A.P. Calculus

Packet 1

Distance Learning Plans:

Packet 1: 4/20/20- 5/11/20

Packet 2: 5/11/20 – 6/3/20

Remember, the A.P. Calculus exam will be Online on May 12th at 2:00 Eastern Standard Time. You will be emailed directions from the College Board prior to this date. Please check your school email regularly for more information.

My email address: hassettj@lakewoodps.org

To receive credit/grade for the course, students must complete the distance-learning plan. For A.P. Calculus, I have decide to use the Progress Checks on the A.P. Classroom website to review material for the A.P. Calculus AB exam. Students can either do this online at the A.P. Classroom site or I have provided the questions in this packet. If you do them online, I will be able to track your progress and you will get immediate feedback on each progress check. Students who do the paper copy may turn it into the school according to the district plan.

Packet 1: Progress Checks for Units 1-4.

Packet 2: Progress Checks for Units 5-7.

Although to receive credit/grade for the class these two packets are required to be finished by May 11th (packet 1) and June 3rd, the A.P. Calculus AB exam is on May 12th. I recommend that anyone still planning on taking the exam finish everything by May 11th. If you have not paid for the exam in full, please do so asap. Contact Mr. Knoth for details about payments.

If you are choosing to not take the A.P. exam, then place contact Mr. Knoth and see about a refund if you have paid. Complete the packets by June 3rd to receive your credit/grade.

Calculus AB		
Test date and time	**Exam Date: May 12** • Hawaii Time: 8 a.m. • Alaska Time: 10 a.m. • Pacific Time: 11 a.m. • Mountain Time: 12 p.m. • Central Time: 1 p.m. • Eastern Time: 2 p.m.	
Exam timing	Students will have 25 minutes to read and respond to Question 1, and then 5 minutes to upload their response. After uploading the response to Question 1, students will have 15 minutes to respond to Question 2, with 5 additional minutes to upload their response to Question 2. Once their response to Question 1 has been submitted, they cannot go back to it.	
Questions	Question 1 (25 mins.)	Question 2 (15 mins.)
% of exam weight (rounded)	60%	40%
Question name	Multi-focus free-response question	Multi-focus free-response question

Question description	This question will assess student knowledge and skills developed in 2 or more of the eligible units and topics.	This question will assess student knowledge and skills developed in 2 or more of the eligible units and topics.
Corresponding free-response question (FRQ) type in the course and exam description binder	Question 1 will consist of similar components to traditional AP Calculus exam questions, with minor modifications to enable students to choose to submit either typed or handwritten responses.	Question 2 will consist of similar components to traditional AP Calculus exam questions, with minor modifications to enable students to choose to submit either typed or handwritten responses.
Units eligible for 2020 exam	Units 1–7	
Units not included in 2020 exam	Unit 8	
Other course-specific info	• For students who prefer to type their responses on a computer rather than handwriting and photographing, we will provide a "keyboarding tip sheet" in advance of the exam, with suggestions on how to enter various mathematical expressions on a regular keyboard. • Questions on the 2020 AP Calculus AB Exam are designed such that a graphing calculator or other calculator is not required. However, use of a calculator is allowed. Simple ("four-function") calculators are freely available as apps for computers and phones (i.e. most or all internet-connected devices), and can be installed beforehand for use on the exam. • No arithmetic or calculations will be required beyond what can readily be done with pencil and paper. As always, AP Calculus AB students are advised to submit "unsimplified" numeric answers, in order to avoid risking arithmetical errors not related to calculus.	
Make-up test date and time	**Date: June 1** • Hawaii Time: 10 a.m. • Alaska Time: 12 p.m. • Pacific Time: 1 p.m. • Mountain Time: 2 p.m. • Central Time: 3 p.m. • Eastern Time: 4 p.m.	

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 1

The function f is given by $f(x) = 0.1x^4 - 0.5x^3 - 3.3x^2 + 7.7x - 1.99$. For how many positive values of b does $\lim_{x \to b} f(x) = 2$?

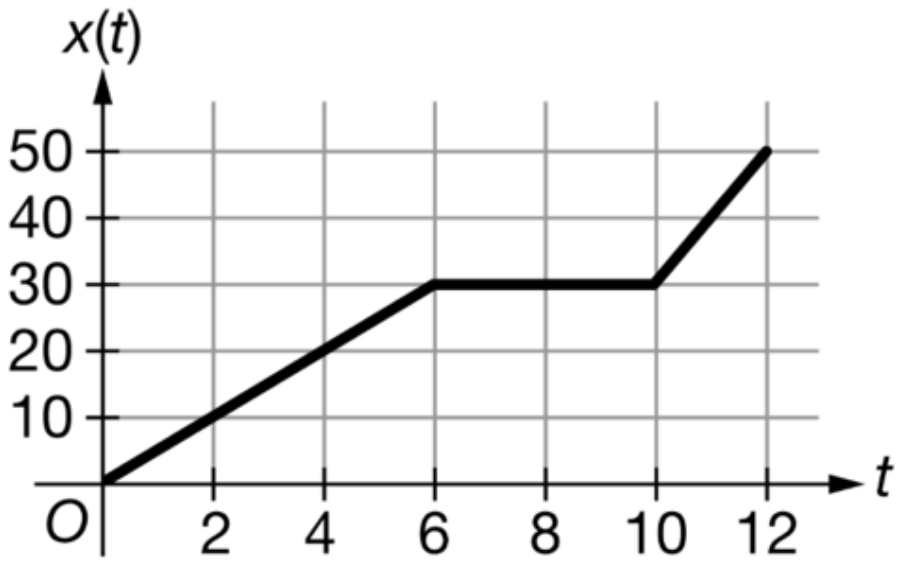
(A) One

(B) Two

(C) Three

(D) Four

Question 2



A particle is moving on the x-axis and the position of the particle at time t is given by $x(t)$, whose graph is given above. Which of the following is the best estimate for the speed of the particle at time $t = 8$?

(A) 0

(B) $\frac{15}{4}$

(C) 5

(D) 30

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 3

t (seconds)	0	100	200	300	400	500	600
$y(t)$ (feet)	0	50	400	1360	3200	6250	10,950

A rocket leaves the surface of Earth at time $t = 0$ and travels straight up from the surface. The height, in feet, of the rocket above the surface of Earth is given by $y(t)$, where t is measured in seconds for $0 \leq t \leq 600$. Values of $y(t)$ for selected values of t are given in the table above. Of the following values of t, at which value would the speed of the rocket most likely be greatest based on the data in the table?

A $t = 100$

B $t = 200$

C $t = 300$

D $t = 400$

Question 4

The position of a particle moving to the right on the x-axis is given by $x(t)$, where $x(t)$ is measured in inches and t is measured in minutes for $0 \leq t \leq 100$. If $y = x(t)$ is a linear function, which of the following would most likely give the best estimate of the speed of the particle, in inches per minute, at time $t = 20$ minutes?

A $x(20)$

B $\frac{x(20)}{20}$

C $x(21) - x(19)$

D The slope of the graph of $y = x(t)$

Question 5

Let f be the function given by $f(x) = \frac{e^{2x}-1}{x}$. Which of the following equations expresses the property that $f(x)$ can be made arbitrarily close to 2 by taking x sufficiently close to 0, but not equal to 0 ?

A $f(0) = 2$

B $f\left(\lim_{x \to 0} x\right) = 2$

C $\lim_{x \to 0} f(x) = 2$

D $\lim_{x \to 2} f(x) = 0$

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 6

The function f has the property that as x gets closer and closer to 4, the values of $f(x)$ get closer and closer to 7. Which of the following statements must be true?

A $f(4) = 7$

B $f(7) = 4$

C $\lim_{x \to 4} f(x) = 7$

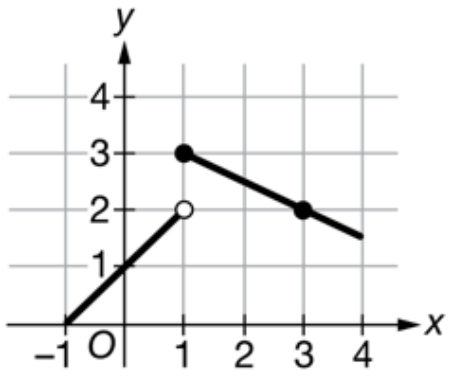
D $\lim_{x \to 7} f(x) = 4$

A.P. Calculus Unit 1 Progress Check: MCQ Part A

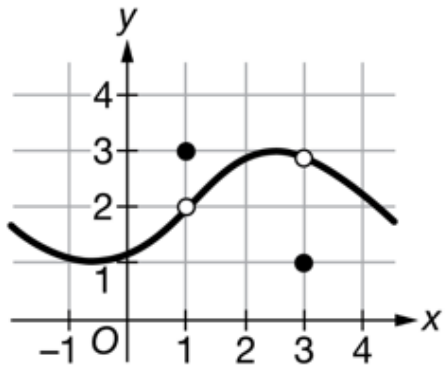
Question 7

A function f satisfies $\lim_{x \to 1} f(x) = 3$. Which of the following could be the graph of f?

(A)



(B)



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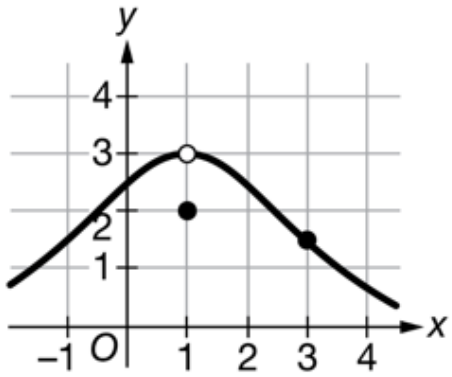

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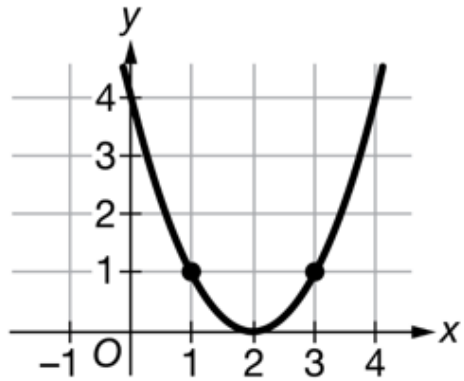
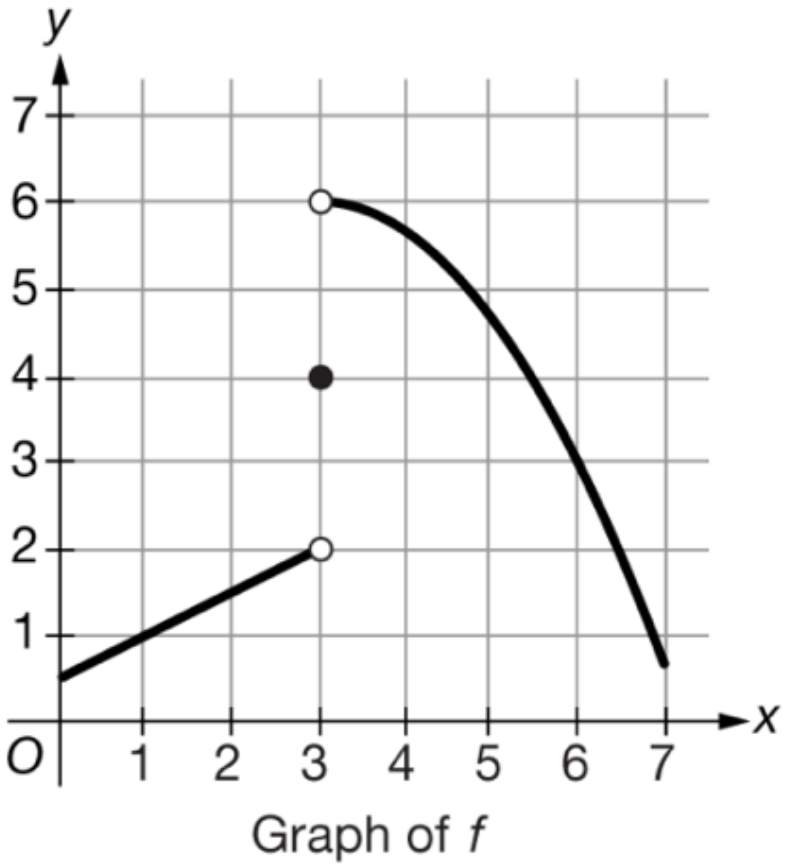


A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 8



Graph of f

The graph of the function f is shown above. Which of the following expressions equals 2 ?

