# **Julia Language Documentation**

Release 0.5.0-dev

Jeff Bezanson, Stefan Karpinski, Viral Shah, Alan Edelman, et al.

January 26, 2016

I	The Julia Manual	1
1	Introduction	3
2	Getting Started	5
3	Variables	9
4	Integers and Floating-Point Numbers	13
5	<b>Mathematical Operations and Elementary Functions</b>	25
6	Complex and Rational Numbers	35
7	Strings	41
8	Functions	55
9	Control Flow	65
10	Scope of Variables	81
11	Types	87
12	Methods	107
13	Constructors	117
14	Conversion and Promotion	127
15	Modules	133
16	Metaprogramming	141
17	Multi-dimensional Arrays	157
18	Linear algebra	169
19	Networking and Streams	173
20	Parallel Computing	179

21	Date and DateTime	193
22	Interacting With Julia	201
23	Running External Programs	207
24	Calling C and Fortran Code	213
25	Embedding Julia	229
26	Packages	237
27	Package Development	245
28	Profiling	257
29	Memory allocation analysis	263
30	Performance Tips	265
31	Style Guide	285
32	Frequently Asked Questions	293
33	Noteworthy Differences from other Languages	305
34	Unicode Input	313
II	The Julie Standard Library	315
	-	
	Essentials  C. H. C. L. D. A. St. L.	317
	Collections and Data Structures	335
	Mathematics	353
	Numbers	377
	Strings	385
	Arrays	393
41	Tasks and Parallel Computing	405
42	Linear Algebra	413
43	Constants	441
44	Filesystem	443
45	I/O and Network	449
46	Punctuation	463
47	Sorting and Related Functions	465
48	Package Manager Functions	471

49 Unit Testing	475
50 C Interface	483
51 LLVM Interface	485
52 C Standard Library	487
53 Dynamic Linker	489
54 Profiling	491
Bibliography	493

# Part I The Julia Manual

#### Introduction

Scientific computing has traditionally required the highest performance, yet domain experts have largely moved to slower dynamic languages for daily work. We believe there are many good reasons to prefer dynamic languages for these applications, and we do not expect their use to diminish. Fortunately, modern language design and compiler techniques make it possible to mostly eliminate the performance trade-off and provide a single environment productive enough for prototyping and efficient enough for deploying performance-intensive applications. The Julia programming language fills this role: it is a flexible dynamic language, appropriate for scientific and numerical computing, with performance comparable to traditional statically-typed languages.

Because Julia's compiler is different from the interpreters used for languages like Python or R, you may find that Julia's performance is unintuitive at first. If you find that something is slow, we highly recommend reading through the *Performance Tips* (page 265) section before trying anything else. Once you understand how Julia works, it's easy to write code that's nearly as fast as C.

Julia features optional typing, multiple dispatch, and good performance, achieved using type inference and just-in-time (JIT) compilation, implemented using LLVM. It is multi-paradigm, combining features of imperative, functional, and object-oriented programming. Julia provides ease and expressiveness for high-level numerical computing, in the same way as languages such as R, MATLAB, and Python, but also supports general programming. To achieve this, Julia builds upon the lineage of mathematical programming languages, but also borrows much from popular dynamic languages, including Lisp, Perl, Python, Lua, and Ruby.

The most significant departures of Julia from typical dynamic languages are:

- The core language imposes very little; the standard library is written in Julia itself, including primitive operations like integer arithmetic
- A rich language of types for constructing and describing objects, that can also optionally be used to make type declarations
- The ability to define function behavior across many combinations of argument types via multiple dispatch
- Automatic generation of efficient, specialized code for different argument types
- Good performance, approaching that of statically-compiled languages like C

Although one sometimes speaks of dynamic languages as being "typeless", they are definitely not: every object, whether primitive or user-defined, has a type. The lack of type declarations in most dynamic languages, however, means that one cannot instruct the compiler about the types of values, and often cannot explicitly talk about types at all. In static languages, on the other hand, while one can — and usually must — annotate types for the compiler, types exist only at compile time and cannot be manipulated or expressed at run time. In Julia, types are themselves run-time objects, and can also be used to convey information to the compiler.

While the casual programmer need not explicitly use types or multiple dispatch, they are the core unifying features of Julia: functions are defined on different combinations of argument types, and applied by dispatching to the most specific matching definition. This model is a good fit for mathematical programming, where it is unnatural for the first

argument to "own" an operation as in traditional object-oriented dispatch. Operators are just functions with special notation — to extend addition to new user-defined data types, you define new methods for the + function. Existing code then seamlessly applies to the new data types.

Partly because of run-time type inference (augmented by optional type annotations), and partly because of a strong focus on performance from the inception of the project, Julia's computational efficiency exceeds that of other dynamic languages, and even rivals that of statically-compiled languages. For large scale numerical problems, speed always has been, continues to be, and probably always will be crucial: the amount of data being processed has easily kept pace with Moore's Law over the past decades.

Julia aims to create an unprecedented combination of ease-of-use, power, and efficiency in a single language. In addition to the above, some advantages of Julia over comparable systems include:

- Free and open source (MIT licensed)
- User-defined types are as fast and compact as built-ins
- No need to vectorize code for performance; devectorized code is fast
- Designed for parallelism and distributed computation
- Lightweight "green" threading (coroutines)
- Unobtrusive yet powerful type system
- Elegant and extensible conversions and promotions for numeric and other types
- Efficient support for Unicode, including but not limited to UTF-8
- Call C functions directly (no wrappers or special APIs needed)
- Powerful shell-like capabilities for managing other processes
- Lisp-like macros and other metaprogramming facilities

# **Getting Started**

Julia installation is straightforward, whether using precompiled binaries or compiling from source. Download and install Julia by following the instructions at http://julialang.org/downloads/.

The easiest way to learn and experiment with Julia is by starting an interactive session (also known as a read-eval-print loop or "repl") by double-clicking the Julia executable or running julia from the command line:

```
$ julia

_____(_)__ | A fresh approach to technical computing
(_) | (_) (_) | Documentation: http://docs.julialang.org
____ | Type "help()" to list help topics
| | | | | | | / _ ` | |
| | | | | | (_| | | Version 0.3.0-prerelease+3690 (2014-06-16 05:11 UTC)
_/ | \_ '_| | Commit 1b73f04* (0 days old master)
| _/ | x86_64-apple-darwin13.1.0

julia> 1 + 2
3

julia> ans
3
```

To exit the interactive session, type  $^D$ — the control key together with the d key or type quit(). When run in interactive mode, julia displays a banner and prompts the user for input. Once the user has entered a complete expression, such as 1+2, and hits enter, the interactive session evaluates the expression and shows its value. If an expression is entered into an interactive session with a trailing semicolon, its value is not shown. The variable ans is bound to the value of the last evaluated expression whether it is shown or not. The ans variable is only bound in interactive sessions, not when Julia code is run in other ways.

To evaluate expressions written in a source file file.jl, write include ("file.jl").

To run code in a file non-interactively, you can give it as the first argument to the julia command:

```
$ julia script.jl arg1 arg2...
```

As the example implies, the following command-line arguments to julia are taken as command-line arguments to the program script.jl, passed in the global constant ARGS. ARGS is also set when script code is given using the -e option on the command line (see the julia help output below). For example, to just print the arguments given to a script, you could do this:

```
$ julia -e 'for x in ARGS; println(x); end' foo bar
foo
bar
```

Or you could put that code into a script and run it:

```
$ echo 'for x in ARGS; println(x); end' > script.jl
$ julia script.jl foo bar
foo
bar
```

The -- delimiter can be used to separate command-line args to the scriptfile from args to Julia:

```
$ julia --color=yes -0 -- foo.jl arg1 arg2..
```

Julia can be started in parallel mode with either the -p or the --machinefile options. -p n will launch an additional n worker processes, while --machinefile file will launch a worker for each line in file file. The machines defined in file must be accessible via a passwordless ssh login, with Julia installed at the same location as the current host. Each machine definition takes the form [count\*][user@]host[:port] [bind\_addr[:port]] . user defaults to current user, port to the standard ssh port. count is the number of workers to spawn on the node, and defaults to 1. The optional bind-to bind\_addr[:port] specifies the ip-address and port that other workers should use to connect to this worker.

If you have code that you want executed whenever julia is run, you can put it in ~/.juliarc.jl:

```
$ echo 'println("Greetings! 好! 안녕하세요?")' > ~/.juliarc.jl
$ julia
Greetings! 好! 안녕하세요?
...
```

There are various ways to run Julia code and provide options, similar to those available for the perl and ruby programs:

```
julia [switches] -- [programfile] [args...]
-v, --version
                          Display version information
-h, --help
                             Print this message
-J, --sysimage <file> Start up with the given system image file 
--precompiled={yes|no} Use precompiled code from system image if available
-- \texttt{compilecache} = \{ \texttt{yes} \, | \, \texttt{no} \} \qquad \texttt{Enable} \, / \, \texttt{disable} \, \, \texttt{incremental} \, \, \texttt{precompilation} \, \, \texttt{of} \, \, \texttt{modules} \\
-H, --home <dir>
                    Set location of julia executable
--startup-file={yes|no} Load ~/.juliarc.jl
-f, --no-startup
                            Don't load ~/.juliarc (deprecated, use --startup-file=no)
                             Load ~/.juliarc (deprecated, use --startup-file=yes)
--handle-signals={yes|no} Enable or disable Julia's default signal handlers
-e, --eval <expr>
                            Evaluate <expr>
-E, --print <expr>
                             Evaluate and show <expr>
-P, --post-boot <expr>
                             Evaluate <expr>, but don't disable interactive mode (deprecated, use -i -c
-L, --load <file>
                             Load <file> immediately on all processors
-p, --procs {N|auto}
                             Integer value N launches N additional local worker processes
                              "auto" launches as many workers as the number of local cores
--machinefile <file>
                             Run processes on hosts listed in <file>
-i
                             Interactive mode; REPL runs and isinteractive() is true
-q, --quiet
                             Quiet startup (no banner)
--color={yes|no}
                            Enable or disable color text
--history-file={yes|no} Load or save history
--no-history-file
                            Don't load history file (deprecated, use --history-file=no)
```

--compile={yes|no|all} Enable or disable compiler, or request exhaustive compilation

-C, --cpu-target <target> Limit usage of cpu features up to <target>

```
-O, --optimize
                          Run time-intensive code optimizations
--inline={yes|no}
                          Control whether inlining is permitted (overrides functions declared as @in
--check-bounds={yes|no}
                          Emit bounds checks always or never (ignoring declarations)
--math-mode={ieee, fast} Disallow or enable unsafe floating point optimizations (overrides @fastmater)
--depwarn={yes|no|error} Enable or disable syntax and method deprecation warnings ("error" turns we
--output-o name
                          Generate an object file (including system image data)
                          Generate a system image data file (.ji)
--output-ji name
--output-bc name
                          Generate LLVM bitcode (.bc)
--output-incremental=no
                        Generate an incremental output file (rather than complete)
--code-coverage={none|user|all}, --code-coverage
                          Count executions of source lines (omitting setting is equivalent to "user"
--track-allocation={none|user|all}, --track-allocation
                          Count bytes allocated by each source line
```

#### 2.1 Resources

In addition to this manual, there are various other resources that may help new users get started with Julia:

- · Julia and IJulia cheatsheet
- · Learn Julia in a few minutes
- Tutorial for Homer Reid's numerical analysis class
- An introductory presentation
- Videos from the Julia tutorial at MIT
- Forio Julia Tutorials

2.1. Resources 7

#### **Variables**

A variable, in Julia, is a name associated (or bound) to a value. It's useful when you want to store a value (that you obtained after some math, for example) for later use. For example:

```
# Assign the value 10 to the variable x
julia> x = 10
10

# Doing math with x's value
julia> x + 1
11

# Reassign x's value
julia> x = 1 + 1
2

# You can assign values of other types, like strings of text
julia> x = "Hello World!"
"Hello World!"
```

Julia provides an extremely flexible system for naming variables. Variable names are case-sensitive, and have no semantic meaning (that is, the language will not treat variables differently based on their names).

```
julia> x = 1.0

1.0

julia> y = -3

-3

julia> Z = "My string"

"My string"

julia> customary_phrase = "Hello world!"

"Hello world!"

julia> UniversalDeclarationOfHumanRightsStart = "人人生而自由, 在尊严和权力上一律平等。"

"人人生而自由, 在尊严和权力上一律平等。"
```

Unicode names (in UTF-8 encoding) are allowed:

```
julia> δ = 0.00001
1.0e-5
julia> 안녕하세요 = "Hello"
"Hello"
```

In the Julia REPL and several other Julia editing environments, you can type many Unicode math symbols by typing the backslashed LaTeX symbol name followed by tab. For example, the variable name  $\delta$  can be entered by typing \delta-tab, or even  $\alpha$ 2 by \alpha-tab-\hat-tab-\\_2-tab. Julia will even let you redefine built-in constants and functions if needed:

```
julia> pi \pi=3.1415926535897... julia> pi = 3 WARNING: imported binding for pi overwritten in module Main 3 julia> pi 3 julia> sqrt(100) 10.0 julia> sqrt = 4 WARNING: imported binding for sqrt overwritten in module Main 4
```

However, this is obviously not recommended to avoid potential confusion.

#### 3.1 Allowed Variable Names

Variable names must begin with a letter (A-Z or a-z), underscore, or a subset of Unicode code points greater than 00A0; in particular, Unicode character categories Lu/Ll/Lt/Lm/Lo/Nl (letters), Sc/So (currency and other symbols), and a few other letter-like characters (e.g. a subset of the Sm math symbols) are allowed. Subsequent characters may also include! and digits (0-9 and other characters in categories Nd/No), as well as other Unicode code points: diacritics and other modifying marks (categories Mn/Mc/Me/Sk), some punctuation connectors (category Pc), primes, and a few other characters.

Operators like + are also valid identifiers, but are parsed specially. In some contexts, operators can be used just like variables; for example (+) refers to the addition function, and (+) = f will reassign it. Most of the Unicode infix operators (in category Sm), such as  $\oplus$ , are parsed as infix operators and are available for user-defined methods (e.g. you can use const  $\otimes$  = kron to define  $\otimes$  as an infix Kronecker product).

The only explicitly disallowed names for variables are the names of built-in statements:

```
julia> else = false
ERROR: syntax: unexpected "else"

julia> try = "No"
ERROR: syntax: unexpected "="
```

# 3.2 Stylistic Conventions

While Julia imposes few restrictions on valid names, it has become useful to adopt the following conventions:

- Names of variables are in lower case.
- Word separation can be indicated by underscores ('\_'), but use of underscores is discouraged unless the name
  would be hard to read otherwise.

- Names of Types and Modules begin with a capital letter and word separation is shown with upper camel case instead of underscores.
- Names of functions and macros are in lower case, without underscores.
- Functions that write to their arguments have names that end in !. These are sometimes called "mutating" or "in-place" functions because they are intended to produce changes in their arguments after the function is called, not just return a value.

# **Integers and Floating-Point Numbers**

Integers and floating-point values are the basic building blocks of arithmetic and computation. Built-in representations of such values are called numeric primitives, while representations of integers and floating-point numbers as immediate values in code are known as numeric literals. For example, 1 is an integer literal, while 1.0 is a floating-point literal; their binary in-memory representations as objects are numeric primitives.

Julia provides a broad range of primitive numeric types, and a full complement of arithmetic and bitwise operators as well as standard mathematical functions are defined over them. These map directly onto numeric types and operations that are natively supported on modern computers, thus allowing Julia to take full advantage of computational resources. Additionally, Julia provides software support for *Arbitrary Precision Arithmetic* (page 21), which can handle operations on numeric values that cannot be represented effectively in native hardware representations, but at the cost of relatively slower performance.

The following are Julia's primitive numeric types:

#### • Integer types:

Туре	Signed?	Number of bits	Smallest value	Largest value
Int8	X	8	-2^7	2^7 - 1
UInt8		8	0	2^8 - 1
Int16	x	16	-2^15	2^15 - 1
UInt16		16	0	2^16 - 1
Int32	x	32	-2^31	2^31 - 1
UInt32		32	0	2^32 - 1
Int64	X	64	-2^63	2^63 - 1
UInt64		64	0	2^64 - 1
Int128	X	128	-2^127	2^127 - 1
UInt128		128	0	2^128 - 1
Bool	N/A	8	false(0)	true(1)

#### Floating-point types:

Туре	Precision	Number of bits
Float16	half	16
Float 32 (page 380)	single	32
Float 64 (page 380)	double	64

Additionally, full support for *Complex and Rational Numbers* (page 35) is built on top of these primitive numeric types. All numeric types interoperate naturally without explicit casting, thanks to a flexible, user-extensible *type promotion system* (page 127).

### 4.1 Integers

Literal integers are represented in the standard manner:

```
julia> 1
1
julia> 1234
1234
```

The default type for an integer literal depends on whether the target system has a 32-bit architecture or a 64-bit architecture:

```
# 32-bit system:
julia> typeof(1)
Int32

# 64-bit system:
julia> typeof(1)
Int64
```

The Julia internal variable WORD\_SIZE (page 441) indicates whether the target system is 32-bit or 64-bit.:

```
# 32-bit system:
julia> WORD_SIZE
32
# 64-bit system:
julia> WORD_SIZE
64
```

Julia also defines the types Int and UInt, which are aliases for the system's signed and unsigned native integer types respectively.:

```
# 32-bit system:
julia> Int
Int32
julia> UInt
UInt32

# 64-bit system:
julia> Int
Int64
julia> UInt
UInt64
```

Larger integer literals that cannot be represented using only 32 bits but can be represented in 64 bits always create 64-bit integers, regardless of the system type:

```
# 32-bit or 64-bit system:
julia> typeof(300000000)
Int64
```

Unsigned integers are input and output using the  $0 \times$  prefix and hexadecimal (base 16) digits 0-9a-f (the capitalized digits A-F also work for input). The size of the unsigned value is determined by the number of hex digits used:

```
julia> 0x1
0x01
```

```
julia> typeof(ans)
UInt8

julia> 0x123

julia> typeof(ans)
UInt16

julia> 0x1234567

0x01234567

julia> typeof(ans)
UInt32

julia> 0x123456789abcdef

0x0123456789abcdef

julia> typeof(ans)
UInt64
```

This behavior is based on the observation that when one uses unsigned hex literals for integer values, one typically is using them to represent a fixed numeric byte sequence, rather than just an integer value.

Recall that the variable ans (page 320) is set to the value of the last expression evaluated in an interactive session. This does not occur when Julia code is run in other ways.

Binary and octal literals are also supported:

```
julia> 0b10
0x02

julia> typeof(ans)
UInt8

julia> 0o10
0x08

julia> typeof(ans)
UInt8
```

The minimum and maximum representable values of primitive numeric types such as integers are given by the typemin() (page 322) and typemax() (page 323) functions:

4.1. Integers 15

The values returned by typemin() (page 322) and typemax() (page 323) are always of the given argument type. (The above expression uses several features we have yet to introduce, including for loops (page 70), Strings (page 41), and Interpolation (page 46), but should be easy enough to understand for users with some existing programming experience.)

#### 4.1.1 Overflow behavior

In Julia, exceeding the maximum representable value of a given type results in a wraparound behavior:

```
julia> x = typemax(Int64)
9223372036854775807

julia> x + 1
-9223372036854775808

julia> x + 1 == typemin(Int64)
true
```

Thus, arithmetic with Julia integers is actually a form of modular arithmetic. This reflects the characteristics of the underlying arithmetic of integers as implemented on modern computers. In applications where overflow is possible, explicit checking for wraparound produced by overflow is essential; otherwise, the BigInt type in *Arbitrary Precision Arithmetic* (page 21) is recommended instead.

#### 4.1.2 Division errors

Integer division (the div function) has two exceptional cases: dividing by zero, and dividing the lowest negative number (typemin() (page 322)) by -1. Both of these cases throw a DivideError (page 330). The remainder and modulus functions (rem and mod) throw a DivideError (page 330) when their second argument is zero.

# 4.2 Floating-Point Numbers

Literal floating-point numbers are represented in the standard formats:

```
julia> 1.0
1.0

julia> 1.
1.0

julia> 0.5
0.5

julia> .5
0.5

julia> -1.23
-1.23

julia> 1e10
1.0e10
```

```
julia> 2.5e-4
0.00025
```

The above results are all Float 64 values. Literal Float 32 values can be entered by writing an f in place of e:

```
julia> 0.5f0
0.5f0

julia> typeof(ans)
Float32

julia> 2.5f-4
0.00025f0
```

Values can be converted to Float 32 easily:

```
julia> Float32(-1.5)
-1.5f0

julia> typeof(ans)
Float32
```

Hexadecimal floating-point literals are also valid, but only as Float 64 values:

```
julia> 0x1p0
1.0

julia> 0x1.8p3
12.0

julia> 0x.4p-1
0.125

julia> typeof(ans)
Float64
```

Half-precision floating-point numbers are also supported (Float16), but only as a storage format. In calculations they'll be converted to Float32:

```
julia> sizeof(Float16(4.))
2
julia> 2*Float16(4.)
8.0f0
```

The underscore \_ can be used as digit separator:

```
julia> 10_000, 0.000_000_005, 0xdead_beef, 0b1011_0010
(10000,5.0e-9,0xdeadbeef,0xb2)
```

#### 4.2.1 Floating-point zero

Floating-point numbers have two zeros, positive zero and negative zero. They are equal to each other but have different binary representations, as can be seen using the bits function: :

```
julia> 0.0 == -0.0
true
julia> bits(0.0)
```

#### 4.2.2 Special floating-point values

There are three specified standard floating-point values that do not correspond to any point on the real number line:

Special value		Name	Description	
Float16	Float32	Float64		
Inf16	Inf32	Inf	positive	a value greater than all finite floating-point values
			infinity	
-Inf16	-Inf32	-Inf	negative	a value less than all finite floating-point values
			infinity	
NaN16	NaN32	NaN	not a number	a value not == to any floating-point value (including
				itself)

For further discussion of how these non-finite floating-point values are ordered with respect to each other and other floats, see *Numeric Comparisons* (page 27). By the IEEE 754 standard, these floating-point values are the results of certain arithmetic operations:

```
julia> 1/Inf
0.0
julia> 1/0
Inf
julia> -5/0
-Inf
julia> 0.000001/0
Inf
julia> 0/0
NaN
julia> 500 + Inf
Inf
julia> 500 - Inf
-Inf
julia> Inf + Inf
Inf
julia> Inf - Inf
NaN
julia> Inf * Inf
julia> Inf / Inf
NaN
julia> 0 * Inf
NaN
```

The typemin () (page 322) and typemax () (page 323) functions also apply to floating-point types:

```
julia> (typemin(Float16), typemax(Float16))
(-Inf16, Inf16)

julia> (typemin(Float32), typemax(Float32))
(-Inf32, Inf32)

julia> (typemin(Float64), typemax(Float64))
(-Inf, Inf)
```

#### 4.2.3 Machine epsilon

Most real numbers cannot be represented exactly with floating-point numbers, and so for many purposes it is important to know the distance between two adjacent representable floating-point numbers, which is often known as machine epsilon.

Julia provides eps() (page 323), which gives the distance between 1.0 and the next larger representable floating-point value:

```
julia> eps(Float32)
1.1920929f-7

julia> eps(Float64)
2.220446049250313e-16

julia> eps() # same as eps(Float64)
2.220446049250313e-16
```

These values are  $2.0^-23$  and  $2.0^-52$  as Float 32 and Float 64 values, respectively. The eps () (page 323) function can also take a floating-point value as an argument, and gives the absolute difference between that value and the next representable floating point value. That is, eps (x) yields a value of the same type as x such that x + eps (x) is the next representable floating-point value larger than x:

```
julia> eps(1.0)
2.220446049250313e-16

julia> eps(1000.)
1.1368683772161603e-13

julia> eps(1e-27)
1.793662034335766e-43

julia> eps(0.0)
5.0e-324
```

The distance between two adjacent representable floating-point numbers is not constant, but is smaller for smaller values and larger for larger values. In other words, the representable floating-point numbers are densest in the real number line near zero, and grow sparser exponentially as one moves farther away from zero. By definition, eps (1.0) is the same as eps (Float64) since 1.0 is a 64-bit floating-point value.

Julia also provides the nextfloat () (page 379) and prevfloat () (page 379) functions which return the next largest or smallest representable floating-point number to the argument respectively:

```
julia> x = 1.25f0
1.25f0
julia> nextfloat(x)
```

This example highlights the general principle that the adjacent representable floating-point numbers also have adjacent binary integer representations.

#### 4.2.4 Rounding modes

If a number doesn't have an exact floating-point representation, it must be rounded to an appropriate representable value, however, if wanted, the manner in which this rounding is done can be changed according to the rounding modes presented in the IEEE 754 standard:

The default mode used is always RoundNearest (page 361), which rounds to the nearest representable value, with ties rounded towards the nearest value with an even least significant bit.

**Warning:** Rounding is generally only correct for basic arithmetic functions (+()) (page 353), -() (page 353), \*() (page 385), /() (page 353) and sqrt() (page 364)) and type conversion operations. Many other functions assume the default RoundNearest (page 361) mode is set, and can give erroneous results when operating under other rounding modes.

#### 4.2.5 Background and References

Floating-point arithmetic entails many subtleties which can be surprising to users who are unfamiliar with the low-level implementation details. However, these subtleties are described in detail in most books on scientific computation, and also in the following references:

- The definitive guide to floating point arithmetic is the IEEE 754-2008 Standard; however, it is not available for free online.
- For a brief but lucid presentation of how floating-point numbers are represented, see John D. Cook's article on the subject as well as his introduction to some of the issues arising from how this representation differs in behavior from the idealized abstraction of real numbers.
- Also recommended is Bruce Dawson's series of blog posts on floating-point numbers.

- For an excellent, in-depth discussion of floating-point numbers and issues of numerical accuracy encountered when computing with them, see David Goldberg's paper What Every Computer Scientist Should Know About Floating-Point Arithmetic.
- For even more extensive documentation of the history of, rationale for, and issues with floating-point numbers, as well as discussion of many other topics in numerical computing, see the collected writings of William Kahan, commonly known as the "Father of Floating-Point". Of particular interest may be An Interview with the Old Man of Floating-Point.

# 4.3 Arbitrary Precision Arithmetic

To allow computations with arbitrary-precision integers and floating point numbers, Julia wraps the GNU Multiple Precision Arithmetic Library (GMP) and the GNU MPFR Library, respectively. The <code>BigInt</code> (page 380) and <code>BigFloat</code> (page 380) types are available in Julia for arbitrary precision integer and floating point numbers respectively.

Constructors exist to create these types from primitive numerical types, and parse() (page 326) can be use to construct them from AbstractStrings. Once created, they participate in arithmetic with all other numeric types thanks to Julia's type promotion and conversion mechanism (page 127):

However, type promotion between the primitive types above and <code>BigInt</code> (page 380)/<code>BigFloat</code> (page 380) is not automatic and must be explicitly stated.

```
julia> x = typemin(Int64)
-9223372036854775808

julia> x = x - 1
9223372036854775807

julia> typeof(x)
Int64

julia> y = BigInt(typemin(Int64))
-9223372036854775808

julia> y = y - 1
-9223372036854775809

julia> typeof(y)
BigInt
```

The default precision (in number of bits of the significand) and rounding mode of BigFloat (page 380) operations can be changed globally by calling setprecision () (page 382) and setrounding () (page 380), and all further

calculations will take these changes in account. Alternatively, the precision or the rounding can be changed only within the execution of a particular block of code by using the same functions with a do block:

#### 4.4 Numeric Literal Coefficients

To make common numeric formulas and expressions clearer, Julia allows variables to be immediately preceded by a numeric literal, implying multiplication. This makes writing polynomial expressions much cleaner:

```
julia> x = 3
3

julia> 2x^2 - 3x + 1
10

julia> 1.5x^2 - .5x + 1
13.0
```

It also makes writing exponential functions more elegant:

```
julia> 2^2x
64
```

The precedence of numeric literal coefficients is the same as that of unary operators such as negation. So  $2^3x$  is parsed as  $2^3x$ , and  $2x^3$  is parsed as  $2^3x$ .

Numeric literals also work as coefficients to parenthesized expressions:

```
julia> 2(x-1)^2 - 3(x-1) + 1
3
```

Additionally, parenthesized expressions can be used as coefficients to variables, implying multiplication of the expression by the variable:

```
julia> (x-1)x
6
```

Neither juxtaposition of two parenthesized expressions, nor placing a variable before a parenthesized expression, however, can be used to imply multiplication:

```
julia> (x-1) (x+1)
ERROR: MethodError: `call` has no method matching call(::Int64, ::Int64)
Closest candidates are:
   Union(!Matched::Any...)
```

```
BoundsError()
BoundsError(!Matched::Any...)
...

julia> x(x+1)
ERROR: MethodError: `call` has no method matching call(::Int64, ::Int64)
Closest candidates are:
   Union(!Matched::Any...)
   BoundsError()
BoundsError(!Matched::Any...)
...
```

Both expressions are interpreted as function application: any expression that is not a numeric literal, when immediately followed by a parenthetical, is interpreted as a function applied to the values in parentheses (see *Functions* (page 55) for more about functions). Thus, in both of these cases, an error occurs since the left-hand value is not a function.

The above syntactic enhancements significantly reduce the visual noise incurred when writing common mathematical formulae. Note that no whitespace may come between a numeric literal coefficient and the identifier or parenthesized expression which it multiplies.

#### 4.4.1 Syntax Conflicts

Juxtaposed literal coefficient syntax may conflict with two numeric literal syntaxes: hexadecimal integer literals and engineering notation for floating-point literals. Here are some situations where syntactic conflicts arise:

- The hexadecimal integer literal expression 0xff could be interpreted as the numeric literal 0 multiplied by the variable xff.
- The floating-point literal expression 1e10 could be interpreted as the numeric literal 1 multiplied by the variable e10, and similarly with the equivalent E form.

In both cases, we resolve the ambiguity in favor of interpretation as a numeric literals:

- Expressions starting with 0x are always hexadecimal literals.
- Expressions starting with a numeric literal followed by e or E are always floating-point literals.

#### 4.5 Literal zero and one

Julia provides functions which return literal 0 and 1 corresponding to a specified type or the type of a given variable.

Function	Description
zero(x) (page 378)	Literal zero of type x or type of variable x
one (x) (page 378)	Literal one of type x or type of variable x

These functions are useful in *Numeric Comparisons* (page 27) to avoid overhead from unnecessary *type conversion* (page 127).

#### Examples:

```
julia> zero(Float32)
0.0f0

julia> zero(1.0)
0.0

julia> one(Int32)
```

# **Mathematical Operations and Elementary Functions**

Julia provides a complete collection of basic arithmetic and bitwise operators across all of its numeric primitive types, as well as providing portable, efficient implementations of a comprehensive collection of standard mathematical functions.

# **5.1 Arithmetic Operators**

The following arithmetic operators are supported on all primitive numeric types:

Expression	Name	Description
+X	unary plus	the identity operation
-x	unary minus	maps values to their additive inverses
х + у	binary plus	performs addition
х - у	binary minus	performs subtraction
х * у	times	performs multiplication
х / у	divide	performs division
х \ у	inverse divide	equivalent to y / x
х ^ у	power	raises x to the yth power
х % у	remainder	equivalent to rem(x,y)

as well as the negation on Bool types:

Expression	Name	Description
! x	negation	changes true to false and vice versa

Julia's promotion system makes arithmetic operations on mixtures of argument types "just work" naturally and automatically. See *Conversion and Promotion* (page 127) for details of the promotion system.

