



1. Input impedance matching:  $|S_{11}| < -12$  dB
2. Center frequency  $f_0 = 28$  GHz
3. Voltage gain  $> 12$  dB at  $f_0$
4. Power consumption  $< 30$  mW (the lower, the better) with  $V_{dd} = 0.9$  V
5. Noise figure  $< 5$  dB
6. IIP3  $> -15$  dBm

Questions:

- a) Simulate the single transistor M1. Plot  $g_m$  vs.  $I_{den1}$  (current density) and  $f_T$  vs.  $I_{den1}$  by sweeping  $I_{ds}$ , M1 with a fixed width, e.g.,  $4\mu\text{m}$ .
- b) Determine  $I_{bias}$  and  $M$  (Multiplier of M1 and M2) that gives the optimal  $f_T$  and  $g_m$  for minimizing NF. What is resulting input resonant circuit Q? (For the size of transistor M3,  $M_b$  is typically very smaller than  $M$ , e.g.,  $M_b = 10\%M$ )
- c) Determine  $L_g$  and  $L_s$  using  $C_{gs}$  and  $f_T$  to provide the required input matching. Assume  $L_s$  and  $L_g$  have a Q of 15 at  $f_0$ , compute the series resistance value and include it in the simulation (use 'ind' and 'res' from 'analogLib').
- d) Determine  $L_d$  such that the output node is tuned at  $f_0$ . Assume  $L_d$  have a Q of 15 at  $f_0$ , compute the series resistance value and include it in the simulation (use 'ind' and 'res' from 'analogLib').
- e) Plot the gain ( $V_{out}/V_{in}$ ), noise figure (NF) and input impedance ( $Z_{in}$ ) vs. frequency from 0.1 to 50 GHz.
- f) Perform a PSS analysis at  $f_0$  to determine the 1-dB compression of this LNA by sweeping the input port power.
- g) Using PSS analysis, perform a two-tone test to determine the IIP3 of the LNA using signals at  $f_0$  and  $f_0 + 800\text{MHz}$ . (Optional: You can use hb and PAC analysis to simulate the IIP3 again and compare results for **bonus credits**).
- h) Using sp analysis, plot the K factor vs. frequency from 0.1 to 50 GHz. Comment on the stability of your designed LNA based on the simulation result.
- i) Using sp analysis, plot noise circles with noise ranges from 1 dB to 10 dB and gain circles with gain ranges from 10 dB to 20 dB, both at  $f_0$ . Comment on the trade-off between gain and noise based on the circles.
- j) Now, add  $C_d$  between the gate and source terminal of M1, plot NF at  $f_0$  vs.  $C_d$ , make sure that you adjust  $L_g$  and  $L_s$  for each value of  $C_d$  to maintain the input matching at  $f_0$ . How much is the improvement in NF? What happened to the amplifier gain?
- k) With the  $C_d$  and the  $L_g$ , perform a sweep of the input device width, is there a new optimal width that gives lower NF? If so, what is it and what is the NF?
- l) Now, add  $L_x$  at the cascode node by first estimating the parasitic capacitance at node X. How much is the improvement in NF? What happened to the amplifier gain?

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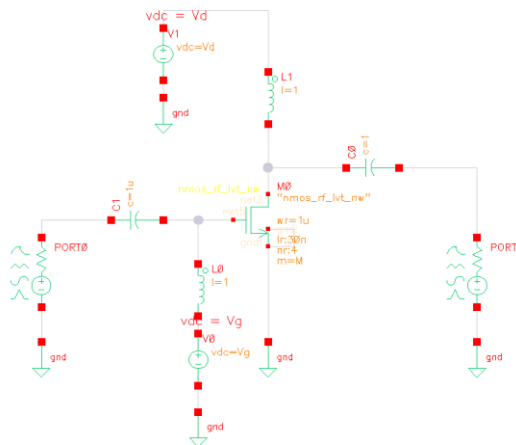
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# ELEC 5280 Homework 2 Answers

## Problem 1

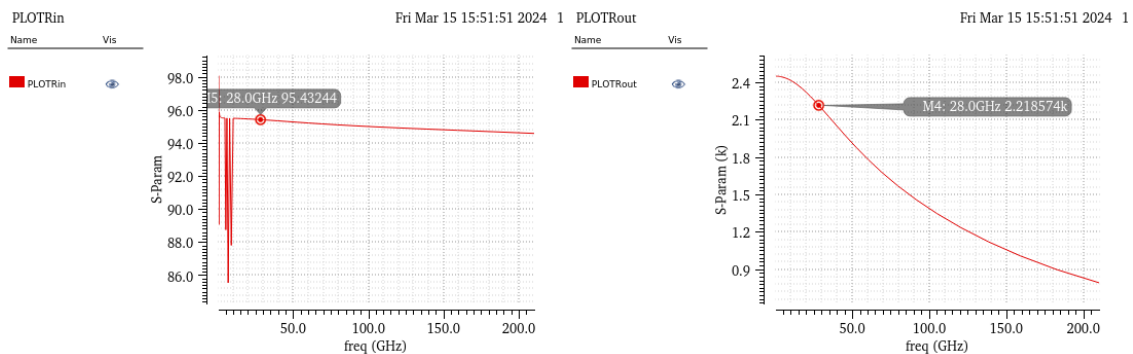
- a) Plot the six equivalent circuit model components listed in Table 1 over the specified frequency range. For the capacitance, you will need to use the derivative function in the Calculator in Spectre.

According to the test bench for an NMOS device given in Problem 1, I build the circuit in Cadence as shown in Figure 1.

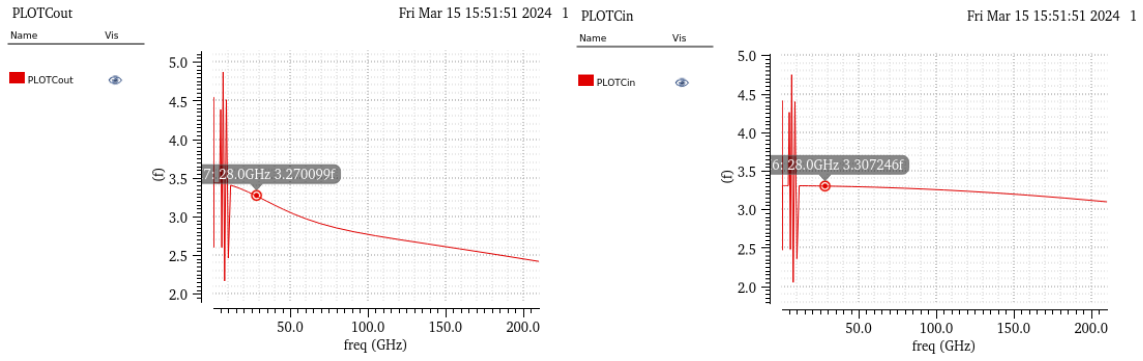


**Figure 1.** Circuit for Problem 1

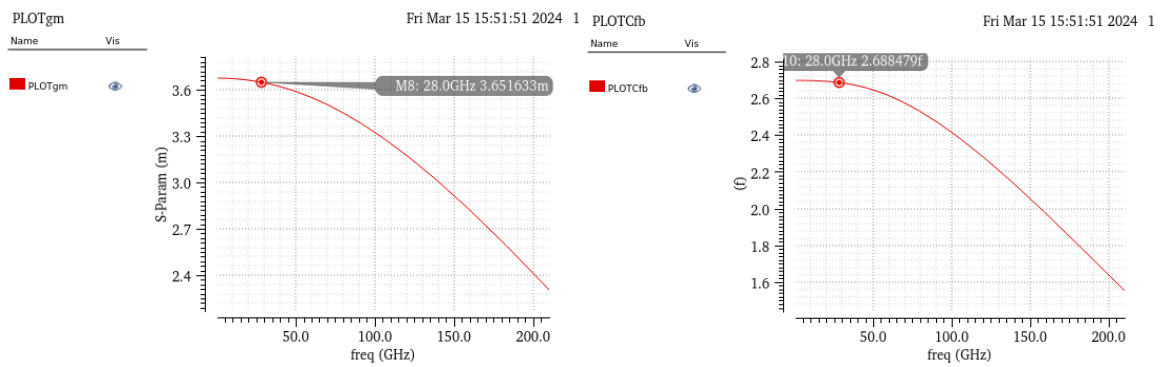
Using the equations listed in Table ‘equivalent circuit model components based on Y-parameters’, I did dc and sp simulation to calculate all the values. The components value over frequency range are shown below.



