Image Processing
lab 4

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Exercise 1 – Skeletons

The morphological skeleton of an image X with structuring element B is defined as:

$$SK(X) = \bigcup_{k=0}^{K} S_k(X)$$

where

$$S_k(X) = (X \ominus kB) - (X \ominus kB) \circ B$$

and $K \in \mathbb{N}$ such that

$$K = max\{k|(X \ominus kB) \neq \emptyset\}.$$

In addition $(X \ominus kB)$ is defined as follows:

$$(X \ominus kB) = \begin{cases} X & \text{if } k = 0 \\ ((\dots(X \ominus B) \ominus B) \ominus \dots) \ominus B) & \text{otherwise} \end{cases}$$

Note that in the second case $\ominus B$ is applied k times to X.

a. Let set $B = \hat{B} = \{0\}$ be given.

Claim. $SK(X) = \emptyset$

Proof. Note that for all $z \in \mathbb{Z}^2$ the following holds:

$$(B)_z = \{c|c = b + z, b \in B\} = \{c|c = z\} = \{z\}$$

Then we have by the definition of the erosion of X by B that

$$\begin{aligned} X \ominus B &= \{z|(B)_z \subseteq X\} \\ &= \{z|\{z\} \subseteq X\} \\ &= X. \end{aligned} \tag{1}$$

Similarly we have by the definition of the dilution of X by B that

$$\begin{aligned} X \oplus B &= \{z|(\hat{B})_z \cap X \neq \emptyset\} \\ &= \{z|\{z\} \cap X \neq \emptyset\} \\ &= X. \end{aligned} \tag{2}$$

The skeleton of image X with structuring element B is:

$$\begin{aligned}
SK(X) &= \bigcup_{k=0}^{K} S_k(X) \\
&= \bigcup_{k=0}^{K} ((X \ominus kB) - (X \ominus kB) \circ B) \\
&= \bigcup_{k=0}^{K} ((X \ominus kB) - ((X \ominus kB) \ominus B) \oplus B)
\end{aligned}$$

Now if we apply the definition of $(X \ominus kB)$ and the results of equation 1 and 2 we get:

$$\begin{aligned}
SK(X) &= \bigcup_{k=0}^{K} ((X \ominus kB) - ((X \ominus kB) \ominus B) \oplus B) \\
&= \bigcup_{k=0}^{K} (X - X) \\
&= \bigcup_{k=0}^{K} \emptyset \\
&= \emptyset
\end{aligned}$$

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b. Let $X \ominus B = \emptyset$ be given.

Claim. *SK(X) = X*

Proof. Consider the definition of $(X \ominus kB)$:

$$(X \ominus kB) = \begin{cases} X & \text{if } k = 0 \\ ((\dots(X \ominus B) \ominus B) \ominus \dots) \ominus B) & \text{otherwise} \end{cases}$$

Note that $X \ominus B = \emptyset$. In addition for $X = \emptyset$ we have that every other erosion in the definition above is

$$X \ominus B = \emptyset \ominus B = \{z | (B)_z \subseteq \emptyset\} = \emptyset. \tag{3}$$

Hence the previous definition $X \ominus kB$ is equal to:

$$(X \ominus kB) = \begin{cases} X & \text{if } k = 0 \\ \emptyset & \text{otherwise} \end{cases} \tag{4}$$

Now the skeleton of image X with structuring element B is:

$$\begin{aligned}
SK(X) &= \bigcup_{k=0}^{K} S_k(X) \\
&= \bigcup_{k=0}^{K} ((X \ominus kB) - (X \ominus kB) \circ B) \\
&= \bigcup_{k=0}^{K} ((X \ominus kB) - ((X \ominus kB) \ominus B) \oplus B) \\
&= ((X \ominus 0B) - ((X \ominus 0B) \ominus B) \oplus B) \cup \bigcup_{k=1}^{K} ((X \ominus kB) - ((X \ominus kB) \ominus B) \oplus B)
\end{aligned}$$

If we use the equation 3, 4 and the definition of $X \oplus B$ we get:

$$\begin{aligned}
SK(X) &= (X - (X \ominus B) \oplus B) \cup \bigcup_{k=1}^{K} (\emptyset - (\emptyset \ominus B) \oplus B) \\
&= (X - (\emptyset \oplus B)) \cup \bigcup_{k=1}^{K} (\emptyset - (\emptyset \oplus B)) \\
&= \left(X - \{z | (\hat{B})_z \cap \emptyset \neq \emptyset\}\right) \cup \bigcup_{k=1}^{K} \left(\emptyset - \{z | (\hat{B})_z \cap \emptyset \neq \emptyset\}\right) \\
&= (X - \{z | \emptyset \neq \emptyset\}) \cup \bigcup_{k=1}^{K} (\emptyset - \{z | \emptyset \neq \emptyset\}) \\
&= (X - \emptyset) \cup \bigcup_{k=1}^{K} (\emptyset - \emptyset) \\
&= X \cup \bigcup_{k=1}^{K} \emptyset \\
&= X \cup \emptyset \\
&= X
\end{aligned}$$

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Exercise 2 – Grey-scale morphology

a.

b.

Exercise 3 – Classification

Task distribution

ex1	design	implementation	answers questions	writing report
Klaas	60%	90%	n.a.	50%
Jan	40%	10%	n.a.	50%

ex2	design	implementation	answers questions	writing report
Klaas	50%	30%	25%	25%
Jan	50%	70%	75%	75%

ex3	design	implementation	answers questions	writing report
Klaas	50%	75%	50%	75%
Jan	50%	25%	50%	25%