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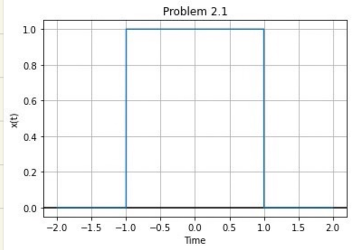
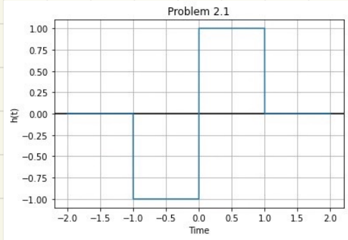
2.1) จาก $x(t)$ คือ

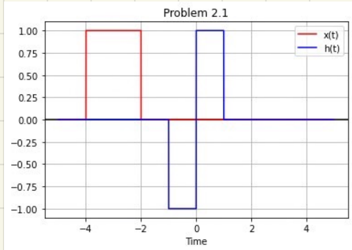


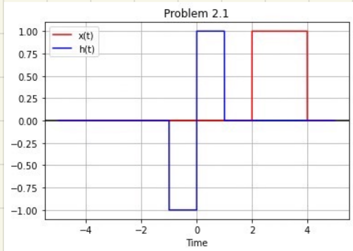
และ $h(t)$ คือ



จะได้ $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau$

<u>กรณี</u> $t < -2$ หรือ $2 \leq t$

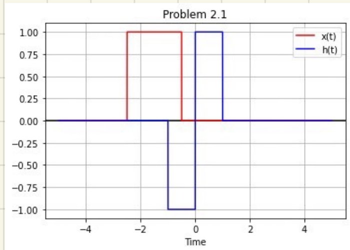




จะเห็นว่ากราฟ $x(t-\tau)$ และ $h(\tau)$ ไม่มีส่วนที่ซ้อนทับกัน

ดังนั้น $\int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau = 0$

<u>กรณี</u> $-2 \leq t < -1$

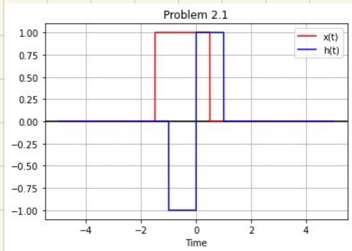


$$\int_{-1}^{t+1} x(t-\tau)h(\tau)\,d\tau = \int_{-1}^{t+1} (1)(-1)\,d\tau$$

$$= -\tau\Big|_{-1}^{t+1} = -t-1-1 = -t-2$$

$$\therefore \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau = -t-2$$

<u>กรณี</u> $-1 \leq t < 0$

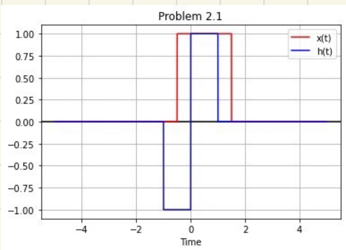


$$\int_{-1}^{0} x(t-\tau)h(\tau)\,d\tau + \int_{0}^{t+1} x(t-\tau)h(\tau)\,d\tau = \int_{-1}^{0}(1)(-1)\,d\tau + \int_{0}^{t+1}(1)(1)\,d\tau$$

$$= -\tau\Big|_{-1}^{0} + \tau\Big|_{0}^{t+1} = -1+t-1 = t$$

$$\therefore \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau = t$$

<u>กรณี</u> $0 \leq t < 1$

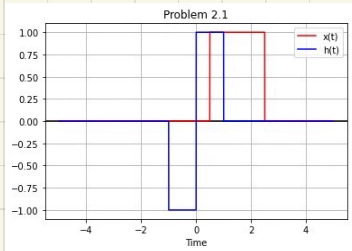


$$\int_{t-1}^{0} x(t-\tau)h(\tau)\,d\tau + \int_{0}^{1} x(t-\tau)h(\tau)\,d\tau = \int_{t-1}^{0}(1)(-1)\,d\tau + \int_{0}^{1}(1)(1)\,d\tau$$

$$= -\tau\Big|_{t-1}^{0} + \tau\Big|_{0}^{1} = t-1+1 = t$$

$$\therefore \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau = t$$

<u>กรณี</u> $1 \leq t < 2$



$$\int_{t-1}^{1} x(t-\tau)h(\tau)\,d\tau = \int_{t-1}^{1}(1)(1)\,d\tau$$

$$= \tau\Big|_{t-1}^{1} = 1-t+1 = 2-t$$

$$\therefore \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau = 2-t$$

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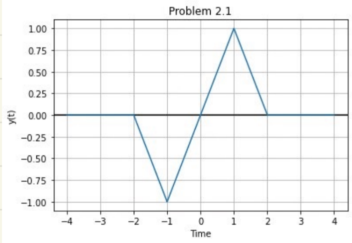
โดยทุกกรณีจะได้ว่า

$$y(t) = \begin{cases} 0 & , \ t < -2 \\ -t-2 & , \ -2 \le t < -1 \\ t & , \ -1 \le t < 1 \\ 2-t & , \ 1 \le t < 2 \\ 0 & , \ 2 \le t \end{cases}$$

