Ch 5

1 หาค่า parameter or statistic of pop. or sample

2. หา prob ที่ $\bar{x}$ หรือ $\bar{x}_1 - \bar{x}_2$ หรือ s^2 หรือ $\frac{s_1^2}{s_2^2}$ จะมากกว่า น้อยกว่า ค่า c นึง เท่าไหร่

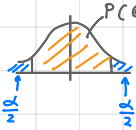
Prob ที่จะหา	Case	σ	n	ค่าทางสถิติที่ใช้
$\bar{x}$	I	known	–	z
	II	unknow	≥ 30	z
	III	unknow	< 30	t
$\bar{x}_1 - \bar{x}_2$		known	$n_1 \geq 30$, $n_2 \geq 30$	z
s^2		know	–	χ^2
$\frac{s_1^2}{s_2^2}$		know	–	F

2 กลุ่มเป็นอิสระต่อกัน + pop เป็น normal dist.

Ch 6

1. เอาค่า statistic จาก sample มาประมาณค่า parameter ของ population. (สถิติอนุมาน)
2. Point Estimation
3. Interval Estimation.

$P(\hat{\Theta}_L < \theta < \hat{\Theta}_U) = 1-\alpha$



Point Estimation

$\bar{x} \rightarrow \mu$

$s \rightarrow \sigma$

$s^2 \rightarrow \sigma$

good point estimator
- unbias $E(\hat{\Theta}) = \theta$
- low variance

Interval Estimator

Mean (ข้อมูลเชิงปริมาณ)
- → 1 กลุ่ม: $\bar{x}$ — z, t
- → 2 กลุ่ม
  - → อิสระต่อกัน: $\bar{x}_1 - \bar{x}_2$ — z, t
  - → ไม่อิสระต่อกัน. $\bar{x}_1 - \bar{x}_2$ — t

Varaince
- → 1 กลุ่ม : s^2 — χ
- → 2 กลุ่ม : $\frac{s_1^2}{s_2^2}$ — f

Proportion (ข้อมูลเชิงคุณภาพ.)
- → 1 กลุ่ม: $\hat{p}$ — z
- → 2 กลุ่ม: $\hat{p}_1 - \hat{p}_2$ — z

α คือระดับความผิดพลาด / ระดับนัยสำคัญ

$1-\alpha$ คือ ความเชื่อมั่น

CH.5 Random sampling and sampling dist.

	Parameter	Statistic
Mean	$\mu = \frac{\Sigma x_i}{N}$	$\bar{x} = \frac{\Sigma x_i}{n}$
Var (ความแปรปรวน)	δ^2 or $\sigma^2 = \frac{\Sigma (x_i-\mu)^2}{N}$	$s^2 = \frac{\Sigma (x_i-\bar{x})^2}{n-1}$
SD (ส่วนเบี่ยงเบนมาตรฐาน)	δ or $\sigma = \sqrt{\frac{\Sigma (x_i-\mu)^2}{N}}$	$s = \sqrt{\frac{\Sigma (x_i-\bar{x})^2}{n-1}}$

$$s^2 = \frac{n\Sigma x_i^2 - (\Sigma x_i)^2}{n(n-1)}$$

$$= \frac{\Sigma x_i^2 - n\bar{x}^2}{n-1}$$

มัธยฐาน $= x_{(\frac{n+1}{2})}$ (n is odd) or $\frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$ (n is even)

ฐานนิยม $=$ x ที่ซ้ำเยอะที่สุด (มีมากกว่า 1 ตัวได้ แต่ไม่เกิน 3)

พิสัย $=$ max − min

SAMPLING DIST. of MEAN

$\mu_{\bar{x}} = E(\bar{x}) = \mu$

$\sigma^2_{\bar{x}} = Var(\bar{x}) = \frac{\sigma^2}{n}$

Note: Popu. เป็น Normal dist

or $n \geq 30$ ถึงใช้ได้

$P(\bar{x} > ■) \dashrightarrow P(z > \frac{■ - \mu}{\sigma/\sqrt{n}})$

$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

SAMPLING DIST. of the DIFF. BETWEEN two MEANs

$\mu_{\bar{x}_1-\bar{x}_2} = E(\bar{x}_1-\bar{x}_2) = \mu_1-\mu_2$

$\sigma^2_{\bar{x}_1-\bar{x}_2} = Var(\bar{x}_1-\bar{x}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

