Complex Numbers

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1 Introduction

Complex Numbers are a very important and useful tool in both Mathematics and Physics(Quantum Mechanics)!
Complex Numbers are a superset of Real Numbers and Imaginary Numbers!

Unlike real or imaginary numbers, complex are analyzed in 2 dimensions! The Real and the Imaginary!
All Scalars can be expressed as a complex Number. Complex numbers are usually denoted by z or w, but we, for now, will stick to z!
let $x, y \in R$ and $i \in \Im$

$$z = x + iy \tag{1}$$

x is called the real part and iy is called the imaginary part!
Here a visual representation that can make it more clear!

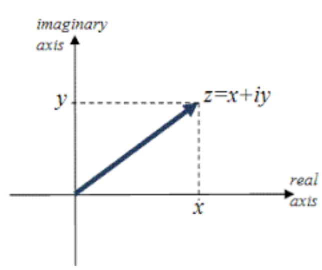
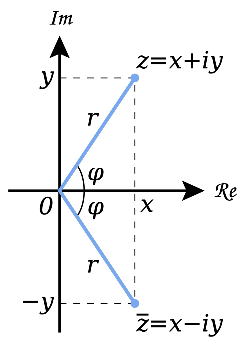


Figure 1: Visual Representation of a Complex Number

2 Complex Numbers

in Figure 1, you can see an arrow in the middle! That is called r!
r is equal to
$r = \sqrt{RealPart^2 + ImaginaryPart^2}$
The angle that is formed between r and the x axis, is called ϕ
Here is a visual that might help you understand everything a bit better!



3 Complex Number Conjugate

Every Complex Number has a conjugate, labelled $\bar{z}$ or z^*

We will stick with the star notation

$$z^* = x - iy \tag{2}$$

As expected we see that the conjugate of z equals to the real part of z, but instead of adding the imaginary, we subtract it!
We can see that from the visual representation on the figure on section 2!

4 Euler's Formula

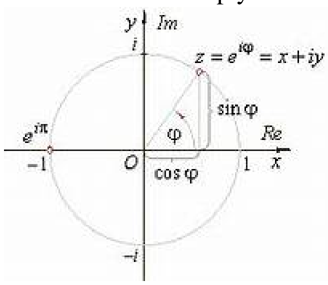
Now i will show you Euler's Formula. It is key formula for Complex Numbers, but we **will not derive it**, since it requires knowledge on Power Series and it is also out of the scope of this chapter!

Euler Found out that

$$e^{i\phi} = \cos\phi + i\sin\phi \tag{3}$$

(This of course is in the Unit circle, meaning that r is equal to 1)

Here is visual to help you:



If combine everything we have learned so far, we get that:

$$z = x + iy = re^{i\phi} = r(\cos\phi + i\sin\phi) \tag{4}$$

and for the conjugate:

$$z^* = x - iy = re^{-i\phi} = r(\cos\phi - i\sin\phi) \tag{5}$$

5 Phase Factor

There is a special complex number called the phase factor or phasor, which has a unique property:

$$z^*z = 1 \tag{6}$$

or

$$z = e^{i\phi} = \cos\phi + i\sin\phi \tag{7}$$