Scales, Chords, and Cadences: Practical Music Theory for MIR Researchers

Scales

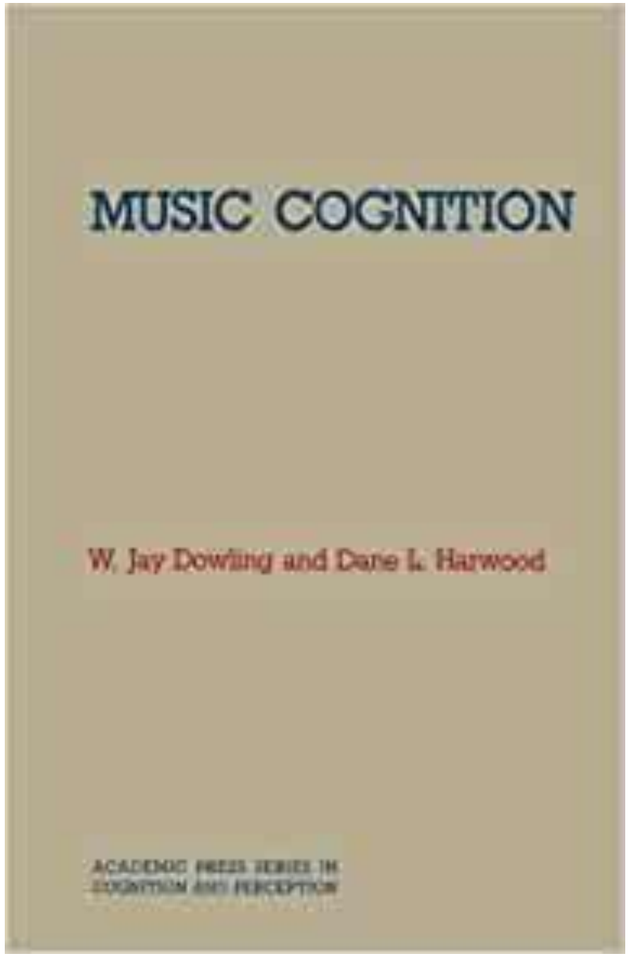
Main Topics

- **References**
- **Terminology**
- **Scale Construction**
  - Constraints
  - Properties
- **Common Scales**
  - 12 TET
  - Beyond 12-TET
- **MIR Tasks**
- **Recent Work!**
- **Main Takeaways**

