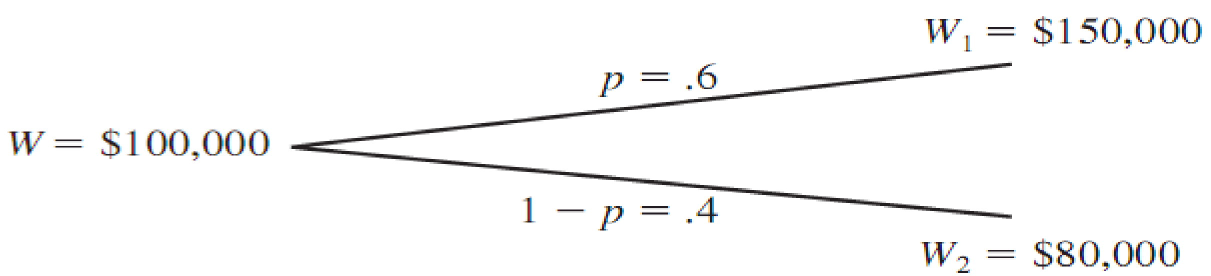
浙江工商大學 ZHEJIANG GONGSHANG UNIVERSITY | 金融学院 School of Finance

Portfolio Management

Portfolio Theory

Risk and Risk Aversion

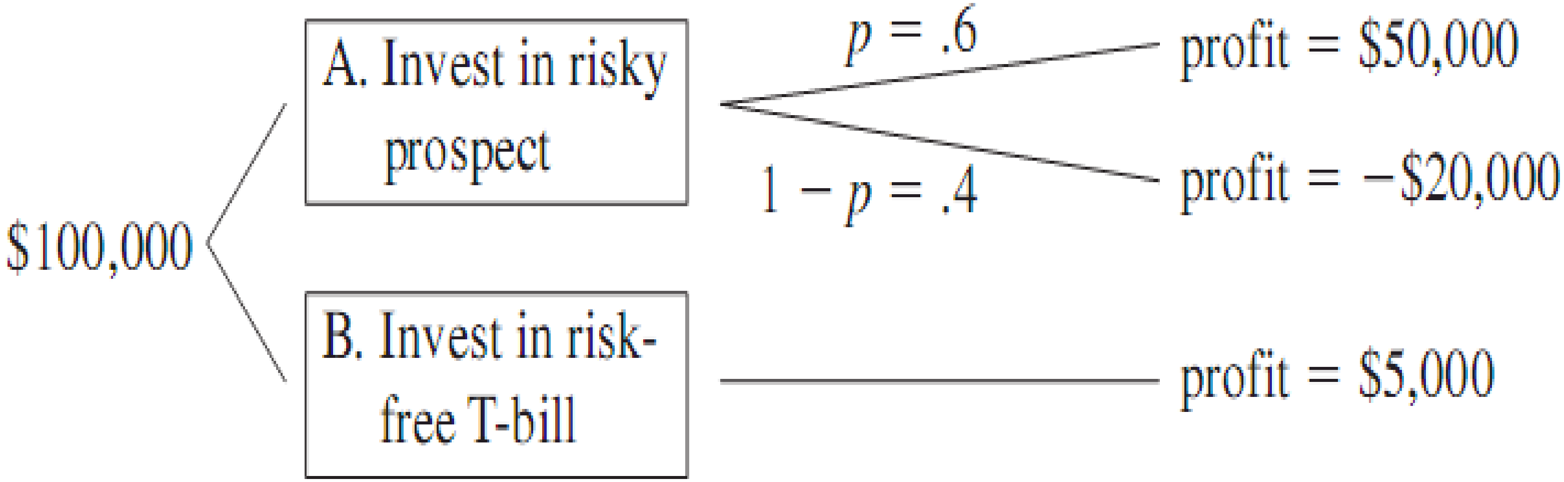
Risk means uncertainty



$$
\begin{aligned}
E(W) &= pW_1 + (1 - p)W_2 \\
&= (.6 \times 150{,}000) + (.4 \times 80{,}000) \\
&= \$122{,}000
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2 &= p[W_1 - E(W)]^2 + (1 - p)\,[W_2 - E(W)]^2 \\
&= .6(150{,}000 - 122{,}000)^2 + .4(80{,}000 - 122{,}000)^2 \\
&= 1{,}176{,}000{,}000
\end{aligned}
$$

Risk and Risk Aversion



$$\$22{,}000 - \$5{,}000 = \$17{,}000$$

Speculation & Gambling

Speculation

"The assumption of considerable business risk in obtaining commensurate gain."

Gambling

" To bet or wager on an uncertain outcome"

Preferable function

Choose from two simply investment strategies

Strategy A		Strategy B	
Res.	Prob.	Res.	Prob.
15	1/3	20	1/3
10	1/3	12	1/3
5	1/3	4	1/3

Preferable function

Who is the winner?

	A	B
Wins	40	45
Ties	20	5
Losses	10	20

Utility function

Utility function

$$U$$

$$U(win) = 2, \qquad U(draw) = 1, \qquad U(lose) = 0$$

Expected utility theorem

$$E(U) = \sum_{W} U(W)P(W)$$

Expected utility theorem

Revisit the first question

Strategy A			Strategy B		
Res.	W.	Prob.	Res.	W.	Prob.
15	1.0	1/3	20	0.9	1/3
10	1.2	1/3	12	1.1	1/3
5	1.4	1/3	4	1.5	1/3

Expected utility theorem

Investment A			Investment B			Investment C		
Outcome	Utility of Outcome	Probability	Outcome	Utility of Outcome	Probability	Outcome	Utility of Outcome	Probability
20	40	3/15	19	39.9	1/5	18	39.6	1/4
18	39.6	5/15	10	30	2/5	16	38.4	1/4
14	36.4	4/15	5	17.5	2/5	12	33.6	1/4
10	30	2/15				8	25.6	1/4
6	20.4	1/15						

Expected utility theorem

Find the potential utility function of traders is of assets managers' great interests.

However, it is a difficult job as traders are not always rational.

Risk attitude

Risk aversion

Risk neutrality

Risk seeking

Risk attitude

An example of fair gamble

Invest			Do Not Invest	
Outcome	Probability		Outcome	Probability
2	1/2		1	1
0	1/2			

Risk attitude

Risk aversion

$$U(1) > \frac{1}{2}U(2) + \frac{1}{2}U(0)$$

$$U(1) - U(0) > U(2) - U(1)$$

Risk neutrality

$$U(1) - U(0) = U(2) - U(1)$$

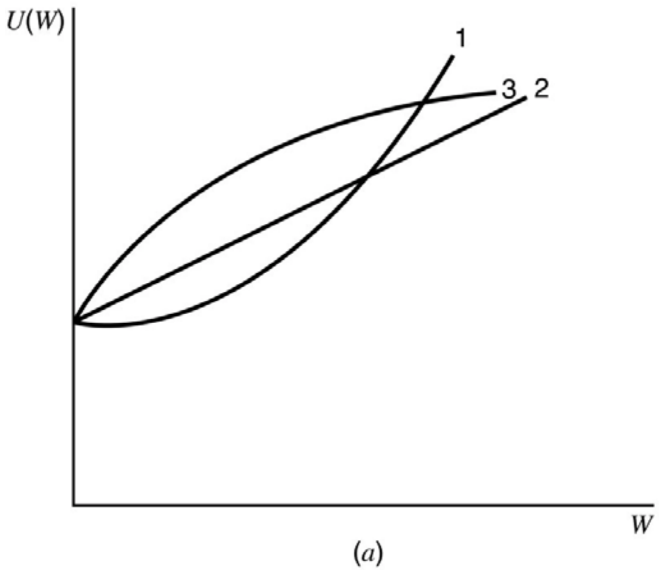
Risk seeking

$$U(1) - U(0) < U(2) - U(1)$$

Risk attitude

Condition	Definition	Implication
1. Risk aversion	Reject fair gamble	$U''(0) < 0$
2. Risk neutrality	Indifferent to fair gamble	$U''(0) = 0$
3. Risk preference	Select a fair gamble	$U''(0) > 0$

Risk attitude



Characteristics of functions with different risk- aversion coefficients. (1) Utility function of a risk-seeking investor. (2) Utility function of a risk-neutral investor. (3) Utility function of a risk-averse investor.