A $f(3)$

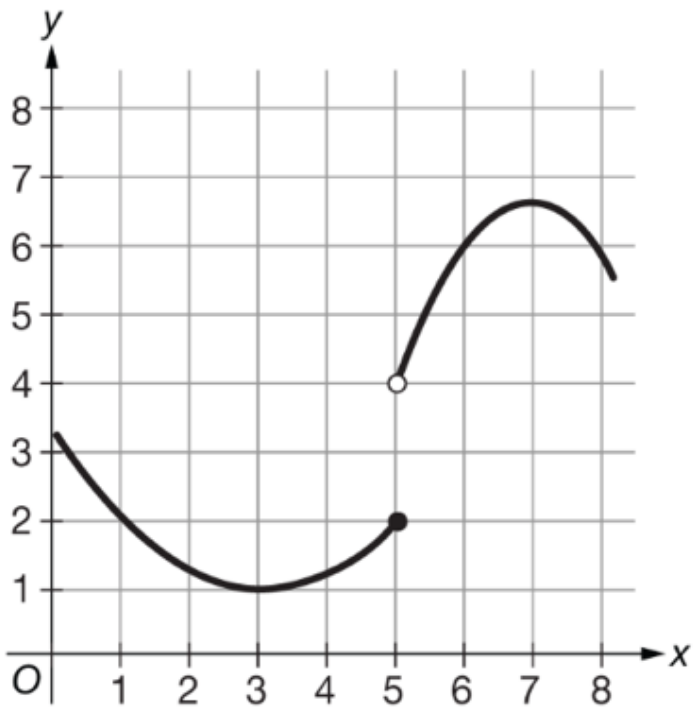
B $\lim_{x \to 3^-} f(x)$

C $\lim_{x \to 3^+} f(x)$

D $\lim_{x \to 3} f(x)$

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 9



Graph of f

The graph of the function f is shown above. The value of $\lim_{x \to 5} f(x)$ is

A 2

B 3

C 4

D nonexistent

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 10

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	5.018	5.007	5.002	4.998	4.982	4.887

The table above gives selected values for a continuous function f. Based on the data in the table, which of the following is the best approximation for $\lim_{x \to 3} f(x)$?

A 0

B 3

C 5

D There is no best approximation, because the limit does not exist.

Question 11

x	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1
$f(x)$	5	-25	125	-625	5.9999	5.999	5.99	5.9

The table above gives values of a function f at selected values of x. Which of the following conclusions is supported by the data in the table?

A $\lim_{x \to 4} f(x) = 6$

B $\lim_{x \to 4^-} f(x) = 6$

C $\lim_{x \to 4^+} f(x) = 6$

D $\lim_{x \to 6^-} f(x) = 4$

Question 12

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
$f(x)$	6.80	6.86	6.90	6.95	2	7.05	7.10	7.14	7.20

The table above gives values of the function f at selected values of x. Which of the following statements must be true?

A $\lim_{x \to 1} f(x) = 2$

B $\lim_{x \to 1} f(x) = 7$

C $\lim_{x \to 1} f(x)$ does not exist.

D $\lim_{x \to 1} f(x)$ cannot be definitively determined from the data in the table.

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 13

$$f(x) = \begin{cases} x+3 & \text{for } x < 1 \\ -2x+7 & \text{for } x > 1 \end{cases}$$

If f is the function defined above, then $\lim_{x \to 1} f(x)$ is

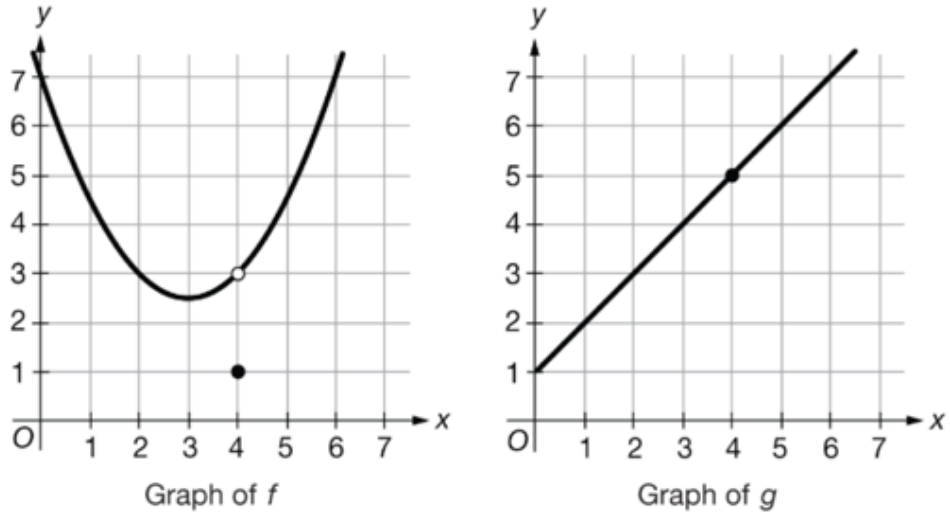
A 2

B 4

C 5

D nonexistent

Question 14



The graphs of the functions f and g are shown above. The value of $\lim_{x \to 4} \frac{f(x)+7}{g(x)}$ is

A $\frac{3}{5}$

B $\frac{8}{5}$

C 2

D nonexistent

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 15

$\lim_{x \to 0} \frac{\cos x + 3e^x}{2e^x}$ is

A $\frac{1}{2}$

B $\frac{3}{2}$

C 2

D nonexistent

Question 16

If f is the function defined by $f(x) = \frac{x-9}{\sqrt{x}-3}$, then $\lim_{x \to 9} f(x)$ is equivalent to which of the following?

A $\lim_{x \to 9} (\sqrt{x} - 3)$

B $\lim_{x \to 9} (\sqrt{x} + 3)$

C $\lim_{x \to 9} \left(\frac{x^2-81}{x-9} \right)$

D $\frac{\lim_{x \to 9}(x-9)}{\lim_{x \to 9}(\sqrt{x}-3)}$

A.P. Calculus Unit 1 Progress Check: MCQ Part A

Question 17

$\lim_{x \to 0} \frac{7x^5+5x^2+12x}{3x^5+4x}$ is

A 0

B $\frac{7}{3}$

C 3

D ∞

Question 18

If $f(x) = \frac{\sin x - 1}{\cos^2 x}$, then $\lim_{x \to \frac{\pi}{2}} f(x)$ is equivalent to which of the following?

A $\lim_{x \to \frac{\pi}{2}} \frac{-1}{1+\sin x}$

B $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{1+\sin^2 x}$

C $\lim_{x \to \frac{\pi}{2}} \sec x$

D $\lim_{x \to \frac{\pi}{2}} (\tan x - \sec x)$

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 1

If f is the function defined by $f(x) = \frac{\frac{1}{x}-1}{x-1}$, then $\lim_{x \to 1} f(x)$ is equivalent to which of the following?

A $\lim_{x \to 1} \left(-\frac{1}{x}\right)$

B $\lim_{x \to 1} \left(\frac{1}{x^2} - 1\right)$

C $\lim_{x \to 1} \left(\frac{x-1}{x-1}\right)$

D $\frac{\lim_{x \to 1} \left(\frac{1}{x} - 1\right)}{\lim_{x \to 1} (x-1)}$

Question 2

Let f and g be functions such that $\lim_{x \to 4} g(x) = 2$ and $\lim_{x \to 4} \frac{f(x)}{g(x)} = \pi$. What is $\lim_{x \to 4} f(x)$?

A $\frac{\pi}{2}$

B $2 + \pi$

C 2π

D The limit cannot be determined from the information given.

Question 3

$$f(x) = \begin{cases} \frac{x}{|x|} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

If f is the function defined above, then $\lim_{x \to 0} f(x)$ is

A -1

B 0

C 1

D nonexistent

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 4

The function f is defined for all x in the interval $4 < x < 6$. Which of the following statements, if true, implies that $\lim_{x \to 5} f(x) = 17$?

A There exists a function g with $f(x) \leq g(x)$ for $4 < x < 6$, and $\lim_{x \to 5} g(x) = 17$.

B There exists a function g with $g(x) \leq f(x)$ for $4 < x < 6$, and $\lim_{x \to 5} g(x) = 17$.

C There exist functions g and h with $f(x) \leq g(x) \leq h(x)$ for $4 < x < 6$, and $\lim_{x \to 5} g(x) = \lim_{x \to 5} h(x) = 17$.

D There exist functions g and h with $g(x) \leq f(x) \leq h(x)$ for $4 < x < 6$, and $\lim_{x \to 5} g(x) = \lim_{x \to 5} h(x) = 17$.

Question 5

The function g is given by $g(x) = \frac{7x-26}{x-5}$. The function h is given by $h(x) = \frac{3x+14}{2x+1}$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $0 < x < 5$, what is $\lim_{x \to 2} f(x)$?

A $\frac{3}{2}$

B 4

C 7

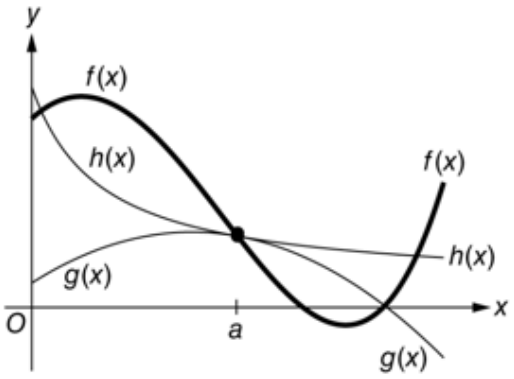
D The limit cannot be determined from the information given.

A.P. Calculus Unit 1 Progress Check: MCQ Part B

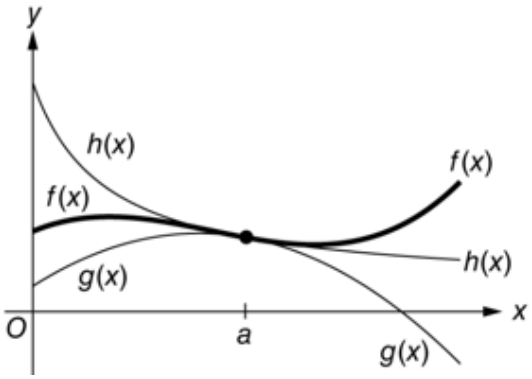
Question 6

Let f be a function of x. The value of $\lim_{x \to a} f(x)$ can be found using the squeeze theorem with the functions g and h. Which of the following could be graphs of f, g, and h ?