Here are some simple examples using arithmetic operators:

```
julia> 1 + 2 + 3
6

julia> 1 - 2
-1

julia> 3*2/12
0.5
```

(By convention, we tend to space less tightly binding operators less tightly, but there are no syntactic constraints.)

# **5.2 Bitwise Operators**

The following bitwise operators are supported on all primitive integer types:

Expression	Name
~X	bitwise not
х & у	bitwise and
х   у	bitwise or
х \$ у	bitwise xor (exclusive or)
х >>> у	logical shift right
х >> у	arithmetic shift right
х << у	logical/arithmetic shift left

Here are some examples with bitwise operators:

```
julia> ~123
-124

julia> 123 & 234
106

julia> 123 | 234
251

julia> 123 $ 234
145

julia> ~UInt32(123)
0xffffff84

julia> ~UInt8(123)
0x84
```

# 5.3 Updating operators

Every binary arithmetic and bitwise operator also has an updating version that assigns the result of the operation back into its left operand. The updating version of the binary operator is formed by placing a = immediately after the operator. For example, writing x += 3 is equivalent to writing x = x + 3:

```
julia> x = 1
1

julia> x += 3
4

julia> x
4
```

The updating versions of all the binary arithmetic and bitwise operators are:

**Note:** An updating operator rebinds the variable on the left-hand side. As a result, the type of the variable may change.

```
julia> x = 0x01; typeof(x)
UInt8

julia> x *= 2 #Same as x = x * 2

julia> isa(x, Int)
true
```

# **5.4 Numeric Comparisons**

Standard comparison operations are defined for all the primitive numeric types:

Operator	Name
== (page 355)	equality
$! = (page 355) \neq (page 355)$	inequality
< (page 356)	less than
$<=$ (page 356) $\leq$ (page 355)	less than or equal to
> (page 356)	greater than
>= (page 356) ≥ (page 356)	greater than or equal to

Here are some simple examples:

```
julia> 1 == 1
true
julia> 1 == 2
false
julia> 1 != 2
true
julia> 1 == 1.0
true
julia> 1 < 2
true
julia> 1.0 > 3
false
julia> 1 >= 1.0
true
julia> -1 <= 1
true
julia> -1 <= -1
julia> -1 <= -2
false
julia> 3 < -0.5
```

Integers are compared in the standard manner — by comparison of bits. Floating-point numbers are compared according to the IEEE 754 standard:

- Finite numbers are ordered in the usual manner.
- Positive zero is equal but not greater than negative zero.
- Inf is equal to itself and greater than everything else except NaN.
- -Inf is equal to itself and less then everything else except NaN.
- NaN is not equal to, not less than, and not greater than anything, including itself.

The last point is potentially surprising and thus worth noting:

```
julia> NaN == NaN
false

julia> NaN != NaN
true

julia> NaN < NaN
false

julia> NaN > NaN
false
```

and can cause especial headaches with Arrays (page 157):

```
julia> [1 NaN] == [1 NaN]
false
```

Julia provides additional functions to test numbers for special values, which can be useful in situations like hash key comparisons:

Function	Tests if
isequal(x, y) (page 320)	x and y are identical
isfinite(x) (page 379)	x is a finite number
isinf(x) (page 379)	x is infinite
isnan(x) (page 379)	x is not a number

isequal () (page 320) considers NaNs equal to each other:

```
julia> isequal(NaN,NaN)
true

julia> isequal([1 NaN], [1 NaN])
true

julia> isequal(NaN,NaN32)
true
```

isequal () (page 320) can also be used to distinguish signed zeros:

```
julia> -0.0 == 0.0
true

julia> isequal(-0.0, 0.0)
false
```

Mixed-type comparisons between signed integers, unsigned integers, and floats can be tricky. A great deal of care has been taken to ensure that Julia does them correctly.

For other types, isequal() (page 320) defaults to calling ==() (page 355), so if you want to define equality for your own types then you only need to add a==() (page 355) method. If you define your own equality function, you should probably define a corresponding hash() (page 321) method to ensure that isequal(x, y) implies hash(x) == hash(y).

#### 5.4.1 Chaining comparisons

Unlike most languages, with the notable exception of Python, comparisons can be arbitrarily chained:

```
julia> 1 < 2 <= 2 < 3 == 3 > 2 >= 1 == 1 < 3 != 5
true</pre>
```

Chaining comparisons is often quite convenient in numerical code. Chained comparisons use the && operator for scalar comparisons, and the & (page 356) operator for elementwise comparisons, which allows them to work on arrays. For example, 0 < A < 1 gives a boolean array whose entries are true where the corresponding elements of A are between 0 and 1.

The operator < (page 356) is intended for array objects; the operation A .< B is valid only if A and B have the same dimensions. The operator returns an array with boolean entries and with the same dimensions as A and B. Such operators are called *elementwise*; Julia offers a suite of elementwise operators: \* (page 385), + (page 353), etc. Some of the elementwise operators can take a scalar operand such as the example 0 .< A .< 1 in the preceding paragraph. This notation means that the scalar operand should be replicated for each entry of the array.

Note the evaluation behavior of chained comparisons:

```
v(x) = (println(x); x)

julia> v(1) < v(2) <= v(3)
2
1
3
true

julia> v(1) > v(2) <= v(3)
2
1
false</pre>
```

The middle expression is only evaluated once, rather than twice as it would be if the expression were written as v(1) < v(2) & v(2) <= v(3). However, the order of evaluations in a chained comparison is undefined. It is strongly recommended not to use expressions with side effects (such as printing) in chained comparisons. If side effects are required, the short-circuit && operator should be used explicitly (see *Short-Circuit Evaluation* (page 68)).

#### 5.4.2 Operator Precedence

Julia applies the following order of operations, from highest precedence to lowest:

Category	Operators
Syntax	. followed by ::
Exponenti-	^ and its elementwise equivalent . ^
ation	
Fractions	// and .//
Multiplica-	* / % & \ and .* ./ .% .\
tion	
Bitshifts	<< >> >>> and .<< .>> .>>>
Addition	+ -   \$ and .+
Syntax	: followed by  >
Compar-	> < >= <= === != !== <: and .> .< .>= .<= .!=
isons	
Control	&& followed by     followed by ?
flow	
Assign-	= += -= *= /= //= \= ^= ÷= %=  = &= \$= <<= >>> = and .+== .*=
ments	./= .//= .\= . <sup>^</sup> = . <sup>÷</sup> = . <sup>%</sup> =

#### 5.5 Numerical Conversions

Julia supports three forms of numerical conversion, which differ in their handling of inexact conversions.

- The notation T(x) or convert (T, x) converts x to a value of type T.
  - If T is a floating-point type, the result is the nearest representable value, which could be positive or negative infinity.
  - If T is an integer type, an InexactError is raised if x is not representable by T.
- x % T converts an integer x to a value of integer type T congruent to x modulo  $2^n$ , where n is the number of bits in T. In other words, the binary representation is truncated to fit.
- The *Rounding functions* (page 31) take a type T as an optional argument. For example, round (Int, x) is a shorthand for Int (round(x)).

The following examples show the different forms.

```
julia> Int8(127)
127
julia> Int8(128)
ERROR: InexactError()
in call at ./essentials.jl:58
in eval at ./boot.jl:263
julia> Int8(127.0)
127
julia> Int8(3.14)
ERROR: InexactError()
in call at ./essentials.jl:58
in eval at ./boot.jl:263
julia> Int8(128.0)
ERROR: InexactError()
in call at ./essentials.jl:58
in eval at ./boot.jl:263
```

```
julia> 127 % Int8
127

julia> 128 % Int8
-128

julia> round(Int8,127.4)
127

julia> round(Int8,127.6)
ERROR: InexactError()
  in trunc at ./float.jl:357
  in round at ./float.jl:177
  in eval at ./boot.jl:263
```

See Conversion and Promotion (page 127) for how to define your own conversions and promotions.

# 5.6 Elementary Functions

Julia provides a comprehensive collection of mathematical functions and operators. These mathematical operations are defined over as broad a class of numerical values as permit sensible definitions, including integers, floating-point numbers, rationals, and complexes, wherever such definitions make sense.

#### 5.6.1 Rounding functions

Function	Description	Return type
round(x) (page 360)	round x to the nearest integer	typeof(x)
round (T, x) (page 360)	round x to the nearest integer	Т
floor(x) (page 362)	round x towards -Inf	typeof(x)
floor(T, x) (page 362)	round x towards -Inf	T
ceil(x) (page 362)	round x towards +Inf	typeof(x)
ceil(T, x) (page 362)	round x towards +Inf	T
trunc(x) (page 362)	round x towards zero	typeof(x)
trunc(T, x) (page 362)	round x towards zero	Т

#### 5.6.2 Division functions

Function	Description		
div(x, y) (page 354)	truncated division; quotient rounded towards zero		
fld(x, y) (page 354)	floored division; quotient rounded towards -Inf		
cld(x, y) (page 354) ceiling division; quotient rounded towards +Inf			
rem(x,y) (page 354)	remainder; satisfies $x == div(x, y) * y + rem(x, y)$ ; sign matches x		
mod(x, y) (page 354)	modulus; satisfies $x == fld(x, y) * y + mod(x, y)$ ; sign matches y		
mod2pi(x) (page 354)	modulus with respect to 2pi; 0 <= mod2pi(x) < 2pi		
divrem(x, y) (page 354)	returns (div(x,y), rem(x,y))		
fldmod(x, y) (page 354)	(page 354) returns (fld(x,y), $mod(x,y)$ )		
gcd(x, y) (page 365)	(page 365) greatest common divisor of x, y,; sign matches x		
1cm(x, y) (page 365)	(page 365) least common multiple of x, y,; sign matches x		

### 5.6.3 Sign and absolute value functions

Function	Description
abs (x) (page 363)	a positive value with the magnitude of x
abs2 (x) (page 363)	the squared magnitude of x
sign(x) (page 364)	indicates the sign of x, returning -1, 0, or +1
signbit(x) (page 364)	indicates whether the sign bit is on (true) or off (false)
copysign(x,y) (page 363)	a value with the magnitude of x and the sign of y
flipsign(x,y) (page 364)	a value with the magnitude of x and the sign of x*y

#### 5.6.4 Powers, logs and roots

Function	Description	
sqrt (x) (page 364) √x	square root of x	
cbrt (x) (page 364) 3/x	cube root of x	
hypot (x, y) (page 360)	hypotenuse of right-angled triangle with other sides of length $x$ and $y$	
exp (x) (page 360)	natural exponential function at x	
expm1 (x) (page 360)	accurate exp (x) -1 for x near zero	
<i>1dexp(x,n)</i> (page 360)	x*2^n computed efficiently for integer values of n	
log(x) (page 360)	natural logarithm of x	
log(b, x) (page 360)	base b logarithm of x	
10g2 (x) (page 360)	base 2 logarithm of x	
log10 (x) (page 360)	base 10 logarithm of x	
log1p(x) (page 360) accurate $log(1+x)$ for x near zero		
exponent (x) (page 378)	binary exponent of x	
significand (x) (page 378) binary significand (a.k.a. mantissa) of a floating-point number		

For an overview of why functions like hypot() (page 360), expm1() (page 360), and log1p() (page 360) are necessary and useful, see John D. Cook's excellent pair of blog posts on the subject: expm1, log1p, erfc, and hypot.

## 5.6.5 Trigonometric and hyperbolic functions

All the standard trigonometric and hyperbolic functions are also defined:

sin	cos	tan	cot	sec	CSC
sinh	cosh	tanh	coth	sech	csch
asin	acos	atan	acot	asec	acsc
asinh	acosh	atanh	acoth	asech	acsch
sinc	cosc	atan2			

These are all single-argument functions, with the exception of atan2, which gives the angle in radians between the x-axis and the point specified by its arguments, interpreted as x and y coordinates.

Additionally, sinpi(x) (page 358) and cospi(x) (page 358) are provided for more accurate computations of sin(pi\*x) (page 358) and cos(pi\*x) (page 358) respectively.

In order to compute trigonometric functions with degrees instead of radians, suffix the function with d. For example, sind(x) (page 358) computes the sine of x where x is specified in degrees. The complete list of trigonometric functions with degree variants is:

sind	cosd	tand	cotd	secd	cscd
asind	acosd	atand	acotd	asecd	acscd

# 5.6.6 Special functions

Function	Description
erf(x) (page 364)	error function at x
erfc(x) (page 364)	complementary error function, i.e. the accurate version
erfinv(x) (page 364)	inverse function to erf() (page 364)
erfcinv(x)	inverse function to erfc() (page 364)
erfi(x) (page 364)	imaginary error function defined as -im * erf(x
erfcx(x) (page 364)	scaled complementary error function, i.e. accurate ex
dawson(x) (page 364)	scaled imaginary error function, a.k.a. Dawson function
gamma (x) (page 366)	gamma function at x
1gamma (x) (page 366)	accurate log(gamma(x)) for large x
Ifact (x) (page 366)	accurate log(factorial(x)) for large x; same
digamma(x) (page 366)	digamma function (i.e. the derivative of 1gamma ()
beta(x, y) (page 367)	beta function at x, y
lbeta(x, y) (page 367)	accurate log(beta(x,y)) for large x or y
eta(x) (page 367)	Dirichlet eta function at x
zeta(x) (page 367)	Riemann zeta function at x
airy(z) (page 366), airyai(z) (page 366), airy(0,z)	Airy Ai function at z
airyprime(z) (page 366), airyaiprime(z) (page 366), airy(1,z)	derivative of the Airy Ai function at z
airybi(z) (page 366), airy(2,z)	Airy Bi function at z
airybiprime(z) (page 366), airy(3,z)	derivative of the Airy Bi function at z
airyx(z) (page 366), airyx(k,z)	scaled Airy AI function and k th derivatives at z
besselj(nu,z) (page 367)	Bessel function of the first kind of order nu at z
besselj0(z) (page 366)	besselj(0,z)
besselj1(z) (page 366)	besselj(1,z)
besseljx(nu,z) (page 367)	scaled Bessel function of the first kind of order nu at
bessely(nu,z) (page 367)	Bessel function of the second kind of order nu at z
bessely0(z) (page 367)	bessely(0,z)
bessely1(z) (page 367)	bessely(1,z)
besselyx(nu,z) (page 367)	scaled Bessel function of the second kind of order nu
besselh(nu, k, z) (page 367)	Bessel function of the third kind (a.k.a. Hankel function
hankelh1 (nu, z) (page 367)	besselh(nu, 1, z)
hankelh1x(nu,z) (page 367)	scaled besselh(nu, 1, z)
hankelh2(nu,z) (page 367)	besselh(nu, 2, z)
hankelh2x(nu,z) (page 367)	scaled besselh(nu, 2, z)
besseli(nu,z) (page 367)	modified Bessel function of the first kind of order nu
besselix(nu,z) (page 367)	scaled modified Bessel function of the first kind of or
besselk(nu,z) (page 367)	modified Bessel function of the second kind of order
besselkx(nu,z) (page 367)	scaled modified Bessel function of the second kind of



# **Complex and Rational Numbers**

Julia ships with predefined types representing both complex and rational numbers, and supports all *standard mathematical operations* (page 25) on them. *Conversion and Promotion* (page 127) are defined so that operations on any combination of predefined numeric types, whether primitive or composite, behave as expected.

# **6.1 Complex Numbers**

The global constant im (page 379) is bound to the complex number i, representing the principal square root of -1. It was deemed harmful to co-opt the name i for a global constant, since it is such a popular index variable name. Since Julia allows numeric literals to be juxtaposed with identifiers as coefficients (page 22), this binding suffices to provide convenient syntax for complex numbers, similar to the traditional mathematical notation:

```
julia> 1 + 2im
1 + 2im
```

You can perform all the standard arithmetic operations with complex numbers:

```
julia > (1 + 2im) * (2 - 3im)
8 + 1im
julia > (1 + 2im) / (1 - 2im)
-0.6 + 0.8im
julia > (1 + 2im) + (1 - 2im)
2 + 0im
julia > (-3 + 2im) - (5 - 1im)
-8 + 3im
julia> (-1 + 2im)^2
-3 - 4im
julia > (-1 + 2im)^2.5
2.7296244647840084 - 6.960664459571898im
julia > (-1 + 2im)^{(1 + 1im)}
-0.27910381075826657 + 0.08708053414102428im
julia > 3(2 - 5im)
6 - 15im
julia > 3(2 - 5im)^2
```

```
-63 - 60im

julia> 3(2 - 5im)^-1.0

0.20689655172413796 + 0.5172413793103449im
```

The promotion mechanism ensures that combinations of operands of different types just work:

```
julia> 2(1 - 1im)
2 - 2im
julia > (2 + 3im) - 1
1 + 3im
julia > (1 + 2im) + 0.5
1.5 + 2.0 im
julia > (2 + 3im) - 0.5im
2.0 + 2.5 \text{im}
julia > 0.75(1 + 2im)
0.75 + 1.5 im
julia> (2 + 3im) / 2
1.0 + 1.5 im
julia > (1 - 3im) / (2 + 2im)
-0.5 - 1.0im
julia> 2im^2
-2 + 0im
julia > 1 + 3/4im
1.0 - 0.75im
```

Note that  $3/4 \text{im} = 3/(4 \times \text{im}) = -(3/4 \times \text{im})$ , since a literal coefficient binds more tightly than division.

Standard functions to manipulate complex values are provided:

```
julia> real(1 + 2im)
1

julia> imag(1 + 2im)
2

julia> conj(1 + 2im)
1 - 2im

julia> abs(1 + 2im)
2.23606797749979

julia> abs2(1 + 2im)
5

julia> angle(1 + 2im)
1.1071487177940904
```

As usual, the absolute value (abs () (page 363)) of a complex number is its distance from zero. abs2 () (page 363) gives the square of the absolute value, and is of particular use for complex numbers where it avoids taking a square root. angle () (page 364) returns the phase angle in radians (also known as the argument or arg function). The full gamut of other *Elementary Functions* (page 31) is also defined for complex numbers:

```
julia> sqrt(1im)
0.7071067811865476 + 0.7071067811865475im

julia> sqrt(1 + 2im)
1.272019649514069 + 0.7861513777574233im

julia> cos(1 + 2im)
2.0327230070196656 - 3.0518977991518im

julia> exp(1 + 2im)
-1.1312043837568135 + 2.4717266720048188im

julia> sinh(1 + 2im)
-0.4890562590412937 + 1.4031192506220405im
```

Note that mathematical functions typically return real values when applied to real numbers and complex values when applied to complex numbers. For example, sqrt() (page 364) behaves differently when applied to -1 versus -1 + 0im even though -1 == -1 + 0im:

```
julia> sqrt(-1)
ERROR: DomainError:
sqrt will only return a complex result if called with a complex argument. Try sqrt(complex(x)).
in sqrt at math.jl:146

julia> sqrt(-1 + 0im)
0.0 + 1.0im
```

The *literal numeric coefficient notation* (page 22) does not work when constructing complex number from variables. Instead, the multiplication must be explicitly written out:

```
julia> a = 1; b = 2; a + b*im
1 + 2im
```

However, this is *not* recommended; Use the *complex()* (page 378) function instead to construct a complex value directly from its real and imaginary parts.:

```
julia> complex(a,b)
1 + 2im
```

This construction avoids the multiplication and addition operations.

Inf (page 379) and NaN (page 379) propagate through complex numbers in the real and imaginary parts of a complex number as described in the *Special floating-point values* (page 18) section:

```
julia> 1 + Inf*im
1.0 + Inf*im

julia> 1 + NaN*im
1.0 + NaN*im
```

#### 6.2 Rational Numbers

Julia has a rational number type to represent exact ratios of integers. Rationals are constructed using the // (page 354) operator:

```
julia> 2//3
2//3
```

6.2. Rational Numbers 37

If the numerator and denominator of a rational have common factors, they are reduced to lowest terms such that the denominator is non-negative:

```
julia> 6//9
2//3

julia> -4//8
-1//2

julia> 5//-15
-1//3

julia> -4//-12
1//3
```

This normalized form for a ratio of integers is unique, so equality of rational values can be tested by checking for equality of the numerator and denominator. The standardized numerator and denominator of a rational value can be extracted using the num() (page 355) and den() (page 355) functions:

```
julia> num(2//3)
2

julia> den(2//3)
3
```

Direct comparison of the numerator and denominator is generally not necessary, since the standard arithmetic and comparison operations are defined for rational values:

```
julia > 2//3 == 6//9
true
julia> 2//3 == 9//27
false
julia> 3//7 < 1//2
true
julia> 3//4 > 2//3
true
julia > 2//4 + 1//6
2//3
julia> 5//12 - 1//4
1//6
julia > 5//8 * 3//12
5//32
julia> 6//5 / 10//7
21//25
```

Rationals can be easily converted to floating-point numbers:

```
julia> float(3//4)
0.75
```

Conversion from rational to floating-point respects the following identity for any integral values of a and b, with the exception of the case a == 0 and b == 0:

```
julia> isequal(float(a//b), a/b)
true
```

#### Constructing infinite rational values is acceptable:

```
julia> 5//0
1//0

julia> -3//0
-1//0

julia> typeof(ans)
Rational{Int64}
```

#### Trying to construct a NaN (page 379) rational value, however, is not:

```
julia> 0//0
ERROR: ArgumentError: invalid rational: zero(Int64)//zero(Int64)
in call at rational.jl:8
in // at rational.jl:22
```

#### As usual, the promotion system makes interactions with other numeric types effortless:

```
julia> 3//5 + 1
8//5
julia > 3//5 - 0.5
0.099999999999998
julia > 2//7 * (1 + 2im)
2//7 + 4//7 * im
julia > 2//7 * (1.5 + 2im)
0.42857142857142855 + 0.5714285714285714im
julia > 3//2 / (1 + 2im)
3//10 - 3//5 * im
julia > 1//2 + 2im
1//2 + 2//1 * im
julia > 1 + 2//3im
1//1 - 2//3 * im
julia > 0.5 == 1//2
true
julia > 0.33 == 1//3
false
julia > 0.33 < 1//3
true
julia > 1//3 - 0.33
0.00333333333333332993
```

6.2. Rational Numbers 39

Julia Language Documentation, Release 0.5.0-dev	
,	

## **Strings**

Strings are finite sequences of characters. Of course, the real trouble comes when one asks what a character is. The characters that English speakers are familiar with are the letters A, B, C, etc., together with numerals and common punctuation symbols. These characters are standardized together with a mapping to integer values between 0 and 127 by the ASCII standard. There are, of course, many other characters used in non-English languages, including variants of the ASCII characters with accents and other modifications, related scripts such as Cyrillic and Greek, and scripts completely unrelated to ASCII and English, including Arabic, Chinese, Hebrew, Hindi, Japanese, and Korean. The Unicode standard tackles the complexities of what exactly a character is, and is generally accepted as the definitive standard addressing this problem. Depending on your needs, you can either ignore these complexities entirely and just pretend that only ASCII characters exist, or you can write code that can handle any of the characters or encodings that one may encounter when handling non-ASCII text. Julia makes dealing with plain ASCII text simple and efficient, and handling Unicode is as simple and efficient as possible. In particular, you can write C-style string code to process ASCII strings, and they will work as expected, both in terms of performance and semantics. If such code encounters non-ASCII text, it will gracefully fail with a clear error message, rather than silently introducing corrupt results. When this happens, modifying the code to handle non-ASCII data is straightforward.

There are a few noteworthy high-level features about Julia's strings:

- AbstractString is an abstraction, not a concrete type many different representations can implement the AbstractString interface, but they can easily be used together and interact transparently. Any string type can be used in any function expecting a AbstractString.
- Like C and Java, but unlike most dynamic languages, Julia has a first-class type representing a single character, called Char. This is just a special kind of 32-bit bitstype whose numeric value represents a Unicode code point.
- As in Java, strings are immutable: the value of a AbstractString object cannot be changed. To construct a different string value, you construct a new string from parts of other strings.
- Conceptually, a string is a *partial function* from indices to characters for some index values, no character value is returned, and instead an exception is thrown. This allows for efficient indexing into strings by the byte index of an encoded representation rather than by a character index, which cannot be implemented both efficiently and simply for variable-width encodings of Unicode strings.
- Julia supports the full range of Unicode characters: literal strings are always ASCII or UTF-8 but other encodings for strings from external sources can be supported.

#### 7.1 Characters

A Char value represents a single character: it is just a 32-bit bitstype with a special literal representation and appropriate arithmetic behaviors, whose numeric value is interpreted as a Unicode code point. Here is how Char values are input and shown:

```
julia> 'x'
'x'
julia> typeof(ans)
Char
```

You can convert a Char to its integer value, i.e. code point, easily:

```
julia> Int('x')
120

julia> typeof(ans)
Int64
```

On 32-bit architectures, typeof (ans) (page 321) will be Int32. You can convert an integer value back to a Char just as easily:

```
julia> Char(120)
```

Not all integer values are valid Unicode code points, but for performance, the Char() conversion does not check that every character value is valid. If you want to check that each converted value is a valid code point, use the <code>isvalid()</code> (page 387) function:

```
julia> Char(0x110000)
'\U110000'

julia> isvalid(Char, 0x110000)
false
```

As of this writing, the valid Unicode code points are U+00 through U+d7ff and U+e000 through U+10ffff. These have not all been assigned intelligible meanings yet, nor are they necessarily interpretable by applications, but all of these values are considered to be valid Unicode characters.

You can input any Unicode character in single quotes using \u followed by up to four hexadecimal digits or \U followed by up to eight hexadecimal digits (the longest valid value only requires six):

```
julia> '\u0'
'\0'

julia> '\u78'
'x'

julia> '\u2200'
'∀'

julia> '\U10ffff'
'\U10fffff'
```

Julia uses your system's locale and language settings to determine which characters can be printed as-is and which must be output using the generic, escaped \u or \U input forms. In addition to these Unicode escape forms, all of C's traditional escaped input forms can also be used:

```
julia> Int('\0')
0

julia> Int('\t')
9

julia> Int('\n')
```

```
julia> Int('\e')
27

julia> Int('\x7f')
127

julia> Int('\177')
127

julia> Int('\xff')
255
```

You can do comparisons and a limited amount of arithmetic with Char values:

```
julia> 'A' < 'a'
true

julia> 'A' <= 'a' <= 'Z'
false

julia> 'A' <= 'X' <= 'Z'
true

julia> 'x' - 'a'
23

julia> 'A' + 1
'B'
```

# 7.2 String Basics

String literals are delimited by double quotes or triple double quotes:

```
julia> str = "Hello, world.\n"
"Hello, world.\n"

julia> """Contains "quote" characters"""
"Contains \"quote\" characters"
```

If you want to extract a character from a string, you index into it:

```
julia> str[1]
'H'

julia> str[6]
','

julia> str[end]
'\n'
```

All indexing in Julia is 1-based: the first element of any integer-indexed object is found at index 1, and the last element is found at index n, when the string has a length of n.

In any indexing expression, the keyword end can be used as a shorthand for the last index (computed by endof(str) (page 337)). You can perform arithmetic and other operations with end, just like a normal value:

7.2. String Basics 43

```
julia> str[end-1]
'.'
julia> str[end÷2]
' '
```

Using an index less than 1 or greater than end raises an error:

```
julia> str[0]
ERROR: BoundsError()
  in getindex at /Users/sabae/src/julia/usr/lib/julia/sys.dylib (repeats 2 times)

julia> str[end+1]
ERROR: BoundsError()
  in getindex at /Users/sabae/src/julia/usr/lib/julia/sys.dylib (repeats 2 times)
```

You can also extract a substring using range indexing:

```
julia> str[4:9]
"lo, wo"
```

Notice that the expressions str[k] and str[k:k] do not give the same result:

```
julia> str[6]
','
julia> str[6:6]
","
```

The former is a single character value of type Char, while the latter is a string value that happens to contain only a single character. In Julia these are very different things.

### 7.3 Unicode and UTF-8

Julia fully supports Unicode characters and strings. As *discussed above* (page 41), in character literals, Unicode code points can be represented using Unicode \u and \U escape sequences, as well as all the standard C escape sequences. These can likewise be used to write string literals:

```
julia> s = "\u2200 x \u2203 y"
"∀ x ∃ y"
```

Whether these Unicode characters are displayed as escapes or shown as special characters depends on your terminal's locale settings and its support for Unicode. Non-ASCII string literals are encoded using the UTF-8 encoding. UTF-8 is a variable-width encoding, meaning that not all characters are encoded in the same number of bytes. In UTF-8, ASCII characters — i.e. those with code points less than 0x80 (128) — are encoded as they are in ASCII, using a single byte, while code points 0x80 and above are encoded using multiple bytes — up to four per character. This means that not every byte index into a UTF-8 string is necessarily a valid index for a character. If you index into a string at such an invalid byte index, an error is thrown:

```
julia> s[1]
'∀'

julia> s[2]
ERROR: UnicodeError: invalid character index
  in next at ./unicode/utf8.jl:65
  in getindex at strings/basic.jl:37
```

```
julia> s[3]
ERROR: UnicodeError: invalid character index
  in next at ./unicode/utf8.jl:65
  in getindex at strings/basic.jl:37

julia> s[4]
' '
```

In this case, the character  $\forall$  is a three-byte character, so the indices 2 and 3 are invalid and the next character's index is 4; this next valid index can be computed by nextind(s, 1) (page 389), and the next index after that by nextind(s, 4) and so on.

Because of variable-length encodings, the number of characters in a string (given by length(s) (page 385)) is not always the same as the last index. If you iterate through the indices 1 through endof(s) (page 337) and index into s, the sequence of characters returned when errors aren't thrown is the sequence of characters comprising the string s. Thus we have the identity that  $length(s) \le endof(s)$ , since each character in a string must have its own index. The following is an inefficient and verbose way to iterate through the characters of s:

The blank lines actually have spaces on them. Fortunately, the above awkward idiom is unnecessary for iterating through the characters in a string, since you can just use the string as an iterable object, no exception handling required:

UTF-8 is not the only encoding that Julia supports, and adding support for new encodings is quite easy. In particular, Julia also provides UTF16String and UTF32String types, constructed by utf16() (page 390) and utf32() (page 390) respectively, for UTF-16 and UTF-32 encodings. It also provides aliases WString and wstring() (page 391) for either UTF-16 or UTF-32 strings, depending on the size of Cwchar\_t. Additional discussion of other encodings and how to implement support for them is beyond the scope of this document for the time being. For further discussion of UTF-8 encoding issues, see the section below on byte array literals (page ??), which goes into some greater detail.

# 7.4 Interpolation

One of the most common and useful string operations is concatenation:

```
julia> greet = "Hello"
"Hello"

julia> whom = "world"
"world"

julia> string(greet, ", ", whom, ".\n")
"Hello, world.\n"
```

Constructing strings like this can become a bit cumbersome, however. To reduce the need for these verbose calls to string () (page 385), Julia allows interpolation into string literals using \$, as in Perl:

```
julia> "$greet, $whom.\n"
"Hello, world.\n"
```

This is more readable and convenient and equivalent to the above string concatenation — the system rewrites this apparent single string literal into a concatenation of string literals with variables.

