Summer project : Hybrid chiral lasers

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- Laser $\rightarrow$ O.K.

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- Chiral ?

Definition (Chirality)
An object is said to be chiral if it is distinguishable from its reflection in a mirror.

Chiral media

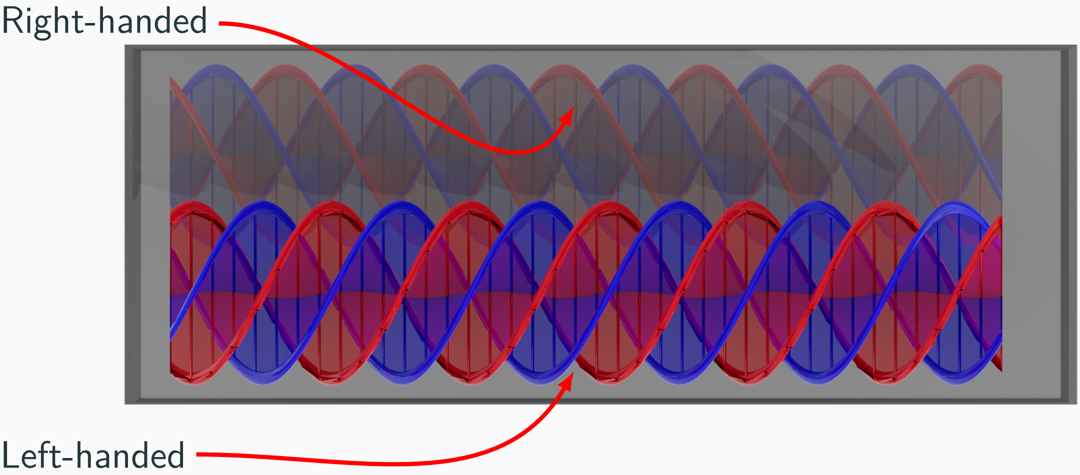


Figure 1: A double helix and its reflection in a mirror

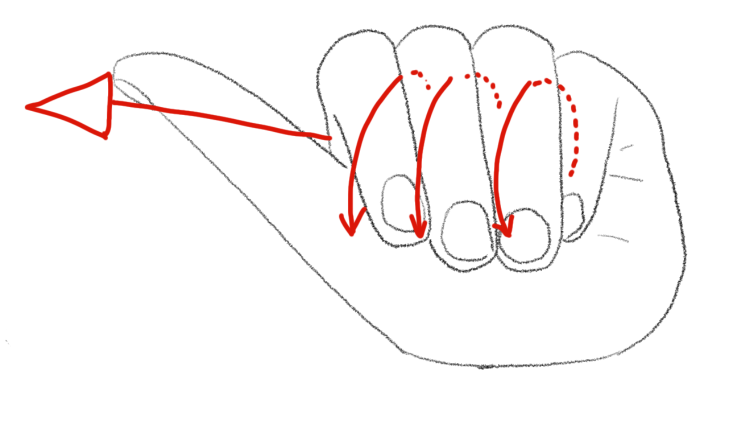


Figure 2: Quick reminder on handedness : left hand

A small subset of chiral media

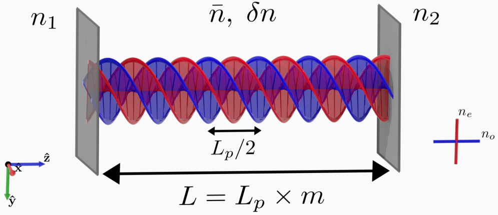


Figure 3: A slab of chiral medium

A small subset of chiral media

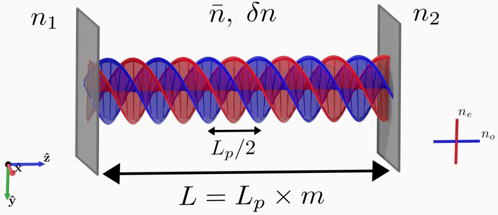
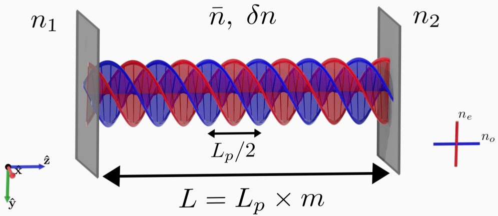


