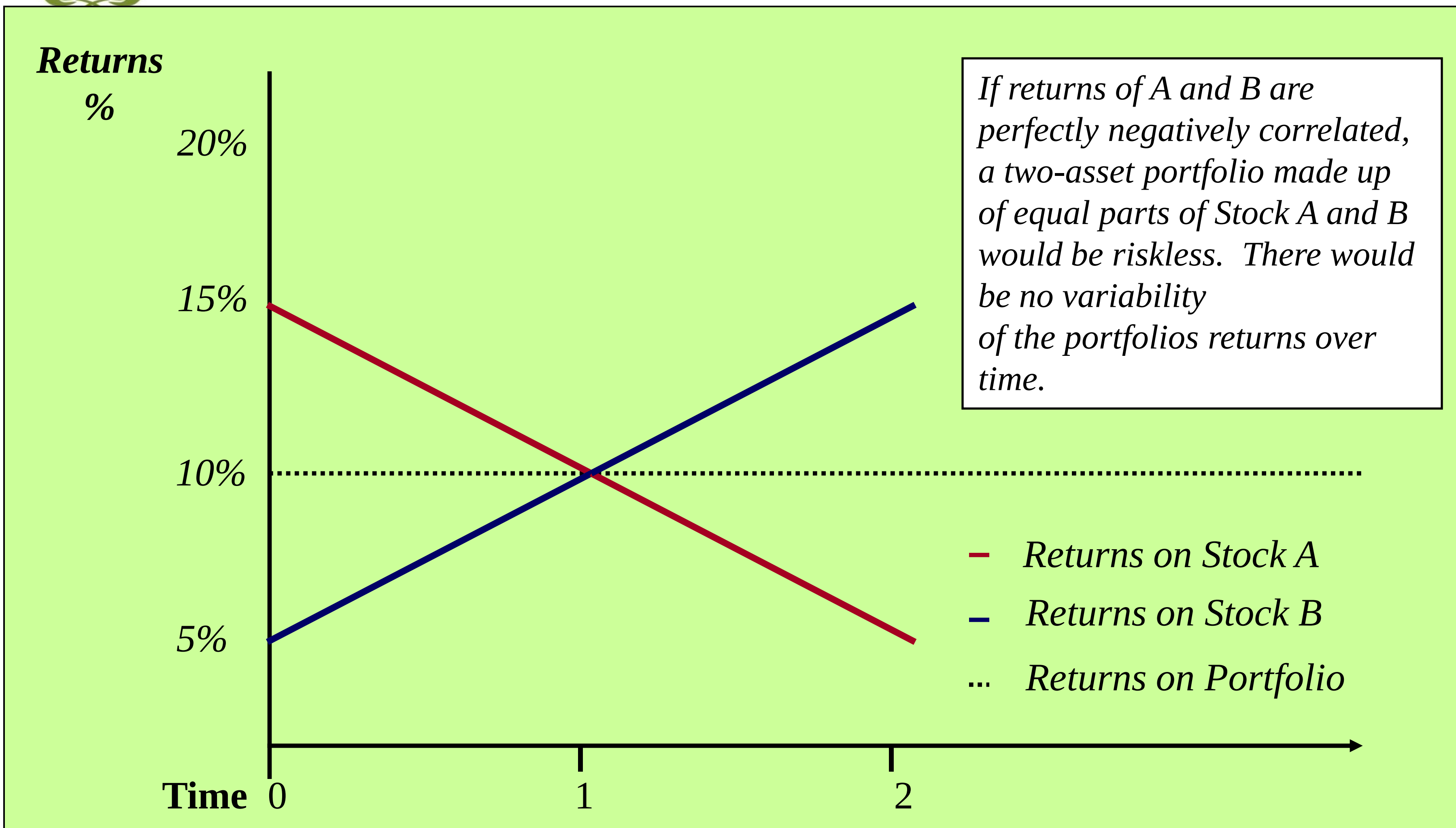
Importance of Correlation

Correlation is important because it affects the degree to which diversification can be achieved using various assets.

Theoretically, if two assets returns are perfectly positively correlated, it is possible to build a riskless portfolio with a return that is greater than the risk-free rate.

