Summer project : Hybrid chiral lasers

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Hybrid chiral lasers

- Laser $\rightarrow$ O.K.
- Hybrid $\rightarrow$ We will come to that later
- Chiral ?

Definition (Chirality)
An object is said to be chiral if it is distinguishable from its reflection in a mirror.

Chiral media

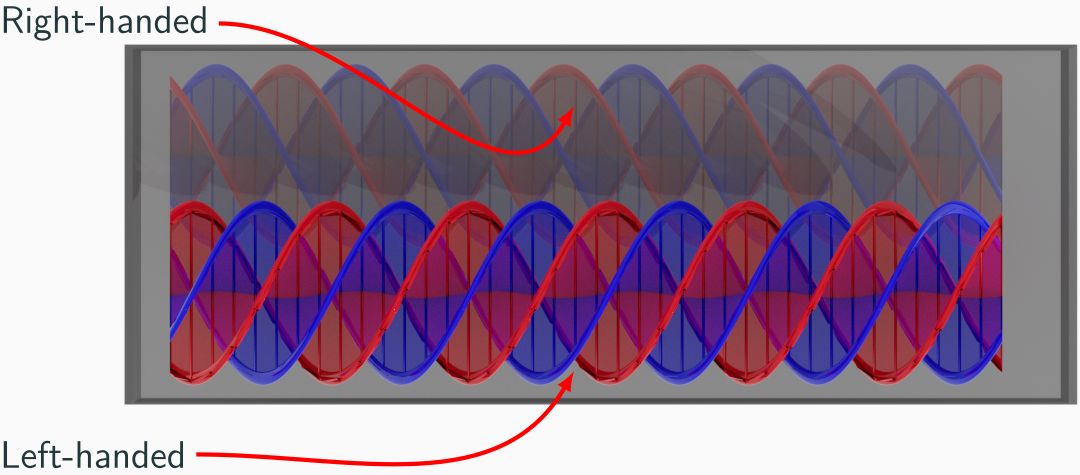


Figure 1: A double helix and its reflection in a mirror

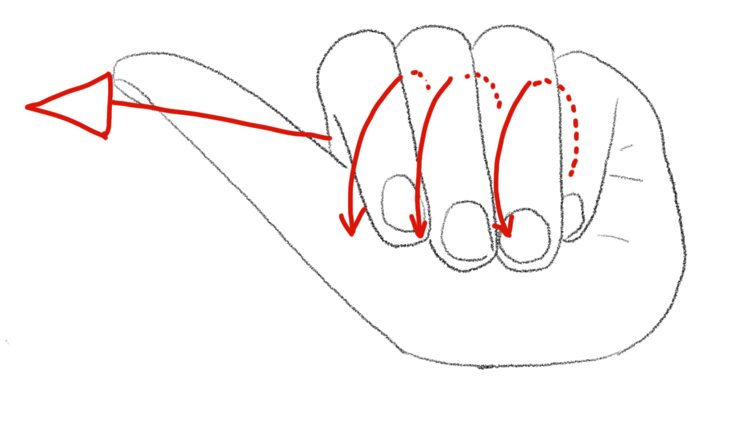


Figure 2: Quick reminder on handedness : left hand

Media studied

A small subset of chiral media

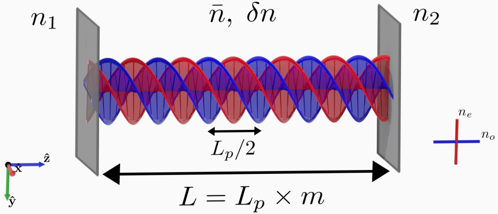


Figure 3: A slab of chiral medium

$$\epsilon = \begin{pmatrix} \boldsymbol{R}^{-1}(z) \cdot \begin{pmatrix} \epsilon_a & 0 \\ 0 & \epsilon_b \end{pmatrix} \cdot \boldsymbol{R}(z) & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & \epsilon_c \end{pmatrix}$$

$$\boldsymbol{R}(z) = \begin{pmatrix} \cos(pz+\psi) & -\sin(pz+\psi) \\ \sin(pz+\psi) & \cos(pz+\psi) \end{pmatrix}$$

Warning
The refractive index does not depend on the position in (x, y) plane.