Figure 3: A slab of chiral medium

$$\epsilon = \begin{pmatrix} \boldsymbol{R}^{-1}(z) \cdot \begin{pmatrix} \epsilon_a & 0 \\ 0 & \epsilon_b \end{pmatrix} \cdot \boldsymbol{R}(z) & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & \epsilon_c \end{pmatrix}$$

$$\boldsymbol{R}(z) = \begin{pmatrix} \cos(pz + \psi) & -\sin(pz + \psi) \\ \sin(pz + \psi) & \cos(pz + \psi) \end{pmatrix}$$

A small subset of chiral media



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$$\boldsymbol{R}(z) = \begin{pmatrix} \cos(pz+\psi) & -\sin(pz+\psi) \\ \sin(pz+\psi) & \cos(pz+\psi) \end{pmatrix}$$

Warning
The refractive index does not depend on the position in (x, y) plane.

Studying propagation of light in this medium

Studying propagation of light in this medium

Maxwell-Thomson

$$\nabla \cdot \boldsymbol{B} = 0$$

Maxwell-Gauss

$$\nabla \cdot \boldsymbol{D} = 0$$

Maxwell-Faraday

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$$

Maxwell-Ampère

$$\nabla \times \boldsymbol{H} = \frac{\partial}{\partial t}\boldsymbol{E}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{\epsilon} \boldsymbol{E}$$

$$\boldsymbol{B} = \mu_0 \boldsymbol{H}$$

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$$\boldsymbol{D} = \epsilon_0 \boldsymbol{\epsilon} \boldsymbol{E} \qquad \boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Use auxiliary field $\boldsymbol{H'}$ and only study planar component of the field.

$$\boldsymbol{H'} = \left(\frac{\mu_0}{\epsilon_0}\right)^{1/2} \boldsymbol{H}$$

$$\begin{aligned} \hat{\boldsymbol{z}}\frac{\mathrm{d}}{\mathrm{d}z} \times \boldsymbol{E}_\perp &= ik_0 \boldsymbol{H'}_\perp \\ \hat{\boldsymbol{z}}\frac{\mathrm{d}}{\mathrm{d}z} \times \boldsymbol{H'}_\perp &= -ik_0 \boldsymbol{\epsilon}_\perp \cdot \boldsymbol{E}_\perp \end{aligned}$$

The $\perp$ sign is omitted for convenience.

Studying propagation of light in this medium

An analytic solution : the Oseen transformation

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Rewrite the field from electromagnetic basis to a more convenient basis.

$$\begin{bmatrix} \boldsymbol{e}(z) \\ \boldsymbol{h}(z) \end{bmatrix} = \begin{pmatrix} \boldsymbol{R}^{-1}(z) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{R}^{-1}(z) \end{pmatrix} \begin{bmatrix} \boldsymbol{E}(z) \\ \boldsymbol{H}'(z) \end{bmatrix}$$

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Maxwell's equation are rewritten

$$\frac{\mathrm{d}}{\mathrm{d}z} \begin{bmatrix} \boldsymbol{e}(z) \\ \boldsymbol{h}(z) \end{bmatrix} = i \underbrace{\begin{pmatrix} 0 & -ip & 0 & k_0 \\ ip & 0 & -k_0 & 0 \\ 0 & -k_0\epsilon_b & 0 & -ip \\ k_0\epsilon_a & 0 & ip & 0 \end{pmatrix}}_{=\boldsymbol{G}} \begin{bmatrix} \boldsymbol{e}(z) \\ \boldsymbol{h}(z) \end{bmatrix}$$

Studying propagation of light in this medium

And this yields a solution.

$$\begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H}' \end{bmatrix}_{z=d} = \underbrace{\begin{pmatrix} \boldsymbol{R}(d) & 0 \\ 0 & \boldsymbol{R}(d) \end{pmatrix} e^{i\boldsymbol{G}d} \begin{pmatrix} \boldsymbol{R}^{-1}(0) & 0 \\ 0 & \boldsymbol{R}^{-1}(0) \end{pmatrix}}_{=\boldsymbol{M_o}} \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H}' \end{bmatrix}_{z=0}$$

Problem solved !

Studying propagation of light in this medium

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Problem solved ! Or is it ?

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Problem solved ! Or is it ?

- Not satisfactory, the transfer matrix is a black-box without any analytic expression of its coefficients

Studying propagation of light in this medium

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Problem solved ! Or is it ?