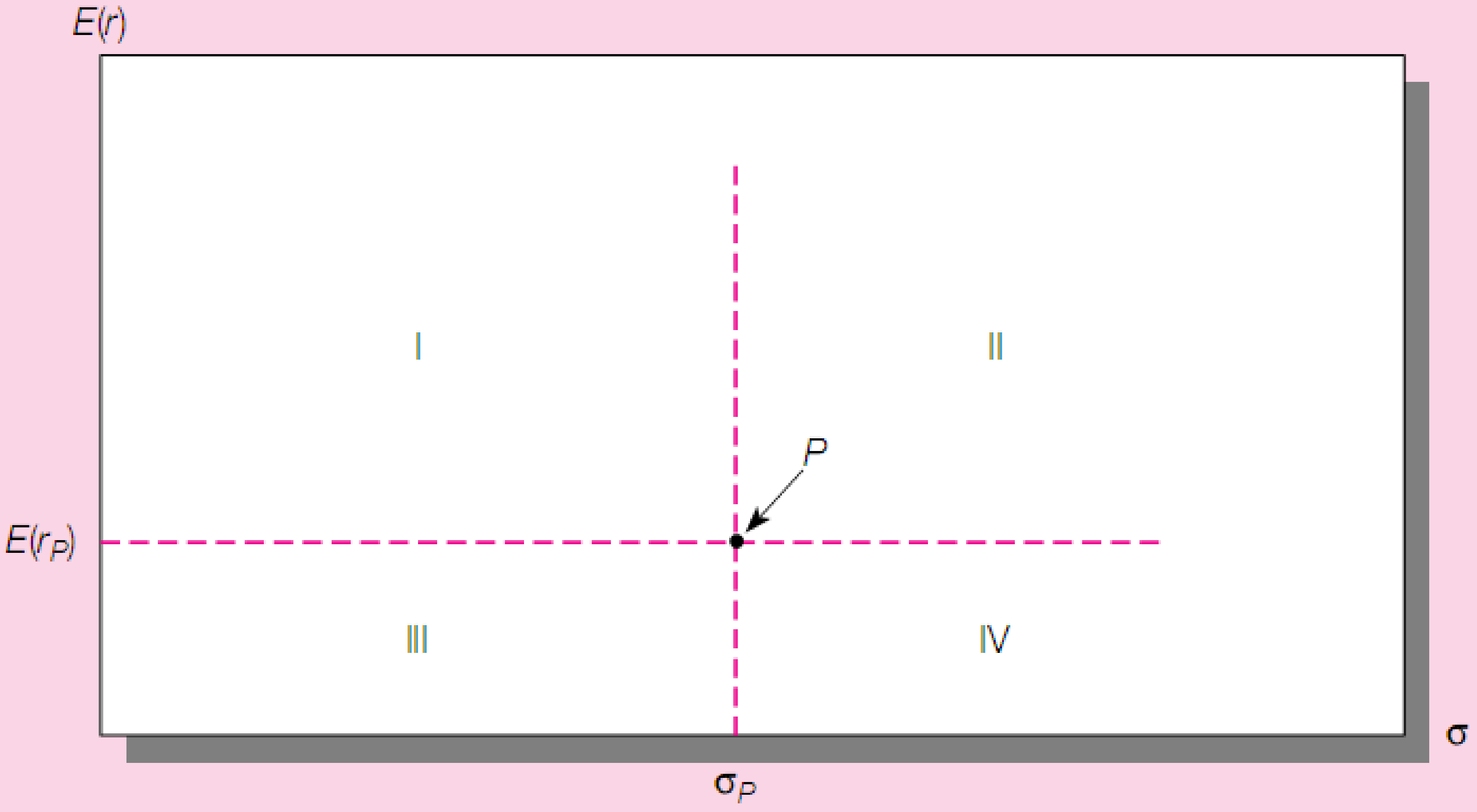
Utility values / scores

Association of Investment Management and Research (AIMA)

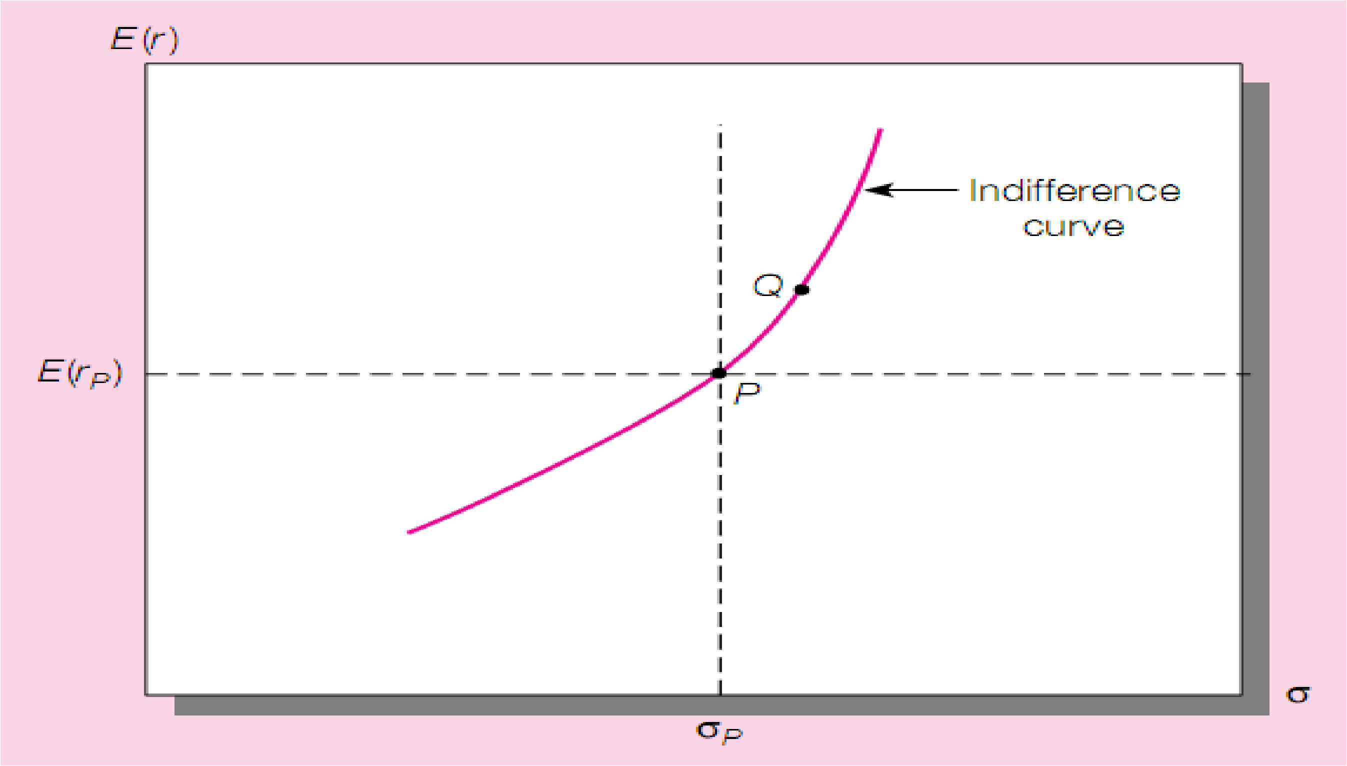
$$U = E(r) - .005A\sigma^2$$

where U is the utility value and A is an index of the investor's risk aversion.

