浙江工商大學 ZHEJIANG GONGSHANG UNIVERSITY | 金融学院 School of Finance

Portfolio Management

Portfolio Theory

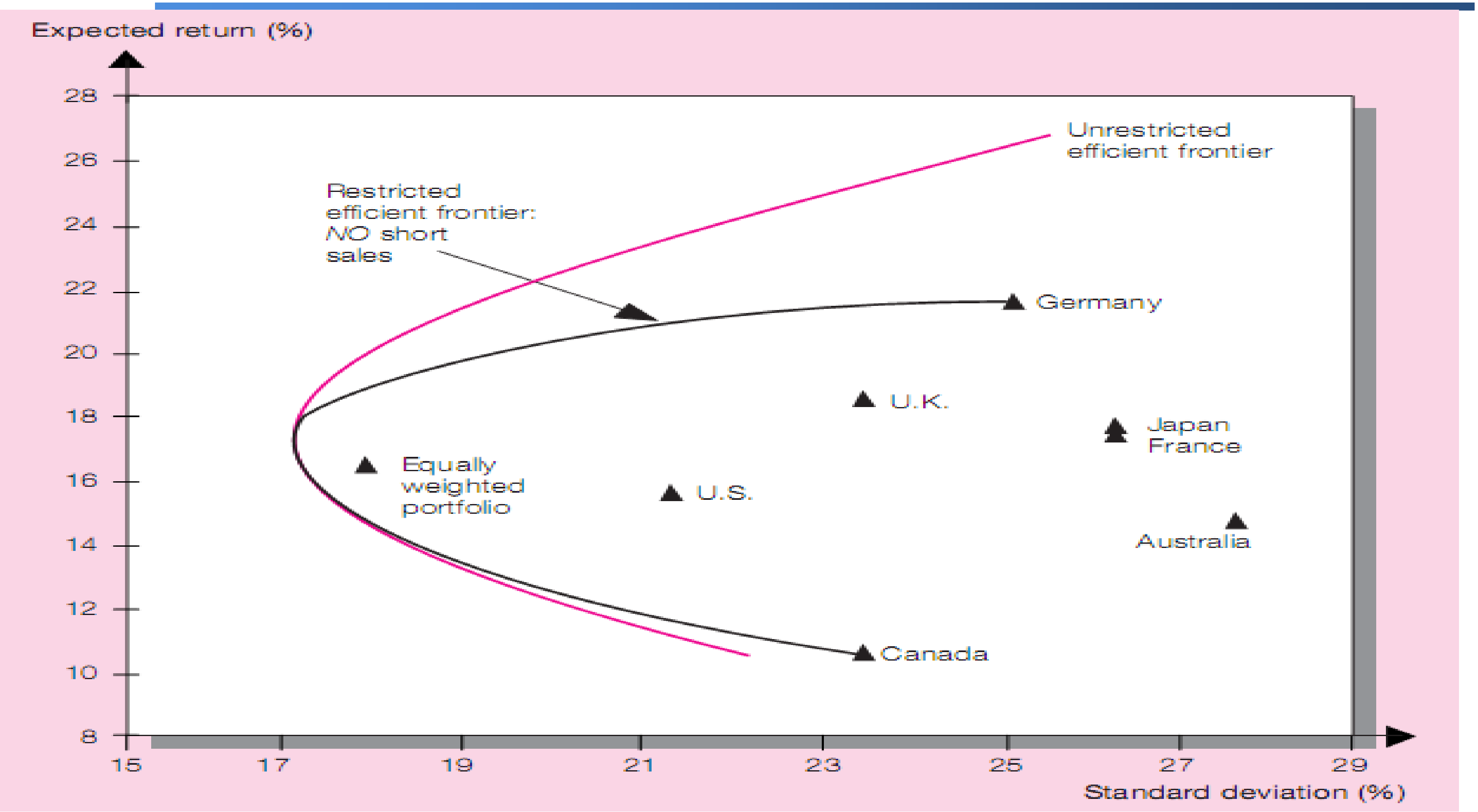
Short sell

Short sell in risky assets

Recall the restriction,