A



B



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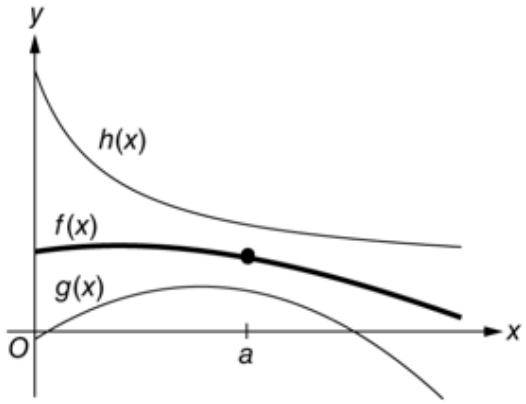
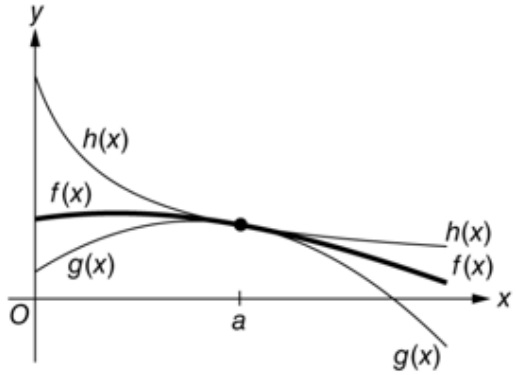


D



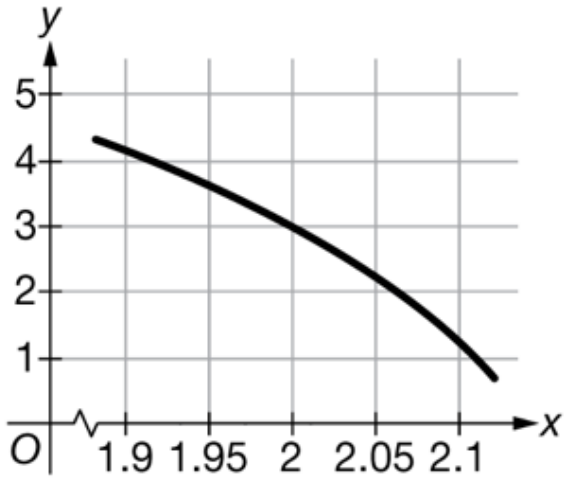
A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 7

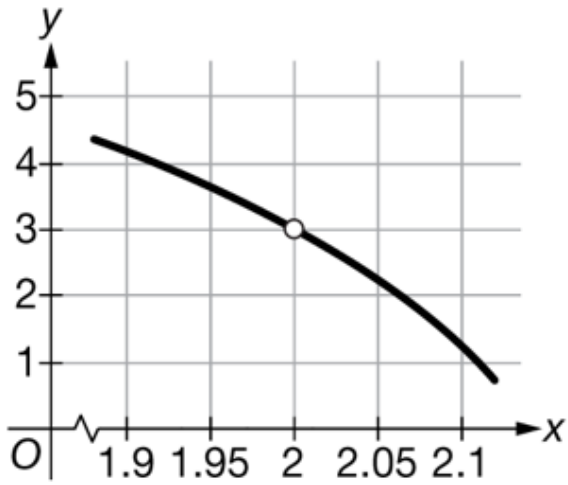
x	1.9	1.95	1.99	1.999	2.001	2.01	2.05	2.1
$f(x)$	4.204	3.671	3.147	3.015	2.985	2.847	2.160	1.113

The table above gives selected values for a function f. Based on the data in the table, which of the following could not be the graph of f on the interval $1.9 \leq x \leq 2.1$?

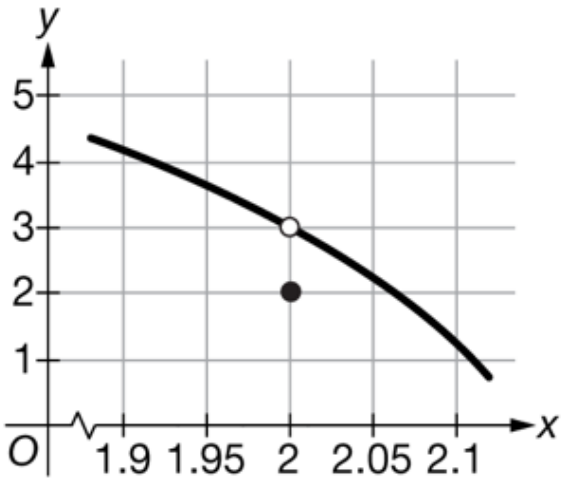
A



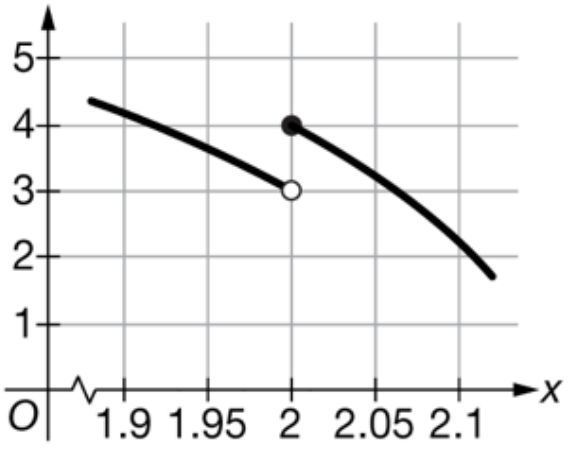
B



C



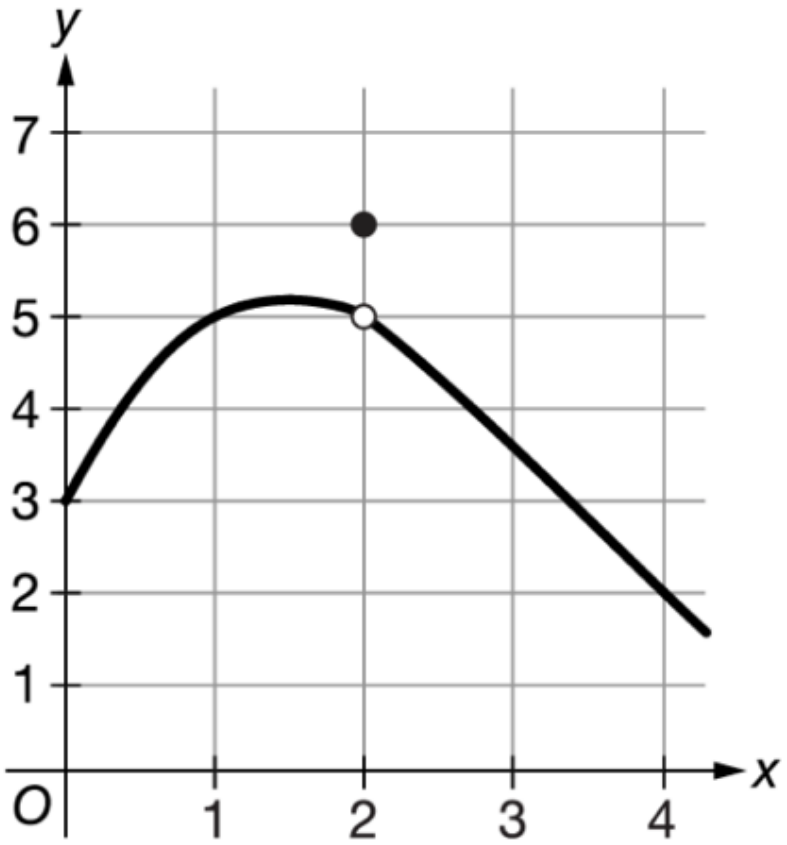
D



A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 8

$$f(x) = \begin{cases} -x^2 + 3x + 3 & \text{for } x < 2 \\ 6 & \text{for } x = 2 \\ 8 - \frac{3}{2}x & \text{for } x > 2 \end{cases}$$



Let f be the piecewise function defined above. Also shown is a portion of the graph of f. What is the value of $\lim_{x \to 2} f(f(x))$?

A -15

B -7

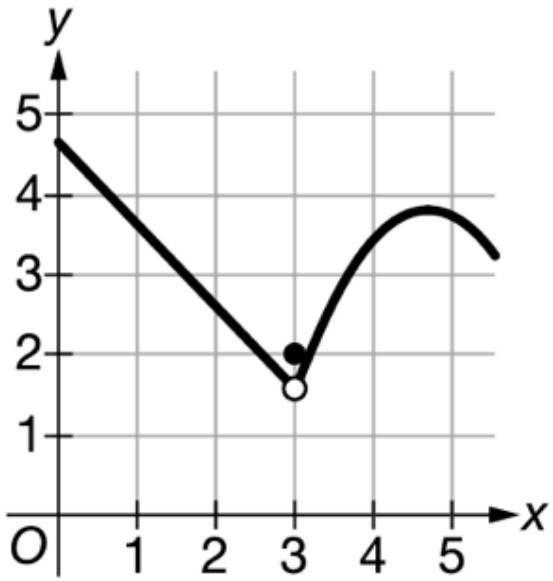
C -1

D $\frac{1}{2}$

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 9

x	2.9	2.95	2.98	2.999	3.001	3.02	3.05	3.1
$f(x)$	1.7	1.65	1.62	1.601	1.603	1.66	1.747	1.89



The table above gives selected values for a function f. Also shown is a portion of the graph of f. The graph consists of a line segment for $x < 3$ and part of a parabola for $x > 3$. What is $\lim_{x \to 3} f(x)$?

A 1.6

B 1.602

C 2

D The limit does not exist.

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 10

$$f(x) = \begin{cases} \frac{2x^2-3x-2}{x-2} & \text{if } x \neq 2 \\ 7 & \text{if } x = 2 \end{cases}$$

The function f is defined above. Which of the following statements is true?

(A) f is continuous at $x = 2$.

(B) f has a removable discontinuity at $x = 2$.

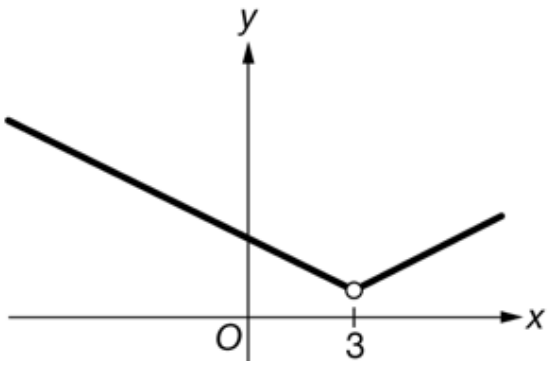
(C) f has a jump discontinuity at $x = 2$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$.

Question 11

The function f has a jump discontinuity at $x = 3$. Which of the following could be the graph of f ?

(A)



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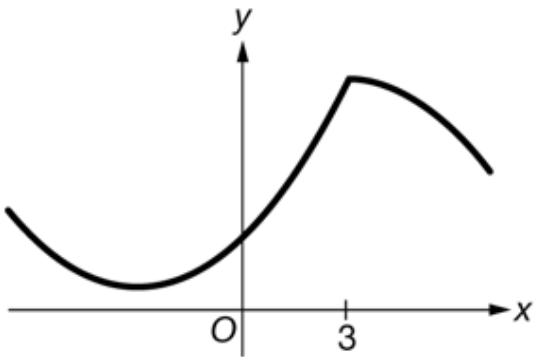
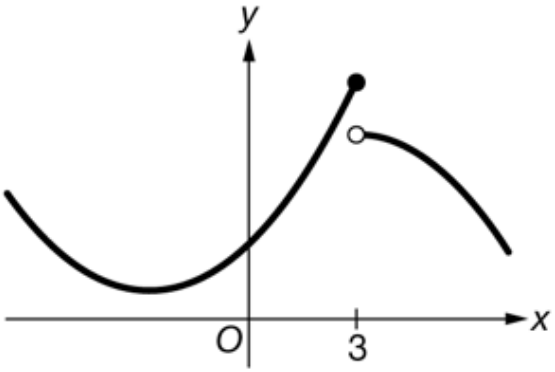
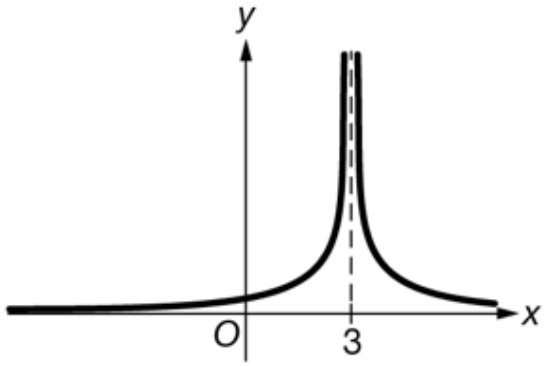


(C)

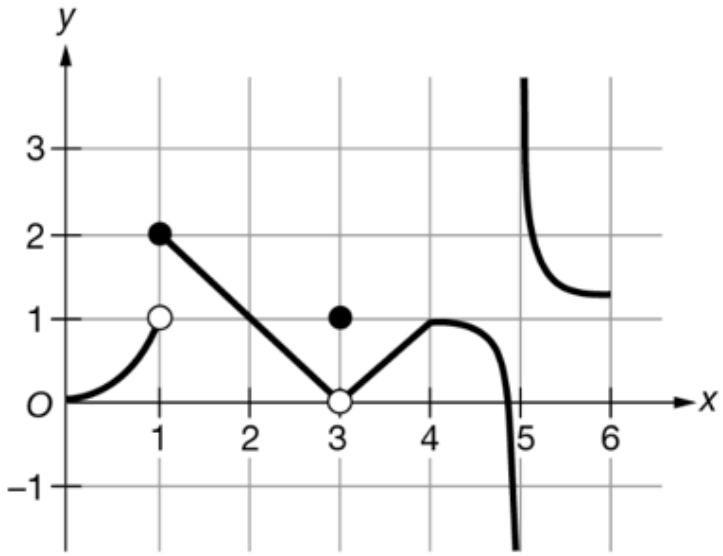


(D)



A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 12



Graph of f

The graph of a function f is shown in the figure above. At what value of x does f have a removable discontinuity?