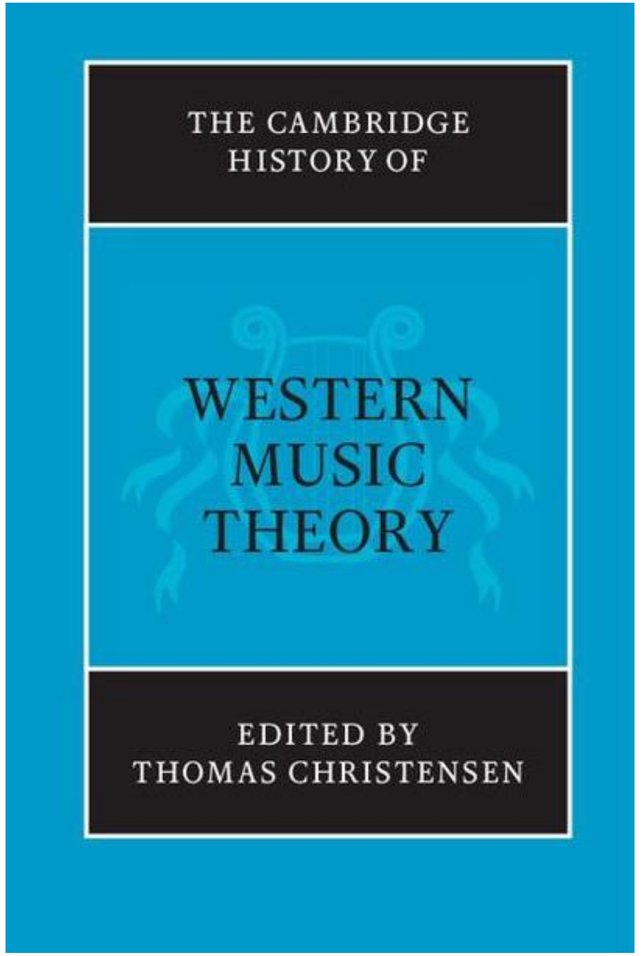
References

References

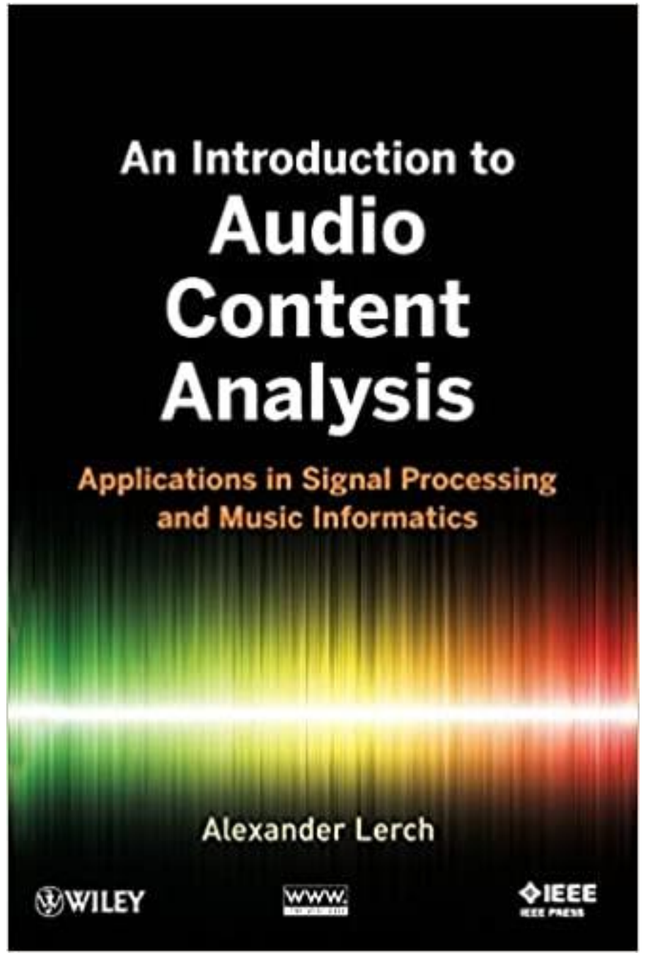
Music Psychology;
Ch. 4



Western Music Theory;
Chs. 11-13, 23



Music Information Retrieval;
Ch. 5



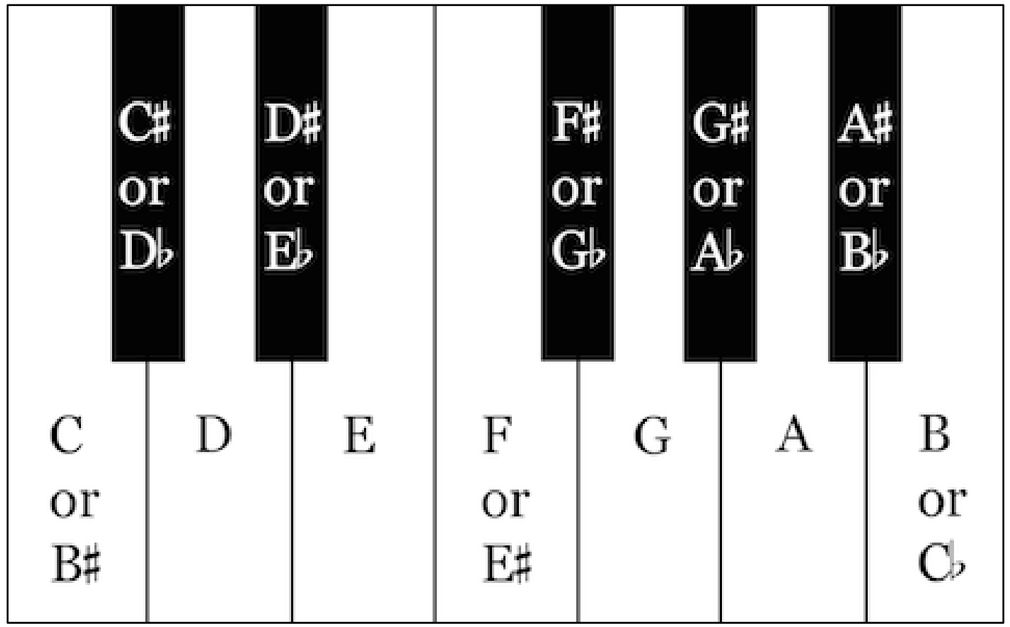
Terminology

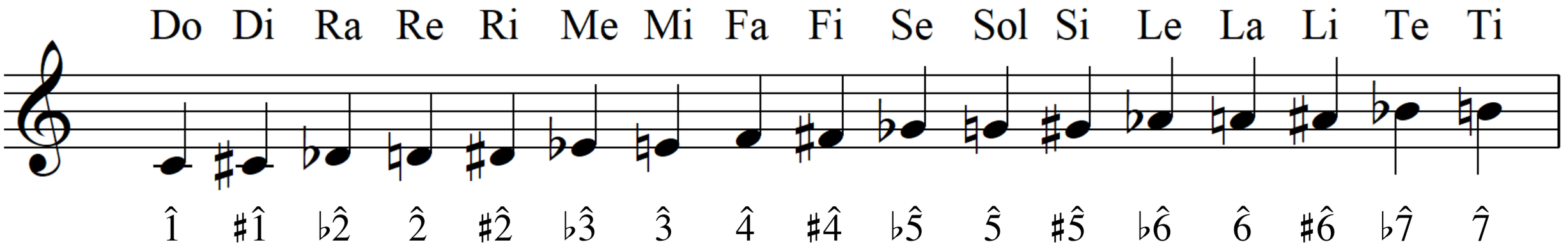
Terminology

- **Pitch**
  - An auditory attribute produced by a sound event consisting of periodic waveforms. In complex tones, the perceived pitch typically corresponds to the fundamental frequency, F_0.
- **Pitch Class (Chroma)**
  - Perceived similarity (or equivalence) between two pitches separated by an octave (i.e., a doubling of F_0).
- **Interval**
  - ΔF_0 (or *distance*) between two tones. Increasing intervals denote an increase in *pitch height*.
- **Scale/Mode**
  - A pitch collection ordered by pitch height (i.e., repetition rate of F_0). Pitches belonging to a scale are *diatonic*.
- **Key**
  - The most stable, salient, or memorable pitch within the scale. Sometimes called the *tonal center*.
- **Tuning vs. Temperament**
  - Tuning refers to the F_0 of a reference pitch (e.g., A_4=440Hz).
  - Temperament refers to the system of frequency ratios that define the intervals encountered in a scale. Examples include Pythagorean, just, meantone, and equal-temperament.

Terminology

Basics -- Pitch Labels





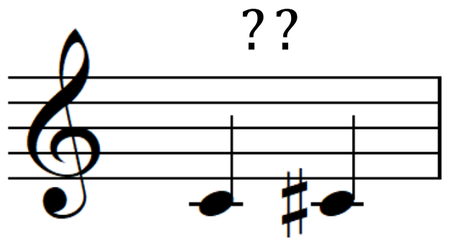
Terminology

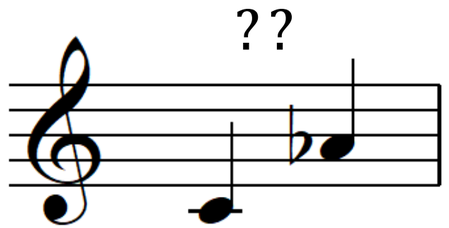
Basics -- Interval Labels

- Size (2nd, 3rd, etc.)
- Quality (dim, minor, major, perfect, augmented)

	unis.	2nd	3rd	4th	5th	6th	7th	oct.
i0	P1	d2						
i1	A1	m2						
i2		M2	d3					
i3		A2	m3					
i4			M3	d4				
i5			A3	P4				
i6				A4	d5			
i7					P5	d6		
i8					A5	m6		
i9						M6	d7	
i10						A6	m7	
i11							M7	d8
i12							A7	P8

http://openmusictheory.com/intervals.html





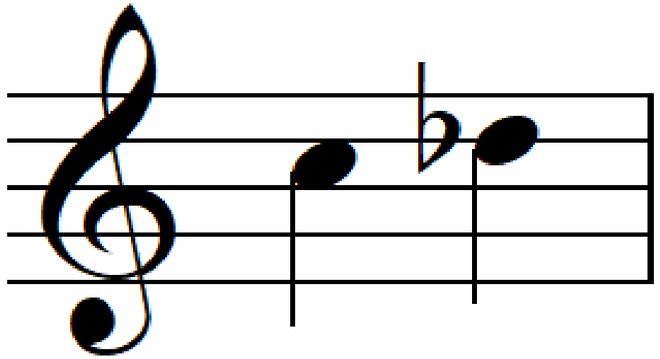
Scale Construction

Scale Construction

Dowling & Harwood (1986)

Constraints

- Discriminability of intervals
- Octave equivalence
- Moderate number of pitches within the octave
  - Miller (1957) 7±2
- The use of a uniform modular pitch interval (the semitone) with which to construct approximations of all the intervals of scales traditionally in use
  - e.g., 12-TET



semitone = 5.9% ΔF_0

JND = ~1% ΔF_0

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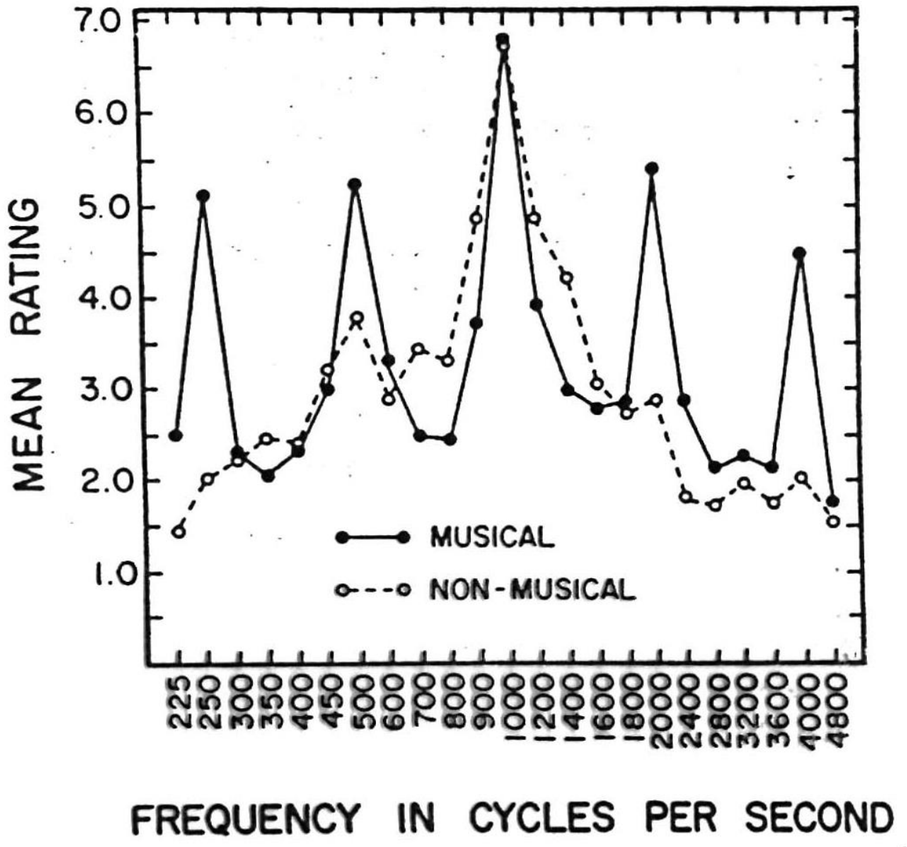

Similarity judgments between a reference pure tone at 1000 Hz and a pure tone of variable frequency. Filled circles: mean similarity ratings given to the variable frequency sound by a group of musicians. Open circles: mean ratings given by a group of nonmusicians.