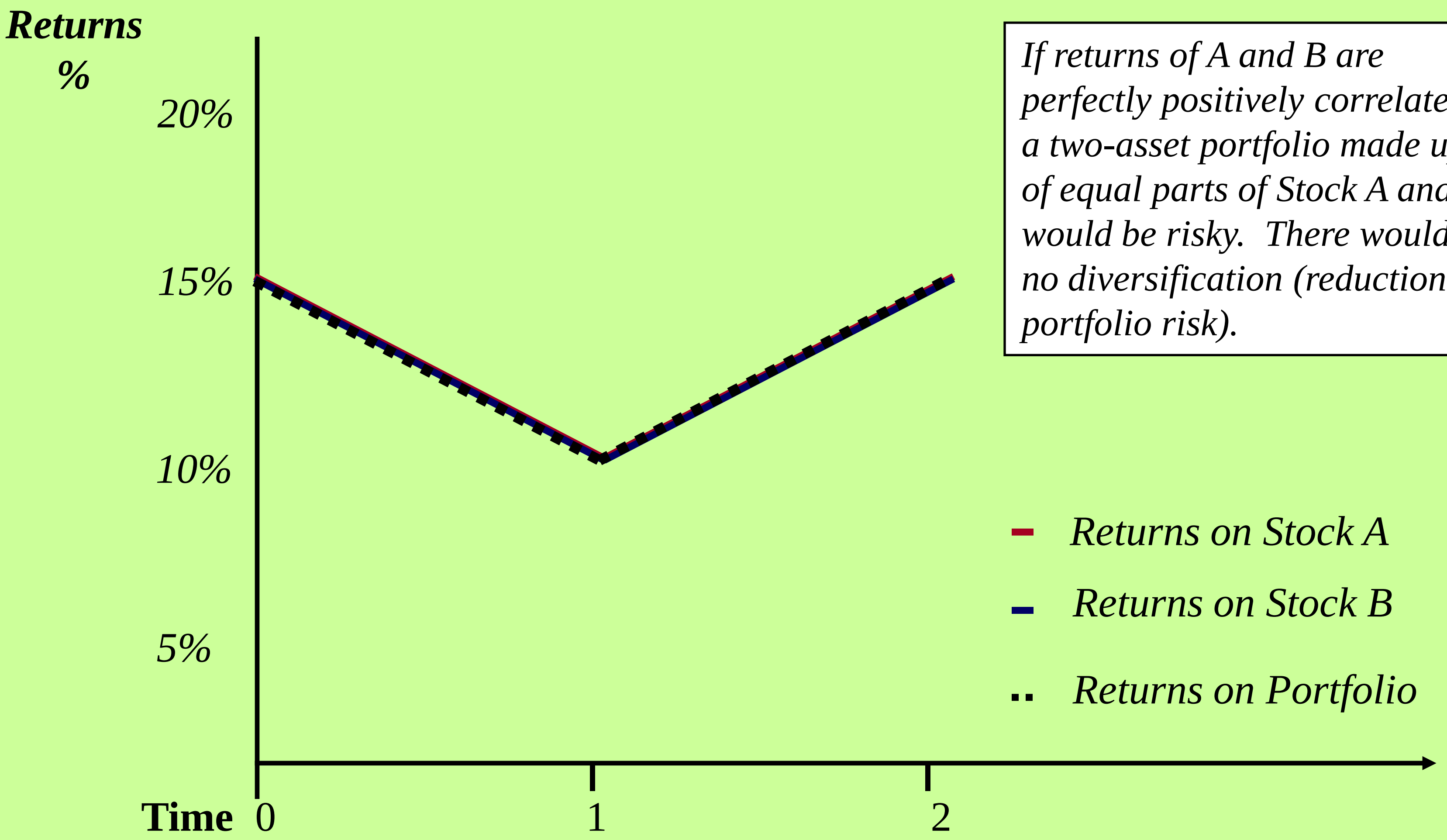
Affect of Perfectly Negatively Correlated Returns

Elimination of Portfolio Risk



Example of Perfectly Positively Correlated Returns

No Diversification of Portfolio Risk



If returns of A and B are perfectly positively correlated, a two-asset portfolio made up of equal parts of Stock A and B would be risky. There would be no diversification (reduction of portfolio risk).

Diversification Potential

The potential of an asset to diversify a portfolio is dependent upon the degree of co-movement of returns of the asset with those other assets that make up the portfolio.

In a simple, two-asset case, if the returns of the two assets are perfectly negatively correlated it is possible (depending on the relative weighting) to eliminate all portfolio risk.