A $x = 1$

B $x = 3$

C $x = 4$

D $x = 5$

Question 13

If $\lim_{x \to 6} f(x)$ exists with $\lim_{x \to 6} f(x) < 5$ and $f(6) = 10$, which of the following statements must be false?

A $\lim_{x \to 6^-} f(x) = 0$

B $\lim_{x \to 6^-} f(x) < 5$

C $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

D f is continuous at $x = 6$.

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 14

$$f(x) = \begin{cases} 2^x & \text{for } 0 < x < 1 \\ \frac{1}{2}x^2 - x + \frac{5}{2} & \text{for } 1 < x < 2 \end{cases}$$

Let f be the function defined above. Which of the following statements is true?

(A) f is continuous at $x = 1$.

(B) f is not continuous at $x = 1$ because $f(1)$ does not exist.

(C) f is not continuous at $x = 1$ because $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$.

(D) f is not continuous at $x = 1$ because $\lim_{x \to 1} f(x)$ does not exist.

Question 15

Which of the following functions is continuous at $x = 3$?

(A) $f(x) = \begin{cases} \frac{x^2+x-12}{x-3} & \text{for } x \neq 3 \\ 8 & \text{for } x = 3 \end{cases}$

(B) $g(x) = \begin{cases} 4x - 7 & \text{for } x < 3 \\ \text{undefined} & \text{for } x = 3 \\ x + 2 & \text{for } x > 3 \end{cases}$

(C) $h(x) = \begin{cases} -8\sin\left(\frac{\pi}{2}x\right) & \text{for } x < 3 \\ 8 & \text{for } x = 3 \\ -8\cos(\pi x) & \text{for } x > 3 \end{cases}$

(D) $k(x) = \begin{cases} 8 + \ln(4 - x) & \text{for } x \leq 3 \\ 8\ln(x - 2) & \text{for } x > 3 \end{cases}$

A.P. Calculus Unit 1 Progress Check: MCQ Part C

Question 1

Let f be the function given by $f(x) = \frac{|x^2-3|\cdot(x+0.5)}{(x^2-3)(x+0.5)}$. On which of the following open intervals is f continuous?

A $(-2, -1)$

B $(-1, 0)$

C $(0, 1)$

D $(1, 2)$

Question 2

$$f(x) = \begin{cases} e^{bx} & \text{for } x \leq 2 \\ 1.5x + b & \text{for } x > 2 \end{cases}$$

Let f be the function defined above. For what values of b is f continuous at $x = 2$?

A 0.508 only

B 0.647 only

C -1.282 and 0.508

D -2.998 and 0.647

Question 3

Let f be the function given by $f(x) = x + \tan\left(\frac{x}{5}\right) - 10$. The Intermediate Value Theorem applied to f on the closed interval $[12, 15]$ guarantees a solution in $[12, 15]$ to which of the following equations?

A $f(x) = -10$

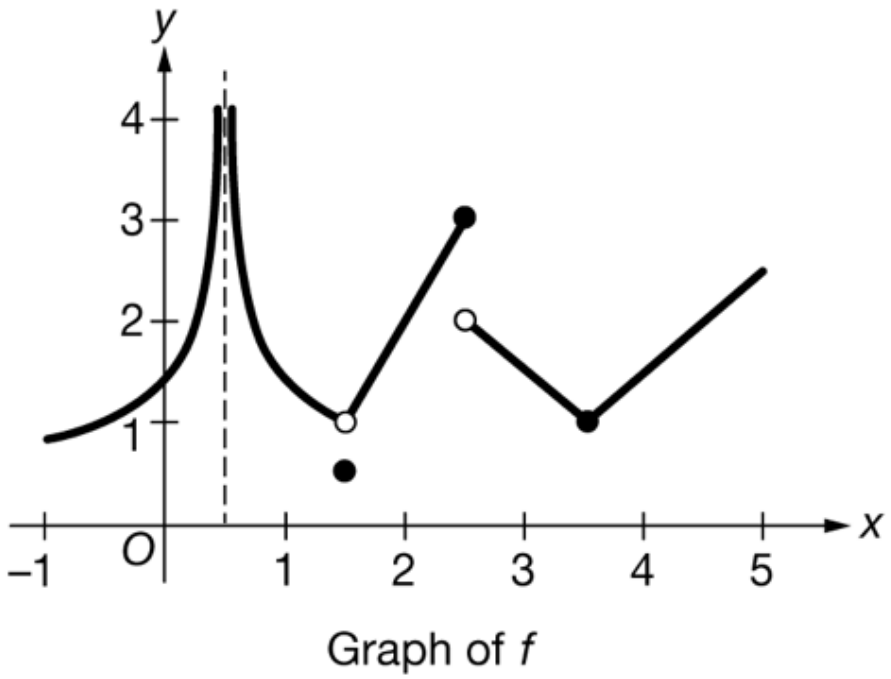
B $f(x) = 0$

C $f(x) = 4$

D $f(x) = 14$

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 4



Graph of f

The graph of the function f is shown above. On which of the following intervals is f continuous?

A $(-1, 1)$

B $(1, 2)$

C $(2, 3)$

D $(3, 5)$

Question 5

The function f is continuous on the interval $-1 < x < 3$ and is not continuous on the interval $-1 \leq x \leq 3$. Which of the following could not be an expression for $f(x)$?

A $\frac{x+1}{x-3}$

B $\frac{x-3}{x+1}$

C $(x+1)(x-3)$

D $\frac{1}{(x+1)(x-3)}$

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 6

$$g(x) = \begin{cases} \frac{x^2-9}{4x+12} & \text{for } x \neq -3 \\ k & \text{for } x = -3 \end{cases}$$

Let g be the function defined above, where k is a constant. For what value of k is g continuous at $x = -3$?

A -3

B $-\frac{3}{2}$

C $-\frac{3}{4}$

D 0

Question 7

$$f(x) = \begin{cases} c + cx - x^2 & \text{for } x < 3 \\ 7 & \text{for } x = 3 \\ 2c + \frac{3}{x-2} & \text{for } x > 3 \end{cases}$$

Let f be the function defined above. For what value of c, if any, is f continuous at $x = 3$?

A 2

B 4

C 6

D There is no such c.

Question 8

The function h is defined by $h(x) = \frac{x^2-7}{x-3}$. Which of the following statements must be true?

A $\lim_{x \to 3^-} h(x) = -\infty$ and $\lim_{x \to 3^+} h(x) = -\infty$

B $\lim_{x \to 3^-} h(x) = +\infty$ and $\lim_{x \to 3^+} h(x) = -\infty$

C $\lim_{x \to 3^-} h(x) = -\infty$ and $\lim_{x \to 3^+} h(x) = +\infty$

D $\lim_{x \to 3} h(x) = +\infty$ and $\lim_{x \to 3^+} h(x) = +\infty$

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 9

Let f be a function such that $\lim_{x \to 5^-} f(x) = \infty$. Which of the following statements must be true?

A $\lim_{x \to 5^+} f(x) = \infty$

B f is undefined at $x = 5$.

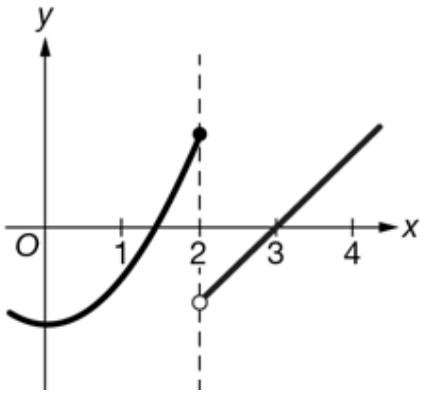
C The graph of f has a vertical asymptote at $x = 5$.

D The graph of f has a vertical asymptote at $x = -5$.

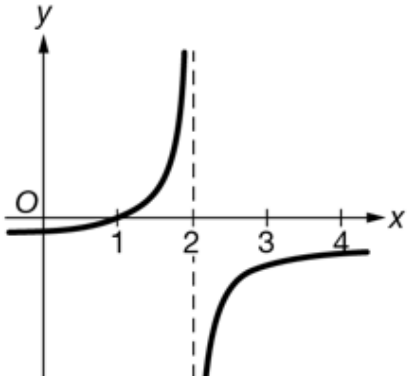
Question 10

Let f be a function of x. If $\lim_{x \to 2^-} f(x) = +\infty$ and $\lim_{x \to 2^+} f(x) = -\infty$, which of the following could be a graph of f ?

A



B



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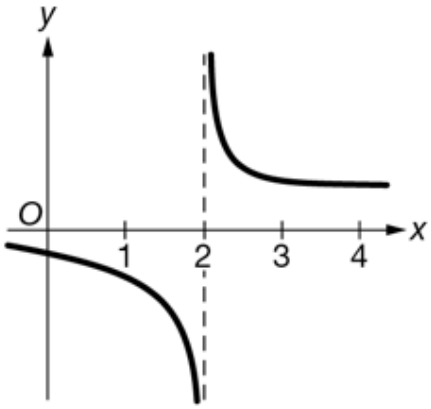
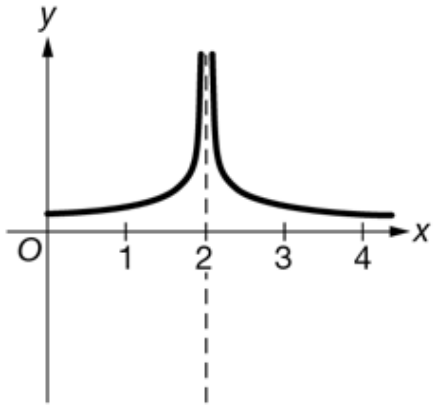


D



A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 11

Let f be the function defined by $f(x) = \frac{2x+3}{x+1}$. Which of the following statements are true?

I. The graph of f has a horizontal asymptote at $y = 2$ because $\lim_{x \to \infty} f(x) = 2$.

II. The graph of f has a horizontal asymptote at $y = 2$ because $\lim_{x \to -\infty} f(x) = 2$.

III. The graph of f has a vertical asymptote at $x = -1$ because $\lim_{x \to -1^+} f(x) = \infty$.

A I only

B III only

C I and II only

D I, II, and III

Question 12

The population on an island is modeled by $P(t) = \frac{6000}{40+60e^{-0.03t}}$ for $t \geq 0$, where $P(t)$ is the number of people on the island after t years. What is $\lim_{t \to \infty} P(t)$?