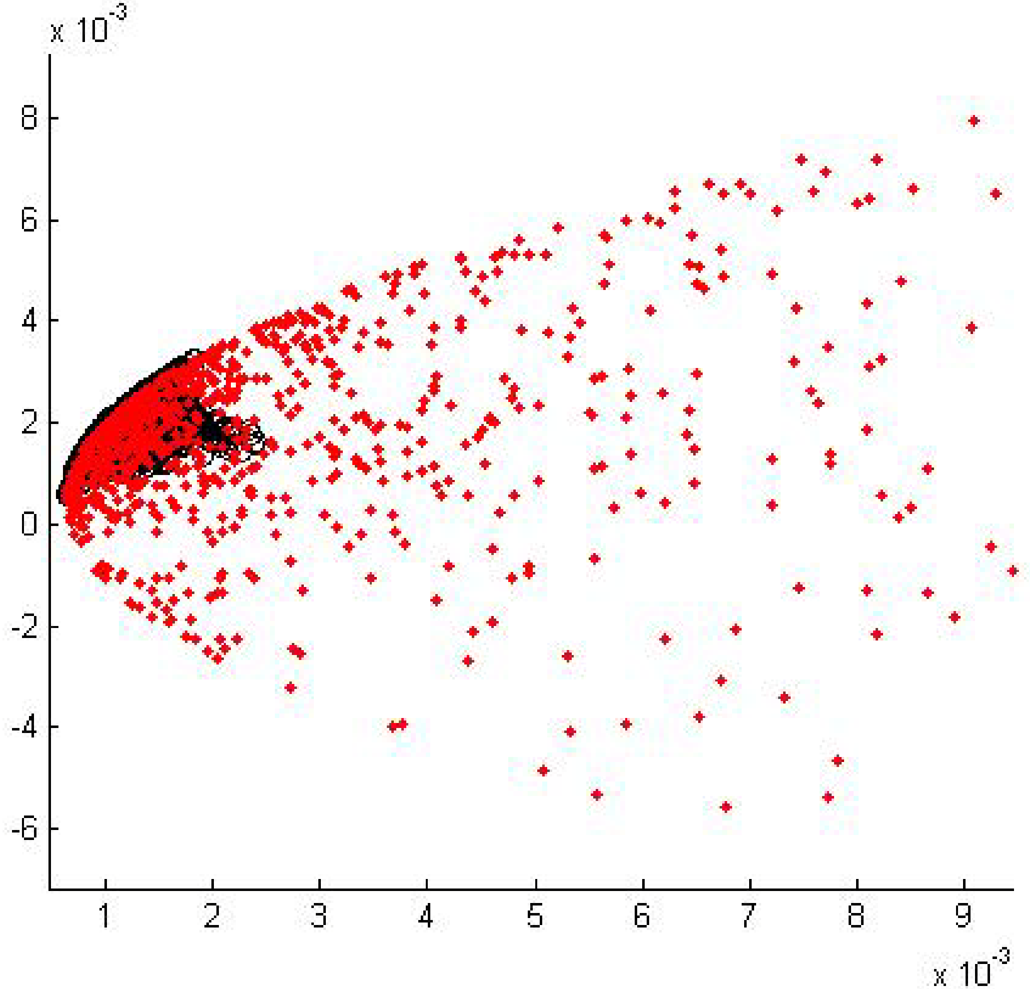
if there is any additional restriction need to be imposed, such as non-negative weights for each risky asset in the portfolio… what will happen?