This is demonstrated through the following series of spreadsheets, and then summarized in graph format.

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	1
B	14.0%	40.0%	

Perfect Positive Correlation – no diversification

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	17.5%
80.00%	20.00%	6.80%	20.0%
70.00%	30.00%	7.70%	22.5%
60.00%	40.00%	8.60%	25.0%
50.00%	50.00%	9.50%	27.5%
40.00%	60.00%	10.40%	30.0%
30.00%	70.00%	11.30%	32.5%
20.00%	80.00%	12.20%	35.0%
10.00%	90.00%	13.10%	37.5%
0.00%	100.00%	14.00%	40.0%

Both portfolio returns and risk are bounded by the range set by the constituent assets when $\rho = +1$

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0.5
B	14.0%	40.0%	

Positive Correlation – weak diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	15.9%
80.00%	20.00%	6.80%	17.4%
70.00%	30.00%	7.70%	19.5%
60.00%	40.00%	8.60%	21.9%
50.00%	50.00%	9.50%	24.6%
40.00%	60.00%	10.40%	27.5%
30.00%	70.00%	11.30%	30.5%
20.00%	80.00%	12.20%	33.6%
10.00%	90.00%	13.10%	36.8%
0.00%	100.00%	14.00%	40.0%

When $\rho = +0.5$ these portfolio combinations have lower risk – expected portfolio return is unaffected.

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0
B	14.0%	40.0%	

No Correlation – some diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	14.1%
80.00%	20.00%	6.80%	14.4%
70.00%	30.00%	7.70%	15.9%
60.00%	40.00%	8.60%	18.4%
50.00%	50.00%	9.50%	21.4%
40.00%	60.00%	10.40%	24.7%
30.00%	70.00%	11.30%	28.4%
20.00%	80.00%	12.20%	32.1%
10.00%	90.00%	13.10%	36.0%
0.00%	100.00%	14.00%	40.0%

Portfolio risk is lower than the risk of either asset A or B.

Example of Portfolio Combinations and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	-0.5
B	14.0%	40.0%	

Negative Correlation – greater diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	12.0%
80.00%	20.00%	6.80%	10.6%
70.00%	30.00%	7.70%	11.3%
60.00%	40.00%	8.60%	13.9%
50.00%	50.00%	9.50%	17.5%
40.00%	60.00%	10.40%	21.6%
30.00%	70.00%	11.30%	26.0%
20.00%	80.00%	12.20%	30.6%
10.00%	90.00%	13.10%	35.3%
0.00%	100.00%	14.00%	40.0%