Studying propagation of light in this medium

Studying propagation of light in this medium

Maxwell-Thomson

$$\nabla \cdot \boldsymbol{B} = 0$$

Maxwell-Gauss

$$\nabla \cdot \boldsymbol{D} = 0$$

Maxwell-Faraday

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$$

Maxwell-Ampère

$$\nabla \times \boldsymbol{H} = \frac{\partial}{\partial t}\boldsymbol{E}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{\epsilon} \boldsymbol{E} \qquad \boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Use auxiliary field $\boldsymbol{H'}$ and only study planar component of the field.

$$\boldsymbol{H'} = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \boldsymbol{H}$$

$$\begin{aligned} \hat{\boldsymbol{z}}\frac{\mathrm{d}}{\mathrm{d}z} \times \boldsymbol{E}_\perp &= i k_0 \boldsymbol{H'}_\perp \\ \hat{\boldsymbol{z}}\frac{\mathrm{d}}{\mathrm{d}z} \times \boldsymbol{H'}_\perp &= -i k_0 \boldsymbol{\epsilon}_\perp \cdot \boldsymbol{E}_\perp \end{aligned}$$

The $\perp$ sign is omitted for convenience.

Studying propagation of light in this medium

An analytic solution : the Oseen transformation

Rewrite the field from electromagnetic basis to a more convenient basis.

$$\begin{bmatrix} \boldsymbol{e}(z) \\ \boldsymbol{h}(z) \end{bmatrix} = \begin{pmatrix} \boldsymbol{R}^{-1}(z) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}^{-1}(z) \end{pmatrix} \begin{bmatrix} \boldsymbol{E}(z) \\ \boldsymbol{H}'(z) \end{bmatrix}$$

Maxwell's equation are rewritten

$$\frac{\mathrm{d}}{\mathrm{d}z} \begin{bmatrix} \boldsymbol{e}(z) \\ \boldsymbol{h}(z) \end{bmatrix} = i \underbrace{\begin{pmatrix} 0 & -ip & 0 & k_0 \\ ip & 0 & -k_0 & 0 \\ 0 & -k_0\epsilon_b & 0 & -ip \\ k_0\epsilon_a & 0 & ip & 0 \end{pmatrix}}_{=\boldsymbol{G}} \begin{bmatrix} \boldsymbol{e}(z) \\ \boldsymbol{h}(z) \end{bmatrix}$$

Studying propagation of light in this medium

And this yields a solution.

$$\begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H'} \end{bmatrix}_{z=d} = \underbrace{\begin{pmatrix} \boldsymbol{R}(d) & 0 \\ 0 & \boldsymbol{R}(d) \end{pmatrix} e^{i\boldsymbol{G}d} \begin{pmatrix} \boldsymbol{R}^{-1}(0) & 0 \\ 0 & \boldsymbol{R}^{-1}(0) \end{pmatrix}}_{=\boldsymbol{M_o}} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H'} \end{bmatrix}_{z=0}$$

Problem solved ! Or is it ?

- Not satisfactory, the transfer matrix is a black-box without any analytic expression of its coefficients
- An approximate solution allowing to grasp the underlying dynamics of the cavity is needed.

Studying propagation of light in this medium

An approximate solution : the Coupled Waves Theory (CWT)

The field is decomposed upon the circular basis as:

$$E^{\pm}_{L,R} = A^{\pm}_{L,R} e^{\pm ikz}$$

Hypothesis

- Slow varying envelope approximation;
- Neglect field component that are not phase matched.

Maxwell equations become:

$$\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{A}_{L,R} = \begin{pmatrix} 0 & i\kappa e^{-2i\varphi(z)} \\ -i\kappa e^{2i\varphi(z)} & 0 \end{pmatrix} \boldsymbol{A}_{L,R}$$

Where,

- $\kappa = \frac{k_0 \delta\epsilon}{2\bar{n}}$
- for a right handed medium, $\varphi(z) = \delta k z/2 - \psi$ and $\delta k = 2(k - p)$;
- for a left handed medium, $\varphi(z) = \delta k z/2 + \psi$ and $\delta k = 2(k + p)$.