**Figure 2.** Input and output resistance



**Figure 3.** Input and output capacitance



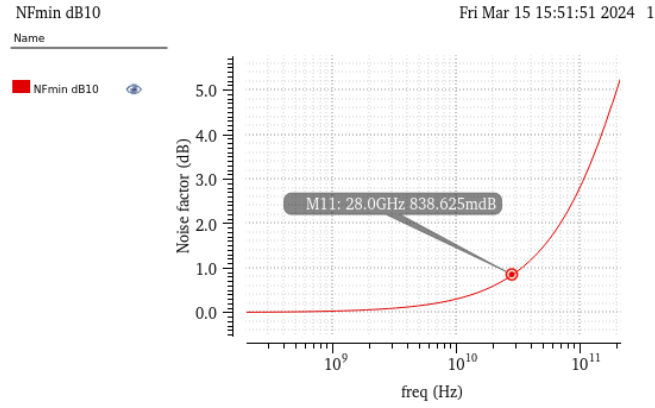
**Figure 4.** Transconductance and Feedback Capacitance

Referring to the second question on frequency point selection, I listed the individual component values at 28 GHz.

$R_{in}$	95.4 $\Omega$
$R_{out}$	2.21k $\Omega$
$C_{in}$	3.27fF
$C_{out}$	3.31fF
$g_m$	3.65mS
$C_{feedback}$	2.69fF

*b) Plot minimum noise figure (in dB) over the specified frequency range.*

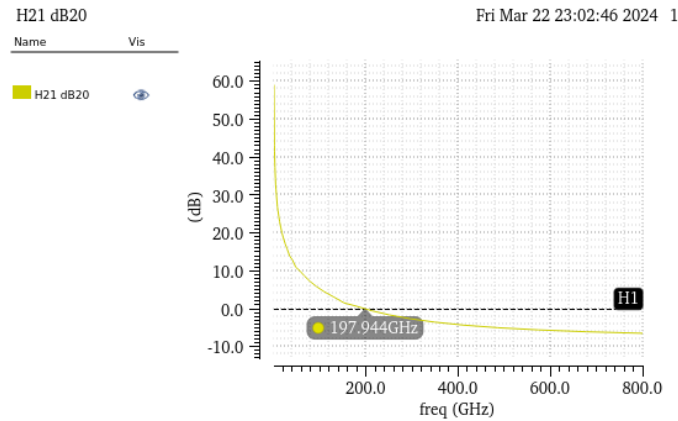
The minimum noise figure of the circuit is shown in Figure 5 from 200MHz to 200GHz, where the value at 28GHz is highlighted.



**Figure 5.**  $NF_{min}$  over frequency range

- c) Determine  $f_T$  from the simulation result ( $|h_{21}|$ ). (You may need to broaden the sweep range to, e.g., **800GHz**)

The value of  $h_{21}$  represents the ratio of output current to input current. The frequency corresponding to when the value of this ratio decreases to 1 is the cutoff frequency of the circuit, which corresponds to the point at which the dB value decreases to 0. Therefore, by applying the cross function in cadence to the  $h_{p21}$  curve, we can obtain  $f_T$ .

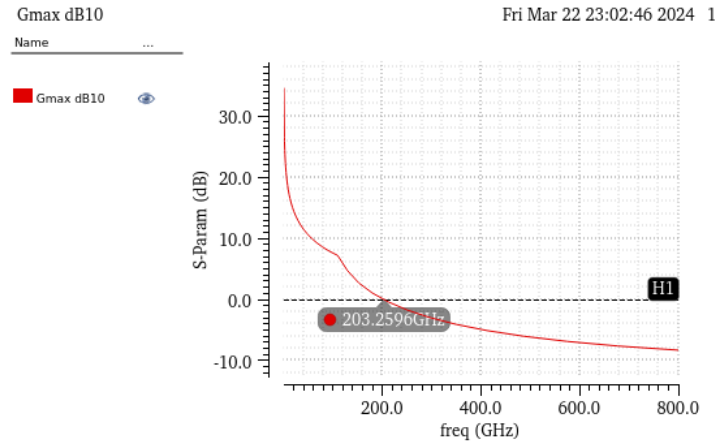


**Figure 6.**  $h_{21}$  over frequency range

After expanding the frequency range to 800GHz, the  $f_T$  can be obtained at 197.9GHz.

- d) Determine  $f_{max}$  by extrapolating the maximum power gain ( $G_{max}$ ). (You may need to broaden the sweep range to, e.g., **800GHz**)

The  $f_{max}$  is defined when  $G_{max}$  drops to 0 dB. In Figure 7, it can be seen that  $f_{max}$  equals to 203.3GHz.