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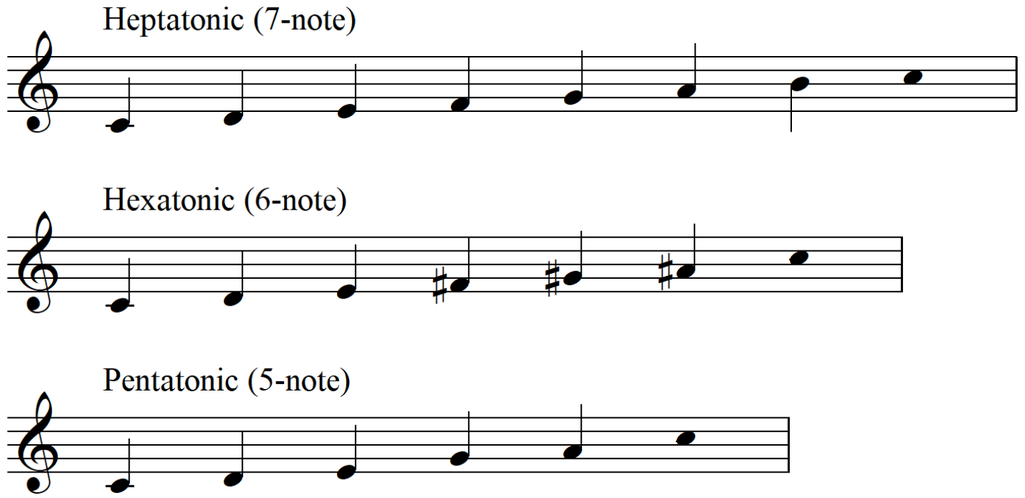

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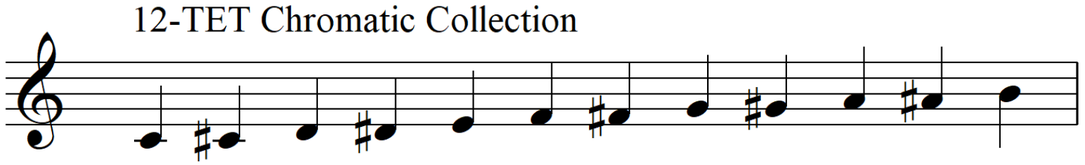
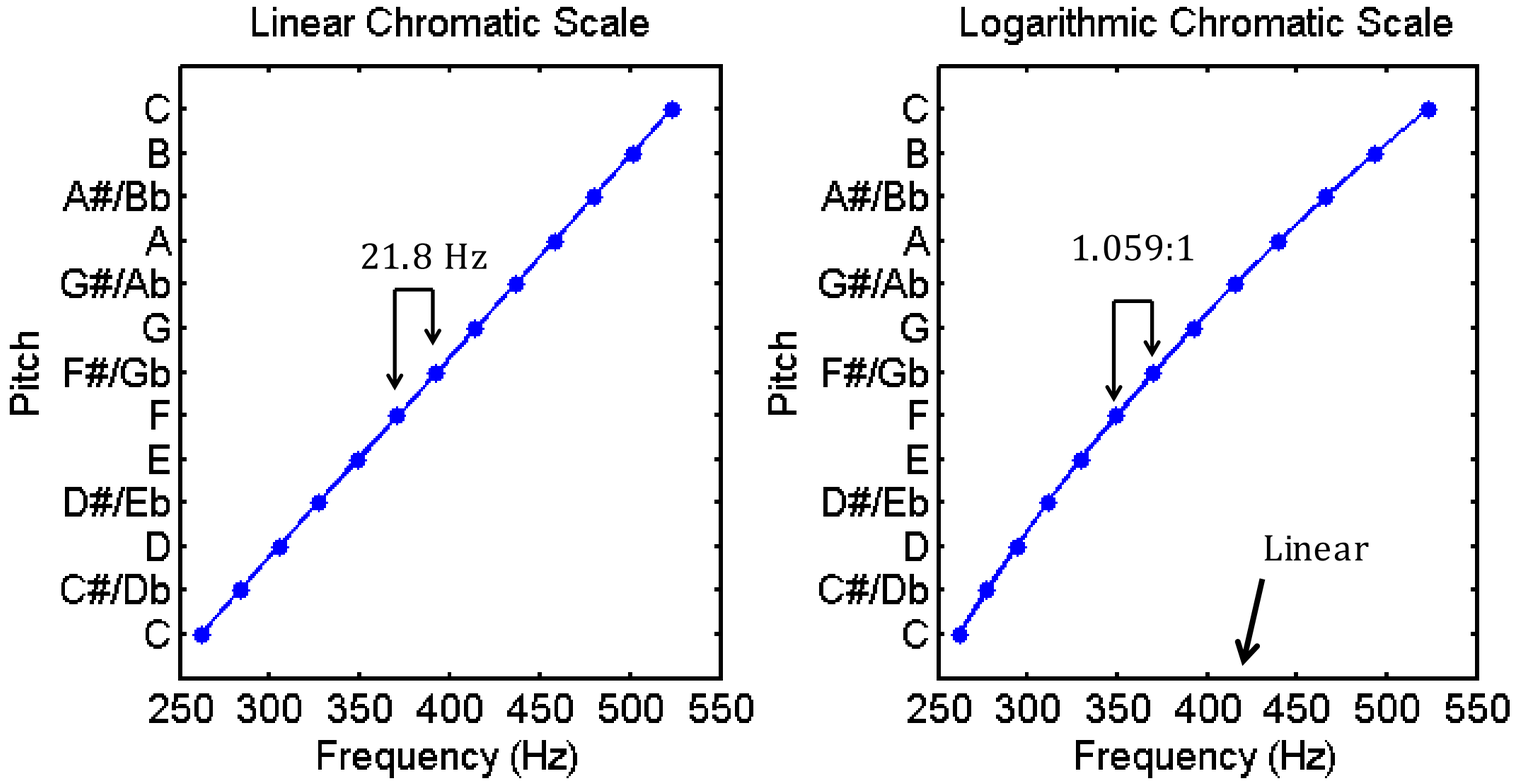


Scale Construction

Constraints

- **Pitch perception is logarithmic**

  Dividing the octave into 12 equal linear (left) or logarithmic (right) steps.



Linear and log chromatic scales

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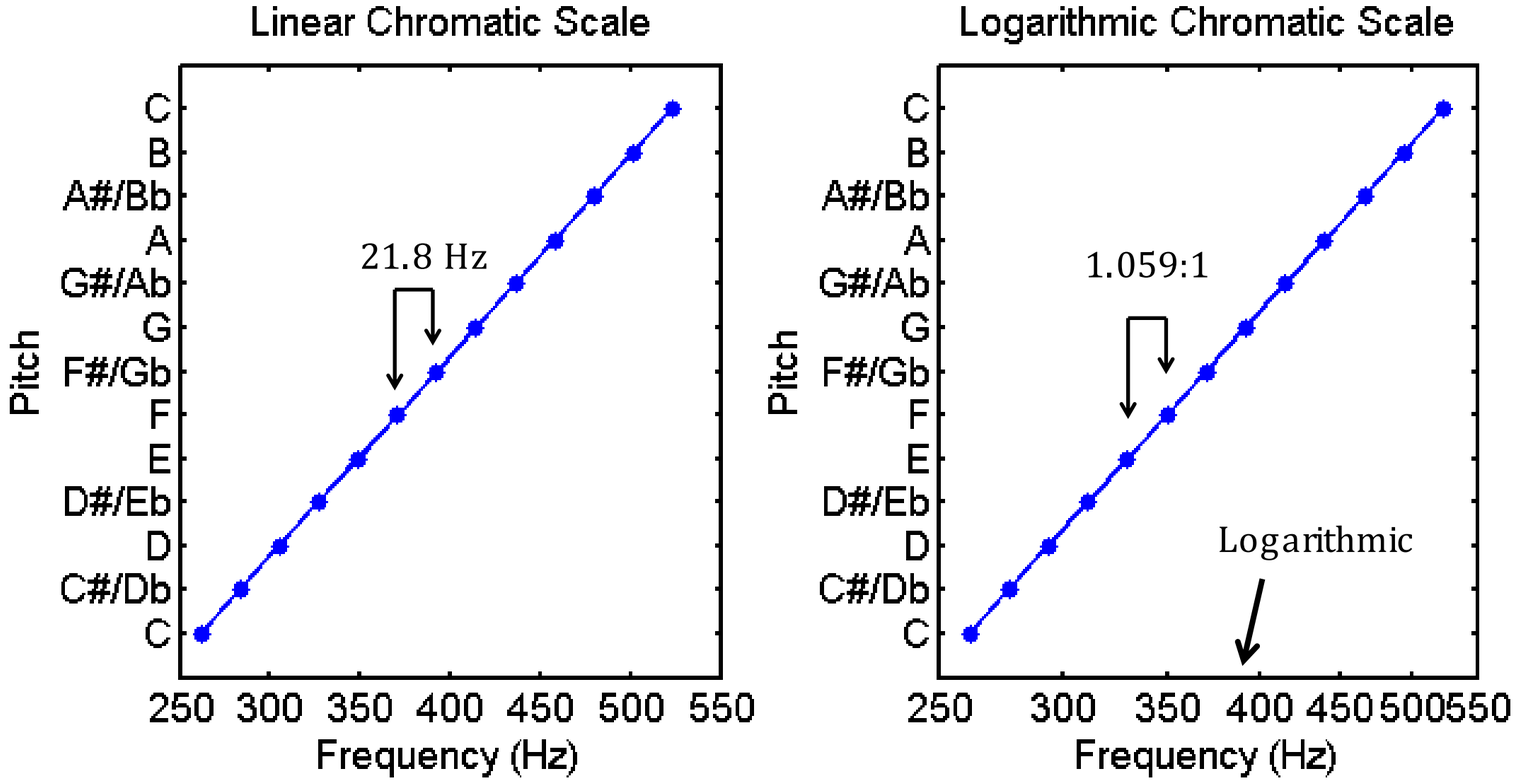