The shortest complete expression after the \$ is taken as the expression whose value is to be interpolated into the string. Thus, you can interpolate any expression into a string using parentheses:

```
julia> "1 + 2 = $(1 + 2)"
"1 + 2 = 3"
```

Both concatenation and string interpolation call *string()* (page 385) to convert objects into string form. Most non-AbstractString objects are converted to strings closely corresponding to how they are entered as literal expressions:

```
julia> v = [1,2,3]
3-element Array{Int64,1}:
    1
    2
    3

julia> "v: $v"
"v: [1,2,3]"
```

string() (page 385) is the identity for AbstractString and Char values, so these are interpolated into strings
as themselves, unquoted and unescaped:

```
julia> c = 'x'
'x'
julia> "hi, $c"
"hi, x"
```

To include a literal \$ in a string literal, escape it with a backslash:

```
julia> print("I have \$100 in my account.\n")
I have \$100 in my account.
```

# 7.5 Triple-Quoted String Literals

When strings are created using triple-quotes ("""..."") they have some special behavior that can be useful for creating longer blocks of text. First, if the opening """ is followed by a newline, the newline is stripped from the resulting string.

```
"""hello"""
```

is equivalent to

```
hello"""
```

but

```
hello"""
```

will contain a literal newline at the beginning. Trailing whitespace is left unaltered. They can contain " symbols without escaping. Triple-quoted strings are also dedented to the level of the least-indented line. This is useful for defining strings within code that is indented. For example:

In this case the final (empty) line before the closing """ sets the indentation level.

Note that line breaks in literal strings, whether single- or triple-quoted, result in a newline (LF) character  $\n$  in the string, even if your editor uses a carriage return  $\n$  (CR) or CRLF combination to end lines. To include a CR in a string, use an explicit escape  $\n$ ; for example, you can enter the literal string "a CRLF line ending  $\n$ ".

# 7.6 Common Operations

You can lexicographically compare strings using the standard comparison operators:

```
julia> "abracadabra" < "xylophone"
true

julia> "abracadabra" == "xylophone"
false

julia> "Hello, world." != "Goodbye, world."
true

julia> "1 + 2 = 3" == "1 + 2 = $(1 + 2)"
true
```

You can search for the index of a particular character using the search () (page 387) function:

```
julia> search("xylophone", 'x')
1
julia> search("xylophone", 'p')
5
```

```
julia> search("xylophone", 'z')
0
```

You can start the search for a character at a given offset by providing a third argument:

```
julia> search("xylophone", 'o')
4

julia> search("xylophone", 'o', 5)
7

julia> search("xylophone", 'o', 8)
0
```

You can use the contains () (page 388) function to check if a substring is contained in a string:

```
julia> contains("Hello, world.", "world")
true

julia> contains("Xylophon", "o")
true

julia> contains("Xylophon", "a")
false

julia> contains("Xylophon", 'o')
ERROR: MethodError: `contains` has no method matching contains(::ASCIIString, ::Char)
Closest candidates are:
   contains(!Matched::Function, ::Any, !Matched::Any)
   contains(::AbstractString, !Matched::AbstractString)
```

The last error is because 'o' is a character literal, and *contains()* (page 388) is a generic function that looks for subsequences. To look for an element in a sequence, you must use *in()* (page 337) instead.

Two other handy string functions are repeat () (page 424) and join () (page 389):

```
julia> repeat(".:Z:.", 10)
".:Z:..:Z:..:Z:..:Z:..:Z:..:Z:..:Z:.."

julia> join(["apples", "bananas", "pineapples"], ", ", " and ")
"apples, bananas and pineapples"
```

Some other useful functions include:

- endof (str) (page 337) gives the maximal (byte) index that can be used to index into str.
- length (str) (page 385) the number of characters in str.
- i = start(str) (page 335) gives the first valid index at which a character can be found in str (typically 1).
- c, j = next(str, i) (page 335) returns next character at or after the index i and the next valid character index following that. With start() (page 335) and endof() (page 337), can be used to iterate through the characters in str.
- ind2chr(str, i) (page 389) gives the number of characters in str up to and including any at index i.
- chr2ind(str, j) (page 389) gives the index at which the jth character in str occurs.

# 7.7 Non-Standard String Literals

There are situations when you want to construct a string or use string semantics, but the behavior of the standard string construct is not quite what is needed. For these kinds of situations, Julia provides *non-standard string literals* (page 151). A non-standard string literal looks like a regular double-quoted string literal, but is immediately prefixed by an identifier, and doesn't behave quite like a normal string literal. Regular expressions, byte array literals and version number literals, as described below, are some examples of non-standard string literals. Other examples are given in the *metaprogramming* (page 151) section.

# 7.8 Regular Expressions

Julia has Perl-compatible regular expressions (regexes), as provided by the PCRE library. Regular expressions are related to strings in two ways: the obvious connection is that regular expressions are used to find regular patterns in strings; the other connection is that regular expressions are themselves input as strings, which are parsed into a state machine that can be used to efficiently search for patterns in strings. In Julia, regular expressions are input using non-standard string literals prefixed with various identifiers beginning with r. The most basic regular expression literal without any options turned on just uses r...":

```
julia> r"^\s*(?:#|$)"
r"^\s*(?:#|$)"
julia> typeof(ans)
Regex
```

To check if a regex matches a string, use *ismatch* () (page 387):

As one can see here, <code>ismatch()</code> (page 387) simply returns true or false, indicating whether the given regex matches the string or not. Commonly, however, one wants to know not just whether a string matched, but also *how* it matched. To capture this information about a match, use the <code>match()</code> (page 387) function instead:

```
julia> match(r"^\s*(?:#|$)", "not a comment")

julia> match(r"^\s*(?:#|$)", "# a comment")
RegexMatch("#")
```

If the regular expression does not match the given string, *match()* (page 387) returns nothing — a special value that does not print anything at the interactive prompt. Other than not printing, it is a completely normal value and you can test for it programmatically:

```
m = match(r"^\s*(?:#|$)", line)
if m == nothing
  println("not a comment")
else
  println("blank or comment")
end
```

If a regular expression does match, the value returned by match () (page 387) is a RegexMatch object. These objects record how the expression matches, including the substring that the pattern matches and any captured substrings,

if there are any. This example only captures the portion of the substring that matches, but perhaps we want to capture any non-blank text after the comment character. We could do the following:

```
julia> m = match(r"^\s*(?:\#\s*(.*?)\s*$|$)", "# a comment ")
RegexMatch("# a comment ", 1="a comment")
```

When calling match () (page 387), you have the option to specify an index at which to start the search. For example:

```
julia> m = match(r"[0-9]", "aaaalaaaa2aaaa3",1)
RegexMatch("1")

julia> m = match(r"[0-9]", "aaaalaaaa2aaaa3",6)
RegexMatch("2")

julia> m = match(r"[0-9]", "aaaalaaaa2aaaa3",11)
RegexMatch("3")
```

You can extract the following info from a RegexMatch object:

- the entire substring matched: m.match
- the captured substrings as an array of strings: m. captures
- the offset at which the whole match begins: m.offset
- the offsets of the captured substrings as a vector: m.offsets

For when a capture doesn't match, instead of a substring, m.captures contains nothing in that position, and m.offsets has a zero offset (recall that indices in Julia are 1-based, so a zero offset into a string is invalid). Here is a pair of somewhat contrived examples:

```
julia> m = match(r"(a|b)(c)?(d)", "acd")
RegexMatch ("acd", 1="a", 2="c", 3="d")
julia> m.match
"acd"
julia> m.captures
3-element Array{Union{SubString{UTF8String}, Void},1}:
 "a"
 "c"
"d"
julia> m.offset
julia> m.offsets
3-element Array{Int64,1}:
2
julia> m = match(r"(a|b)(c)?(d)", "ad")
RegexMatch("ad", 1="a", 2=nothing, 3="d")
julia> m.match
"ad"
julia> m.captures
3-element Array{Union{SubString{UTF8String}, Void},1}:
 "a"
```

```
nothing
"d"

julia> m.offset

julia> m.offsets
3-element Array{Int64,1}:
1
0
2
```

It is convenient to have captures returned as an array so that one can use destructuring syntax to bind them to local variables:

```
julia> first, second, third = m.captures; first
"a"
```

Captures can also be accessed by indexing the RegexMatch object with the number or name of the capture group:

```
julia> m=match(r"(?<hour>\d+):(?<minute>\d+)","12:45")
RegexMatch("12:45", hour="12", minute="45")
julia> m[:minute]
"45"
julia> m[2]
"45"
```

Captures can be referenced in a substitution string when using replace() (page 388) by using \n to refer to the nth capture group and prefixing the substitution string with s. Capture group 0 refers to the entire match object. Named capture groups can be referenced in the substitution with g<groupname>. For example:

```
julia> replace("first second", r"(\w+) (?<agroup>\w+)", s"\g<agroup> \1")
julia> "second first"
```

Numbered capture groups can also be referenced as \g<n> for disambiguation, as in:

```
julia> replace("a", r".", "\g<0>1")
julia> a1
```

You can modify the behavior of regular expressions by some combination of the flags i, m, s, and x after the closing double quote mark. These flags have the same meaning as they do in Perl, as explained in this excerpt from the perlre manpage:

```
i Do case-insensitive pattern matching.

If locale matching rules are in effect, the case map is taken from the current locale for code points less than 255, and from Unicode rules for larger code points. However, matches that would cross the Unicode rules/non-Unicode rules boundary (ords 255/256) will not succeed.

m Treat string as multiple lines. That is, change "^" and "$" from matching the start or end of the string to matching the start or end of any line anywhere within the string.

s Treat string as single line. That is, change "." to match any character whatsoever, even a newline, which normally it would not match.

Used together, as r""ms, they let the "." match any character
```

```
whatsoever, while still allowing "^" and "$" to match,
respectively, just after and just before newlines within the
string.

x Tells the regular expression parser to ignore most whitespace
that is neither backslashed nor within a character class. You
can use this to break up your regular expression into
(slightly) more readable parts. The '#' character is also
treated as a metacharacter introducing a comment, just as in
ordinary code.
```

For example, the following regex has all three flags turned on:

```
julia> r"a+.*b+.*?d$"ism
r"a+.*b+.*?d$"ims

julia> match(r"a+.*b+.*?d$"ism, "Goodbye,\nOh, angry,\nBad world\n")
RegexMatch("angry,\nBad world")
```

Triple-quoted regex strings, of the form r""", are also supported (and may be convenient for regular expressions containing quotation marks or newlines).

# 7.9 Byte Array Literals

Another useful non-standard string literal is the byte-array string literal: b"...". This form lets you use string notation to express literal byte arrays — i.e. arrays of UInt8 values. The convention is that non-standard literals with uppercase prefixes produce actual string objects, while those with lowercase prefixes produce non-string objects like byte arrays or compiled regular expressions. The rules for byte array literals are the following:

- ASCII characters and ASCII escapes produce a single byte.
- $\xspace \times$  and octal escape sequences produce the *byte* corresponding to the escape value.
- Unicode escape sequences produce a sequence of bytes encoding that code point in UTF-8.

There is some overlap between these rules since the behavior of  $\xspace x$  and octal escapes less than 0x80 (128) are covered by both of the first two rules, but here these rules agree. Together, these rules allow one to easily use ASCII characters, arbitrary byte values, and UTF-8 sequences to produce arrays of bytes. Here is an example using all three:

```
julia> b"DATA\xff\u2200"
8-element Array{UInt8,1}:
    0x44
    0x41
    0x54
    0x41
    0xff
    0xe2
    0x88
    0x80
```

The ASCII string "DATA" corresponds to the bytes 68, 65, 84, 65. \xff produces the single byte 255. The Unicode escape \u2200 is encoded in UTF-8 as the three bytes 226, 136, 128. Note that the resulting byte array does not correspond to a valid UTF-8 string — if you try to use this as a regular string literal, you will get a syntax error:

```
julia> "DATA\xff\u2200"
ERROR: syntax: invalid UTF-8 sequence
```

Also observe the significant distinction between \xff and \uff: the former escape sequence encodes the *byte* 255, whereas the latter escape sequence represents the *code point* 255, which is encoded as two bytes in UTF-8:

In character literals, this distinction is glossed over and  $\xff$  is allowed to represent the code point 255, because characters *always* represent code points. In strings, however,  $\xff$  escapes always represent bytes, not code points, whereas  $\xff$  u and  $\xff$  escapes always represent code points, which are encoded in one or more bytes. For code points less than  $\xfff$  as that the UTF-8 encoding of each code point is just the single byte produced by the corresponding  $\xfff$  escape, so the distinction can safely be ignored. For the escapes  $\xffff$  as compared to  $\xffff$  as compared to  $\xffff$  however, there is a major difference: the former escapes all encode single bytes, which — unless followed by very specific continuation bytes — do not form valid UTF-8 data, whereas the latter escapes all represent Unicode code points with two-byte encodings.

If this is all extremely confusing, try reading "The Absolute Minimum Every Software Developer Absolutely, Positively Must Know About Unicode and Character Sets". It's an excellent introduction to Unicode and UTF-8, and may help alleviate some confusion regarding the matter.

#### 7.10 Version Number Literals

Version numbers can easily be expressed with non-standard string literals of the form v"...". Version number literals create VersionNumber objects which follow the specifications of semantic versioning, and therefore are composed of major, minor and patch numeric values, followed by pre-release and build alpha-numeric annotations. For example, v"0.2.1-rc1+win64" is broken into major version 0, minor version 2, patch version 1, pre-release rc1 and build win64. When entering a version literal, everything except the major version number is optional, therefore e.g. v"0.2" is equivalent to v"0.2.0" (with empty pre-release/build annotations), v"2" is equivalent to v"2.0.0", and so on.

VersionNumber objects are mostly useful to easily and correctly compare two (or more) versions. For example, the constant VERSION holds Julia version number as a VersionNumber object, and therefore one can define some version-specific behavior using simple statements as:

```
if v"0.2" <= VERSION < v"0.3-"
    # do something specific to 0.2 release series
end</pre>
```

Note that in the above example the non-standard version number v"0.3-" is used, with a trailing -: this notation is a Julia extension of the standard, and it's used to indicate a version which is lower than any 0.3 release, including all of its pre-releases. So in the above example the code would only run with stable 0.2 versions, and exclude such versions as v"0.3.0-rc1". In order to also allow for unstable (i.e. pre-release) 0.2 versions, the lower bound check should be modified like this: v"0.2-" <= VERSION.

Another non-standard version specification extension allows one to use a trailing + to express an upper limit on build versions, e.g. VERSION > "v"0.2-rc1+" can be used to mean any version above 0.2-rc1 and any of its builds: it will return false for version v"0.2-rc1+win64" and true for v"0.2-rc2".

It is good practice to use such special versions in comparisons (particularly, the trailing – should always be used on upper bounds unless there's a good reason not to), but they must not be used as the actual version number of anything, as they are invalid in the semantic versioning scheme.

Besides being used for the *VERSION* (page 441) constant, *VersionNumber* objects are widely used in the *Pkg* (page 469) module, to specify packages versions and their dependencies.

### **Functions**

In Julia, a function is an object that maps a tuple of argument values to a return value. Julia functions are not pure mathematical functions, in the sense that functions can alter and be affected by the global state of the program. The basic syntax for defining functions in Julia is:

```
function f(x,y)
   x + y
end
```

There is a second, more terse syntax for defining a function in Julia. The traditional function declaration syntax demonstrated above is equivalent to the following compact "assignment form":

```
f(x,y) = x + y
```

In the assignment form, the body of the function must be a single expression, although it can be a compound expression (see *Compound Expressions* (page 65)). Short, simple function definitions are common in Julia. The short function syntax is accordingly quite idiomatic, considerably reducing both typing and visual noise.

A function is called using the traditional parenthesis syntax:

```
julia> f(2,3)
5
```

Without parentheses, the expression f refers to the function object, and can be passed around like any value:

```
julia> g = f;
julia> g(2,3)
5
```

As with variables, Unicode can also be used for function names:

```
julia> \sum (x,y) = x + y
\sum (generic function with 1 method)
```

# 8.1 Argument Passing Behavior

Julia function arguments follow a convention sometimes called "pass-by-sharing", which means that values are not copied when they are passed to functions. Function arguments themselves act as new variable *bindings* (new locations that can refer to values), but the values they refer to are identical to the passed values. Modifications to mutable values (such as Arrays) made within a function will be visible to the caller. This is the same behavior found in Scheme, most Lisps, Python, Ruby and Perl, among other dynamic languages.

# 8.2 The return Keyword

The value returned by a function is the value of the last expression evaluated, which, by default, is the last expression in the body of the function definition. In the example function, f, from the previous section this is the value of the expression f. As in C and most other imperative or functional languages, the return keyword causes a function to return immediately, providing an expression whose value is returned:

```
function g(x,y)
  return x * y
  x + y
end
```

Since function definitions can be entered into interactive sessions, it is easy to compare these definitions:

```
f(x,y) = x + y
function g(x,y)
  return x * y
  x + y
end

julia> f(2,3)
5

julia> g(2,3)
6
```

Of course, in a purely linear function body like g, the usage of return is pointless since the expression x + y is never evaluated and we could simply make x \* y the last expression in the function and omit the return. In conjunction with other control flow, however, return is of real use. Here, for example, is a function that computes the hypotenuse length of a right triangle with sides of length x and y, avoiding overflow:

```
function hypot(x,y)
  x = abs(x)
  y = abs(y)
  if x > y
    r = y/x
    return x*sqrt(1+r*r)
  end
  if y == 0
    return zero(x)
  end
  r = x/y
  return y*sqrt(1+r*r)
end
```

There are three possible points of return from this function, returning the values of three different expressions, depending on the values of x and y. The return on the last line could be omitted since it is the last expression.

# 8.3 Operators Are Functions

In Julia, most operators are just functions with support for special syntax. (The exceptions are operators with special evaluation semantics like && and ||. These operators cannot be functions since *short-circuit evaluation* (page 68) requires that their operands are not evaluated before evaluation of the operator.) Accordingly, you can also apply them using parenthesized argument lists, just as you would any other function:

```
julia> 1 + 2 + 3
6

julia> +(1,2,3)
6
```

The infix form is exactly equivalent to the function application form — in fact the former is parsed to produce the function call internally. This also means that you can assign and pass around operators such as +() (page 353) and +() (page 385) just like you would with other function values:

```
julia> f = +;
julia> f(1,2,3)
6
```

Under the name f, the function does not support infix notation, however.

# 8.4 Operators With Special Names

A few special expressions correspond to calls to functions with non-obvious names. These are:

Expression	Calls
[A B C]	hcat () (page 397)
[A, B, C,]	vcat () (page 397)
[A B; C D;]	hvcat () (page 397)
A'	ctranspose() (page 426)
A.'	transpose() (page 425)
1:n	colon() (page 355)
A[i]	getindex() (page 342)
A[i]=x	setindex!()(page 342)
A(x)	call() (page 325)

These functions are included in the Base. Operators module even though they do not have operator-like names.

# 8.5 Anonymous Functions

Functions in Julia are first-class objects: they can be assigned to variables, called using the standard function call syntax from the variable they have been assigned to. They can be used as arguments, and they can be returned as values. They can also be created anonymously, without being given a name, using either of these syntaxes:

This creates an unnamed function taking one argument x and returning the value of the polynomial  $x^2 + 2x - 1$  at that value. The primary use for anonymous functions is passing them to functions which take other functions as arguments. A classic example is map() (page 340), which applies a function to each value of an array and returns a new array containing the resulting values:

```
julia> map(round, [1.2,3.5,1.7])
3-element Array{Float64,1}:
    1.0
    4.0
    2.0
```

This is fine if a named function effecting the transform one wants already exists to pass as the first argument to map () (page 340). Often, however, a ready-to-use, named function does not exist. In these situations, the anonymous function construct allows easy creation of a single-use function object without needing a name:

```
julia> map(x -> x^2 + 2x - 1, [1,3,-1])
3-element Array{Int64,1}:
   2
   14
   -2
```

An anonymous function accepting multiple arguments can be written using the syntax  $(x, y, z) \rightarrow 2x+y-z$ . A zero-argument anonymous function is written as ()  $\rightarrow$  3. The idea of a function with no arguments may seem strange, but is useful for "delaying" a computation. In this usage, a block of code is wrapped in a zero-argument function, which is later invoked by calling it as f().

## 8.6 Multiple Return Values

In Julia, one returns a tuple of values to simulate returning multiple values. However, tuples can be created and destructured without needing parentheses, thereby providing an illusion that multiple values are being returned, rather than a single tuple value. For example, the following function returns a pair of values:

```
julia> function foo(a,b)
        a+b, a*b
    end;
```

If you call it in an interactive session without assigning the return value anywhere, you will see the tuple returned:

```
julia> foo(2,3)
(5,6)
```

A typical usage of such a pair of return values, however, extracts each value into a variable. Julia supports simple tuple "destructuring" that facilitates this:

```
julia> x, y = foo(2,3);
julia> x
5
julia> y
6
```

You can also return multiple values via an explicit usage of the return keyword:

```
function foo(a,b)
  return a+b, a*b
end
```

This has the exact same effect as the previous definition of foo.

# 8.7 Varargs Functions

It is often convenient to be able to write functions taking an arbitrary number of arguments. Such functions are traditionally known as "varargs" functions, which is short for "variable number of arguments". You can define a varargs function by following the last argument with an ellipsis:

```
julia> bar(a,b,x...) = (a,b,x)
bar (generic function with 1 method)
```

The variables a and b are bound to the first two argument values as usual, and the variable x is bound to an iterable collection of the zero or more values passed to bar after its first two arguments:

```
julia> bar(1,2)
(1,2,())

julia> bar(1,2,3)
(1,2,(3,))

julia> bar(1,2,3,4)
(1,2,(3,4))

julia> bar(1,2,3,4,5,6)
(1,2,(3,4,5,6))
```

In all these cases, x is bound to a tuple of the trailing values passed to bar.

On the flip side, it is often handy to "splice" the values contained in an iterable collection into a function call as individual arguments. To do this, one also uses . . . but in the function call instead:

```
julia> x = (3,4)
(3,4)

julia> bar(1,2,x...)
(1,2,(3,4))
```

In this case a tuple of values is spliced into a varargs call precisely where the variable number of arguments go. This need not be the case, however:

```
julia> x = (2,3,4)
(2,3,4)

julia> bar(1,x...)
(1,2,(3,4))

julia> x = (1,2,3,4)
(1,2,3,4)

julia> bar(x...)
(1,2,(3,4))
```

Furthermore, the iterable object spliced into a function call need not be a tuple:

```
julia> x = [3,4]
2-element Array{Int64,1}:
3
4

julia> bar(1,2,x...)
(1,2,(3,4))
```

```
julia> x = [1,2,3,4]
4-element Array{Int64,1}:
    1
    2
    3
    4

julia> bar(x...)
(1,2,(3,4))
```

Also, the function that arguments are spliced into need not be a varargs function (although it often is):

```
baz(a,b) = a + b

julia> args = [1,2]
2-element Array{Int64,1}:
    1
    2

julia> baz(args...)
3

julia> args = [1,2,3]
3-element Array{Int64,1}:
    1
    2
    3

julia> baz(args...)
no method baz(Int64,Int64,Int64)
```

As you can see, if the wrong number of elements are in the spliced container, then the function call will fail, just as it would if too many arguments were given explicitly.

# 8.8 Optional Arguments

In many cases, function arguments have sensible default values and therefore might not need to be passed explicitly in every call. For example, the library function parse (type, num, base) (page 326) interprets a string as a number in some base. The base argument defaults to 10. This behavior can be expressed concisely as:

```
function parse(type, num, base=10)
    ###
end
```

With this definition, the function can be called with either two or three arguments, and 10 is automatically passed when a third argument is not specified:

```
julia> parse(Int, "12", 10)

12

julia> parse(Int, "12", 3)

julia> parse(Int, "12")

12
```

Optional arguments are actually just a convenient syntax for writing multiple method definitions with different numbers of arguments (see *Note on Optional and keyword Arguments* (page 115)).

# 8.9 Keyword Arguments

Some functions need a large number of arguments, or have a large number of behaviors. Remembering how to call such functions can be difficult. Keyword arguments can make these complex interfaces easier to use and extend by allowing arguments to be identified by name instead of only by position.

For example, consider a function plot that plots a line. This function might have many options, for controlling line style, width, color, and so on. If it accepts keyword arguments, a possible call might look like plot (x, y, width=2), where we have chosen to specify only line width. Notice that this serves two purposes. The call is easier to read, since we can label an argument with its meaning. It also becomes possible to pass any subset of a large number of arguments, in any order.

Functions with keyword arguments are defined using a semicolon in the signature:

```
function plot(x, y; style="solid", width=1, color="black")
    ###
end
```

When the function is called, the semicolon is optional: one can either call plot(x, y, width=2) or plot(x, y; width=2), but the former style is more common. An explicit semicolon is required only for passing varargs or computed keywords as described below.

Keyword argument default values are evaluated only when necessary (when a corresponding keyword argument is not passed), and in left-to-right order. Therefore default expressions may refer to prior keyword arguments.

The types of keyword arguments can be made explicit as follows:

```
function f(;x::Int64=1) ###
```

Extra keyword arguments can be collected using . . . , as in varargs functions:

```
function f(x; y=0, kwargs...)
    ###
end
```

Inside f, kwargs will be a collection of (key, value) tuples, where each key is a symbol. Such collections can be passed as keyword arguments using a semicolon in a call, e.g. f(x, z=1; kwargs...). Dictionaries can also be used for this purpose.

One can also pass (key, value) tuples, or any iterable expression (such as a => pair) that can be assigned to such a tuple, explicitly after a semicolon. For example, plot(x, y; (:width,2)) and plot(x, y; :width => 2) are equivalent to plot(x, y, width=2). This is useful in situations where the keyword name is computed at runtime.

# 8.10 Evaluation Scope of Default Values

Optional and keyword arguments differ slightly in how their default values are evaluated. When optional argument default expressions are evaluated, only *previous* arguments are in scope. In contrast, *all* the arguments are in scope when keyword arguments default expressions are evaluated. For example, given this definition:

the b in a=b refers to a b in an outer scope, not the subsequent argument b. However, if a and b were keyword arguments instead, then both would be created in the same scope and the b in a=b would refer the the subsequent argument b (shadowing any b in an outer scope), which would result in an undefined variable error (since the default expressions are evaluated left-to-right, and b has not been assigned yet).

## 8.11 Do-Block Syntax for Function Arguments

Passing functions as arguments to other functions is a powerful technique, but the syntax for it is not always convenient. Such calls are especially awkward to write when the function argument requires multiple lines. As an example, consider calling map() (page 340) on a function with several cases:

```
map(x->begin
    if x < 0 && iseven(x)
        return 0
    elseif x == 0
        return 1
    else
        return x
    end
    end,
    [A, B, C])</pre>
```

Julia provides a reserved word do for rewriting this code more clearly:

```
map([A, B, C]) do x
    if x < 0 && iseven(x)
        return 0
    elseif x == 0
        return 1
    else
        return x
    end
end</pre>
```

The do x syntax creates an anonymous function with argument x and passes it as the first argument to map() (page 340). Similarly, do a, b would create a two-argument anonymous function, and a plain do would declare that what follows is an anonymous function of the form () -> ....

How these arguments are initialized depends on the "outer" function; here, map() (page 340) will sequentially set x to A, B, C, calling the anonymous function on each, just as would happen in the syntax map(func, [A, B, C]).

This syntax makes it easier to use functions to effectively extend the language, since calls look like normal code blocks. There are many possible uses quite different from map() (page 340), such as managing system state. For example, there is a version of open() (page 449) that runs code ensuring that the opened file is eventually closed:

```
open("outfile", "w") do io
    write(io, data)
end
```

This is accomplished by the following definition:

```
function open(f::Function, args...)
io = open(args...)
```

```
try
     f(io)
    finally
     close(io)
    end
end
```

In contrast to the <code>map()</code> (page 340) example, here io is initialized by the <code>result</code> of open("outfile", "w"). The stream is then passed to your anonymous function, which performs the writing; finally, the <code>open()</code> (page 449) function ensures that the stream is closed after your function exits. The <code>try/finally</code> construct will be described in <code>Control Flow</code> (page 65).

With the do block syntax, it helps to check the documentation or implementation to know how the arguments of the user function are initialized.

# 8.12 Further Reading

We should mention here that this is far from a complete picture of defining functions. Julia has a sophisticated type system and allows multiple dispatch on argument types. None of the examples given here provide any type annotations on their arguments, meaning that they are applicable to all types of arguments. The type system is described in *Types* (page 87) and defining a function in terms of methods chosen by multiple dispatch on run-time argument types is described in *Methods* (page 107).

### **Control Flow**

Julia provides a variety of control flow constructs:

- Compound Expressions (page 65): begin and (;).
- Conditional Evaluation (page 66): if-elseif-else and ?: (ternary operator).
- Short-Circuit Evaluation (page 68): &&, | | and chained comparisons.
- Repeated Evaluation: Loops (page 70): while and for.
- Exception Handling (page 73): try-catch, error() (page 329) and throw() (page 329).
- Tasks (aka Coroutines) (page 77): yieldto() (page 405).

The first five control flow mechanisms are standard to high-level programming languages. *Task* (page 405)s are not so standard: they provide non-local control flow, making it possible to switch between temporarily-suspended computations. This is a powerful construct: both exception handling and cooperative multitasking are implemented in Julia using tasks. Everyday programming requires no direct usage of tasks, but certain problems can be solved much more easily by using tasks.

# 9.1 Compound Expressions

Sometimes it is convenient to have a single expression which evaluates several subexpressions in order, returning the value of the last subexpression as its value. There are two Julia constructs that accomplish this: begin blocks and (;) chains. The value of both compound expression constructs is that of the last subexpression. Here's an example of a begin block:

Since these are fairly small, simple expressions, they could easily be placed onto a single line, which is where the (;) chain syntax comes in handy:

```
julia> z = (x = 1; y = 2; x + y)
3
```

This syntax is particularly useful with the terse single-line function definition form introduced in *Functions* (page 55). Although it is typical, there is no requirement that begin blocks be multiline or that (;) chains be single-line:

#### 9.2 Conditional Evaluation

Conditional evaluation allows portions of code to be evaluated or not evaluated depending on the value of a boolean expression. Here is the anatomy of the if-elseif-else conditional syntax:

```
if x < y
  println("x is less than y")
elseif x > y
  println("x is greater than y")
else
  println("x is equal to y")
end
```

If the condition expression x < y is true, then the corresponding block is evaluated; otherwise the condition expression x > y is evaluated, and if it is true, the corresponding block is evaluated; if neither expression is true, the else block is evaluated. Here it is in action:

The elseif and else blocks are optional, and as many elseif blocks as desired can be used. The condition expressions in the if-elseif-else construct are evaluated until the first one evaluates to true, after which the associated block is evaluated, and no further condition expressions or blocks are evaluated.

if blocks are "leaky", i.e. they do not introduce a local scope. This means that new variables defined inside the if clauses can be used after the if block, even if they weren't defined before. So, we could have defined the test function above as

```
julia> function test(x,y)
    if x < y
        relation = "less than"</pre>
```

```
elseif x == y
    relation = "equal to"
else
    relation = "greater than"
end
    println("x is ", relation, " y.")
end
test (generic function with 1 method)
```

The variable relation is declared inside the if block, but used outside. However, when depending on this behavior, make sure all possible code paths define a value for the variable. The following change to the above function results in a runtime error

if blocks also return a value, which may seem unintuitive to users coming from many other languages. This value is simply the return value of the last executed statement in the branch that was chosen, so

Note that very short conditional statements (one-liners) are frequently expressed using Short-Circuit Evaluation in Julia, as outlined in the next section.