A 60

B 100

C 150

D 6000

A.P. Calculus Unit 1 Progress Check: MCQ Part B

Question 13

Let f be the function defined by $f(x) = \frac{3x^{20}}{4e^x + 8x^{20}}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?

A $y = 0$

B $y = \frac{3}{8}$

C $y = \frac{3}{4}$

D There is no horizontal asymptote to the graph of f.

Question 14

Let f be a function such that $f(5) < 6 < f(7)$. Which of the following statements provides sufficient additional information to conclude that there is a value $x = c$ in the interval $[5, 7]$ such that $f(c) = 6$?

A f is defined for all x.

B f is increasing for all x.

C f is continuous for all x.

D There is a value $x = c$ in the interval $[5, 7]$ such that $\lim_{x \to c} f(x) = 6$.

Question 15

Let f be a function of x. Which of the following statements, if true, would guarantee that there is a number c in the interval $[-2, 3]$ such that $f(c) = 10$?

A f is increasing on the interval $[-2, 3]$, where $f(-2) = 0$ and $f(3) = 20$.

B f is increasing on the interval $[-2, 3]$, where $f(-2) = 15$ and $f(3) = 30$.

C f is continuous on the interval $[-2, 3]$, where $f(-2) = 0$ and $f(3) = 20$.

D f is continuous on the interval $[-2, 3]$, where $f(-2) = 15$ and $f(3) = 30$.

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 1

The derivative of a function f is given by $f'(x) = 0.1x + e^{0.25x}$. At what value of x for $x > 0$ does the line tangent to the graph of f at x have slope 2 ?

A 0.512

B 1.849

C 2.287

D 8.113

Question 2

x	0	1
$f(x)$	1	2

Let f be the function given by $f(x) = 2^{x^3}$. Selected values of f are given in the table above. If the values in the table are used to approximate $f'(0.5)$, what is the difference between the approximation and the actual value of $f'(0.5)$?

A 0

B 0.433

C 0.567

D 1

Question 3

Let f be the function given by $f(x) = \frac{1}{7}x^7 + \frac{1}{2}x^6 - x^5 - \frac{15}{4}x^4 + \frac{4}{3}x^3 + 6x^2$. Which of the following statements is true?

A $f'(-3.1) < f'(-1.5) < f'(0.4)$

B $f'(-3.1) < f'(0.4) < f'(-1.5)$

C $f'(-1.5) < f'(0.4) < f'(-3.1)$

D $f'(0.4) < f'(-1.5) < f'(-3.1)$

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 4

x	1	2	3	4	5
$f(x)$	2	3	5	6	14

Selected values of a function f are shown in the table above. What is the average rate of change of f over the interval $[1, 5]$?

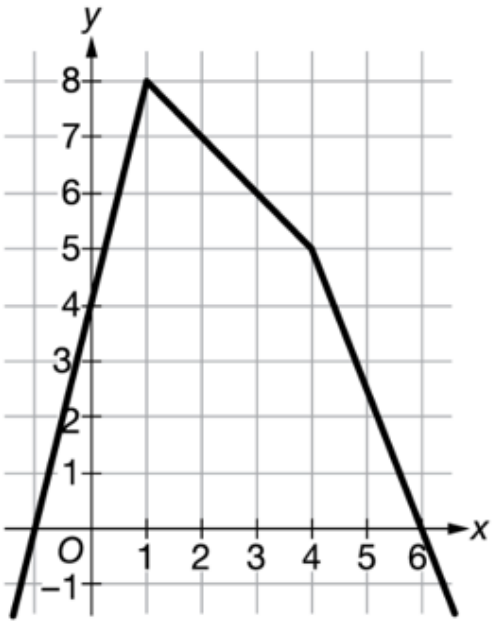
A $\frac{5-1}{14-2}$

B $\frac{14+2}{5+1}$

C $\frac{14-2}{5-1}$

D $\frac{2+3+5+6+14}{5}$

Question 5



The graph of the function f, shown above, consists of three line segments. What is the average rate of change of f over the interval $-1 \leq x \leq 6$?

A $-\frac{5}{2}$

B 0

C $\frac{1}{6}$

D 4

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 6

The function f is given by $f(x) = 1 + 3\cos x$. What is the average rate of change of f over the interval $[0, \pi]$?

A $-\frac{6}{\pi}$

B $-\frac{2}{\pi}$

C $\frac{2}{\pi}$

D 1

Question 7

The derivative of the function f is given by $f'(x) = -3x + 4$ for all x, and $f(-1) = 6$. Which of the following is an equation of the line tangent to the graph of f at $x = -1$?

A $y = -3x + 3$

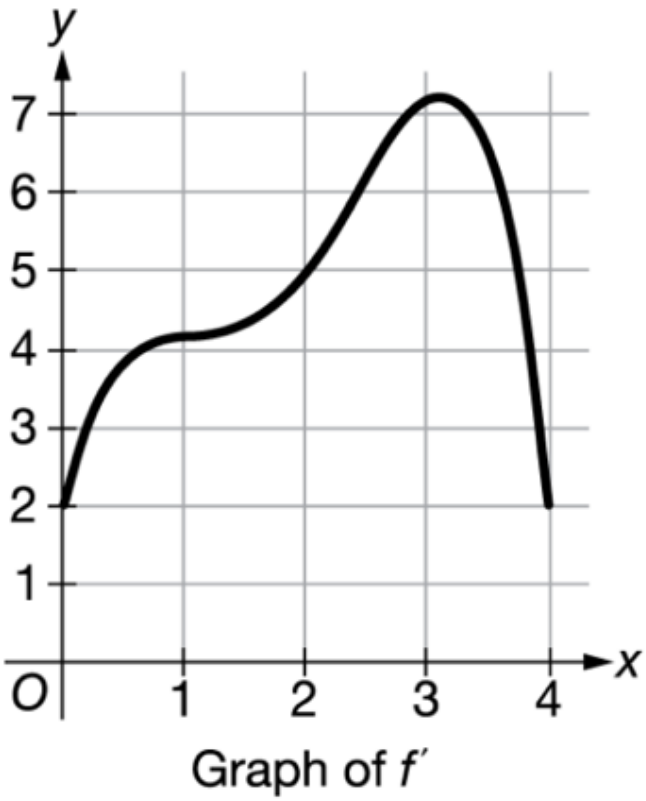
B $y = -3x + 4$

C $y = 7x + 6$

D $y = 7x + 13$

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 8



Graph of f'

The graph of f', the derivative of a function f, is shown above. The points $(2, 6)$ and $(4, 18)$ are on the graph of f. Which of the following is an equation of the line tangent to the graph of f at $x = 2$?

(A) $y = 2x + 1$

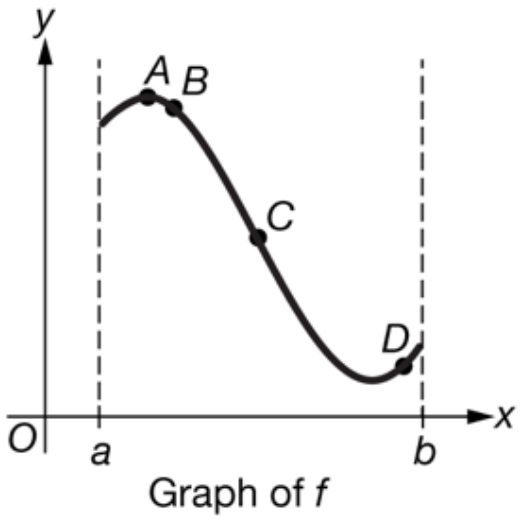
(B) $y = 5x - 4$

(C) $y = 5x - 10$

(D) $y = 6x - 6$

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 9



Graph of f

The graph of the trigonometric function f is shown above for $a \leq x \leq b$. At which of the following points on the graph of f could the instantaneous rate of change of f equal the average rate of change of f on the interval $[a, b]$?

(A) A

(B) B

(C) C

(D) D

Question 10

Which of the following statements, if true, can be used to conclude that $f(2)$ exists?

I. $\lim_{x \to 2} f(x)$ exists.
II. f is continuous at $x = 2$.
III. f is differentiable at $x = 2$.

(A) I only

(B) II only

(C) II and III only

(D) I, II, and III

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 11

$$f(x) = \begin{cases} 3x + 1 & \text{for } x \leq 2 \\ 5x - 3 & \text{for } x > 2 \end{cases}$$

Let f be the function defined above. Which of the following statements is true?

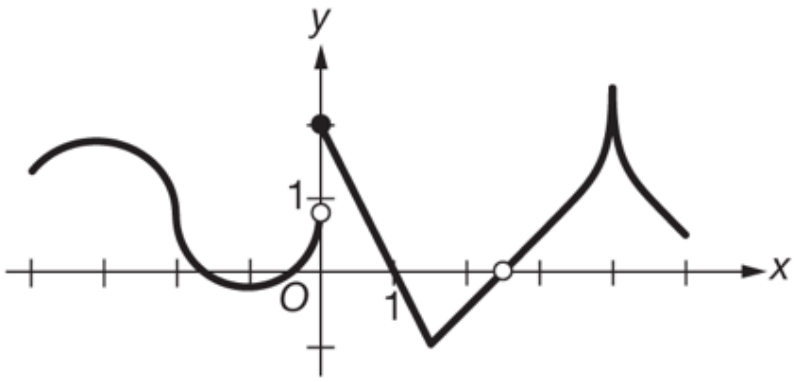
(A) f is neither continuous nor differentiable at $x = 2$.

(B) f is continuous but not differentiable at $x = 2$.

(C) f is differentiable but not continuous at $x = 2$.

(D) f is both continuous and differentiable at $x = 2$.

Question 12



Graph of f

The graph of the function f, shown above, has a vertical tangent at $x = -2$ and horizontal tangents at $x = -3$ and $x = -1$. Which of the following statements is false?

(A) f is not differentiable at $x = -2$ because the graph of f has a vertical tangent at $x = -2$.

(B) f is not differentiable at $x = 0$ and $x = 2.5$ because f is not continuous at $x = 0$ and $x = 2.5$.

(C) f is not differentiable at $x = 1.5$ and $x = 4$ because the graph of f has sharp corners at $x = 1.5$ and $x = 4$.

(D) f is not differentiable at $x = -3$ and $x = -1$ because the graph of f has horizontal tangents at $x = -3$ and $x = -1$.

A.P. Calculus Unit 2 Progress Check: MCQ Part A

Question 13

If $f(x) = x^5$, then $f'(x) =$

A x^4

B $4x^4$

C $5x^4$

D $5x^5$

Question 14

If $f(x) = \frac{1}{x^7}$, then $f'(x) =$

A $\frac{1}{7x^6}$

B $-\frac{7}{x^6}$

C $-\frac{1}{7x^8}$

D $-\frac{7}{x^8}$

Question 15

If f is the function defined by $f(x) = \sqrt[4]{x}$, what is $f'(x)$?

A $\frac{1}{4}x^{\frac{1}{4}}$

B $x^{-\frac{3}{4}}$

C $\frac{1}{4}x^{-\frac{3}{4}}$

D $4 \cdot \sqrt[3]{x}$

A.P. Calculus Unit 2 Progress Check: MCQ Part B

Question 1

Let g be the function given by $g(x) = x^4 - 3x^3 - x$. What are all values of x such that $g'(x) = \frac{1}{2}$?

A -2.750

B 2.297

C 2.320

D -0.353 and 3.119

Question 2

Let f be the function given by $f(x) = x^3 + 3x^2 - 4$. What is the value of $f'(2)$?

A 48

B 24

C 20

D 10

Question 3

If $f(x) = 4x^6 - 3x^4 + 2x^3 + e^2$, then $f'(x) =$

A $4x^5 - 3x^3 + 2x^2$

B $24x^5 - 12x^3 + 6x^2$

C $24x^5 - 12x^3 + 6x^2 + 2e$

D $24x^6 - 12x^4 + 6x^3$

A.P. Calculus Unit 2 Progress Check: MCQ Part B

Question 4

If $g(x) = 4\cos x + 2\sin x + 1$, then $g'\left(\frac{\pi}{6}\right) =$

A $-2+\sqrt{3}$

B $2-\sqrt{3}$

C $2+\sqrt{3}$

D $2+2\sqrt{3}$

Question 5

Let g be the function given by $g(x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$. What is the instantaneous rate of change of g with respect to x at $x = \frac{\pi}{3}$?