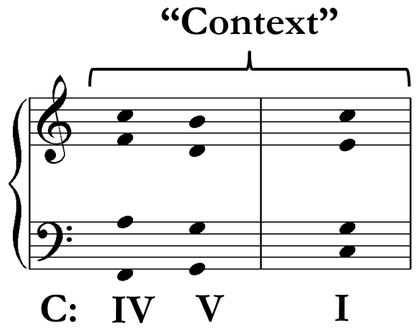
Linear and log chromatic scales

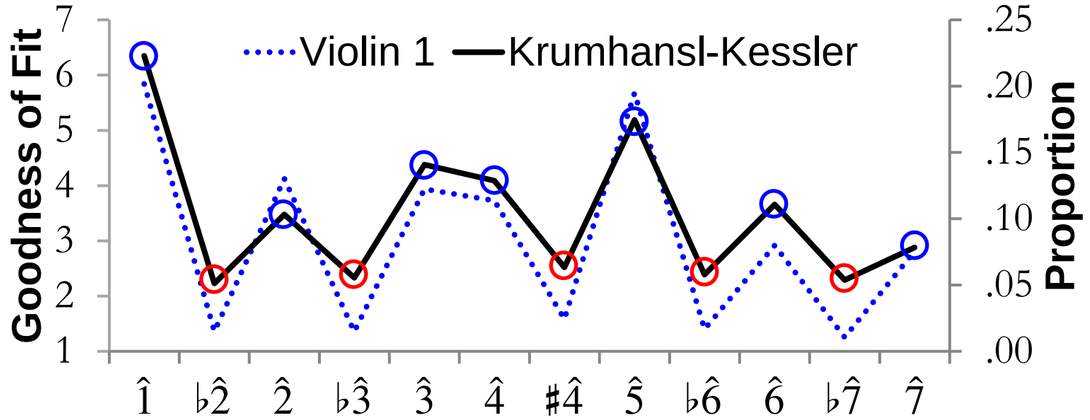
Scale Construction

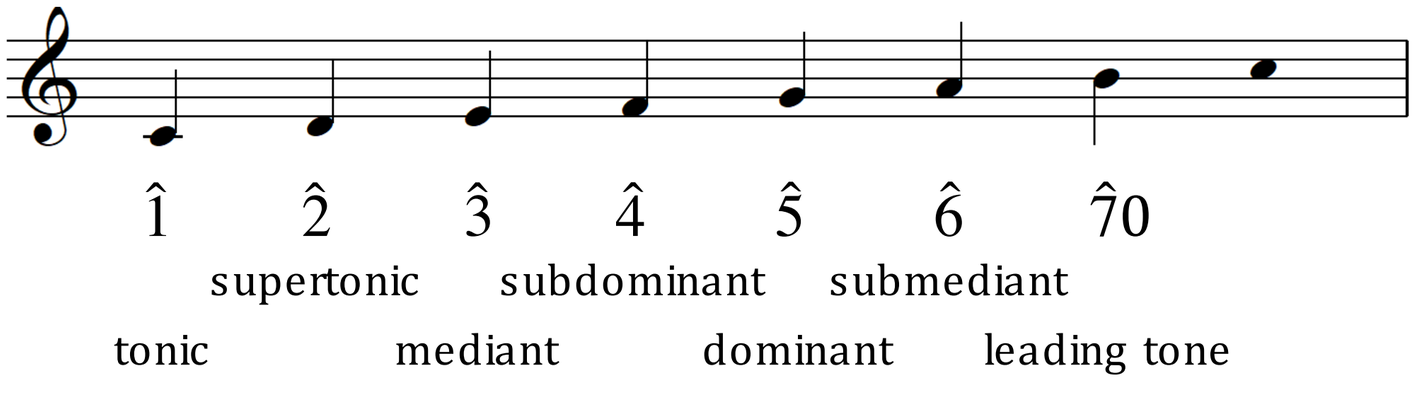
McAdams(1989)

Properties of Scale Structure

- Modal hierarchies (like the major scale) are established by:
  - *focal pitches*
  - *asymmetrical pattern of large and small intervals*
  - *distinctive intervals*
  - *Pattern repeated at the octave throughout the frequency range*







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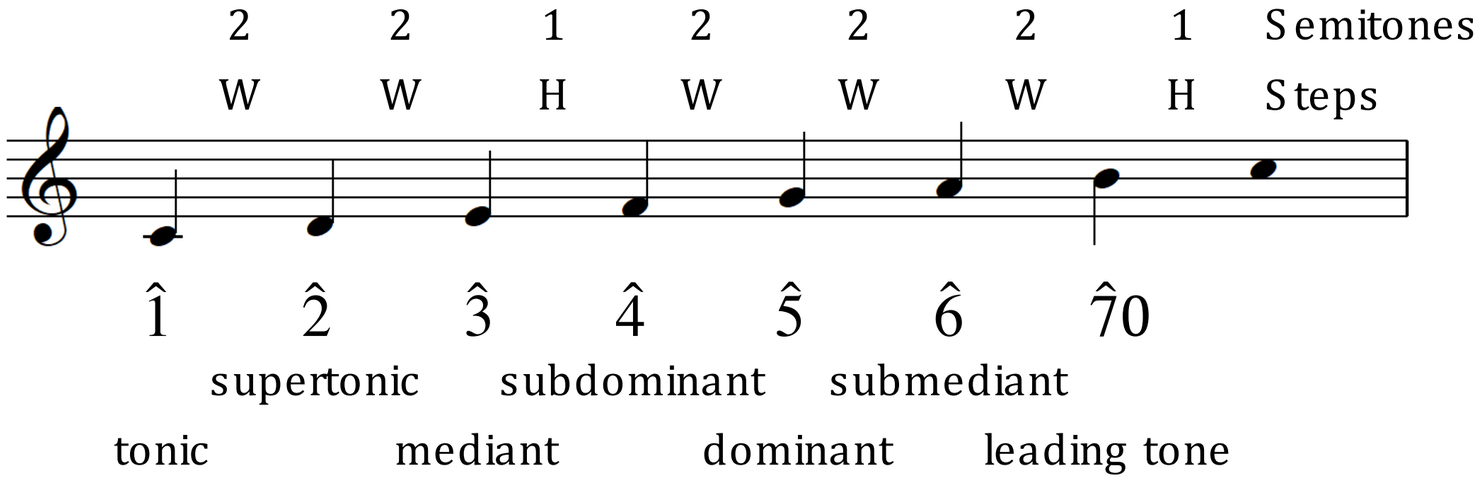

