HW2 Complex Data

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Exercise 1

1. Page 3

- $\mathbf{l} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$.
- $H_0 : \beta_3 - \beta_4 = 0$, $\mathbf{l} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$.

2. Page 9

- $r = 3$.

3. Page 10

- REML eliminates the regression parameters from the likelihood.

4. Page 15

```
pchisq(3*1.159, df = 3, lower.tail = FALSE)
```

```
## [1] 0.3237574
```

- Multivariate Wald test is used. P-value > 0.5. We cannot reject the null hypothesis for this significance level

5. Page 16

- The week effect is statistically significant, whereas the diet effect is not.

Exercise 2

```
moo.all.cat <- gls(protein ~ factor(diet)*factor(week),
                   correlation = corSymm(form = ~1 | id),
                   weights = varIdent(form = ~1 | week),
                   data = moo.all.uni, na.action = na.omit)
```

```
summary(moo.all.cat)
```

```
## Generalized least squares fit by REML
##   Model: protein ~ factor(diet) * factor(week)
##   Data: moo.all.uni
##        AIC     BIC    logLik
##   2880.931 2940.35 -1424.465
##
## Unitialized correlation structure of class corSymm
## Variance function:
##  Structure: Different standard deviations per stratum
##  Formula: ~1 | week
##  Parameter estimates:
##         1         2         3         4
```

```
## 1.0000000 0.7757842 0.7217998 0.6925371
##
## Coefficients:
##                                            Value Std.Error   t-value
## (Intercept)                             89.52000  6.327496 14.147778
## factor(diet)barley+lupins               -1.96444  8.781155 -0.223711
## factor(diet)lupins                      -9.59407  8.781155 -1.092575
## factor(week)2                          -17.02000  8.070764 -2.108846
## factor(week)3                          -30.52000  7.803613 -3.911009
## factor(week)4                          -41.68000  7.696710 -5.415301
## factor(diet)barley+lupins:factor(week)2  -8.38741 11.158847 -0.751637
## factor(diet)lupins:factor(week)2        -10.35037 11.158847 -0.927548
## factor(diet)barley+lupins:factor(week)3 -11.51704 10.829677 -1.063470
## factor(diet)lupins:factor(week)3         -0.96148 10.829677 -0.088782
## factor(diet)barley+lupins:factor(week)4  -6.43111 10.681320 -0.602090
## factor(diet)lupins:factor(week)4          2.75407 10.681320  0.257840
##                                         p-value
## (Intercept)                              0.0000
## factor(diet)barley+lupins                0.8231
## factor(diet)lupins                       0.2754
## factor(week)2                            0.0358
## factor(week)3                            0.0001
## factor(week)4                            0.0000
## factor(diet)barley+lupins:factor(week)2  0.4529
## factor(diet)lupins:factor(week)2         0.3544
## factor(diet)barley+lupins:factor(week)3  0.2884
## factor(diet)lupins:factor(week)3         0.9293
## factor(diet)barley+lupins:factor(week)4  0.5476
## factor(diet)lupins:factor(week)4         0.7967
##
##  Correlation:
##                                         (Intr) fct()+ fctr() fct()2 fct()3
## factor(diet)barley+lupins               -0.721
## factor(diet)lupins                      -0.721  0.519
## factor(week)2                           -0.784  0.565  0.565
## factor(week)3                           -0.811  0.584  0.584  0.636
## factor(week)4                           -0.822  0.592  0.592  0.645  0.667
## factor(diet)barley+lupins:factor(week)2  0.567 -0.787 -0.409 -0.723 -0.460
## factor(diet)lupins:factor(week)2         0.567 -0.409 -0.787 -0.723 -0.460
## factor(diet)barley+lupins:factor(week)3  0.584 -0.811 -0.421 -0.458 -0.721
## factor(diet)lupins:factor(week)3         0.584 -0.421 -0.811 -0.458 -0.721
## factor(diet)barley+lupins:factor(week)4  0.592 -0.822 -0.427 -0.464 -0.480
## factor(diet)lupins:factor(week)4         0.592 -0.427 -0.822 -0.464 -0.480
##                                         fct()4 f()+:()2 f():()2 f()+:()3
## factor(diet)barley+lupins
## factor(diet)lupins
## factor(week)2
## factor(week)3
## factor(week)4
## factor(diet)barley+lupins:factor(week)2 -0.466
## factor(diet)lupins:factor(week)2        -0.466  0.523
## factor(diet)barley+lupins:factor(week)3 -0.480  0.638    0.331
## factor(diet)lupins:factor(week)3        -0.480  0.331    0.638   0.519
## factor(diet)barley+lupins:factor(week)4 -0.721  0.647    0.336   0.667
```

```
## factor(diet)lupins:factor(week)4        -0.721  0.336    0.647   0.346
##                                         f():()3 f()+:()4
## factor(diet)barley+lupins
## factor(diet)lupins
## factor(week)2
## factor(week)3
## factor(week)4
## factor(diet)barley+lupins:factor(week)2
## factor(diet)lupins:factor(week)2
## factor(diet)barley+lupins:factor(week)3
## factor(diet)lupins:factor(week)3
## factor(diet)barley+lupins:factor(week)4  0.346
## factor(diet)lupins:factor(week)4         0.667   0.519
##
## Standardized residuals:
##         Min          Q1         Med          Q3         Max
## -2.49469716 -0.73173931  0.05294355  0.71035843  2.78410055
##
## Residual standard error: 31.63748
## Degrees of freedom: 315 total; 303 residual
```

a)

