Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

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5 State-Space Model

5.1 Controllability and Observability

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6 Nyquist Plot

6.1 Polar plots

6.1. Plot the polar plot of

$$G(s) = \frac{1}{(s^2)(s+1)(s+2)}. \tag{6.1.1}$$

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Solution: For polar plot we have to plot magnitude of $G(s)$ versus its phase by varying ω from 0 to ∞.
First substitute,

$$s = j\omega \tag{6.1.2}$$

Now the magnitude will be

$$|G(j\omega)| = \frac{1}{(\omega^2)(\sqrt{1+\omega^2})(\sqrt{1+4\omega^2})} \tag{6.1.3}$$

Similarly phase ϕ can be determined by,

$$\phi = -\tan^{-1}(0) - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{6.1.4}$$

The phase of first term is π or can be $-\pi$ since it is a negative real number.

$$\implies \phi = 180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \tag{6.1.5}$$

Now we have to sweep ω from 0 to ∞.
So at $\omega = 0$,

$$|G(j\omega)| \xrightarrow{0} \infty \tag{6.1.6}$$

And phase,

$$\angle G(j\omega) = 180^\circ \tag{6.1.7}$$

At $\omega = \infty$

$$|G(j\omega)| \xrightarrow{\infty} 0 \tag{6.1.8}$$

And phase,

$$\angle G(j\omega) = 0^\circ \tag{6.1.9}$$

For a complete plot we have to put various values of ω in eq. 6.1.3 and eq. 6.1.3 for magnitude and phase respectively. Thus the polar plot looks like,
The following python code generates Fig. 6.1

```
codes/ee18btech11028.py
```

Utility of polar plot in control systems
Note that in Fig. 6.1 (a) the point $(-1, 0)$ is enclosed by the polar plot, which implies system is not stable.
Polar plots make it easy to determine the Phase margin (PM) and gain margin (GM) of the system. This two quantities are substantial for determining the stability of the system. Please refer to the sections of Gain Margin and Phase margin for definations.
From the plot it's really easy to find GM,

$$GM = \frac{1}{|G(j\omega_{pc})|} \tag{6.1.10}$$

and PM is ϕ_{pm} in anti-clockwise direction considered as positive.
So from (6.1.6), (6.1.7) and (6.1.10),

$$GM = 0 \tag{6.1.11}$$

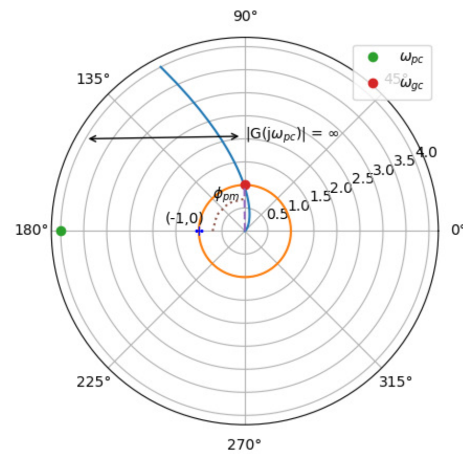

Fig. 6.1

And it intersects unity circle as in Fig. 6.1 with ϕ_{pm} in clockwise direction, which implies it is negative.
Now we can deduce the stability of the system by,

- If the gain margin GM is greater than one and the phase margin PM is positive, then the control system is **stable**.
- If the gain margin GM is equal to one and the phase margin PM is zero degrees, then the control system is **marginally stable**.
- If the gain margin GM is less than one and (OR) the phase margin PM is negative, then the control system is **unstable**.

Therefore, our system is unstable.
We can find phase cross over frequency (ω_{gc}) and gain cross over frequency (ω_{gc}) by putting the magnitudes or phases as mentioned in the legend of 6.1 in (6.1.3) and (6.1.5) respectively.

7 Compensators

7.1 Phase Lead

7.2 Example

8 Gain Margin

8.1 Introduction

8.2 Example

9 Phase Margin

10 Oscillator

10.1 Introduction

10.2 Example