- Not satisfactory, the transfer matrix is a black-box without any analytic expression of its coefficients
- An approximate solution allowing to grasp the underlying dynamics of the cavity is needed.

Studying propagation of light in this medium

An approximate solution : the Coupled Waves Theory (CWT)

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The field is decomposed upon the circular basis as:

$$E^{\pm}_{L,R} = A^{\pm}_{L,R} e^{\pm ikz}$$

Hypothesis

- Slow varying envelope approximation;
- Neglect field component that are not phase matched.

Studying propagation of light in this medium

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Maxwell equations become:

$$\frac{\mathrm{d}}{\mathrm{d}z}\boldsymbol{A}_{L,R} = \begin{pmatrix} 0 & i\kappa e^{-2i\varphi(z)} \\ -i\kappa e^{2i\varphi(z)} & 0 \end{pmatrix} \boldsymbol{A}_{L,R}$$

Where,

- $\kappa = \frac{k_0 \delta\epsilon}{2\bar{n}}$
- for a right handed medium, $\varphi(z) = \delta kz/2 - \psi$ and $\delta k = 2(k-p)$;
- for a left handed medium, $\varphi(z) = \delta kz/2 + \psi$ and $\delta k = 2(k+p)$.

Studying propagation of light in this medium

Solution to CWT equation for *e.g.* right-handed media

$$\begin{bmatrix} E_L^+ \\ E_R^+ \\ E_L^- \\ E_R^- \end{bmatrix}_{z=d} = \underbrace{\begin{pmatrix} e^{ikd} & 0 & 0 & 0 \\ 0 & \mathcal{P}_R^+ & 0 & \mathcal{Q}_R^+ \\ 0 & 0 & e^{-ikd} & 0 \\ 0 & \mathcal{Q}_R^- & 0 & \mathcal{P}_R^- \end{pmatrix}}_{\boldsymbol{M_{cwt}}} \begin{bmatrix} E_L^+ \\ E_R^+ \\ E_L^- \\ E_R^- \end{bmatrix}_{z=0}$$

with

$$\begin{aligned} \mathcal{P}_R^\pm &= \left[\cosh(\Delta d) \pm i\frac{\delta k}{2\Delta}\sinh(\Delta d)\right] e^{\pm ipd} \\ \mathcal{Q}_R^\pm &= \pm i\frac{\kappa}{\Delta}\sinh(\Delta d) e^{\pm i(pd+2\psi)} \end{aligned}$$

Past results with chiral media

Past results with chiral media

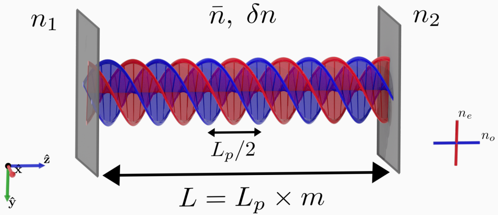


Figure 4

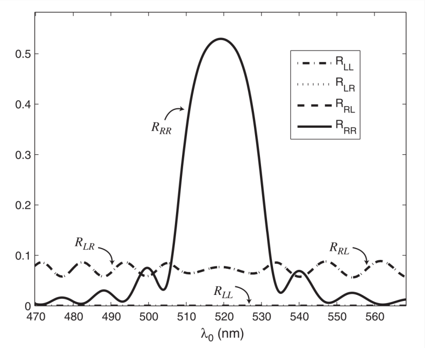
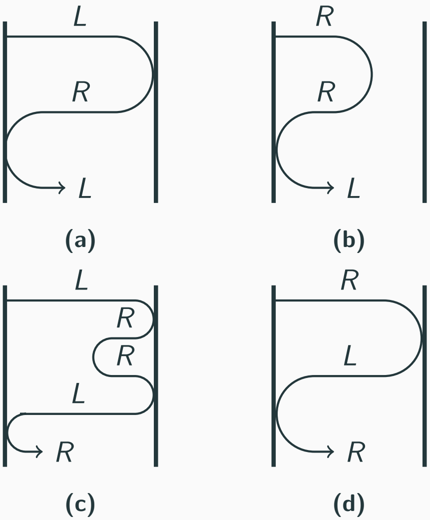


Figure 5: Reflectivity of a simulated chiral medium Martin W McCall. "Simplified theory of axial propagation through structurally chiral media". In: *Journal of Optics A: Pure and Applied Optics* 11.7 (July 1, 2009), p. 074006

This kind of cavity acts like a Bragg reflector for light polarised with the same handedness as the medium.