Trade-off risk vs return

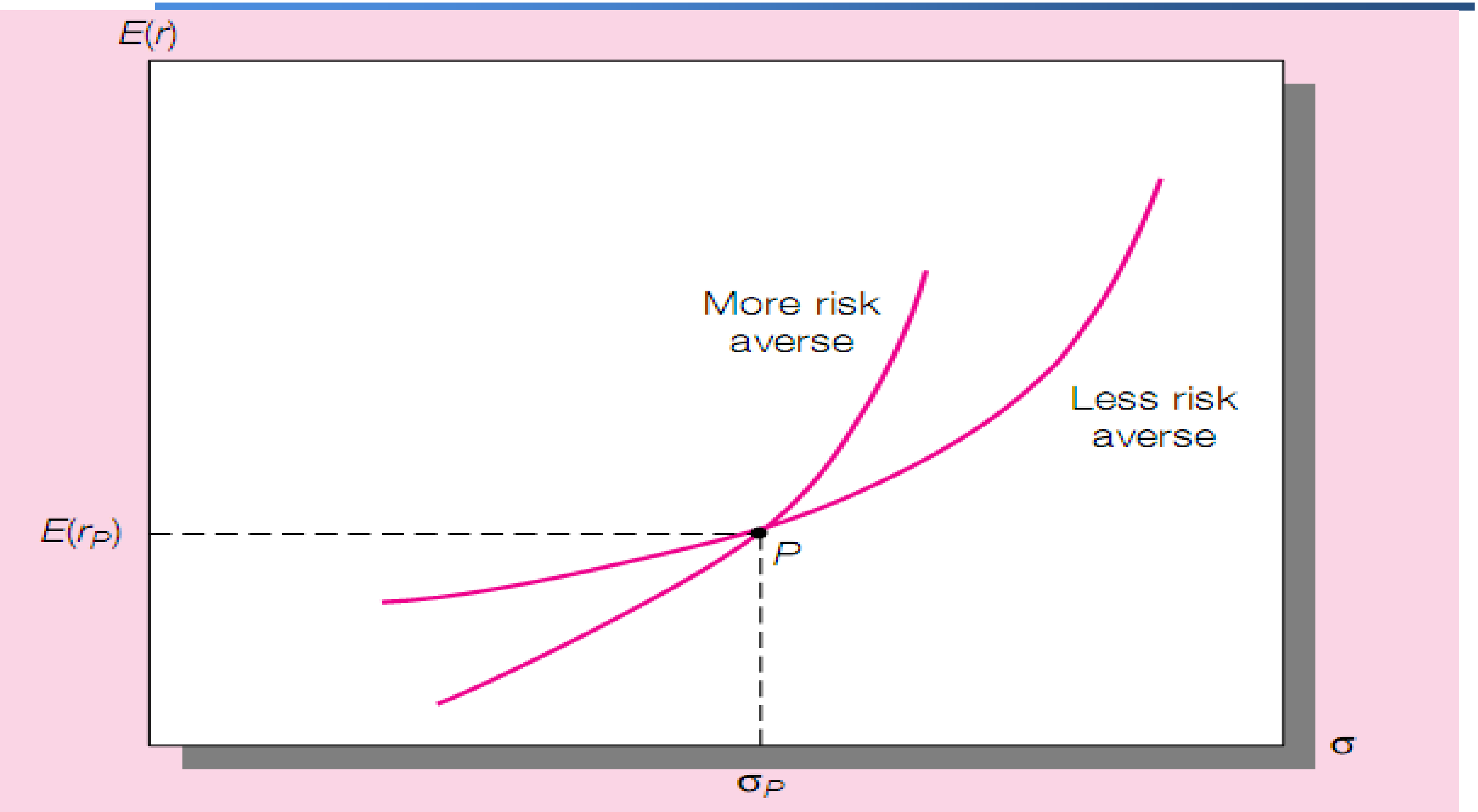


Trade-off risk vs return



same utility values

Trade-off risk vs return



Portfolio risk

Hedging

Investing in an asset with a payoff pattern that offsets exposure to a particular source of risk

Diversification

Investing in a wide variety of assets so that exposure to the risk of any particular security is limited

Review of portfolio maths

Rule 1 The mean or **expected return** of an asset is a probability-weighted average of its return in all scenarios. Calling $\Pr(s)$ the probability of scenario s and $r(s)$ the return in scenario s, we may write the expected return, $E(r)$, as

$$E(r) = \sum_s \Pr(s) r(s) \tag{6.2}$$

Rule 2 The **variance** of an asset's returns is the expected value of the squared deviations from the expected return. Symbolically,

$$\sigma^2 = \sum_s \Pr(s)[r(s) - E(r)]^2 \tag{6.3}$$

Review of portfolio maths

Rule 3 The rate of return on a portfolio is a weighted average of the rates of return of each asset comprising the portfolio, with portfolio proportions as weights. This implies that the *expected* rate of return on a portfolio is a weighted average of the *expected* rate of return on each component asset.

Rule 4 When a risky asset is combined with a risk-free asset, the portfolio standard deviation equals the risky asset's standard deviation multiplied by the portfolio proportion invested in the risky asset.

Review of portfolio maths

Rule 5 When two risky assets with variances σ_1^2 and σ_2^2, respectively, are combined into a portfolio with portfolio weights w_1 and w_2, respectively, the portfolio variance σ_p^2 is given by

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}(r_1, r_2)$$

Risk free asset

Default free perfectly price indexed bond

Money Market Fund

One risk & one risk free

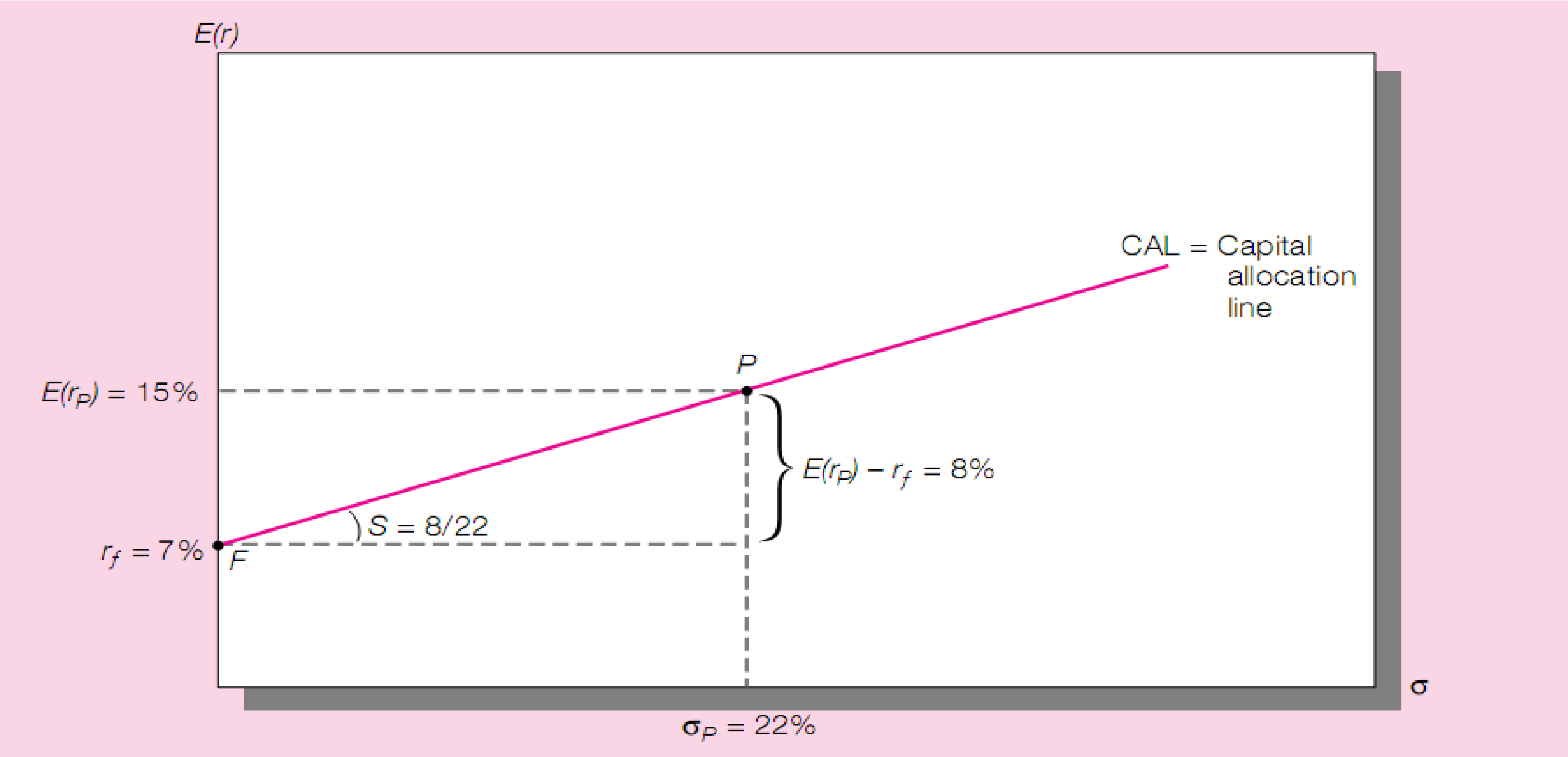
Denote:

$$r_C = y r_P + (1 - y) r_f$$

$$\begin{aligned} E(r_C) &= y E(r_P) + (1 - y) r_f \\ &= r_f + y[E(r_P) - r_f] \end{aligned}$$

