Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```


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2.1 Introduction

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3.1 Damping

3.1. The transfer function of causal L.T.I system is

$$H(s) = \frac{1}{s}. \tag{3.1.1}$$

If the input to the system is

$$x(t) = (\frac{sin(t)}{\pi * t})u(t) \tag{3.1.2}$$

, where u(t) is a unit step function, the system output $y(t)$ as $t \to \infty$ is ?

Solution: let

$$f(t) = sin(t)u(t) \tag{3.1.3}$$

We know that,

$$\mathcal{L}f(t) = F(s) = \frac{1}{1+s^2} \tag{3.1.4}$$

(by u.v rule of integration.)
By using,

$$\mathcal{L}\frac{f(t)}{t} = \int_s^\infty F(s)\,\mathrm{d}s \tag{3.1.5}$$

$$\implies X(s) = (1/\pi)(\frac{\pi}{2} - \tan^{-1}(s)) \tag{3.1.6}$$

$$\implies Y(s) = (1/\pi s)(\frac{\pi}{2} - \tan^{-1}(s)) \tag{3.1.7}$$

since

$$Y(s) = X(s)H(s) \tag{3.1.8}$$

Then using Final Value theorem,

$$y(\infty) = \lim_{s\to 0} sY(s) = (\frac{1}{\pi})(\frac{\pi}{2} - \tan^{-1}) = \frac{1}{2} \tag{3.1.9}$$

3.2. Proof of Final Value Theorem:
Solution: To prove :

$$\lim_{s\to\ 0}(s*F(s)) = \lim_{t\to\infty} f(t) \tag{3.2.1}$$

We know,

$$\mathcal{L}\{\frac{\partial f(t)}{\partial t}\} = \int_{0^-}^{\infty} \frac{\partial f(t)}{\partial t} e^{-st}\, dt = s*F(s) - f(0^-) \tag{3.2.2}$$

Now applying $s \to 0$ We have,

$$RHS = \int_{0^-}^{\infty} \frac{\partial f}{\partial t}\, dt = \lim_{s\to\ 0}(f(\infty) - f(0^-)) \tag{3.2.3}$$

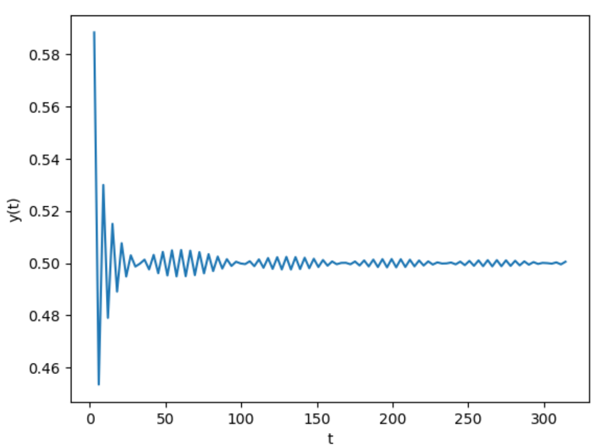
And

$$LHS = \lim_{s\to\ 0}(s.F(s) - f(0^-)) \tag{3.2.4}$$

Hence proved

3.3. Plotting y(t) in time domain.

```
https://github.com/Kkuntal990/Control−theory
    −course/blob/master/codes/
    EE18BTECH11028.py
```



This shows as t goes to infinity $y(t)$ tends to 0.5.

3.2 Example

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4.3 Stability

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