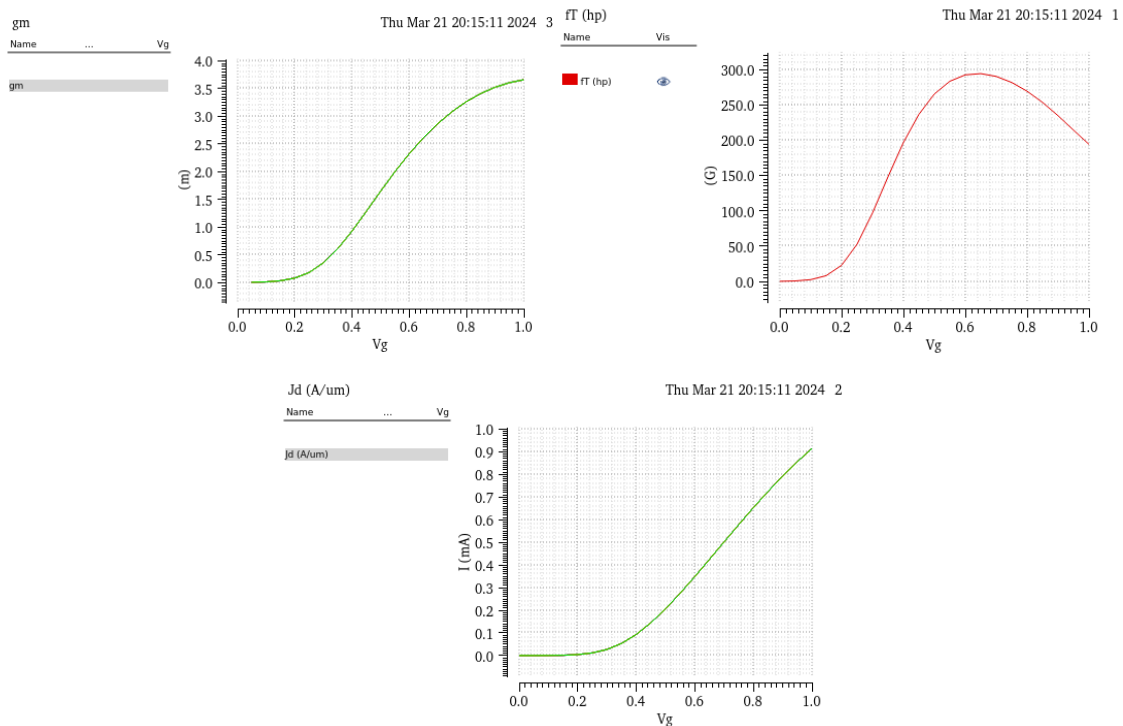


**Figure 7.**  $G_{\max}$  over frequency range

## Problem 2

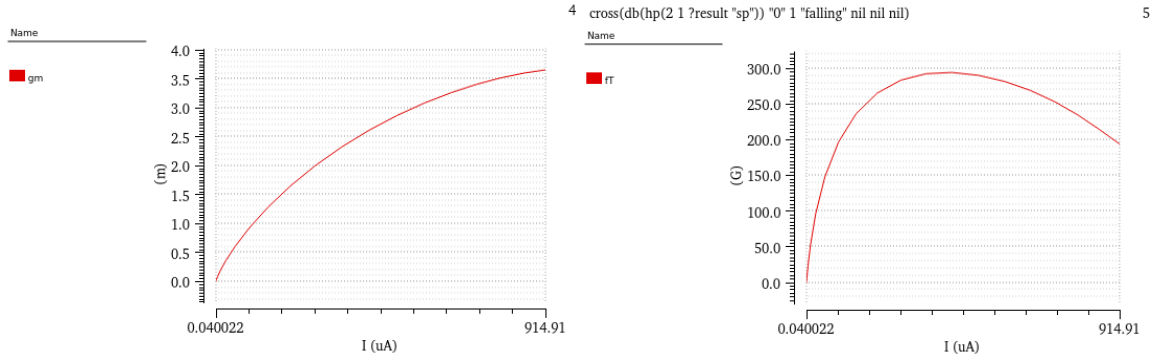
- a) Simulate the single transistor M1. Plot  $g_m$  vs.  $I_{\text{den1}}$  (current density) and  $f_T$  vs.  $I_{\text{den1}}$  by sweeping  $I_{\text{ds}}$ , M1 with a fixed width, e.g.,  $4\mu\text{m}$ .

The simulation circuit for single transistor is the same as the circuit in problem 1. During the simulation, I swept  $V_g$  in dc simulator from 0 to 1 V. After the simulation,  $g_m$ ,  $f_T$  and  $I_{\text{den}}$  versus  $V_g$  are plotted in Figure 8.



**Figure 8.**  $g_m$ ,  $f_T$  and  $I_{\text{den}}$  of single transistor

After changing the x axis of  $g_m$  and  $f_T$  to  $I_{den}$ , the required  $g_m$  vs.  $I_{den1}$  (current density) and  $f_T$  vs.  $I_{den1}$  can be plotted in Figure 9.

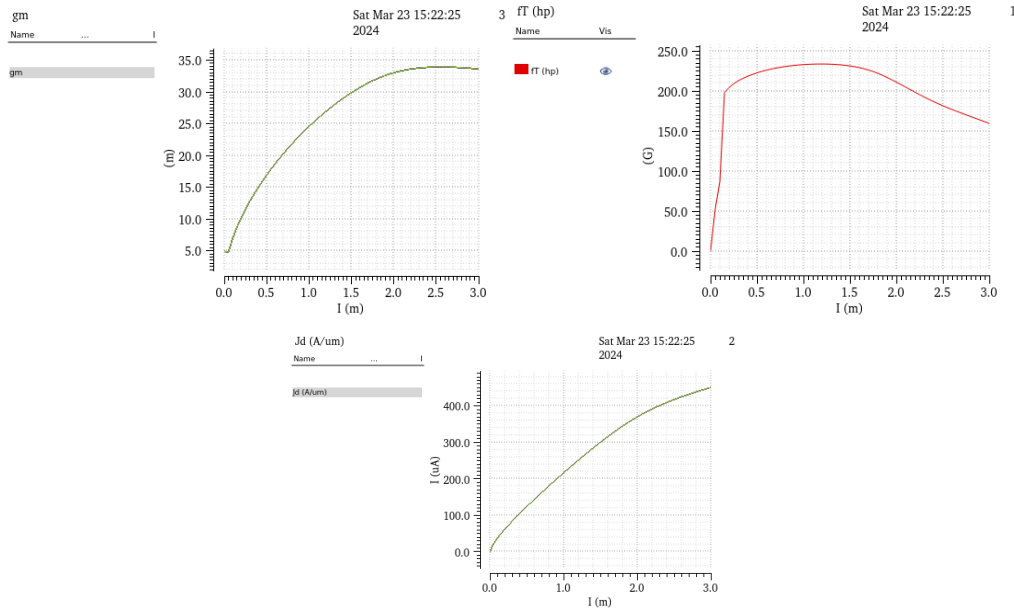


**Figure 9.**  $g_m, f_T$  vs  $I_{den}$  of single transistor

However, in the given single-ended LNA, the  $g_m, f_T$  vs  $I_{den}$  can also be obtained by sweeping  $I_{bias}$ . This is due to the topology of a current sink with M3 which serves as the bias for M1. When M1 and M3 are both in saturation region, the current flows through M1 can be determined simply by the ratio of multipliers, which is shown below:

$$\frac{I_{dsM1}}{I_{dsM3}} = \frac{M}{M_b} \quad (1)$$

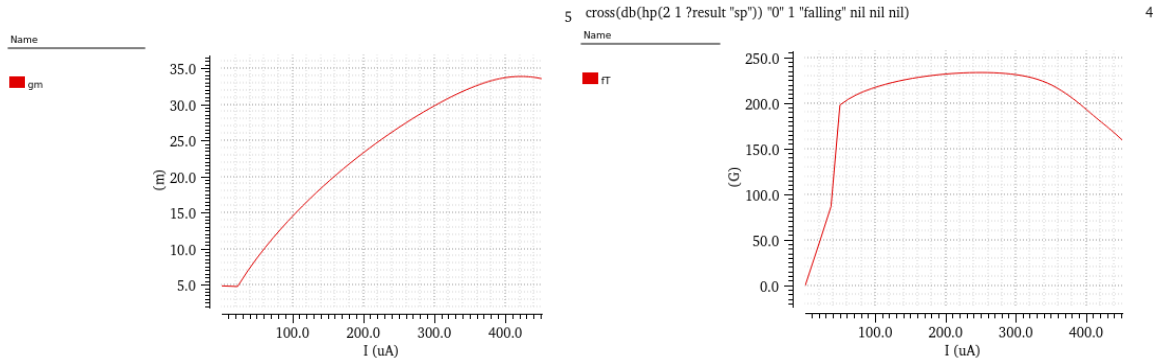
Considering the power consumption specification, the drain current of single ended LNA should be less than 33mA. So I firstly set M to be 10 and  $M_b$  to be 1, and do the dc simulation by sweeping  $I_{bias}$  from 0 to 3mA. It is worth mentioning that since it is the current density that is being observed, the value of M takes no effect at this point.



**Figure 10.**  $g_m, f_T$  vs  $I_{den}$  of single-ended LNA



After changing the x axis of  $g_m$  and  $f_T$  to  $I_{den}$ , the required  $g_m$  vs.  $I_{den}$  (current density) and  $f_T$  vs.  $I_{den}$  can be plotted in Figure 11.