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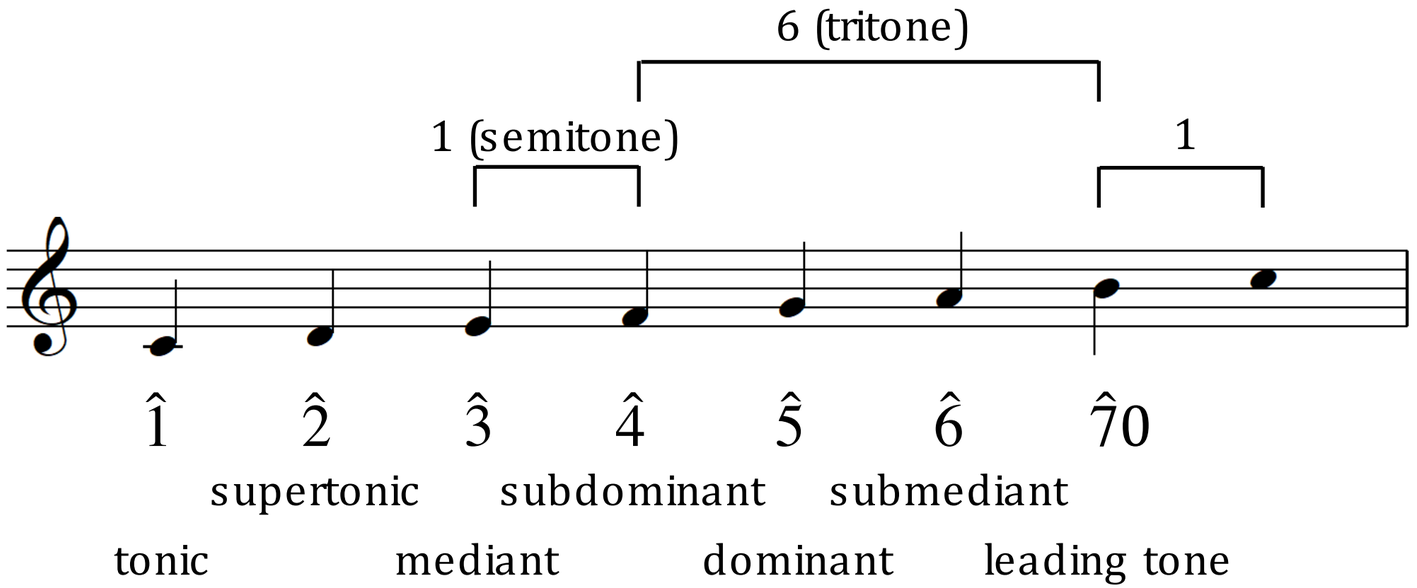

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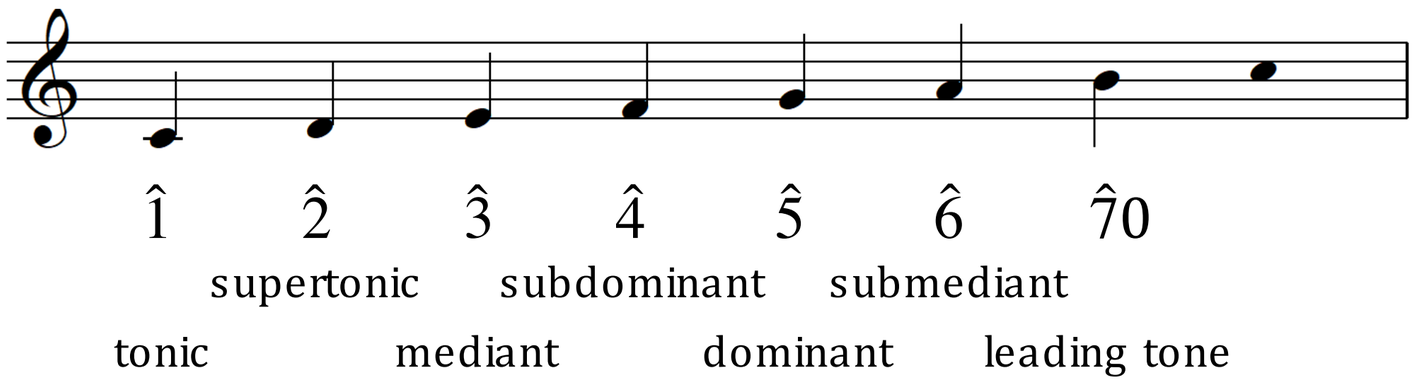
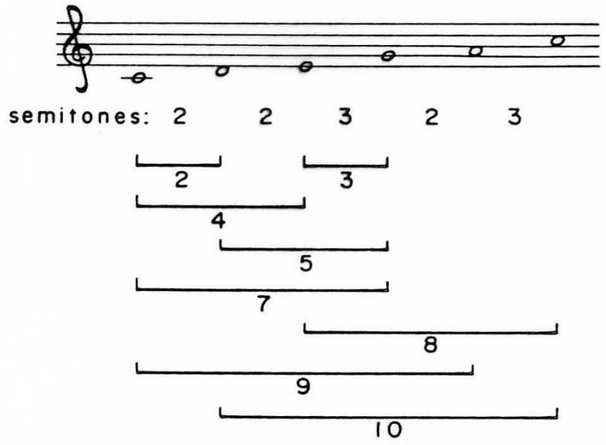


Scale Construction

Balzano (1980)

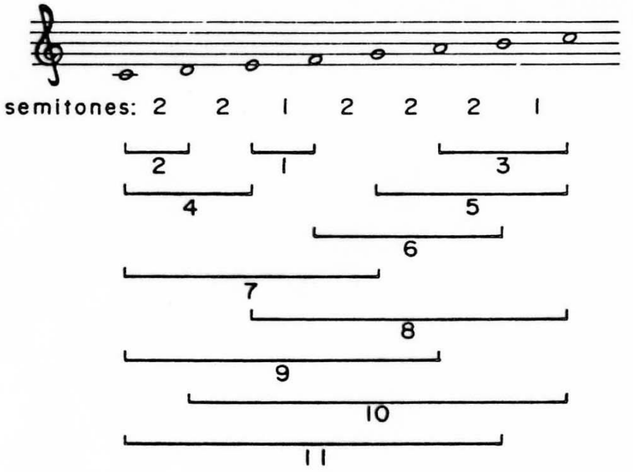
Properties of Scale Structure

- *Completeness* - diatonic scales give maximal variety of interval sizes
  - smallest number of pitches that provide all of the possible intervals is 7.
- *Coherence* - desirable property of a melodic scale
  - Any interval of *n* steps > any interval of *n*-1 steps (in semitones)



coherent

Figure 4.6 A pentatonic scale, with instances of its possible interval sizes within the octave.



complete and coherent

Figure 4.7 The C-major scale, with instances of its possible interval sizes.

