Hidden Markov Models

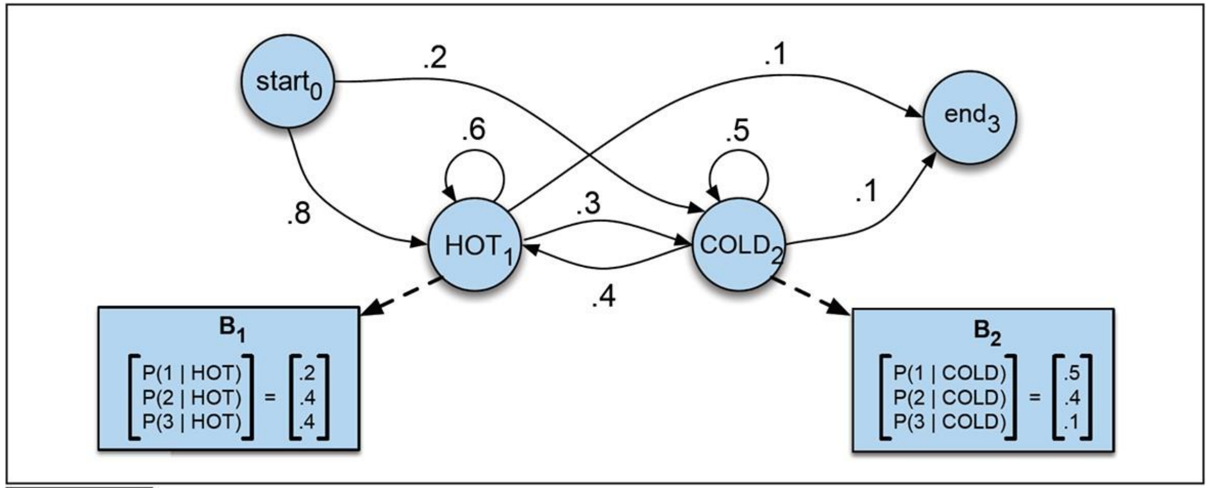
Lab 9

Exercise 1: The Forward & Viterbi Algorithm

1. Implement the Forward Algorithm for the Hidden Markov Model (shown on the next slide) to compute the probability of the observation sequence *3 1 3*.
2. Implement the Viterbi Algorithm to compute the most likely weather sequence for the observation sequence *3 1 3*.

Use the file `hidden_markov_models.py`, it contains the incomplete functions `compute_forward` and `compute_viterbi`.

