



Beyond CAPM

----- Arbitrage pricing theory



Idea

In finance, arbitrage pricing theory (APT) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient.

- the asset price should equal the expected end of period price discounted at the rate implied by the model
- If the price diverges, arbitrage should bring it back into line



Facts

- The arbitrage pricing theory (APT) describes the price where a mispriced asset is expected to be. It is often viewed as an alternative to the capital asset pricing model (CAPM), since the APT has more flexible assumption requirements.
- Arbitrageurs use the APT model to profit by taking advantage of mispriced securities, which have prices that differ from the theoretical price predicted by the model.
- By shorting an overpriced security, while concurrently going long in the portfolio the APT calculations were based on, the arbitrageur is in a position to make a theoretically risk-free profit.

History



The theory predicts a relationship between the returns of a portfolio and the returns of a single asset through a linear combination of many independent macroeconomic variables.

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Reference

1. Ross, Stephen (1976). "The arbitrage theory of capital asset pricing". *Journal of Economic Theory*. 13 (3): 341–360
2. Roll, Richard; Ross, Stephen (1980). "An empirical investigation of the arbitrage pricing theory". *Journal of Finance*. 35 (5): 1073–1103.
3. Chen, N. F.; Ingersoll, E. (1983). "Exact Pricing in Linear Factor Models with Finitely Many Assets: A Note". *Journal of Finance*. 38 (3): 985–988



Assumption

- It operates with a pricing model that factors in many sources of risk and uncertainty.
- It looks at several macroeconomic factors that, according to the theory, determine the risk and return of the specific asset.
- It suggests that investors will diversify their portfolios, but that they will also choose their own individual profile of risk and returns based on the premiums and sensitivity of the macroeconomic risk factors.

Mathematical Model



The Arbitrage Pricing Theory can be expressed as a mathematical model:

$$E(r_i) = r_f + \sum_{k=1}^n \beta_k RP_k$$

where

$E(r_j)$ – Expected return on asset

r_f – Risk-free rate

β_k – Sensitivity of the asset price to macroeconomic factor k

RP_k – Risk premium associated with factor k

The beta coefficients in the APT are estimated by using linear regression.



Factors in the APT

The APT provides analysts and investors with a high degree of flexibility regarding the factors that can be applied to the model.

There is no consensus agreement of finance professionals and academics on which factors are best for predicting returns on securities

Ross suggests that there are some specific macroeconomic factors that have proven most reliable as price predictors. These include sudden changes in inflation , GDP , GNP , corporate bond premiums , commodities prices , market indices levels , and currency exchange rates .



Arbitrage in the APT

The APT suggests that the returns on assets follow a linear pattern. An investor can leverage deviations in returns from the linear pattern using the arbitrage strategy.

Arbitrage is a practice of the simultaneous purchase and sale of an asset, taking advantage of slight pricing discrepancies to lock in a risk-free profit for the trade.



Arbitrage

In the APT context, arbitrage consists of trading in two assets – with at least one being mispriced. The arbitrageur sells the asset which is relatively too expensive and uses the proceeds to buy one which is relatively too cheap.

Under the APT, an asset is mispriced if its current price diverges from the price predicted by the model. The expected return of the asset is a linear function of various factors, and sensitivity to changes in each factor is represented by a factor-specific beta coefficient.

A correctly priced asset here may be in fact a synthetic asset - a portfolio consisting of other correctly priced assets. This portfolio has the same exposure to each of the macroeconomic factors as the mispriced asset.



Arbitrage

The arbitrageur creates the portfolio by identifying some correctly priced assets and then weighting the assets such that portfolio beta per factor is the same as for the mispriced asset.

if the price is too low (undervalue), the arbitrageur could therefore:

First:

1. short sell the portfolio
2. buy the mispriced asset with the proceeds.

Then:

1. sell the mispriced asset
2. use the proceeds to buy back the portfolio
3. pocket the difference.



Implementation of APT

The implementation of APT involves three steps:

1. Identify the factors
2. Estimate factor loadings of assets
3. Estimate factor premia.



Sample

Suppose that there are two factors represented by:

- return on the market portfolio r_M
- return on Treasury bond portfolio r_N

$$R_i - r_f = \beta_{iM}(r_M - r_f) + \beta_{iN}(r_N - r_F)$$

if we have: $r_f = 5\%$, $r_M - r_f = 8\%$, $r_N - r_f = 2\%$

and for asset α its $\beta_{\alpha M} = \beta_{\alpha N} = 1$

How can we make profit? if $R_\alpha = 10\%$



Solution

according to APT

$$R_a = 0.05 + (1.0)(0.08) + (1.0)(0.02) = 15\%$$

We would then have an arbitrage:

- (1) buy \$100 of market portfolio
- (2) buy \$100 of bond portfolio
- (3) sell \$100 of risk-free asset

sell \$100 of asset α



Solution

- requires zero initial investment (an arbitrage portfolio)
- bears no factor risk
- profits $(13 + 7 - 10 - 5) = \$5$ surely.



