

2. 01g $\times_{c(j\omega)} = \text{rect}[(c\omega - 1)/2]$

2.1) ඔබ

$x(t) \leftrightarrow X(j\omega)$

time shifting : $x(t+4) \leftrightarrow e^{-j4\omega} X(j\omega)$

time & freq scaling : $x(-2t+4) \leftrightarrow \frac{1}{2} e^{-j2\omega} X\left(\frac{-j\omega}{2}\right) = \frac{e^{-j2\omega}}{2} \text{rect}\left(\frac{-\omega-1}{2}\right)$

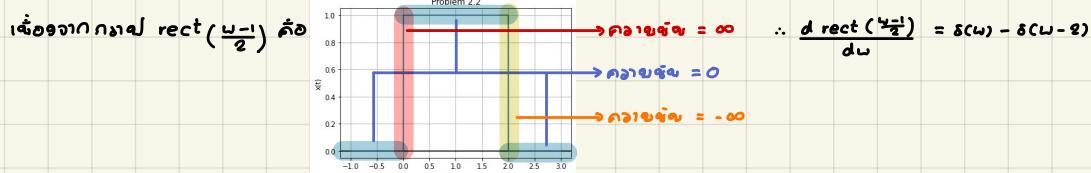
 \therefore fourier transform ලබා $\frac{e^{-j2\omega}}{2} \text{rect}\left(\frac{-\omega-1}{2}\right)$ ✗

2.2) ඔබ

$x(t) \leftrightarrow X(j\omega)$

differentiation in freq. : $t \times x(t) \leftrightarrow j \frac{dX(j\omega)}{d\omega}$

time shifting : $(t-1)x(t-1) \leftrightarrow e^{-j\omega} j \frac{dX(j\omega)}{d\omega} = e^{-j\omega} j \frac{d \text{rect}(\frac{\omega-1}{2})}{d\omega}$

 \therefore fourier transform ලබා $e^{-j\omega} j (\delta(\omega) - \delta(\omega-2))$ ✗

2.3) ඔබ

$x(t) \leftrightarrow X(j\omega)$

differentiation in time : $t \frac{dx(t)}{dt} \leftrightarrow j\omega X(j\omega)$

differentiation in freq. : $t \frac{dx(t)}{dt} \leftrightarrow j \frac{d(j\omega X(j\omega))}{d\omega} = - \frac{d\omega X(j\omega)}{d\omega} = -x(j\omega) - \omega \frac{dX(j\omega)}{d\omega}$
 $= -\text{rect}(\frac{\omega-1}{2}) - \omega \delta(\omega) + \omega \delta(\omega-2)$

 \therefore fourier transform ලබා $-\text{rect}(\frac{\omega-1}{2}) - \omega \delta(\omega) + \omega \delta(\omega-2)$ ✗

2.4) ඔබ

$x(t) \leftrightarrow X(j\omega)$

time shifting : $x(t-1) \leftrightarrow e^{-j\omega} X(j\omega)$

time & freq scaling : $x(2t-1) \leftrightarrow \frac{e^{-j\omega/2}}{2} X(j\omega/2)$

freq shifting : $x(2t-1)e^{-j2t} \leftrightarrow \frac{e^{-j(2\omega+2)/2}}{2} X(j(\omega+2)/2) = \frac{e^{-j(\omega+2)/2}}{2} \text{rect}(\frac{\omega}{4})$

 \therefore fourier transform ලබා $\frac{e^{-j(\omega+2)/2}}{2} \text{rect}(\frac{\omega}{4})$ ✗

2.5) ඔබ

$x(t) \leftrightarrow X(j\omega) \quad \text{--- (1)}$

වෙනු t තැන් t-1 (shift) : $x(t-1) \leftrightarrow e^{-j\omega} X(j\omega) \quad \text{--- (2)}$

නො (1), (2) එකිනෙක් $x(t) * x(t-1) \leftrightarrow e^{-j\omega} (X(j\omega))^2 = e^{-j\omega} (\text{rect}[(c\omega-1)/2])^2$

