3. Principles of Quantum Mechanics

Stavros Klaoudatos

Contents

Prerequisites: Linear Algebra, Vector Spaces, Complex Numbers and Hermitian Operators

1 The principles

1.1 Principle 1:

The observable or measurable quantities of Quantum Mechanics are represented by linear operators L, that also are Hermitian Operators.

1.2 Principle 2:

The possible outcomes of a measurement are the eigenvalues of the operator of that observable. We call them λ_i. (If the system is prepared in a state $|\lambda_i\rangle$, the result is guaranteed to be λ_i)

1.3 Principle 3:

Unambiguously distinguishable states are represented by orthogonal vectors.(e.g. up and down)

1.4 Principle 4:

If $|A\rangle$ is the state-vector of a system, and the observable L is measured, the probability for the result of a measurement to be λ_i, is given by:

$$P(\lambda_i) = \langle A|\lambda_i\rangle \langle\lambda_i|A\rangle \tag{1}$$

2 Explanation of the Principles

The first principle implies that each observable, σ_x,σ_y and σ_z, is identified with a specific linear operator in the two dimensional space of states describing the spin.

The second one defines the relation between measurement and the eigenvalues of that operator, which is that all the possible outcomes of the measurement, are eigenvalues of the operator.

Principle 3 speaks for unambiguously distinct states and their relation as in the position of the vectors that represent them. For example, $|u\rangle$ and $|d\rangle$ can be distinguished by measuring σ_z. That is true for the pairs, up and down, left and right in and out! The **inner product** of two states is a measure of the **inability** to distinguish them with certainty. Since the state-vectors are normalized, the maximum value of that inability is 1 while the lowest is 0.

Finally, principle 4 tells us that if we prepared a system in state $|A\rangle$, and then we measured L, the outcome will be one of the eigenvalues of L,
with each having a probability : $P(\lambda_i) = \langle A|\lambda_i\rangle \langle\lambda_i|A\rangle$ or $P(\lambda_i) = |\langle A|\lambda_i\rangle|^2$

3 Spin Operators

We call σ a 3-vector operator
A spin operator provides information about the spin component in a specific direction.
There is a spin Operator for every direction in which the Apparatus A can be oriented along!

Overall, there are certain states, $|u\rangle$ and $|d\rangle$, in which the measurement of σ_z gives unambiguous results: ±1. From the previous principle, we know that $|u\rangle$ and $|d\rangle$ are orthogonal and are eigenvectors of the linear operator σ_z. That means that the eigenvalues of σ_z correspond to ±1

The same applies for σ_x and $_y$, with $|r\rangle$ and $|l\rangle$, and $|i\rangle$ and $|o\rangle$ respectively.
By definition, and also easy to prove, the linear operators are:

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{3}$$

and finally

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{4}$$

These matrices are also known as the *Pauli Matrices*

4 The Spin Polarization

We will notate σ as $\overrightarrow{\sigma}$ but when talking about components we will keep the σ_i notation

Generally, any state-vector of a single spin, is an eigenvector of some component of the spin. or, in other words:
Given any state $|A\rangle$,

$$|A\rangle = a_u |u\rangle + a_d |d\rangle \tag{5}$$

, and $\hat{n}$, such that

$$\overrightarrow{\sigma} \cdot \hat{n} |A\rangle = |A\rangle \tag{6}$$

or simply:

$$\overrightarrow{\sigma} \cdot \hat{n} = \sigma_n \tag{7}$$