Studying propagation of light in this medium

Solution to CWT equation for *e.g.* right-handed media

$$\begin{bmatrix} E_L^+ \\ E_R^+ \\ E_L^- \\ E_R^- \end{bmatrix}_{z=d} = \underbrace{\begin{pmatrix} e^{ikd} & 0 & 0 & 0 \\ 0 & \mathcal{P}_R^+ & 0 & \mathcal{Q}_R^+ \\ 0 & 0 & e^{-ikd} & 0 \\ 0 & \mathcal{Q}_R^- & 0 & \mathcal{P}_R^- \end{pmatrix}}_{\boldsymbol{M_{cwt}}} \begin{bmatrix} E_L^+ \\ E_R^+ \\ E_L^- \\ E_R^- \end{bmatrix}_{z=0}$$

with

$$\begin{aligned} \mathcal{P}_R^\pm &= \left[\cosh(\Delta d) \pm i\frac{\delta k}{2\Delta}\sinh(\Delta d)\right] e^{\pm ipd} \\ \mathcal{Q}_R^\pm &= \pm i\frac{\kappa}{\Delta}\sinh(\Delta d) e^{\pm i(pd+2\psi)} \end{aligned}$$

Past results with chiral media

Past results with chiral media

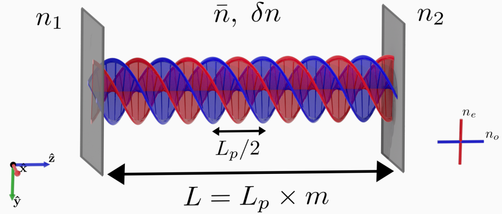


Figure 4

This kind of cavity acts like a Bragg reflector for light polarised with the same handedness as the medium.

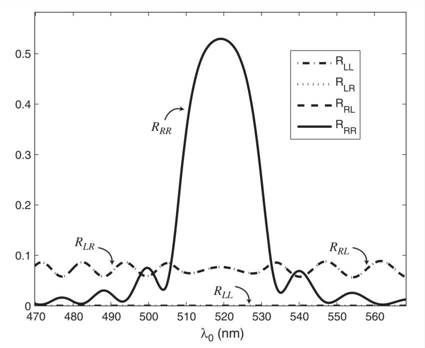
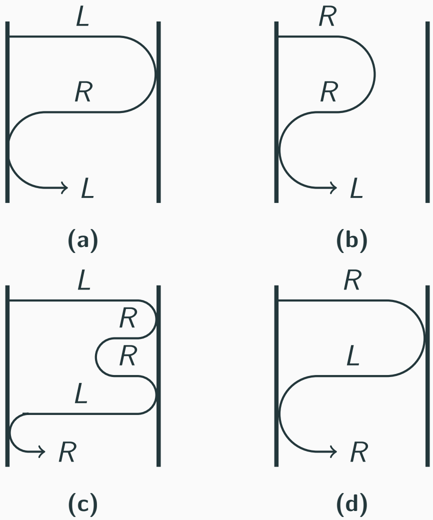


Figure 5: Reflectivity of a simulated chiral medium Martin W McCall. "Simplified theory of axial propagation through structurally chiral media". In: *Journal of Optics A: Pure and Applied Optics* 11.7 (July 1, 2009), p. 074006

Past results with chiral media

This gives complex behaviour when combined to Fresnel reflections at the interfaces. (René D. M. Topf and Martin W. McCall. “Modes of structurally chiral lasers”. In: *Physical Review A* 90.5 (Nov. 12, 2014), p. 053824)



Past results with chiral media

When pumped this creates a relatively un-pure circularly polarised light[1].

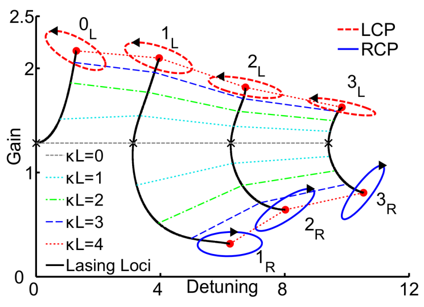


Figure 7

[1] René D. M. Topf and Martin W. McCall. "Modes of structurally chiral lasers". In: *Physical Review A* 90.5 (Nov. 12, 2014), p. 053824.

Objectives

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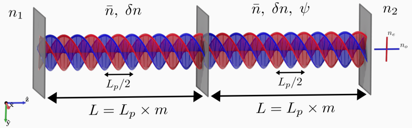
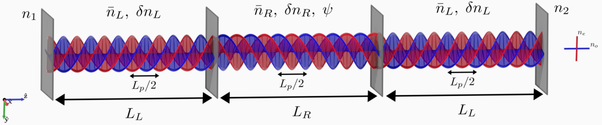
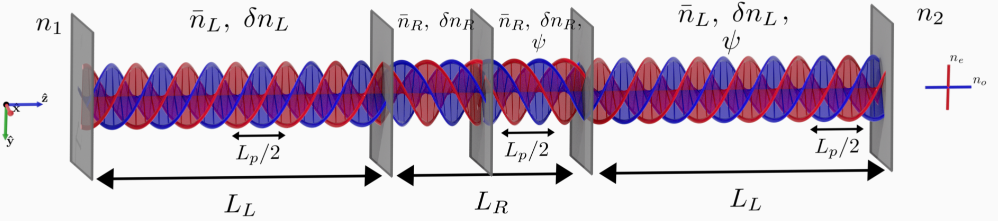