Unlike C, MATLAB, Perl, Python, and Ruby — but like Java, and a few other stricter, typed languages — it is an error if the value of a conditional expression is anything but true or false:

```
julia> if 1
          println("true")
         end
ERROR: TypeError: non-boolean (Int64) used in boolean context
```

This error indicates that the conditional was of the wrong type: Int64 ' rather than the required Bool.

The so-called "ternary operator", ?:, is closely related to the if-elseif-else syntax, but is used where a conditional choice between single expression values is required, as opposed to conditional execution of longer blocks of code. It gets its name from being the only operator in most languages taking three operands:

```
a ? b : c
```

The expression a, before the ?, is a condition expression, and the ternary operation evaluates the expression b, before the :, if the condition a is true or the expression c, after the :, if it is false.

The easiest way to understand this behavior is to see an example. In the previous example, the println call is shared by all three branches: the only real choice is which literal string to print. This could be written more concisely using the ternary operator. For the sake of clarity, let's try a two-way version first:

```
julia> x = 1; y = 2;

julia> println(x < y ? "less than" : "not less than")
less than

julia> x = 1; y = 0;

julia> println(x < y ? "less than" : "not less than")
not less than</pre>
```

If the expression x < y is true, the entire ternary operator expression evaluates to the string "less than" and otherwise it evaluates to the string "not less than". The original three-way example requires chaining multiple uses of the ternary operator together:

To facilitate chaining, the operator associates from right to left.

It is significant that like if-elseif-else, the expressions before and after the : are only evaluated if the condition expression evaluates to true or false, respectively:

```
julia> v(x) = (println(x); x)
v (generic function with 1 method)

julia> 1 < 2 ? v("yes") : v("no")
yes
"yes"

julia> 1 > 2 ? v("yes") : v("no")
no
"no"
```

### 9.3 Short-Circuit Evaluation

Short-circuit evaluation is quite similar to conditional evaluation. The behavior is found in most imperative programming languages having the && and  $|\cdot|$  boolean operators: in a series of boolean expressions connected by these

operators, only the minimum number of expressions are evaluated as are necessary to determine the final boolean value of the entire chain. Explicitly, this means that:

- In the expression a && b, the subexpression b is only evaluated if a evaluates to true.
- In the expression a || b, the subexpression b is only evaluated if a evaluates to false.

The reasoning is that a && b must be false if a is false, regardless of the value of b, and likewise, the value of a  $| \cdot |$  b must be true if a is true, regardless of the value of b. Both && and  $| \cdot |$  associate to the right, but && has higher precedence than  $| \cdot |$  does. It's easy to experiment with this behavior:

```
julia > t(x) = (println(x); true)
t (generic function with 1 method)
julia> f(x) = (println(x); false)
f (generic function with 1 method)
julia> t(1) && t(2)
true
julia> t(1) && f(2)
false
julia> f(1) && t(2)
false
julia> f(1) && f(2)
false
julia> t(1) || t(2)
true
julia> t(1) || f(2)
true
julia> f(1) \mid \mid t(2)
true
julia> f(1) \mid \mid f(2)
2
```

You can easily experiment in the same way with the associativity and precedence of various combinations of & & and  $| \cdot |$  operators.

This behavior is frequently used in Julia to form an alternative to very short if statements. Instead of if <cond> <statement> end, one can write <cond> && <statement> (which could be read as: <cond> and then <statement>). Similarly, instead of if ! <cond> <statement> end, one can write <cond> | | <statement> (which could be read as: <cond> or else <statement>).

For example, a recursive factorial routine could be defined like this:

Boolean operations *without* short-circuit evaluation can be done with the bitwise boolean operators introduced in *Mathematical Operations and Elementary Functions* (page 25): & and |. These are normal functions, which happen to support infix operator syntax, but always evaluate their arguments:

```
julia> f(1) & t(2)
1
2
false

julia> t(1) | t(2)
1
2
true
```

Just like condition expressions used in if, elseif or the ternary operator, the operands of && or || must be boolean values (true or false). Using a non-boolean value anywhere except for the last entry in a conditional chain is an error:

```
julia> 1 && true
ERROR: TypeError: non-boolean (Int64) used in boolean context
```

On the other hand, any type of expression can be used at the end of a conditional chain. It will be evaluated and returned depending on the preceding conditionals:

```
julia> true && (x = rand(2,2))
2x2 Array{Float64,2}:
    0.768448    0.673959
    0.940515    0.395453

julia> false && (x = rand(2,2))
false
```

# 9.4 Repeated Evaluation: Loops

There are two constructs for repeated evaluation of expressions: the while loop and the for loop. Here is an example of a while loop:

The while loop evaluates the condition expression (i  $\leq$  5 in this case), and as long it remains true, keeps also evaluating the body of the while loop. If the condition expression is false when the while loop is first reached, the body is never evaluated.

The for loop makes common repeated evaluation idioms easier to write. Since counting up and down like the above while loop does is so common, it can be expressed more concisely with a for loop:

Here the 1:5 is a Range object, representing the sequence of numbers 1, 2, 3, 4, 5. The for loop iterates through these values, assigning each one in turn to the variable i. One rather important distinction between the previous while loop form and the for loop form is the scope during which the variable is visible. If the variable i has not been introduced in an other scope, in the for loop form, it is visible only inside of the for loop, and not afterwards. You'll either need a new interactive session instance or a different variable name to test this:

See Scope of Variables (page 81) for a detailed explanation of variable scope and how it works in Julia.

In general, the for loop construct can iterate over any container. In these cases, the alternative (but fully equivalent) keyword in or  $\in$  is typically used instead of =, since it makes the code read more clearly:

```
println(s)
  end
foo
bar
baz
```

Various types of iterable containers will be introduced and discussed in later sections of the manual (see, e.g., *Multi-dimensional Arrays* (page 157)).

It is sometimes convenient to terminate the repetition of a while before the test condition is falsified or stop iterating in a for loop before the end of the iterable object is reached. This can be accomplished with the break keyword:

```
julia > i = 1;
julia> while true
         println(i)
         if i >= 5
           break
          end
          i += 1
       end
1
3
4
5
julia> for i = 1:1000
         println(i)
         if i >= 5
            break
          end
       end
2
3
4
```

The above while loop would never terminate on its own, and the for loop would iterate up to 1000. These loops are both exited early by using the break keyword.

In other circumstances, it is handy to be able to stop an iteration and move on to the next one immediately. The continue keyword accomplishes this:

```
julia> for i = 1:10
    if i % 3 != 0
        continue
    end
    println(i)
    end
3
6
9
```

This is a somewhat contrived example since we could produce the same behavior more clearly by negating the condition and placing the println call inside the if block. In realistic usage there is more code to be evaluated after the continue, and often there are multiple points from which one calls continue.

Multiple nested for loops can be combined into a single outer loop, forming the cartesian product of its iterables:

A break statement inside such a loop exits the entire nest of loops, not just the inner one.

# 9.5 Exception Handling

When an unexpected condition occurs, a function may be unable to return a reasonable value to its caller. In such cases, it may be best for the exceptional condition to either terminate the program, printing a diagnostic error message, or if the programmer has provided code to handle such exceptional circumstances, allow that code to take the appropriate action.

#### 9.5.1 Built-in ExceptionS

Exceptions are thrown when an unexpected condition has occurred. The built-in Exceptions listed below all interrupt the normal flow of control.

•
Exception
ArgumentError (page 330)
BoundsError (page 330)
CompositeException
DivideError (page 330)
DomainError (page 330)
EOFError (page 330)
ErrorException (page 330)
InexactError (page 330)
InitError (page 331)
InterruptException (page 330)
InvalidStateException
KeyError (page 330)
LoadError (page 330)
OutOfMemoryError (page 330)
ReadOnlyMemoryError (page 330)
RemoteException
MethodError (page 330)
OverflowError (page 330)
ParseError (page 330)
SystemError (page 331)
TypeError (page 331)
UndefRefError (page 331)
UndefVarError (page 331)
UnicodeError

For example, the <code>sqrt()</code> (page 364) function throws a <code>DomainError</code> (page 330) if applied to a negative real value:

```
julia> sqrt(-1)
ERROR: DomainError:
```

```
sqrt will only return a complex result if called with a complex argument. Try sqrt(complex(x)).

in sqrt at math.jl:146
```

You may define your own exceptions in the following way:

```
julia> type MyCustomException <: Exception end
```

#### 9.5.2 The throw() function

Exceptions can be created explicitly with throw() (page 329). For example, a function defined only for nonnegative numbers could be written to throw() (page 329) a DomainError (page 330) if the argument is negative:

```
julia> f(x) = x>=0 ? exp(-x) : throw(DomainError())
f (generic function with 1 method)

julia> f(1)
0.36787944117144233

julia> f(-1)
ERROR: DomainError:
in f at none:1
```

Note that *DomainError* (page 330) without parentheses is not an exception, but a type of exception. It needs to be called to obtain an Exception object:

```
julia> typeof(DomainError()) <: Exception
true

julia> typeof(DomainError) <: Exception
false</pre>
```

Additionally, some exception types take one or more arguments that are used for error reporting:

```
julia> throw(UndefVarError(:x))
ERROR: UndefVarError: x not defined
```

This mechanism can be implemented easily by custom exception types following the way *UndefVarError* (page 331) is written:

#### **9.5.3 Errors**

74

The error() (page 329) function is used to produce an ErrorException (page 330) that interrupts the normal flow of control.

Suppose we want to stop execution immediately if the square root of a negative number is taken. To do this, we can define a fussy version of the sqrt () (page 364) function that raises an error if its argument is negative:

```
julia> fussy_sqrt(x) = x >= 0 ? sqrt(x) : error("negative x not allowed")
fussy_sqrt (generic function with 1 method)
julia> fussy_sqrt(2)
1.4142135623730951
```

```
julia> fussy_sqrt(-1)
ERROR: negative x not allowed
in fussy_sqrt at none:1
```

If fussy\_sqrt is called with a negative value from another function, instead of trying to continue execution of the calling function, it returns immediately, displaying the error message in the interactive session:

#### 9.5.4 Warnings and informational messages

Julia also provides other functions that write messages to the standard error I/O, but do not throw any Exceptions and hence do not interrupt execution.:

```
julia> info("Hi"); 1+1
INFO: Hi
2

julia> warn("Hi"); 1+1
WARNING: Hi
2

julia> error("Hi"); 1+1
ERROR: Hi
in error at ./error.jl:21
```

#### 9.5.5 The try/catch statement

The try/catch statement allows for Exceptions to be tested for. For example, a customized square root function can be written to automatically call either the real or complex square root method on demand using Exceptions:

```
1.0

julia> f(-1)

0.0 + 1.0im
```

It is important to note that in real code computing this function, one would compare x to zero instead of catching an exception. The exception is much slower than simply comparing and branching.

try/catch statements also allow the Exception to be saved in a variable. In this contrived example, the following example calculates the square root of the second element of x if x is indexable, otherwise assumes x is a real number and returns its square root:

```
julia> sqrt_second(x) = try
         sqrt(x[2])
       catch y
         if isa(y, DomainError)
           sqrt(complex(x[2], 0))
         elseif isa(y, BoundsError)
           sqrt(x)
         end
       end
sqrt_second (generic function with 1 method)
julia> sqrt_second([1 4])
2.0
julia> sqrt_second([1 -4])
0.0 + 2.0 im
julia> sqrt_second(9)
3.0
julia> sgrt_second(-9)
ERROR: DomainError:
in sqrt_second at none:7
```

Note that the symbol following catch will always be interpreted as a name for the exception, so care is needed when writing try/catch expressions on a single line. The following code will *not* work to return the value of x in case of an error:

```
try bad() catch x end
```

Instead, use a semicolon or insert a line break after catch:

```
try bad() catch; x end

try bad()
catch
   x
end
```

The catch clause is not strictly necessary; when omitted, the default return value is nothing.

```
julia> try error() end #Returns nothing
```

The power of the try/catch construct lies in the ability to unwind a deeply nested computation immediately to a much higher level in the stack of calling functions. There are situations where no error has occurred, but the ability to unwind the stack and pass a value to a higher level is desirable. Julia provides the rethrow() (page 329), backtrace() (page 329) and catch\_backtrace() (page 329) functions for more advanced error handling.

#### 9.5.6 finally Clauses

In code that performs state changes or uses resources like files, there is typically clean-up work (such as closing files) that needs to be done when the code is finished. Exceptions potentially complicate this task, since they can cause a block of code to exit before reaching its normal end. The finally keyword provides a way to run some code when a given block of code exits, regardless of how it exits.

For example, here is how we can guarantee that an opened file is closed:

```
f = open("file")
try
    # operate on file f
finally
    close(f)
end
```

When control leaves the try block (for example due to a return, or just finishing normally), close (f) will be executed. If the try block exits due to an exception, the exception will continue propagating. A catch block may be combined with try and finally as well. In this case the finally block will run after catch has handled the error.

## 9.6 Tasks (aka Coroutines)

Tasks are a control flow feature that allows computations to be suspended and resumed in a flexible manner. This feature is sometimes called by other names, such as symmetric coroutines, lightweight threads, cooperative multitasking, or one-shot continuations.

When a piece of computing work (in practice, executing a particular function) is designated as a *Task* (page 405), it becomes possible to interrupt it by switching to another *Task* (page 405). The original *Task* (page 405) can later be resumed, at which point it will pick up right where it left off. At first, this may seem similar to a function call. However there are two key differences. First, switching tasks does not use any space, so any number of task switches can occur without consuming the call stack. Second, switching among tasks can occur in any order, unlike function calls, where the called function must finish executing before control returns to the calling function.

This kind of control flow can make it much easier to solve certain problems. In some problems, the various pieces of required work are not naturally related by function calls; there is no obvious "caller" or "callee" among the jobs that need to be done. An example is the producer-consumer problem, where one complex procedure is generating values and another complex procedure is consuming them. The consumer cannot simply call a producer function to get a value, because the producer may have more values to generate and so might not yet be ready to return. With tasks, the producer and consumer can both run as long as they need to, passing values back and forth as necessary.

Julia provides the functions produce () (page 405) and consume () (page 405) for solving this problem. A producer is a function that calls produce () (page 405) on each value it needs to produce:

```
julia> function producer()
    produce("start")
    for n=1:4
        produce(2n)
    end
    produce("stop")
end;
```

To consume values, first the producer is wrapped in a Task (page 405), then consume () (page 405) is called repeatedly on that object:

```
julia> p = Task(producer);
```

```
julia> consume(p)
"start"

julia> consume(p)
2

julia> consume(p)
4

julia> consume(p)
6

julia> consume(p)
8

julia> consume(p)
"stop"
```

One way to think of this behavior is that producer was able to return multiple times. Between calls to produce () (page 405), the producer's execution is suspended and the consumer has control.

A Task can be used as an iterable object in a for loop, in which case the loop variable takes on all the produced values:

Note that the *Task ()* (page 405) constructor expects a 0-argument function. A common pattern is for the producer to be parameterized, in which case a partial function application is needed to create a 0-argument *anonymous function* (page 57). This can be done either directly or by use of a convenience macro:

```
function mytask(myarg)
...
end

taskHdl = Task(() -> mytask(7))
# or, equivalently
taskHdl = @task mytask(7)
```

produce () (page 405) and consume () (page 405) do not launch threads that can run on separate CPUs. True kernel threads are discussed under the topic of *Parallel Computing* (page 179).

#### 9.6.1 Core task operations

While produce() (page 405) and consume() (page 405) illustrate the essential nature of tasks, they are actually implemented as library functions using a more primitive function, yieldto() (page 405). yieldto(task, value) suspends the current task, switches to the specified task, and causes that task's last yieldto() (page 405) call to return the specified value. Notice that yieldto() (page 405) is the only operation required to use task-style control flow; instead of calling and returning we are always just switching to a different

task. This is why this feature is also called "symmetric coroutines"; each task is switched to and from using the same mechanism.

yieldto() (page 405) is powerful, but most uses of tasks do not invoke it directly. Consider why this might be. If you switch away from the current task, you will probably want to switch back to it at some point, but knowing when to switch back, and knowing which task has the responsibility of switching back, can require considerable coordination. For example, produce() (page 405) needs to maintain some state to remember who the consumer is. Not needing to manually keep track of the consuming task is what makes produce() (page 405) easier to use than yieldto() (page 405).

In addition to yieldto() (page 405), a few other basic functions are needed to use tasks effectively.

- current\_task() (page 405) gets a reference to the currently-running task.
- istaskdone() (page 405) queries whether a task has exited.
- istaskstarted() (page 405) queries whether a task has run yet.
- task\_local\_storage() (page 405) manipulates a key-value store specific to the current task.

#### 9.6.2 Tasks and events

Most task switches occur as a result of waiting for events such as I/O requests, and are performed by a scheduler included in the standard library. The scheduler maintains a queue of runnable tasks, and executes an event loop that restarts tasks based on external events such as message arrival.

The basic function for waiting for an event is wait() (page 408). Several objects implement wait() (page 408); for example, given a Process object, wait() (page 408) will wait for it to exit. wait() (page 408) is often implicit; for example, a wait() (page 408) can happen inside a call to read() (page 450) to wait for data to be available.

In all of these cases, wait () (page 408) ultimately operates on a Condition (page 405) object, which is in charge of queueing and restarting tasks. When a task calls wait () (page 408) on a Condition (page 405), the task is marked as non-runnable, added to the condition's queue, and switches to the scheduler. The scheduler will then pick another task to run, or block waiting for external events. If all goes well, eventually an event handler will call notify () (page 406) on the condition, which causes tasks waiting for that condition to become runnable again.

A task created explicitly by calling Task (page 405) is initially not known to the scheduler. This allows you to manage tasks manually using yieldto() (page 405) if you wish. However, when such a task waits for an event, it still gets restarted automatically when the event happens, as you would expect. It is also possible to make the scheduler run a task whenever it can, without necessarily waiting for any events. This is done by calling schedule() (page 406), or using the @schedule(page 406) or @async (page 410) macros (see Parallel Computing (page 179) for more details).

#### 9.6.3 Task states

Tasks have a state field that describes their execution status. A task state is one of the following symbols:

Symbol	Meaning
:runnable	Currently running, or available to be switched to
:waiting	Blocked waiting for a specific event
:queued	In the scheduler's run queue about to be restarted
:done	Successfully finished executing
:failed	Finished with an uncaught exception

## **Scope of Variables**

The *scope* of a variable is the region of code within which a variable is visible. Variable scoping helps avoid variable naming conflicts. The concept is intuitive: two functions can both have arguments called x without the two x's referring to the same thing. Similarly there are many other cases where different blocks of code can use the same name without referring to the same thing. The rules for when the same variable name does or doesn't refer to the same thing are called scope rules; this section spells them out in detail.

Certain constructs in the language introduce *scope blocks*, which are regions of code that are eligible to be the scope of some set of variables. The scope of a variable cannot be an arbitrary set of source lines; instead, it will always line up with one of these blocks. The constructs introducing such blocks are:

- function bodies (either syntax (page 55))
- while loops
- for loops
- try blocks
- catch blocks
- finally blocks
- let blocks
- type blocks.

Notably missing from this list are *begin blocks* (page 65) and *if blocks* (page 66), which do *not* introduce new scope blocks.

Certain constructs introduce new variables into the current innermost scope. When a variable is introduced into a scope, it is also inherited by all inner scopes unless one of those inner scopes explicitly overrides it.

Julia uses lexical scoping, meaning that a function's scope does not inherit from its caller's scope, but from the scope in which the function was defined. For example, in the following code the x inside  $f \circ 0$  is found in the global scope (and if no global variable x existed, an undefined variable error would be raised):

```
function foo()
   x
end

function bar()
   x = 1
   foo()
end

x = 2
```

```
julia> bar()
2
```

If foo is instead defined inside bar, then it accesses the local x present in that function:

```
function bar()
  function foo()
    x
  end
  x = 1
  foo()
end

x = 2
julia> bar()
```

The constructs that introduce new variables into the current scope are as follows:

- A declaration local x or const x introduces a new local variable.
- A declaration global x makes x in the current scope and inner scopes refer to the global variable of that name.
- · A function's arguments are introduced as new local variables into the function's body scope.
- An assignment x = y introduces a new local variable x only if x is neither declared global nor introduced as local by any enclosing scope before *or after* the current line of code.

In the following example, there is only one x assigned both inside and outside the for loop:

```
function foo(n)
    x = 0
    for i = 1:n
        x = x + 1
    end
    x
end
julia> foo(10)
10
```

In the next example, the loop has a separate x and the function always returns zero:

```
function foo(n)
    x = 0
    for i = 1:n
        local x
        x = i
    end
        x
end
julia> foo(10)
0
```

In this example, an x exists only inside the loop, and the function encounters an undefined variable error on its last line (unless there is a global variable x):

```
function foo(n)
  for i = 1:n
      x = i
  end
  x
end
julia> foo(10)
in foo: x not defined
```

A variable that is not assigned to or otherwise introduced locally defaults to global, so this function would return the value of the global x if there were such a variable, or produce an error if no such global existed. As a consequence, the only way to assign to a global variable inside a non-top-level scope is to explicitly declare the variable as global within some scope, since otherwise the assignment would introduce a new local rather than assigning to the global. This rule works out well in practice, since the vast majority of variables assigned inside functions are intended to be local variables, and using global variables should be the exception rather than the rule, and assigning new values to them even more so.

One last example shows that an outer assignment introducing x need not come before an inner usage:

```
function foo(n)
  f = y -> n + x + y
  x = 1
  f(2)
end
julia> foo(10)
13
```

This behavior may seem slightly odd for a normal variable, but allows for named functions — which are just normal variables holding function objects — to be used before they are defined. This allows functions to be defined in whatever order is intuitive and convenient, rather than forcing bottom up ordering or requiring forward declarations, both of which one typically sees in C programs. As an example, here is an inefficient, mutually recursive way to test if positive integers are even or odd:

```
even(n) = n == 0 ? true : odd(n-1)
odd(n) = n == 0 ? false : even(n-1)

julia> even(3)
false

julia> odd(3)
true
```

Julia provides built-in, efficient functions to test for oddness and evenness called *iseven()* (page 382) and *isodd()* (page 382) so the above definitions should only be taken as examples.

Since functions can be used before they are defined, as long as they are defined by the time they are actually called, no syntax for forward declarations is necessary, and definitions can be ordered arbitrarily.

At the interactive prompt, variable scope works the same way as anywhere else. The prompt behaves as if there is scope block wrapped around everything you type, except that this scope block is identified with the global scope. This is especially evident in the case of assignments:

```
julia> for i = 1:1; y = 10; end

julia> y
ERROR: UndefVarError: y not defined
```

```
julia> y = 0
0

julia> for i = 1:1; y = 10; end

julia> y
10
```

In the former case, y only exists inside of the for loop. In the latter case, an outer y has been introduced and so is inherited within the loop. Due to the special identification of the prompt's scope block with the global scope, it is not necessary to declare global y inside the loop. However, in code not entered into the interactive prompt this declaration would be necessary in order to modify a global variable.

Multiple variables can be declared global using the following syntax:

```
function foo()
    global x=1, y="bar", z=3
end

julia> foo()
3

julia> x
1

julia> y
"bar"

julia> z
3
```

The let statement provides a different way to introduce variables. Unlike assignments to local variables, let statements allocate new variable bindings each time they run. An assignment modifies an existing value location, and let creates new locations. This difference is usually not important, and is only detectable in the case of variables that outlive their scope via closures. The let syntax accepts a comma-separated series of assignments and variable names:

```
let var1 = value1, var2, var3 = value3
    code
end
```

The assignments are evaluated in order, with each right-hand side evaluated in the scope before the new variable on the left-hand side has been introduced. Therefore it makes sense to write something like let x = x since the two x variables are distinct and have separate storage. Here is an example where the behavior of let is needed:

```
Fs = cell(2)
i = 1
while i <= 2
Fs[i] = ()->i
i += 1
end

julia> Fs[1]()
3

julia> Fs[2]()
```

Here we create and store two closures that return variable i. However, it is always the same variable i, so the two

closures behave identically. We can use let to create a new binding for i:

```
Fs = cell(2)
i = 1
while i <= 2
let i = i
   Fs[i] = ()->i
end
i += 1
end

julia> Fs[1]()
1

julia> Fs[2]()
```

Since the begin construct does not introduce a new scope, it can be useful to use a zero-argument let to just introduce a new scope block without creating any new bindings:

```
julia> let
            local x = 1
            let
            local x = 2
            end
            x
            end
1
```

Since let introduces a new scope block, the inner local x is a different variable than the outer local x.

# 10.1 For Loops and Comprehensions

for loops and *comprehensions* (page 159) have a special additional behavior: any new variables introduced in their body scopes are freshly allocated for each loop iteration. Therefore these constructs are similar to while loops with let blocks inside:

```
Fs = cell(2)
for i = 1:2
   Fs[i] = ()->i
end

julia> Fs[1]()
1

julia> Fs[2]()
```

for loops will reuse existing variables for iteration:

```
i = 0
for i = 1:3
end
i  # here equal to 3
```

However, comprehensions do not do this, and always freshly allocate their iteration variables:

```
x = 0
[ x for x=1:3 ]
x # here still equal to 0
```

#### 10.2 Constants

A common use of variables is giving names to specific, unchanging values. Such variables are only assigned once. This intent can be conveyed to the compiler using the const keyword:

```
const e = 2.71828182845904523536
const pi = 3.14159265358979323846
```

The const declaration is allowed on both global and local variables, but is especially useful for globals. It is difficult for the compiler to optimize code involving global variables, since their values (or even their types) might change at almost any time. If a global variable will not change, adding a const declaration solves this performance problem.

Local constants are quite different. The compiler is able to determine automatically when a local variable is constant, so local constant declarations are not necessary for performance purposes.

Special top-level assignments, such as those performed by the function and type keywords, are constant by default.

Note that const only affects the variable binding; the variable may be bound to a mutable object (such as an array), and that object may still be modified.

## **Types**

Type systems have traditionally fallen into two quite different camps: static type systems, where every program expression must have a type computable before the execution of the program, and dynamic type systems, where nothing is known about types until run time, when the actual values manipulated by the program are available. Object orientation allows some flexibility in statically typed languages by letting code be written without the precise types of values being known at compile time. The ability to write code that can operate on different types is called polymorphism. All code in classic dynamically typed languages is polymorphic: only by explicitly checking types, or when objects fail to support operations at run-time, are the types of any values ever restricted.

Julia's type system is dynamic, but gains some of the advantages of static type systems by making it possible to indicate that certain values are of specific types. This can be of great assistance in generating efficient code, but even more significantly, it allows method dispatch on the types of function arguments to be deeply integrated with the language. Method dispatch is explored in detail in *Methods* (page 107), but is rooted in the type system presented here.

The default behavior in Julia when types are omitted is to allow values to be of any type. Thus, one can write many useful Julia programs without ever explicitly using types. When additional expressiveness is needed, however, it is easy to gradually introduce explicit type annotations into previously "untyped" code. Doing so will typically increase both the performance and robustness of these systems, and perhaps somewhat counterintuitively, often significantly simplify them.

Describing Julia in the lingo of type systems, it is: dynamic, nominative and parametric. Generic types can be parameterized, and the hierarchical relationships between types are explicitly declared, rather than implied by compatible structure. One particularly distinctive feature of Julia's type system is that concrete types may not subtype each other: all concrete types are final and may only have abstract types as their supertypes. While this might at first seem unduly restrictive, it has many beneficial consequences with surprisingly few drawbacks. It turns out that being able to inherit behavior is much more important than being able to inherit structure, and inheriting both causes significant difficulties in traditional object-oriented languages. Other high-level aspects of Julia's type system that should be mentioned up front are:

- There is no division between object and non-object values: all values in Julia are true objects having a type that belongs to a single, fully connected type graph, all nodes of which are equally first-class as types.
- There is no meaningful concept of a "compile-time type": the only type a value has is its actual type when the program is running. This is called a "run-time type" in object-oriented languages where the combination of static compilation with polymorphism makes this distinction significant.
- Only values, not variables, have types variables are simply names bound to values.
- Both abstract and concrete types can be parameterized by other types. They can also be parameterized by symbols, by values of any type for which <code>isbits()</code> (page 324) returns true (essentially, things like numbers and bools that are stored like C types or structs with no pointers to other objects), and also by tuples thereof. Type parameters may be omitted when they do not need to be referenced or restricted.

Julia's type system is designed to be powerful and expressive, yet clear, intuitive and unobtrusive. Many Julia programmers may never feel the need to write code that explicitly uses types. Some kinds of programming, however, become clearer, simpler, faster and more robust with declared types.

### 11.1 Type Declarations

The :: operator can be used to attach type annotations to expressions and variables in programs. There are two primary reasons to do this:

- 1. As an assertion to help confirm that your program works the way you expect,
- 2. To provide extra type information to the compiler, which can then improve performance in some cases

When appended to an expression computing a *value*, the :: operator is read as "is an instance of". It can be used anywhere to assert that the value of the expression on the left is an instance of the type on the right. When the type on the right is concrete, the value on the left must have that type as its implementation — recall that all concrete types are final, so no implementation is a subtype of any other. When the type is abstract, it suffices for the value to be implemented by a concrete type that is a subtype of the abstract type. If the type assertion is not true, an exception is thrown, otherwise, the left-hand value is returned:

```
julia> (1+2)::AbstractFloat
ERROR: TypeError: typeassert: expected AbstractFloat, got Int64
julia> (1+2)::Int
3
```

This allows a type assertion to be attached to any expression in-place. The most common usage of :: as an assertion is in function/methods signatures, such as f(x::Int8) = ... (see *Methods* (page 107)).

When appended to a *variable* in a statement context, the :: operator means something a bit different: it declares the variable to always have the specified type, like a type declaration in a statically-typed language such as C. Every value assigned to the variable will be converted to the declared type using *convert* () (page 321):

```
julia> function foo()
         x::Int8 = 100
         x
        end
foo (generic function with 1 method)

julia> foo()
100

julia> typeof(ans)
Int8
```

This feature is useful for avoiding performance "gotchas" that could occur if one of the assignments to a variable changed its type unexpectedly.

The "declaration" behavior only occurs in specific contexts:

```
x::Int8  # a variable by itself
local x::Int8  # in a local declaration
x::Int8 = 10  # as the left-hand side of an assignment
```

and applies to the whole current scope, even before the declaration. Currently, type declarations cannot be used in global scope, e.g. in the REPL, since Julia does not yet have constant-type globals. Note that in a function return statement, the first two of the above expressions compute a value and then :: is a type assertion and not a declaration.

## 11.2 Abstract Types

Abstract types cannot be instantiated, and serve only as nodes in the type graph, thereby describing sets of related concrete types: those concrete types which are their descendants. We begin with abstract types even though they have no instantiation because they are the backbone of the type system: they form the conceptual hierarchy which makes Julia's type system more than just a collection of object implementations.

Recall that in *Integers and Floating-Point Numbers* (page 13), we introduced a variety of concrete types of numeric values: Int8, UInt8, Int16, UInt16, Int32, UInt32, Int64, UInt64, Int128, UInt128, Float16, Float32 (page 380), and Float64 (page 380). Although they have different representation sizes, Int8, Int16, Int32, Int64 and Int128 all have in common that they are signed integer types. Likewise UInt8, UInt16, UInt32, UInt64 and UInt128 are all unsigned integer types, while Float16, Float32 (page 380) and Float64 (page 380) are distinct in being floating-point types rather than integers. It is common for a piece of code to make sense, for example, only if its arguments are some kind of integer, but not really depend on what particular kind of integer. For example, the greatest common denominator algorithm works for all kinds of integers, but will not work for floating-point numbers. Abstract types allow the construction of a hierarchy of types, providing a context into which concrete types can fit. This allows you, for example, to easily program to any type that is an integer, without restricting an algorithm to a specific type of integer.