$$\sigma_C = y \sigma_P$$

Capital allocation line

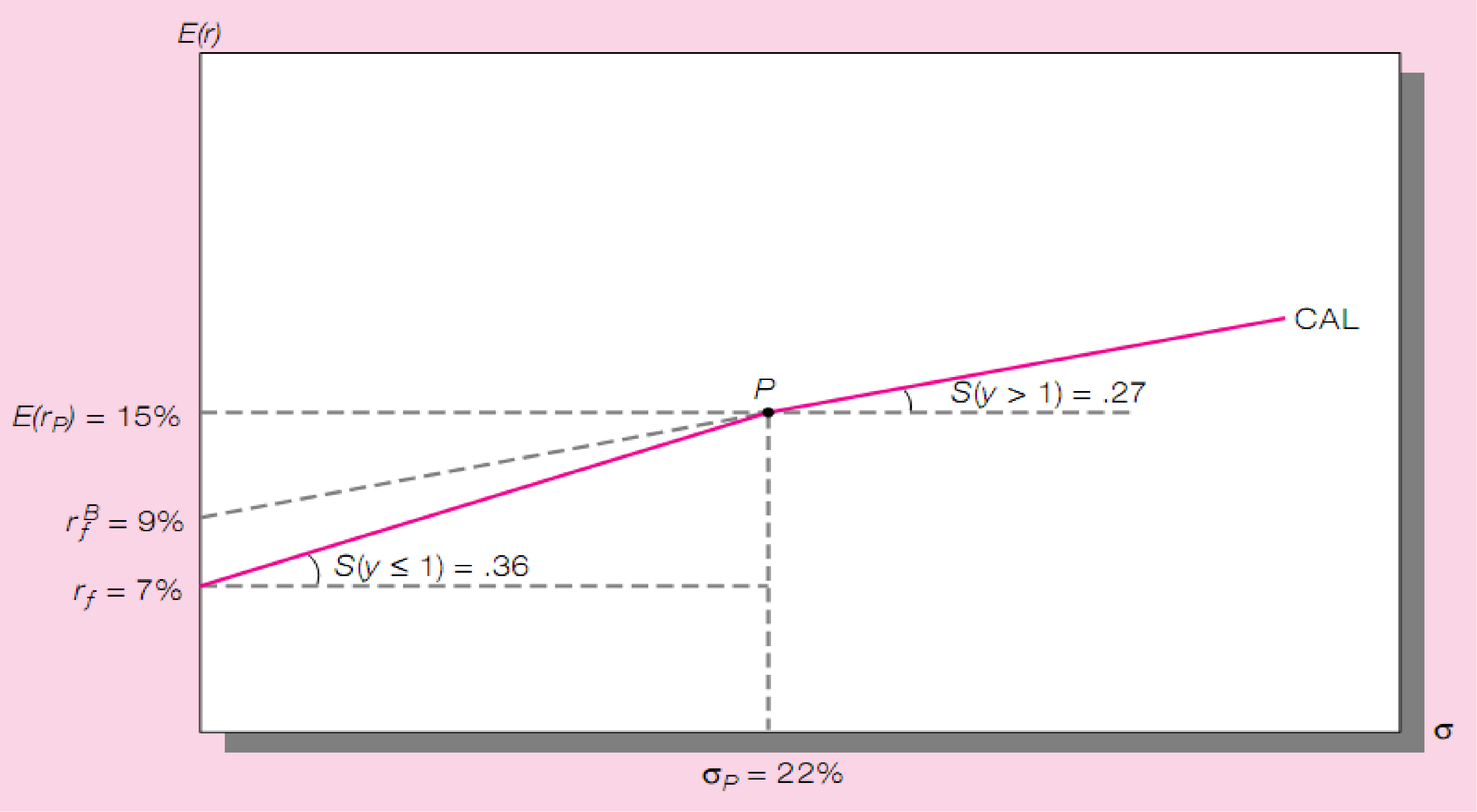


Leverage position

Borrowing risk free assets

Having more return while bearing more risk (standard deviation)

Leverage position



Risk tolerance and asset allocation

$$U = E(r) - .005A\sigma^2$$

$$E(r_C) = r_f + y[E(r_P) - r_f] \qquad \sigma_C^2 = y^2\sigma_P^2$$

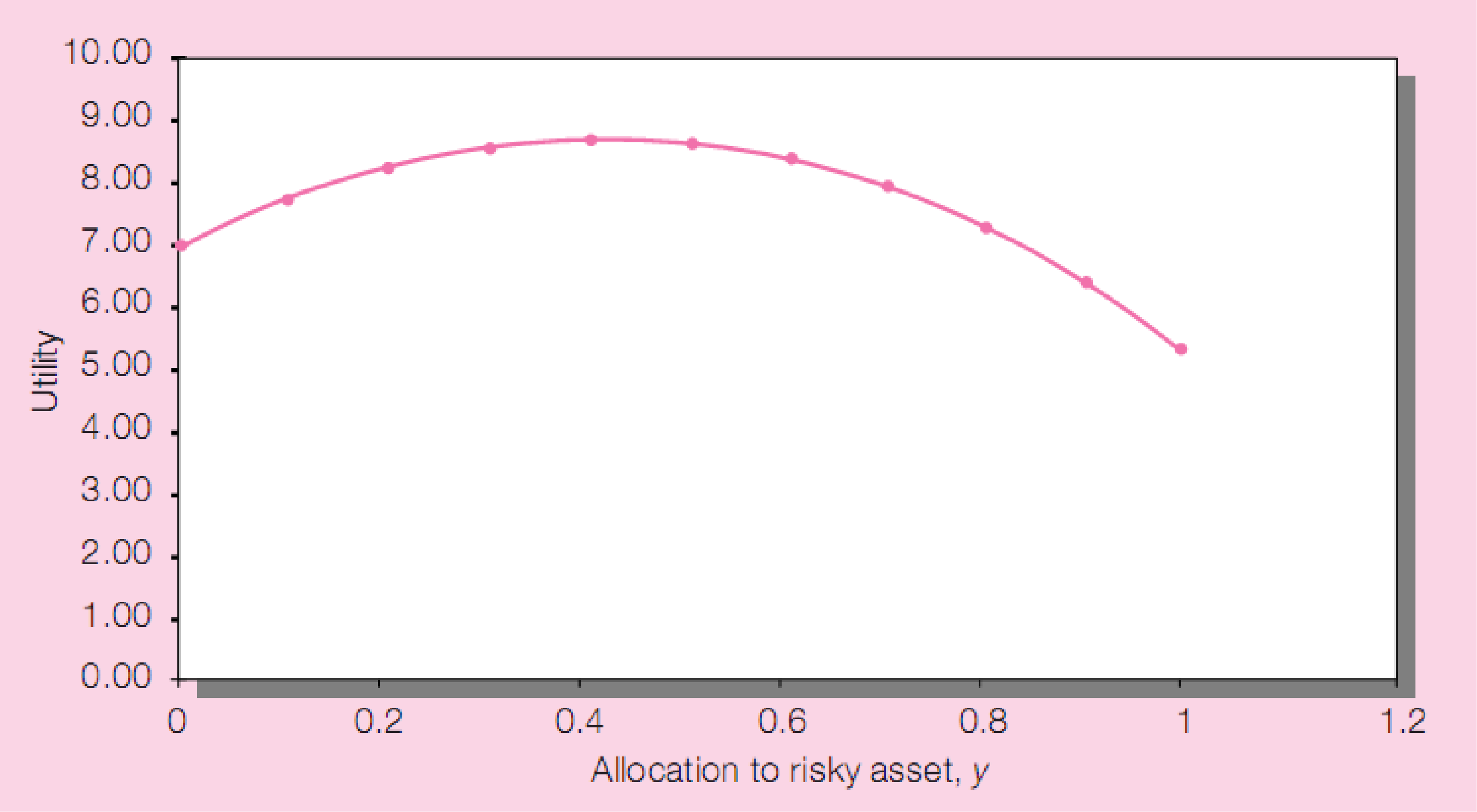
$$\max_y U = E(r_C) - .005A\sigma_C^2 = r_f + y[E(r_P) - r_f] - .005Ay^2\sigma_P^2$$

$$y^* = \frac{E(r_P) - r_f}{.01A\sigma_P^2}$$

Utility values when A=4

(1) y	(2) $E(r_C)$	(3) σ_C	(4) U
0	7	0	7.00
0.1	7.8	2.2	7.70
0.2	8.6	4.4	8.21
0.3	9.4	6.6	8.53
0.4	10.2	8.8	8.65
0.5	11.0	11.0	8.58
0.6	11.8	13.2	8.32
0.7	12.6	15.4	7.86
0.8	13.4	17.6	7.20
0.9	14.2	19.8	6.36
1.0	15.0	22.0	5.32

Plot


10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
Utility
0
0.2
0.4
0.6
0.8
1
1.2
Allocation to risky asset, y