(a) Cavity with a defect



(b) Hybrid cavity



(c) Hybrid defect cavity

Figure 8

Method

Method

How to calculate reflectivities ?

Partial inverse of a matrix allows determining the reflection and transmission matrices

$$\begin{bmatrix} E_a^+ \\ E_b^+ \\ E_a^- \\ E_b^- \end{bmatrix}_1 = \begin{pmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{pmatrix} \begin{bmatrix} E_a^+ \\ E_b^+ \\ E_a^- \\ E_b^- \end{bmatrix}_0$$

As

$$\begin{pmatrix} t_{aa} & t_{ab} \\ t_{ba} & t_{bb} \end{pmatrix} = \boldsymbol{M}_{11} - \boldsymbol{M}_{12}\boldsymbol{M}_{22}^{-1}\boldsymbol{M}_{21}$$

$$\begin{pmatrix} r_{aa} & r_{ab} \\ r_{ba} & r_{bb} \end{pmatrix} = -\boldsymbol{M}_{22}^{-1}\boldsymbol{M}_{21}$$

Method

How to characterise laser action ?

For a cavity of length L.

$$\begin{bmatrix} E_a^+ \\ E_b^+ \\ 0 \\ 0 \end{bmatrix}_{L^+} = \begin{pmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ E_a^- \\ E_b^- \end{bmatrix}_{0^-}$$

That means

$$|\boldsymbol{M}_{22}| = 0$$

The output mode can be retrieved we the eigen-vector in the kernel of $\boldsymbol{M}_{22}$.

Results

Results : Defect cavity

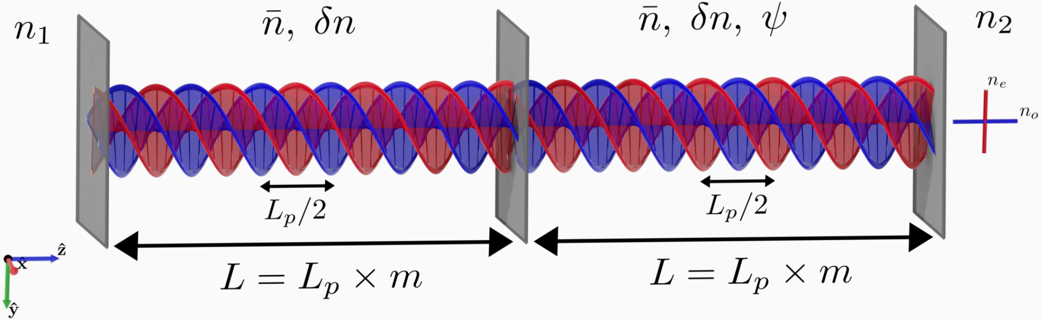
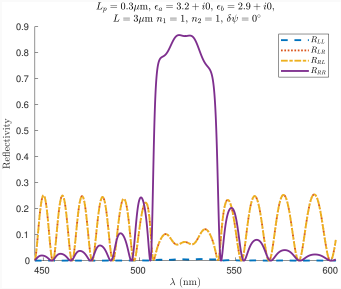
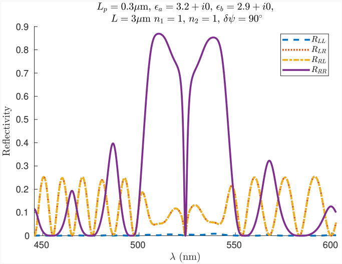