Abstract types are declared using the abstract keyword. The general syntaxes for declaring an abstract type are:

```
abstract «name»
abstract «name» <: «supertype»
```

The abstract keyword introduces a new abstract type, whose name is given by «name». This name can be optionally followed by <: and an already-existing type, indicating that the newly declared abstract type is a subtype of this "parent" type.

When no supertype is given, the default supertype is Any — a predefined abstract type that all objects are instances of and all types are subtypes of. In type theory, Any is commonly called "top" because it is at the apex of the type graph. Julia also has a predefined abstract "bottom" type, at the nadir of the type graph, which is written as Union{}. It is the exact opposite of Any: no object is an instance of Union{} and all types are supertypes of Union{}.

Let's consider some of the abstract types that make up Julia's numerical hierarchy:

```
abstract Number
abstract Real <: Number
abstract AbstractFloat <: Real
abstract Integer <: Real
abstract Signed <: Integer
abstract Unsigned <: Integer
```

The Number type is a direct child type of Any, and Real is its child. In turn, Real has two children (it has more, but only two are shown here; we'll get to the others later): Integer and AbstractFloat, separating the world into representations of integers and representations of real numbers. Representations of real numbers include, of course, floating-point types, but also include other types, such as rationals. Hence, AbstractFloat is a proper subtype of Real, including only floating-point representations of real numbers. Integers are further subdivided into Signed and Unsigned varieties.

The <: operator in general means "is a subtype of", and, used in declarations like this, declares the right-hand type to be an immediate supertype of the newly declared type. It can also be used in expressions as a subtype operator which returns true when its left operand is a subtype of its right operand:

```
julia> Integer <: Number
true

julia> Integer <: AbstractFloat
false</pre>
```

An important use of abstract types is to provide default implementations for concrete types. To give a simple example, consider:

```
function myplus(x,y)
  x+y
end
```

The first thing to note is that the above argument declarations are equivalent to x::Any and y::Any. When this function is invoked, say as myplus(2,5), the dispatcher chooses the most specific method named myplus that matches the given arguments. (See *Methods* (page 107) for more information on multiple dispatch.)

Assuming no method more specific than the above is found, Julia next internally defines and compiles a method called myplus specifically for two Int arguments based on the generic function given above, i.e., it implicitly defines and compiles:

```
function myplus(x::Int, y::Int)
  x+y
end
```

and finally, it invokes this specific method.

Thus, abstract types allow programmers to write generic functions that can later be used as the default method by many combinations of concrete types. Thanks to multiple dispatch, the programmer has full control over whether the default or more specific method is used.

An important point to note is that there is no loss in performance if the programmer relies on a function whose arguments are abstract types, because it is recompiled for each tuple of argument concrete types with which it is invoked. (There may be a performance issue, however, in the case of function arguments that are containers of abstract types; see *Performance Tips* (page 265).)

## 11.3 Bits Types

A bits type is a concrete type whose data consists of plain old bits. Classic examples of bits types are integers and floating-point values. Unlike most languages, Julia lets you declare your own bits types, rather than providing only a fixed set of built-in bits types. In fact, the standard bits types are all defined in the language itself:

```
bitstype 16 Float16 <: AbstractFloat</pre>
bitstype 32 Float32 <: AbstractFloat</pre>
bitstype 64 Float64 <: AbstractFloat</pre>
bitstype 8 Bool <: Integer
bitstype 32 Char
bitstype 8 Int8 <: Signed
bitstype 8 UInt8 <: Unsigned</pre>
bitstype 16 Int16 <: Signed
bitstype 16 UInt16 <: Unsigned</pre>
bitstype 32 Int32 <: Signed</pre>
bitstype 32 UInt32 <: Unsigned
bitstype 64 Int64
                     <: Signed
bitstype 64 UInt64
                     <: Unsigned
bitstype 128 Int128 <: Signed
bitstype 128 UInt128 <: Unsigned
```

The general syntaxes for declaration of a bitstype are:

```
bitstype «bits» «name»
bitstype «bits» «name» <: «supertype»
```

The number of bits indicates how much storage the type requires and the name gives the new type a name. A bits type can optionally be declared to be a subtype of some supertype. If a supertype is omitted, then the type defaults to having Any as its immediate supertype. The declaration of Bool above therefore means that a boolean value takes eight bits to store, and has Integer as its immediate supertype. Currently, only sizes that are multiples of 8 bits are supported. Therefore, boolean values, although they really need just a single bit, cannot be declared to be any smaller than eight bits.

The types Bool, Int8 and UInt8 all have identical representations: they are eight-bit chunks of memory. Since Julia's type system is nominative, however, they are not interchangeable despite having identical structure. Another fundamental difference between them is that they have different supertypes: Bool's direct supertype is Integer, Int8's is Signed, and UInt8's is Unsigned. All other differences between Bool, Int8, and UInt8 are matters of behavior — the way functions are defined to act when given objects of these types as arguments. This is why a nominative type system is necessary: if structure determined type, which in turn dictates behavior, then it would be impossible to make Bool behave any differently than Int8 or UInt8.

## 11.4 Composite Types

Composite types are called records, structures (structs in C), or objects in various languages. A composite type is a collection of named fields, an instance of which can be treated as a single value. In many languages, composite types are the only kind of user-definable type, and they are by far the most commonly used user-defined type in Julia as well.

In mainstream object oriented languages, such as C++, Java, Python and Ruby, composite types also have named functions associated with them, and the combination is called an "object". In purer object-oriented languages, such as Ruby or Smalltalk, all values are objects whether they are composites or not. In less pure object oriented languages, including C++ and Java, some values, such as integers and floating-point values, are not objects, while instances of user-defined composite types are true objects with associated methods. In Julia, all values are objects, but functions are not bundled with the objects they operate on. This is necessary since Julia chooses which method of a function to use by multiple dispatch, meaning that the types of *all* of a function's arguments are considered when selecting a method, rather than just the first one (see *Methods* (page 107) for more information on methods and dispatch). Thus, it would be inappropriate for functions to "belong" to only their first argument. Organizing methods into function objects rather than having named bags of methods "inside" each object ends up being a highly beneficial aspect of the language design.

Since composite types are the most common form of user-defined concrete type, they are simply introduced with the type keyword followed by a block of field names, optionally annotated with types using the :: operator:

Fields with no type annotation default to Any, and can accordingly hold any type of value.

New objects of composite type Foo are created by applying the Foo type object like a function to values for its fields:

```
julia> foo = Foo("Hello, world.", 23, 1.5)
Foo("Hello, world.",23,1.5)

julia> typeof(foo)
Foo
```

When a type is applied like a function it is called a *constructor*. Two constructors are generated automatically (these are called *default constructors*). One accepts any arguments and calls *convert()* (page 321) to convert them to the types of the fields, and the other accepts arguments that match the field types exactly. The reason both of these are generated is that this makes it easier to add new definitions without inadvertently replacing a default constructor.

Since the bar field is unconstrained in type, any value will do. However, the value for baz must be convertible to Int:

```
julia> Foo((), 23.5, 1)
ERROR: InexactError()
in call at none:2
```

You may find a list of field names using the fieldnames function.

```
julia> fieldnames(foo)
3-element Array{Symbol,1}:
   :bar
   :baz
   :qux
```

You can access the field values of a composite object using the traditional foo.bar notation:

```
julia> foo.bar
"Hello, world."

julia> foo.baz
23

julia> foo.qux
1.5
```

You can also change the values as one would expect:

```
julia> foo.qux = 2
2.0

julia> foo.bar = 1//2
1//2
```

Composite types with no fields are singletons; there can be only one instance of such types:

```
type NoFields
end

julia> is(NoFields(), NoFields())
true
```

The is function confirms that the "two" constructed instances of NoFields are actually one and the same. Singleton types are described in further detail *below* (page 100).

There is much more to say about how instances of composite types are created, but that discussion depends on both *Parametric Types* (page 94) and on *Methods* (page 107), and is sufficiently important to be addressed in its own section: *Constructors* (page 117).

# 11.5 Immutable Composite Types

It is also possible to define *immutable* composite types by using the keyword immutable instead of type:

```
immutable Complex
  real::Float64
  imag::Float64
end
```

Such types behave much like other composite types, except that instances of them cannot be modified. Immutable types have several advantages:

- They are more efficient in some cases. Types like the Complex example above can be packed efficiently into arrays, and in some cases the compiler is able to avoid allocating immutable objects entirely.
- It is not possible to violate the invariants provided by the type's constructors.
- Code using immutable objects can be easier to reason about.

An immutable object might contain mutable objects, such as arrays, as fields. Those contained objects will remain mutable; only the fields of the immutable object itself cannot be changed to point to different objects.

A useful way to think about immutable composites is that each instance is associated with specific field values — the field values alone tell you everything about the object. In contrast, a mutable object is like a little container that might hold different values over time, and so is not identified with specific field values. In deciding whether to make a type immutable, ask whether two instances with the same field values would be considered identical, or if they might need to change independently over time. If they would be considered identical, the type should probably be immutable.

To recap, two essential properties define immutability in Julia:

- An object with an immutable type is passed around (both in assignment statements and in function calls) by copying, whereas a mutable type is passed around by reference.
- It is not permitted to modify the fields of a composite immutable type.

It is instructive, particularly for readers whose background is C/C++, to consider why these two properties go hand in hand. If they were separated, i.e., if the fields of objects passed around by copying could be modified, then it would become more difficult to reason about certain instances of generic code. For example, suppose x is a function argument of an abstract type, and suppose that the function changes a field: x.isprocessed = true. Depending on whether x is passed by copying or by reference, this statement may or may not alter the actual argument in the calling routine. Julia sidesteps the possibility of creating functions with unknown effects in this scenario by forbidding modification of fields of objects passed around by copying.

# 11.6 Declared Types

The three kinds of types discussed in the previous three sections are actually all closely related. They share the same key properties:

- They are explicitly declared.
- · They have names.
- They have explicitly declared supertypes.
- They may have parameters.

Because of these shared properties, these types are internally represented as instances of the same concept, DataType, which is the type of any of these types:

```
julia> typeof(Real)
DataType

julia> typeof(Int)
DataType
```

A DataType may be abstract or concrete. If it is concrete, it has a specified size, storage layout, and (optionally) field names. Thus a bits type is a DataType with nonzero size, but no field names. A composite type is a DataType that has field names or is empty (zero size).

Every concrete value in the system is an instance of some DataType.

# 11.7 Type Unions

A type union is a special abstract type which includes as objects all instances of any of its argument types, constructed using the special Union function:

```
julia> IntOrString = Union{Int, AbstractString}
Union{AbstractString, Int64}

julia> 1 :: IntOrString

julia> "Hello!" :: IntOrString
"Hello!"

julia> 1.0 :: IntOrString
ERROR: type: typeassert: expected Union{AbstractString, Int64}, got Float64
```

The compilers for many languages have an internal union construct for reasoning about types; Julia simply exposes it to the programmer.

# 11.8 Parametric Types

An important and powerful feature of Julia's type system is that it is parametric: types can take parameters, so that type declarations actually introduce a whole family of new types — one for each possible combination of parameter values. There are many languages that support some version of generic programming, wherein data structures and algorithms to manipulate them may be specified without specifying the exact types involved. For example, some form of generic programming exists in ML, Haskell, Ada, Eiffel, C++, Java, C#, F#, and Scala, just to name a few. Some of these languages support true parametric polymorphism (e.g. ML, Haskell, Scala), while others support ad-hoc, template-based styles of generic programming (e.g. C++, Java). With so many different varieties of generic programming and parametric types in various languages, we won't even attempt to compare Julia's parametric types to other languages, but will instead focus on explaining Julia's system in its own right. We will note, however, that because Julia is a dynamically typed language and doesn't need to make all type decisions at compile time, many traditional difficulties encountered in static parametric type systems can be relatively easily handled.

All declared types (the DataType variety) can be parameterized, with the same syntax in each case. We will discuss them in the following order: first, parametric composite types, then parametric abstract types, and finally parametric bits types.

#### 11.8.1 Parametric Composite Types

Type parameters are introduced immediately after the type name, surrounded by curly braces:

```
type Point{T}
    x::T
    y::T
end
```

This declaration defines a new parametric type, Point {T}, holding two "coordinates" of type T. What, one may ask, is T? Well, that's precisely the point of parametric types: it can be any type at all (or a value of any bits type, actually, although here it's clearly used as a type). Point {Float64} is a concrete type equivalent to the type defined by replacing T in the definition of Point with Float64 (page 380). Thus, this single declaration actually declares an unlimited number of types: Point {Float64}, Point {AbstractString}, Point {Int64}, etc. Each of these is now a usable concrete type:

```
julia> Point{Float64}
Point{Float64}

julia> Point{AbstractString}
Point{AbstractString}
```

The type Point{Float64} is a point whose coordinates are 64-bit floating-point values, while the type Point{AbstractString} is a "point" whose "coordinates" are string objects (see *Strings* (page 41)). However, Point itself is also a valid type object:

```
julia> Point
Point{T}
```

Here the T is the dummy type symbol used in the original declaration of Point. What does Point by itself mean? It is an abstract type that contains all the specific instances Point {Float64}, Point {AbstractString}, etc.:

```
julia> Point{Float64} <: Point
true

julia> Point{AbstractString} <: Point
true</pre>
```

Other types, of course, are not subtypes of it:

```
julia> Float64 <: Point
false

julia> AbstractString <: Point
false</pre>
```

Concrete Point types with different values of T are never subtypes of each other:

```
julia> Point{Float64} <: Point{Int64}
false

julia> Point{Float64} <: Point{Real}
false</pre>
```

This last point is very important:

• Even though Float 64 <: Real we DO NOT have Point {Float 64} <: Point {Real}.

In other words, in the parlance of type theory, Julia's type parameters are *invariant*, rather than being covariant (or even contravariant). This is for practical reasons: while any instance of Point {Float64} may conceptually be like an instance of Point {Real} as well, the two types have different representations in memory:

- An instance of Point {Float64} can be represented compactly and efficiently as an immediate pair of 64-bit values:
- An instance of Point{Real} must be able to hold any pair of instances of Real. Since objects that are instances of Real can be of arbitrary size and structure, in practice an instance of Point{Real} must be represented as a pair of pointers to individually allocated Real objects.

The efficiency gained by being able to store Point {Float64} objects with immediate values is magnified enormously in the case of arrays: an Array{Float64} can be stored as a contiguous memory block of 64-bit floating-point values, whereas an Array{Real} must be an array of pointers to individually allocated Real objects — which may well be boxed 64-bit floating-point values, but also might be arbitrarily large, complex objects, which are declared to be implementations of the Real abstract type.

Since Point {Float64} is not a subtype of Point {Real}, the following method can't be applied to arguments of type Point {Float64}:

```
function norm(p::Point{Real})
    sqrt(p.x^2 + p.y^2)
end
```

The correct way to define a method that accepts all arguments of type  $Point\{T\}$  where T is a subtype of Real is:

```
function norm{T<:Real} (p::Point{T})
    sqrt (p.x^2 + p.y^2)
end</pre>
```

More examples will be discussed later in *Methods* (page 107).

How does one construct a Point object? It is possible to define custom constructors for composite types, which will be discussed in detail in *Constructors* (page 117), but in the absence of any special constructor declarations, there are two default ways of creating new composite objects, one in which the type parameters are explicitly given and the other in which they are implied by the arguments to the object constructor.

Since the type Point {Float 64} is a concrete type equivalent to Point declared with Float 64 (page 380) in place of T, it can be applied as a constructor accordingly:

```
julia> Point{Float64}(1.0,2.0)
Point{Float64}(1.0,2.0)

julia> typeof(ans)
Point{Float64}
```

For the default constructor, exactly one argument must be supplied for each field:

```
julia> Point{Float64}(1.0)
ERROR: MethodError: `convert` has no method matching convert(::Type{Point{Float64}}}, :: | float64)
This may have arisen from a call to the constructor Point{Float64}(...),
since type constructors fall back to convert methods.
Closest candidates are:
 Point{T} (::Any, !Matched::Any)
 call{T}(::Type{T}, ::Any)
 convert{T}(::Type{T}, !Matched::T)
in call at essentials.jl:56
julia > Point {Float 64} (1.0, 2.0, 3.0)
ERROR: MethodError: `convert` has no method matching convert(::Type{Point{Float64}}}, ::rloat64, ::rloat64,
This may have arisen from a call to the constructor Point\{Float64\}(...),
since type constructors fall back to convert methods.
Closest candidates are:
  Point(T)(::Any, ::Any)
 call{T}(::Type{T}, ::Any)
 convert{T}(::Type{T}, !Matched::T)
in call at essentials.jl:57
```

Only one default constructor is generated for parametric types, since overriding it is not possible. This constructor accepts any arguments and converts them to the field types.

In many cases, it is redundant to provide the type of Point object one wants to construct, since the types of arguments to the constructor call already implicitly provide type information. For that reason, you can also apply Point itself as a constructor, provided that the implied value of the parameter type T is unambiguous:

```
julia> Point(1.0,2.0)
Point(Float64)(1.0,2.0)

julia> typeof(ans)
Point(Float64)
```

```
julia> Point(1,2)
Point{Int64}(1,2)

julia> typeof(ans)
Point{Int64}
```

In the case of Point, the type of T is unambiguously implied if and only if the two arguments to Point have the same type. When this isn't the case, the constructor will fail with a MethodError (page 330):

```
julia> Point(1,2.5)
ERROR: MethodError: `convert` has no method matching convert(::Type{Point{T}}, ::Int64,
This may have arisen from a call to the constructor Point{T}(...),
since type constructors fall back to convert methods.
Closest candidates are:
  Point{T}(::T, !Matched::T)
  call{T}(::Type{T}, ::Any)
  convert{T}(::Type{T}, !Matched::T)
  in call at essentials.jl:57
```

Constructor methods to appropriately handle such mixed cases can be defined, but that will not be discussed until later on in *Constructors* (page 117).

#### 11.8.2 Parametric Abstract Types

Parametric abstract type declarations declare a collection of abstract types, in much the same way:

```
abstract Pointy{T}
```

With this declaration,  $Pointy\{T\}$  is a distinct abstract type for each type or integer value of T. As with parametric composite types, each such instance is a subtype of Pointy:

```
julia> Pointy{Int64} <: Pointy
true

julia> Pointy{1} <: Pointy
true</pre>
```

Parametric abstract types are invariant, much as parametric composite types are:

```
julia> Pointy{Float64} <: Pointy{Real}
false

julia> Pointy{Real} <: Pointy{Float64}
false</pre>
```

Much as plain old abstract types serve to create a useful hierarchy of types over concrete types, parametric abstract types serve the same purpose with respect to parametric composite types. We could, for example, have declared  $Point{T}$  to be a subtype of  $Pointy{T}$  as follows:

```
type Point{T} <: Pointy{T}
    x::T
    y::T
end</pre>
```

Given such a declaration, for each choice of T, we have  $Point\{T\}$  as a subtype of  $Pointy\{T\}$ :

```
julia> Point{Float64} <: Pointy{Float64}
true</pre>
```

```
julia> Point{Real} <: Pointy{Real}
true

julia> Point{AbstractString} <: Pointy{AbstractString}
true</pre>
```

This relationship is also invariant:

```
julia> Point{Float64} <: Pointy{Real}
false</pre>
```

What purpose do parametric abstract types like Pointy serve? Consider if we create a point-like implementation that only requires a single coordinate because the point is on the diagonal line x = y:

```
type DiagPoint{T} <: Pointy{T}
    x::T
end</pre>
```

Now both Point {Float64} and DiagPoint {Float64} are implementations of the Pointy {Float64} abstraction, and similarly for every other possible choice of type T. This allows programming to a common interface shared by all Pointy objects, implemented for both Point and DiagPoint. This cannot be fully demonstrated, however, until we have introduced methods and dispatch in the next section, *Methods* (page 107).

There are situations where it may not make sense for type parameters to range freely over all possible types. In such situations, one can constrain the range of  $\mathbb{T}$  like so:

```
abstract Pointy{T<:Real}</pre>
```

With such a declaration, it is acceptable to use any type that is a subtype of Real in place of T, but not types that are not subtypes of Real:

```
julia> Pointy{Float64}
Pointy{Float64}

julia> Pointy{Real}

pointy{Real}

julia> Pointy{AbstractString}
ERROR: TypeError: Pointy: in T, expected T<:Real, got Type{AbstractString}

julia> Pointy{1}
ERROR: TypeError: Pointy: in T, expected T<:Real, got Int64</pre>
```

Type parameters for parametric composite types can be restricted in the same manner:

```
type Point{T<:Real} <: Pointy{T}
    x::T
    y::T
end</pre>
```

To give a real-world example of how all this parametric type machinery can be useful, here is the actual definition of Julia's Rational immutable type (except that we omit the constructor here for simplicity), representing an exact ratio of integers:

```
immutable Rational{T<:Integer} <: Real
  num::T
  den::T
end</pre>
```

It only makes sense to take ratios of integer values, so the parameter type T is restricted to being a subtype of Integer, and a ratio of integers represents a value on the real number line, so any Rational is an instance of the Real abstraction.

#### 11.8.3 Tuple Types

Tuples are an abstraction of the arguments of a function — without the function itself. The salient aspects of a function's arguments are their order and their types. Therefore a tuple type is similar to a parameterized immutable type where each parameter is the type of one field. For example, a 2-element tuple type resembles the following immutable type:

```
immutable Tuple2{A,B}
  a::A
  b::B
end
```

However, there are three key differences:

- Tuple types may have any number of parameters.
- Tuple types are *covariant* in their parameters: Tuple{Int} is a subtype of Tuple{Any}. Therefore Tuple{Any} is considered an abstract type, and tuple types are only concrete if their parameters are.
- Tuples do not have field names; fields are only accessed by index.

Tuple values are written with parentheses and commas. When a tuple is constructed, an appropriate tuple type is generated on demand:

```
julia> typeof((1,"foo",2.5))
Tuple{Int64, ASCIIString, Float64}
```

Note the implications of covariance:

```
julia> Tuple{Int, AbstractString} <: Tuple{Real, Any}
true

julia> Tuple{Int, AbstractString} <: Tuple{Real, Real}
false

julia> Tuple{Int, AbstractString} <: Tuple{Real,}
false</pre>
```

Intuitively, this corresponds to the type of a function's arguments being a subtype of the function's signature (when the signature matches).

#### 11.8.4 Vararg Tuple Types

The last parameter of a tuple type can be the special type Vararq, which denotes any number of trailing elements:

```
julia> isa(("1",), Tuple{AbstractString, Vararg{Int}})
true

julia> isa(("1",1), Tuple{AbstractString, Vararg{Int}})
true

julia> isa(("1",1,2), Tuple{AbstractString, Vararg{Int}})
true
```

```
julia> isa(("1",1,2,3.0), Tuple{AbstractString, Vararg{Int}})
false
```

Notice that Vararg{T} matches zero or more elements of type T. Vararg tuple types are used to represent the arguments accepted by varargs methods (see *Varargs Functions* (page 59)).

#### **Singleton Types**

There is a special kind of abstract parametric type that must be mentioned here: singleton types. For each type, T, the "singleton type"  $Type\{T\}$  is an abstract type whose only instance is the object T. Since the definition is a little difficult to parse, let's look at some examples:

```
julia> isa(Float64, Type{Float64})
true

julia> isa(Real, Type{Float64})
false

julia> isa(Real, Type{Real})
true

julia> isa(Float64, Type{Real})
false
```

In other words,  $isa(A, Type\{B\})$  (page 320) is true if and only if A and B are the same object and that object is a type. Without the parameter, Type is simply an abstract type which has all type objects as its instances, including, of course, singleton types:

```
julia> isa(Type{Float64}, Type)
true

julia> isa(Float64, Type)
true

julia> isa(Real, Type)
true
```

Any object that is not a type is not an instance of Type:

```
julia> isa(1,Type)
false

julia> isa("foo",Type)
false
```

Until we discuss *Parametric Methods* (page 113) and *conversions* (page 128), it is difficult to explain the utility of the singleton type construct, but in short, it allows one to specialize function behavior on specific type *values*. This is useful for writing methods (especially parametric ones) whose behavior depends on a type that is given as an explicit argument rather than implied by the type of one of its arguments.

A few popular languages have singleton types, including Haskell, Scala and Ruby. In general usage, the term "singleton type" refers to a type whose only instance is a single value. This meaning applies to Julia's singleton types, but with that caveat that only type objects have singleton types.

#### 11.8.5 Parametric Bits Types

Bits types can also be declared parametrically. For example, pointers are represented as boxed bits types which would be declared in Julia like this:

```
# 32-bit system:
bitstype 32 Ptr{T}

# 64-bit system:
bitstype 64 Ptr{T}
```

The slightly odd feature of these declarations as compared to typical parametric composite types, is that the type parameter T is not used in the definition of the type itself — it is just an abstract tag, essentially defining an entire family of types with identical structure, differentiated only by their type parameter. Thus, Ptr{Float64} and Ptr{Int64} are distinct types, even though they have identical representations. And of course, all specific pointer types are subtype of the umbrella Ptr type:

```
julia> Ptr{Float64} <: Ptr
true

julia> Ptr{Int64} <: Ptr
true</pre>
```

## 11.9 Type Aliases

Sometimes it is convenient to introduce a new name for an already expressible type. For such occasions, Julia provides the typealias mechanism. For example, UInt is type aliased to either UInt32 or UInt64 as is appropriate for the size of pointers on the system:

```
# 32-bit system:
julia> UInt
UInt32
# 64-bit system:
julia> UInt
UInt64
```

This is accomplished via the following code in base/boot.jl:

```
if is(Int,Int64)
    typealias UInt UInt64
else
    typealias UInt UInt32
end
```

Of course, this depends on what Int is aliased to — but that is predefined to be the correct type — either Int 32 or Int 64.

For parametric types, typealias can be convenient for providing names for cases where some of the parameter choices are fixed. Julia's arrays have type Array{T,N} where T is the element type and N is the number of array dimensions. For convenience, writing Array{Float64} allows one to specify the element type without specifying the dimension:

```
julia> Array{Float64,1} <: Array{Float64} <: Array
true</pre>
```

11.9. Type Aliases 101

However, there is no way to equally simply restrict just the dimension but not the element type. Yet, one often needs to ensure an object is a vector or a matrix (imposing restrictions on the number of dimensions). For that reason, the following type aliases are provided:

```
typealias Vector{T} Array{T,1}
typealias Matrix{T} Array{T,2}
```

Writing Vector {Float64} is equivalent to writing Array {Float64, 1}, and the umbrella type Vector has as instances all Array objects where the second parameter — the number of array dimensions — is 1, regardless of what the element type is. In languages where parametric types must always be specified in full, this is not especially helpful, but in Julia, this allows one to write just Matrix for the abstract type including all two-dimensional dense arrays of any element type.

This declaration of Vector creates a subtype relation Vector {Int} <: Vector. However, it is not always the case that a parametric typealias statement creates such a relation; for example, the statement:

```
typealias AA{T} Array{Array{T,1},1}
```

does not create the relation AA{Int} <: AA. The reason is that  $Array{Array{T,1},1}$  is not an abstract type at all; in fact, it is a concrete type describing a 1-dimensional array in which each entry is an object of type  $Array{T,1}$  for some value of T.

# 11.10 Operations on Types

Since types in Julia are themselves objects, ordinary functions can operate on them. Some functions that are particularly useful for working with or exploring types have already been introduced, such as the <: operator, which indicates whether its left hand operand is a subtype of its right hand operand.

The isa function tests if an object is of a given type and returns true or false:

```
julia> isa(1,Int)
true

julia> isa(1,AbstractFloat)
false
```

The typeof() (page 321) function, already used throughout the manual in examples, returns the type of its argument. Since, as noted above, types are objects, they also have types, and we can ask what their types are:

```
julia> typeof(Rational)
DataType

julia> typeof(Union{Real, Float64, Rational})
DataType

julia> typeof(Union{Real, ASCIIString})
Union
```

What if we repeat the process? What is the type of a type of a type? As it happens, types are all composite values and thus all have a type of DataType:

```
julia> typeof(DataType)
DataType

julia> typeof(UnionType)
DataType
```

DataType is its own type.

Another operation that applies to some types is *supertype()* (page 322), which reveals a type's supertype. Only declared types (DataType) have unambiguous supertypes:

```
julia> supertype(Float64)
AbstractFloat

julia> supertype(Number)
Any

julia> supertype(AbstractString)
Any

julia> supertype(Any)
Any
```

If you apply *supertype()* (page 322) to other type objects (or non-type objects), a *MethodError* (page 330) is raised:

```
julia> supertype(Union{Float64, Int64})
ERROR: `supertype` has no method matching supertype(::Type{Union{Float64, Int64}})
```

### 11.11 "Value types"

As one application of these ideas, Julia includes a parametric type, Val{T}, designated for dispatching on bits-type *values*. For example, if you pass a boolean to a function, you have to test the value at run-time:

```
function firstlast(b::Bool)
    return b ? "First" : "Last"
end
println(firstlast(true))
```

You can instead cause the conditional to be evaluated during function compilation by using the Val trick:

```
firstlast(::Type{Val{true}}) = "First"
firstlast(::Type{Val{false}}) = "Last"
println(firstlast(Val{true}))
```

Any legal type parameter (Types, Symbols, Integers, floating-point numbers, tuples, etc.) can be passed via Val.

For consistency across Julia, the call site should always pass a Val type rather than creating an instance, i.e., use foo (Val{:bar}) rather than foo (Val{:bar}).

## 11.12 Nullable Types: Representing Missing Values

In many settings, you need to interact with a value of type T that may or may not exist. To handle these settings, Julia provides a parametric type called Nullable{T}, which can be thought of as a specialized container type that can contain either zero or one values. Nullable{T} provides a minimal interface designed to ensure that interactions with missing values are safe. At present, the interface consists of four possible interactions:

- Construct a Nullable (page 326) object.
- Check if a Nullable (page 326) object has a missing value.

- Access the value of a *Nullable* (page 326) object with a guarantee that a *NullException* (page 330) will be thrown if the object's value is missing.
- Access the value of a *Nullable* (page 326) object with a guarantee that a default value of type T will be returned if the object's value is missing.

### 11.12.1 Constructing Nullable objects

To construct an object representing a missing value of type T, use the Nullable {T} () function:

```
julia> x1 = Nullable{Int64}()
Nullable{Int64}()

julia> x2 = Nullable{Float64}()
Nullable{Float64}()

julia> x3 = Nullable{Vector{Int64}}()
Nullable{Array{Int64,1}}()
```

To construct an object representing a non-missing value of type T, use the Nullable (x::T) function:

```
julia> x1 = Nullable(1)
Nullable(1)

julia> x2 = Nullable(1.0)
Nullable(1.0)

julia> x3 = Nullable([1, 2, 3])
Nullable([1,2,3])
```

Note the core distinction between these two ways of constructing a *Nullable* (page 326) object: in one style, you provide a type, T, as a function parameter; in the other style, you provide a single value of type T as an argument.

