Estimation of Discrete Game and its Realization in R

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Abstract

In this paper, we will discuss the estimation of static, discrete games of complete information in simultaneous situation. And we will use R to simulate and realize it.

Introduction

Discrete game is a generalization of a standard discrete model where utility depends not only on itself but also on action of other players. The decisions involve strategic interactions: none of subjects act in a vacuum.

A critical consideration when formatting a discrete game involves specifying each player's information set and relevant time horizon. With regard to the players' information sets, there are two main approaches: complete information and incomplete information. Under the complete information setting, the researcher assumes that the players observe everything about each other's payoffs and therefore face no uncertainty regarding the payoffs of their rivals. Turning to the palyer's relevant time horizon, there are again two choices: one-shot static game or infinite horizon dynamic game. For the purpose of this article, we will focus exclusively on the static, discrete game with complete information.

Entry game

The model

To illustrate these, we can take an entry game as an example. Support firm A and B compete in many local markets. Like the model of Ellicken and Misra(2011), focusing on small markets enables us to ignore the existence of Target, which mainly serves more urban locations.

Besides, we consider a much more simplified version of model. These two firms make entry decisions across a collection of local markets.Each firm chooses either "enter" or "not enter". Furthermore, we set up that each firm can only open at most one store in one market. By using R, we simulate and generate 2000 relatively small and isolated markets. In this process, we assume that Xm is a vector containing market characteristics common to both firms, and Zm represents firm characteristics that are specific and don't impact the profits of its rival. In our generating process, we let X_m follows normal distribution N($\left(\begin{smallmatrix}3\\3\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$). similarly, the Z_{Am} ~ N($\left(\begin{smallmatrix}3.5\\3\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$) and Z_{Bm} ~ N($\left(\begin{smallmatrix}3\\3\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$). Let the profit function be:

- When the firm chooses to enter,

$$\pi_{im} = \alpha' X_m + \beta' Z_m + \delta y_{-im} + \epsilon_{im}$$

  where y_{-im} is the action of its rivals, ϵ_{im} is a component of profits that is unobservable.

- When the firm chooses not to enter,

$$\pi_{im} = 0$$

In our model, we assume the true value of parameters $\{\alpha_1, \alpha_2, \beta_1, \beta_2, \delta, \sigma\}$ are {1, -1, 1, -1, -1, 1}. Thus, expected profits (net of ϵ_{im}) are a function of only the common market characteristics, the firm's own characteristics, and its rival's chosen action.

Assuming the firms make choices simultaneously, the complete information Nash Equilibrium can be expressed as

$$y_{Am} = 1[\alpha' X_m + \beta' Z_m + \delta y_{Bm} + \epsilon_{Am} \geq 0]$$

$$y_{Bm} = 1[\alpha' X_m + \beta' Z_m + \delta y_{Am} + \epsilon_{Bm} \geq 0]$$

The result can be showed as:

Payoff	$\pi_B < 0$	$\pi_B > 0$ & $\pi_{By} < 0$	$\pi_{By} > 0$
$\pi_A < 0$	*(0, 0)*	*(0, 1)*	*(0, 1)*
$\pi_A > 0$ & $\pi_{Ay} < 0$	*(1, 0)*	*(1, 0)*	*(0, 1)*
$\pi_{Ay} > 0$	*(1, 0)*	*(1, 0)*	*(1, 1)*

where 1 means the firm will enter and 0 means the firm won't enter.

In particular, at the center there exists a situation where for a given set of parameters there may be more than one possible vector of equilibrium outcomes. In order to solve the multiplicity problem, we set the characteristic variable Z_{m1} larger which has a positive effect on its profit so that firm A has a little bit more power, and specify an equilibrium selection rule: when both firms will enter without the other firm's entry while won't enter with the other's entry, only firm A can enter the market.

```
library(MASS)
set.seed(123)
alpha <- beta <- c(1,-1)
delta <- -1
ea <- rnorm(2000)
eb <- rnorm(2000)
sigma2 <- matrix(c(1,0,0,1),nrow=2)
x <- mvrnorm(2000,c(3,3),sigma2)
za <- mvrnorm(2000,c(3.5,3),sigma2)
zb <- mvrnorm(2000,c(3,3),sigma2)
sigma <- 1
parameter <- c(alpha,beta,delta,sigma)


game <- function(parameter){
  market <- matrix(1,nrow=2000,ncol=2)
  alpha = parameter[1:2]
  beta = parameter[3:4]
  delta = parameter[5]
  sigma = parameter[6]
  paia <- x%*%alpha+za%*%beta+ea
  paiay <- x%*%alpha+za%*%beta+delta+ea
  paib <- x%*%alpha+zb%*%beta+eb
  paiby <- x%*%alpha+zb%*%beta+delta+eb
  for (i in 1:2000) {
    if (paia[i] < 0 & paib[i] <0){
      market[i,] <- c(0,0)
    }
    else if (paiay[i] >=0 & paiby[i] >= 0){
      market[i,] <- c(paiay[i],paiby[i])
    }
    else if (paia[i] >=0 & paiby[i] < 0){
      market[i,] <- c(paia[i],0)
```

```
    }
    else {
      market[i,] <- c(0,paib[i])
    }

  }
  return(market)
}


pai <- game(parameter)
market <- matrix(pai!=0,nrow = 2000)
```

```
#The true market structure, with firm A in the left, firm B in the top

table(market[,1],market[,2])
```

```
##
##          FALSE TRUE
##   FALSE   507  397
##   TRUE    697  399
```

From the table we can see that firm A has a slightly market share, which is consistent with our settings.