Note: Both popu. เป็น Normal dist

or n_1 & $n_2 \geq 30$ ถึงใช้ได้

ข้อมูลต้องเป็นอิสระต่อกัน.

$Z = \frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{(\sigma_1^2/n_1)+(\sigma_2^2/n_2)}} \sim N(0,1)$

$P(\bar{x}_1-\bar{x}_2 > ■) \rightarrow P(z > \frac{■ - (\mu_1-\mu_2)}{\sqrt{(\sigma_1^2/n_1)+(\sigma_2^2/n_2)}})$

SAMPLING DIST. of $\frac{(n-1)S^2}{\sigma^2}$

s^2 กลุ่มเดียว

$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \sum_{i=1}^{n} \frac{(X_i-\bar{X})^2}{\sigma^2}$

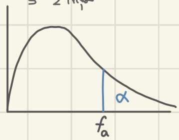
df = n-1

Note: popu เป็น Normal dist

$P(s^2 > ■) \rightarrow P(\chi^2_{n-1} > \frac{(n-1)■}{\sigma^2})$

F-DIST

s^2 2 กลุ่ม



$f_{1-\alpha}(\nu_1,\nu_2) = \frac{1}{f_\alpha(\nu_2,\nu_1)}$

$F = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$; $\nu_1 = n_1-1$, $\nu_2 = n_2-2$

t-DIST

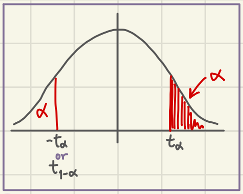
คล้าย Z // ถ้า n ≥ 30 ใช้ Z ได้เลย

ไม่รู้ σ and n<30

$T = \frac{Z}{\sqrt{V/\nu}}$ (V: χ^2)

$E(T) = 0$

$Var(T) = \frac{\nu}{(\nu-2)}$; $\nu \geq 3$



if pop is Normal dist

or $n \geq 30$

use $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

if don't know σ^2

$n \geq 30$ $Z = \frac{\bar{X}-\mu}{S/\sqrt{n}}$

$n < 30$ $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$; $\nu = n-1$

$t_{1-\alpha} = -t_\alpha$

CH.6 Estimation *of parameter* Main Idea: เอาค่า Sample มาประมาณค่าของ Population จริงๆ

Point Estimation

Let $\hat{\Theta}$ be statistic (ค่าที่คิดมาจาก Sample) $\bar{x}$, s^2

for estimating parameter θ (ค่าที่คิดมาจาก Population) μ, σ^2

Good estimator requires to be

1. unbiased

   if $E(\hat{\Theta}) == \theta$ = unbias

   $E(\bar{x}) == \mu$ = unbias

2. Variance ต่ำ ที่สุดที่เป็นไปได้

   Cramer-Rao Inequality

   - give the lowest possible value of Variance

$$Var(\hat{\Theta}) \geq \frac{1}{n \cdot E\left[\frac{d}{d\theta} \ln(f(x;\theta))\right]^2}$$

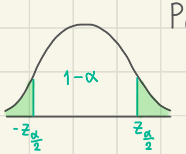
Ex. $Var(\bar{x}) \geq \frac{\sigma^2}{n}$ ← จากการคำนวณแล้ว