Differences between APT and CAPM

- APT is based on the factor model of returns and **arbitrage**
- CAPM is based on investors' portfolio demand and equilibrium



Review CAPM (Implications)

1. Every investor puts their money into two pots:
 - the riskless asset
 - a single portfolio of risky assets – the tangent portfolio.
2. All investors hold the risky assets in same proportions
 - they hold the same risky portfolio, the tangent portfolio.
3. The tangent portfolio is the market portfolio



Review CAPM

- β_{iM} gives a measure of asset i 's systematic risk
- $r_M - r_f$ gives the premium per unit of systematic risk
- The risk premium on asset i equals the amount of its systematic risk times the premium per unit of the risk.

Three lines: CAL, CML, SML



Review CAPM

Suppose that CAPM holds. The expected market return is 14% and T-bill rate is 5%.

1. What should be the expected return on a stock with $\beta = 0$?
2. What should be the expected return on a stock with $\beta = 1$?
3. What should be the expected return on a portfolio made up of 50% T-bills and 50% market portfolio?



Review CAPM

We can decompose an asset's return into three pieces:

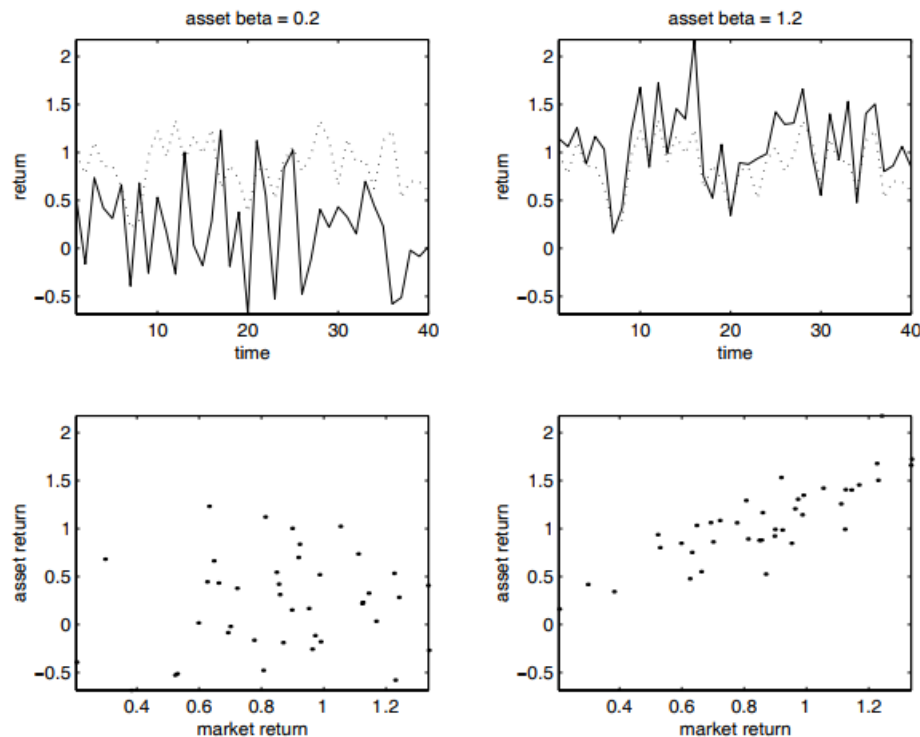
$$r_i - r_f = \alpha_i + \beta_{iM}(r_M - r_f) + \sigma_i$$

Three characteristics of an asset:

- Beta
- Alpha
- Sigma = StD (σ_i).

Beta

- Beta measures an asset's systematic risk.
- Assets with higher betas are more sensitive to the market



(Market premium = 8%, market volatility = 25%, asset volatility = 40%.)

Solid lines – asset returns. Dotted lines – market returns.



Alpha

- According to CAPM, α should be zero for all assets.
- α measures an asset's return in excess of its risk-adjusted award according to CAPM.

What to do with an asset with a positive α ?