Graphical way

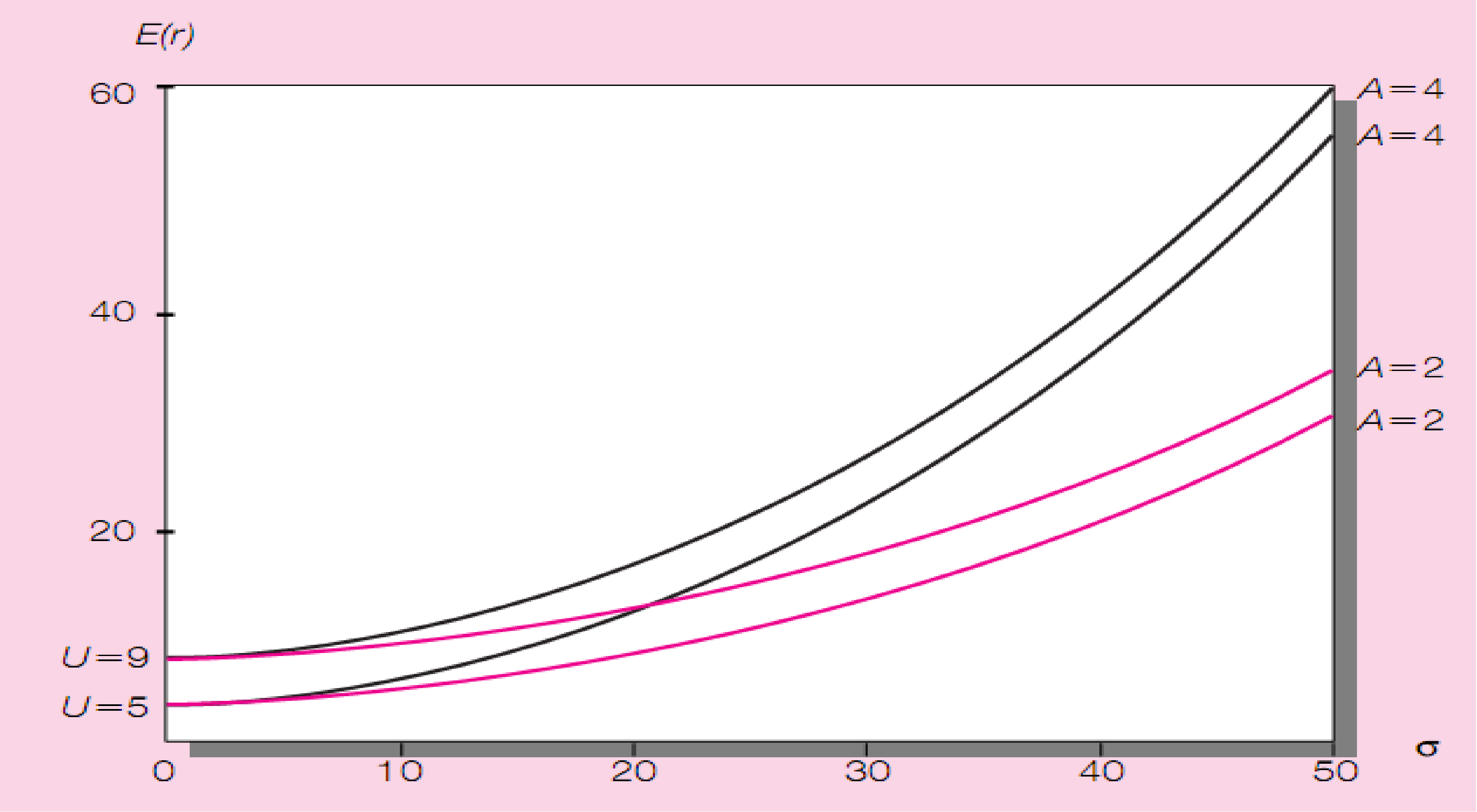
Indifference curve

find out expected return and std. deviation pairs that maintain same level of utility.

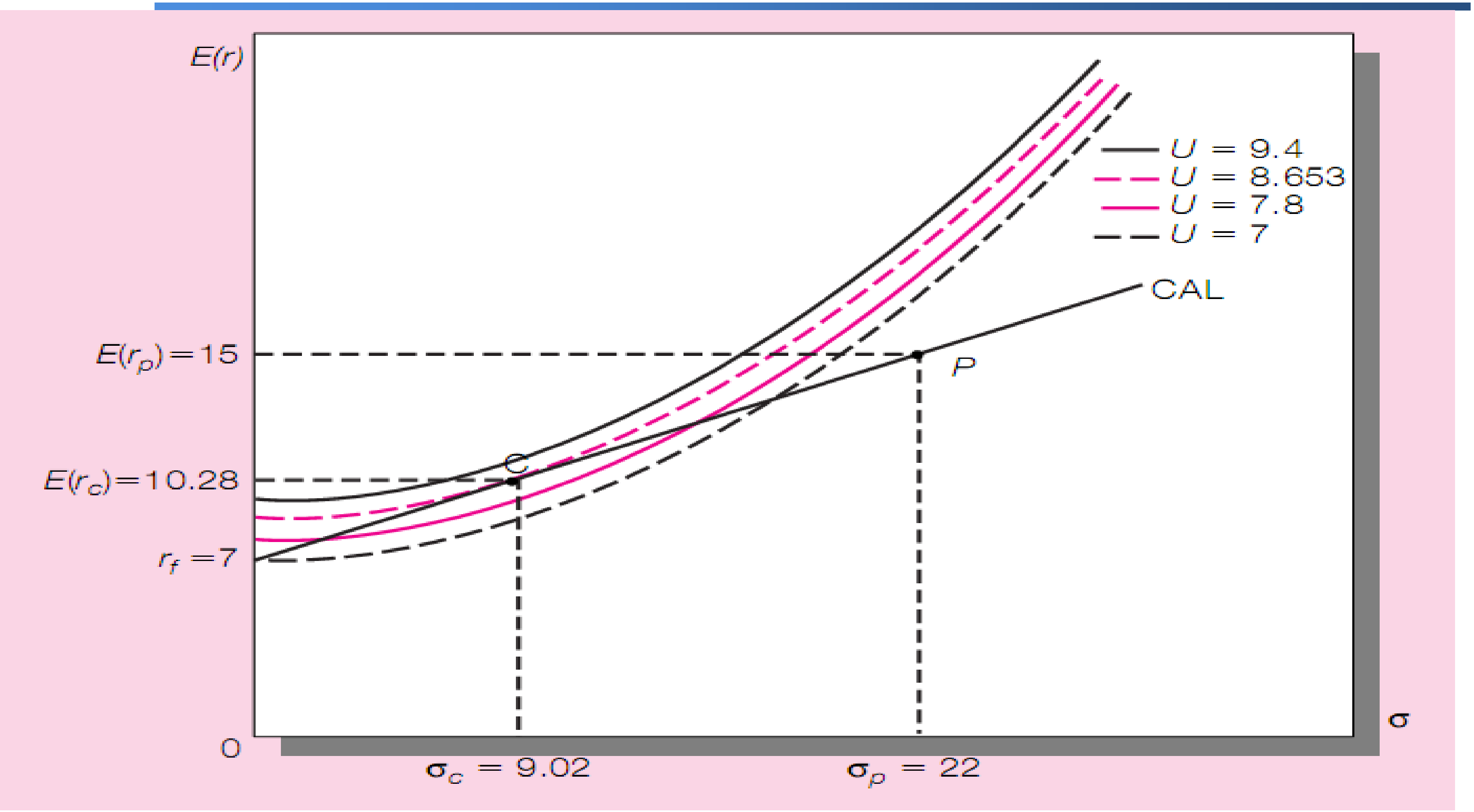
Graphical way

σ	$A = 2$		$A = 4$	
	$U = 5$	$U = 9$	$U = 5$	$U = 9$
0	5.000	9.000	5.000	9.000
5	5.250	9.250	5.500	9.500
10	6.000	10.000	7.000	11.000
15	7.250	11.250	9.500	13.500
20	9.000	13.000	13.000	17.000
25	11.250	15.250	17.500	21.500
30	14.000	18.000	23.000	27.000
35	17.250	21.250	29.500	33.500
40	21.000	25.000	37.000	41.000
45	25.250	29.250	45.500	49.500
50	30.000	34.000	55.000	59.000

Graphical way



Graphical way



Passive Strategies

"neutral" diversification strategy

e.g. S&P 500

Capital market line (CML) refers to the CAL provided by one month T-bills and a broad index of common stocks.