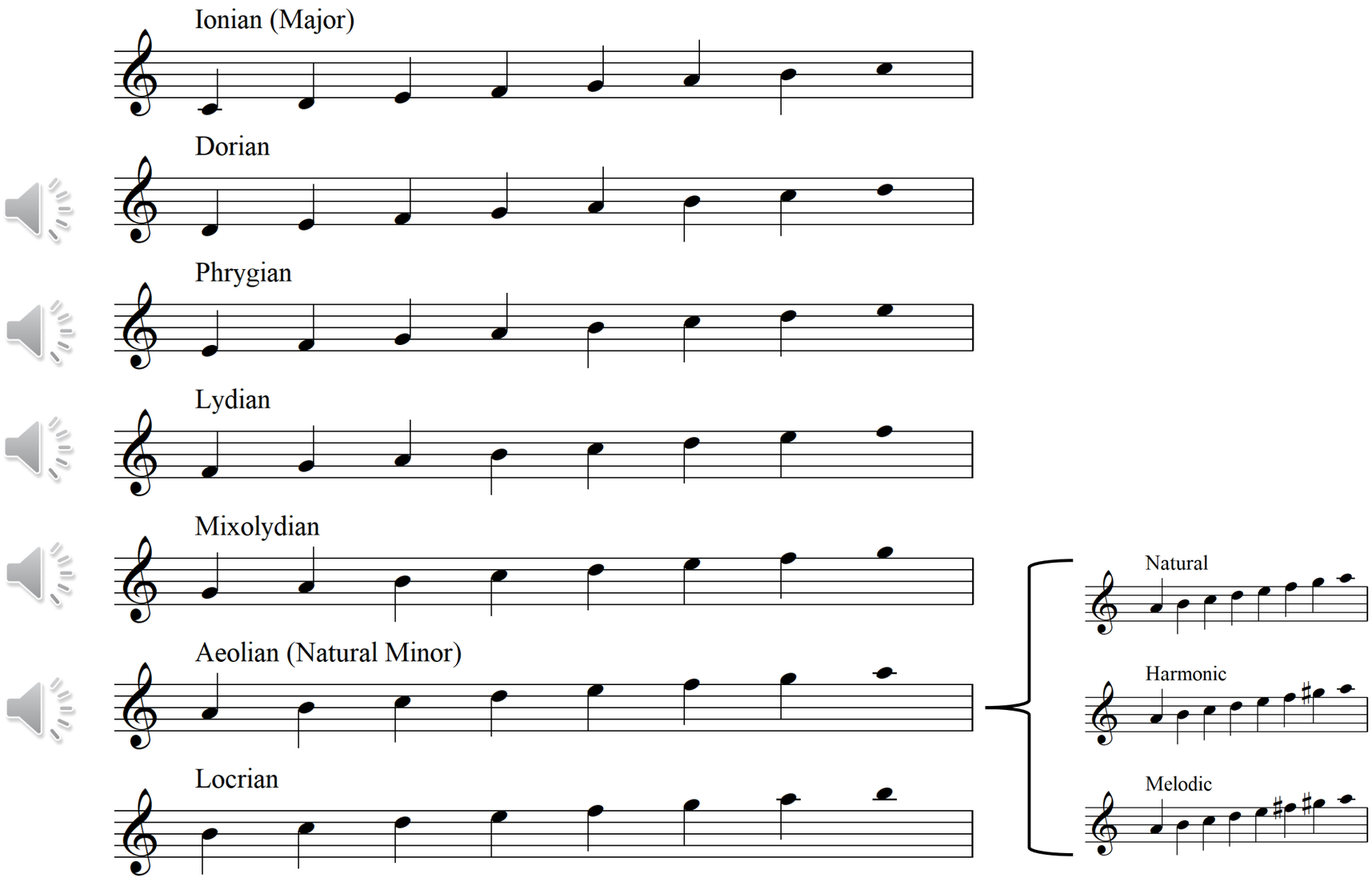
Common Scales

Common Scales

12-TET

- Heptatonic
- Hexatonic
- Pentatonic

Diatonic -- Asymmetrical



Mixed modes: Lydian-Mixolydian (jazz)

Common Scales

12-TET

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- Hexatonic
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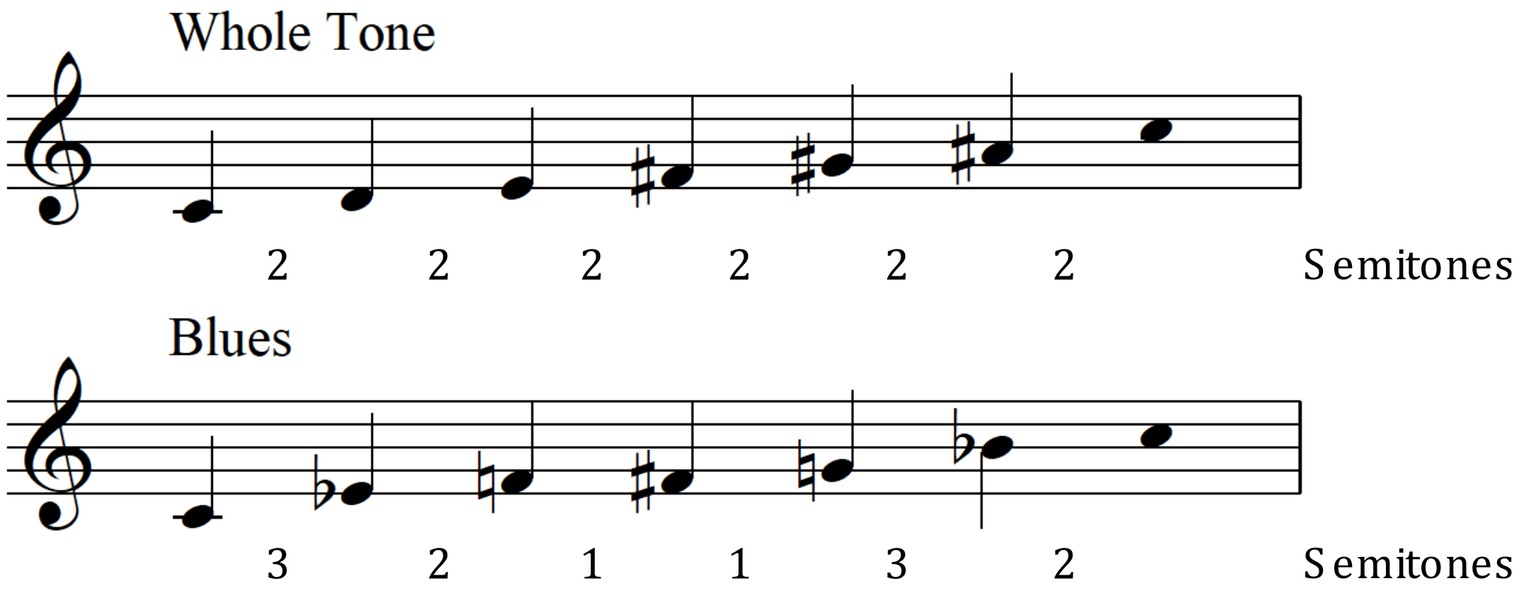

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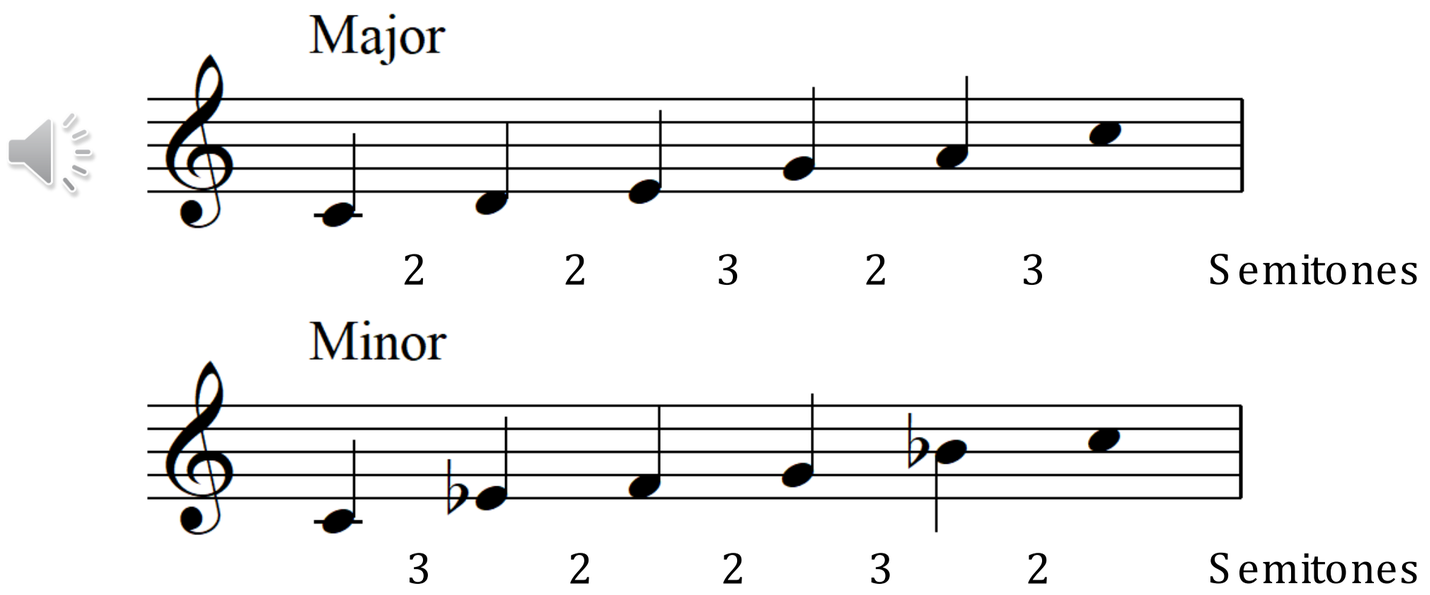
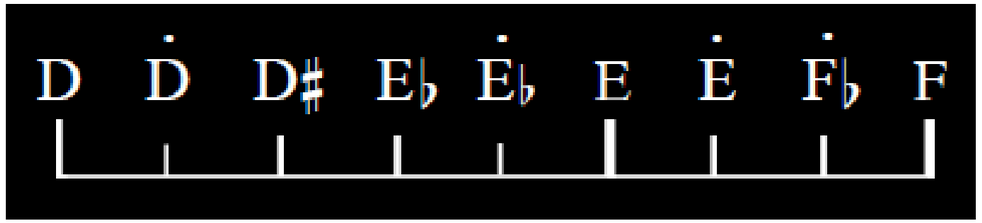


Common Scales

Wild (2008)

Beyond 12-TET

- **31 TET**
  - Nicola Vicentino (1555)
  - Vicentino divided the octave into 31 tones.
  - He selected several scale structures from the 31TET chromatic scale.