Portfolio risk for more combinations is lower than the risk of either asset

Example of Portfolio Combinations and Correlation

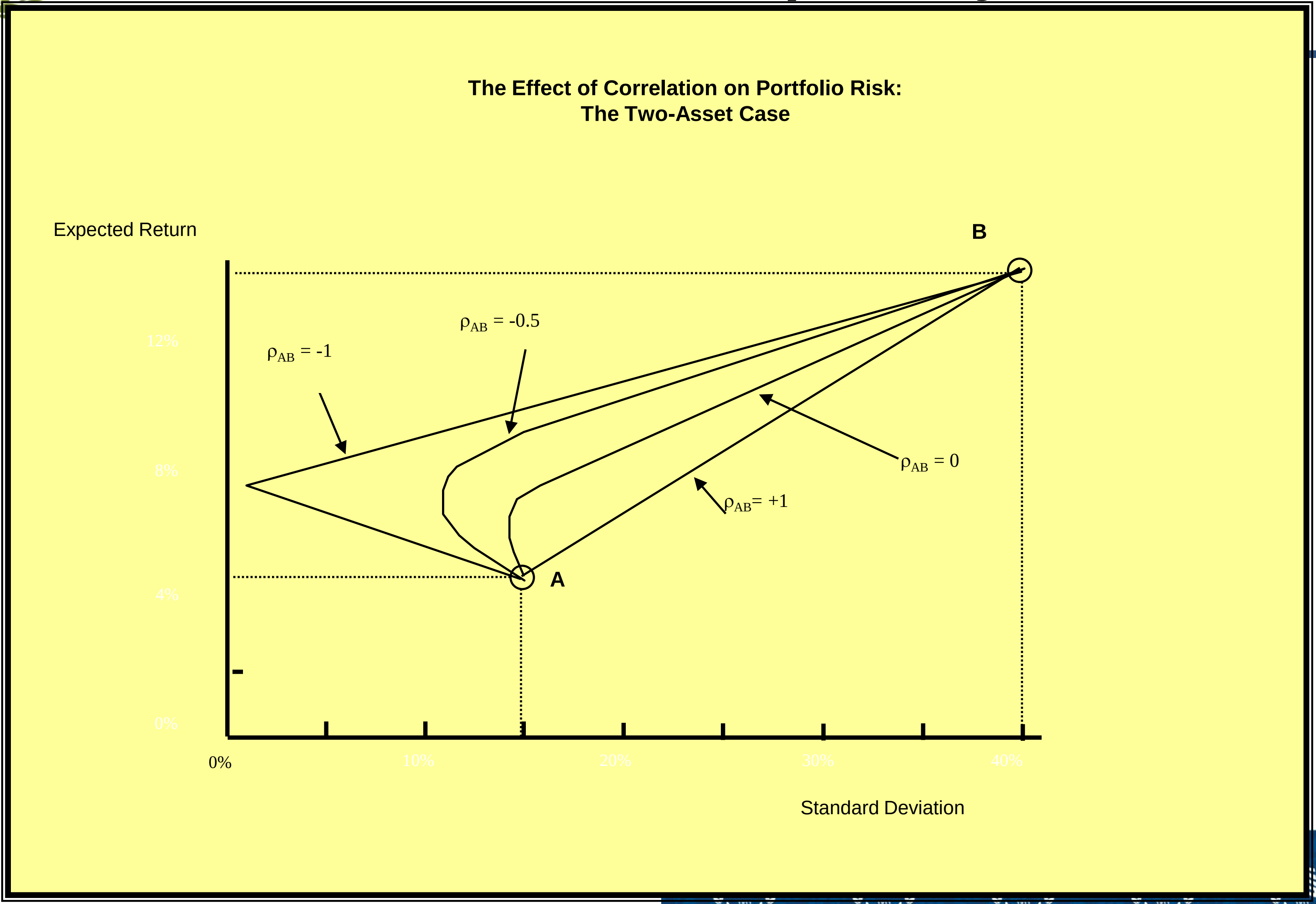
Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	-1
B	14.0%	40.0%	

Perfect Negative Correlation – greatest diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	9.5%
80.00%	20.00%	6.80%	4.0%
70.00%	30.00%	7.70%	1.5%
60.00%	40.00%	8.60%	7.0%
50.00%	50.00%	9.50%	12.5%
40.00%	60.00%	10.40%	18.0%
30.00%	70.00%	11.30%	23.5%
20.00%	80.00%	12.20%	29.0%
10.00%	90.00%	13.10%	34.5%
0.00%	100.00%	14.00%	40.0%