3. be CONSISTENT ESTIMATOR

   ☑ unbiased

   ☑ $Var(\hat{\Theta}) \to 0$ when $n \to \infty$

Interval Estimation



$$P(-z_{\frac{\alpha}{2}} < Z < z_{\frac{\alpha}{2}}) = 1-\alpha$$

where $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ we'll get

popu is Normal Dist or $n \geq 30$

$$P\left(\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

Estimating the MEAN

if σ is Known

$$\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$$

if σ is UNknown, $n \geq 30$

$$\bar{x} - z_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{e}\right)^2$$

n ที่รับประกัน $(1-\alpha)100\%$ ว่าค่า error จะไม่เกินค่า e

ถ้าเป็นทศนิยมให้ปัดเลขขึ้น

if σ is UNknown, $n < 30$

$$\bar{x} - t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}} < \mu < \bar{x} + t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$$

Estimating the diff. between Two MEANs

if σ_1, σ_2 are Known

$$(\bar{x}_1 - \bar{x}_2) - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

if σ_1, σ_2 are UNknown, $n_1, n_2 \geq 30$

$$(\bar{x}_1 - \bar{x}_2) - z_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

if σ_1, σ_2 are UNknown, n_1 or $n_2 < 30$

Case 1: $\sigma_1^2 = \sigma_2^2$

$$S_p = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)}$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\frac{\alpha}{2}} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\frac{\alpha}{2}} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\nu = n_1 + n_2 - 2$$

Case 2: $\sigma_1^2 \neq \sigma_2^2$

$$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{S_1^2}{n_1}\right)^2\left(\frac{1}{n_1-1}\right) + \left(\frac{S_2^2}{n_2}\right)^2\left(\frac{1}{n_2-1}\right)}$$

$$(\bar{x}_1 - \bar{x}_2) - t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

if 2 group are is dependent; $\bar{d} - t_{\frac{\alpha}{2}}\frac{S_d}{\sqrt{n}} < \mu_D < \bar{d} + t_{\frac{\alpha}{2}}\frac{S_d}{\sqrt{n}}$

Estimating the Proportion

$$\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$\hat{p}$ คือ อัตราส่วนของสิ่งที่สนใจ กับ จำนวนทั้งหมด ที่สุ่มมา

$\hat{q} = 1 - \hat{p}$

$$n = \frac{z_{\frac{\alpha}{2}}^2 \hat{p}\hat{q}}{e^2} \approx \frac{z_{\frac{\alpha}{2}}^2}{4e^2}$$

Estimating the diff proportion between two group

$$(\hat{p}_1 - \hat{p}_2) - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Estimating the Varaince

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}$$

Estimation the ratio between two variance

$$\frac{s_1^2}{s_2^2} \cdot \frac{1}{f_{\frac{\alpha}{2}}(\nu_1, \nu_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} f_{\frac{\alpha}{2}}(\nu_2, \nu_1)$$

CH.7 Test of Hypotheses

Test of Hypotheses on a SINGLE MEAN

Case 1: Known σ^2, normal dist or $n \geq 30$

1) $H_0 : \mu = \mu_0$
2) H_1 : a) $\mu > \mu_0$
   b) $\mu < \mu_0$
   c) $\mu \neq \mu_0$
3) Significance level: α
4) The test statistic $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$

   Critical region: a) $Z > Z_\alpha$
   b) $Z < -Z_\alpha$
   c) $Z < -Z_{\frac{\alpha}{2}}$ and $Z > Z_{\frac{\alpha}{2}}$
5) Compute $Z_0 = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ from the sample data
6) Accept H_1 if Z_0 falls in critical region.
   If not Accept H_0

Case 2: unknown σ^2, $n \geq 30$

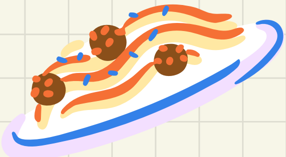
1) $H_0 : \mu = \mu_0$
2) H_1 : a) $\mu > \mu_0$
   b) $\mu < \mu_0$
   c) $\mu \neq \mu_0$
3) Significance level: α
4) The test statistic $Z = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim N(0,1)$

   Critical region: a) $Z > Z_\alpha$
   b) $Z < -Z_\alpha$
   c) $Z < -Z_{\frac{\alpha}{2}}$ and $Z > Z_{\frac{\alpha}{2}}$
5) Compute $Z_0 = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ from the sample data
6) Accept H_1 if Z_0 falls in critical region.
   If not Accept H_0

Case 3: unknown σ^2, $n < 30$, normal dist.

