

# Control Systems

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$$G(s) = \frac{1}{(s^2)(s+1)(s+2)}. \quad (6.1.1)$$

**Solution:** For polar plot we have to plot magnitude of  $G(s)$  versus its phase by varying  $\omega$  from 0 to  $\infty$ .

First substitute,

$$s = j\omega \quad (6.1.2)$$

Now the magnitude will be

$$|G(j\omega)| = \frac{1}{(\omega^2)(\sqrt{1+\omega^2})(\sqrt{1+4\omega^2})} \quad (6.1.3)$$

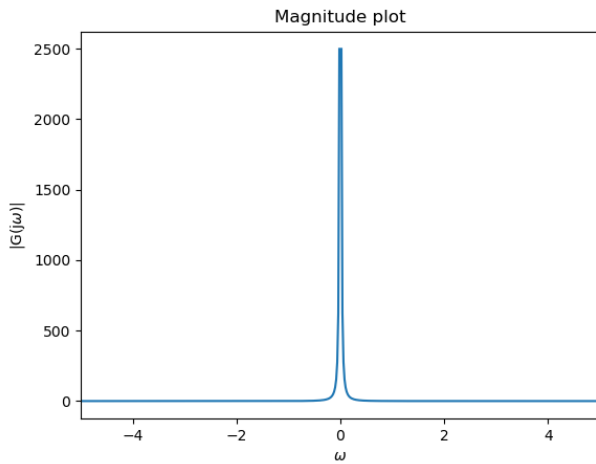


Fig. 6.1: (a)

Similarly phase  $\phi$  can be determined by,

$$\phi = -\tan^{-1}(0) - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \quad (6.1.4)$$

The phase of first term is  $\pi$  or can be  $-\pi$  since it is a negative real number.

$$\Rightarrow \phi = 180^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega) \quad (6.1.5)$$

Now we have to sweep  $\omega$  from 0 to  $\infty$ .

So at  $\omega = 0$ ,

$$|G(j\omega)| \xrightarrow{0} \infty \quad (6.1.6)$$

And phase,

$$\angle G(j\omega) = 180^\circ \quad (6.1.7)$$

At  $\omega = \infty$

$$|G(j\omega)| \xrightarrow{\infty} 0 \quad (6.1.8)$$

And phase,

$$\angle G(j\omega) = 0^\circ \quad (6.1.9)$$

For a complete plot we have to put various values of  $\omega$  in eq. 6.1.5 and eq. 6.1.3 for magnitude and phase respectively. Thus the polar plot looks like,

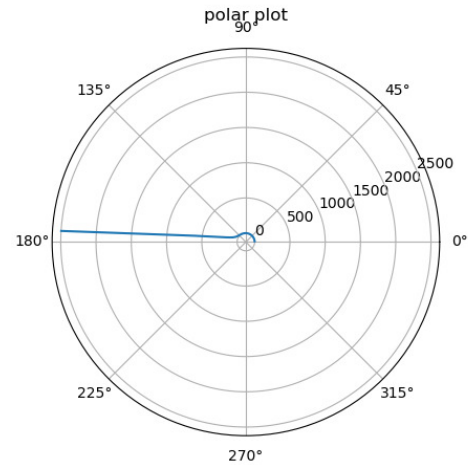


Fig. 6.1: (b)

To take a closer look at how phase is changing in smaller ranges of  $|G(j\omega)|$ .

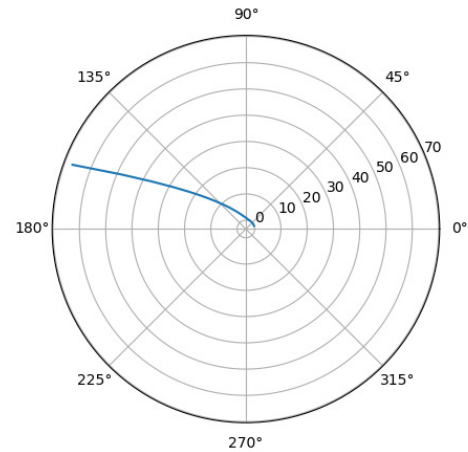


Fig. 6.1: (c)

The following python code generates Fig . 6.1 (a), Fig . 6.1 (b) and Fig. 6.1 (c)

```
codes/ee18btech11028.py
```

### Utility of polar plot in control systems

Polar plots make it easy to determine the Phase margin (PM) and gain margin (GM) of the system. This two quantities are substantial for determining the stability of the system. Please

refer to the sections of Gain Margin and Phase margin for definitions.

As it is seen in polar plot of (6.1.1) the graph is not cutting the  $180^\circ$  line so we will use another transfer function to see the usage of polar plots.

Let's take

$$G(s) = \frac{5}{(s)(s+1)(s+3)}. \quad (6.1.10)$$

The polar plot for this transfer function using the above method looks like,

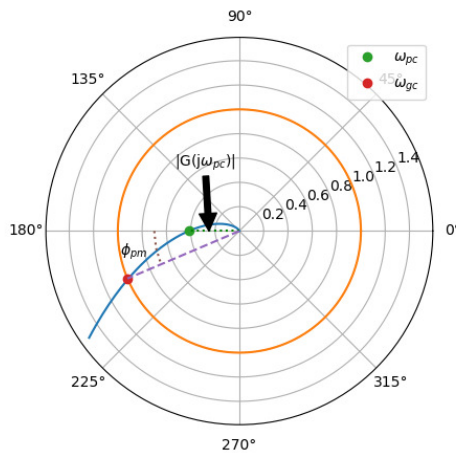


Fig. 6.1: (d)

From the plot it's really easy to find GM,

$$GM = \frac{1}{|G(j\omega_{gc})|} \quad (6.1.11)$$

and PM is  $\phi_{pm}$  in anti-clockwise direction considered as positive. We can find phase cross over frequency ( $\omega_{gc}$ ) and gain cross over frequency ( $\omega_{gc}$ ) by putting the magnitudes or phases as mentioned in the legend of 6.1 in (6.1.3) and (6.1.5) respectively. Since the polar plot of (6.1.1) does not intersect  $180^\circ$  line it's GM is considered  $\infty$ .

Now we can deduce the stability of the system by,

- If the gain margin GM is greater than one and the phase margin PM is positive, then the control system is **stable**.
- If the gain margin GM is equal to one and the phase margin PM is zero degrees, then the control system is marginally **stable**.
- If the gain margin GM is less than one and

(OR) the phase margin PM is negative, then the control system is **unstable**.

We can find phase cross over frequency ( $\omega_{gc}$ ) and gain cross over frequency ( $\omega_{gc}$ ) by putting the magnitudes or phases as mentioned in the legend of 6.1 in (6.1.3) and (6.1.5) respectively. The following python code generates Fig . 6.1 (d),

codes/example.py

## 7 COMPENSATORS

### 7.1 Phase Lead

### 7.2 Example

## 8 GAIN MARGIN

### 8.1 Introduction

### 8.2 Example

## 9 PHASE MARGIN

## 10 OSCILLATOR

### 10.1 Introduction

### 10.2 Example