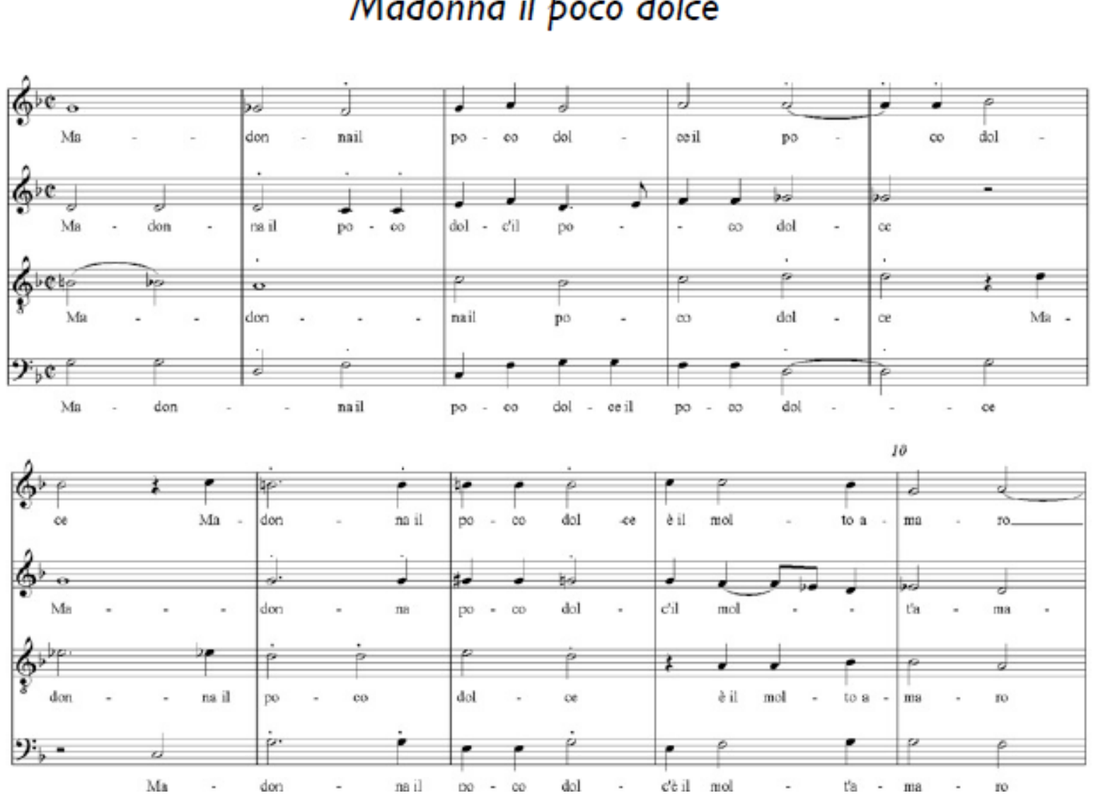


Replica of Vicentino's *archicembalo*, which has keys for each of the 31 pitches per octave.

Common Scales

Wild (2008)

Beyond 12-TET

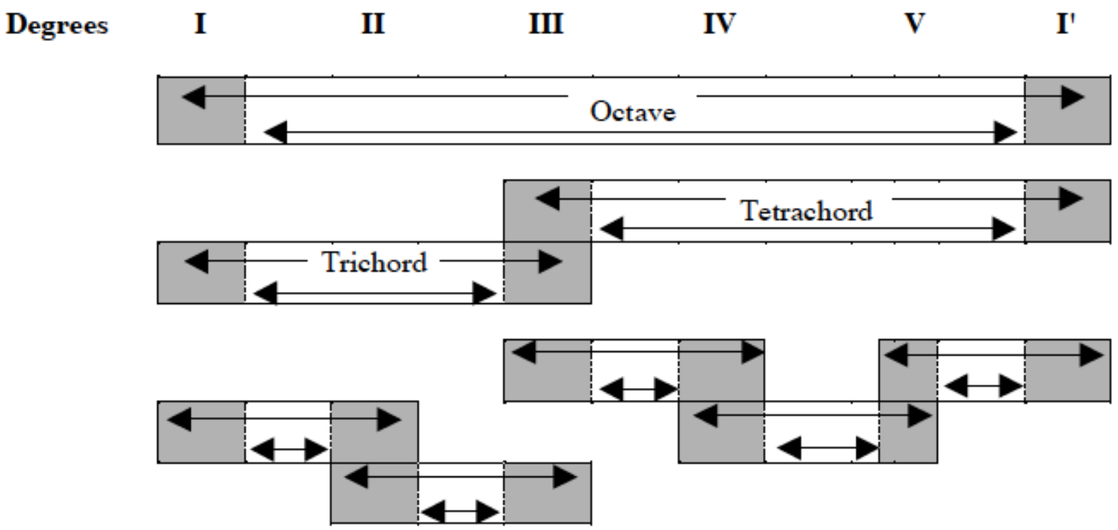


Common Scales

Arom, Fernando & Marandola (2005)

Beyond 12-TET

- **Variable-interval scale system**

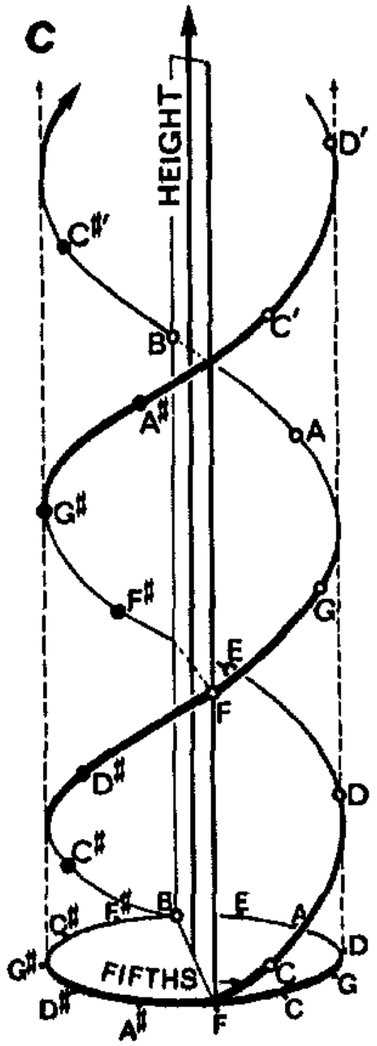


MIR Tasks

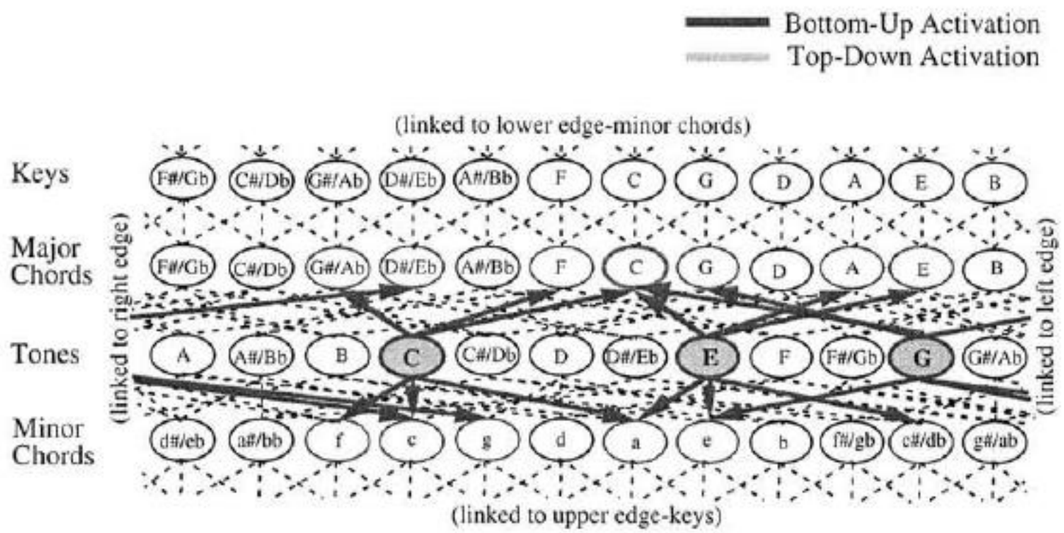
MIR Tasks

- **Key/Scale Identification (i.e., Pitch-Centricity Algorithms)**
  - Identify the most focal pitch, or *tonal center* (i.e., pitch centricity)
    - Approaches
      - *Template* (predicted modal hierarchy)
      - *Supervised* (human annotations)
      - *Unsupervised* (human annotations)
    - Representations
      - *0^{th}-order PC/Chroma distributions* (Structural Accounts)
      - *Interval distributions* (Functional Accounts; e.g., rare intervals like A4/d5)
    - Models
      - Associations (*template*; correlations, distance measures, etc.)
      - Classifiers (*supervised/unsupervised*; clustering, logistic regression, neural networks, etc.)