1) $H_0 : \mu = \mu_0$
2) H_1 : a) $\mu > \mu_0$
   b) $\mu < \mu_0$
   c) $\mu \neq \mu_0$
3) Significance level: α
4) The test statistic $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$, $\nu = n-1$

   Critical region: a) $t > t_\alpha$
   b) $t < -t_\alpha$
   c) $t < -t_{\frac{\alpha}{2}}$ and $t > t_{\frac{\alpha}{2}}$
5) Compute $t_0 = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ from the sample data
6) Accept H_1 if t_0 falls in critical region.
   If not Accept H_0



Tests of Hypotheses on TWO MEAN

case 1: known σ_1^2, σ_2^2, normal dist or $n_1, n_2 \geq 30$

1) $H_0 : \mu_1 - \mu_2 = d_0$
2) H_1 : a) $\mu_1 - \mu_2 > d_0$
   b) $\mu_1 - \mu_2 < d_0$
   c) $\mu_1 - \mu_2 \neq d_0$
3) Significance level: α
4) The test statistic $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim n(z;0,1)$

   Critical region: a) $z > z_\alpha$
   b) $z < -z_\alpha$
   c) $z < -z_{\frac{\alpha}{2}}$ and $z > z_{\frac{\alpha}{2}}$
5) Compute $z_0 = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ from the sample data
6) Accept H_1 if z_0 falls in critical region.
   If not Accept H_0

case 2: unknown σ_1^2, σ_2^2, $n_1, n_2 \geq 30$

1) $H_0 : \mu_1 - \mu_2 = d_0$
2) H_1 : a) $\mu_1 - \mu_2 > d_0$
   b) $\mu_1 - \mu_2 < d_0$
   c) $\mu_1 - \mu_2 \neq d_0$
3) Significance level: α
4) The test statistic $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim n(z;0,1)$

   Critical region: a) $z > z_\alpha$
   b) $z < -z_\alpha$
   c) $z < -z_{\frac{\alpha}{2}}$ and $z > z_{\frac{\alpha}{2}}$
5) Compute $z_0 = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$ from the sample data
6) Accept H_1 if z_0 falls in critical region.
   If not Accept H_0

case 3: unknown σ_1^2, σ_2^2, $n_1, n_2 < 30$ // $\sigma_1^2 = \sigma_2^2$

1) $H_0 : \mu_1 - \mu_2 = d_0$
2) H_1 : a) $\mu_1 - \mu_2 > d_0$
   b) $\mu_1 - \mu_2 < d_0$
   c) $\mu_1 - \mu_2 \neq d_0$
3) Significance level: α
4) The test statistic $T = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$; $\nu = n_1 + n_2 - 2$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

   Critical region: a) $t > t_\alpha$
   b) $t < -t_\alpha$
   c) $t < -t_{\frac{\alpha}{2}}$ and $t > t_{\frac{\alpha}{2}}$
5) Compute $t_0 = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ from the sample data
6) Accept H_1 if z_0 falls in critical region.
   If not Accept H_0

case 4: unknown σ_1^2, σ_2^2, $n_1, n_2 < 30$ // $\sigma_1^2 \neq \sigma_2^2$

1) $H_0 : \mu_1 - \mu_2 = d_0$
2) H_1 : a) $\mu_1 - \mu_2 > d_0$
   b) $\mu_1 - \mu_2 < d_0$
   c) $\mu_1 - \mu_2 \neq d_0$
3) Significance level: α
4) The test statistic $T = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$

$$\boxed{S^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}}$$

$$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\left(\frac{S_1^2}{n_1}\right)^2\left(\frac{1}{n_1 - 1}\right) + \left(\frac{S_2^2}{n_2}\right)^2\left(\frac{1}{n_2 - 1}\right)}$$

