Gate problem

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Problem statement

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Problem

The transfer function of causal LTI system is $H(s) = \frac{1}{s}$. If the input to the system is $x(t) = [\frac{sin(t)}{\pi t}]u(t)$, where u(t) is a unit step function, the system output $y(t)$ as $t \to \infty$ is ?

Solution

let $f(t) = sin(t)u(t)$

We know that,$L\{f(t)\} = F(s) = \frac{1}{1+s^2}$
(We can prove the above by uv rule of integration.)

By using, $\mathcal{L}\{\frac{f(t)}{t}\} = \int_s^\infty F(s)ds$

We can say, $L\{f(t)\} = F(s) = \int_s^\infty \frac{1}{1+s^2}ds$

$\implies$ X(s) = [1/ π] $[\frac{\pi}{2} - \tan^{-1}(s)]$

$\implies$ Y(s) = [1/ πs] $[\frac{\pi}{2} - \tan^{-1}(s)]$, since $Y(s) = X(s)H(s)$

$\implies$ y(∞) = $\lim_{s\to} sY(s)$ =[1/ π] $[\frac{\pi}{2} - \tan^{-1}(s)] = \frac{1}{2}$

Proof of Final Value Theorem:

The last statement is implied by final value theorem.

To prove: $\lim_{s\rightarrow\ 0}(sF(s)) = \lim_{t\rightarrow\infty} f(t)$

Proof:
We know, $L\{\frac{df(t)}{dt}\} = \int_{0^-}^{\infty} \frac{df(t)}{dt} e^{-st} dt = sF(s) - f(0^-)$

Now applying s $\rightarrow$ 0,

We have, $RHS = \int_{0^-}^{\infty} \frac{df(t)}{dt} dt = \lim_{s\rightarrow\ 0}(f(\infty) - f(0^-))$

And , $LHS = \lim_{s\rightarrow\ 0}(sF(s) - f(0^-)$

Hence proved