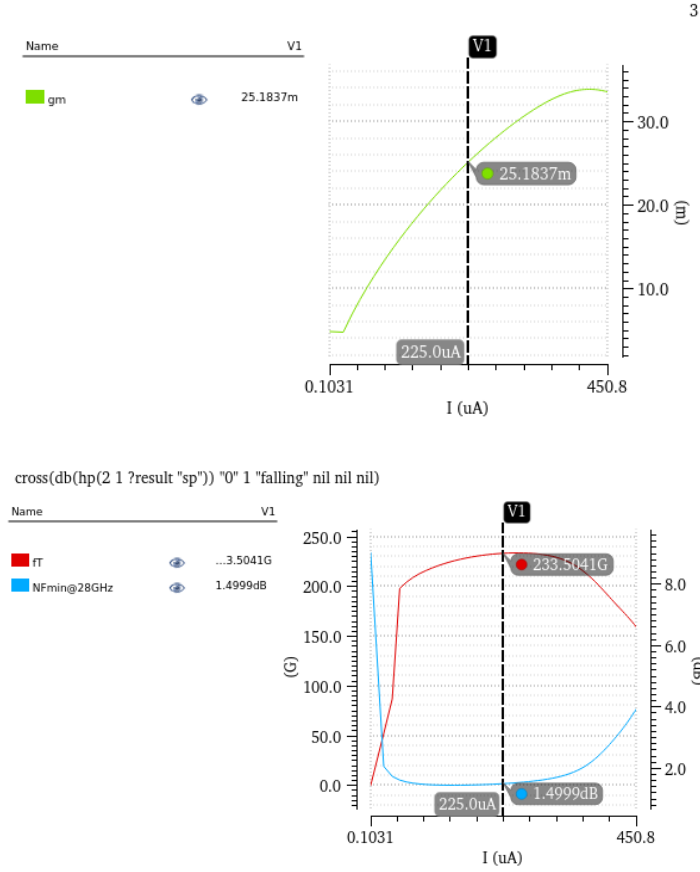
**Figure 11.**  $g_m, f_T$  vs  $I_{den}$  of single-ended LNA

- b) Determine  $I_{bias}$  and  $M$  (Multiplier of  $M1$  and  $M2$ ) that gives the optimal  $f_T$  and  $g_m$  for minimizing NF. What is resulting input resonant circuit  $Q$ ? (For the size of transistor  $M3$ ,  $M_b$  is typically very smaller than  $M$ , e.g.,  $M_b = 10\%M$ )

To determine the optimal value of all three important parameters, the first step is to figure out the relationship between them.

The  $g_m$  is the transconductance of the transistor. When drain current is large enough for the transistor to enter saturation region, velocity saturation takes over which leads to the saturation of  $g_m$ . In order to get minimum noise figure, we choose the suitable  $I_{den}$  to make sure  $g_m$  won't saturate. Thus  $I_{den}$  should be smaller than 400  $\mu A/\mu m$ . The next step is to illustrate  $f_T$  and  $NF_{min}$  vs  $I_{den}$ . By setting  $M$  to be 10 and  $M_b$  to be 1, the dc simulation by sweeping  $I_{bias}$  is done from 0 to 3mA just the same as question (a).

What matters next is how to do sp analysis over the current sweeping range. By using parametric analysis, we can get the value of  $f_T$  and  $NF_{min}$  at different current biasing point. The required working frequency is 28GHz, so  $NF_{min}$  of 28GHz is extracted.



**Figure 12.**  $g_m, f_T$  and  $NF_{min}$  vs  $I_{den}$  of LNA

Based on the current density range determined previously from  $g_m$ , I choose 225uA/um as the current density of single transistor, which gives the optimal  $g_m=25.2mS$ ,  $f_T=233.5GHz$  and  $NF_{min}=1.5dB$  as can be seen in Figure 12. So the biasing current equals to the product of width of M3(4um) and  $I_{den}$  of 225uA/um.

Up to now,  $I_{bias}$  and  $M$  of this differential LNA is chosen to be **900uA** and **15**, fulfilling all the requirements including optimal parameters and power consumptions at same time.

The resulting input resonant circuit  $Q$  can be calculated as below:

$$C_{gs} = \frac{g_m}{2\pi f_T} = \frac{25.2 \times 10^{-3}}{6.28 \times 233.5 \times 10^9} = 17.2 fF$$

$$Q = \frac{1}{2\omega_0 C_{gs} R_0} = \frac{1}{2 \times 6.28 \times 28 \times 10^9 \times 17.2 \times 10^{-15} \times 50} = 3.31 \quad (2)$$

Input resonant circuit  $Q$  is 3.31, which is not quite good.

- c) Determine  $L_g$  and  $L_s$  using  $C_{gs}$  and  $f_T$  to provide the required input matching. Assume  $L_s$  and  $L_g$  have a  $Q$  of 15 at  $f_0$ , compute the series resistance value and include it in the simulation (use 'ind' and 'res' from 'analogLib').

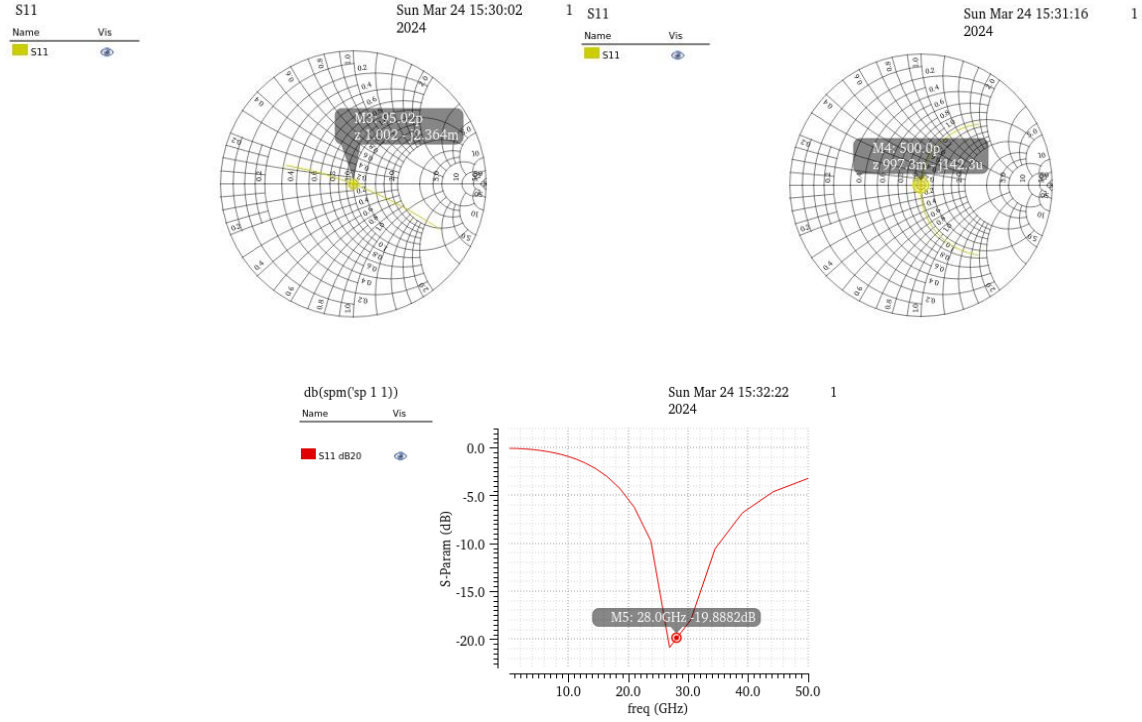
The topology of given single-ended LNA is source degeneration LNA. The source degeneration inductor is added for negative feedback. With  $L_s$  contributing to the real part of the input impedance, the conjugately power match can be easily achieved. However, the trade-off between better power match and lower noise figure leave much to discuss when choosing the degeneration inductance. It is still worth mentioning that  $L_s$  makes it easier to satisfy  $S_{11}$  and  $NF_{min}$  requirments. Below equation gives the calculation of single ended LNA input impedance.

$$Z_{in} = \frac{g_m L_s}{C_{gs}} + j[\omega(L_s + L_g) - \frac{1}{\omega_0 C_{gs}}] \quad (3)$$

Basically, there are two ways to calculate the required  $L_s$  and  $L_g$  for input matching. In this work, I initially use formulas to calcula the theoretical value of them and then made some adjustments to the  $S_{11}$  based on the input Smith Chart. The calculation of series resistance is also shown below.

$$\begin{aligned} L_s &= \frac{R_0}{2\pi f_T} \\ \omega_0 &= \sqrt{\frac{1}{(L_s + L_g)C_{gs}}} \\ R &= \frac{\omega L}{Q} \end{aligned} \quad (4)$$

In the second step, I use sp simulator to sweep  $L_s$  and  $L_g$  at 28GHz separately. Then, the conjugately matched  $S_{11}$  is plotted in Figure 13. However, this is only the perfect match without considering the output part of the circuit. After trying to tune the output node at 28GHz in (d),  $S_{11}$  will inevitably deviate from the initial value. At this point,  $L_s$  and  $L_g$  are chosen to be 95pH and 500pH respectively.