Short sell



Expected return (%)
28
26
24
22
20
18
16
14
12
10
8
15
17
19
21
23
25
27
29
Standard deviation (%)
Unrestricted
efficient frontier
Restricted
efficient frontier:
NO short
sales
Germany
U.K.
Japan
France
Equally
weighted
portfolio
U.S.
Australia
Canada

Short sell



Separation property

Optimal risky portfolio (EF)

--- Technicial

Complete portfolio with risk-free assets (CAL – a straight line)

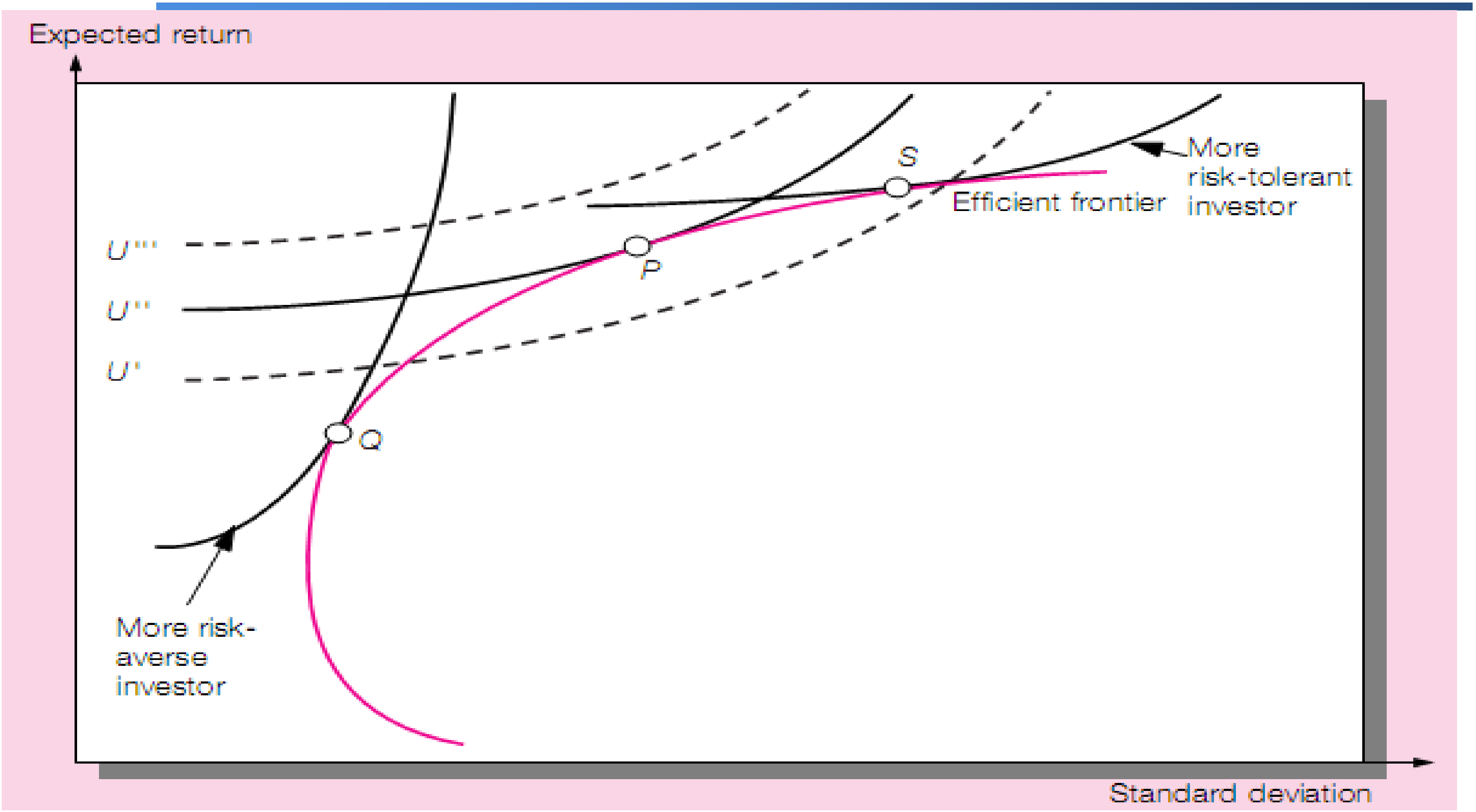
--- Risk aversion

Portfolio without risk-free assets

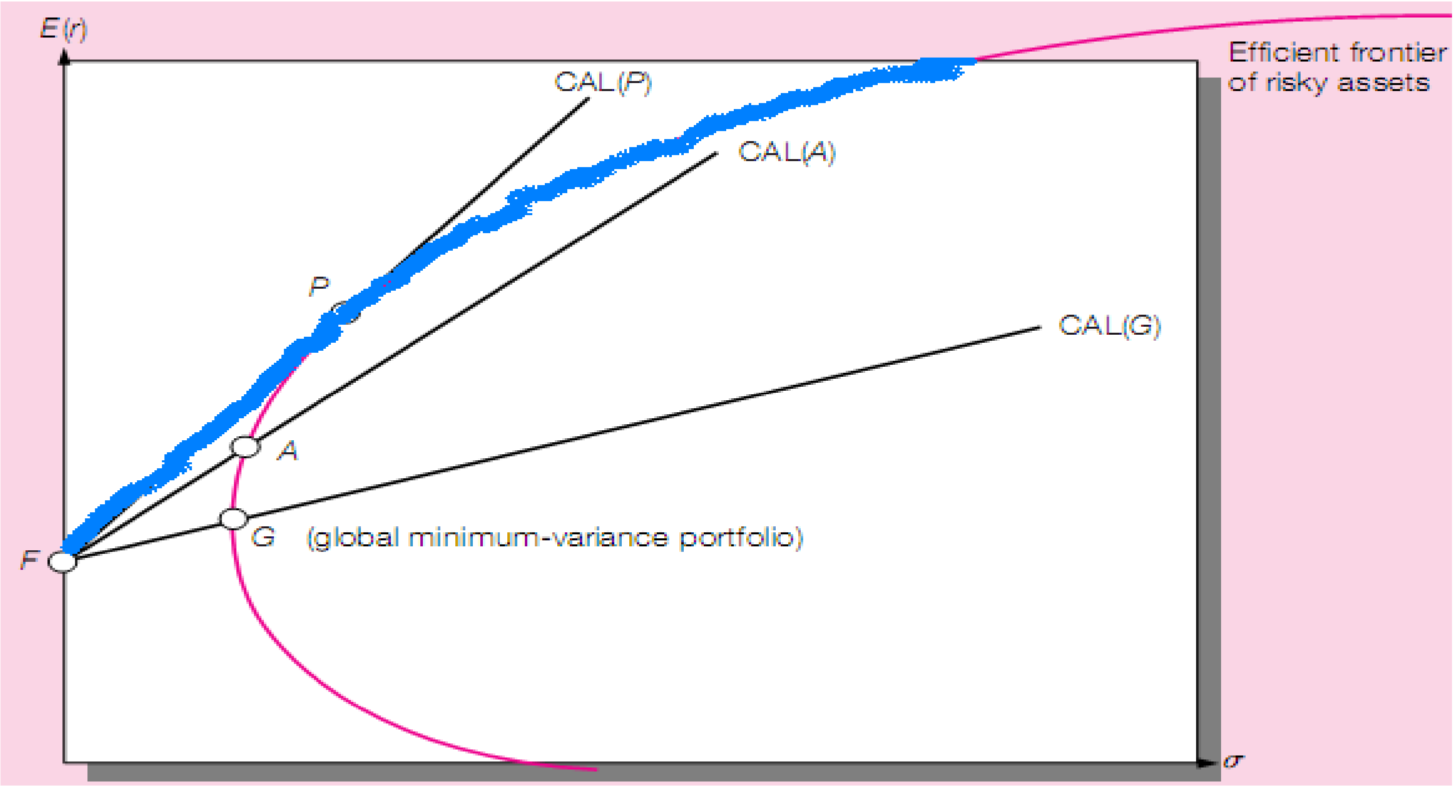
Choose an optimal risky portfolio by clients' perference in risk tolerance.

The complete portfolio will locate on the efficient frontier.