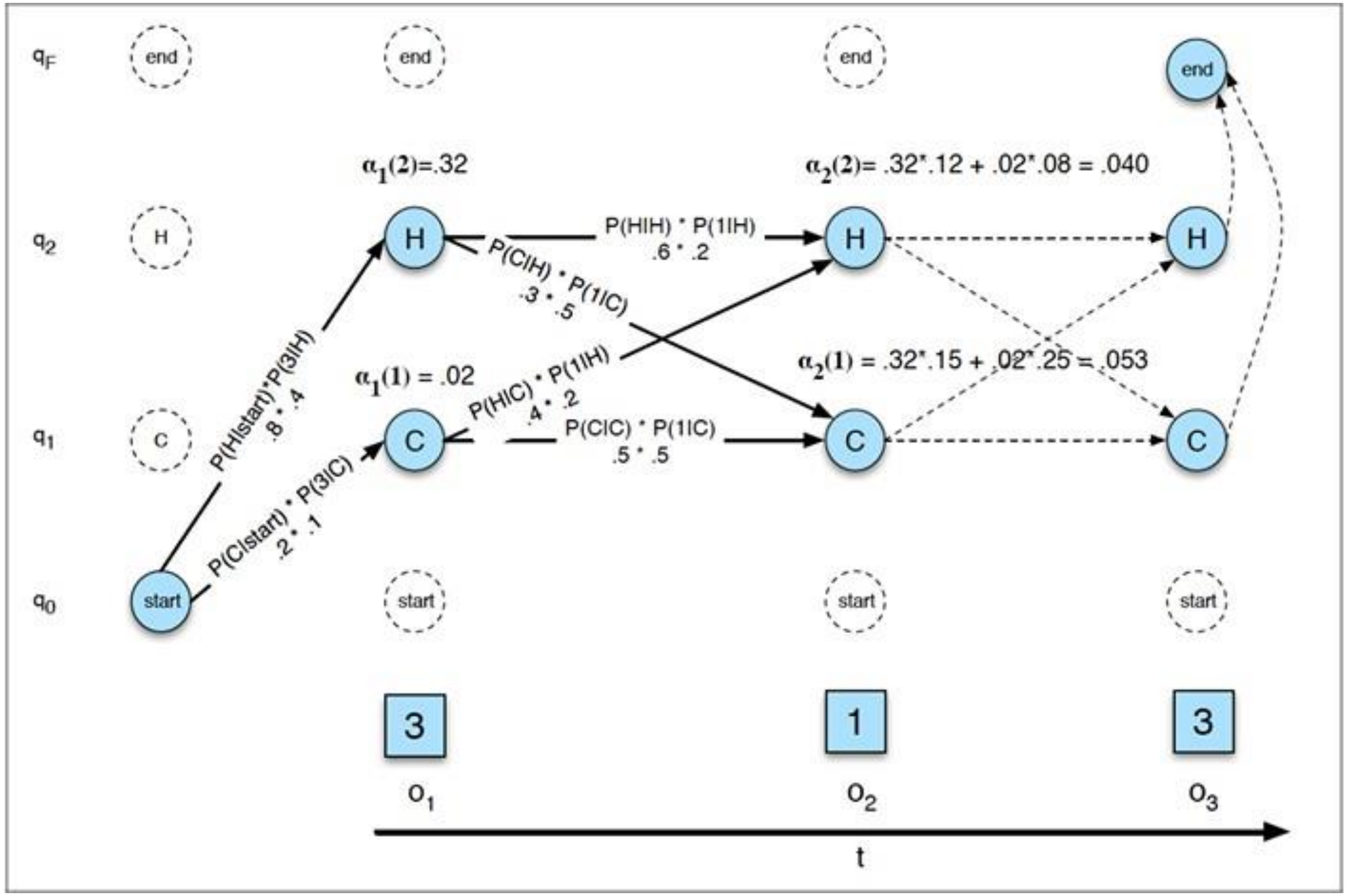
Exercise: The Forward & Viterbi Algorithm



The Hidden Markov Model above shows the number of ice creams eaten by Jason (the observations) related to the weather (HOT or COLD, the hidden variables).

Visit https://web.stanford.edu/~jurafsky/slp3/A.pdf for further details.