\therefore fourier transform ලබා $e^{-j\omega} (\text{rect}[(c\omega-1)/2])^2$ ✗

3. 3.1) నగ $X(j\omega) = u(\omega) - u(\omega - \omega_0)$

॥॥॥: $\times(t) \longleftrightarrow X(j\omega)$

duality : $\times(t) \longleftrightarrow 2\pi \times(-j\omega)$

$u(t) - u(t - \omega_0) \longleftrightarrow 2\pi \times(-j\omega)$ ————— (G)

ప్రశ్నలకు $u(t) \longleftrightarrow \frac{1}{j\omega} + \pi \delta(\omega)$ ————— (1)

time shifting : $u(t - \omega_0) \longleftrightarrow e^{-j\omega_0} \left(\frac{1}{j\omega} + \pi \delta(\omega) \right)$ ————— (2)

linearity నుండి (1) నుండి (2) : $u(t) - u(t - \omega_0) \longleftrightarrow (1 - e^{-j\omega_0}) \left(\frac{1}{j\omega} + \pi \delta(\omega) \right)$ ————— (3)

మొత్త (3) ||||| (3) ద్వారా $2\pi \times(-j\omega) = (1 - e^{-j\omega_0}) \left(\frac{1}{j\omega} + \pi \delta(\omega) \right)$

$\times(-j\omega) = (1 - e^{-j\omega_0}) \left(\frac{1}{j2\pi\omega} + \frac{\pi \delta(\omega)}{2} \right)$

$\times(j\omega) = (1 - e^{j\omega_0}) \left(\frac{\delta(-\omega)}{2} - \frac{1}{j2\pi\omega} \right)$

$\therefore \times(t) = (1 - e^{j\omega_0 t}) \left(\frac{\delta(-t)}{2} - \frac{1}{j2\pi t} \right)$ *

3.2) నగ $\times(j\omega) = \cos(\omega_0 t) \sin(\frac{\omega}{2})$

॥॥॥: $\times(t) \longleftrightarrow X(j\omega)$

duality : $\times(t) \longleftrightarrow 2\pi \times(-j\omega)$

$\cos(\omega_0 t) \sin(\frac{\omega}{2}) \longleftrightarrow 2\pi \times(-j\omega)$ ————— (4)

మరింత $\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$ ||||| $\sin(\frac{\omega}{2}) = \frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}$

మొత్త $\cos(\omega_0 t) \sin(\frac{\omega}{2}) = \frac{1}{4j} (e^{j\omega_0 t/2} + e^{-j\omega_0 t/2} - e^{j\omega_0 t/2} - e^{-j\omega_0 t/2})$ ————— (5)

ప్రశ్నలకు $e^{j\omega_0 t} \longleftrightarrow 2\pi \delta(\omega - \omega_0)$ ————— (6)

linearity నుండి (5) ||||| (6) : $\cos(\omega_0 t) \sin(\frac{\omega}{2}) \longleftrightarrow \frac{2\pi}{4j} \left[\delta(\omega - \frac{\omega_0}{2}) + \delta(\omega + \frac{\omega_0}{2}) - \delta(\omega - \frac{3\omega_0}{2}) - \delta(\omega + \frac{3\omega_0}{2}) \right]$ ————— (7)

మొత్త (6) ||||| (7) ద్వారా $2\pi \times(-j\omega) = \frac{2\pi}{4j} \left[\delta(\omega - \frac{\omega_0}{2}) + \delta(\omega + \frac{\omega_0}{2}) - \delta(\omega - \frac{3\omega_0}{2}) - \delta(\omega + \frac{3\omega_0}{2}) \right]$

$\times(-j\omega) = \frac{1}{4j} \left[\delta(\omega - \frac{\omega_0}{2}) + \delta(\omega + \frac{\omega_0}{2}) - \delta(\omega - \frac{3\omega_0}{2}) - \delta(\omega + \frac{3\omega_0}{2}) \right]$

$\times(j\omega) = -\frac{j}{4} \left[\delta(-\omega - \frac{\omega_0}{2}) + \delta(-\omega + \frac{\omega_0}{2}) - \delta(-\omega - \frac{3\omega_0}{2}) - \delta(-\omega + \frac{3\omega_0}{2}) \right]$

$\therefore \times(t) = -\frac{j}{4} \left[\delta(-t - \frac{\omega_0}{2}) + \delta(-t + \frac{\omega_0}{2}) - \delta(-t - \frac{3\omega_0}{2}) - \delta(-t + \frac{3\omega_0}{2}) \right]$ *