Past results with chiral media

This gives complex behaviour when combined to Fresnel reflections at the interfaces. (René D. M. Topf and Martin W. McCall. "Modes of structurally chiral lasers". In: *Physical Review A* 90.5 (Nov. 12, 2014), p. 053824)



Past results with chiral media

When pumped this creates a relatively un-pure circularly polarised light[1].

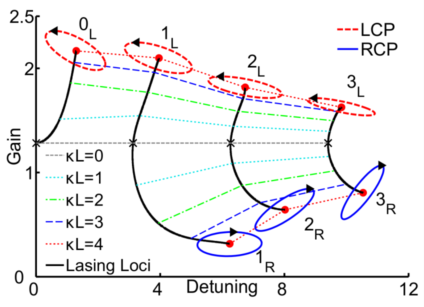


Figure 7

[1] René D. M. Topf and Martin W. McCall. "Modes of structurally chiral lasers". In: *Physical Review A* 90.5 (Nov. 12, 2014), p. 053824.

Objectives

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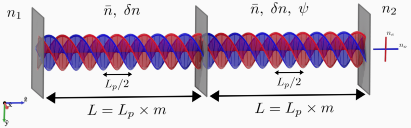
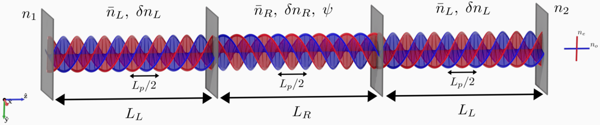
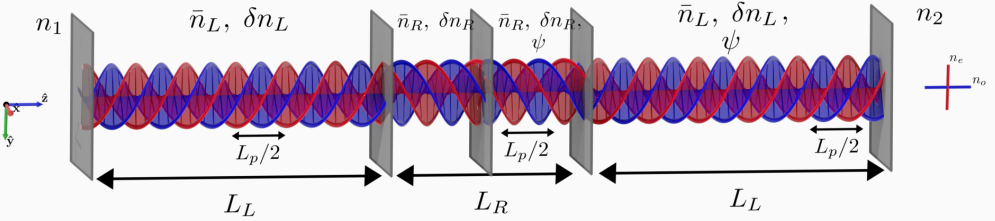


(a) Cavity with a defect



(b) Hybrid cavity



(c) Hybrid defect cavity

Figure 8

Method

Method : Calculate reflectivities

Partial inverse of a matrix gives

$$\begin{bmatrix} E_a^+ \\ E_b^+ \\ E_a^- \\ E_b^- \end{bmatrix}_1 = \begin{pmatrix} \boldsymbol{M_{11}} & \boldsymbol{M_{12}} \\ \boldsymbol{M_{21}} & \boldsymbol{M_{22}} \end{pmatrix} \begin{bmatrix} E_a^+ \\ E_b^+ \\ E_a^- \\ E_b^- \end{bmatrix}_0$$

$$\begin{aligned} \begin{pmatrix} t_{aa} & t_{ab} \\ t_{ba} & t_{bb} \end{pmatrix} &= \boldsymbol{M_{11}} - \boldsymbol{M_{12}} \boldsymbol{M_{22}^{-1}} \boldsymbol{M_{21}} \\ \begin{pmatrix} r_{aa} & r_{ab} \\ r_{ba} & r_{bb} \end{pmatrix} &= -\boldsymbol{M_{22}^{-1}} \boldsymbol{M_{21}} \end{aligned}$$

Method : Characterize laser action

For a cavity of length L.

$$\begin{bmatrix} E_a^+ \\ E_b^+ \\ 0 \\ 0 \end{bmatrix}_{L^+} = \begin{pmatrix} \boldsymbol{M_{11}} & \boldsymbol{M_{12}} \\ \boldsymbol{M_{21}} & \boldsymbol{M_{22}} \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ E_a^- \\ E_b^- \end{bmatrix}_{0^-}$$

That means

$$|\boldsymbol{M_{22}}| = 0$$

The output mode can be retrieved we the eigen-vector in the kernel of $\boldsymbol{M_{22}}$.