#### 11.12.2 Checking if a Nullable object has a value

You can check if a Nullable (page 326) object has any value using isnull() (page 326):

```
julia> isnull(Nullable{Float64}())
true
julia> isnull(Nullable(0.0))
false
```

#### 11.12.3 Safely accessing the value of a Nullable object

You can safely access the value of a Nullable (page 326) object using get () (page 326):

```
julia> get(Nullable{Float64}())
ERROR: NullException()
  in get at nullable.jl:30

julia> get(Nullable(1.0))
1.0
```

104 Chapter 11. Types

If the value is not present, as it would be for  $Nullable\{Float64\}$ , a NullException (page 330) error will be thrown. The error-throwing nature of the get() (page 326) function ensures that any attempt to access a missing value immediately fails.

In cases for which a reasonable default value exists that could be used when a *Nullable* (page 326) object's value turns out to be missing, you can provide this default value as a second argument to *get* () (page 326):

```
julia> get(Nullable{Float64}(), 0)
0.0

julia> get(Nullable(1.0), 0)
1.0
```

Note that this default value will automatically be converted to the type of the Nullable (page 326) object that you attempt to access using the get() (page 326) function. For example, in the code shown above the value 0 would be automatically converted to a Float64 (page 380) value before being returned. The presence of default replacement values makes it easy to use the get() (page 326) function to write type-stable code that interacts with sources of potentially missing values.

106 Chapter 11. Types

### **Methods**

Recall from *Functions* (page 55) that a function is an object that maps a tuple of arguments to a return value, or throws an exception if no appropriate value can be returned. It is common for the same conceptual function or operation to be implemented quite differently for different types of arguments: adding two integers is very different from adding two floating-point numbers, both of which are distinct from adding an integer to a floating-point number. Despite their implementation differences, these operations all fall under the general concept of "addition". Accordingly, in Julia, these behaviors all belong to a single object: the + function.

To facilitate using many different implementations of the same concept smoothly, functions need not be defined all at once, but can rather be defined piecewise by providing specific behaviors for certain combinations of argument types and counts. A definition of one possible behavior for a function is called a *method*. Thus far, we have presented only examples of functions defined with a single method, applicable to all types of arguments. However, the signatures of method definitions can be annotated to indicate the types of arguments in addition to their number, and more than a single method definition may be provided. When a function is applied to a particular tuple of arguments, the most specific method applicable to those arguments is applied. Thus, the overall behavior of a function is a patchwork of the behaviors of its various method definitions. If the patchwork is well designed, even though the implementations of the methods may be quite different, the outward behavior of the function will appear seamless and consistent.

The choice of which method to execute when a function is applied is called *dispatch*. Julia allows the dispatch process to choose which of a function's methods to call based on the number of arguments given, and on the types of all of the function's arguments. This is different than traditional object-oriented languages, where dispatch occurs based only on the first argument, which often has a special argument syntax, and is sometimes implied rather than explicitly written as an argument. <sup>1</sup> Using all of a function's arguments to choose which method should be invoked, rather than just the first, is known as multiple dispatch. Multiple dispatch is particularly useful for mathematical code, where it makes little sense to artificially deem the operations to "belong" to one argument more than any of the others: does the addition operation in x + y belong to x any more than it does to y? The implementation of a mathematical operator generally depends on the types of all of its arguments. Even beyond mathematical operations, however, multiple dispatch ends up being a powerful and convenient paradigm for structuring and organizing programs.

## 12.1 Defining Methods

Until now, we have, in our examples, defined only functions with a single method having unconstrained argument types. Such functions behave just like they would in traditional dynamically typed languages. Nevertheless, we have used multiple dispatch and methods almost continually without being aware of it: all of Julia's standard functions and operators, like the aforementioned + function, have many methods defining their behavior over various possible combinations of argument type and count.

<sup>&</sup>lt;sup>1</sup> In C++ or Java, for example, in a method call like obj.meth(arg1, arg2), the object obj "receives" the method call and is implicitly passed to the method via the this keyword, rather than as an explicit method argument. When the current this object is the receiver of a method call, it can be omitted altogether, writing just meth(arg1, arg2), with this implied as the receiving object.

When defining a function, one can optionally constrain the types of parameters it is applicable to, using the :: type-assertion operator, introduced in the section on *Composite Types* (page 91):

```
julia> f(x::Float64, y::Float64) = 2x + y;
```

This function definition applies only to calls where x and y are both values of type Float 64:

```
julia> f(2.0, 3.0)
7.0
```

Applying it to any other types of arguments will result in a MethodError:

```
julia> f(2.0, 3)
ERROR: MethodError: `f` has no method matching f(::Float64, ::Int64)
Closest candidates are:
   f(::Float64, !Matched::Float64)

julia> f(Float32(2.0), 3.0)
ERROR: MethodError: `f` has no method matching f(::Float32, ::Float64)
Closest candidates are:
   f(!Matched::Float64, ::Float64)

julia> f(2.0, "3.0")
ERROR: MethodError: `f` has no method matching f(::Float64, ::ASCIIString)
Closest candidates are:
   f(::Float64, !Matched::Float64)

julia> f("2.0", "3.0")
ERROR: MethodError: `f` has no method matching f(::ASCIIString, ::ASCIIString)
```

As you can see, the arguments must be precisely of type Float64. Other numeric types, such as integers or 32-bit floating-point values, are not automatically converted to 64-bit floating-point, nor are strings parsed as numbers. Because Float64 is a concrete type and concrete types cannot be subclassed in Julia, such a definition can only be applied to arguments that are exactly of type Float64. It may often be useful, however, to write more general methods where the declared parameter types are abstract:

```
julia> f(x::Number, y::Number) = 2x - y;
julia> f(2.0, 3)
1.0
```

This method definition applies to any pair of arguments that are instances of Number. They need not be of the same type, so long as they are each numeric values. The problem of handling disparate numeric types is delegated to the arithmetic operations in the expression 2x - y.

To define a function with multiple methods, one simply defines the function multiple times, with different numbers and types of arguments. The first method definition for a function creates the function object, and subsequent method definitions add new methods to the existing function object. The most specific method definition matching the number and types of the arguments will be executed when the function is applied. Thus, the two method definitions above, taken together, define the behavior for f over all pairs of instances of the abstract type Number — but with a different behavior specific to pairs of Float64 values. If one of the arguments is a 64-bit float but the other one is not, then the f (Float64, Float64) method cannot be called and the more general f (Number, Number) method must be used:

```
julia> f(2.0, 3.0)
7.0

julia> f(2, 3.0)
1.0
```

```
julia> f(2.0, 3)
1.0

julia> f(2, 3)
1
```

The 2x + y definition is only used in the first case, while the 2x - y definition is used in the others. No automatic casting or conversion of function arguments is ever performed: all conversion in Julia is non-magical and completely explicit. *Conversion and Promotion* (page 127), however, shows how clever application of sufficiently advanced technology can be indistinguishable from magic. *[Clarke61]* (page 493)

For non-numeric values, and for fewer or more than two arguments, the function f remains undefined, and applying it will still result in a MethodError:

```
julia> f("foo", 3)
ERROR: MethodError: `f` has no method matching f(::ASCIIString, ::Int64)
Closest candidates are:
  f(!Matched::Number, ::Number)

julia> f()
ERROR: MethodError: `f` has no method matching f()
```

You can easily see which methods exist for a function by entering the function object itself in an interactive session:

```
julia> f
f (generic function with 2 methods)
```

This output tells us that f is a function object with two methods. To find out what the signatures of those methods are, use the methods () function:

```
julia> methods(f)
# 2 methods for generic function "f":
f(x::Float64, y::Float64) at none:1
f(x::Number, y::Number) at none:1
```

which shows that f has two methods, one taking two Float64 arguments and one taking arguments of type Number. It also indicates the file and line number where the methods were defined: because these methods were defined at the REPL, we get the apparent line number none: 1.

In the absence of a type declaration with ::, the type of a method parameter is Any by default, meaning that it is unconstrained since all values in Julia are instances of the abstract type Any. Thus, we can define a catch-all method for f like so:

```
julia> f(x,y) = println("Whoa there, Nelly.");
julia> f("foo", 1)
Whoa there, Nelly.
```

This catch-all is less specific than any other possible method definition for a pair of parameter values, so it is only be called on pairs of arguments to which no other method definition applies.

Although it seems a simple concept, multiple dispatch on the types of values is perhaps the single most powerful and central feature of the Julia language. Core operations typically have dozens of methods:

```
julia> methods(+)
# 139 methods for generic function "+":
+(x::Bool) at bool.jl:33
+(x::Bool,y::Bool) at bool.jl:36
+(y::AbstractFloat,x::Bool) at bool.jl:46
```

```
+(x::Int64,y::Int64) at int.jl:14
+(x::Int8,y::Int8) at int.jl:14
+(x::UInt8,y::UInt8) at int.jl:14
+(x::Int16,y::Int16) at int.jl:14
+(x::UInt16,y::UInt16) at int.jl:14
+(x::Int32,y::Int32) at int.jl:14
+(x::UInt32,y::UInt32) at int.jl:14
+(x::UInt64,y::UInt64) at int.jl:14
+(x::Int128,y::Int128) at int.jl:14
+(x::UInt128,y::UInt128) at int.jl:14
+(x::Float32, y::Float32) at float.jl:192
+(x::Float64, y::Float64) at float.j1:193
+(z::Complex{T<:Real},w::Complex{T<:Real}) at complex.jl:96
+(x::Real,z::Complex{T<:Real}) at complex.jl:106
+(z::Complex{T<:Real},x::Real) at complex.jl:107
+(x::Rational{T<:Integer},y::Rational{T<:Integer}) at rational.jl:167
+(a::Float16,b::Float16) at float16.jl:136
+(x::Base.GMP.BigInt,y::Base.GMP.BigInt) at gmp.jl:243
+ (a::Base.GMP.BigInt,b::Base.GMP.BigInt,c::Base.GMP.BigInt) at gmp.jl:266
+ (a::Base.GMP.BigInt,b::Base.GMP.BigInt,c::Base.GMP.BigInt,d::Base.GMP.BigInt) at gmp.j1:272
+ (a::Base.GMP.BigInt,b::Base.GMP.BigInt,c::Base.GMP.BigInt,d::Base.GMP.BigInt,e::Base.GMP.BigInt) at
+(x::Base.GMP.BigInt,c::Union{UInt32,UInt16,UInt8,UInt64}) at gmp.jl:291
+(c::Union{UInt32,UInt16,UInt8,UInt64},x::Base.GMP.BigInt) at gmp.jl:295
+(x::Base.GMP.BigInt,c::Union{Int16,Int32,Int8,Int64}) at gmp.jl:307
+(c::Union{Int16,Int32,Int8,Int64},x::Base.GMP.BigInt) at gmp.jl:308
+(x::Base.MPFR.BigFloat,y::Base.MPFR.BigFloat) at mpfr.jl:206
+(x::Base.MPFR.BigFloat,c::Union{UInt32,UInt16,UInt8,UInt64}) at mpfr.jl:213
+(c::Union{UInt32,UInt16,UInt8,UInt64},x::Base.MPFR.BigFloat) at mpfr.jl:217
+(x::Base.MPFR.BigFloat,c::Union{Int16,Int32,Int8,Int64}) at mpfr.jl:221
+(c::Union{Int16, Int32, Int8, Int64}, x::Base.MPFR.BigFloat) at mpfr.jl:225
+(x::Base.MPFR.BigFloat,c::Union{Float16,Float64,Float32}) at mpfr.jl:229
+(c::Union{Float16, Float64, Float32}, x::Base.MPFR.BigFloat) at mpfr.jl:233
+(x::Base.MPFR.BigFloat,c::Base.GMP.BigInt) at mpfr.jl:237
+(c::Base.GMP.BigInt,x::Base.MPFR.BigFloat) at mpfr.jl:241
+ (a::Base.MPFR.BigFloat,b::Base.MPFR.BigFloat,c::Base.MPFR.BigFloat) at mpfr.jl:318
+(a::Base.MPFR.BigFloat,b::Base.MPFR.BigFloat,c::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat) at mpfr.j.
+ (a::Base.MPFR.BigFloat,b::Base.MPFR.BigFloat,c::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFloat,d::Base.MPFR.BigFlo
+(x::Irrational{sym},y::Irrational{sym}) at constants.jl:71
+{T<:Number}(x::T<:Number,y::T<:Number) at promotion.jl:205
+{T<:AbstractFloat}(x::Bool,y::T<:AbstractFloat) at bool.jl:43
+(x::Number, y::Number) at promotion.jl:167
+(x::Integer,y::Ptr{T}) at pointer.jl:70
+(x::Bool, A::AbstractArray(Bool, N)) at array.jl:829
+(x::Integer,y::Char) at char.jl:41
+(x::Number) at operators.jl:72
+(r1::OrdinalRange{T,S},r2::OrdinalRange{T,S}) at operators.jl:325
+{T<:AbstractFloat}(r1::FloatRange{T<:AbstractFloat},r2::FloatRange{T<:AbstractFloat}) at operators.
+(r1::FloatRange{T<:AbstractFloat}, r2::FloatRange{T<:AbstractFloat}) at operators.j1:34$
+(r1::FloatRange{T<:AbstractFloat},r2::OrdinalRange{T,S}) at operators.j1:349
+(r1::OrdinalRange{T,S},r2::FloatRange{T<:AbstractFloat}) at operators.j1:350
+(x::Ptr{T},y::Integer) at pointer.jl:68
+{S,T}(A::Range{S},B::Range{T}) at array.jl:773
+{S,T}(A::Range{S},B::AbstractArray{T,N}) at array.jl:791
+(A::AbstractArray{Bool, N}, x::Bool) at array.jl:828
+(A::BitArray{N},B::BitArray{N}) at bitarray.jl:926
+ (A::Union{DenseArray{Bool, N}, SubArray{Bool, N, A<:DenseArray{T, N}, I<:Tuple{Vararg{Union{¢olon,Range{In
+(A::Base.LinAlg.SymTridiagonal{T},B::Base.LinAlg.SymTridiagonal{T}) at linalg/tridiag. 1:59
+(A::Base.LinAlg.Tridiagonal{T},B::Base.LinAlg.Tridiagonal{T}) at linalg/tridiag.jl:254
```

110 Chapter 12. Methods

```
+(A::Base.LinAlg.Tridiagonal{T},B::Base.LinAlg.SymTridiagonal{T}) at linalg/special.jl:113
+(A::Base.LinAlg.SymTridiagonal{T},B::Base.LinAlg.Tridiagonal{T}) at linalg/special.jl:12
+(A::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UpperTriangular
+(A::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.LowerTriangular
+(A::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,3}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,3}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,3}},R::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,3}},R::Base.
+(A::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,2}},B::Base.LinAlg.UnitLowerTri
+(A::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}}},B::Base.LinAlg.UpperTriangular{T,S<:AbstractArray{T,2}}}
+(A::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}}},B::Base.LinAlg.LowerTriangular{T,S<:AbstractArray{T,2}}}
+(A::Base.LinAlg.UnitUpperTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitUpperTriangular{T,S
+(A::Base.LinAlg.UnitLowerTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.UnitLowerTriangular{T,S-
+(A::Base.LinAlg.AbstractTriangular{T,S<:AbstractArray{T,2}},B::Base.LinAlg.AbstractTriangular{T,S<:A
+ (Da::Base.LinAlg.Diagonal{T}, Db::Base.LinAlg.Diagonal{T}) at linalg/diagonal.jl:50
+(A::Base.LinAlq.Bidiagonal{T},B::Base.LinAlq.Bidiagonal{T}) at linalq/bidiag.jl:111
+{T}(B::BitArray{2},J::Base.LinAlg.UniformScaling{T}) at linalg/uniformscaling.jl:28
+(A::Base.LinAlg.Diagonal{T},B::Base.LinAlg.Bidiagonal{T}) at linalg/special.jl:103
+(A::Base.LinAlg.Bidiagonal{T},B::Base.LinAlg.Diagonal{T}) at linalg/special.jl:104
+(A::Base.LinAlg.Diagonal{T},B::Base.LinAlg.Tridiagonal{T}) at linalg/special.jl:103
+(A::Base.LinAlg.Tridiagonal{T},B::Base.LinAlg.Diagonal{T}) at linalg/special.jl:104
+(A::Base.LinAlq.Diagonal{T},B::Array{T,2}) at linalq/special.jl:103
+(A::Array{T,2},B::Base.LinAlg.Diagonal{T}) at linalg/special.jl:104
+(A::Base.LinAlg.Bidiagonal{T},B::Base.LinAlg.Tridiagonal{T}) at linalg/special.jl:103
+(A::Base.LinAlg.Tridiagonal{T},B::Base.LinAlg.Bidiagonal{T}) at linalg/special.jl:104
+(A::Base.LinAlg.Bidiagonal{T},B::Array{T,2}) at linalg/special.jl:103
+(A::Array{T,2},B::Base.LinAlg.Bidiagonal{T}) at linalg/special.jl:104
+(A::Base.LinAlg.Tridiagonal{T},B::Array{T,2}) at linalg/special.jl:103
+(A::Array{T,2},B::Base.LinAlg.Tridiagonal{T}) at linalg/special.jl:104
+(A::Base.LinAlg.SymTridiagonal{T},B::Array{T,2}) at linalg/special.jl:112
+(A::Array{T,2},B::Base.LinAlg.SymTridiagonal{T}) at linalg/special.jl:113
+(A::Base.LinAlg.Diagonal{T},B::Base.LinAlg.SymTridiagonal{T}) at linalg/special.jl:121
+(A::Base.LinAlg.SymTridiagonal{T},B::Base.LinAlg.Diagonal{T}) at linalg/special.jl:122
+(A::Base.LinAlg.Bidiagonal{T},B::Base.LinAlg.SymTridiagonal{T}) at linalg/special.jl:121
+(A::Base.LinAlg.SymTridiagonal{T},B::Base.LinAlg.Bidiagonal{T}) at linalg/special.jl:122
+{Tv1,Ti1,Tv2,Ti2}(A_1::Base.SparseMatrix.SparseMatrixCSC{Tv1,Ti1},A_2::Base.SparseMatrix.SparseMatr
+(A::Base.SparseMatrix.SparseMatrixCSC{Tv,Ti<:Integer},B::Array{T,N}) at sparse/sparsematrix.jl:885
+(A::Array{T,N},B::Base.SparseMatrix.SparseMatrixCSC{Tv,Ti<:Integer}) at sparse/sparsematrix.jl:887
+{P<:Base.Dates.Period}(Y::Union{SubArray{P<:Base.Dates.Period,N,A<:DenseArray{T,N},I<:Tuple{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{Vararg{
+{T<:Base.Dates.TimeType}(r::Range{T<:Base.Dates.TimeType},x::Base.Dates.Period) at dates/ranges.jl:
+{T<:Number}(x::AbstractArray{T<:Number,N}) at abstractarray.jl:442
+{S,T}(A::AbstractArray{S,N},B::Range{T}) at array.jl:782
+{S,T}(A::AbstractArray{S,N},B::AbstractArray{T,N}) at array.jl:800
+(A::AbstractArray{T,N},x::Number) at array.jl:832
+(x::Number, A::AbstractArray(T, N)) at array.jl:833
+(x::Char,y::Integer) at char.jl:40
+{N}(index1::Base.IteratorsMD.CartesianIndex{N},index2::Base.IteratorsMD.CartesianIndex{N}) at multic
+(J1::Base.LinAlg.UniformScaling{T<:Number}, J2::Base.LinAlg.UniformScaling{T<:Number}) at linalg/uni
+(J::Base.LinAlq.UniformScaling{T<:Number},B::BitArray{2}) at linalg/uniformscaling.jl:29
+(J::Base.LinAlg.UniformScaling{T<:Number}, A::AbstractArray{T,2}) at linalg/uniformscaling.jl:30
+(J::Base.LinAlg.UniformScaling{T<:Number},x::Number) at linalg/uniformscaling.jl:31
+(x::Number, J::Base.LinAlg.UniformScaling{T<:Number}) at linalg/uniformscaling.jl:32
+{TA,TJ}(A::AbstractArray{TA,2},J::Base.LinAlg.UniformScaling{TJ}) at linalg/uniformscaling.jl:35
+{T}(a::Base.Pkg.Resolve.VersionWeights.HierarchicalValue{T},b::Base.Pkg.Resolve.VersionWeights.Hier
+ (a::Base.Pkg.Resolve.VersionWeights.VWPreBuildItem,b::Base.Pkg.Resolve.VersionWeights.VWPreBuildItem
+ (a::Base.Pkg.Resolve.VersionWeights.VWPreBuild,b::Base.Pkg.Resolve.VersionWeights.VWPreBuild) at pkg
+ (a::Base.Pkg.Resolve.VersionWeights.VersionWeight,b::Base.Pkg.Resolve.VersionWeights.VersionWeight)
+ (a::Base.Pkg.Resolve.MaxSum.FieldValues.FieldValue,b::Base.Pkg.Resolve.MaxSum.FieldValues.FieldValue
+{P<:Base.Dates.Period}(x::P<:Base.Dates.Period,y::P<:Base.Dates.Period) at dates/periods.jl:43
+{P<:Base.Dates.Period}(x::P<:Base.Dates.Period,Y::Union{SubArray{P<:Base.Dates.Period,№,A<:DenseArra
+(x::Base.Dates.Period,y::Base.Dates.Period) at dates/periods.jl:196
```

```
+(x::Base.Dates.CompoundPeriod,y::Base.Dates.Period) at dates/periods.jl:197
+(y::Base.Dates.Period,x::Base.Dates.CompoundPeriod) at dates/periods.jl:198
+(x::Base.Dates.CompoundPeriod,y::Base.Dates.CompoundPeriod) at dates/periods.jl:199
+(dt::Base.Dates.DateTime,y::Base.Dates.Year) at dates/arithmetic.jl:13
+(dt::Base.Dates.Date,y::Base.Dates.Year) at dates/arithmetic.jl:17
+(dt::Base.Dates.DateTime,z::Base.Dates.Month) at dates/arithmetic.jl:37
+(dt::Base.Dates.Date,z::Base.Dates.Month) at dates/arithmetic.j1:43
+(x::Base.Dates.Date,y::Base.Dates.Week) at dates/arithmetic.jl:60
+(x::Base.Dates.Date,y::Base.Dates.Day) at dates/arithmetic.jl:62
+(x::Base.Dates.DateTime,y::Base.Dates.Period) at dates/arithmetic.jl:64
+(a::Base.Dates.TimeType,b::Base.Dates.Period,c::Base.Dates.Period) at dates/periods.jl 210
+ (a::Base.Dates.TimeType,b::Base.Dates.Period,c::Base.Dates.Period,d::Base.Dates.Period...) at dates
+(x::Base.Dates.TimeType,y::Base.Dates.CompoundPeriod) at dates/periods.jl:216
+(x::Base.Dates.CompoundPeriod,y::Base.Dates.TimeType) at dates/periods.jl:221
+(x::Base.Dates.Instant) at dates/arithmetic.jl:4
+(x::Base.Dates.TimeType) at dates/arithmetic.jl:8
+(y::Base.Dates.Period,x::Base.Dates.TimeType) at dates/arithmetic.jl:66
+{T<:Base.Dates.TimeType}(x::Base.Dates.Period,r::Range{T<:Base.Dates.TimeType}) at dates/ranges.jl:
+(a,b,c) at operators.jl:83
+(a,b,c,xs...) at operators.jl:84
```

Multiple dispatch together with the flexible parametric type system give Julia its ability to abstractly express high-level algorithms decoupled from implementation details, yet generate efficient, specialized code to handle each case at run time.

## 12.2 Method Ambiguities

It is possible to define a set of function methods such that there is no unique most specific method applicable to some combinations of arguments:

```
julia> g(x::Float64, y) = 2x + y;

julia> g(x, y::Float64) = x + 2y;

WARNING: New definition
      g(Any, Float64) at none:1
is ambiguous with:
      g(Float64, Any) at none:1.

To fix, define
      g(Float64, Float64)
before the new definition.

julia> g(2.0, 3)
7.0

julia> g(2, 3.0)
8.0

julia> g(2.0, 3.0)
7.0
```

Here the call g(2.0, 3.0) could be handled by either the g(Float64, Any) or the g(Any, Float64) method, and neither is more specific than the other. In such cases, Julia warns you about this ambiguity, but allows you to proceed, arbitrarily picking a method. You should avoid method ambiguities by specifying an appropriate method for the intersection case:

```
julia> g(x::Float64, y::Float64) = 2x + 2y;
julia> g(x::Float64, y) = 2x + y;
julia> g(x, y::Float64) = x + 2y;
julia> g(2.0, 3)
7.0

julia> g(2, 3.0)
8.0

julia> g(2.0, 3.0)
10.0
```

To suppress Julia's warning, the disambiguating method must be defined first, since otherwise the ambiguity exists, if transiently, until the more specific method is defined.

### 12.3 Parametric Methods

Method definitions can optionally have type parameters immediately after the method name and before the parameter tuple:

```
julia> same_type{T}(x::T, y::T) = true;
julia> same_type(x,y) = false;
```

The first method applies whenever both arguments are of the same concrete type, regardless of what type that is, while the second method acts as a catch-all, covering all other cases. Thus, overall, this defines a boolean function that checks whether its two arguments are of the same type:

```
julia> same_type(1, 2)
true

julia> same_type(1, 2.0)
false

julia> same_type(1.0, 2.0)
true

julia> same_type("foo", 2.0)
false

julia> same_type("foo", "bar")
true

julia> same_type("foo", "bar")
true
```

This kind of definition of function behavior by dispatch is quite common — idiomatic, even — in Julia. Method type parameters are not restricted to being used as the types of parameters: they can be used anywhere a value would be in the signature of the function or body of the function. Here's an example where the method type parameter T is used as the type parameter to the parametric type  $Vector\{T\}$  in the method signature:

```
julia> myappend{T} (v::Vector{T}, x::T) = [v..., x]
myappend (generic function with 1 method)
```

```
julia > myappend([1,2,3],4)
4-element Array{Int64,1}:
2
3
4
julia> myappend([1,2,3],2.5)
ERROR: MethodError: `myappend` has no method matching myappend(::Array{Int64,1}, ::Float64)
Closest candidates are:
myappend{T}(::Array{T,1}, !Matched::T)
julia> myappend([1.0,2.0,3.0],4.0)
4-element Array{Float64,1}:
1.0
2.0
3.0
4.0
julia > myappend([1.0,2.0,3.0],4)
ERROR: MethodError: `myappend` has no method matching myappend(::Array{Float64,1}, ::Int64)
Closest candidates are:
 myappend{T}(::Array{T,1}, !Matched::T)
```

As you can see, the type of the appended element must match the element type of the vector it is appended to, or else a MethodError is raised. In the following example, the method type parameter T is used as the return value:

```
julia> mytypeof{T}(x::T) = T
mytypeof (generic function with 1 method)

julia> mytypeof(1)
Int64

julia> mytypeof(1.0)
Float64
```

Just as you can put subtype constraints on type parameters in type declarations (see *Parametric Types* (page 94)), you can also constrain type parameters of methods:

```
same_type_numeric(T<:Number) (x::T, y::T) = true
same_type_numeric(x::Number, y::Number) = false

julia> same_type_numeric(1, 2)
true

julia> same_type_numeric(1, 2.0)
false

julia> same_type_numeric(1.0, 2.0)
true

julia> same_type_numeric("foo", 2.0)
no method same_type_numeric(ASCIIString,Float64)

julia> same_type_numeric("foo", "bar")
no method same_type_numeric(ASCIIString,ASCIIString)

julia> same_type_numeric(Int32(1), Int64(2))
false
```

The same\_type\_numeric function behaves much like the same\_type function defined above, but is only defined for pairs of numbers.

### 12.4 Note on Optional and keyword Arguments

As mentioned briefly in *Functions* (page 55), optional arguments are implemented as syntax for multiple method definitions. For example, this definition:

```
f(a=1,b=2) = a+2b
```

translates to the following three methods:

```
f(a,b) = a+2b

f(a) = f(a,2)

f() = f(1,2)
```

This means that calling f() is equivalent to calling f(1,2). In this case the result is 5, because f(1,2) invokes the first method of f above. However, this need not always be the case. If you define a fourth method that is more specialized for integers:

```
f(a::Int,b::Int) = a-2b
```

then the result of both f() and f(1,2) is -3. In other words, optional arguments are tied to a function, not to any specific method of that function. It depends on the types of the optional arguments which method is invoked. When optional arguments are defined in terms of a global variable, the type of the optional argument may even change at run-time.

Keyword arguments behave quite differently from ordinary positional arguments. In particular, they do not participate in method dispatch. Methods are dispatched based only on positional arguments, with keyword arguments processed after the matching method is identified.

## 12.5 Call overloading and function-like objects

For any arbitrary Julia object x other than Function objects (defined via function syntax), x(args...) is equivalent to call (x, args...), where call () is a generic function in the Julia Base module. By adding new methods to call, you can add a function-call syntax to arbitrary Julia types. (Such "callable" objects are sometimes called "functors.")

For example, if you want to make x (arg) equivalent to x \* arg for x:: Number, you can define:

```
Base.call(x::Number, arg) = x * arg
```

at which point you can do:

```
\begin{bmatrix} x = 7 \\ x (10) \end{bmatrix}
```

to get 70.

call overloading is also used extensively for type constructors in Julia, discussed *later in the manual* (page 124).

# 12.6 Empty generic functions

Occasionally it is useful to introduce a generic function without yet adding methods. This can be used to separate interface definitions from implementations. It might also be done for the purpose of documentation or code readability. The syntax for this is an empty function block without a tuple of arguments:

 $\begin{array}{ll} \textbf{function} & \texttt{emptyfunc} \\ \textbf{end} & \end{array}$ 

### **Constructors**

Constructors <sup>1</sup> are functions that create new objects — specifically, instances of *Composite Types* (page 91). In Julia, type objects also serve as constructor functions: they create new instances of themselves when applied to an argument tuple as a function. This much was already mentioned briefly when composite types were introduced. For example:

```
type Foo
  bar
  baz
end

julia> foo = Foo(1,2)
Foo(1,2)

julia> foo.bar
1

julia> foo.baz
2
```

For many types, forming new objects by binding their field values together is all that is ever needed to create instances. There are, however, cases where more functionality is required when creating composite objects. Sometimes invariants must be enforced, either by checking arguments or by transforming them. Recursive data structures, especially those that may be self-referential, often cannot be constructed cleanly without first being created in an incomplete state and then altered programmatically to be made whole, as a separate step from object creation. Sometimes, it's just convenient to be able to construct objects with fewer or different types of parameters than they have fields. Julia's system for object construction addresses all of these cases and more.

### 13.1 Outer Constructor Methods

A constructor is just like any other function in Julia in that its overall behavior is defined by the combined behavior of its methods. Accordingly, you can add functionality to a constructor by simply defining new methods. For example, let's say you want to add a constructor method for Foo objects that takes only one argument and uses the given value for both the bar and baz fields. This is simple:

```
Foo(x) = Foo(x, x)
```

<sup>&</sup>lt;sup>1</sup> Nomenclature: while the term "constructor" generally refers to the entire function which constructs objects of a type, it is common to abuse terminology slightly and refer to specific constructor methods as "constructors". In such situations, it is generally clear from context that the term is used to mean "constructor method" rather than "constructor function", especially as it is often used in the sense of singling out a particular method of the constructor from all of the others.

```
julia> Foo(1)
Foo(1,1)
```

You could also add a zero-argument Foo constructor method that supplies default values for both of the bar and baz fields:

```
Foo() = Foo(0)

julia> Foo()
Foo(0,0)
```

Here the zero-argument constructor method calls the single-argument constructor method, which in turn calls the automatically provided two-argument constructor method. For reasons that will become clear very shortly, additional constructor methods declared as normal methods like this are called *outer* constructor methods. Outer constructor methods can only ever create a new instance by calling another constructor method, such as the automatically provided default ones.