Portfolio without risk-free assets

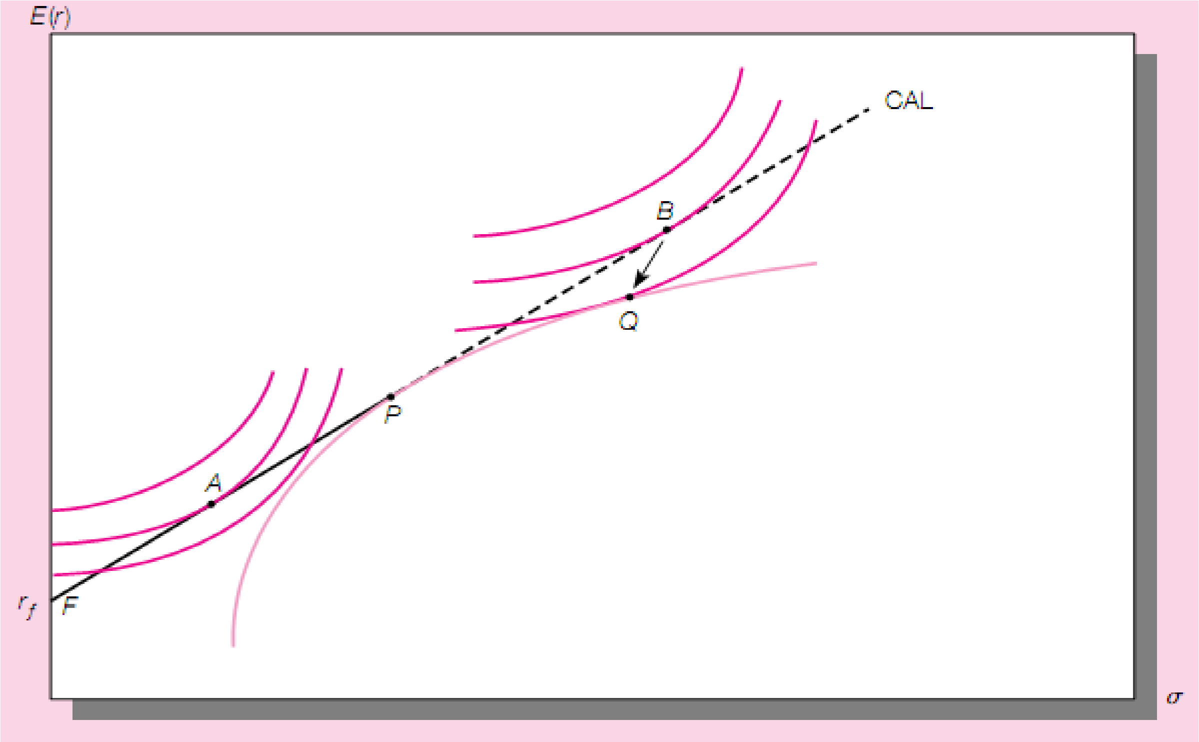


Portfolio with risk-free assets



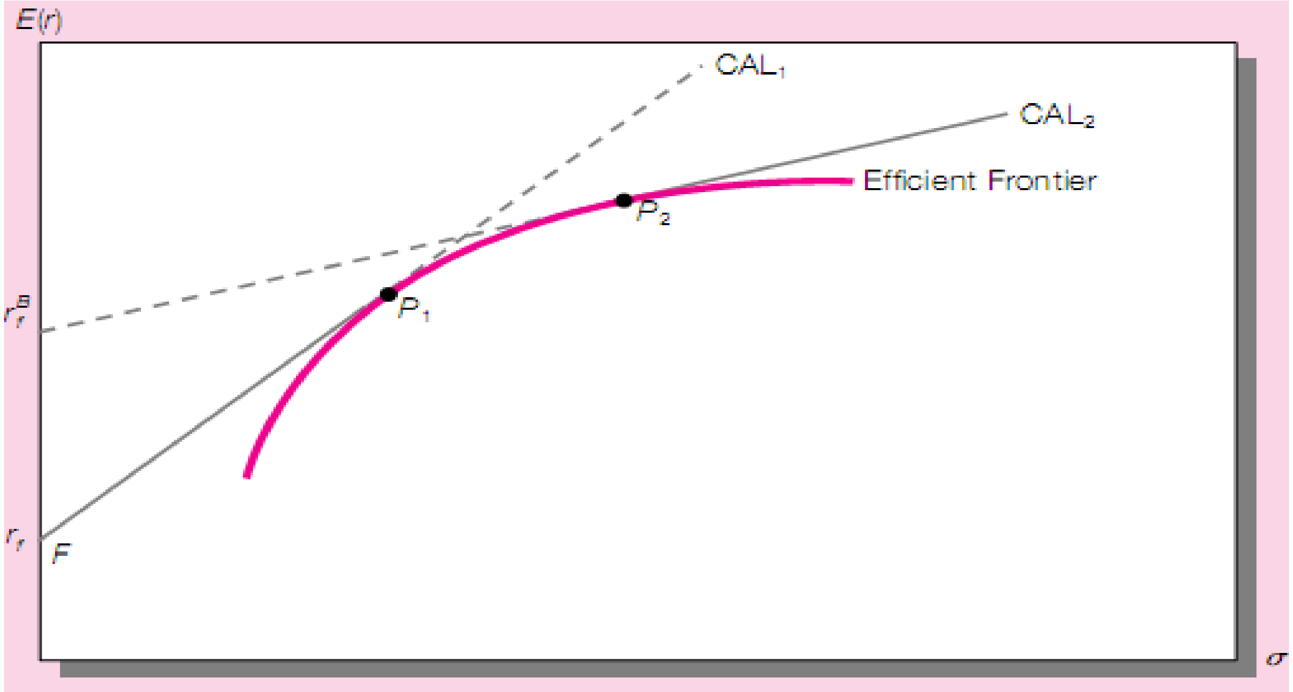
Ristriction on risk-free assets

When borrowing is prohibit



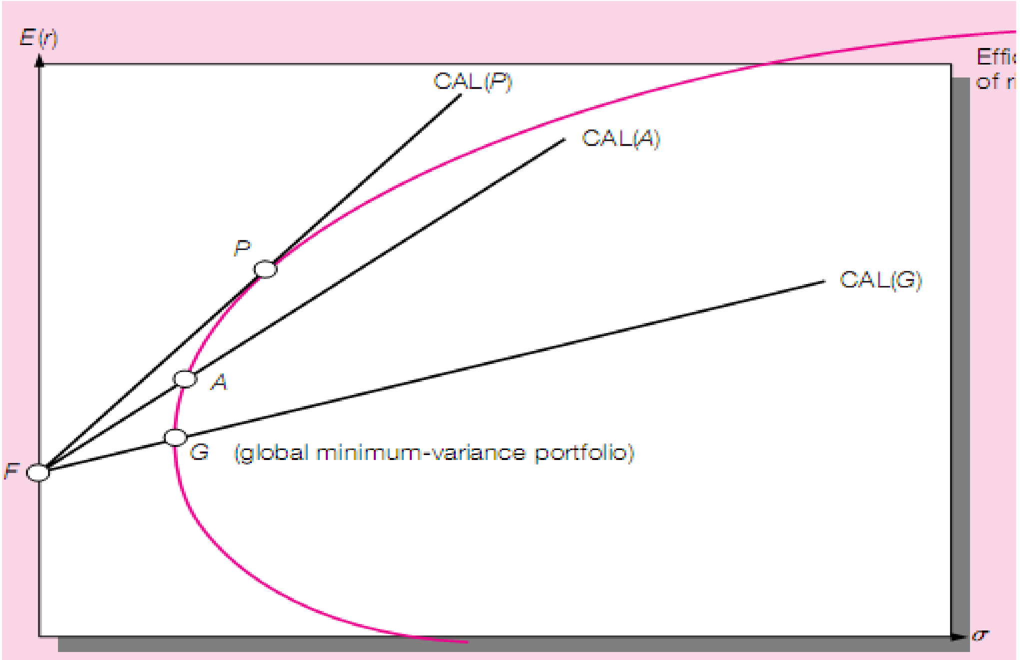
Ristriction on risk-free assets

Where borrowing incurs a higher rate



Find the efficient frontier

Short sell and risk free borrow allowed



Find the efficient frontier

Objective function - to be maximized

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

with the restriction

$$\sum_{i=1}^{N} W_i = 1$$

Find the efficient frontier

Maximize

$$S_p = \frac{\sum_{i=1}^{N} W_i (E(r_i) - r_f)}{[\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} W_i W_j \sigma_{ij}]^{1/2}}$$

$$\frac{dS_p}{dW_1} = 0; \qquad \frac{dS_p}{dW_2} = 0; \qquad \ldots \qquad \frac{dS_p}{dW_N} = 0$$

