# Control Systems

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Contents



***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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## 1 Mason's Gain Formula

## 2 Bode Plot

### 2.1 Introduction

### 2.2 Example

## 3 Second order System

### 3.1 Damping

3.1. The transfer function of causal L.T.I system is

$$H(s) = \frac{1}{s}. \tag{3.1.1}$$

If the input to the system is

$$x(t) = (\frac{sin(t)}{\pi t})u(t) \tag{3.1.2}$$

, where u(t) is a unit step function, the system output $y(t)$ as $t \to \infty$ is ?
**Solution:** let

$$f(t) = sin(t)u(t) \tag{3.1.3}$$

We know that,

$$\mathcal{L}f(t) = F(s) = \frac{1}{1+s^2} \tag{3.1.4}$$

(by u.v rule of integration.)
By using,

$$\mathcal{L}\frac{f(t)}{t} = \int_s^\infty F(s)\,\mathrm{d}s \tag{3.1.5}$$

$$\implies X(s) = (1/\pi)(\frac{\pi}{2} - \tan^{-1}(s)) \tag{3.1.6}$$

$$\implies Y(s) = (1/\pi s)(\frac{\pi}{2} - \tan^{-1}(s)) \tag{3.1.7}$$

since

$$Y(s) = X(s)H(s) \tag{3.1.8}$$

Then using Final Value theorem,

$$y(\infty) = \lim_{s\to 0} sY(s) = (\frac{1}{\pi})(\frac{\pi}{2} - \tan^{-1}) = \frac{1}{2} \tag{3.1.9}$$

**Proof of Final Value Theorem:**
To prove :

$$\lim_{s\to 0}(sF(s)) = \lim_{t\to\infty} f(t) \tag{3.1.10}$$

We know,

$$\mathcal{L}\{\frac{\partial f(t)}{\partial t}\} = \int_{0^-}^{\infty} \frac{\partial f(t)}{\partial t} e^{-st}\,\mathrm{d}t = s * F(s) - f(0^-) \tag{3.1.11}$$

Now applying $s \to 0$ We have,

$$RHS = \int_{0^-}^{\infty} \frac{\partial f}{\partial t}\,\mathrm{d}t = \lim_{s\to 0}(f(\infty) - f(0^-)) \tag{3.1.12}$$

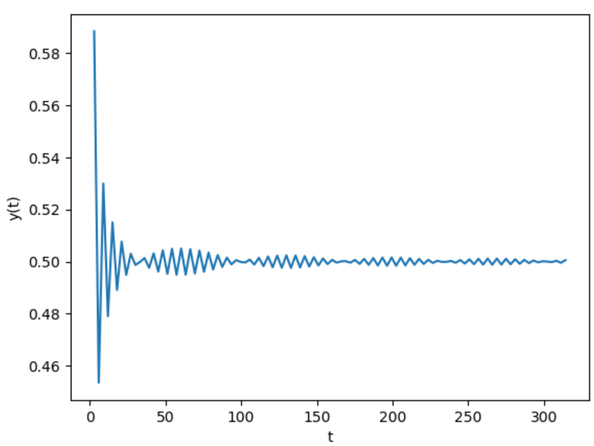
And

$$LHS = \lim_{s\to 0}(sF(s) - f(0^-)) \tag{3.1.13}$$

Hence proved

**Plotting y(t) in time domain.**

```
https://github.com/Kkuntal990/Control−theory
    −course/blob/master/codes/
    EE18BTECH11028.py
```




This shows as $t$ goes to infinity $y(t)$ tends to 0.5.

*3.2 Example*

## 4 Routh Hurwitz Criterion

*4.1 Routh Array*

*4.2 Marginal Stability*

*4.3 Stability*

## 5 State-Space Model

*5.1 Controllability and Observabiity*

*5.2 Second Order System*

## 6 Nyquist Plot

## 7 Phase Margin

## 8 Gain Margin

## 9 Compensators

*9.1 Phase Lead*

## 10 Oscillator