### 13.2 Inner Constructor Methods

While outer constructor methods succeed in addressing the problem of providing additional convenience methods for constructing objects, they fail to address the other two use cases mentioned in the introduction of this chapter: enforcing invariants, and allowing construction of self-referential objects. For these problems, one needs *inner* constructor methods. An inner constructor method is much like an outer constructor method, with two differences:

- 1. It is declared inside the block of a type declaration, rather than outside of it like normal methods.
- 2. It has access to a special locally existent function called new that creates objects of the block's type.

For example, suppose one wants to declare a type that holds a pair of real numbers, subject to the constraint that the first number is not greater than the second one. One could declare it like this:

```
type OrderedPair
  x::Real
  y::Real
  OrderedPair(x,y) = x > y ? error("out of order") : new(x,y)
end
```

Now OrderedPair objects can only be constructed such that x <= y:

```
julia> OrderedPair(1,2)
OrderedPair(1,2)

julia> OrderedPair(2,1)
ERROR: out of order
  in call at none:5
```

You can still reach in and directly change the field values to violate this invariant, but messing around with an object's internals uninvited is considered poor form. You (or someone else) can also provide additional outer constructor methods at any later point, but once a type is declared, there is no way to add more inner constructor methods. Since outer constructor methods can only create objects by calling other constructor methods, ultimately, some inner constructor must be called to create an object. This guarantees that all objects of the declared type must come into existence by a call to one of the inner constructor methods provided with the type, thereby giving some degree of enforcement of a type's invariants.

Of course, if the type is declared as immutable, then its constructor-provided invariants are fully enforced. This is an important consideration when deciding whether a type should be immutable.

If any inner constructor method is defined, no default constructor method is provided: it is presumed that you have supplied yourself with all the inner constructors you need. The default constructor is equivalent to writing your own inner constructor method that takes all of the object's fields as parameters (constrained to be of the correct type, if the corresponding field has a type), and passes them to new, returning the resulting object:

```
type Foo
bar
baz

Foo(bar,baz) = new(bar,baz)
end
```

This declaration has the same effect as the earlier definition of the Foo type without an explicit inner constructor method. The following two types are equivalent — one with a default constructor, the other with an explicit constructor:

```
type T1
    x::Int64
end

type T2
    x::Int64
    T2(x) = new(x)
end

julia> T1(1)
T1(1)

julia> T2(1)

julia> T2(1)

julia> T1(1.0)
T1(1)

julia> T2(1.0)
T2(1)
```

It is considered good form to provide as few inner constructor methods as possible: only those taking all arguments explicitly and enforcing essential error checking and transformation. Additional convenience constructor methods, supplying default values or auxiliary transformations, should be provided as outer constructors that call the inner constructors to do the heavy lifting. This separation is typically quite natural.

## 13.3 Incomplete Initialization

The final problem which has still not been addressed is construction of self-referential objects, or more generally, recursive data structures. Since the fundamental difficulty may not be immediately obvious, let us briefly explain it. Consider the following recursive type declaration:

```
type SelfReferential
  obj::SelfReferential
end
```

This type may appear innocuous enough, until one considers how to construct an instance of it. If a is an instance of SelfReferential, then a second instance can be created by the call:

```
b = SelfReferential(a)
```

But how does one construct the first instance when no instance exists to provide as a valid value for its obj field? The only solution is to allow creating an incompletely initialized instance of SelfReferential with an unassigned obj field, and using that incomplete instance as a valid value for the obj field of another instance, such as, for example, itself.

To allow for the creation of incompletely initialized objects, Julia allows the new function to be called with fewer than the number of fields that the type has, returning an object with the unspecified fields uninitialized. The inner constructor method can then use the incomplete object, finishing its initialization before returning it. Here, for example, we take another crack at defining the SelfReferential type, with a zero-argument inner constructor returning instances having obj fields pointing to themselves:

```
type SelfReferential
  obj::SelfReferential

SelfReferential() = (x = new(); x.obj = x)
end
```

We can verify that this constructor works and constructs objects that are, in fact, self-referential:

```
julia> x = SelfReferential();
julia> is(x, x)
true

julia> is(x, x.obj)
true

julia> is(x, x.obj.obj)
true
```

Although it is generally a good idea to return a fully initialized object from an inner constructor, incompletely initialized objects can be returned:

While you are allowed to create objects with uninitialized fields, any access to an uninitialized reference is an immediate error:

```
julia> z.xx
ERROR: UndefRefError: access to undefined reference
```

This avoids the need to continually check for null values. However, not all object fields are references. Julia considers some types to be "plain data", meaning all of their data is self-contained and does not reference other objects. The plain data types consist of bits types (e.g. Int) and immutable structs of other plain data types. The initial contents of a plain data type is undefined:

Arrays of plain data types exhibit the same behavior.

You can pass incomplete objects to other functions from inner constructors to delegate their completion:

```
type Lazy
    xx

Lazy(v) = complete_me(new(), v)
end
```

As with incomplete objects returned from constructors, if complete\_me or any of its callees try to access the xx field of the Lazy object before it has been initialized, an error will be thrown immediately.

### 13.4 Parametric Constructors

Parametric types add a few wrinkles to the constructor story. Recall from *Parametric Types* (page 94) that, by default, instances of parametric composite types can be constructed either with explicitly given type parameters or with type parameters implied by the types of the arguments given to the constructor. Here are some examples:

```
julia> type Point {T<:Real}</pre>
         x::T
         y::T
       end
## implicit T ##
julia> Point(1,2)
Point { Int 64 } (1, 2)
julia > Point (1.0, 2.5)
Point (Float 64) (1.0, 2.5)
julia > Point(1, 2.5)
ERROR: MethodError: `convert` has no method matching convert(::Type{Point{T<:Real}}, ::Int64, ::Float
This may have arisen from a call to the constructor Point\{T < : Real\} (...),
since type constructors fall back to convert methods.
Closest candidates are:
  Point{T<:Real}(::T<:Real, !Matched::T<:Real)</pre>
  call{T}(::Type{T}, ::Any)
  convert{T}(::Type{T}, !Matched::T)
in call at essentials.jl:57
## explicit T ##
julia> Point{Int64} (1,2)
Point { Int 64 } (1,2)
julia > Point { Int 64 } (1.0, 2.5)
ERROR: InexactError()
in call at none:2
julia > Point {Float 64} (1.0, 2.5)
Point {Float 64} (1.0, 2.5)
julia> Point {Float64} (1,2)
Point {Float 64} (1.0, 2.0)
```

As you can see, for constructor calls with explicit type parameters, the arguments are converted to the implied field types: Point {Int64} (1,2) works, but Point {Int64} (1.0,2.5) raises an InexactError when convert-

ing 2.5 to Int64. When the type is implied by the arguments to the constructor call, as in Point (1, 2), then the types of the arguments must agree — otherwise the T cannot be determined — but any pair of real arguments with matching type may be given to the generic Point constructor.

What's really going on here is that Point, Point {Float64} and Point {Int64} are all different constructor functions. In fact, Point {T} is a distinct constructor function for each type T. Without any explicitly provided inner constructors, the declaration of the composite type Point {T<:Real} automatically provides an inner constructor, Point {T}, for each possible type T<:Real, that behaves just like non-parametric default inner constructors do. It also provides a single general outer Point constructor that takes pairs of real arguments, which must be of the same type. This automatic provision of constructors is equivalent to the following explicit declaration:

```
type Point{T<:Real}
    x::T
    y::T
    Point(x,y) = new(x,y)
end
Point{T<:Real}(x::T, y::T) = Point{T}(x,y)</pre>
```

Some features of parametric constructor definitions at work here deserve comment. First, inner constructor declarations always define methods of Point {T} rather than methods of the general Point constructor function. Since Point is not a concrete type, it makes no sense for it to even have inner constructor methods at all. Thus, the inner method declaration Point (x,y) = new(x,y) provides an inner constructor method for each value of T. It is this method declaration that defines the behavior of constructor calls with explicit type parameters like Point {Int 64} (1,2) and Point {Float 64} (1.0,2.0). The outer constructor declaration, on the other hand, defines a method for the general Point constructor which only applies to pairs of values of the same real type. This declaration makes constructor calls without explicit type parameters, like Point (1,2) and Point (1.0,2.5), work. Since the method declaration restricts the arguments to being of the same type, calls like Point (1,2.5), with arguments of different types, result in "no method" errors.

Suppose we wanted to make the constructor call Point (1, 2.5) work by "promoting" the integer value 1 to the floating-point value 1.0. The simplest way to achieve this is to define the following additional outer constructor method:

```
julia> Point(x::Int64, y::Float64) = Point(convert(Float64,x),y);
```

This method uses the <code>convert()</code> (page 321) function to explicitly convert x to <code>Float64</code> (page 380) and then delegates construction to the general constructor for the case where both arguments are <code>Float64</code> (page 380). With this method definition what was previously a <code>MethodError</code> (page 330) now successfully creates a point of type <code>Point{Float64}</code>:

```
julia> Point(1,2.5)
Point{Float64}(1.0,2.5)

julia> typeof(ans)
Point{Float64}
```

However, other similar calls still don't work:

```
julia> Point(1.5,2)
ERROR: MethodError: `convert` has no method matching convert(::Type{Point{T<:Real}}, ::Float64, ::Int
This may have arisen from a call to the constructor Point{T<:Real}(...),
since type constructors fall back to convert methods.
Closest candidates are:
   Point{T<:Real}(::T<:Real, !Matched::T<:Real)
   call{T}(::Type{T}, ::Any)
   convert{T}(::Type{T}, !Matched::T)</pre>
```

```
in call at essentials.jl:57
```

For a much more general way of making all such calls work sensibly, see *Conversion and Promotion* (page 127). At the risk of spoiling the suspense, we can reveal here that all it takes is the following outer method definition to make all calls to the general Point constructor work as one would expect:

```
julia> Point(x::Real, y::Real) = Point(promote(x,y)...);
```

The promote function converts all its arguments to a common type — in this case Float 64 (page 380). With this method definition, the Point constructor promotes its arguments the same way that numeric operators like + (page 353) do, and works for all kinds of real numbers:

```
julia> Point(1.5,2)
Point(Float64)(1.5,2.0)

julia> Point(1,1//2)
Point(Rational(Int64))(1//1,1//2)

julia> Point(1.0,1//2)
Point(Float64)(1.0,0.5)
```

Thus, while the implicit type parameter constructors provided by default in Julia are fairly strict, it is possible to make them behave in a more relaxed but sensible manner quite easily. Moreover, since constructors can leverage all of the power of the type system, methods, and multiple dispatch, defining sophisticated behavior is typically quite simple.

## 13.5 Case Study: Rational

Perhaps the best way to tie all these pieces together is to present a real world example of a parametric composite type and its constructor methods. To that end, here is beginning of rational.jl, which implements Julia's *Rational Numbers* (page 37):

```
immutable Rational{T<:Integer} <: Real</pre>
    num··T
    den::T
    function Rational(num::T, den::T)
        if num == 0 && den == 0
            error("invalid rational: 0//0")
        end
        g = gcd(den, num)
        num = div(num, g)
        den = div(den, g)
        new(num, den)
    end
end
Rational{T<:Integer}(n::T, d::T) = Rational{T}(n,d)
Rational(n::Integer, d::Integer) = Rational(promote(n,d)...)
Rational(n::Integer) = Rational(n,one(n))
//(n::Integer, d::Integer) = Rational(n,d)
//(x::Rational, y::Integer) = x.num // (x.den*y)
//(x::Integer, y::Rational) = (x*y.den) // y.num
//(x::Complex, y::Real) = complex(real(x)//y, imag(x)//y)
//(x::Real, y::Complex) = x*y'//real(y*y')
function //(x::Complex, y::Complex)
```

```
xy = x*y'
yy = real(y*y')
complex(real(xy)//yy, imag(xy)//yy)
end
```

The first line — immutable Rational {T<:Int} <: Real — declares that Rational takes one type parameter of an integer type, and is itself a real type. The field declarations num::T and den::T indicate that the data held in a Rational {T} object are a pair of integers of type T, one representing the rational value's numerator and the other representing its denominator.

Now things get interesting. Rational has a single inner constructor method which checks that both of num and den aren't zero and ensures that every rational is constructed in "lowest terms" with a non-negative denominator. This is accomplished by dividing the given numerator and denominator values by their greatest common divisor, computed using the gcd function. Since gcd returns the greatest common divisor of its arguments with sign matching the first argument (den here), after this division the new value of den is guaranteed to be non-negative. Because this is the only inner constructor for Rational, we can be certain that Rational objects are always constructed in this normalized form.

Rational also provides several outer constructor methods for convenience. The first is the "standard" general constructor that infers the type parameter T from the type of the numerator and denominator when they have the same type. The second applies when the given numerator and denominator values have different types: it promotes them to a common type and then delegates construction to the outer constructor for arguments of matching type. The third outer constructor turns integer values into rationals by supplying a value of 1 as the denominator.

Following the outer constructor definitions, we have a number of methods for the // (page 354) operator, which provides a syntax for writing rationals. Before these definitions, // (page 354) is a completely undefined operator with only syntax and no meaning. Afterwards, it behaves just as described in *Rational Numbers* (page 37) — its entire behavior is defined in these few lines. The first and most basic definition just makes a//b construct a Rational by applying the Rational constructor to a and b when they are integers. When one of the operands of // (page 354) is already a rational number, we construct a new rational for the resulting ratio slightly differently; this behavior is actually identical to division of a rational with an integer. Finally, applying // (page 354) to complex integral values creates an instance of Complex {Rational} — a complex number whose real and imaginary parts are rationals:

```
julia> (1 + 2im) // (1 - 2im)
-3//5 + 4//5*im

julia> typeof(ans)
Complex{Rational{Int64}}

julia> ans <: Complex{Rational}
false</pre>
```

Thus, although the // (page 354) operator usually returns an instance of Rational, if either of its arguments are complex integers, it will return an instance of Complex {Rational} instead. The interested reader should consider perusing the rest of rational.jl: it is short, self-contained, and implements an entire basic Julia type.

## 13.6 Constructors, Call, and Conversion

Technically, constructors T(args...) in Julia are implemented by defining new methods  $Base.call(::Type{T}, args...)$  for the call() (page 325) function. That is, Julia types are not functions, but they can be called as if they were functions (functors) via call overloading, just like any other Julia object. This also means that you can declare more flexible constructors, e.g. constructors for abstract types, by instead explicitly defining Base.call methods using function syntax.

However, in some cases you could consider adding methods to Base.convert *instead* of defining a constructor, because defining a *convert()* (page 321) method *automatically* defines a corresponding constructor, while the

reverse is not true. That is, defining Base.convert(::Type{T}, args...) = ... automatically defines a constructor T(args...) = ....

convert is used extensively throughout Julia whenever one type needs to be converted to another (e.g. in assignment, ccall, etcetera), and should generally only be defined (or successful) if the conversion is lossless. For example, convert (Int, 3.0) produces 3, but convert (Int, 3.2) throws an InexactError. If you want to define a constructor for a lossless conversion from one type to another, you should probably define a convert method instead.

On the other hand, if your constructor does not represent a lossless conversion, or doesn't represent "conversion" at all, it is better to leave it as a constructor rather than a convert method. For example, the Array (Int) constructor creates a zero-dimensional Array of the type Int, but is not really a "conversion" from Int to an Array.

## 13.7 Outer-only constructors

As we have seen, a typical parametric type has inner constructors that are called when type parameters are known; e.g. they apply to Point {Int} but not to Point. Optionally, outer constructors that determine type parameters automatically can be added, for example constructing a Point {Int} from the call Point (1, 2). Outer constructors call inner constructors to do the core work of making an instance. However, in some cases one would rather not provide inner constructors, so that specific type parameters cannot be requested manually.

For example, say we define a type that stores a vector along with an accurate representation of its sum:

```
type SummedArray{T<:Number,S<:Number}
   data::Vector{T}
   sum::S
end</pre>
```

The problem is that we want S to be a larger type than T, so that we can sum many elements with less information loss. For example, when T is Int32, we would like S to be Int64. Therefore we want to avoid an interface that allows the user to construct instances of the type SummedArray{Int32, Int32}. One way to do this is to provide only an outer constructor for SummedArray. This can be done using explicit call overloading:

```
type SummedArray{T<:Number, S<:Number}
    data::Vector{T}
    sum::S

function call{T}(::Type{SummedArray}, a::Vector{T})
    S = widen(T)
    new{T,S}(a, sum(S, a))
end
end</pre>
```

This constructor will be invoked by the syntax SummedArray(a). The syntax new{T,S} allows specifying parameters for the type to be constructed, i.e. this call will return a SummedArray{T,S}.

### **Conversion and Promotion**

Julia has a system for promoting arguments of mathematical operators to a common type, which has been mentioned in various other sections, including *Integers and Floating-Point Numbers* (page 13), *Mathematical Operations and Elementary Functions* (page 25), *Types* (page 87), and *Methods* (page 107). In this section, we explain how this promotion system works, as well as how to extend it to new types and apply it to functions besides built-in mathematical operators. Traditionally, programming languages fall into two camps with respect to promotion of arithmetic arguments:

- Automatic promotion for built-in arithmetic types and operators. In most languages, built-in numeric types, when used as operands to arithmetic operators with infix syntax, such as +, -, \*, and /, are automatically promoted to a common type to produce the expected results. C, Java, Perl, and Python, to name a few, all correctly compute the sum 1 + 1.5 as the floating-point value 2.5, even though one of the operands to + is an integer. These systems are convenient and designed carefully enough that they are generally all-but-invisible to the programmer: hardly anyone consciously thinks of this promotion taking place when writing such an expression, but compilers and interpreters must perform conversion before addition since integers and floating-point values cannot be added as-is. Complex rules for such automatic conversions are thus inevitably part of specifications and implementations for such languages.
- No automatic promotion. This camp includes Ada and ML very "strict" statically typed languages. In these languages, every conversion must be explicitly specified by the programmer. Thus, the example expression 1 + 1.5 would be a compilation error in both Ada and ML. Instead one must write real(1) + 1.5, explicitly converting the integer 1 to a floating-point value before performing addition. Explicit conversion everywhere is so inconvenient, however, that even Ada has some degree of automatic conversion: integer literals are promoted to the expected integer type automatically, and floating-point literals are similarly promoted to appropriate floating-point types.

In a sense, Julia falls into the "no automatic promotion" category: mathematical operators are just functions with special syntax, and the arguments of functions are never automatically converted. However, one may observe that applying mathematical operations to a wide variety of mixed argument types is just an extreme case of polymorphic multiple dispatch — something which Julia's dispatch and type systems are particularly well-suited to handle. "Automatic" promotion of mathematical operands simply emerges as a special application: Julia comes with pre-defined catch-all dispatch rules for mathematical operators, invoked when no specific implementation exists for some combination of operand types. These catch-all rules first promote all operands to a common type using user-definable promotion rules, and then invoke a specialized implementation of the operator in question for the resulting values, now of the same type. User-defined types can easily participate in this promotion system by defining methods for conversion to and from other types, and providing a handful of promotion rules defining what types they should promote to when mixed with other types.

#### 14.1 Conversion

Conversion of values to various types is performed by the convert function. The convert function generally takes two arguments: the first is a type object while the second is a value to convert to that type; the returned value is the value converted to an instance of given type. The simplest way to understand this function is to see it in action:

```
julia> x = 12

julia> typeof(x)
Int64

julia> convert(UInt8, x)
0x0c

julia> typeof(ans)
UInt8

julia> convert(AbstractFloat, x)
12.0

julia> typeof(ans)
```

Conversion isn't always possible, in which case a no method error is thrown indicating that convert doesn't know how to perform the requested conversion:

```
julia> convert(AbstractFloat, "foo")
ERROR: MethodError: `convert` has no method matching convert(::Type{AbstractFloat}, ::ASCIIString)
This may have arisen from a call to the constructor AbstractFloat(...),
since type constructors fall back to convert methods.
Closest candidates are:
   call{T}(::Type{T}, ::Any)
   convert(::Type{AbstractFloat}, !Matched::Bool)
   convert(::Type{AbstractFloat}, !Matched::Int8)
...
```

Some languages consider parsing strings as numbers or formatting numbers as strings to be conversions (many dynamic languages will even perform conversion for you automatically), however Julia does not: even though some strings can be parsed as numbers, most strings are not valid representations of numbers, and only a very limited subset of them are. Therefore in Julia the dedicated parse() function must be used to perform this operation, making it more explicit.

### 14.1.1 Defining New Conversions

To define a new conversion, simply provide a new method for convert(). That's really all there is to it. For example, the method to convert a real number to a boolean is this:

```
convert(::Type{Bool}, x::Real) = x==0 ? false : x==1 ? true : throw(InexactError())
```

The type of the first argument of this method is a *singleton type* (page 100), Type {Bool}, the only instance of which is Bool. Thus, this method is only invoked when the first argument is the type value Bool. Notice the syntax used for the first argument: the argument name is omitted prior to the :: symbol, and only the type is given. This is the syntax in Julia for a function argument whose type is specified but whose value is never used in the function body. In this example, since the type is a singleton, there would never be any reason to use its value within the body. When invoked, the method determines whether a numeric value is true or false as a boolean, by comparing it to one and zero:

```
julia> convert(Bool, 1)
true

julia> convert(Bool, 0)
false

julia> convert(Bool, 1im)
ERROR: InexactError()
  in convert at complex.jl:18

julia> convert(Bool, 0im)
false
```

The method signatures for conversion methods are often quite a bit more involved than this example, especially for parametric types. The example above is meant to be pedagogical, and is not the actual julia behaviour. This is the actual implementation in julia:

### 14.1.2 Case Study: Rational Conversions

To continue our case study of Julia's Rational type, here are the conversions declared in rational.jl, right after the declaration of the type and its constructors:

```
convert{T<:Integer}(::Type{Rational{T}}, x::Rational) = Rational(convert(T,x.num),convert(T,x.den))
convert{T<:Integer}(::Type{Rational{T}}, x::Integer) = Rational(convert(T,x), convert(T,1))</pre>
function convert{T<:Integer}(::Type{Rational{T}}, x::AbstractFloat, tol::Real)</pre>
    if isnan(x); return zero(T)//zero(T); end
    if isinf(x); return sign(x)//zero(T); end
    a = d = one(T)
    b = c = zero(T)
    while true
        f = convert(T, round(y)); y -= f
        a, b, c, d = f*a+c, f*b+d, a, b
        if y == 0 \mid \mid abs(a/b-x) \le tol
            return a//b
        end
        y = 1/y
    end
end
convert{T<:Integer}(rt::Type{Rational{T}}, x::AbstractFloat) = convert(rt,x,eps(x))</pre>
convert{T<:AbstractFloat}(::Type{T}, x::Rational) = convert(T,x.num)/convert(T,x.den)</pre>
convert{T<:Integer}(::Type{T}, x::Rational) = div(convert(T,x.num),convert(T,x.den))</pre>
```

The initial four convert methods provide conversions to rational types. The first method converts one type of rational to another type of rational by converting the numerator and denominator to the appropriate integer type. The second method does the same conversion for integers by taking the denominator to be 1. The third method implements a standard algorithm for approximating a floating-point number by a ratio of integers to within a given tolerance, and

14.1. Conversion 129

the fourth method applies it, using machine epsilon at the given value as the threshold. In general, one should have  $a//b = convert(Rational{Int64}, a/b)$ .

The last two convert methods provide conversions from rational types to floating-point and integer types. To convert to floating point, one simply converts both numerator and denominator to that floating point type and then divides. To convert to integer, one can use the div operator for truncated integer division (rounded towards zero).

### 14.2 Promotion

Promotion refers to converting values of mixed types to a single common type. Although it is not strictly necessary, it is generally implied that the common type to which the values are converted can faithfully represent all of the original values. In this sense, the term "promotion" is appropriate since the values are converted to a "greater" type — i.e. one which can represent all of the input values in a single common type. It is important, however, not to confuse this with object-oriented (structural) super-typing, or Julia's notion of abstract super-types: promotion has nothing to do with the type hierarchy, and everything to do with converting between alternate representations. For instance, although every Int32 value can also be represented as a Float 64 value, Int32 is not a subtype of Float 64.

Promotion to a common "greater" type is performed in Julia by the promote function, which takes any number of arguments, and returns a tuple of the same number of values, converted to a common type, or throws an exception if promotion is not possible. The most common use case for promotion is to convert numeric arguments to a common type:

```
julia> promote(1, 2.5)
(1.0,2.5)

julia> promote(1, 2.5, 3)
(1.0,2.5,3.0)

julia> promote(2, 3//4)
(2//1,3//4)

julia> promote(1, 2.5, 3, 3//4)
(1.0,2.5,3.0,0.75)

julia> promote(1.5, im)
(1.5 + 0.0im,0.0 + 1.0im)

julia> promote(1 + 2im, 3//4)
(1//1 + 2//1*im,3//4 + 0//1*im)
```

Floating-point values are promoted to the largest of the floating-point argument types. Integer values are promoted to the larger of either the native machine word size or the largest integer argument type. Mixtures of integers and floating-point values are promoted to a floating-point type big enough to hold all the values. Integers mixed with rationals are promoted to rationals. Rationals mixed with floats are promoted to floats. Complex values mixed with real values are promoted to the appropriate kind of complex value.

That is really all there is to using promotions. The rest is just a matter of clever application, the most typical "clever" application being the definition of catch-all methods for numeric operations like the arithmetic operators +, -, \* and /. Here are some of the the catch-all method definitions given in promotion.jl:

```
+(x::Number, y::Number) = +(promote(x,y)...)
-(x::Number, y::Number) = -(promote(x,y)...)
*(x::Number, y::Number) = *(promote(x,y)...)
/(x::Number, y::Number) = /(promote(x,y)...)
```

In certain cases, the result type also depends on the operator; how to handle such scenarios is described elsewhere.

These method definitions say that in the absence of more specific rules for adding, subtracting, multiplying and dividing pairs of numeric values, promote the values to a common type and then try again. That's all there is to it: nowhere else does one ever need to worry about promotion to a common numeric type for arithmetic operations — it just happens automatically. There are definitions of catch-all promotion methods for a number of other arithmetic and mathematical functions in promotion.jl, but beyond that, there are hardly any calls to promote required in the Julia standard library. The most common usages of promote occur in outer constructors methods, provided for convenience, to allow constructor calls with mixed types to delegate to an inner type with fields promoted to an appropriate common type. For example, recall that rational.jl provides the following outer constructor method:

```
Rational(n::Integer, d::Integer) = Rational(promote(n,d)...)
```

This allows calls like the following to work:

```
julia> Rational(Int8(15), Int32(-5))
-3//1

julia> typeof(ans)
Rational{Int32}
```

For most user-defined types, it is better practice to require programmers to supply the expected types to constructor functions explicitly, but sometimes, especially for numeric problems, it can be convenient to do promotion automatically.

### 14.2.1 Defining Promotion Rules

Although one could, in principle, define methods for the promote function directly, this would require many redundant definitions for all possible permutations of argument types. Instead, the behavior of promote is defined in terms of an auxiliary function called promote\_rule, which one can provide methods for. The promote\_rule function takes a pair of type objects and returns another type object, such that instances of the argument types will be promoted to the returned type. Thus, by defining the rule:

```
promote_rule(::Type{Float64}, ::Type{Float32} ) = Float64
```

one declares that when 64-bit and 32-bit floating-point values are promoted together, they should be promoted to 64-bit floating-point. The promotion type does not need to be one of the argument types, however; the following promotion rules both occur in Julia's standard library:

```
promote_rule(::Type{UInt8}, ::Type{Int8}) = Int
promote_rule(::Type{BigInt}, ::Type{Int8}) = BigInt
```

In the latter case, the result type is <code>BigInt</code> since <code>BigInt</code> is the only type large enough to hold integers for arbitrary-precision integer arithmetic. Also note that one does not need to define both <code>promote\_rule(::Type{A}, ::Type{B})</code> and <code>promote\_rule(::Type{B}, ::Type{A})</code> — the symmetry is implied by the way <code>promote\_rule(::Type{B}, ::Type{A})</code> is used in the promotion process.

The promote\_rule function is used as a building block to define a second function called promote\_type, which, given any number of type objects, returns the common type to which those values, as arguments to promote should be promoted. Thus, if one wants to know, in absence of actual values, what type a collection of values of certain types would promote to, one can use promote\_type:

```
julia> promote_type(Int8, UInt16)
Int64
```

Internally, promote\_type is used inside of promote to determine what type argument values should be converted to for promotion. It can, however, be useful in its own right. The curious reader can read the code in promotion.jl, which defines the complete promotion mechanism in about 35 lines.

14.2. Promotion 131

### 14.2.2 Case Study: Rational Promotions

Finally, we finish off our ongoing case study of Julia's rational number type, which makes relatively sophisticated use of the promotion mechanism with the following promotion rules:

```
promote_rule{T<:Integer}(::Type{Rational{T}}, ::Type{T}) = Rational{T}
promote_rule{T<:Integer, S<:Integer}(::Type{Rational{T}}, ::Type{S}) = Rational{promote_type(T,S)}
promote_rule{T<:Integer, S<:Integer}(::Type{Rational{T}}, ::Type{Rational{S}}) = Rational{promote_type}(T,S)
promote_rule{T<:Integer, S<:AbstractFloat}(::Type{Rational{T}}, ::Type{S}) = promote_type(T,S)</pre>
```

The first rule asserts that promotion of a rational number with its own numerator/denominator type, simply promotes to itself. The second rule says that promoting a rational number with any other integer type promotes to a rational type whose numerator/denominator type is the result of promotion of its numerator/denominator type with the other integer type. The third rule applies the same logic to two different types of rational numbers, resulting in a rational of the promotion of their respective numerator/denominator types. The fourth and final rule dictates that promoting a rational with a float results in the same type as promoting the numerator/denominator type with the float.

This small handful of promotion rules, together with the *conversion methods discussed above* (page 129), are sufficient to make rational numbers interoperate completely naturally with all of Julia's other numeric types — integers, floating-point numbers, and complex numbers. By providing appropriate conversion methods and promotion rules in the same manner, any user-defined numeric type can interoperate just as naturally with Julia's predefined numerics.

### **Modules**

Modules in Julia are separate global variable workspaces. They are delimited syntactically, inside module Name . . . end. Modules allow you to create top-level definitions without worrying about name conflicts when your code is used together with somebody else's. Within a module, you can control which names from other modules are visible (via importing), and specify which of your names are intended to be public (via exporting).

The following example demonstrates the major features of modules. It is not meant to be run, but is shown for illustrative purposes:

```
module MyModule
using Lib

using BigLib: thing1, thing2

import Base.show

importall OtherLib

export MyType, foo

type MyType
    x
end

bar(x) = 2x
foo(a::MyType) = bar(a.x) + 1

show(io, a::MyType) = print(io, "MyType $(a.x)")
end
```

Note that the style is not to indent the body of the module, since that would typically lead to whole files being indented.

This module defines a type MyType, and two functions. Function foo and type MyType are exported, and so will be available for importing into other modules. Function bar is private to MyModule.