ซึ่งสามารถนำมา plot graph ได้ดังนี้



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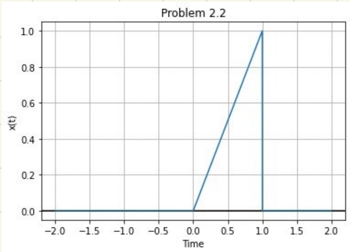
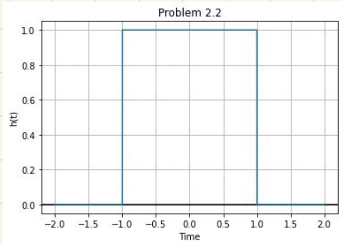
2.2) จาก x(t) คือ

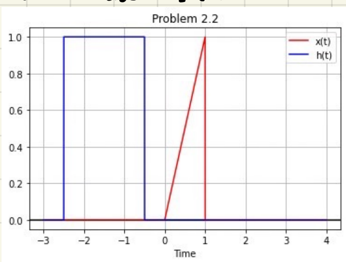


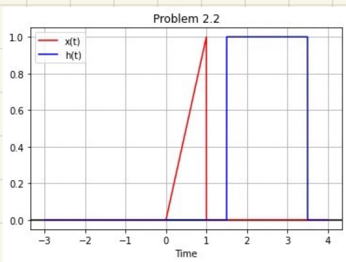
และ h(t) คือ



จะได้ $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$

<u>กรณี</u> $t < -1$ หรือ $2 \le t$

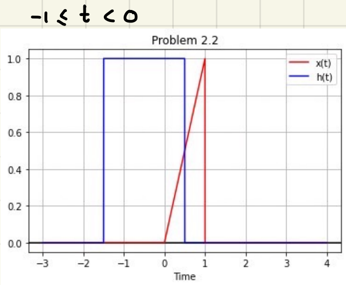




จะเห็นว่ากราฟ $x(\tau)$ และ $h(t-\tau)$ ไม่มีส่วนที่ซ้อนทับกัน

นั่นคือ $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = 0$

<u>กรณี</u> $-1 \le t < 0$

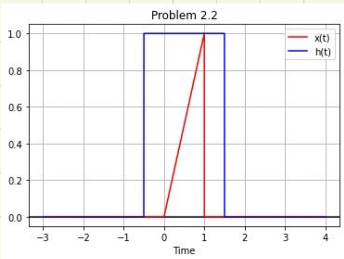


$$\int_{0}^{t+1} x(\tau)h(t-\tau)\,d\tau = \int_{0}^{t+1} (\tau)(1)\,d\tau$$

$$= \frac{\tau^2}{2}\Big|_{0}^{t+1} = \frac{(t+1)^2}{2}$$

$$\therefore \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = (t+1)^2/2$$

<u>กรณี</u> $0 \le t < 1$

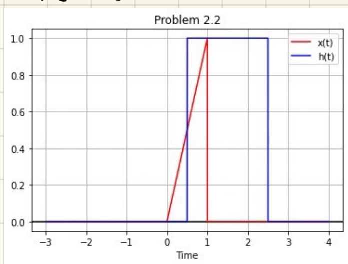


$$\int_{0}^{1} x(\tau)h(t-\tau)\,d\tau = \int_{0}^{1} (\tau)(1)\,d\tau$$

$$= \frac{\tau^2}{2}\Big|_{0}^{1} = 1/2$$

$$\therefore \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = 1/2$$

<u>กรณี</u> $1 \le t < 2$



$$\int_{t-1}^{1} x(\tau)h(t-\tau)\,d\tau = \int_{t-1}^{1} (\tau)(1)\,d\tau$$

$$= \frac{\tau^2}{2}\Big|_{t-1}^{1} = \frac{1}{2} - \frac{(t-1)^2}{2} = \frac{2t-t^2}{2}$$

$$\therefore \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = (2t-t^2)/2$$

