Quantum Mechanics - Systems and Experiments

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1 Introduction

Quantum Mechanics is different! Quantum Mechanics Deals with the behavior of objects so small, that humans with their technology can barely observe!
In Quantum Mechanics, **states are represented by mathematical objects** and have a completely different structure in comparison with Classical Mechanics.
States and Measurements: In Classical Mechanics, you can simply perform an experiment to determine the state of a system. That simply isn't the case in Quantum Mechanics. The relation between states and measurements is counter-intuitive and subtle.

2 Spins and Qubits

Particles have properties in addition to their location in space. They have intrinsic properties like mass, which they have by nature and because they are the specific particles they are, and extrinsic, like weight, which depends on the gravitational pull!
Elementary particles, have an extra intrinsic property called ***Spin***! This property provides an extra degree of freedom to the particle.
Visualizing and attempting to represent spin is really hard, counter-intuitive and **is not** the point!
Quantum Spin Visualization Video

What we are going to do for now, is to abstract the idea of spin and isolate it forgetting its relation to particles.
The isolated quantum Spin is an example of the general class of simple systems we call qubits or quantum bits.
All systems can be build by combining qubits!
Note that Spin is either 1 or -1, nothing in between!!!

3 An Experiment

For now it would be way easier if you thought of it as a 2 Dimensional Space, just so you can understand it easier, later on we will return in the 3D space!

In classical Mechanics, we called , σ the degrees of freedom. We also talked about dynamical laws and laws of evolution like $\sigma(n+1) = \sigma(n)$, or $\sigma(n+1) = \sigma(-n)$
In Quantum Mechanics we have an Apparatus A, which is used to record the results of the measurements!
Apparatus A can be though of as a black box that we can't see inside, with an arrow showing us how it is oriented in space, or in other words what component of the spin it measures, and with a screen on it, displaying the results!

Initially we don't know whether $\sigma = 1 \quad or \quad \sigma = -1$.

The purpose of the Experiment is to find the value of σ.

Before interacting with the two-state system, the screen is blank, with a question mark indicating it hasn't recorded anything yet(notation in diagrams).
Lets now do the actual experiment!
Suppose we think of spin as a typical vector(which it isn't), and that it is oriented in space along the z axis. Now suppose the Apparatus is also oriented along the z direction. If A records a measurement while the above are true, well it is obvious the result will be +1. It is also intuitive and follows the laws of nature and determinism, that if we don't change anything, the result will be again +1!

Now, without affecting the spin, **rotate the Apparatus 180**°.
Again, we can easily predict with intuition the result, which in this case is -1, and again if we repeat the experiment without changing anything, following the same operations, the result will always be -1

So far, there isn't anything new or counter-intuitive, from what we know from Classical

Mechanics!
The weird things start to happen when we turn the Apparatus an arbitrary angle! Lets see how!
Suppose the Apparatus is oriented along the Spin along the z axis, and as we found out earlier, the expected result is +1! Now, without measuring or changing the spin at all, suppose we rotate the Apparatus by **90**° or $\frac{\pi}{2} rad$
Now we are measuring the σ_x component of the spin!
If spin is a normal vector, and it is oriented along the z axis,
meaning its x component =0, we except the result of the measurement to be 0!
Here is where we start to see a difference!
When we make the measurement, the result will be weird:
The result is either 1 or -1! Never 0. no matter how many time or how many times we repeat this, or how we orient the Apparatus, the result will be $\sigma = \pm 1$

Now if we repeat the operation many times (The whole operation, including the initial orientation, and measurement):

Determinism is Broken Down!

The Number of $\sigma = +1$ events, are statistically equal to the events of $\sigma = -1$!

The component along the a-axis is not 0, The Average of these measurements are 0!

In General, if the Apparatus is oriented along the $\hat{m}$ direction, and the spin is prepared so it will measure +1, meaning it is along $\hat{m}$, if then rotate the Apparatus to the direction $\hat{n}$ without disturbing the spin, **The Average Value will be $\cos\theta$, where θ is the angle $\hat{m}$ and $\hat{n}$ form!** The General Notation for this is:

$$< \sigma >= \hat{m} \cdot \hat{n}$$

($\hat{m} \cdot \hat{n} = n \cdot m \cdot \cos\theta$ Which is equal to $\cos\theta$, since $\hat{m}$,$\hat{n}$ are unit vectors of the directions n and m so they are equal to 1!)

Quantum Mechanical Systems are NOT Deterministic and the results of the experiments can be statistically random, however, the average quantities can follow the expectations of Classical Mechanics, up to a point

4 The problem with Experiments

Every Experiment requires an outside system to record the measurements, namely an Apparatus. In order to make a measurement, the Apparatus needs to Interact with the System ! That is a big Problem for Quantum Mechanics. In Q.M., any interaction that is enough to measure an aspect of the system, is strong enough to disrupt the other aspects of the system. Measuring the spin on one component, destroys the information about any other component! Will later see about this in more depth

Before moving to Quantum States, which is the 2 chapter, you should make sure you cover all the math prerequisites!