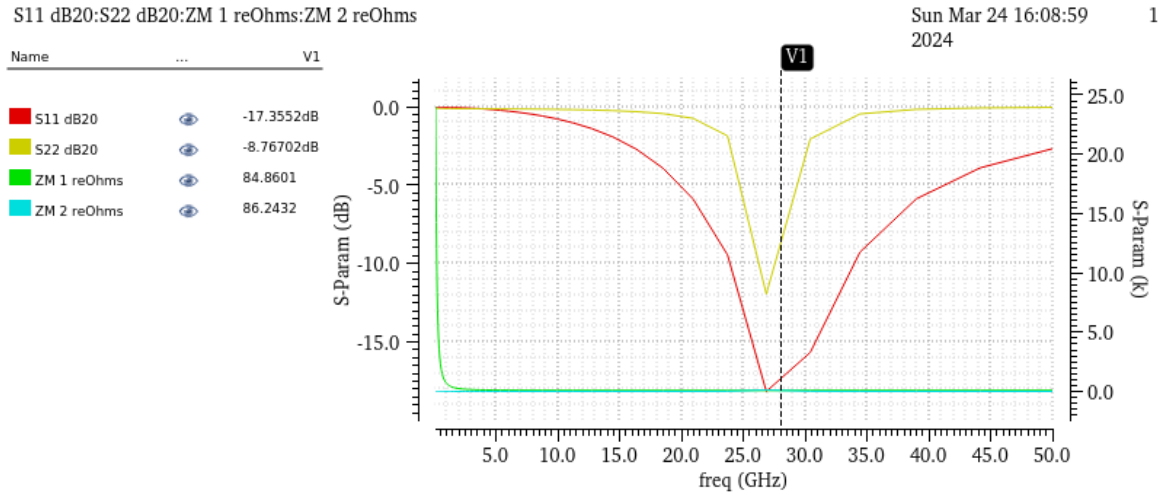
**Figure 13.** a)first step of sweeping  $L_s$  ;b)second step of sweeping  $L_g$  ;c) $S_{11}$  of LNA after doing input matching

- d) Determine  $L_d$  such that the output node is tuned at  $f_0$ . Assume  $L_d$  have a  $Q$  of 15 at  $f_0$ , compute the series resistance value and include it in the simulation (use 'ind' and 'res' from 'analogLib').

In order to tune the output node at  $f_0$ , I need to select output inductance, such that it resonates with the transistor output capacitance at 28GHz. The output inductance can be approximately estimated through  $C_{gd}$ . The  $L_d$  can be calculated as below.

$$L_d = \frac{1}{\omega^2 C_{gd}} \quad (5)$$

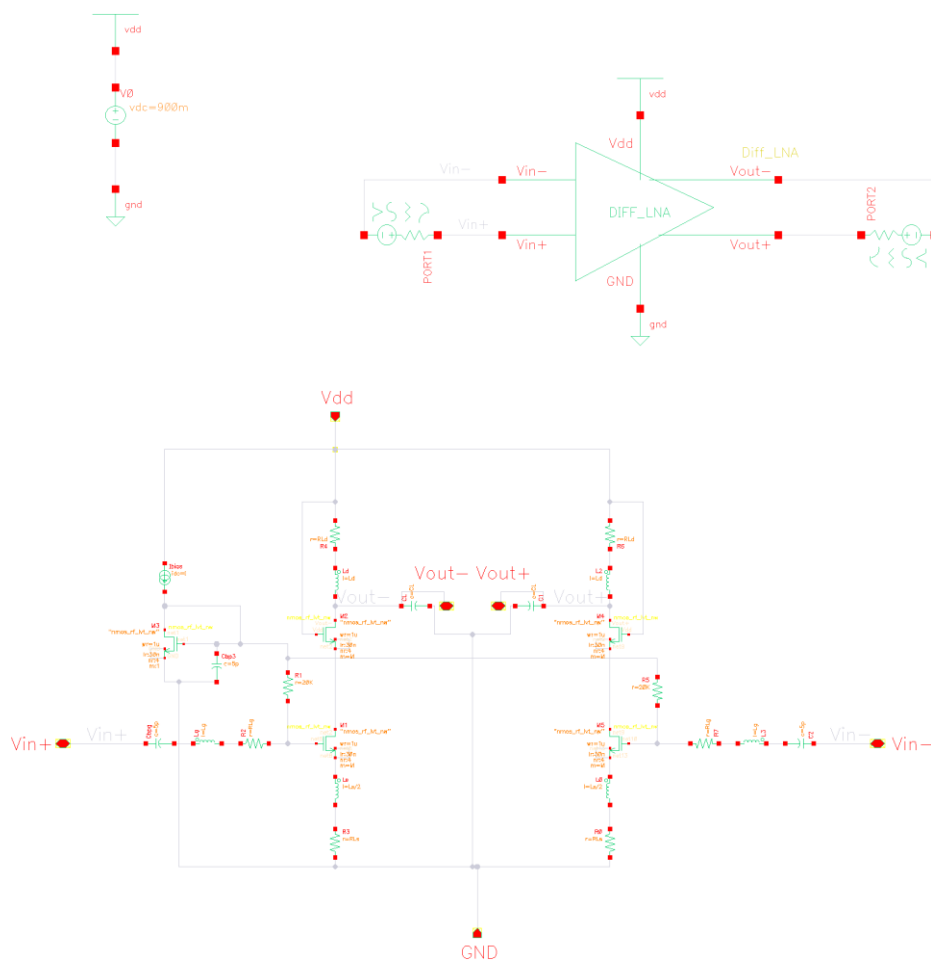
After calculation, smith chart of  $S_{11}$  is used for further tuning. Thus,  $L_d$  is chosen to be 33.6pH. However, after deciding the drain inductance of LNA, the whole circuit seen from input port is different. In order to get required impedance match and S parameters, all the three inductance should be tuned again to maximize the satisfaction of the indicators. During the adjustments of  $L_s$ ,  $L_g$  and  $L_d$ , it is obvious that  $S_{11}$  is much more easier to be tuned while  $S_{22}$  can hardly change to lower than -12dB. So the only way to meet the requirements of output tuning is to sacrifice  $S_{11}$  to some extent. The final results of tuning are shown in Figure 14 at  $L_s=74\text{pH}$ ,  $L_g=500\text{pH}$  and  $L_d=32.5\text{pH}$ .