Passive Strategies

The alternative is not free

The free-rider benefit

Diversification

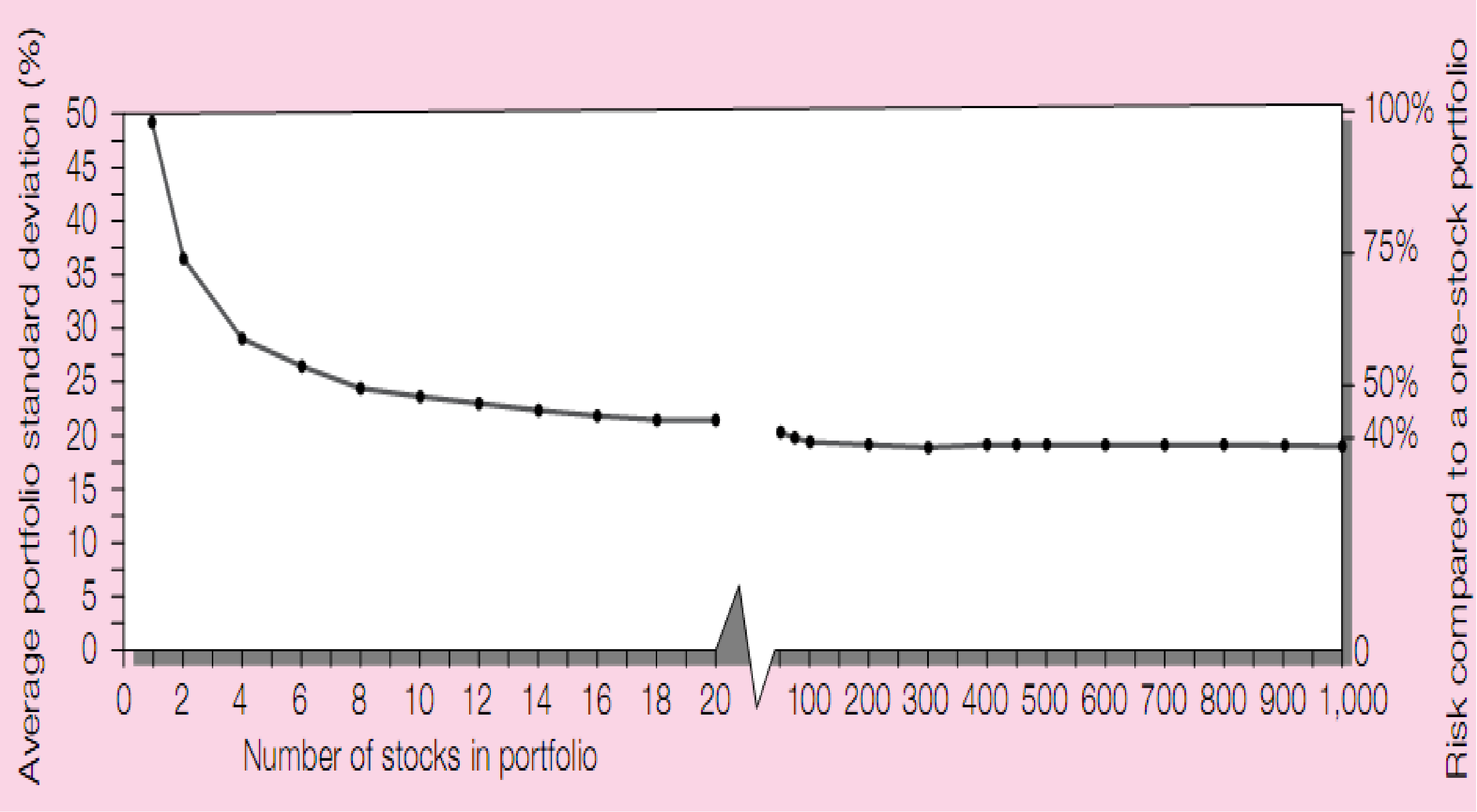
Systematic risk

market risk, nondiversifiable risk

Nonsystematic risk

unique risk, firm-specific risk, diversifiable risk

Diversification



Portfolio (two assets)

Expected return

$$r_p = \sum_{i=1}^{N}(W_i r_i) = W_1 r_1 + (1-W_1) r_2$$

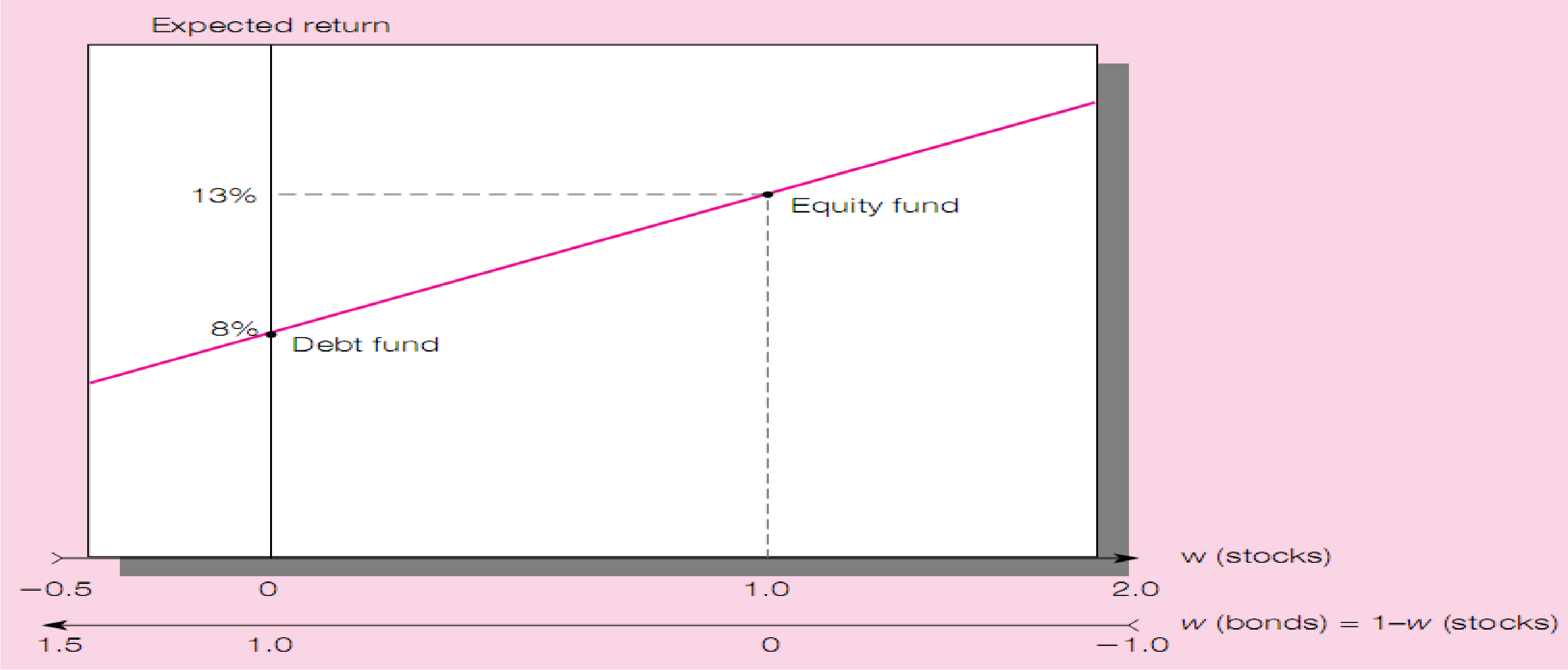
Variance

$$\sigma_p^2 = \sum_{i=1}^{N}(W_i^2 \sigma_i^2) + \sum_{j=1}^{N}\sum_{\substack{k=1 \\ k\neq j}}^{N}(W_j W_k \sigma_j \sigma_k \rho_{jk})$$

$$= W_1^2 \sigma_1^2 + (1-W_1)^2 \sigma_2^2 + 2W_1(1-W_1)\sigma_1 \sigma_2 \rho_{12}$$

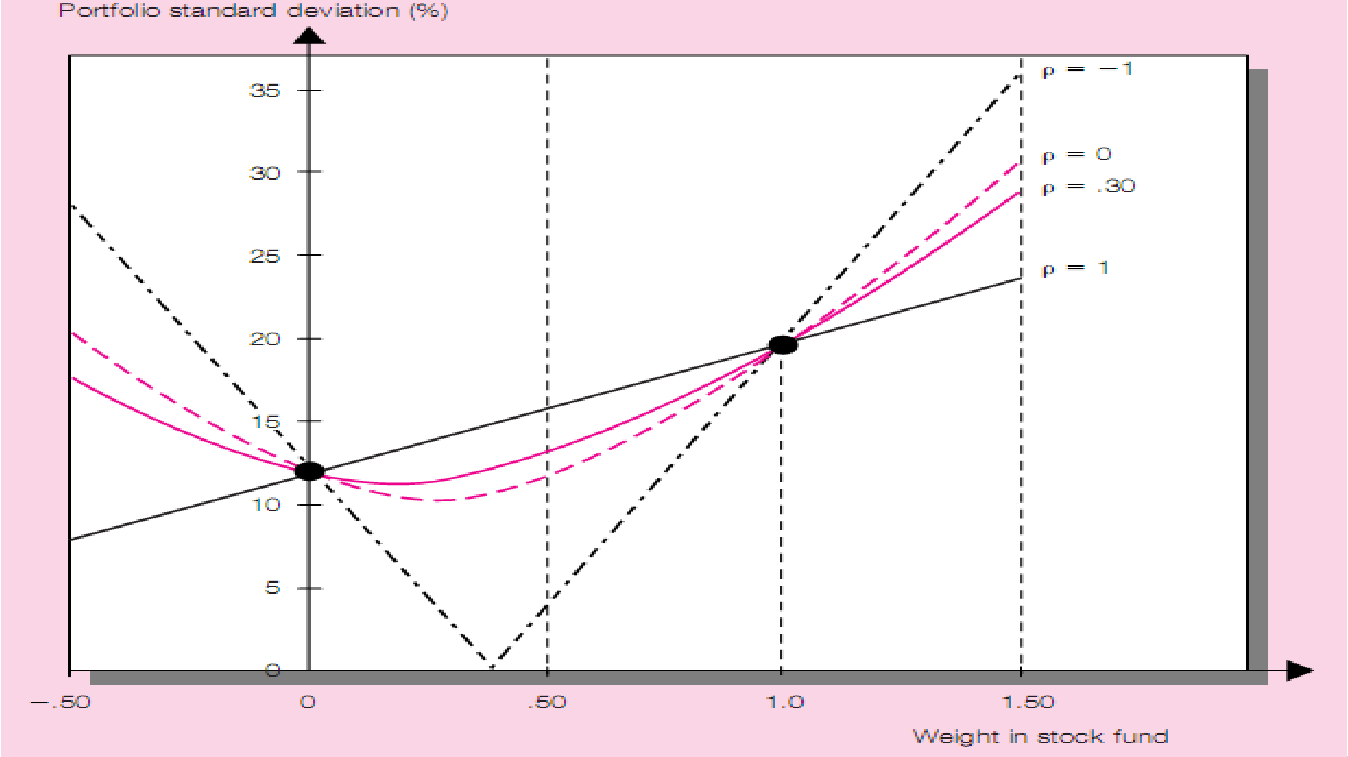
Portfolio (two assets)