Risk of the portfolio is almost eliminated at 70% invested in asset A

Diversification of a Two Asset Demonstrated Graphically



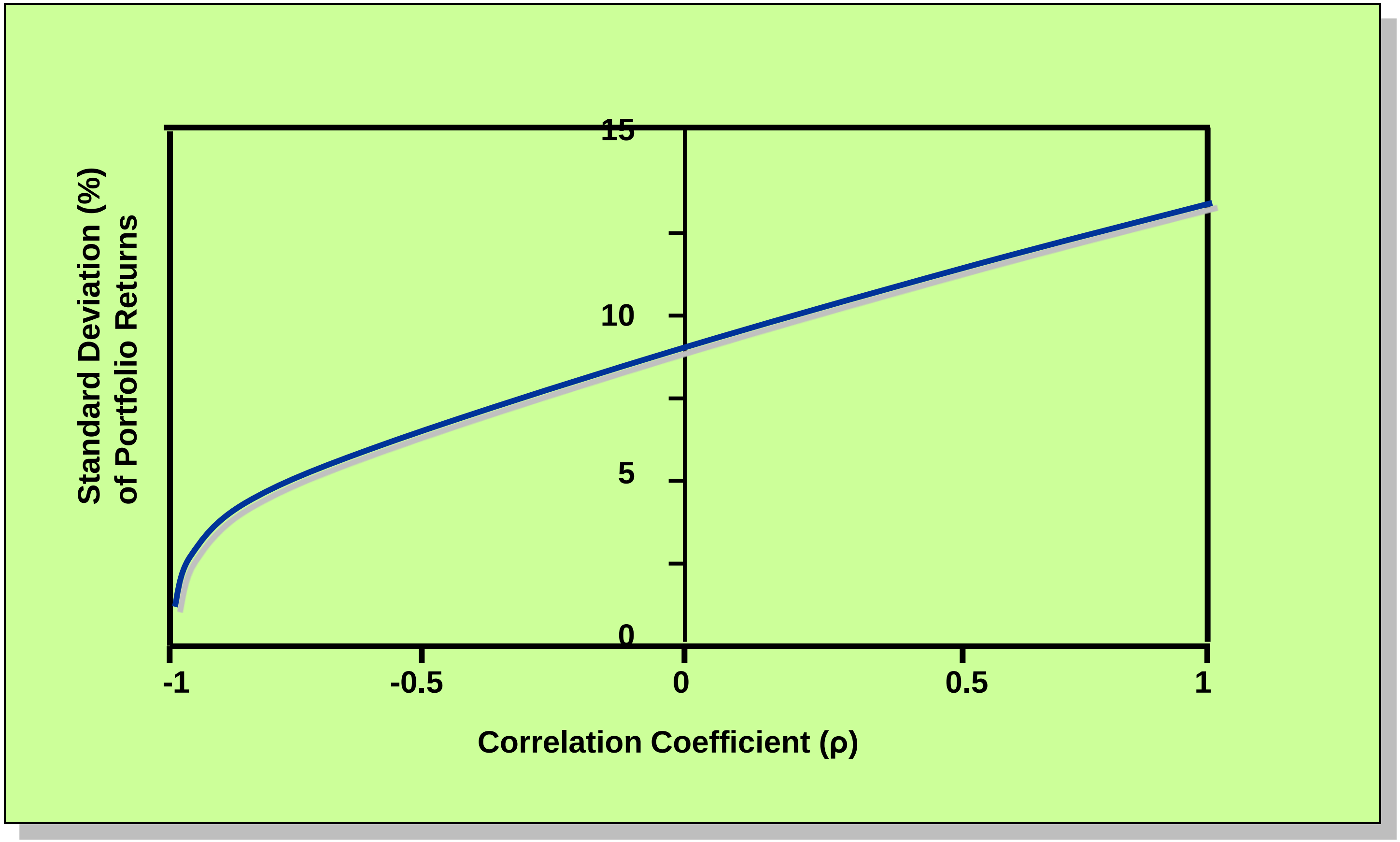
Impact of the Correlation Coefficient

Relationship between portfolio risk (σ) and the correlation coefficient

- The slope is not linear a significant amount of diversification is possible with assets with no correlation (it is not necessary, nor is it possible to find, perfectly negatively correlated securities in the real world)
- With perfect negative correlation, the variability of portfolio returns is reduced to nearly zero.

Expected Portfolio Return

Impact of the Correlation Coefficient



Zero Risk Portfolio

We can calculate the portfolio that removes all risk.

When ρ = -1, then

$$\sigma_p = \sqrt{(w_A)^2(\sigma_A)^2 + (w_B)^2(\sigma_B)^2 + 2(w_A)(w_B)(\rho_{A,B})(\sigma_A)(\sigma_B)}$$

Becomes:

$$\sigma_p = w\sigma_A - (1-w)\sigma_B$$

Review of portfolio maths

Rule 1 The mean or **expected return** of an asset is a probability-weighted average of its return in all scenarios. Calling $\Pr(s)$ the probability of scenario s and $r(s)$ the return in scenario s, we may write the expected return, $E(r)$, as

$$E(r) = \sum_s \Pr(s) r(s) \tag{6.2}$$

Rule 2 The **variance** of an asset's returns is the expected value of the squared deviations from the expected return. Symbolically,

$$\sigma^2 = \sum_s \Pr(s)[r(s) - E(r)]^2 \tag{6.3}$$

Review of portfolio maths

Rule 3 The rate of return on a portfolio is a weighted average of the rates of return of each asset comprising the portfolio, with portfolio proportions as weights. This implies that the *expected* rate of return on a portfolio is a weighted average of the *expected* rate of return on each component asset.

Rule 4 When a risky asset is combined with a risk-free asset, the portfolio standard deviation equals the risky asset's standard deviation multiplied by the portfolio proportion invested in the risky asset.