   Critical region: a) $t > t_\alpha$
   b) $t < -t_\alpha$
   c) $t < -t_{\frac{\alpha}{2}}$ and $t > t_{\frac{\alpha}{2}}$
5) Compute $t_0 = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$ from the sample data
6) Accept H_1 if z_0 falls in critical region.
   If not Accept H_0

case 5: Paired Observation

1) $H_0 : \mu_D = d_0$
2) H_1 : a) $\mu_D > d_0$
   b) $\mu_D < d_0$
   c) $\mu_D \neq d_0$
3) Significance level : α
4) The test statistic is $T = \dfrac{\bar{D} - d_0}{S_D/\sqrt{n}}$; $\nu = n-1$

   critical region : a) $t > t_\alpha$
   b) $t < -t_\alpha$
   c) $t < -t_{\frac{\alpha}{2}}$ and $t > t_{\frac{\alpha}{2}}$
5) Compute $t_0 = \dfrac{\bar{d} - d_0}{S_d/\sqrt{n}}$ from the sample data.

$$\boxed{\bar{d} = \sum_{i=1}^{n} \frac{d_i}{n} \quad , \quad S_d = \sqrt{\sum_{i=1}^{n} \frac{(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} d_i^2 - n\bar{d}^2}{n-1}}}$$

6) Accept H_1 if z_0 falls in critical region.
   If not Accept H_0

Tests of Hypotheses on a SINGLE PROPORTION

case 1: small sample size

1. to test $H_0 : p \geq p_0$
   $H_1 : p < p_0$

   use binomial dist to compute P-value.

   $P = P(X \leq x$ when $p = p_0)$

   x = # of successes in our sample

   $\therefore$ if $P \leq \alpha \rightarrow$ reject H_0

2. To test $H_0 : p \leq p_0$
   $H_1 : p < p_0$

   $P = P(X \geq x$ when $p = p_0)$

   $\therefore$ if $P \leq \alpha \rightarrow$ reject H_0

3. To test $H_0 : p = p_0$
   $H_1 : p \neq p_0$

   $P = P(X \leq x$ when $p = p_0)$ if $x < np_0$

   $P = P(X \geq x$ when $p = p_0)$ if $x > np_0$

   $\therefore$ if $P \leq \dfrac{\alpha}{2} \rightarrow$ reject H_0

Steps :

1) $H_0 : p = p_0$
2) $H_1 : p < p_0$
   $p > p_0$
   $p \neq p_0$
3) Significance level: α
4) Test statistic : Binomial variable X with $p = p_0$
5) Find x, the number of successes, and compute the appropriate P-value
6) Draw appropriate conclusions based on the P-valve.

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}$$

case 2: Large sample size

1) $H_0 : p = p_0$
2) H_1 : a) $p > p_0$
   b) $p < p_0$
   c) $p \neq p_0$
3) Significance level : α
4) The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{X - np_0}{\sqrt{np_0 q_0}}$$

   critical region a) $Z > Z_\alpha$
   b) $Z < -Z_\alpha$
   c) $Z < -Z_{\frac{\alpha}{2}}$ and $Z > Z_{\frac{\alpha}{2}}$

5) Compute $Z_0 = \dfrac{x - np_0}{\sqrt{np_0 q_0}}$

6) Accept H_1 if z_0 falls in critical region
   If not Accept H_0

Tests of hypotheses of TWO PROPORTION

case 1 : $p_1 = p_2$

1) $H_0: p_1 - p_2 = 0$
2) H_1: a) $p_1 - p_2 > 0$
   b) $p_1 - p_2 < 0$
   c) $p_1 - p_2 \neq 0$
3) Significance level : α
4) The test statistic: $Z = \dfrac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}, \quad \hat{q} = 1 - \hat{p}$$

critical region : a) $Z > Z_\alpha$
b) $Z < -Z_\alpha$
c) $Z < -Z_{\frac{\alpha}{2}}$ and $Z > Z_{\frac{\alpha}{2}}$