A $-\frac{\sqrt{3}}{2}$

B $-\frac{1}{2}$

C $\frac{1}{2}$

D $\frac{\sqrt{3}}{2}$

Question 6

$\lim_{h\to 0}\frac{5e^x-5e^{x+h}}{3h} =$

A $-5e^x$

B $5e^x$

C $-\frac{5}{3}e^x$

D $\frac{5}{3}e^x$

A.P. Calculus Unit 2 Progress Check: MCQ Part B

Question 7

The function f is given by $f(x) = (x^3 + bx + 6)g(x)$, where b is a constant and g is a differentiable function satisfying $g(2) = 3$ and $g'(2) = -1$. For what value of b is $f'(2) = 0$?

A -7

B -10

C -12

D -22

Question 8

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
4	2	7	3	5

The table above gives the values of the differentiable functions f and g and their derivatives at $x = 4$. What is the value of $\frac{d}{dx}(f(x)g(x))$ at $x = 4$?

A 11

B 29

C 31

D 35

Question 9

If $f(x) = \sqrt{x}\cos x$, then $f'(x) =$

A $-\frac{\sin x}{2\sqrt{x}}$

B $\frac{1-2\sqrt{x}\sin x}{2\sqrt{x}}$

C $\frac{\cos x-2x\sin x}{2\sqrt{x}}$

D $\frac{\cos x+2x\sin x}{2\sqrt{x}}$

A.P. Calculus Unit 2 Progress Check: MCQ Part B

Question 10

If $f(x) = \frac{2x^2-1}{5x+3}$, then $f'(-1) =$

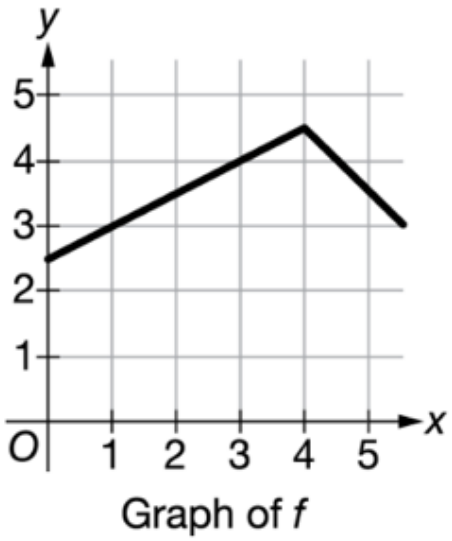
A $-\frac{3}{2}$

B $-\frac{4}{5}$

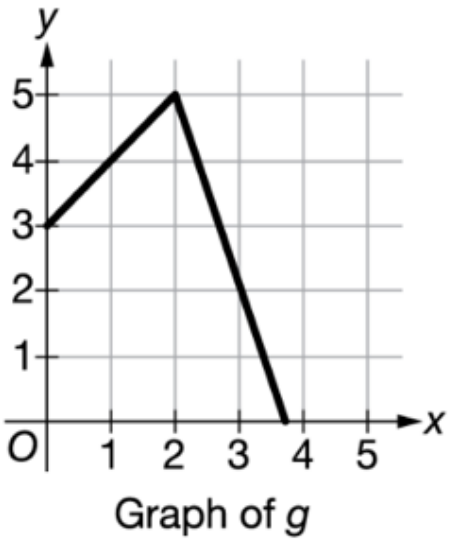
C $\frac{3}{4}$

D $\frac{13}{4}$

Question 11



Graph of f



Graph of g

The graphs of the functions f and g are shown above. If $h(x) = \frac{f(x)+4}{g(x)+2x}$, then $h'(3) =$

A $-\frac{1}{2}$

B $-\frac{1}{16}$

C $\frac{3}{16}$

D $\frac{3}{2}$

A.P. Calculus Unit 2 Progress Check: MCQ Part B

Question 12

What is the slope of the line tangent to the graph of $y = \frac{9x^2}{x+2}$ at $x = 1$?

A 3

B 5

C 7

D 18

Question 13

$\frac{d}{dx}(\tan x) =$

A $-\cot x$

B $-\csc^2 x$

C $\cot x$

D $\sec^2 x$

A.P. Calculus Unit 2 Progress Check: MCQ Part B

Question 14

$\frac{d}{dx}(\csc x) =$

A $\sec x$

B $-\sec x$

C $\csc x \cot x$

D $-\csc x \cot x$

Question 15

Below is an attempt to derive the derivative of $\sec x$ using the product rule, where x is in the domain of $\sec x$. In which step, if any, does an error first appear?

Step 1: $\sec x \cdot \cos x = 1$
Step 2: $\frac{d}{dx}(\sec x \cdot \cos x) = 0$
Step 3: $\frac{d}{dx}(\sec x) \cdot \cos x - \sec x \cdot \sin x = 0$
Step 4: $\frac{d}{dx}(\sec x) = \frac{\sec x \cdot \sin x}{\cos x} = \sec x \cdot \frac{\sin x}{\cos x} = \sec x \cdot \tan x$

A Step 1

B Step 2

C Step 3

D There is no error.

A.P. Calculus Unit 3 Progress Check: MCQ

Question 1

If $g(x) = \ln x$ and f is a differentiable function of x, which of the following is equivalent to the derivative of $f(g(x))$ with respect to x ?

A $f'\left(\frac{1}{x}\right)$

B $\frac{f'(x)}{x}$

C $f'(\ln x)$

D $\frac{f'(\ln x)}{x}$

Question 2

For which of the following functions is the chain rule an appropriate method to find the derivative with respect to x ?

I. $y = \sin\left(3x^2\right)$
II. $y = e^x \tan x$
III. $y = \frac{1}{8x^4 - 2x}$

A I only

B II only

C III only

D I and III only

Question 3

Let f be a differentiable function. If $h(x) = (1 + f(3x))^2$, which of the following gives a correct process for finding $h'(x)$?

A $h'(x) = 2(1 + f(3x))$

B $h'(x) = 2(1 + f(3x)) \cdot f'(3x)$

C $h'(x) = 2(1 + f(3x)) \cdot f'(x)$

D $h'(x) = 2(1 + f(3x)) \cdot f'(3x) \cdot 3$

A.P. Calculus Unit 3 Progress Check: MCQ

Question 4

What is the slope of the line tangent to the curve $y^3 - xy^2 + x^3 = 5$ at the point $(1, 2)$?

A $\frac{1}{10}$

B $\frac{1}{8}$

C $\frac{5}{12}$

D $\frac{11}{4}$

Question 5

If $\sin(x + y) = 3x - 2y$, then $\frac{dy}{dx} =$

A $\frac{3-\cos(x+y)}{2}$

B $\frac{1-\cos(x+y)}{\cos(x+y)}$

C $\frac{3}{2+\cos(x+y)}$

D $\frac{3-\cos(x+y)}{2+\cos(x+y)}$

Question 6

$f(-2) = 3$	$f'(-2) = 4$	$g(4) = 5$	$g'(4) = 2$

The point $(-2, 4)$ lies on the curve in the xy-plane given by the equation $f(x)g(y) = 17 - x - y$, where f is a differentiable function of x and g is a differentiable function of y. Selected values of f, f', g, and g' are given in the table above. What is the value of $\frac{dy}{dx}$ at the point $(-2, 4)$?

A -27

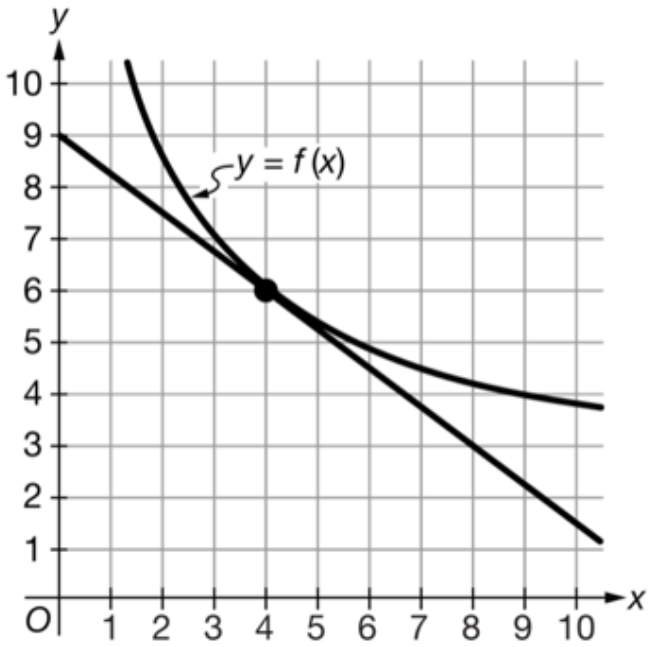
B $-\frac{11}{3}$

C -3

D $-\frac{4}{7}$

A.P. Calculus Unit 3 Progress Check: MCQ

Question 7



The graph of the decreasing differentiable function f is shown above. Also shown is the line tangent to the graph of f at the point $(4, 6)$. Let g be the inverse of f. Which of the following statements about g' is true?

A $g'(4) = -\frac{4}{3}$

B $g'(4) = -\frac{3}{4}$

C $g'(6) = -\frac{4}{3}$

D $g'(6) = -\frac{3}{4}$

Question 8

Let f be the increasing function defined by $f(x) = x^3 + 2x^2 + 4x + 5$, where $f(-1) = 2$. If g is the inverse function of f, which of the following is a correct expression for $g'(2)$?

A $g'(2) = \frac{1}{f'(2)}$

B $g'(2) = \frac{1}{f'(-1)}$

C $g'(2) = f'(-1)$

D $g'(2) = f'(2)$

A.P. Calculus Unit 3 Progress Check: MCQ

Question 9

x	0	2	4
$f(x)$	8	5	2
$f'(x)$	-1	-2	-5

The table above gives selected values for a differentiable and decreasing function f and its derivative. If $g(x) = f^{-1}(x)$ for all x, which of the following is a correct expression for $g'(2)$?

A $g'(2) = f'(2) = -2$

B $g'(2) = \frac{1}{f'(2)} = -\frac{1}{2}$

C $g'(2) = \frac{1}{f'(4)} = -\frac{1}{5}$

D $g'(2) = -\frac{f'(2)}{(f(2))^2} = \frac{2}{25}$

Question 10

$\frac{d}{dx}\left(\sin^{-1}x\right)\Big|_{x=\frac{1}{2}} =$

A $\frac{1}{1+\left(\frac{1}{2}\right)^2}$

B $\frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$

C $\cos^{-1}\left(\frac{1}{2}\right)$

D $-\csc\left(\frac{1}{2}\right)\cot\left(\frac{1}{2}\right)$

Question 11

$\frac{d}{dx}\left(\cos^{-1}x\right) =$

A $-\frac{1}{\sqrt{1-x^2}}$

B $\frac{1}{\sqrt{1-x^2}}$

C $-\sin^{-1}x$

D $-\cos^{-2}x$

A.P. Calculus Unit 3 Progress Check: MCQ

Question 12

Which of the following methods can be used to find the derivative of $y = \arcsin x$ with respect to x ?

I. Use the quotient rule to differentiate $\frac{1}{\sin x}$.
II. Use the chain rule to differentiate $\sin(\arcsin x) = x$.
III. Use implicit differentiation to differentiate the function y in the relation $\sin y = x$ with respect to x.

A. I only
B. III only
C. II and III only
D. I, II, and III

Question 13

Which of the following expressions can be differentiated using the product rule?