Review of portfolio maths

Rule 5 When two risky assets with variances σ_1^2 and σ_2^2, respectively, are combined into a portfolio with portfolio weights w_1 and w_2, respectively, the portfolio variance σ_p^2 is given by

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}(r_1, r_2)$$

Diversification

We have demonstrated that risk of a portfolio can be reduced by spreading the value of the portfolio across, two, three, four or more assets.

The key to efficient diversification is to choose assets whose returns are less than perfectly positively correlated.

Even with random or naive diversification, risk of the portfolio can be reduced.

As the portfolio is divided across more and more securities, the risk of the portfolio falls rapidly at first, until a point is reached where, further division of the portfolio does not result in a reduction in risk.

Going beyond this point is known as superfluous diversification.

Diversification

General case

$$\sigma_p^2 = \sum_{i=1}^{N}(W_i^2\sigma_i^2) + \sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}(W_j W_k \sigma_{jk})$$

In case of independent and equal split

$$\sigma_p^2 = \sum_{i=1}^{N}(W_i^2\sigma_i^2) = \sum_{i=1}^{N}(\frac{1}{N})^2\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_i^2}{N}$$

Diversification

Equal weights in general cases

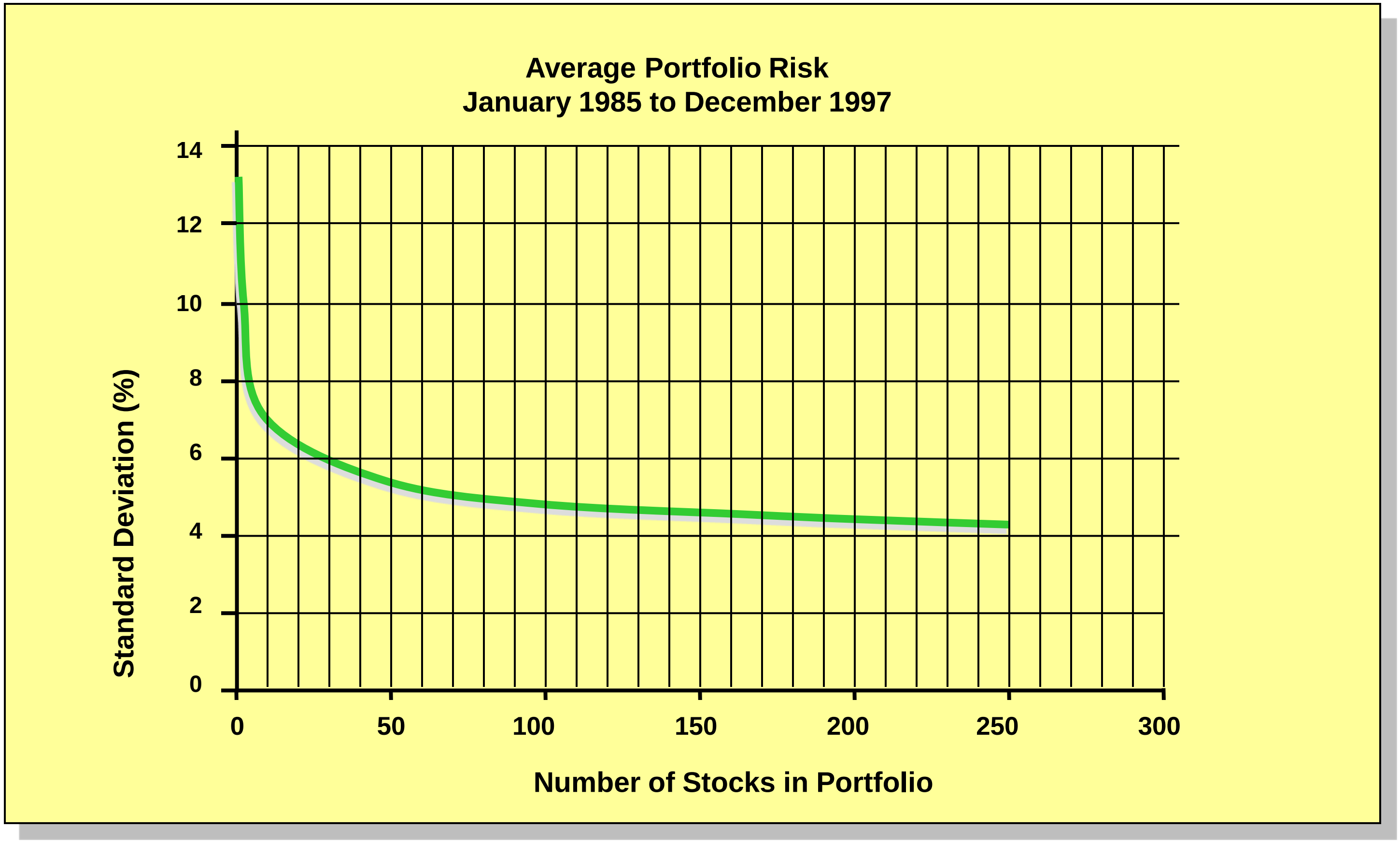
$$\sigma_p^2 = \sum_{i=1}^{N}(W_i^{\,2}\sigma_i^2) + \sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}(W_j W_k \sigma_{jk})$$