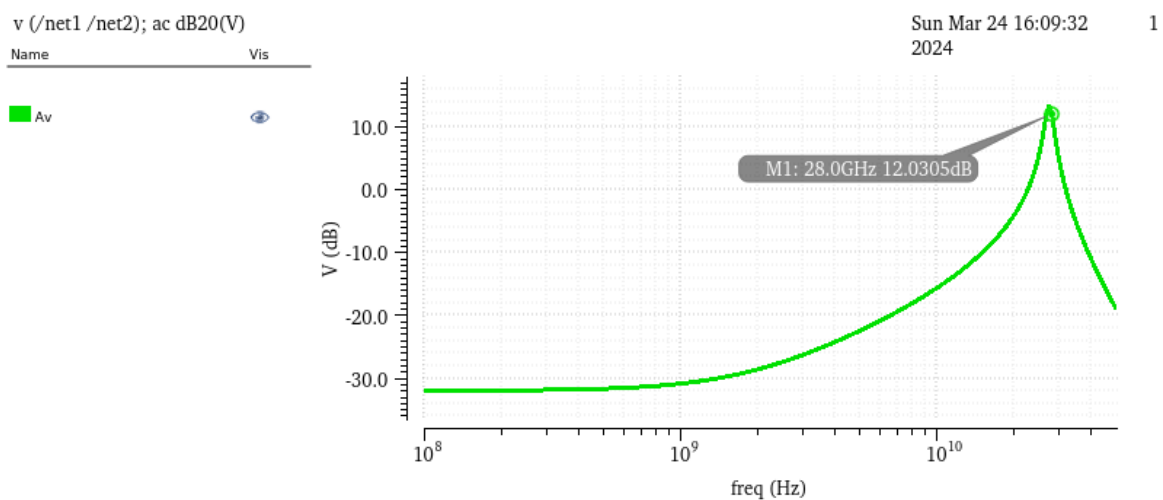
**Figure 14.** Final results of  $S_{11}$ ,  $S_{22}$  and impedance after input and output tuning

e) Plot the gain ( $V_{out}/V_{in}$ ), noise figure (NF) and input impedance ( $Z_{in}$ ) vs. frequency from 0.1 to 50 GHz.

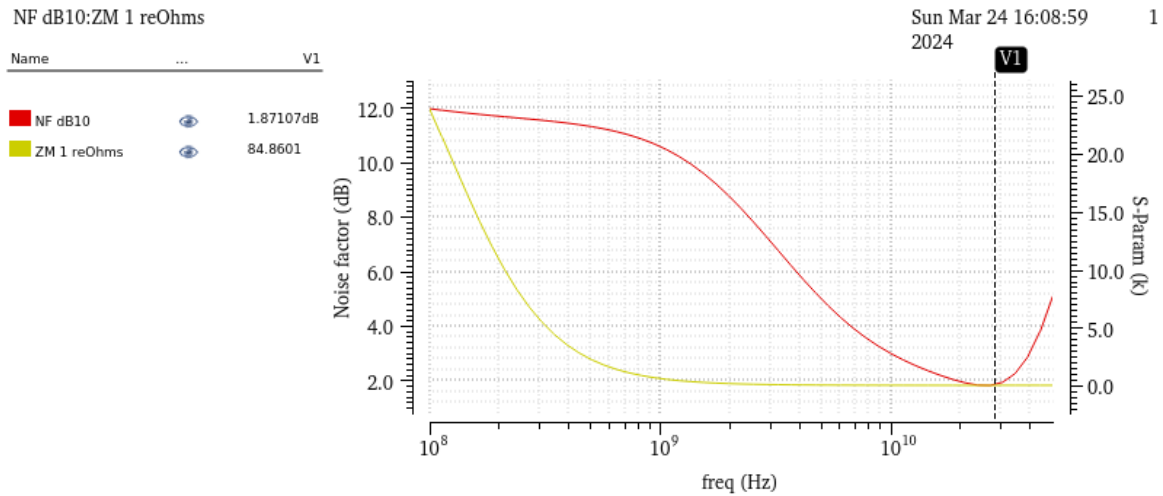
Up to now, the circuit is determined with certain values of components. The whole circuit is shown in Figure 15. After creating symbol based on the differential LNA, Voltage gain, NF and  $Z_{in}$  are plotted separately in Figure 16 and 17.



**Figure 15.** The whole simulation circuit: top part and symbol part



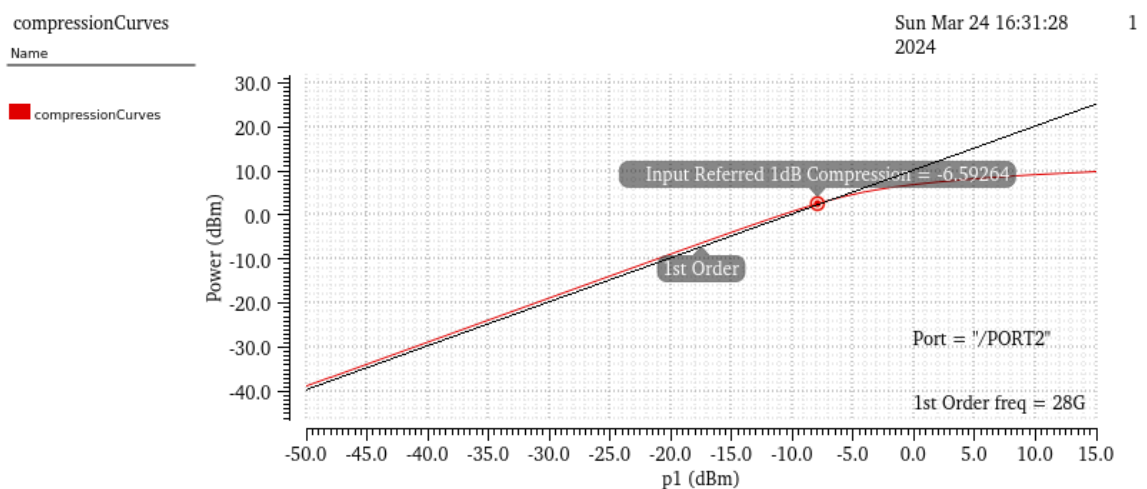
**Figure 16.** Voltage gain of the circuit



**Figure 17.** NF and  $Z_{in}$  of the circuit over the frequency range

- f) Perform a PSS analysis at  $f_0$  to determine the 1-dB compression of this LNA by sweeping the input port power.

The 1-dB compression characteristic of differential LNA is a large signal parameter. It tells LNA's ability of handling weak input signal without output distortion. The 1dB compression point of LNA is the input power level at which the output power of the amplifier is compressed by 1dB, where non-linearity of the circuit takes in charge. By sweeping input power of port 1 from -50dBm to 10dBm,  $P_{1dB}$  of -6.59dBm can easily be extracted as shown in Figure 18, which is quite outstanding.



**Figure 18.** 1-dB compression point of the designed LNA

- g) Using PSS analysis, perform a two-tone test to determine the IIP3 of the LNA using signals at  $f_0$  and  $f_0+800\text{MHz}$ . (Optional: You can use hb and PAC analysis to simulate the IIP3 again and compare results for **bonus credits**).

IIP3 is a measure of the linearity of the amplifier, indicating how well it can handle strong input signals without introducing distortion.

Firstly, I use PSS and PAC analysis to perform two-tone test. The first frequency is set to be 28GHz while the second one is 28.8GHz. The second frequency is one interferer for the working frequency. In this kind of analysis, the second frequency is added through PAC, causing the possible distortion at the output port. The PSS and PAC analysis settings are shown in Figure 19, and the obtained IIP3 is 3.98dBm as shown in Figure 20.