Method : Purity of the output polarisation ellipse

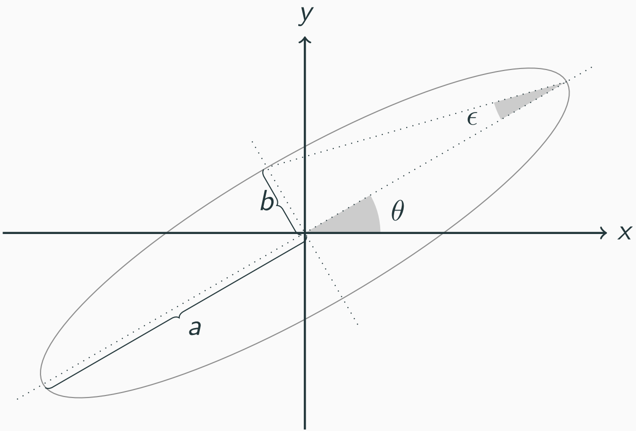


Figure 9: Parameters of the ellipse. θ is the rotation of the axes, a is the semi-major axis, b the semi-minor axis and ϵ relates to a and b as $\frac{b}{a} = \tan(\epsilon)$.

Results

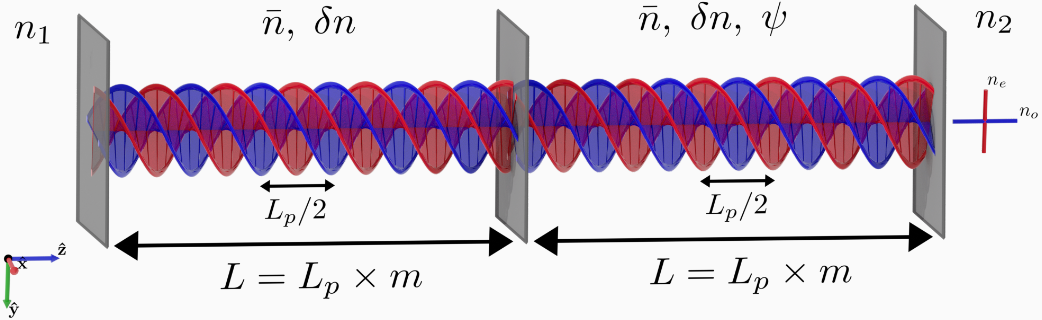
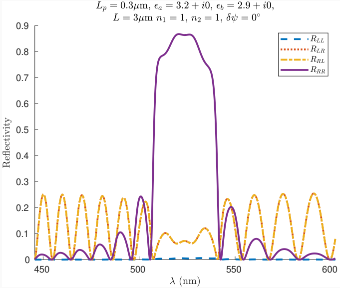
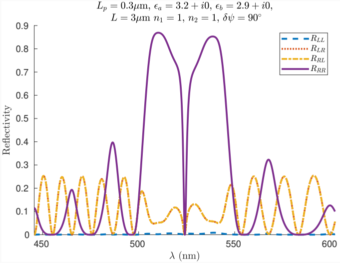


