Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

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1 MASON'S GAIN FORMULA

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.1. The transfer function of causal L.T.I system is

$$H(s) = \frac{1}{s}. \qquad (3.1.1)$$

If the input to the system is

$$x(t) = (\frac{sin(t)}{\pi * t})u(t) \qquad (3.1.2)$$

, where u(t) is a unit step function, the system output $y(t)$ as $t \to \infty$ is ?
Solution: let

$$f(t) = sin(t)u(t) \qquad (3.1.3)$$

We know that,

$$\mathcal{L}f(t) = F(s) = \frac{1}{1 + s^2} \qquad (3.1.4)$$

(by u.v rule of integration.)
By using,

$$\mathcal{L}\frac{f(t)}{t} = \int_s^\infty F(s)\,ds \qquad (3.1.5)$$

$$\implies X(s) = (1/\pi)(\frac{\pi}{2} - \tan^{-1}(s)) \qquad (3.1.6)$$

$$\implies Y(s) = (1/\pi s)(\frac{\pi}{2} - \tan^{-1}(s)) \qquad (3.1.7)$$

since

$$Y(s) = X(s)H(s) \qquad (3.1.8)$$

Then using Final Value theorem,

$$y(\infty) = \lim_{s \to 0} sY(s) = (\frac{1}{\pi})(\frac{\pi}{2} - \tan^{-1}) = \frac{1}{2} \qquad (3.1.9)$$

Proof of Final Value Theorem:

To prove :

$$\lim_{s \to 0}(s * F(s)) = \lim_{t \to \infty} f(t) \qquad (3.1.10)$$

We know,

$$\mathcal{L}\{\frac{\partial f(t)}{\partial t}\} = \int_{0^-}^{\infty} \frac{\partial f(t)}{\partial t} e^{-st}\, \mathrm{d}t = s * F(s) - f(0^-)$$
$$(3.1.11)$$

Now applying $s \to 0$ We have,

$$RHS = \int_{0^-}^{\infty} \frac{\partial f}{\partial t}\, \mathrm{d}t = \lim_{s \to 0}(f(\infty) - f(0^-))$$
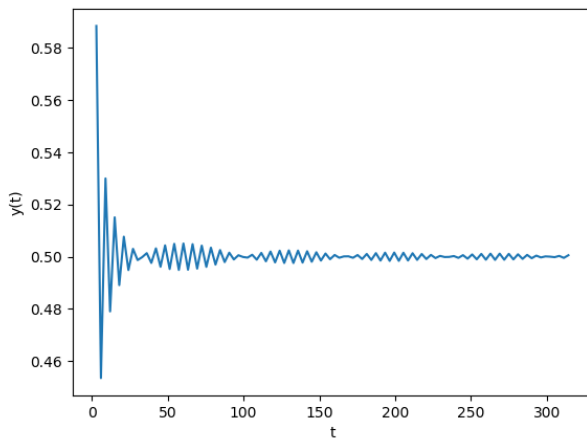$$(3.1.12)$$

And

$$LHS = \lim_{s \to 0}(s.F(s) - f(0^-)) \qquad (3.1.13)$$

Hence proved

Plotting y(t) in time domain.

https://github.com/Kkuntal990/Control−theory
−course/blob/master/codes/
EE18BTECH11028.py

This shows as t goes to infinity $y(t)$ tends to 0.5.