A. $\cos(\sqrt{x})$
B. $x^2 \tan^{-1} x$
C. $x^4 + \arcsin x$
D. $(8x^3 - 5x + 2)^\pi$

Question 14

Which of the following requires the use of implicit differentiation to find $\frac{dy}{dx}$?

A. $y - x^2 - 3x + 5 = 0$
B. $y = \ln(3 + x) + x^2$
C. $y = \ln(y + x) + x^2$
D. $y = \frac{x^3 - 4}{3x + 2}$

A.P. Calculus Unit 3 Progress Check: MCQ

Question 15

For which of the following functions would the quotient rule be considered the best method for finding the derivative?

A $y = (2x+1)^{-\frac{1}{2}}$

B $y = \frac{2x+1}{x}$

C $y = \sin^{-1}(2x+1)$

D $y = \frac{\sin(2x+1)}{2x+1}$

Question 16

If $y = 2\ln x$, then $\frac{d^4y}{dx^4} =$

A $\frac{2}{x}$

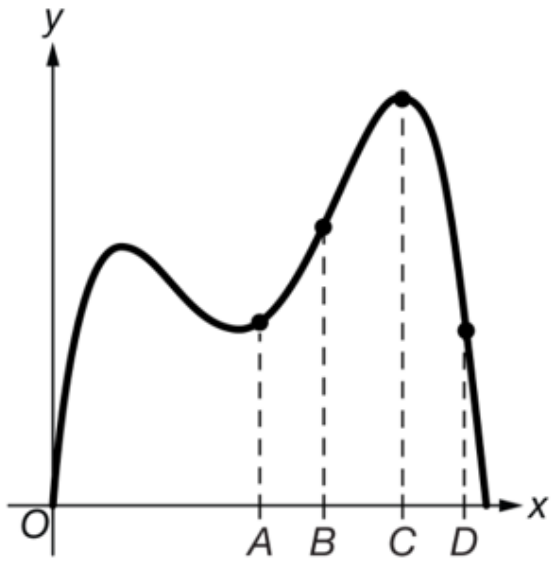
B $-\frac{12}{x^4}$

C $\frac{16}{x^4}$

D $\frac{48}{x^5}$

A.P. Calculus Unit 3 Progress Check: MCQ

Question 17



Graph of f'

The figure above shows the graph of f', the derivative of the function f. At which of the four indicated values of x is $f''(x)$ greatest?

(A) A

(B) B

(C) C

(D) D

Question 18

Let $y = f(x)$ be a twice-differentiable function such that $f(1) = 3$ and $\frac{dy}{dx} = 4\sqrt{y^2 + 7x^2}$. What is the value of $\frac{d^2y}{dx^2}$ at $x = 1$?

(A) 10

(B) 23

(C) 55

(D) 160

A.P. Calculus Unit 3 Progress Check: FRQ Part A

Question 1

A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	-3	0	3	6
$f(x)$	-5	4	1	7
$f'(x)$	-1	2	-2	4

The table above gives values of a twice-differentiable function f and its first derivative f' for selected values of x. Let g be the function defined by $g(x) = f(x^2 - x)$.

(a) What is the value of $g'(3)$?

(b) It is known that $g''(0) = -1$. What is the value of $f''(0)$?

(c) Is there a value c, for $0 < c < 3$, such that $g(c) = 5$? Justify your answer.

(d) Let h be the function with derivative given by $h'(x) = 4e^{\cos x}$. At what value of x in the interval $-3 \leq x \leq 0$ does the instantaneous rate of change of h equal the average rate of change of f over the interval $-3 \leq x \leq 0$?

A.P. Calculus Unit 3 Progress Check: FRQ Part B

Question 1

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the curve given by the equation $(2y+1)^3 - 24x = -3$.

(a) Show that $\frac{dy}{dx} = \frac{4}{(2y+1)^2}$.

(b) Write an equation for the line tangent to the curve at the point $(-1, -2)$.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$.

(d) The point $\left(\frac{1}{6}, 0\right)$ is on the curve. Find the value of $\left(y^{-1}\right)'(0)$.

A.P. Calculus Unit 3 Progress Check: FRQ Part B

Question 2

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	-3	-2	-1	1
$f(x)$	$-\frac{5}{2}$	-3	-2	$\frac{2}{3}$
$f'(x)$	-1	$\frac{1}{3}$	$\frac{6}{5}$	$\frac{4}{3}$

The table above gives values of the differentiable function f and its derivative for selected values of x.

(a) Let g be the function defined by $g(x) = \frac{f(x^2)}{e^x}$. Find $g'(-1)$.

Let h be the function defined by $h(x) = f(f(-2x))$. Find $h'(1)$.

Let k be the function defined by $k(x) = f(x) \cdot \arcsin\left(\frac{x}{2}\right)$. Find $k'(-1)$.

A.P. Calculus Unit 4 Progress Check: MCQ

Question 1

The mass of a colony of bacteria, in grams, is modeled by the function P given by $P(t) = 2 + 5\tan^{-1}\left(\frac{t}{2}\right)$, where t is measured in days. What is the instantaneous rate of change of the mass of the colony, in grams per day, at the moment the colony reaches a mass of 6 grams?

A -0.606

B 0.250

C 1.214

D 1.942

Question 2

The derivative of the function A is given by $A'(t) = 2 + 9e^{0.4\sin t}$, and $A(1.2) = 7.5$. If the linear approximation to $A(t)$ at $t = 1.2$ is used to estimate $A(t)$, at what value of t does the linear approximation estimate that $A(t) = 15$?

A 0.498

B 1.166

C 1.698

D 2.400

Question 3

Oil is spilled onto a kitchen floor. The area covered by the oil at time t is given by the function A, where $A(t)$ is measured in square centimeters and t is measured in seconds. Which of the following gives the rate at which the area covered by the oil is changing at time $t = 10$?

A $A(10)$

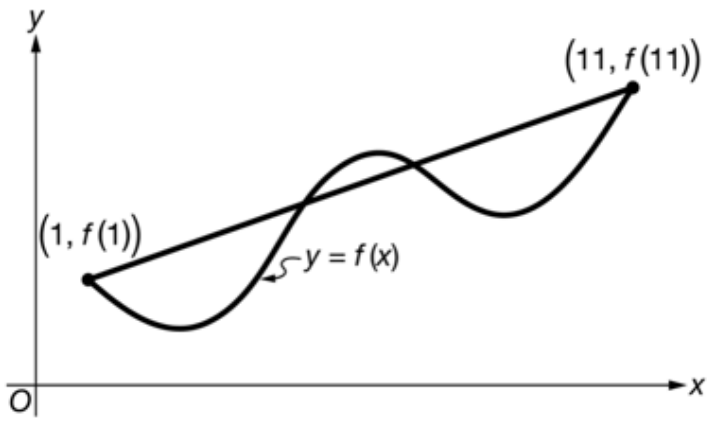
B $A(11) - A(9)$

C $\frac{A(10)}{10}$

D $A'(10)$

A.P. Calculus Unit 4 Progress Check: MCQ

Question 4



The figure above shows the graph of the differentiable function f for $1 \leq x \leq 11$ and the secant line through the points $(1, f(1))$ and $(11, f(11))$. For how many values of x in the closed interval $[1, 11]$ does the instantaneous rate of change of f at x equal the average rate of change of f over that interval?

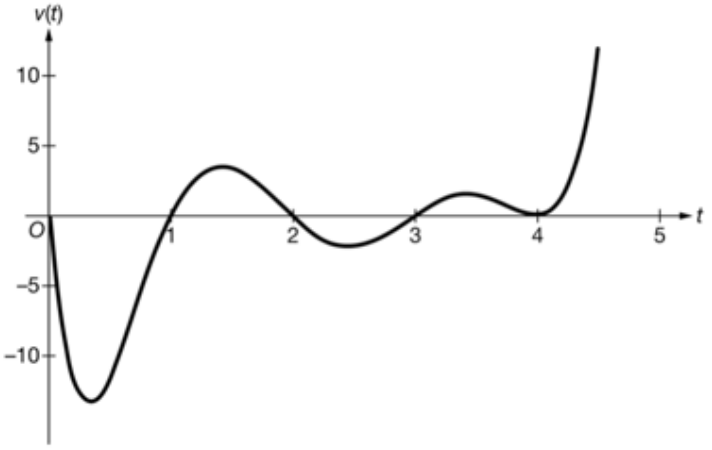
(A) Zero

(B) Two

(C) Three

(D) Four

Question 5



A particle moves along the y-axis. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 < t < 4.5$. How many times does the particle change direction over the time interval $0 < t < 4.5$?

(A) Three

(B) Four

(C) Five

(D) Eight

A.P. Calculus Unit 4 Progress Check: MCQ

Question 6

A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 + 3t^2 - 36t + 50$. What is the total distance traveled by the particle over the time interval $0 \leq t \leq 5$?

A 145

B 180

C 195

D 233

Question 7

A particle moves along the y-axis so that at time $t \geq 0$ its position is given by $y(t) = t^3 - 6t^2 + 9t$. Over the time interval $0 < t < 4$, for what values of t is the speed of the particle increasing?

A $2 < t < 4$

B $3 < t < 4$ only

C $0 < t < 1$ and $3 < t < 4$

D $1 < t < 2$ and $3 < t < 4$

Question 8

Charles's law states that if the pressure of a dry gas is held constant, then the volume V of the gas and its temperature T, measured in degrees Kelvin, satisfy the relationship $V = cT$, where c is a constant. Which of the following best describes the relationship between the rate of change, with respect to time t, of the volume and the rate of change, with respect to time t, of the temperature?

A $\frac{dV}{dt} = T\frac{dc}{dt}$

B $\frac{dV}{dt} = c\frac{dT}{dt}$

C $\frac{dV}{dT} = c$

D $1 = c\frac{dT}{dV}$

A.P. Calculus Unit 4 Progress Check: MCQ

Question 9

A rectangle has width w inches and height h inches, where the width is twice the height. Both w and h are functions of time t, measured in seconds. If A represents the area of the rectangle, which of the following gives the rate of change of A with respect to t ?

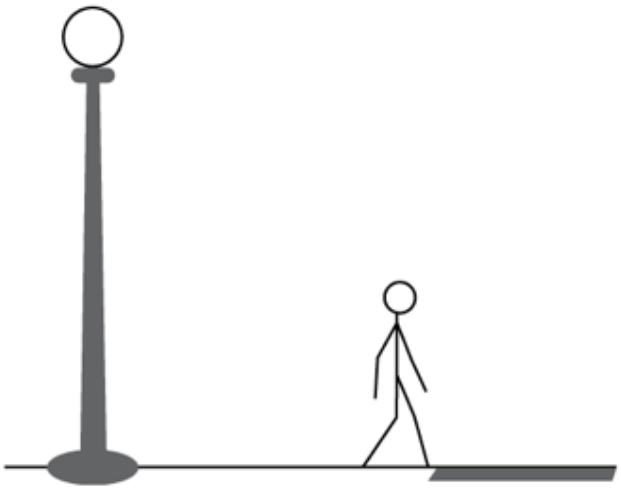
(A) $\frac{dA}{dt} = 4h \text{ in/ sec}$

(B) $\frac{dA}{dt} = 3h\frac{dh}{dt} \text{ in}^2\text{/ sec}$

(C) $\frac{dA}{dt} = 4h\frac{dh}{dt} \text{ in/ sec}$

(D) $\frac{dA}{dt} = 4h\frac{dh}{dt} \text{ in}^2\text{/ sec}$

Question 10



A person whose height is M feet is walking away from the base of a streetlight along a straight road, as shown in the figure above. The height of the streetlight is L feet. At time t seconds, the person is x feet from the streetlight, and the length of the person's shadow is s feet. The quantities are related by the equation $\frac{1}{L}(x+s) = \frac{1}{M}s$, where L and M are constants. Which of the following best describes the relationship between the rate of change of x with respect to time and the rate of change of s with respect to time?