Visual Representation of the Forward Algorithm



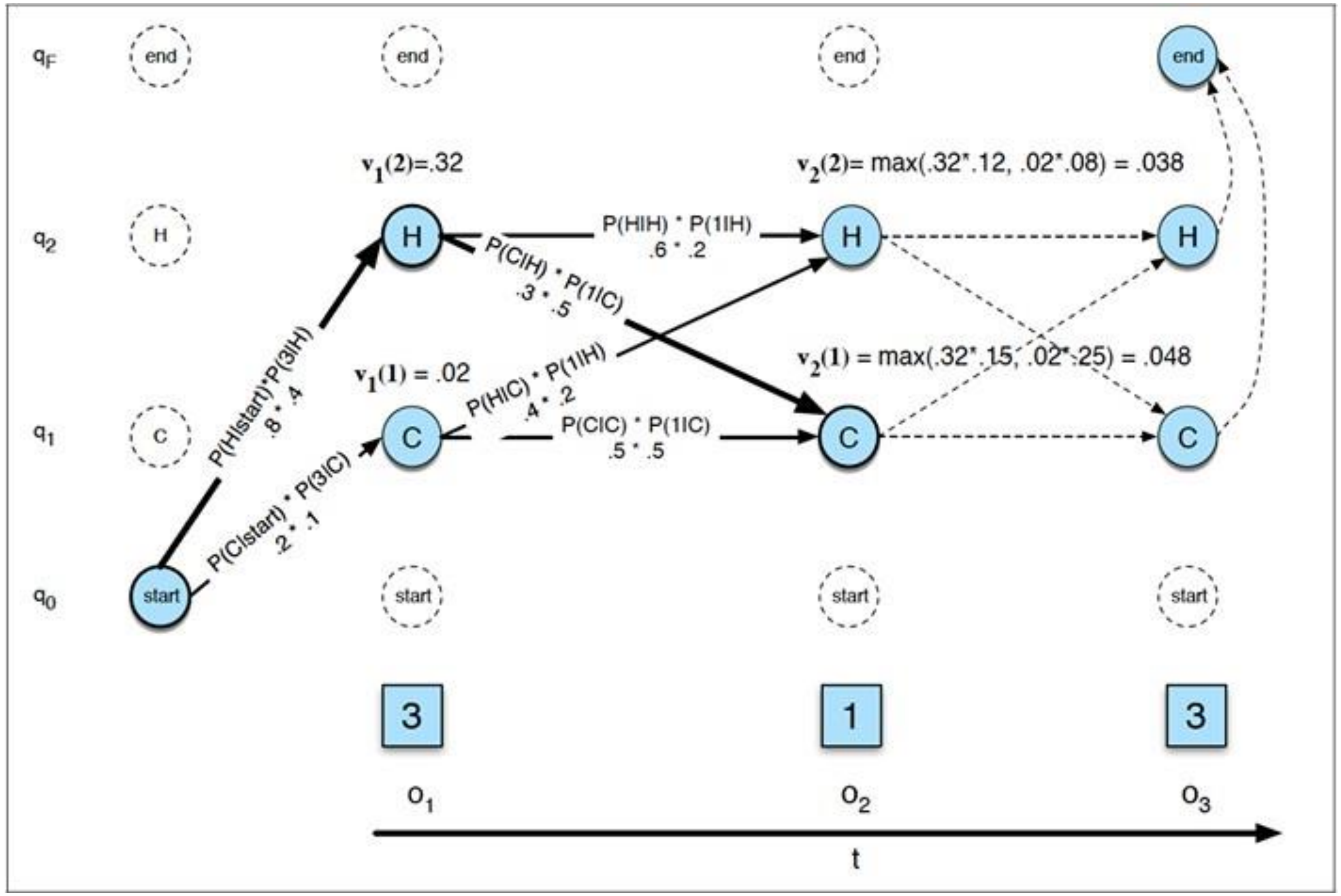
Pseudocode for the Forward Algorithm

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
$$forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$$
for each time step t **from** 2 **to** T **do** ; recursion step
for each state s **from** 1 **to** N **do**
$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$
$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step
return $forward[q_F,T]$

Note that in the code, the transition matrix corresponds to a, whereas the emissions matrix corresponds to b.

Visual Representation of the Viterbi Algorithm



Pseudocode for the Viterbi Algorithm

function VITERBI(*observations* of len T, *state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
  $viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$
  $backpointer[s,1] \leftarrow 0$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**
    $viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$
    $backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$
$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$; termination step
$backpointer[q_F,T] \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$; termination step
return the backtrace path by following backpointers to states back in
  time from $backpointer[q_F,T]$

Note that in the code, the transition matrix corresponds to a, whereas the emissions matrix corresponds to b.

Exercise 2:

1) Find the probability of the following observation sequences:

- 3, 3, 1, 1, 2, 2, 3, 1, 3.
- 3, 3, 1, 1, 2, 3, 3, 1, 2.

2) Also find the most likely weather sequences for the two observation sequences.