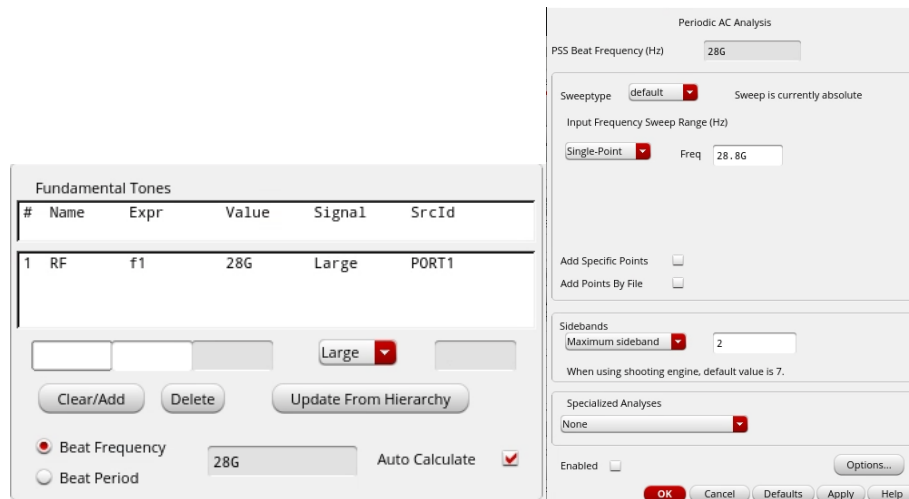


Figure 19. PSS and PAC analysis settings

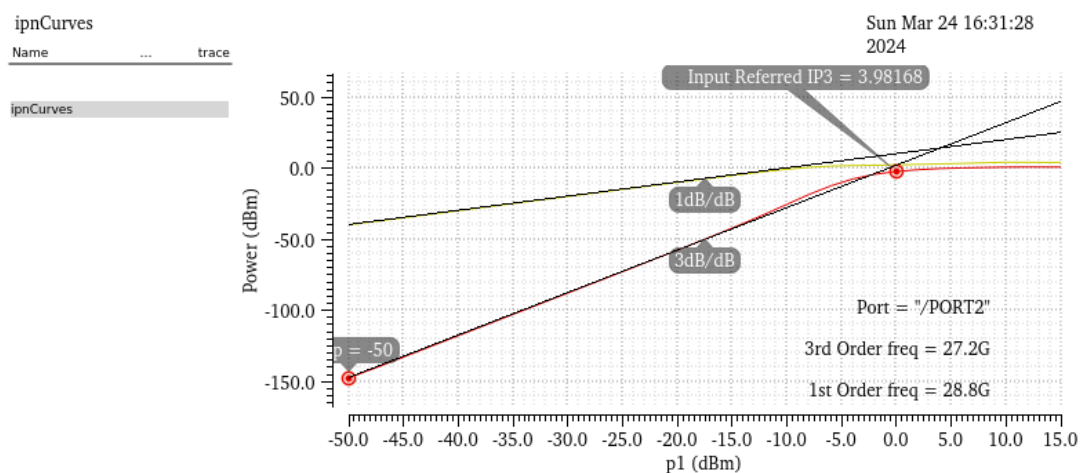
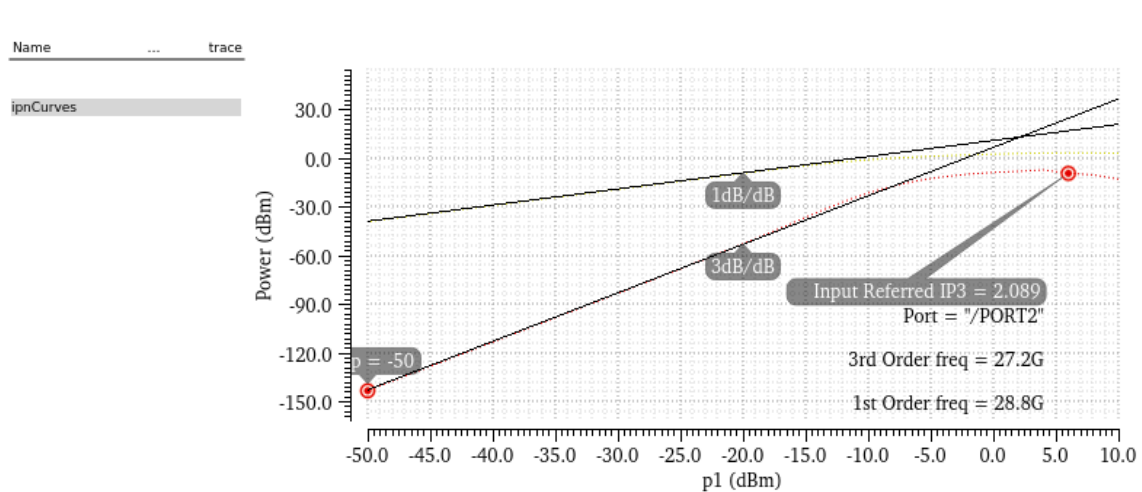


Figure 20. IIP3 with PSS and PAC analysis



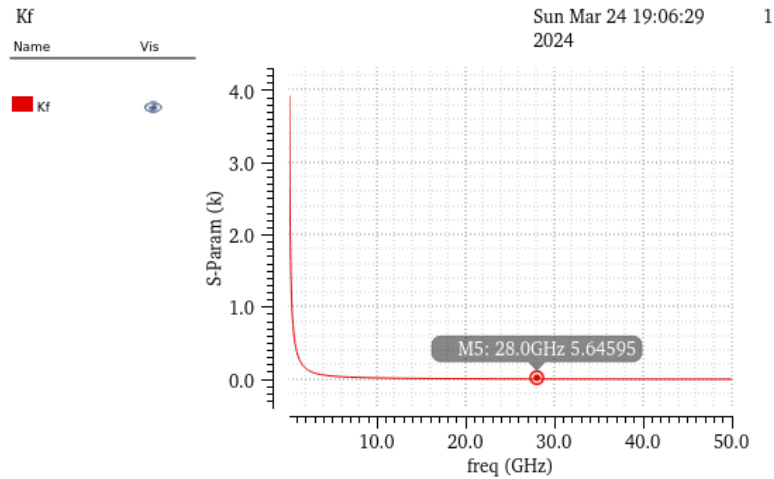
Secondly, by conducting HB(harmonic balance) analysis alone, IIP3 can also be easily obtained. The different part is that I need to add another sinusoidal signal at 28.8GHz in port 1 of the circuit. Thus, two-tone simulation will be performed in the circuit. HB settings and IIP3 in this simulation are shown in Figure 21. The results of IIP3 are completely the same.



**Figure 21. IIP3 with HB analysis**

h) Using *sp* analysis, plot the *K* factor vs. frequency from 0.1 to 50 GHz. Comment on the stability of your designed LNA based on the simulation result.

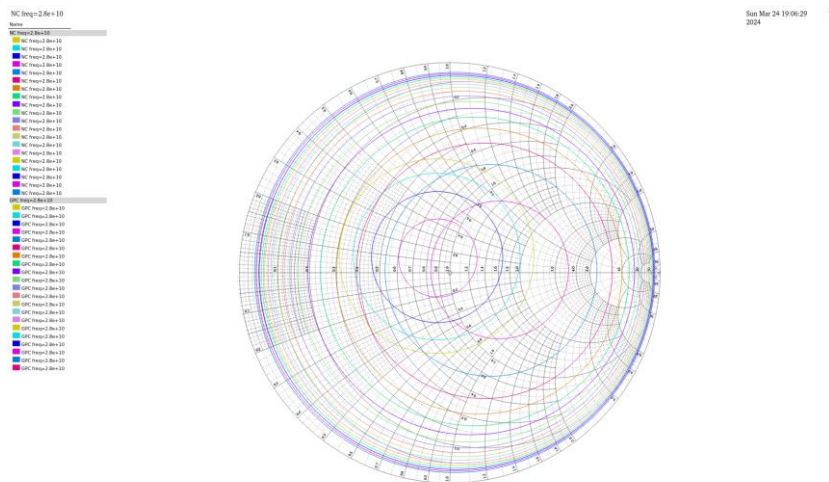
It is known that when  $|K_r| > 1$ , the system is stable. In Figure 222, we can know that the system is always free from oscillation and distortion from 0.1GHz to 50GHz, which shows extraordinary stability. Also, as the frequency increase more, the negative feedback may cause oscillation at some point.



**Figure 22.**  $K_f$  over the frequency range

- i) Using *sp* analysis, plot noise circles with noise ranges from 1 dB to 10 dB and gain circles with gain ranges from 10 dB to 20 dB, both at  $f_0$ . Comment on the trade-off between gain and noise based on the circles.

