Control Systems

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Contents

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

3.2 Example

4 Routh Hurwitz Criterion

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

5 State-Space Model

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

6 Nyquist Plot

6.1 Polar plots

6.1. Plot the polar plot of

$$G(s) = \frac{1}{(s^2)(s+1)(s+2)}. \tag{6.1.1}$$

Solution: For polar plot we have to plot magnitude of $G(s)$ versus its phase by varying ω from 0 to ∞.
First substitute,

$$s = j\omega \tag{6.1.2}$$

Now the magnitude will be

$$|G(j\omega)| = \frac{1}{(\omega^2)(\sqrt{1+\omega^2})(\sqrt{1+4\omega^2})} \tag{6.1.3}$$

Similarly phase ϕ can be determined by,

$$\phi = -\tan^{-1}(0) - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{6.1.4}$$

The phase of first term is π or can be $-\pi$ since it is a negative real number.

$$\implies \phi = 180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{6.1.5}$$

Now we have to sweep ω from 0 to ∞.
So at $\omega = 0$,

$$|G(j\omega)| \xrightarrow{0} \infty \tag{6.1.6}$$

And phase,

$$\angle G(j\omega) = 180^\circ \tag{6.1.7}$$

At $\omega = \infty$

$$|G(j\omega)| \xrightarrow{\infty} 0 \tag{6.1.8}$$

And phase,

$$\angle G(j\omega) = 0^\circ \tag{6.1.9}$$

For a complete plot we have to put various values of ω in eq. 6.1.5 and eq. 6.1.3 for magnitude and phase respectively. Thus the polar plot looks like,
The following python code generates Fig. 6.1 (a)

```
codes/ee18btech11028.py
```

Also note that in Fig. 6.1 (a) the point $(-1, 0)$ is enclosed by the polar plot, which implies system is not stable.

Utility of polar plot in control systems

Polar plots make it easy to determine the Phase margin (PM) and gain margin (GM) of the system. This two quantities are substantial for determining the stability of the system. Please

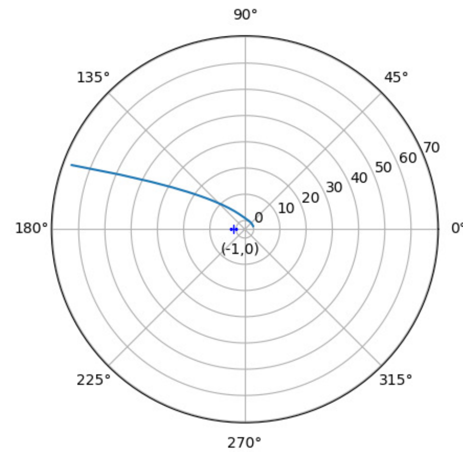

Fig. 6.1: (a)

refer to the sections of Gain Margin and Phase margin for definations.
As it is seen in polar plot of (6.1.1) the graph is not intersecting the 180 ° line in finite range we will use another transfer function to see the usage of polar plots.
Let's take

$$G(s) = \frac{5}{(s)(s+1)(s+3)}. \tag{6.1.10}$$

The polar plot for this transfer function using the above method looks like,

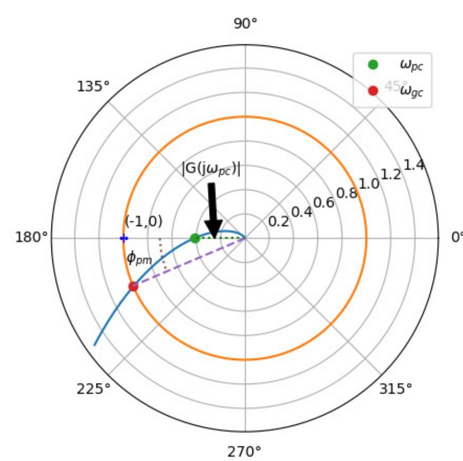

Fig. 6.1: (b)

From the Fig. **??** (b) it is apparent that it does not enclose the point $(-1, 0)$. So the system is stable. And also, From the plot it's really easy to find GM,

$$GM = \frac{1}{|G(j\omega_{pc})|} \tag{6.1.11}$$

and PM is ϕ_{pm} in anti-clockwise direction considered as positive.
Since the polar plot of (6.1.1) has magnitude of ∞ at 180 $^\circ$ line it's GM is considered 0.
Now we can deduce the stability of the system by,

- If the gain margin GM is greater than one and the phase margin PM is positive, then the control system is **stable**.
- If the gain margin GM is equal to one and the phase margin PM is zero degrees, then the control system is **marginally stable**.
- If the gain margin GM is less than one and (OR) the phase margin PM is negative, then the control system is **unstable**.

We can find phase cross over frequency (ω_{gc}) and gain cross over frequency (ω_{gc}) by putting the magnitudes or phases as mentioned in the legend of **??** in (6.1.3) and (6.1.5) respectively.
The following python code generates Fig . 6.1 (b),

```
codes/example.py
```

7 Compensators

7.1 Phase Lead

7.2 Example

8 Gain Margin

8.1 Introduction

8.2 Example

9 Phase Margin

10 Oscillator

10.1 Introduction

10.2 Example