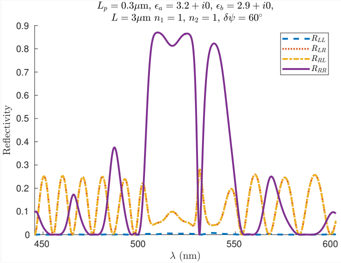
Figure 10: Cavity with a defect



(a) Cavity without a defect

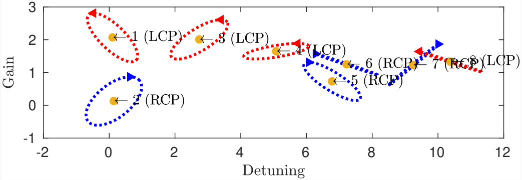


(b) Cavity with a $\frac{\pi}{2}$ defect

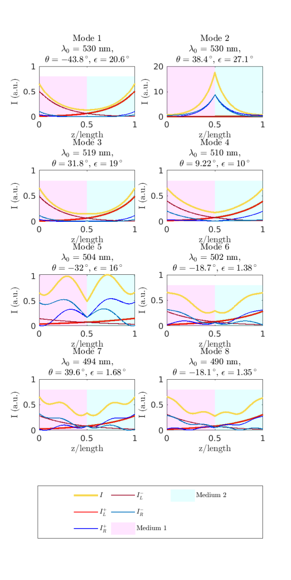


(c) Cavity with a $\frac{\pi}{3}$ defect

Results : Defect cavity



(a) Modes found for $\pi/2$ defect



Results : Defect cavity

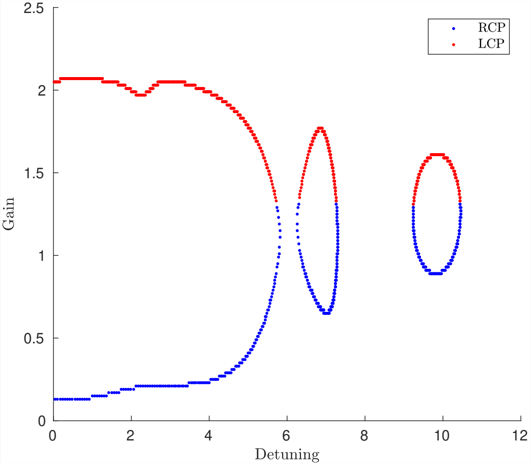


Figure 13: Tuning of the cavity

Results : Hybrid cavity

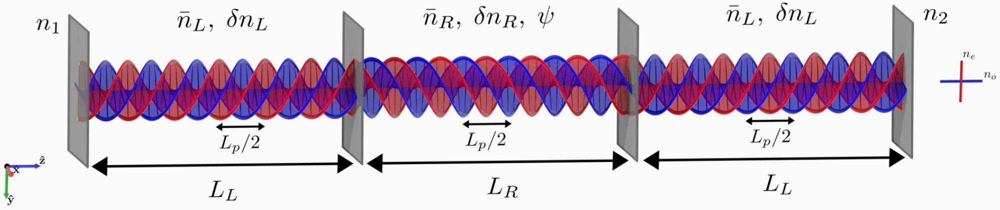
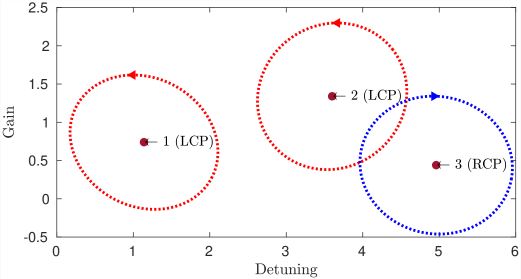
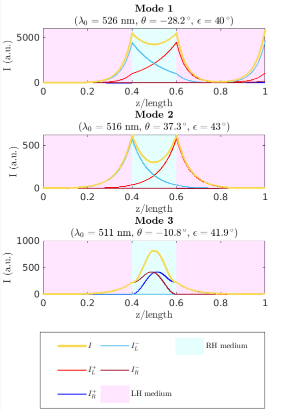


Figure 14: Hybrid cavity

Results : Hybrid cavity





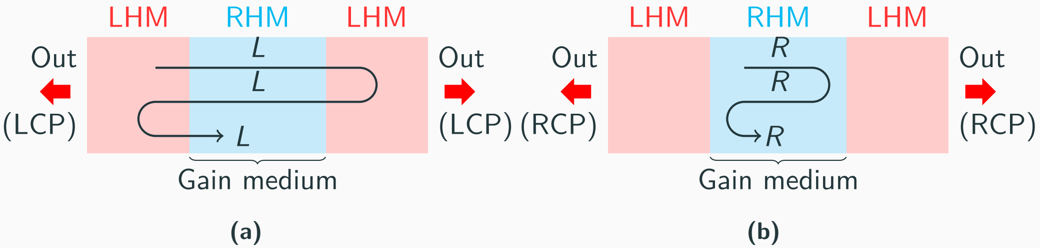


Figure 16: Mechanisms leading to laser action in an hybrid cavity. 16a Reflections of left-handed light on the reflectors. 16b Reflections of right-handed light inside the gain medium.

Results : Hybrid defect cavity

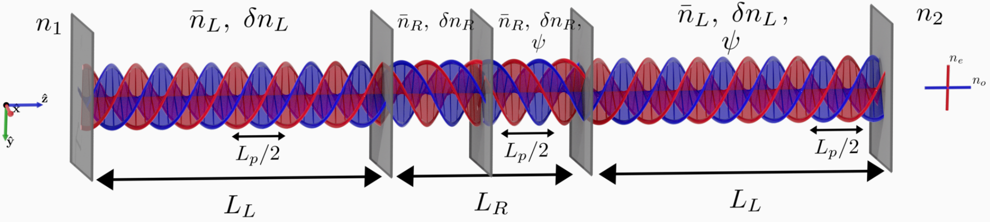
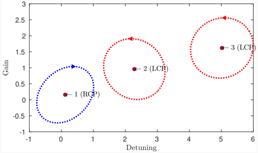
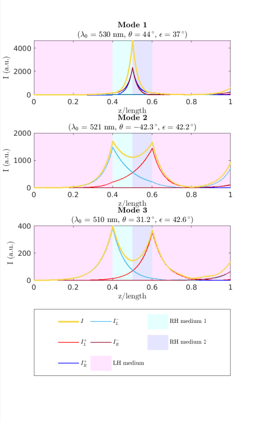