Simulation

Now we will use R to simulate it and use maximum likelihood estimation(MLE) to find those coefficients following the approach of Bresnahan and Reiss (1990, 1991). Since the ϵ follows normal distribution, then profit π also follows normal distribution. And we should also standalize it in order to calculate cdf more easily.

Thus, the likelihood of observing $\mathrm{n_m}$ firms in a given market m can be computed in closed form. For example, the probability of seeing a duopoly is simply

$$Pr(n_m = 2) = \prod_i Pr(\alpha' X_m + \beta' Z_m + \delta y_{im} + \epsilon_{im} \geq 0)$$

The sample likelihood function is then

$$lnL = \sum_m^{2000} \sum_{i=0}^{2} \sum_{j=0}^{2} 1(A = i)1(B = j)[lnP(A = i) + lnP(B = j)]$$

Results for complete information games are presented in the follows:

Table 1: Estimation Results

Variable	Value
α_1	0.9559076
α_2	-0.9669120
β_1	0.9639258
β_2	-0.9159532
δ	-1.0577803
σ	0.9303945

```r
probability <- function(hat=c(1,-1,1,-1,-1,1)){
  prob <- rep(0,2000)
  halpha = hat[1:2]
  hbeta = hat[3:4]
  hdelta = hat[5]
  hsigma = hat[6]
  for (i in 1:2000) {
    if (market[i,1]==1 & market[i,2]==1){
      prob[i] <- (1-pnorm(-(x[i,]%*%halpha+za[i,]%*%hbeta+hdelta)/hsigma)
                 )*(1-pnorm(-(x[i,]%*%halpha+zb[i,]%*%hbeta+hdelta)/hsigma))
    }
    else if (market[i,1]==1 & market[i,2]==0){
      prob[i] <- (1-pnorm(-(x[i,]%*%halpha+za[i,]%*%hbeta)/hsigma)
                 )*(pnorm(-(x[i,]%*%halpha+zb[i,]%*%hbeta+hdelta)/hsigma))
    }
    else if (market[i,1]==0 & market[i,2]==0){
      prob[i] <- (pnorm(-(x[i,]%*%halpha+za[i,]%*%hbeta)/hsigma)
                 )*(pnorm(-(x[i,]%*%halpha+zb[i,]%*%hbeta)/hsigma))
    }
    else {
      prob[i] <- ((pnorm(-(x[i,]%*%halpha+za[i,]%*%hbeta)/hsigma)
                  )*(1-pnorm(-(x[i,]%*%halpha+zb[i,]%*%hbeta)/hsigma)
                     )+(pnorm(-(x[i,]%*%halpha+za[i,]%*%hbeta+hdelta)/hsigma)
                        -pnorm(-(x[i,]%*%halpha+za[i,]%*%hbeta)/hsigma)
                        )*(1-pnorm(-(x[i,]%*%halpha+zb[i,]%*%hbeta+hdelta)/hsigma)))
    }
  }
  return(prob)
}

L <- function(hat){
  sum(log(probability(hat)))
}

negativeL <- function(hat){
  -sum(log(probability(hat)))
}

maxL <- optim(par=c(1,-1,1,-1,-1,1),negativeL)
maxL
```

```
## $par
## [1]  0.9559076 -0.9669120  0.9639258 -0.9159532 -1.0577803  0.9303945
##
## $value
## [1] 1248.418
##
## $counts
## function gradient
##      267       NA
##
## $convergence
## [1] 0
##
```

```
## $message
## NULL
```

```r
hat <- maxL$par
```

Discussion of results

The results are presented above. All parameter estimates' signs that are consistent both with intuition and previous results. The negative sign of δ shows that facing competition reduces firm's profit; $\delta \neq 0$ also means that each firm cannot depend on its own characteristics(X and Z), but should also consider the rival's choice.

Furthermore, we test whether the MLE process is appropriate for this model:

```r
#The True-estimated table, with estimate result in the left, true results in the right
paifit <- game(hat)
marketfit <- matrix(paifit!=0,nrow = 2000)
table(marketfit,market)
```

```
##          market
## marketfit FALSE TRUE
##     FALSE  2053   15
##     TRUE     55 1877
```

where the lower left corner represents the false positive rate equal to $\frac{55}{2000} = 2.75\%$, and the upper right corner represents the false negative rate equal to $\frac{15}{2000} = 0.75\%$. Then we can get a conclusion that MLE is feasible.

```r
#The estimated market structure, with firm A in the left, firm B in the top

table(marketfit[,1],marketfit[,2])
```

```
##
##         FALSE TRUE
##   FALSE   477  393
##   TRUE    721  409
```

Like the table in the front, our estimation still shows firm A has a larger market share.

we would like to be clear that it's not a general result and just based on our generated data. Different number of players and different power can make significant difference. It's also important that the choice of the payoff specification is key. If we allow the players to have different coefficients (like δ or β), it will have significant impact on estimates.

While we have focused our attention on simple static, complete information games, there are also new developments and challenges in dynamic games. This article provides a critical overview of the estimation of static discrete games, aimed at providing a brief introduction and simulation. There still exists many imperfections and inaccurations in this article, and we hope we can continue to make it better in the future and encourage more people to research in this field.

Reference

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