Figure 9: Cavity with a defect

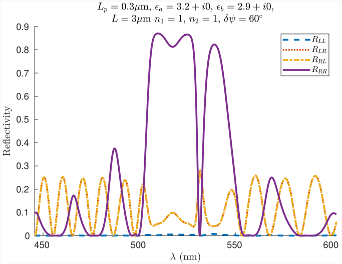
Results : Defect cavity



(a) Cavity without a defect

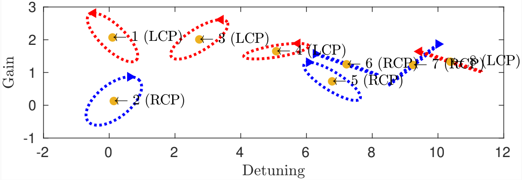


(b) Cavity with a $\frac{\pi}{2}$ defect

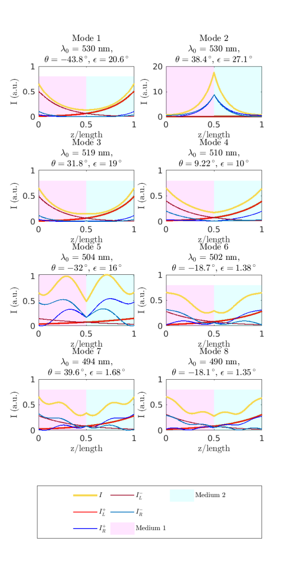


(c) Cavity with a $\frac{\pi}{3}$ defect

Results : Defect cavity



(a) Modes found for $\pi/2$ defect



Results : Defect cavity

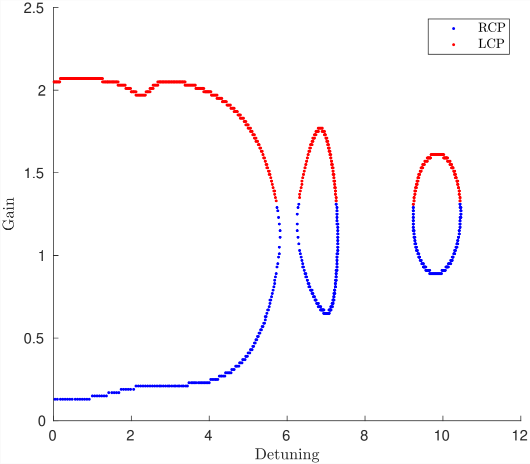


Figure 12: Tuning of the cavity

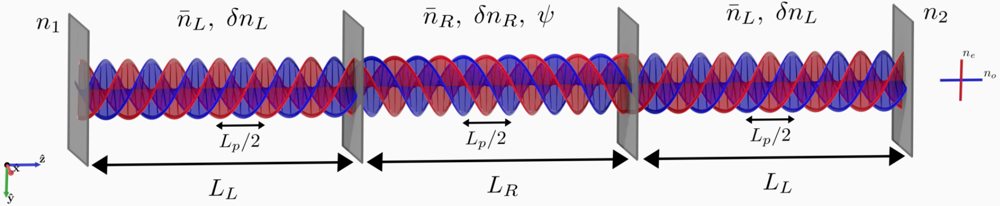
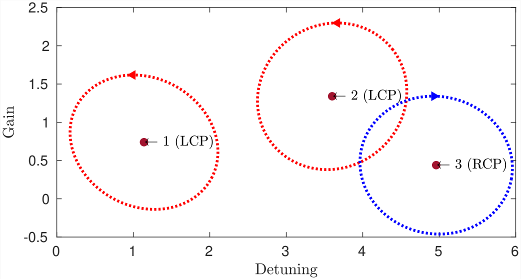
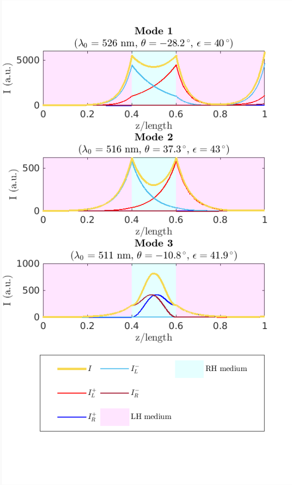


Figure 13: Hybrid cavity

Results : Hybrid cavity





Results : Hybrid defect cavity

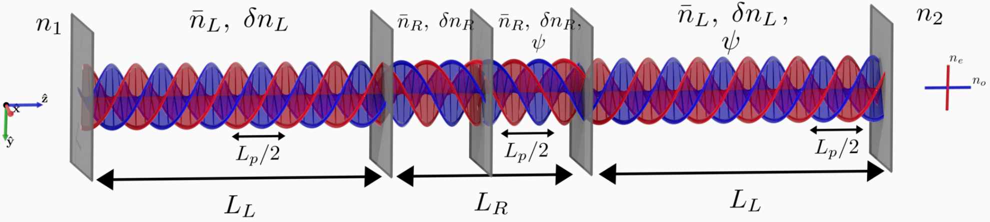
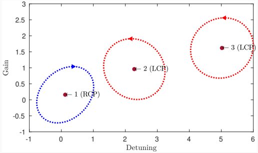
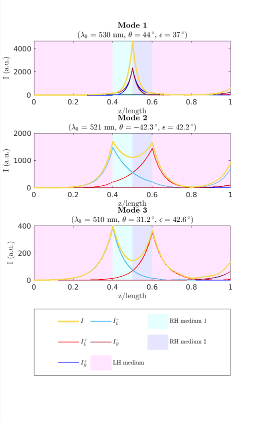