Expected return as a function of weight



Portfolio (two assets)

Std. deviation as a function of weight

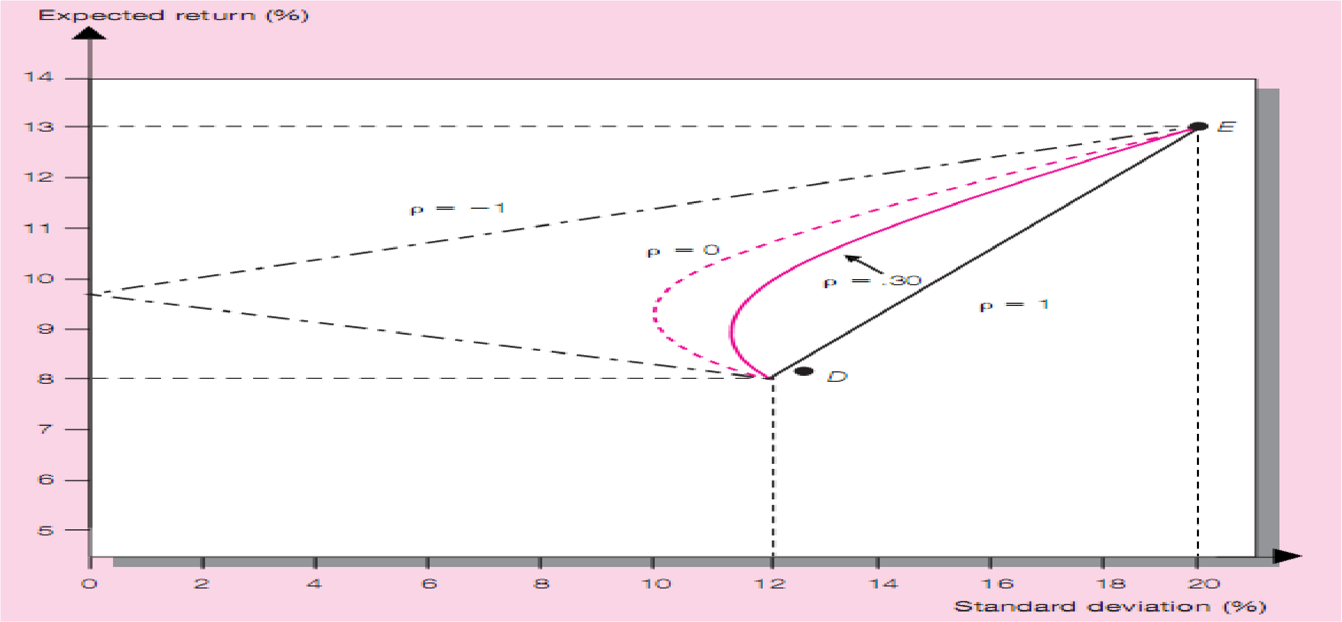


Portfolio (two assets)

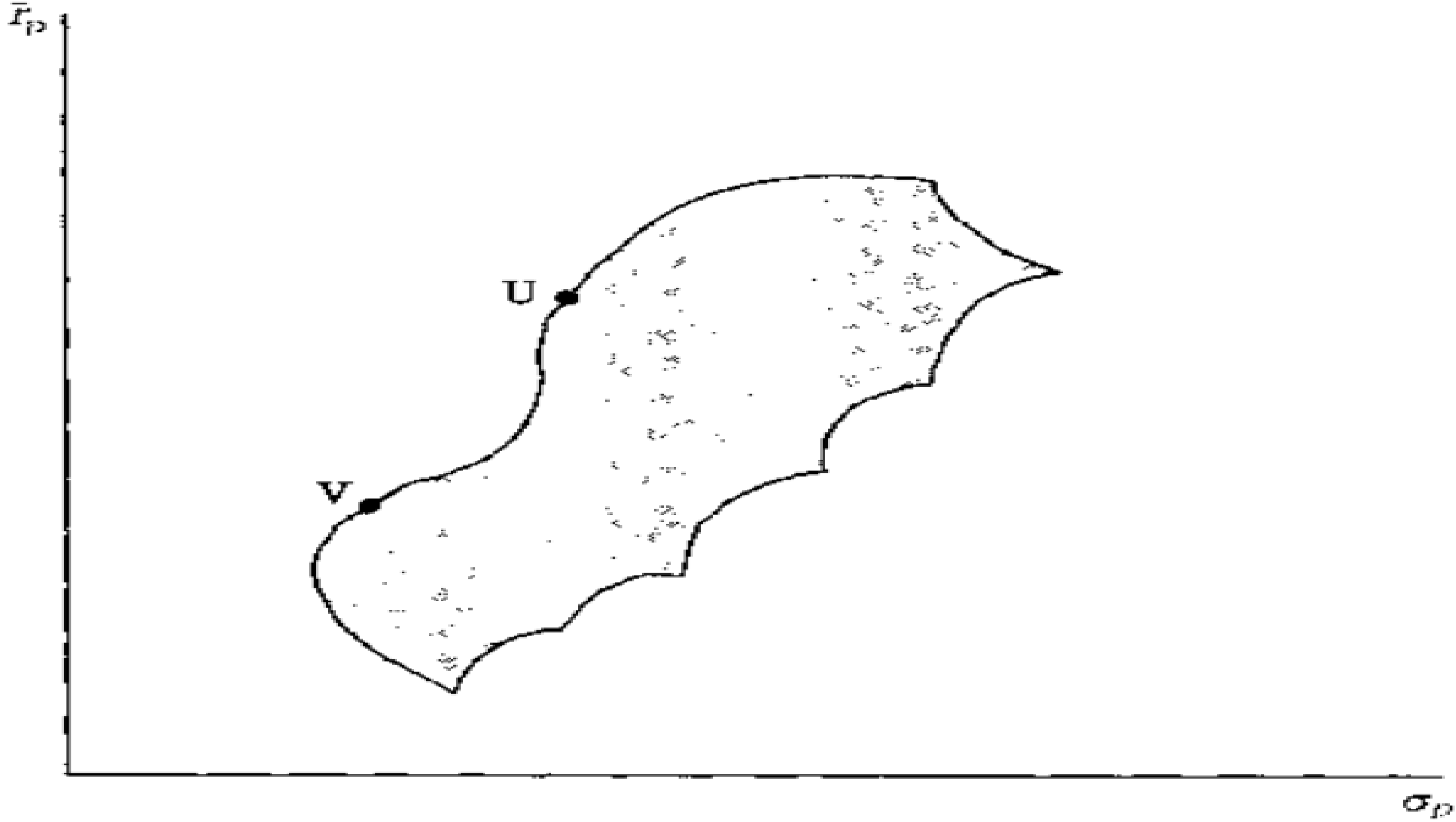
w_D	w_E	$E(r_P)$	Portfolio Standard Deviation for Given Correlation $\rho = -1$	$\rho = 0$	$\rho = .30$	$\rho = 1$
0.00	1.00	13.00	20.00	20.00	20.00	20.00
0.10	0.90	12.50	16.80	18.04	18.40	19.20
0.20	0.80	12.00	13.60	16.18	16.88	18.40
0.30	0.70	11.50	10.40	14.46	15.47	17.60
0.40	0.60	11.00	7.20	12.92	14.20	16.80
0.50	0.50	10.50	4.00	11.66	13.11	16.00
0.60	0.40	10.00	0.80	10.76	12.26	15.20
0.70	0.30	9.50	2.40	10.32	11.70	14.40
0.80	0.20	9.00	5.60	10.40	11.45	13.60
0.90	0.10	8.50	8.80	10.98	11.56	12.80
1.00	0.00	8.00	12.00	12.00	12.00	12.00
			Minimum Variance Portfolio			
		w_D	0.6250	0.7353	0.8200	—
		w_E	0.3750	0.2647	0.1800	—
		$E(r_P)$	9.8750	9.3235	8.9000	—
		σ_P	0.0000	10.2899	11.4473	—