(A) $\frac{dx}{dt} = \frac{L}{M}s - s$

(B) $\frac{dx}{dt} = \frac{L}{M}s - \frac{ds}{dt}$

(C) $\frac{dx}{dt} = \frac{L}{M}\frac{ds}{dt} - s$

(D) $\frac{dx}{dt} = \frac{L}{M}\frac{ds}{dt} - \frac{ds}{dt}$

A.P. Calculus Unit 4 Progress Check: MCQ

Question 11

A particle moves on the circle $x^2 + y^2 = 25$ in the xy-plane for time $t \geq 0$. At the time when the particle is at the point $(3, 4)$, $\frac{dx}{dt} = 6$. What is the value of $\frac{dy}{dt}$ at this time?

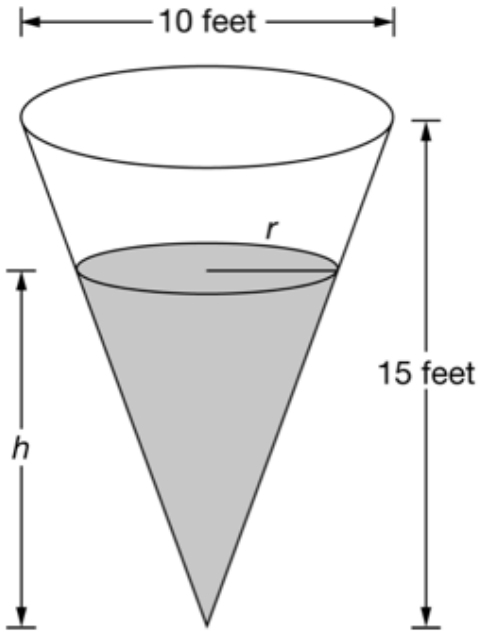
A $\frac{dy}{dt} = \frac{19}{8}$

B $\frac{dy}{dt} = -\frac{3}{4}$

C $\frac{dy}{dt} = -\frac{11}{8}$

D $\frac{dy}{dt} = -\frac{9}{2}$

Question 12



A water tank is in the shape of a right circular cone as shown above. The diameter of the cone is 10 feet, and the height is 15 feet. The shape of the water in the tank is conical with radius r feet and height h feet. At noon, water is leaking from the bottom of the tank at a rate of 12 cubic feet per hour, and the volume of water in the tank is 27π cubic feet. At noon, what is the rate at which the height of the water in the tank is changing? (The volume V of a right circular cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

A Decreasing at $\frac{4}{9\pi}$ feet per hour

B Increasing at $\frac{4}{9\pi}$ feet per hour

C Increasing at $\frac{4}{3\pi}$ feet per hour

D Decreasing at $\frac{4}{3\pi}$ feet per hour

A.P. Calculus Unit 4 Progress Check: MCQ

Question 13

Two straight roads intersect at right angles. A car is traveling south toward the intersection while a radar-monitored-speed sign is positioned $\frac{1}{4}$ mile east of the intersection. The speed sign provides both the distance from the sign to an approaching car and the rate at which the distance between the sign and the car is changing. At a certain time, the speed sign shows that the approaching car is exactly $\frac{1}{2}$ mile away from the sign and that the rate at which the distance between the car and the sign is decreasing is 60 miles per hour. Which of the following is true about the distance between the car and the intersection at this instant?

(A) The distance is increasing at a rate of $\frac{120}{\sqrt{3}}$ miles per hour.

(B) The distance is decreasing at a rate of $\frac{120}{\sqrt{3}}$ miles per hour.

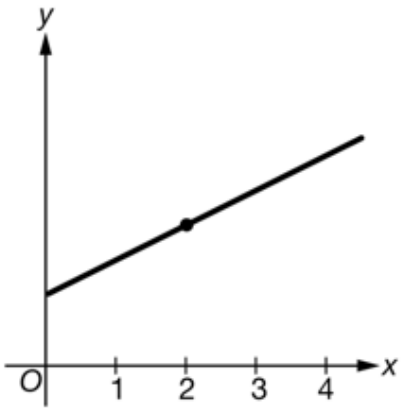
(C) The distance is increasing at a rate of $\frac{30}{\sqrt{3}}$ miles per hour.

(D) The distance is decreasing at a rate of $\frac{30}{\sqrt{3}}$ miles per hour.

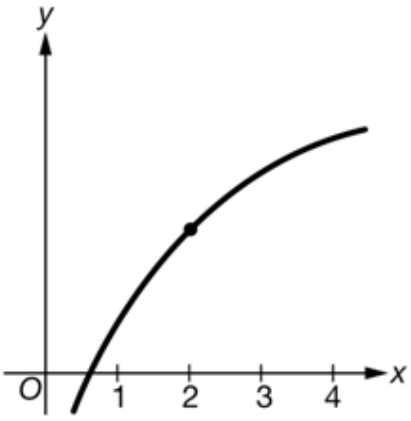
Question 14

The locally linear approximation of the differentiable function f at $x = 2$ is used to approximate the value of $f(2.3)$. The approximation at $x = 2.3$ is an underestimate of the corresponding function value at $x = 2.3$. Which of the following could be the graph of f?

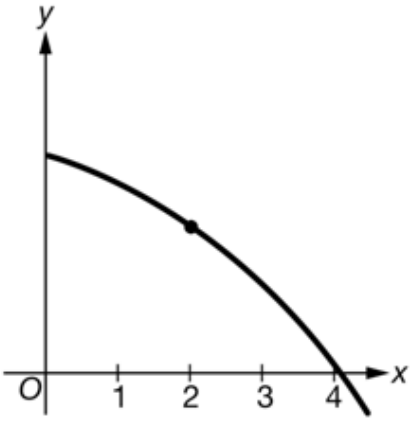
(A)



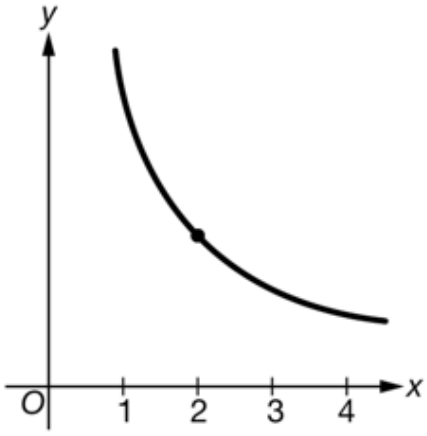
(B)



(C)



(D)



A.P. Calculus Unit 4 Progress Check: MCQ

Question 15

The line tangent to the graph of the twice-differentiable function f at the point $x = 5$ is used to approximate the value of $f(5.25)$. Which of the following statements guarantees that the tangent line approximation at $x = 5.25$ is an overestimate of $f(5.25)$?

(A) The function f is decreasing on the interval $5 \leq x \leq 5.25$.

(B) The function f is increasing on the interval $5 \leq x \leq 5.25$.

(C) The graph of the function f is concave down on the interval $5 \leq x \leq 5.25$.

(D) The graph of the function f is concave up on the interval $5 \leq x \leq 5.25$.

Question 16

Which of the following limits does not yield an indeterminate form?

(A) $\lim_{x \to 0} \frac{3x^2}{x - \sin x}$

(B) $\lim_{x \to 2} \frac{\ln\left(\frac{x}{2}\right)}{x^2 - 5x + 6}$

(C) $\lim_{x \to \pi} \frac{x - \pi}{\cos x}$

(D) $\lim_{x \to \infty} \frac{e^{3x}}{x^{100}}$

A.P. Calculus Unit 4 Progress Check: MCQ

Question 17

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
3	2	-4	1	2

Selected values of the twice-differentiable functions f and g and their derivatives are given in the table above. The value of $\lim_{x \to 3} \frac{x^3 f(x) - 54}{g(x) - 1}$ is

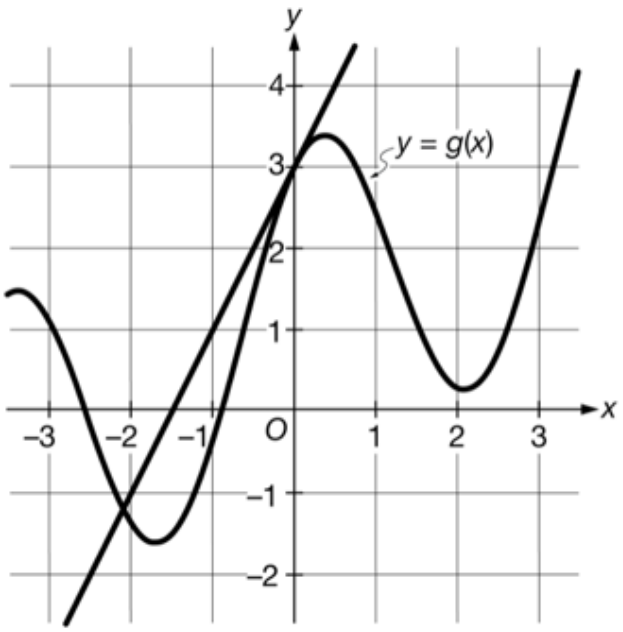
(A) -108

(B) -54

(C) -27

(D) nonexistent

Question 18



The figure above shows the graph of the twice-differentiable function g and the line tangent to the graph of g at the point $(0, 3)$. The value of $\lim_{x \to 0} \frac{g(x)e^{-x} - 3}{x^2 - 2x}$ is

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{5}{2}$

(D) nonexistent

A.P. Calculus Unit 4 Progress Check: FRQ Part A

Question 1

A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

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Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

For time $0 \le t \le 10$, water is flowing into a small tub at a rate given by the function F defined by $F(t) = \arctan\left(\frac{\pi}{2} - \frac{t}{10}\right)$. For time $5 \le t \le 10$, water is leaking from the tub at a rate given by the function L defined by $L(t) = 0.03\left(20t - t^2 - 75\right)$. Both $F(t)$ and $L(t)$ are measured in cubic feet per minute, and t is measured in minutes. The volume of water in the tub, in cubic feet, at time t minutes is given by $W(t)$.

(a) At time $t = 3$, there are 2.5 cubic feet of water in the tub. Write an equation for the locally linear approximation of W at $t = 3$, and use it to approximate the volume of water in the tub at time $t = 3.5$.

(b) Find $W''(8)$. Using correct units, interpret the meaning of $W''(8)$ in the context of the problem.

(c) Is there a time t, for $5 < t < 10$, at which the rate of change of the volume of water in the tub changes from positive to negative? Give a reason for your answer.

(d) The tub is in the shape of a rectangular box that is 0.5 foot wide, 4 feet long, and 3 feet deep. What is the rate of change of the depth of the water in the tub at time $t = 6$?

A.P. Calculus Unit 4 Progress Check: FRQ Part B

Question 1

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$W(t) = \begin{cases} \frac{17}{2} - \frac{3}{2}\cos\left(\frac{\pi t}{6}\right) & \text{for } 0 \le t \le 6 \\ 10 - \frac{1}{5}(t-6)^2 & \text{for } 6 < t \le 10 \end{cases}$$

The depth of a river at a certain point is modeled by the function W defined above, where $W(t)$ is measured in feet and time t is measured in hours.

(a) Find $W'(8)$. Using correct units, explain the meaning of $W'(8)$ in the context of the problem.

(b) The graph of W is concave down for $3 \le t \le 3.5$. Use the line tangent to the graph of W at $t = 3$ to show that $W(3.5) \le 9$.

(c) Find $\lim_{t \to 2} \frac{W(t) - t^3 + \frac{1}{4}}{t-2}$.

A.P. Calculus Unit 4 Progress Check: FRQ Part B

Question 2

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t) = \frac{t^2-9}{3t^2+8}$.

(a) Show that the velocity of the particle at time t is given by $v(t) = \frac{70t}{(3t^2+8)^2}$.

(b) Is the particle moving toward the origin or away from the origin at time $t = 2$? Give a reason for your answer.

(c) The acceleration of the particle is given by $a(t)$. Write an expression for $a(t)$, and find the value of $a(2)$.

(d) What position does the particle approach as t approaches infinity?