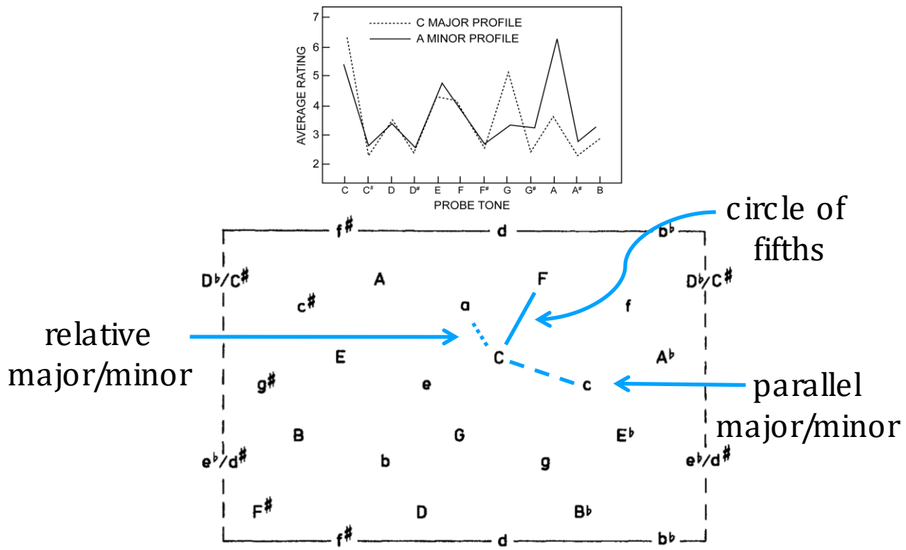
MIR Tasks



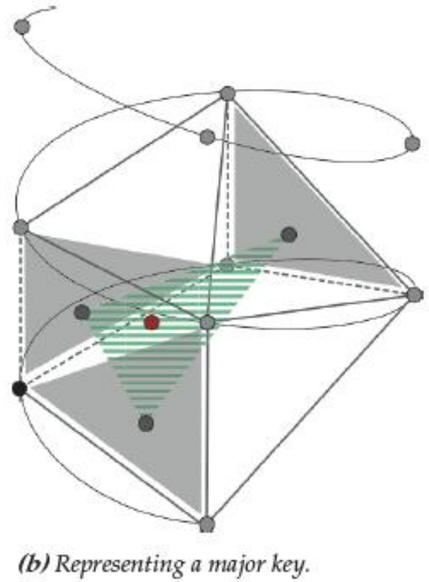
Double Helix (Shepard, 1982)



MUSACT (Tillmann et al., 2000)



4D Torus (Krumhansl & Kessler, 1982)



(b) Representing a major key.

Spiral Array (Chew, 2007)

MIR Tasks

Albrecht & Shanahan (2013)

TABLE 1. *The Accuracy Ratings for Key-Finding Methods Compared for Major, Minor, and Overall.*

	Entire Piece			1st and last 8 measures		
Algorithm	Major	Minor	Overall	Major	Minor	Overall
Krumhansl-Schmuckler	69.0%	83.2%	74.2%	85.3%	79.3%	83.1%
Temperley (Krumhansl-Schmuckler algorithm)	96.8%	74.3%	88.6%	94.6%	67.6%	84.8%
Bellman-Budge	94.9%	84.4%	91.1%	94.2%	86.6%	91.5%
Aarden-Essen	90.7%	93.3%	91.7%	94.9%	84.9%	89.8%
Sapp Simple Weightings	92.3%	87.2%	90.4%	95.2%	88.9%	92.9%
Proposed model (Krumhansl-Schmuckler algorithm)	92.7%	85.5%	90.0%	96.5%	83.8%	91.9%
Proposed model (Euclidean distance)	89.1%	95.0%	91.3%	94.2%	91.1%	93.1%

Recent Work!

Recent Work!

- **Weiss & Habryka (2014); Weiss (2017); Weiss et al. (ISMIR; 2020)**
  - Key identification of major/minor diatonic scales in music audio using template matching for chroma vectors.

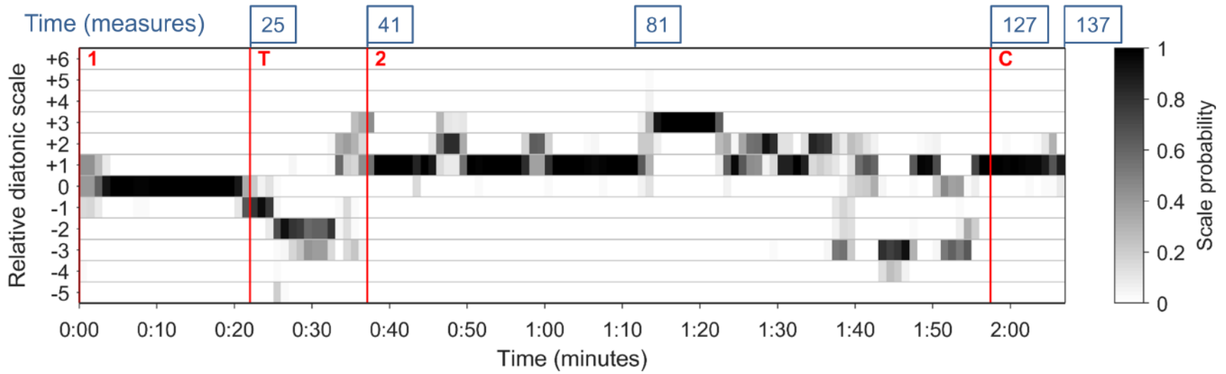


Figure 1. L. v. Beethoven, piano sonata Op. 7 in E♭ major, 1st mvmt. *Allegro molto e con brio*, exposition. Computational tonal analysis with a window size of $w = 4$ seconds.

- **Harasim et al. (2021)**
  - Key/Scale identification of heptatonic scales in a large data set of midi files using unsupervised learning for PC distributions (i.e., chroma vectors).

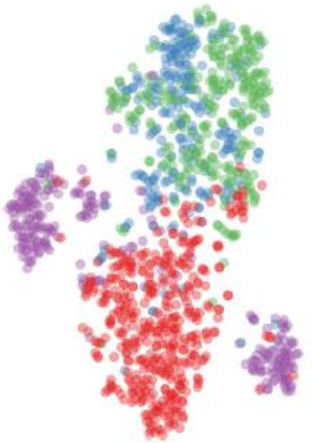


Fig. 6 Mode classification for four modes in the Renaissance epoch according to the Bayesian model. The clustering reveals one distinct mode (red), two very similar modes (blue, green), and another mode distinct from the others (violet). Note that the latter is separated into two subclusters after dimensionality reduction. In the original mode space, the pieces in the two violet clusters are all close to each other.

Main Takeaways

Main Takeaways

- **Approach**
  - Template-matching has hit a ceiling (Albrecht & Shanahan, 2013), and restricts itself to 12-TET generally, and the major/minor modes specifically.
- **Representation**
  - Structural accounts (i.e., 0^{th}-order distributions) dominate the field, but richer representations may improve performance.
- **Models**
  - Geometric models for key-finding are incredibly popular but inherently symmetric. How do we model the asymmetric properties of a given scale system?
  - A piece of music may feature two or more scale systems simultaneously (modal mixture, mixed scale systems, supermode, etc.), but researchers rarely employ fuzzy classifiers.