Minimum-variance portfolio

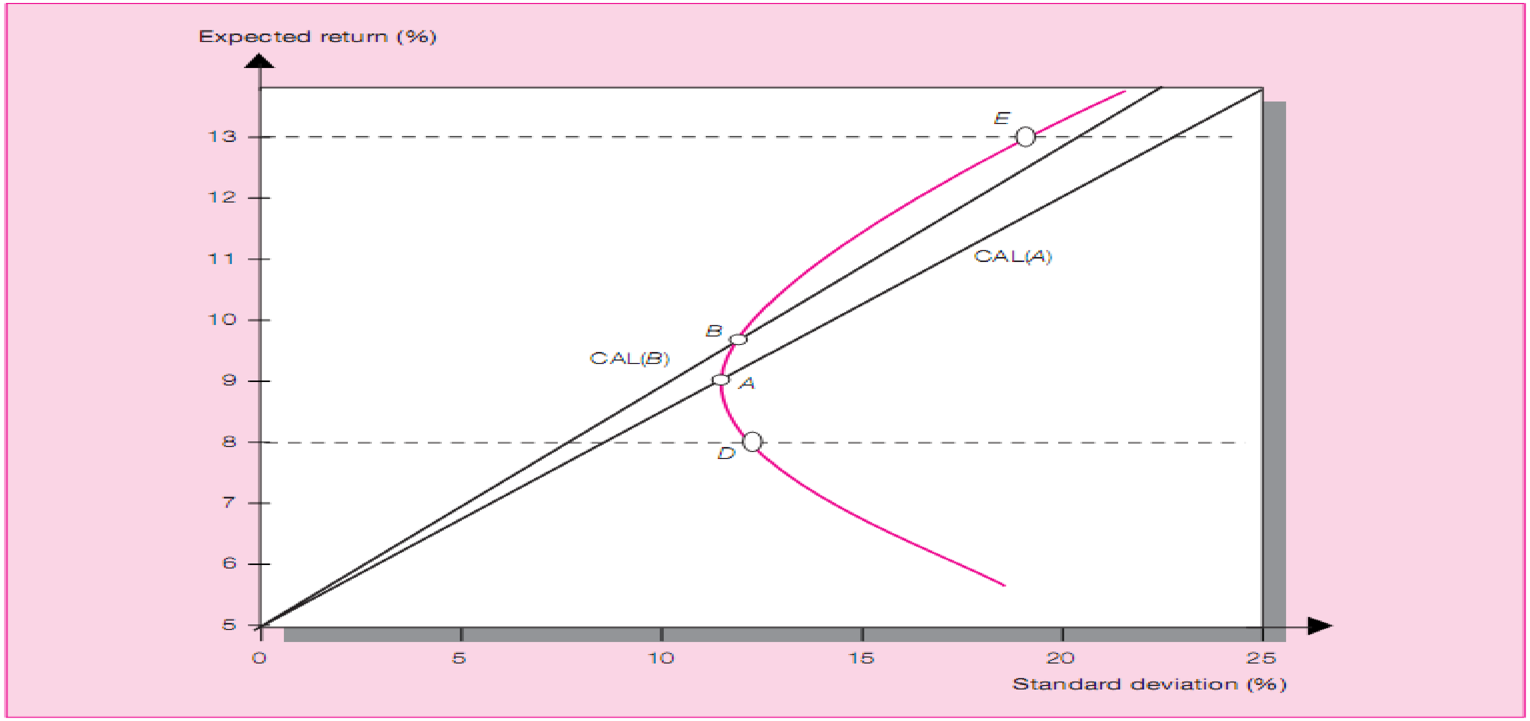
Smaller than that of either of the individual component assets.



Efficient frontier

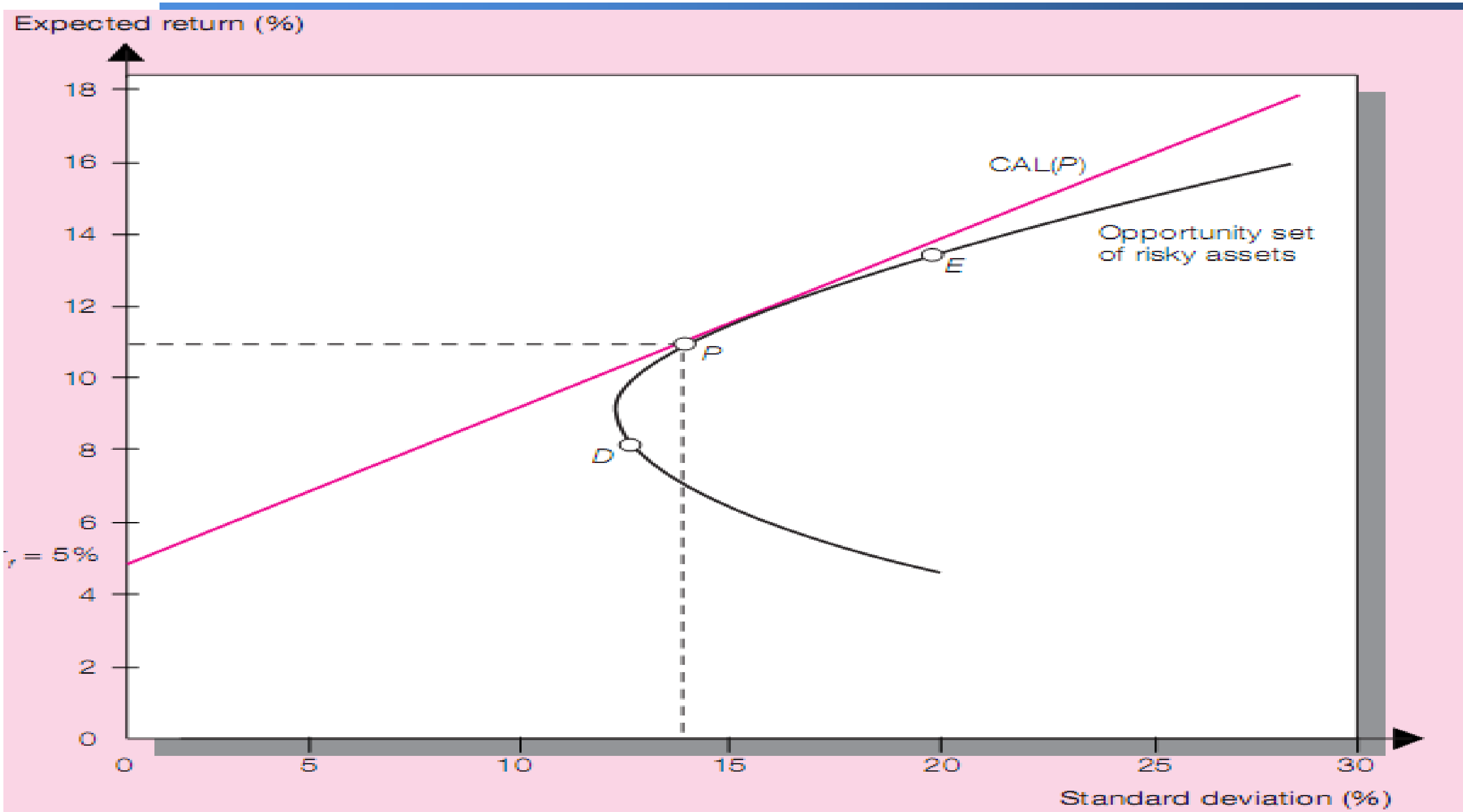


Assets allocation with stocks, bonds and bills



Look at the reward-to-variability ratio

Assets allocation with stocks, bonds and bills



Assets allocation with stocks, bonds and bills

Objective function

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

$$E(r_p) = w_D E(r_D) + w_E E(r_E)$$

$$\sigma_p = [w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \mathrm{Cov}(r_D, r_E)]^{1/2}$$

$$\underset{w_i}{\mathrm{Max}}\ S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

subject to $\Sigma w_i = 1$. This is a standard problem in optimization.

The optimal overall portfolio