Figure 17: Hybrid defect cavity

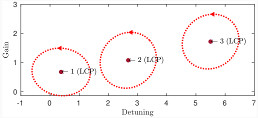
Results : Hybrid defect cavity



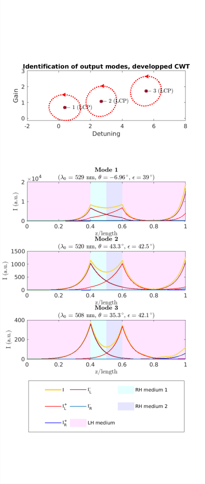
(a) $\pi/2$ defect



(b) $\pi/2$ defect

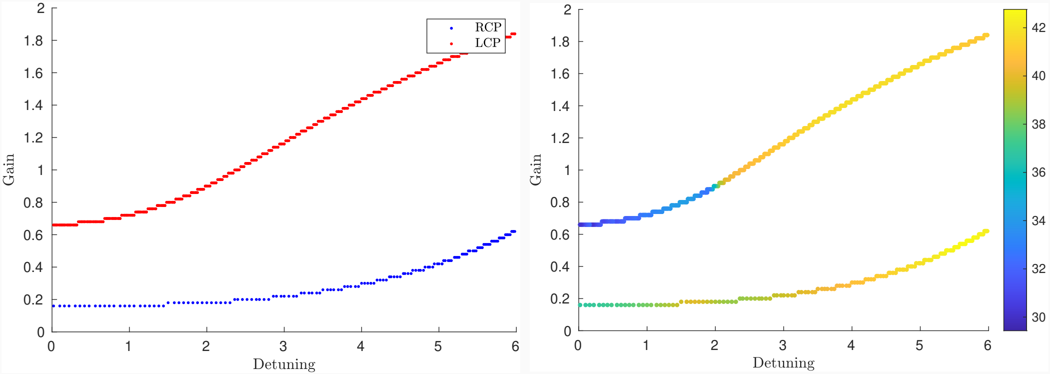


(c) $\pi/3$ defect

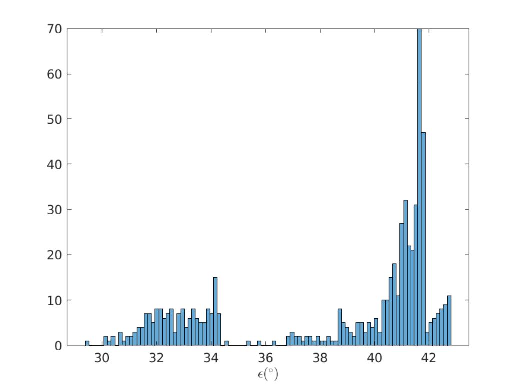


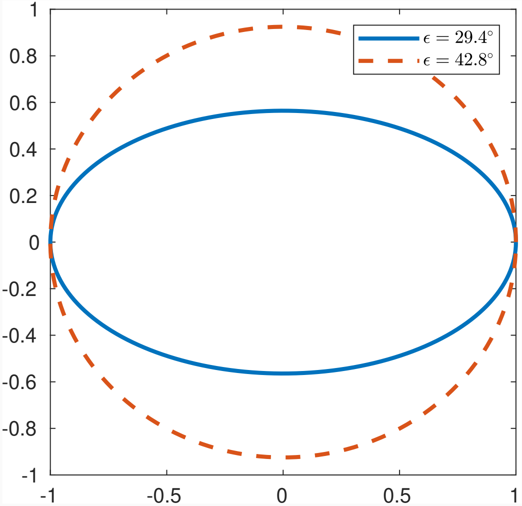
(d) $\pi/3$ defect

Results : Hybrid defect cavity



Results : Hybrid defect cavity





Conclusion

Thank you for your attention !

Do you have any question ?