

# Control Systems

G V V Sharma\*

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

## 1 MASON'S GAIN FORMULA

### 2 BODE PLOT

#### 2.1 Introduction

#### 2.2 Example

## 3 SECOND ORDER SYSTEM

### 3.1 Damping

3.1. The transfer function of causal L.T.I system is

$$H(s) = \frac{1}{s}. \quad (3.1.1)$$

If the input to the system is

$$x(t) = \left(\frac{\sin(t)}{\pi * t}\right)u(t) \quad (3.1.2)$$

, where  $u(t)$  is a unit step function, the system output  $y(t)$  as  $t \rightarrow \infty$  is ?

**Solution:** let

$$f(t) = \sin(t)u(t) \quad (3.1.3)$$

We know that,

$$\mathcal{L}f(t) = F(s) = \frac{1}{1 + s^2} \quad (3.1.4)$$

(by u.v rule of integration.)

By using,

$$\mathcal{L}\frac{f(t)}{t} = \int_s^\infty F(s) ds \quad (3.1.5)$$

$$\Rightarrow X(s) = (1/\pi)\left(\frac{\pi}{2} - \tan^{-1}(s)\right) \quad (3.1.6)$$

$$\Rightarrow Y(s) = (1/\pi s)\left(\frac{\pi}{2} - \tan^{-1}(s)\right) \quad (3.1.7)$$

since

$$Y(s) = X(s)H(s) \quad (3.1.8)$$

Then using Final Value theorem,

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = \left(\frac{1}{\pi}\right)\left(\frac{\pi}{2} - \tan^{-1}(s)\right) = \frac{1}{2} \quad (3.1.9)$$

**Proof of Final Value Theorem:**

To prove :

$$\lim_{s \rightarrow 0} (s * F(s)) = \lim_{t \rightarrow \infty} f(t) \quad (3.1.10)$$

We know,

$$\mathcal{L}\left\{\frac{\partial f(t)}{\partial t}\right\} = \int_{0^-}^{\infty} \frac{\partial f(t)}{\partial t} e^{-st} dt = s * F(s) - f(0^-) \quad (3.1.11)$$

Now applying  $s \rightarrow 0$  We have,

$$RHS = \int_{0^-}^{\infty} \frac{\partial f}{\partial t} dt = \lim_{s \rightarrow 0} (f(\infty) - f(0^-)) \quad (3.1.12)$$

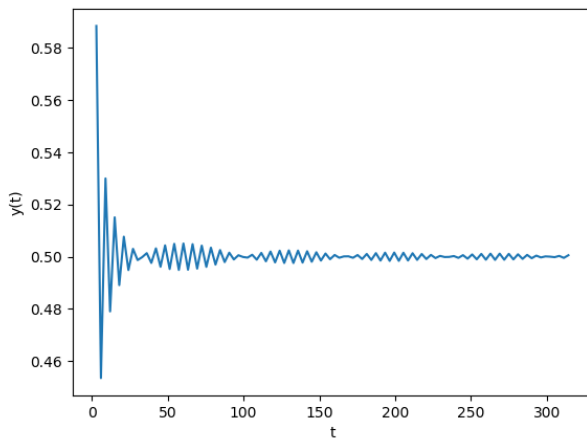
And

$$LHS = \lim_{s \rightarrow 0} (s.F(s) - f(0^-)) \quad (3.1.13)$$

Hence proved

**Plotting y(t) in time domain.**

<https://github.com/Kkuntal990/Control-theory-course/blob/master/codes/EE18BTECH11028.py>



This shows as  $t$  goes to infinity  $y(t)$  tends to 0.5.

*3.2 Example***4 ROUTH HURWITZ CRITERION***4.1 Routh Array**4.2 Marginal Stability**4.3 Stability***5 STATE-SPACE MODEL***5.1 Controllability and Observability**5.2 Second Order System***6 NYQUIST PLOT****7 PHASE MARGIN****8 GAIN MARGIN****9 COMPENSATORS***9.1 Phase Lead***10 OSCILLATOR**