Figure 15: Hybrid defect cavity

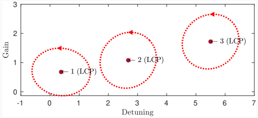
Results : Hybrid defect cavity



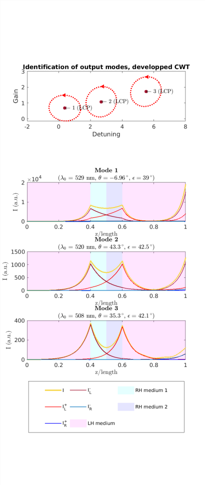
(a) $\pi/2$ defect



(b) $\pi/2$ defect

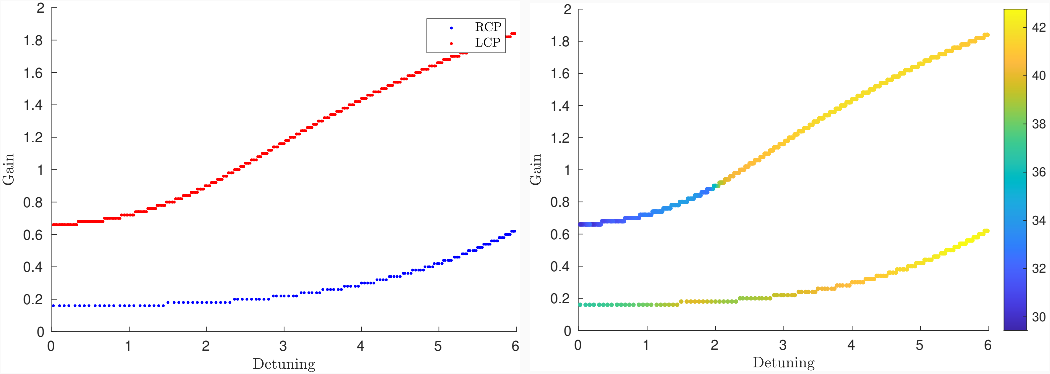


(c) $\pi/3$ defect

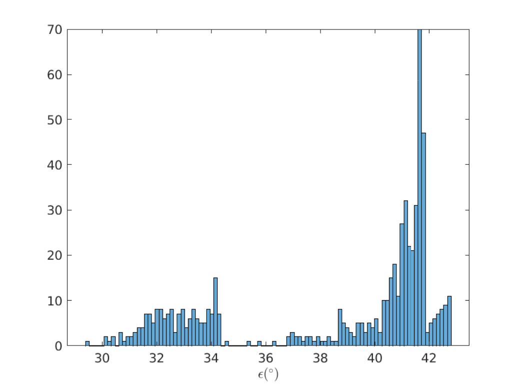


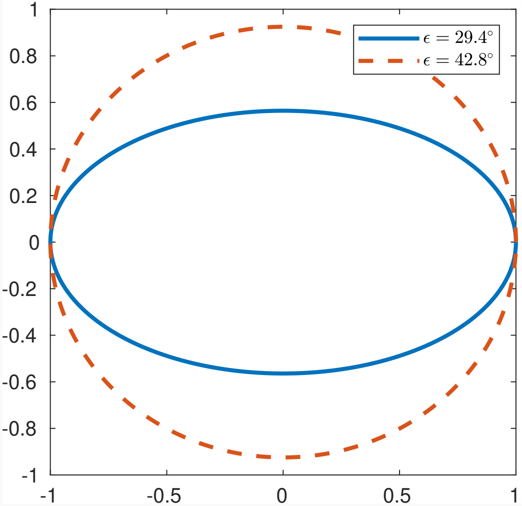
(d) $\pi/3$ defect

Results : Hybrid defect cavity



Results : Hybrid defect cavity





Conclusion

Thank you for your attention !

Do you have any question ?