MATRIX – BASICS (LINEAR ALGEBRA)

Math for Machine Learning

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 7 & 3 \\ 8 & -3 & 1 \end{bmatrix}$$

MATRIX - BASICS

1. Scalars; Vectors; Matrix
2. Shape of a Matrix
3. Different Types of Matrix
4. Transpose of a Matrix
5. Role of Matrix in Machine Learning

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 7 & 3 \\ 8 & -3 & 1 \end{bmatrix}$$

SCALARS; VECTORS; MATRIX

Matrix

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 7 & 3 \\ 8 & -3 & 1 \end{bmatrix}$$

SHAPE OF A MATRIX

$$\begin{bmatrix} 2 & 5 \\ 4 & 8 \end{bmatrix}$$

2 x 2 Matrix

$$\begin{bmatrix} 8 & 6 & 1 \\ 2 & 9 & 2 \\ 3 & 4 & 3 \end{bmatrix}$$

3 x 3 Matrix

$$\begin{bmatrix} 2 & 3 \\ 6 & 4 \\ 7 & 8 \end{bmatrix}$$

3 x 2 Matrix

General Matrix Notation :

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

m x n Matrix

a_{ij} → Matrix element

i → Row number

j → Column number

DIFFERENT TYPES OF MATRICES

Null Matrix or Zero Matrix :

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2 x 2

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3 x 3

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4 x 4

Identity Matrix :

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2 x 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3 x 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 x 4

TRANSPOSE OF A MATRIX

Transpose of a matrix is formed by turning all the rows of a given matrix into columns and vice-versa

$$A = \begin{bmatrix} 2 & 5 \\ 4 & 8 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 4 \\ 5 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 8 & 6 & 1 \\ 2 & 9 & 2 \\ 3 & 4 & 3 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 8 & 2 & 3 \\ 6 & 9 & 4 \\ 1 & 2 & 3 \end{bmatrix}$$

MATRIX OPERATIONS

1. Matrix Addition
2. Matrix Subtraction
3. Multiplying a Matrix by a Scalar
4. Multiplying 2 Matrices

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 7 & 3 \\ 8 & -3 & 1 \end{bmatrix}$$

Matrix Addition

Rule : Two Matrices can be added only if the have the same shape, that is, both the matrix should have the same number of rows and columns

$$\begin{bmatrix} 2 & 3 \\ 10 & 5 \end{bmatrix} + \begin{bmatrix} 10 & 5 \\ 20 & 4 \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ 30 & 9 \end{bmatrix}$$

2 x 2 2 x 2 2 x 2

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 3 & 6 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 7 & 8 \\ 8 & 8 \end{bmatrix}$$

3 x 2 3 x 2 3 x 2

Matrix Subtraction

Rule : Two Matrices can be subtracted only if they have the same shape, that is, both the matrix should have the same number of rows and columns

$$\begin{bmatrix} 2 & 3 \\ 10 & 5 \end{bmatrix} - \begin{bmatrix} 10 & 5 \\ 20 & 4 \end{bmatrix} = \begin{bmatrix} -8 & -2 \\ -10 & 1 \end{bmatrix}$$

2 x 2 2 x 2 2 x 2

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 2 \\ 3 & 6 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 1 & -4 \\ 4 & -2 \end{bmatrix}$$

3 x 2 3 x 2 3 x 2

MULTIPLYING A MATRIX BY A SCALAR

$$5 \times \underset{3 \times 1}{\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}} = \begin{bmatrix} 5 \times 2 \\ 5 \times 4 \\ 5 \times 6 \end{bmatrix} = \underset{3 \times 1}{\begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}}$$

$$5 \times \underset{3 \times 2}{\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{bmatrix}} = \underset{3 \times 2}{\begin{bmatrix} 10 & 5 \\ 20 & 10 \\ 30 & 15 \end{bmatrix}}$$

Note : Vectors are a type of Matrix with either one row or one column

MULTIPLYING 2 MATRICES

Rule : The number of columns in the First matrix should be equal to the number of rows in the Second Matrix

The resultant matrix will have the same number of rows as the first matrix & the same number of columns as the Second Matrix

$$\begin{bmatrix} 2 & 3 \\ 10 & 5 \end{bmatrix} \times \begin{bmatrix} 10 & 5 \\ 20 & 4 \end{bmatrix}$$

2 x 2 2 x 2

Can be multiplied.
Resultant matrix will have the shape 2 x 2

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & 2 \\ 3 & 6 \\ 2 & 5 \end{bmatrix}$$

3 x 2 3 x 2

Cannot be multiplied.

Multiplying 2 Matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$\underset{2 \times \boxed{2}}{\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}} \times \underset{\boxed{2} \times 2}{\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}} = \begin{bmatrix} 2x5 + 4x3 & 2x6 + 4x4 \\ 3x5 + 6x3 & 3x6 + 6x4 \end{bmatrix} = \underset{2 \times 2}{\begin{bmatrix} 22 & 28 \\ 33 & 42 \end{bmatrix}}$$

$$\begin{bmatrix} -6 \\ -4 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 5 \\ 20 & 10 \\ 30 & 15 \end{bmatrix} + \begin{bmatrix} 10 & 5 \\ 20 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 3 & 6 \\ 2 & 5 \end{bmatrix} = ? \quad \begin{bmatrix} 7 & 3 \\ 8 & 7 \\ 5 & 11 \end{bmatrix}$$

$$\begin{matrix} 1 & 5 & 6 \\ 3 & 2 & 4 \\ 0 & 1 & 2 \\ 5 & 4 & 6 \end{matrix} \quad \times \quad \begin{bmatrix} 5 & 2 & 4 \\ 1 & 0 & 3 \\ 2 & 7 & 1 \end{bmatrix} \quad = \quad ?$$

$$\begin{bmatrix} 22 & 44 & 20 \\ 22 & 14 & 22 \\ 15 & 18 & 25 \\ 41 & 40 & 16 \end{bmatrix}$$

Multiplying 2 Matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$$\underset{2\times 2}{\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}} \quad \text{x} \quad \underset{2\times 2}{\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}} = \begin{bmatrix} 2x5 + 4x3 & 2x6 + 4x4 \\ 3x5 + 6x3 & 3x6 + 6x4 \end{bmatrix} = \underset{2\times 2}{\begin{bmatrix} 22 & 28 \\ 33 & 42 \end{bmatrix}}$$

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