Find the efficient frontier

Continued

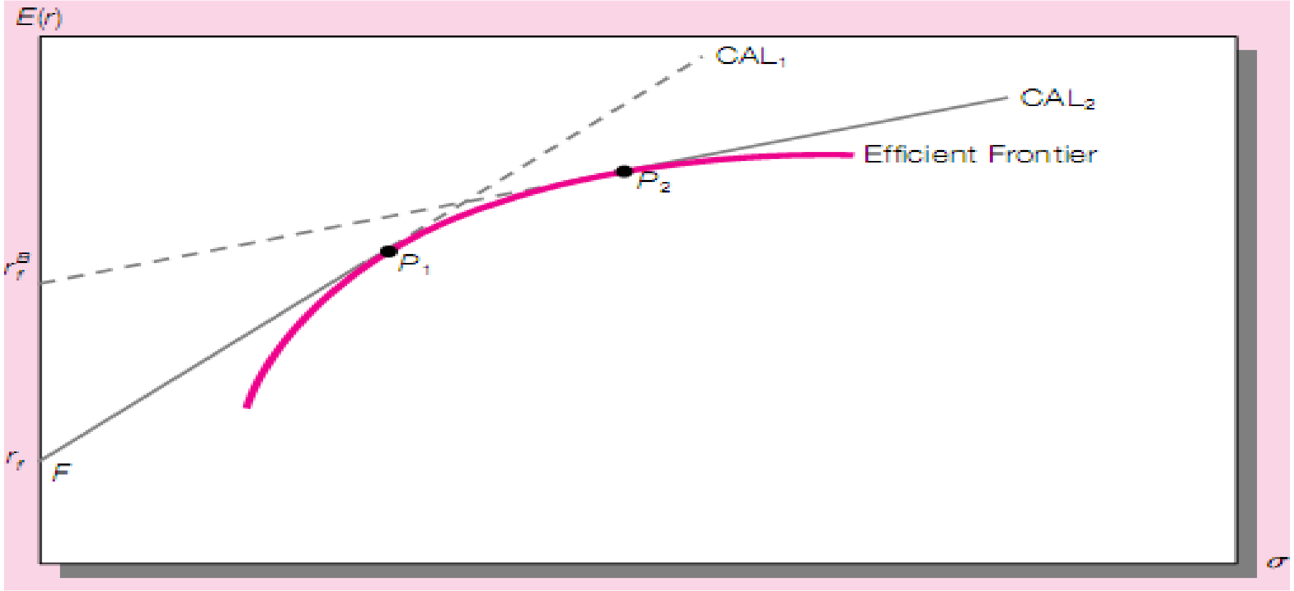
$$\frac{dS_p}{dW_i} = -(\lambda W_1 \sigma_{1i} + \lambda W_2 \sigma_{2i} + \ldots + \lambda W_N \sigma_{Ni}) + E(r_i) - r_f = 0$$

$$E(r_i) - r_f = Z_1 \sigma_{1i} + Z_2 \sigma_{2i} + \ldots + Z_N \sigma_{Ni}$$

$$W_i = \frac{Z_i}{\sum_{j=1}^{N} Z_j}$$

Find the efficient frontier

Short sell allowed but no risk free borrow



Find the efficient frontier

No short sell but with risk-free borrow

Objective function - to be maximized

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

with restrictions

$$\sum_{i=1}^{N} W_i = 1; \qquad W_i \geq 0$$

Find the efficient frontier

Neither short sell nor risk-free borrow

Objective function – to be minimized

$$\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} W_i W_j \sigma_{ij}$$

With restrictions

$$\sum_{i=1}^{N} W_i = 1; \qquad W_i \geq 0; \qquad \sum_{i=1}^{N} (W_i E(r_i)) = E(r_p)$$