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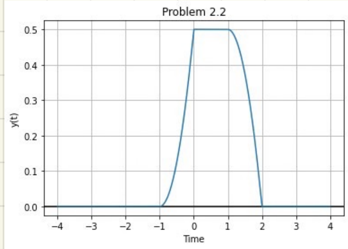
โดยทุกกรณีจะได้ว่า

$$y(t) = \begin{cases} 0 & , \quad t < -1 \\ (t+1)^2/2 & , \quad -1 \le t < 0 \\ 1/2 & , \quad 0 \le t < 1 \\ (2t - t^2)/2 & , \quad 1 \le t < 2 \\ 0 & , \quad 2 \le t \end{cases}$$

ซึ่งสามารถนำมา plot graph ได้ดังนี้



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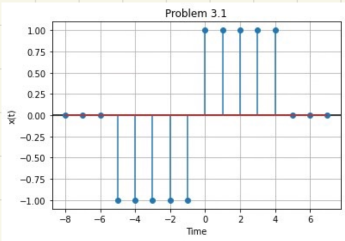
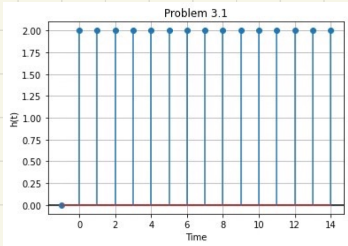
3.1) จาก x[n] คือ



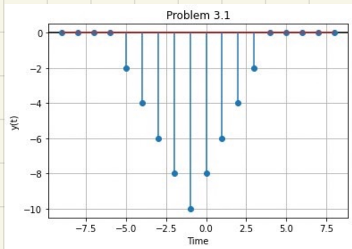
และ h[n] คือ



ใช้ Tabular method

x[n]										-1	-1	-1	-1	-1	1	1	1	1	1	×
h[n]										2	2	2	2	2	2	2	2	2	2	...
										-2	-2	-2	-2	-2	2	2	2	2	2	
									-2	-2	-2	-2	-2	2	2	2	2	2		
								-2	-2	-2	-2	-2	2	2	2	2	2			
							-2	-2	-2	-2	-2	2	2	2	2	2				
						-2	-2	-2	-2	-2	2	2	2	2	2					
					-2	-2	-2	-2	-2	2	2	2	2	2						
				-2	-2	-2	-2	-2	2	2	2	2	2							
			-2	-2	-2	-2	-2	2	2	2	2	2								
		-2	-2	-2	-2	-2	2	2	2	2	2									
	-2	-2	-2	-2	-2	2	2	2	2	2										
	-2	-4	-6	-8	-10	-8	-6	-4	-2	0	0	0	0	0	0	0	0	0	0	...

ซึ่งสามารถนำมา plot graph ได้ดังนี้



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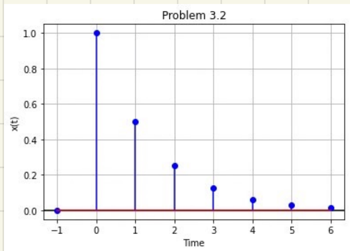
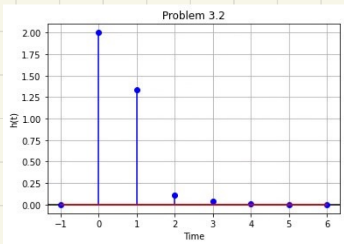
3.2) จาก x[n] คือ



และ h[n] คือ



จะได้ $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$$= \sum_{k=-\infty}^{\infty} \left(\left(\tfrac{1}{2}\right)^k u[k]\right)\left(\delta[n-k] + \delta[n-1-k] + \left(\tfrac{1}{3}\right)^{n-k} u[n-k]\right)$$

$$= \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u[k]\,\delta[n-k] + \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u[k]\,\delta[n-1-k] + \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u[k] \left(\tfrac{1}{3}\right)^{n-k} u[n-k]$$

$$= \frac{u[n]}{2^n} + \frac{u[n-1]}{2^{n-1}} + \frac{1}{3^n}\left(\sum_{k=0}^{n} \left(\tfrac{3}{2}\right)^k\right) u[n]$$

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$$y[n] = \frac{u[n]}{2^n} + \frac{u[n-1]}{2^{n-1}} + \frac{1}{3^n}\left(\frac{(3/2)^{n+1}-1}{3/2-1}\right)u[n] = \begin{cases} 2 & , n=0 \\ \frac{6}{2^n} - \frac{2}{3^n} & , 1 \le n \end{cases}$$

ซึ่งสามารถนำมา plot graph ได้ดังนี้