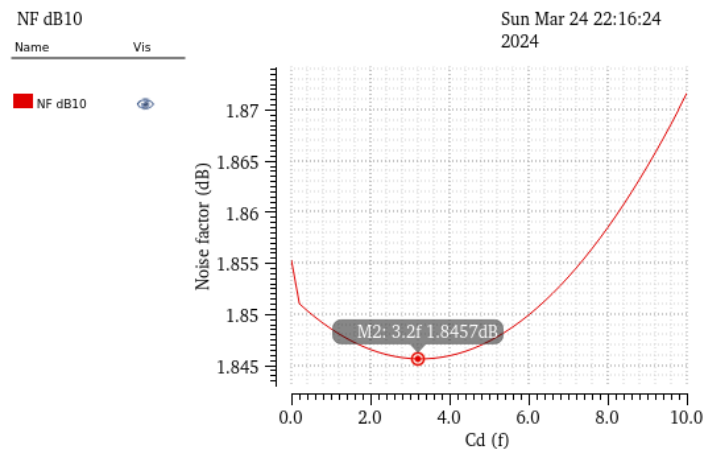
For noise and gain circles, the center of each circle represents the ideal noise figure or gain, and the radius of each circle represents the deviation from the ideal. As the gain of the amplifier increases, the gain circle expands, and the noise circle also expands due to the increase in noise figure. Thus the noise and gain of the system show an inverse relation. Using the noise and gain circles, the trade-off can be easily achieved by choosing the intercept points and tuning along the circles. Theoretically, perfect power match won't give good noise figure. So trade-off between noise and gain should be at the cost of mismatch.



**Figure 23.** Noise and gain circles

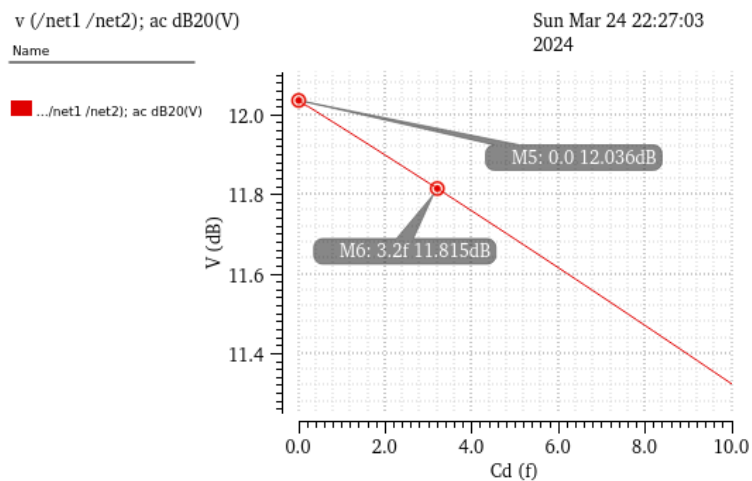
- j) Now, add  $C_d$  between the gate and source terminal of M1, plot NF at  $f_0$  vs.  $C_d$ , make sure that you adjust  $L_g$  and  $L_s$  for each value of  $C_d$  to maintain the input matching at  $f_0$ . How much is the improvement in NF? What happened to the amplifier gain?

Adding  $C_d$  between gate and source in parallel with  $C_{gs}$  is an effective way to improve noise figure. After sweeping  $C_d$  in sp analysis, the change of NF can be seen from Figure 24, where 3.2fF  $C_d$  stands out when considering NF improvement. The NF changes from 1.856dB to 1.846dB by 0.54%.



**Figure 24.** Noise Figure improvement

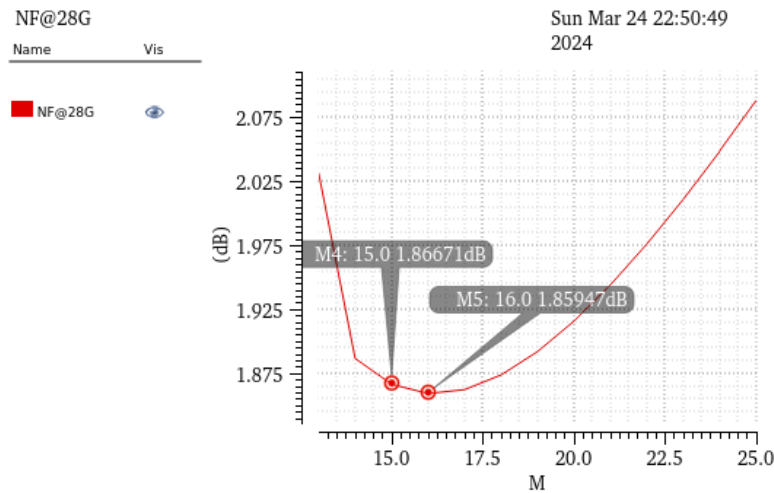
In the meantime, the amplifier gain goes through decrease. This is due to the inverse relation between noise and gain. Voltage gain decreases from 12.036dB to 11.815dB by 1.84%.



**Figure 25.** Amplifier gain variations

k) With the  $C_d$  and the  $L_g$ , perform a sweep of the input device width, is there a new optimal width that gives lower NF? If so, what is it and what is the NF?

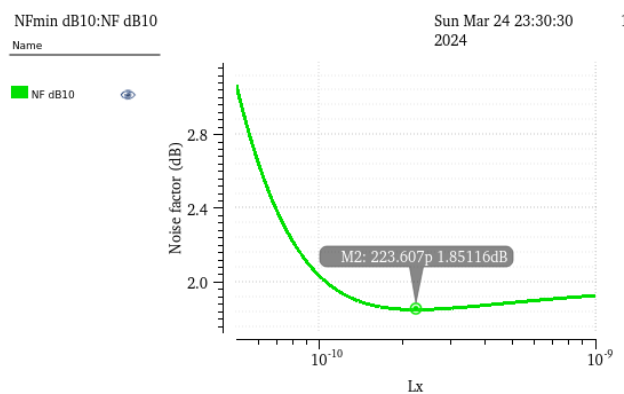
In the question, I choose to sweep multiplier of  $M_1$ , which is initially 15. Figure 26 shows that  $M=16$  gives a better NF. In this case, the total width of input device is  $64\mu\text{m}$ . NF changes from  $1.867\text{dB}$  to  $1.859\text{dB}$ , improved by  $0.43\%$ .



**Figure 26.** NF with sweeping width

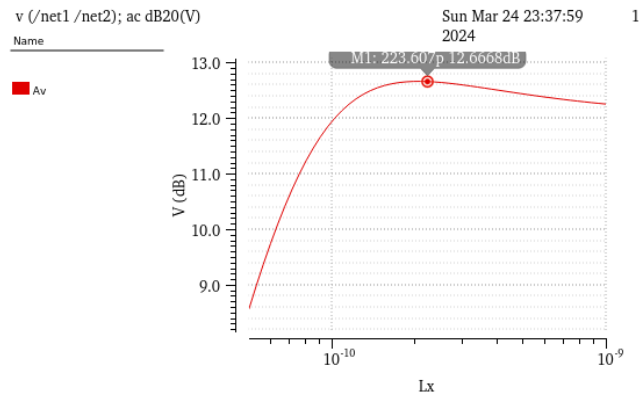
l) Now, add  $L_x$  at the cascode node by first estimating the parasitic capacitance at node  $X$ . How much is the improvement in NF? What happened to the amplifier gain?

Firstly estimating  $L_x$  based on the parallel parasitic capacitance, I choose the sweep range of  $L_x$  to be from  $50\text{pH}$  to  $1\text{nH}$ . Theoretically,  $L_x$  will eliminate the influence of parasitic capacitance on input impedance. However, the NF improvements is not obvious in my design.



**Figure 27.** NF with sweeping  $L_x$

It can be seen that NF improves from 1.859dB to 1.851dB by 0.4% when choosing  $L_s$  to be 223.6pH. At the same time, voltage gain improves from 11.815dB to 12.667dB by 7.2%, which is significant.



**Figure 28.** NF with sweeping  $L_x$

Homework 2 finishes at this moment, while the exploration of High Frequency Circuits has just begun.