$\hat{p}_1 = \frac{x_1}{n_1}$

5) Compute $Z_0 = \dfrac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

6) Accept H_1 if Z_0 falls in critical region
If not accept H_0

case 2 : $p_1 \neq p_2$

1) $H_0: p_1 - p_2 = d_0$
2) H_1: a) $p_1 - p_2 > d_0$
   b) $p_1 - p_2 < d_0$
   c) $p_1 - p_2 \neq d_0$
3) Significance level : α
4) The test statistic: $Z = \dfrac{(\hat{p}_1 - \hat{p}_2) - d_0}{\sqrt{\left(\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}\right)}}$

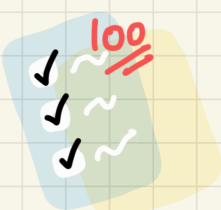
$$\hat{p}_1 = \frac{x_1}{n_1}, \quad \hat{q}_1 = 1 - \hat{p}_1$$

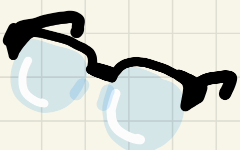
$$\hat{p}_2 = \frac{x_2}{n_2}, \quad \hat{q}_2 = 1 - \hat{p}_2$$

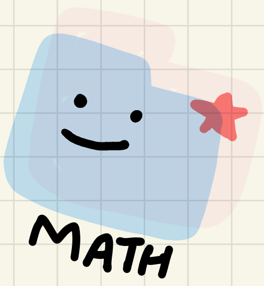
critical region : a) $Z > Z_\alpha$
b) $Z < -Z_\alpha$
c) $Z < -Z_{\frac{\alpha}{2}}$ and $Z > Z_{\frac{\alpha}{2}}$

5) Compute $Z_0 = \dfrac{(\hat{p}_1 - \hat{p}_2) - d_0}{\sqrt{\left(\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}\right)}}$

6) Accept H_1 if Z_0 falls in critical region
If not accept H_0







Tests of hypotheses on a SINGLE VARIANCE

1) $H_0: \sigma^2 = \sigma_0^2$

2) H_1: a) $\sigma^2 > \sigma_0^2$

   b) $\sigma^2 < \sigma_0^2$

   c) $\sigma^2 \neq \sigma_0^2$

3) Significance level: α

4) The test statistic is $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$; $\nu = n-1$

Critical region a) $\chi^2 > \chi^2_{\alpha}$

b) $\chi^2 < \chi^2_{1-\alpha}$

c) $\chi^2 < \chi^2_{1-\frac{\alpha}{2}}$ and $\chi^2 > \chi^2_{\frac{\alpha}{2}}$

5) Compute $\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$

6) Accept H_1 if χ_0^2 falls in critical region

If not Accept H_0

Tests of hypotheses on TWO VARIANCES

1) $H_0: \sigma_1^2 = \sigma_2^2$

2) H_1: a) $\sigma_1^2 > \sigma_2^2$

   b) $\sigma_1^2 < \sigma_2^2$

   c) $\sigma_1^2 \neq \sigma_2^2$

3) Significance level : α

4) the test statistic $F = \frac{S_1^2 \cdot \sigma_2^2}{S_2^2 \cdot \sigma_1^2} = \frac{S_1^2 \cdot \sigma^2}{S_2^2 \cdot \sigma^2} = \frac{S_1^2}{S_2^2}$

$\nu_1 = n_1 - 1$, $\nu_2 = n_2 - 1$

critical region: a) $f > f_{\alpha}(\nu_1, \nu_2)$

b) $f < f_{1-\alpha}(\nu_1, \nu_2)$

c) $f < f_{1-\frac{\alpha}{2}}(\nu_1, \nu_2)$ and $f > f_{\frac{\alpha}{2}}(\nu_1, \nu_2)$

5) compute $f_0 = \frac{S_1^2}{S_2^2}$

6) Accept H_1 if f_0 falls in critical region

If not accept H_0