Comlex Data - HW #5

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Exercise 1

1) In our case $\beta_3 + \beta_5$ represents the log ratio of odds of success (remission) for the new drug versus the standard drug at time = 1, for a particular individual. Namely:

$$\beta_3 + \beta_5 = log\left(\frac{p_{ND}(1-p_{ND})}{p_S(1-p_S)}\right)$$

   Where p_{ND} is the probability of remission on the new drug, and p_S is the probability of remission on the standard drug.
   Taking the exponent we get that $e^{\beta_3+\beta_5}$ is the odds ratio. The treatment with the new drug is more effective than the treatment with the standard drug, if that ratio is greater than 1. Since $\beta_3 = -0.05967$, and $\beta_5 = 1.01817$, we have $e^{-0.05967+1.01817} = e^{0.9585} = 2.607782$, and so we conclude that the odds of remission on the new drug are 2.607782 times higher than on the standard drug at time = 1.

2) Similarly the ratio of odds at time = 2, is equal to $e^{\beta_3+2\beta_5} = e^{-0.05967+2*1.01817} = 7.218665$. So the odds of remission on the new drug are 7.218665 higher than on the standard drug.

3) For time = 0, we have:

$$logit\{Pr(Y_{ij} = 1|b_{i1})\} = \beta_1 + \beta_2 severe_i + b_{i1}$$

   For time = 1, we have:

$$logit\{Pr(Y_{ij} = 1|b_{i1})\} = \beta_1 + \beta_2 severe_i + \beta_4 + b_{i1}$$

   For time = 2, we have:

$$logit\{Pr(Y_{ij} = 1|b_{i1})\} = \beta_1 + \beta_2 severe_i + 2\beta_4 + b_{i1}$$

Therefore for individuals with the same random effect and the same severity, the log ratio of odds is β_4. So the odds of remission are $e^{0.48274} = 1.620509$ times higher at time = 1, than at time = 0, and 1.620509 times higher at time = 2, than at time = 1.

We can also calculate the values of the logit function for all individuals, using the random effect estimates provided by the function ranef.

The table contains the logit function for each time point for groups of individuals with the same random effect and severity:

```
##           ids drug severe     time0     time1     time2
## 1      1 - 16    0      0 -0.024177  0.458563  0.941303
## 2     17 - 38    0      0 -0.027401  0.455339  0.938079
## 3     39 - 41    0      0 -0.030624  0.452116  0.934856
## 4     42 - 55    0      0 -0.027401  0.455339  0.938079
## 5     56 - 74    0      0 -0.030624  0.452116  0.934856
## 6     75 - 80    0      0 -0.033848  0.448892  0.931632
## 10 151 - 152    0      1 -1.336115 -0.853375 -0.370635
## 11 153 - 162    0      1 -1.339339 -0.856599 -0.373859
```

```
## 12 163 - 171    0       1 -1.342563 -0.859823 -0.377083
## 13 172 - 180    0       1 -1.339339 -0.856599 -0.373859
## 14 181 - 222    0       1 -1.342563 -0.859823 -0.377083
## 15 223 - 250    0       1 -1.345788 -0.863048 -0.380308
```

4) For time = 0, we have:

$$logit\{Pr(Y_{ij} = 1|b_{i1})\} = \beta_1 + \beta_2 severe_i + \beta_3 + b_{i1}$$

For time = 1, we have:

$$logit\{Pr(Y_{ij} = 1|b_{i1})\} = \beta_1 + \beta_2 severe_i + \beta_3 + \beta_4 + \beta_5 + b_{i1}$$

For time = 2, we have:

$$logit\{Pr(Y_{ij} = 1|b_{i1})\} = \beta_1 + \beta_2 severe_i + \beta_3 + 2(\beta_4 + \beta_5) + b_{i1}$$

For individuals with the same random effect and the same severity, the log ratio of odds is $\beta_4 + \beta_5$. So the odds of remission are $e^{0.48274+1.01817} = 4.485769$ times higher at time = 1, than at time = 0, and 4.485769 times higher at time = 2, than at time = 1.

Values of the logit function for groups of individuals with the same random effect and severity:

```
##          ids drug severe     time0    time1    time2
## 7   81 - 111    1      0 -0.085159 1.415751 2.916661
## 8  112 - 139    1      0 -0.088385 1.412525 2.913435
## 9  140 - 150    1      0 -0.091611 1.409299 2.910209
## 16 251 - 257    1      1 -1.397856 0.103054 1.603964
## 17 258 - 264    1      1 -1.401081 0.099829 1.600739
## 18 265 - 266    1      1 -1.404306 0.096604 1.597514
## 19 267 - 297    1      1 -1.401081 0.099829 1.600739
## 20 298 - 334    1      1 -1.404306 0.096604 1.597514
## 21 335 - 339    1      1 -1.407531 0.093379 1.594289
```

Exercise 2

Let $severe_i = severe_k = s$, and $b_{i1} = b_{k1} = b$, then:

- $time = 0$

Patient on a standard treatment: $logit\{Pr(Y_{ij} = 1)|b_{i1}\} = \beta_1 + \beta_2 s + b$

Patient on a new treatment: $logit\{Pr(Y_{kj} = 1)|b_{k1}\} = \beta_1 + \beta_2 s + \beta_3 + b$

The log odds ratio is equal to β_3. The 95% confidence interval for the odds ratio is:

$$e^{\beta_3 \pm 1.96 SE_{\beta_3}} = e^{-0.05967 \pm 1.96 * 0.22240} = (0.6092211, 1.456788)$$

- $time = 1$

The log odds ratio is equal to $\beta_3 + \beta_5$. The standard error is equal to: $\sqrt{SE^2_{\beta_3} + SE^2_{\beta_5} + 2cov(\beta_3, \beta_5)}$.

```
vcov(depress.glmer)
```

```
## 5 x 5 Matrix of class "dpoMatrix"
##              (Intercept)        severe          drug         time   drug:time
## (Intercept)  0.026917970 -0.0097407782 -0.0224107779 -0.012769015  0.01452528
## severe      -0.009740778  0.0232897759 -0.0001755429 -0.002178549 -0.00353706
## drug        -0.022410778 -0.0001755429  0.0494598077  0.013477141 -0.03161830
## time        -0.012769015 -0.0021785490  0.0134771412  0.013377365 -0.01245581
## drug:time    0.014525280 -0.0035370599 -0.0316182968 -0.012455814  0.03667169
```

The 95% confidence interval for the odds ratio:

$$e^{\beta_3+\beta_5 \pm 1.96 SE_{\beta_3+\beta_5}} = e^{-0.05967+1.01817 \pm 1.96 * \sqrt{(-0.05967)^2+1.01817^2-2*0.03162}} = (0.3757544, 18.09833)$$

- time = 2

The log odds ratio is equal to $\beta_3 + 2\beta_5$. The standard error is equal to: $\sqrt{SE_{\beta_3}^2 + 4SE_{\beta_5}^2 + 4cov(\beta_3, \beta_5)}$.

The 95% confidence interval for the odds ratio:

$$e^{\beta_3+2\beta_5 \pm 1.96 SE_{\beta_3+2\beta_5}} = e^{-0.05967+2*1.01817 \pm 1.96 * \sqrt{(-0.05967)^2+4*1.01817^2-4*0.03162}} = (0.1415707, 368.0783)$$