Gate problem

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Problem statement

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Problem

The transfer function of causal L.T.I system is $H(s) = \frac{1}{s}$. If the input to the system is $x(t) = (\frac{sin(t)}{\pi * t})u(t)$, where u(t) is a unit step function, the system output $y(t)$ as $t \to \infty$ is ?

Solution

let $f(t) = sin(t)u(t)$
We know that, $\mathcal{L}\{f(t)\} = F(s) = \frac{1}{1+s^2}$ (u.v rule of integration.)
By using, $\mathcal{L}\{\frac{f(t)}{t}\} = \int_s^{\infty} F(s)\,\mathrm{d}s$

$\implies X(s) = (1/\pi)(\frac{\pi}{2} - \text{tan inverse}(s))$

$\implies Y(s) = (1/\pi s)(\frac{\pi}{2} - \text{tan inverse}(s))$, (since $Y(s) = X(s)H(s)$)

$$y(\infty) = \lim_{s\to\ 0} sY(s) = (\frac{1}{\pi})(\frac{\pi}{2} - \text{tan inverse}(s)) = \frac{1}{2}$$

(Using Final value theorem.)

Proof of Final Value Theorem:

The last statement is implied by final value theorem.

To prove: $\lim_{s\to\ 0}(s * F(s)) = \lim_{t\to\infty} f(t)$

Proof:

We know, $\mathcal{L}\{\frac{\partial f(t)}{\partial t}\} = \int_{0^-}^{\infty} \frac{\partial f(t)}{\partial t} e^{-st}\,\mathrm{d}t = s * F(s) - f(0^-)$

Now applying $s \to 0$,

We have, RHS $= \int_{0^-}^{\infty} \frac{\partial f}{\partial t}\,\mathrm{d}t = \lim_{s\to\ 0}(f(\infty) - f(0^-))$

And , $LHS = \lim_{s\to\ 0}(s.F(s) - f(0^-))$
Hence proved

Verification