$X_{1ij} = 1$ for all measurements
$X_{2ij} = 1$ if the jth measurement was taken at time = 2 weeks, 0 otherwise
$X_{3ij} = 1$ if the jth measurement was taken at time = 3 weeks, 0 otherwise
$X_{4ij} = 1$ if the jth measurement was taken at time = 4 weeks, 0 otherwise
$X_{5ij} = 1$ if the ith cow ate barley+lupins, 0 if it ate lupins or barley
$X_{6ij} = 1$ if the ith cow ate lupins, 0 if it ate barley or barley+lupins
$X_{7ij} = 1$ if the ith cow ate barley+lupins & its jth measurement is at time = 2 weeks, 0 otherwise
$X_{8ij} = 1$ if the ith cow ate lupins & its jth measurement is at time = 2 weeks, 0 otherwise
$X_{9ij} = 1$ if the ith cow ate barley+lupins & its jth measurement is at time = 3 weeks, 0 otherwise
$X_{10ij} = 1$ if the ith cow ate lupins & its jth measurement is at time = 3 weeks, 0 otherwise
$X_{11ij} = 1$ if the ith cow ate barley+lupins & its jth measurement is at time = 4 weeks, 0 otherwise
$X_{12ij} = 1$ if the ith cow ate lupins & its jth measurement is at time = 4 weeks, 0 otherwise Model:

$$Y_{ij} = \beta_1+\beta_2 X_{2ij}+\beta_3 X_{3ij}+\beta_4 X_{4ij}+\beta_5 X_{5ij}+\beta_6 X_{6ij}+\beta_7 X_{7ij}+\beta_8 X_{8ij}+\beta_9 X_{9ij}+\beta_{10} X_{10ij}+\beta_{11} X_{11ij}+\beta_{12} X_{12ij}+\epsilon_{ij}$$

$\mu_{b1} = \beta_1$
$\mu_{b2} = \beta_1 + \beta_2$
$\mu_{b3} = \beta_1 + \beta_3$
$\mu_{b4} = \beta_1 + \beta_4$
$\mu_{l1} = \beta_1 + \beta_6$
$\mu_{l2} = \beta_1 + \beta_2 + \beta_6 + \beta_8$
$\mu_{l3} = \beta_1 + \beta_3 + \beta_6 + \beta_{10}$
$\mu_{l4} = \beta_1 + \beta_4 + \beta_6 + \beta_{12}$
$\mu_{lb1} = \beta_1 + \beta_5$
$\mu_{lb2} = \beta_1 + \beta_2 + \beta_5 + \beta_7$
$\mu_{lb3} = \beta_1 + \beta_3 + \beta_5 + \beta_9$
$\mu_{lb4} = \beta_1 + \beta_4 + \beta_5 + \beta_{11}$

b)
$H_0 : \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = \beta_{12} = 0$, H_1 : at least one $\neq 0$.

```
anova(moo.all.cat)
```

```
## Denom. DF: 303
##                            numDF   F-value p-value
## (Intercept)                    1 1658.4799  <.0001
## factor(diet)                   2    6.3256  0.0020
## factor(week)                   3   35.7751  <.0001
## factor(diet):factor(week)      6    0.6744  0.6704
```

P-value $= 0.6704 > 0.05$, so for significance level 0.05 we cannot reject the null hypothesis.

c) Hypothesis for the diet effect:
   $H_0^1 : \beta_5 = \beta_6 = 0$, H_1^1 : at least one $\neq 0$
   Hypothesis for the week effect:
   $H_0^2 : \beta_2 = \beta_3 = \beta_4 = 0$, H_1^2 : at least one $\neq 0$

```
moo.all.cat.main <- gls(protein ~ factor(diet) + factor(week),
                        correlation = corSymm(form = ~1 | id),
                        weights = varIdent(form = ~1 | week),
                        data = moo.all.uni, na.action = na.omit)
```

```
anova(moo.all.cat.main)
```

```
## Denom. DF: 309
##              numDF   F-value p-value
## (Intercept)      1 1668.2783  <.0001
## factor(diet)     2    6.3402   0.002
## factor(week)     3   36.1463  <.0001
```

The week factor is statistically significant, but the diet factor is not. Therefore we cannot reject the diet null hypothesis.