$$= \sum_{i=1}^{N}(\sigma_i^2 / N^2) + \sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}(1/N)(1/N)\sigma_{jk}$$

$$= (1/N)\sum_{i=1}^{N}(\sigma_i^2 / N) + \frac{N-1}{N}\sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}\frac{\sigma_{jk}}{N(N-1)}$$

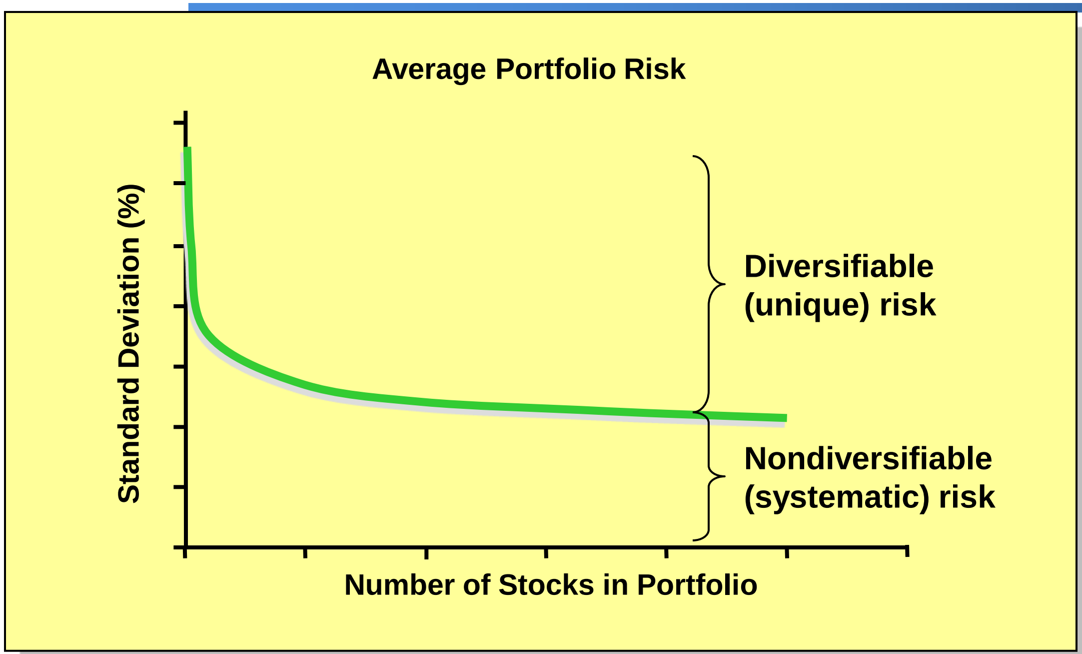
Diversification

Domestic Diversification



Total Risk of an Individual Asset

Equals the Sum of Market and Unique Risk



This graph illustrates that total risk of a stock is made up of market risk (that cannot be diversified away because it is a function of the economic 'system') and unique, company-specific risk that is eliminated from the portfolio through diversification.

$$\text{Total risk} = \text{Market (systematic) risk} + \text{Unique (non-systematic) risk}$$

International Diversification

- Clearly, diversification adds value to a portfolio by reducing risk while not reducing the return on the portfolio significantly.
- Most of the benefits of diversification can be achieved by investing in 40 – 50 different ‘positions’ (investments)
- However, if the investment universe is expanded to include investments beyond the domestic capital markets, additional risk reduction is possible.