The statement using Lib means that a module called Lib will be available for resolving names as needed. When a global variable is encountered that has no definition in the current module, the system will search for it among variables exported by Lib and import it if it is found there. This means that all uses of that global within the current module will resolve to the definition of that variable in Lib.

The statement using BigLib: thing1, thing2 is a syntactic shortcut for using BigLib.thing1, BigLib.thing2.

The import keyword supports all the same syntax as using, but only operates on a single name at a time. It does not add modules to be searched the way using does. import also differs from using in that functions must be

imported using import to be extended with new methods.

In MyModule above we wanted to add a method to the standard show function, so we had to write import Base. show. Functions whose names are only visible via using cannot be extended.

The keyword importall explicitly imports all names exported by the specified module, as if import were individually used on all of them.

Once a variable is made visible via using or import, a module may not create its own variable with the same name. Imported variables are read-only; assigning to a global variable always affects a variable owned by the current module, or else raises an error.

## 15.1 Summary of module usage

To load a module, two main keywords can be used: using and import. To understand their differences, consider the following example:

```
module MyModule

export x, y

x() = "x"
y() = "y"
p() = "p"

end
```

In this module we export the x and y functions (with the keyword export), and also have the non-exported function p. There are several different ways to load the Module and its inner functions into the current workspace:

Import Command	What is brought into scope	Available for method extension
using MyModule	All exported names (x and y), MyModule.x,	MyModule.x,
	MyModule.y and MyModule.p	MyModule.y and
		MyModule.p
using MyModule.x,	x and p	
MyModule.p		
using MyModule:	x and p	
x, p		
import MyModule	MyModule.x, MyModule.y and MyModule.p	MyModule.x,
		MyModule.y and
		MyModule.p
import	x and p	x and p
MyModule.x,		
MyModule.p		
import MyModule:	x and p	x and p
x, p		
importall	All exported names (x and y)	x and y
MyModule		

#### 15.1.1 Modules and files

Files and file names are mostly unrelated to modules; modules are associated only with module expressions. One can have multiple files per module, and multiple modules per file:

```
module Foo

include("file1.jl")
include("file2.jl")
end
```

Including the same code in different modules provides mixin-like behavior. One could use this to run the same code with different base definitions, for example testing code by running it with "safe" versions of some operators:

```
module Normal
include("mycode.jl")
end

module Testing
include("safe_operators.jl")
include("mycode.jl")
end
```

#### 15.1.2 Standard modules

There are three important standard modules: Main, Core, and Base.

Main is the top-level module, and Julia starts with Main set as the current module. Variables defined at the prompt go in Main, and whos () lists variables in Main.

Core contains all identifiers considered "built in" to the language, i.e. part of the core language and not libraries. Every module implicitly specifies using Core, since you can't do anything without those definitions.

Base is the standard library (the contents of base/). All modules implicitly contain using Base, since this is needed in the vast majority of cases.

### 15.1.3 Default top-level definitions and bare modules

In addition to using Base, modules also perform import Base.call by default, to facilitate adding constructors to new types. A new module also automatically contains a definition of the eval function, which evaluates expressions within the context of that module.

If these default definitions are not wanted, modules can be defined using the keyword baremodule instead (note: Core is still imported, as per above). In terms of baremodule, a standard module looks like this:

```
baremodule Mod
using Base
import Base.call
eval(x) = Core.eval(Mod, x)
eval(m,x) = Core.eval(m, x)
```

#### 15.1.4 Relative and absolute module paths

Given the statement using Foo, the system looks for Foo within Main. If the module does not exist, the system attempts to require ("Foo"), which typically results in loading code from an installed package.

However, some modules contain submodules, which means you sometimes need to access a module that is not directly available in Main. There are two ways to do this. The first is to use an absolute path, for example using Base.Sort. The second is to use a relative path, which makes it easier to import submodules of the current module or any of its enclosing modules:

```
module Parent

module Utils
...
end

using .Utils
...
end
```

Here module Parent contains a submodule Utils, and code in Parent wants the contents of Utils to be visible. This is done by starting the using path with a period. Adding more leading periods moves up additional levels in the module hierarchy. For example using ..Utils would look for Utils in Parent's enclosing module rather than in Parent itself.

Note that relative-import qualifiers are only valid in using and import statements.

### 15.1.5 Module file paths

The global variable LOAD\_PATH contains the directories Julia searches for modules when calling require. It can be extended using push!:

```
push! (LOAD_PATH, "/Path/To/My/Module/")
```

Putting this statement in the file ~/.juliarc.jl will extend LOAD\_PATH on every Julia startup. Alternatively, the module load path can be extended by defining the environment variable JULIA\_LOAD\_PATH.

#### 15.1.6 Namespace miscellanea

If a name is qualified (e.g. Base.sin), then it can be accessed even if it is not exported. This is often useful when debugging.

Macro names are written with @ in import and export statements, e.g. import Mod.@mac. Macros in other modules can be invoked as Mod.@mac or @Mod.mac.

The syntax M.x = y does not work to assign a global in another module; global assignment is always module-local.

A variable can be "reserved" for the current module without assigning to it by declaring it as global x at the top level. This can be used to prevent name conflicts for globals initialized after load time.

### 15.1.7 Module initialization and precompilation

Large modules can take several second to load because executing all of the statements in a module often involves compiling a large amount of code. Julia provides the ability to create precompiled versions of modules to reduce this time.

There are two mechanisms that can achieve this: incremental compile and custom system image.

To create a custom system image that can be used to start julia with the -J option, recompile Julia after modifying the file base/userimg.jl to require the desired modules.

To create an incremental precompiled module file, add \_\_precompile\_\_() at the top of your module file (before the module starts). This will cause it to be automatically compiled the first time it is imported. Alternatively, you can manually call Base.compilecache (modulename). The resulting cache files will be stored in Base.LOAD\_CACHE\_PATH[1]. Subsequently, the module is automatically recompiled upon import whenever any of its dependencies change; dependencies are modules it imports, the Julia build, files it includes, or explicit dependencies declared by include\_dependency (path) in the module file(s). Precompiling a module also recursively precompiles any modules that are imported therein. If you know that it is not safe to precompile your module (for the reasons described below), you should put \_\_precompile\_\_(false) in the module file to cause Base.compilecache to throw an error (and thereby prevent the module from being imported by any other precompiled module).

\_\_precompile\_\_() should *not* be used in a module unless all of its dependencies are also using \_\_precompile\_\_(). Failure to do so can result in a runtime error when loading the module.

In order to make your module work with precompilation, however, you may need to change your module to explicitly separate any initialization steps that must occur at *runtime* from steps that can occur at *compile time*. For this purpose, Julia allows you to define an \_\_\_init\_\_\_() function in your module that executes any initialization steps that must occur at runtime.

In particular, if you define a function \_\_\_init\_\_\_() in a module, then Julia will call \_\_init\_\_\_() immediately after the module is loaded (e.g., by import, using, or require) at runtime for the first time (i.e., \_\_init\_\_ is only called once, and only after all statements in the module have been executed). Because it is called after the module is fully imported, any submodules or other imported modules have their \_\_init\_\_ functions called before the \_\_init\_\_ of the enclosing module.

Two typical uses of \_\_init\_\_ are calling runtime initialization functions of external C libraries and initializing global constants that involve pointers returned by external libraries. For example, suppose that we are calling a C library libfoo that requires us to call a foo\_init() initialization function at runtime. Suppose that we also want to define a global constant foo\_data\_ptr that holds the return value of a void \*foo\_data() function defined by libfoo — this constant must be initialized at runtime (not at compile time) because the pointer address will change from run to run. You could accomplish this by defining the following \_\_init\_\_ function in your module:

```
function __init__()
    ccall((:foo_init,:libfoo), Void, ())
    global const foo_data_ptr = ccall((:foo_data,:libfoo), Ptr{Void}, ())
end
```

Notice that it is perfectly possible to define a global inside a function like \_\_init\_\_; this is one of the advantages of using a dynamic language. Obviously, any other globals in your module that depends on foo\_data\_ptr would also have to be initialized in \_\_init\_\_.

Constants involving most Julia objects that are not produced by ccall do not need to be placed in \_\_init\_\_: their definitions can be precompiled and loaded from the cached module image. This includes complicated heap-allocated objects like arrays. However, any routine that returns a raw pointer value must be called at runtime for precompilation to work (Ptr objects will turn into null pointers unless they are hidden inside an isbits object). This includes the return values of the Julia functions cfunction and pointer.

Dictionary and set types, or in general anything that depends on the output of a hash (key) method, are a trickier case. In the common case where the keys are numbers, strings, symbols, ranges, Expr, or compositions of these types (via arrays, tuples, sets, pairs, etc.) they are safe to precompile. However, for a few other key types, such as Function or DataType and generic user-defined types where you haven't defined a hash method, the fallback hash method depends on the memory address of the object (via its object\_id) and hence may change from run to run. If you have one of these key types, or if you aren't sure, to be safe you can initialize this dictionary from

within your \_\_init\_\_ function. Alternatively, you can use the ObjectIdDict dictionary type, which is specially handled by precompilation so that it is safe to initialize at compile-time.

When using precompilation, it is important to keep a clear sense of the distinction between the compilation phase and the execution phase. In this mode, it will often be much more clearly apparent that Julia is a compiler which allows execution of arbitrary Julia code, not a standalone interpreter that also generates compiled code.

Other known potential failure scenarios include:

1. Global counters (for example, for attempting to unique identifying objects) Consider the following code snippet:

```
type UniquedById
    myid::Int
    let counter = 0
        UniquedById() = new(counter += 1)
    end
end
```

while the intent of this code was to give every instance a unique id, the counter value is recorded at the end of compilation. All subsequent usages of this incrementally compiled module will start from that same counter value.

Note that object\_id (which works by hashing the memory pointer) has similar issues (see notes on Dict usage below).

One alternative is to store both current\_module() and the current counter value, however, it may be better to redesign the code to not depend on this global state.

- 2. Associative collections (such as Dict and Set) need to be re-hashed in \_\_init\_\_. (In the future, a mechanism may be provided to register an initializer function.)
- 3. Depending on compile-time side-effects persisting through load-time. Example include: modifying arrays or other variables in other Julia modules; maintaining handles to open files or devices; storing pointers to other system resources (including memory);
- 4. Creating accidental "copies" of global state from another module, by referencing it directly instead of via its lookup path. For example, (in global scope):

```
#mystdout = Base.STDOUT #= will not work correctly, since this will copy Base.STDOUT into this m
# instead use accessor functions:
getstdout() = Base.STDOUT #= best option =#
# or move the assignment into the runtime:
__init__() = global mystdout = Base.STDOUT #= also works =#
```

Several additional restrictions are placed on the operations that can be done while precompiling code to help the user avoid other wrong-behavior situations:

- 1. Calling eval to cause a side-effect in another module. This will also cause a warning to be emitted when the incremental precompile flag is set.
- 2. global const statements from local scope after \_\_init\_\_() has been started (see issue #12010 for plans to add an error for this)
- 3. Replacing a module (or calling workspace ()) is a runtime error while doing an incremental precompile.

A few other points to be aware of:

- 1. No code reload / cache invalidation is performed after changes are made to the source files themselves, (including by Pkg.update), and no cleanup is done after Pkg.rm
- 2. The memory sharing behavior of a reshaped array is disregarded by precompilation (each view gets its own copy)

- 3. Expecting the filesystem to be unchanged between compile-time and runtime e.g. @\_\_FILE\_\_/source\_path() to find resources at runtime, or the BinDeps @checked\_lib macro. Sometimes this is unavoidable. However, when possible, it can be good practice to copy resources into the module at compile-time so they won't need to be found at runtime.
- 4. WeakRef objects and finalizers are not currently handled properly by the serializer (this will be fixed in an upcoming release).

It is sometimes helpful during module development to turn off incremental precompilation. The command line flag --compilecache={yes|no} enables you to toggle module precompilation on and off. When Julia is started with --compilecache=no the serialized modules in the compile cache are ignored when loading modules and module dependencies. Base.compilecache() can still be called manually and it will respect \_\_precompile\_\_() directives for the module. The state of this command line flag is passed to Pkg.build() to disable automatic precompilation triggering when installing, updating, and explicitly building packages.

140

# Metaprogramming

The strongest legacy of Lisp in the Julia language is its metaprogramming support. Like Lisp, Julia represents its own code as a data structure of the language itself. Since code is represented by objects that can be created and manipulated from within the language, it is possible for a program to transform and generate its own code. This allows sophisticated code generation without extra build steps, and also allows true Lisp-style macros operating at the level of abstract syntax trees. In contrast, preprocessor "macro" systems, like that of C and C++, perform textual manipulation and substitution before any actual parsing or interpretation occurs. Because all data types and code in Julia are represented by Julia data structures, powerful reflection capabilities are available to explore the internals of a program and its types just like any other data.

# 16.1 Program representation

Every Julia program starts life as a string:

```
julia> prog = "1 + 1"
"1 + 1"
```

#### What happens next?

The next step is to parse each string into an object called an expression, represented by the Julia type Expr:

```
julia> ex1 = parse(prog)
:(1 + 1)

julia> typeof(ex1)
Expr
```

Expr objects contain three parts:

a Symbol identifying the kind of expression. A symbol is an interned string identifier (more discussion below).

```
julia> ex1.head
:call
```

• the expression arguments, which may be symbols, other expressions, or literal values:

```
julia> ex1.args
3-element Array{Any,1}:
   :+
   1
   1
```

• finally, the expression result type, which may be annotated by the user or inferred by the compiler (and may be ignored completely for the purposes of this chapter):

```
julia> ex1.typ
Any
```

Expressions may also be constructed directly in prefix notation:

```
julia> ex2 = Expr(:call, :+, 1, 1)
:(1 + 1)
```

The two expressions constructed above – by parsing and by direct construction – are equivalent:

```
julia> ex1 == ex2
true
```

The key point here is that Julia code is internally represented as a data structure that is accessible from the language itself.

The dump () (page 454) function provides indented and annotated display of Expr objects:

```
julia> dump(ex2)
Expr
head: Symbol call
args: Array(Any,(3,))
1: Symbol +
2: Int64 1
3: Int64 1
typ: Any
```

Expr objects may also be nested:

```
julia> ex3 = parse("(4 + 4) / 2")
:((4 + 4) / 2)
```

Another way to view expressions is with Meta.show\_sexpr, which displays the S-expression form of a given Expr, which may look very familiar to users of Lisp. Here's an example illustrating the display on a nested Expr:

```
julia> Meta.show_sexpr(ex3)
(:call, :/, (:call, :+, 4, 4), 2)
```

### **16.1.1 Symbols**

The: character has two syntactic purposes in Julia. The first form creates a Symbol, an interned string used as one building-block of expressions:

```
julia> :foo
:foo

julia> typeof(ans)
Symbol
```

Symbols can also be created using symbol () (page 390), which takes any number of arguments and creates a new symbol by concatenating their string representations together:

```
julia> :foo == symbol("foo")
true
julia> symbol("func",10)
```

```
:func10

julia> symbol(:var,'_',"sym")
:var_sym
```

In the context of an expression, symbols are used to indicate access to variables; when an expression is evaluated, a symbol is replaced with the value bound to that symbol in the appropriate *scope* (page 81).

Sometimes extra parentheses around the argument to: are needed to avoid ambiguity in parsing.:

```
julia> :(:)
:(:)
julia> :(::)
:(::)
```

# 16.2 Expressions and evaluation

### **16.2.1 Quoting**

The second syntactic purpose of the : character is to create expression objects without using the explicit Expr constructor. This is referred to as *quoting*. The : character, followed by paired parentheses around a single statement of Julia code, produces an Expr object based on the enclosed code. Here is example of the short form used to quote an arithmetic expression:

```
julia> ex = :(a+b*c+1)
:(a + b * c + 1)

julia> typeof(ex)
Expr
```

(to view the structure of this expression, try ex. head and ex. args, or use dump () (page 454) as above)

Note that equivalent expressions may be constructed using parse () (page 326) or the direct Expr form:

```
julia> :(a + b*c + 1) ==
    parse("a + b*c + 1") ==
    Expr(:call, :+, :a, Expr(:call, :*, :b, :c), 1)
true
```

Expressions provided by the parser generally only have symbols, other expressions, and literal values as their args, whereas expressions constructed by Julia code can have arbitrary run-time values without literal forms as args. In this specific example, + and a are symbols, \*(b, c) is a subexpression, and 1 is a literal 64-bit signed integer.

There is a second syntactic form of quoting for multiple expressions: blocks of code enclosed in quote ... end. Note that this form introduces QuoteNode elements to the expression tree, which must be considered when directly manipulating an expression tree generated from quote blocks. For other purposes, : ( ... ) and quote .. end blocks are treated identically.

```
x + y
end

julia> typeof(ex)
Expr
```

### 16.2.2 Interpolation

Direct construction of Expr objects with value arguments is powerful, but Expr constructors can be tedious compared to "normal" Julia syntax. As an alternative, Julia allows "splicing" or interpolation of literals or expressions into quoted expressions. Interpolation is indicated by the \$ prefix.

In this example, the literal value of a is interpolated:

```
julia> a = 1;
julia> ex = :($a + b)
:(1 + b)
```

Interpolating into an unquoted expression is not supported and will cause a compile-time error:

```
julia> $a + b
ERROR: unsupported or misplaced expression $
```

In this example, the tuple (1, 2, 3) is interpolated as an expression into a conditional test:

```
julia> ex = :(a in $:((1,2,3)) )
:($(Expr(:in, :a, :((1,2,3)))))
```

Interpolating symbols into a nested expression requires enclosing each symbol in an enclosing quote block:

The use of \$ for expression interpolation is intentionally reminiscent of *string interpolation* (page 46) and *command interpolation* (page 208). Expression interpolation allows convenient, readable programmatic construction of complex Julia expressions.

#### 16.2.3 eval() and effects

Given an expression object, one can cause Julia to evaluate (execute) it at global scope using eval () (page 325):

```
julia> :(1 + 2)
:(1 + 2)

julia> eval(ans)

julia> ex = :(a + b)
:(a + b)

julia> eval(ex)
ERROR: UndefVarError: b not defined

julia> a = 1; b = 2;
```

```
julia> eval(ex)
3
```

Every *module* (page 133) has its own eval() (page 325) function that evaluates expressions in its global scope. Expressions passed to eval() (page 325) are not limited to returning values — they can also have side-effects that alter the state of the enclosing module's environment:

```
julia> ex = :(x = 1)
:(x = 1)

julia> x
ERROR: UndefVarError: x not defined

julia> eval(ex)
1

julia> x
1
```

Here, the evaluation of an expression object causes a value to be assigned to the global variable x.

Since expressions are just Expr objects which can be constructed programmatically and then evaluated, it is possible to dynamically generate arbitrary code which can then be run using eval () (page 325). Here is a simple example:

```
julia> a = 1;
julia> ex = Expr(:call, :+, a, :b)
:(1 + b)

julia> a = 0; b = 2;

julia> eval(ex)
3
```

The value of a is used to construct the expression ex which applies the + function to the value 1 and the variable b. Note the important distinction between the way a and b are used:

- The value of the *variable* a at expression construction time is used as an immediate value in the expression. Thus, the value of a when the expression is evaluated no longer matters: the value in the expression is already 1, independent of whatever the value of a might be.
- On the other hand, the *symbol*: b is used in the expression construction, so the value of the variable b at that time is irrelevant—: b is just a symbol and the variable b need not even be defined. At expression evaluation time, however, the value of the symbol: b is resolved by looking up the value of the variable b.

# **16.2.4 Functions on Expressions**

As hinted above, one extremely useful feature of Julia is the capability to generate and manipulate Julia code within Julia itself. We have already seen one example of a function returning Expr objects: the parse() (page 326) function, which takes a string of Julia code and returns the corresponding Expr. A function can also take one or more Expr objects as arguments, and return another Expr. Here is a simple, motivating example:

```
:(1 + 4*5)

julia> eval(ex)
21
```

As another example, here is a function that doubles any numeric argument, but leaves expressions alone:

### 16.3 Macros

Macros provide a method to include generated code in the final body of a program. A macro maps a tuple of arguments to a returned *expression*, and the resulting expression is compiled directly rather than requiring a runtime eval() (page 325) call. Macro arguments may include expressions, literal values, and symbols.

#### 16.3.1 Basics

Here is an extraordinarily simple macro:

```
julia> macro sayhello()
          return :( println("Hello, world!") )
          end
```

Macros have a dedicated character in Julia's syntax: the @ (at-sign), followed by the unique name declared in a macro NAME ... end block. In this example, the compiler will replace all instances of @sayhello with:

```
:( println("Hello, world!") )
```

When @sayhello is given at the REPL, the expression executes immediately, thus we only see the evaluation result:

```
julia> @sayhello()
"Hello, world!"
```

Now, consider a slightly more complex macro:

```
julia> macro sayhello(name)
          return :( println("Hello, ", $name) )
        end
```

This macro takes one argument: name. When @sayhello is encountered, the quoted expression is *expanded* to interpolate the value of the argument into the final expression:

```
julia> @sayhello("human")
Hello, human
```

We can view the quoted return expression using the function macroexpand() (page 332) (**important note:** this is an extremely useful tool for debugging macros):

### 16.3.2 Hold up: why macros?

We have already seen a function f(::Expr...) -> Expr in a previous section. In fact, macroexpand() (page 332) is also such a function. So, why do macros exist?

Macros are necessary because they execute when code is parsed, therefore, macros allow the programmer to generate and include fragments of customized code *before* the full program is run. To illustrate the difference, consider the following example:

The first call to <code>println()</code> (page 454) is executed when <code>macroexpand()</code> (page 332) is called. The resulting expression contains <code>only</code> the second <code>println</code>:

```
julia> typeof(ex)
Expr

julia> ex
:(println("I execute at runtime. The argument is: ",$(Expr(:copyast, :(:((1,2,3)))))))

julia> eval(ex)
I execute at runtime. The argument is: (1,2,3)
```

#### 16.3.3 Macro invocation

Macros are invoked with the following general syntax:

```
@name expr1 expr2 ...
@name(expr1, expr2, ...)
```

Note the distinguishing @ before the macro name and the lack of commas between the argument expressions in the first form, and the lack of whitespace after @name in the second form. The two styles should not be mixed. For example, the following syntax is different from the examples above; it passes the tuple (expr1, expr2, ...) as one argument to the macro:

16.3. Macros 147

```
@name (expr1, expr2, ...)
```

It is important to emphasize that macros receive their arguments as expressions, literals, or symbols. One way to explore macro arguments is to call the show() (page 453) function within the macro body:

```
julia> macro showarg(x)
    show(x)
    # ... remainder of macro, returning an expression
end

julia> @showarg(a)
(:a,)

julia> @showarg(1+1)
:(1 + 1)

julia> @showarg(println("Yo!"))
:(println("Yo!"))
```

### 16.3.4 Building an advanced macro

Here is a simplified definition of Julia's @assert macro:

```
macro assert(ex)
    return :( $ex ? nothing : throw(AssertionError($(string(ex)))) )
end
```

This macro can be used like this:

```
julia> @assert 1==1.0

julia> @assert 1==0
ERROR: AssertionError: 1 == 0
```

In place of the written syntax, the macro call is expanded at parse time to its returned result. This is equivalent to writing:

```
1==1.0 ? nothing : throw(AssertionError("1==1.0"))
1==0 ? nothing : throw(AssertionError("1==0"))
```

That is, in the first call, the expression : (1==1.0) is spliced into the test condition slot, while the value of string(:(1==1.0)) is spliced into the assertion message slot. The entire expression, thus constructed, is placed into the syntax tree where the @assert macro call occurs. Then at execution time, if the test expression evaluates to true, then nothing is returned, whereas if the test is false, an error is raised indicating the asserted expression that was false. Notice that it would not be possible to write this as a function, since only the *value* of the condition is available and it would be impossible to display the expression that computed it in the error message.

The actual definition of @assert in the standard library is more complicated. It allows the user to optionally specify their own error message, instead of just printing the failed expression. Just like in functions with a variable number of arguments, this is specified with an ellipses following the last argument:

```
macro assert(ex, msgs...)
    msg_body = isempty(msgs) ? ex : msgs[1]
    msg = string(msg_body)
    return :($ex ? nothing : throw(AssertionError($msg)))
end
```

Now @assert has two modes of operation, depending upon the number of arguments it receives! If there's only one argument, the tuple of expressions captured by msgs will be empty and it will behave the same as the simpler definition above. But now if the user specifies a second argument, it is printed in the message body instead of the failing expression. You can inspect the result of a macro expansion with the aptly named macroexpand() (page 332) function:

There is yet another case that the actual @assert macro handles: what if, in addition to printing "a should equal b," we wanted to print their values? One might naively try to use string interpolation in the custom message, e.g., @assert a==b "a (a) should equal b (a)!", but this won't work as expected with the above macro. Can you see why? Recall from *string interpolation* (page 46) that an interpolated string is rewritten to a call to string() (page 385). Compare:

```
julia> typeof(:("a should equal b"))
ASCIIString

julia> typeof(:("a ($a) should equal b ($b)!"))
Expr

julia> dump(:("a ($a) should equal b ($b)!"))
Expr
 head: Symbol string
  args: Array(Any, (5,))
  1: ASCIIString "a ("
  2: Symbol a
  3: ASCIIString ") should equal b ("
  4: Symbol b
  5: ASCIIString ")!"
  typ: Any
```

So now instead of getting a plain string in msg\_body, the macro is receiving a full expression that will need to be evaluated in order to display as expected. This can be spliced directly into the returned expression as an argument to the string() (page 385) call; see error.jl for the complete implementation.

The @assert macro makes great use of splicing into quoted expressions to simplify the manipulation of expressions inside the macro body.

## 16.3.5 Hygiene

An issue that arises in more complex macros is that of hygiene. In short, macros must ensure that the variables they introduce in their returned expressions do not accidentally clash with existing variables in the surrounding code they expand into. Conversely, the expressions that are passed into a macro as arguments are often *expected* to evaluate in the context of the surrounding code, interacting with and modifying the existing variables. Another concern arises from the fact that a macro may be called in a different module from where it was defined. In this case we need to

16.3. Macros 149

ensure that all global variables are resolved to the correct module. Julia already has a major advantage over languages with textual macro expansion (like C) in that it only needs to consider the returned expression. All the other variables (such as msg in @assert above) follow the *normal scoping block behavior* (page 81).

To demonstrate these issues, let us consider writing a @time macro that takes an expression as its argument, records the time, evaluates the expression, records the time again, prints the difference between the before and after times, and then has the value of the expression as its final value. The macro might look like this:

```
macro time(ex)
  return quote
  local t0 = time()
  local val = $ex
  local t1 = time()
   println("elapsed time: ", t1-t0, " seconds")
  val
  end
end
```

Here, we want t0, t1, and val to be private temporary variables, and we want time to refer to the time() (page 328) function in the standard library, not to any time variable the user might have (the same applies to println). Imagine the problems that could occur if the user expression ex also contained assignments to a variable called t0, or defined its own time variable. We might get errors, or mysteriously incorrect behavior.

Julia's macro expander solves these problems in the following way. First, variables within a macro result are classified as either local or global. A variable is considered local if it is assigned to (and not declared global), declared local, or used as a function argument name. Otherwise, it is considered global. Local variables are then renamed to be unique (using the <code>gensym()</code> (page 325) function, which generates new symbols), and global variables are resolved within the macro definition environment. Therefore both of the above concerns are handled; the macro's locals will not conflict with any user variables, and <code>time</code> and <code>println</code> will refer to the standard library definitions.

One problem remains however. Consider the following use of this macro:

```
module MyModule
import Base.@time

time() = ... # compute something

@time time()
end
```

Here the user expression ex is a call to time, but not the same time function that the macro uses. It clearly refers to MyModule.time. Therefore we must arrange for the code in ex to be resolved in the macro call environment. This is done by "escaping" the expression with esc () (page 325):

```
macro time(ex)
    ...
    local val = $(esc(ex))
    ...
end
```

An expression wrapped in this manner is left alone by the macro expander and simply pasted into the output verbatim. Therefore it will be resolved in the macro call environment.

This escaping mechanism can be used to "violate" hygiene when necessary, in order to introduce or manipulate user variables. For example, the following macro sets x to zero in the call environment:

```
macro zerox()
  return esc(:(x = 0))
end
```

```
function foo()
    x = 1
    @zerox
    x # is zero
end
```

This kind of manipulation of variables should be used judiciously, but is occasionally quite handy.

### 16.4 Code Generation

When a significant amount of repetitive boilerplate code is required, it is common to generate it programmatically to avoid redundancy. In most languages, this requires an extra build step, and a separate program to generate the repetitive code. In Julia, expression interpolation and eval () (page 325) allow such code generation to take place in the normal course of program execution. For example, the following code defines a series of operators on three arguments in terms of their 2-argument forms:

```
for op = (:+, :*, :&, :|, :$)
  eval(quote
     ($op)(a,b,c) = ($op)(($op)(a,b),c)
  end)
end
```

In this manner, Julia acts as its own preprocessor, and allows code generation from inside the language. The above code could be written slightly more tersely using the : prefix quoting form:

```
for op = (:+, :*, :&, :|, :$)
  eval(:(($op)(a,b,c) = ($op)(($op)(a,b),c)))
end
```

This sort of in-language code generation, however, using the eval (quote (...)) pattern, is common enough that Julia comes with a macro to abbreviate this pattern:

```
for op = (:+, :*, :&, :|, :$)
  @eval ($op)(a,b,c) = ($op)(($op)(a,b),c)
end
```

The @eval (page 325) macro rewrites this call to be precisely equivalent to the above longer versions. For longer blocks of generated code, the expression argument given to @eval (page 325) can be a block:

```
@eval begin
  # multiple lines
end
```

# 16.5 Non-Standard String Literals

Recall from *Strings* (page 49) that string literals prefixed by an identifier are called non-standard string literals, and can have different semantics than un-prefixed string literals. For example:

- r"^\s\* (?: #|\$) " produces a regular expression object rather than a string
- b"DATA\xff\u2200" is a byte array literal for [68, 65, 84, 65, 255, 226, 136, 128].

Perhaps surprisingly, these behaviors are not hard-coded into the Julia parser or compiler. Instead, they are custom behaviors provided by a general mechanism that anyone can use: prefixed string literals are parsed as calls to specially-named macros. For example, the regular expression macro is just the following:

16.4. Code Generation 151

```
macro r_str(p)
  Regex(p)
end
```

That's all. This macro says that the literal contents of the string literal  $r"^\s*(?:\#|\$)$ " should be passed to the @r\_str macro and the result of that expansion should be placed in the syntax tree where the string literal occurs. In other words, the expression  $r"^\s*(?:\#|\$)$ " is equivalent to placing the following object directly into the syntax tree:

```
Regex("^\\s*(?:#|\$)")
```

Not only is the string literal form shorter and far more convenient, but it is also more efficient: since the regular expression is compiled and the Regex object is actually created *when the code is compiled*, the compilation occurs only once, rather than every time the code is executed. Consider if the regular expression occurs in a loop:

```
for line = lines
  m = match(r"^\s*(?:#|$)", line)
  if m == nothing
    # non-comment
  else
    # comment
  end
end
```

Since the regular expression  $r"^{\ }s*(?;\#|\$)$ " is compiled and inserted into the syntax tree when this code is parsed, the expression is only compiled once instead of each time the loop is executed. In order to accomplish this without macros, one would have to write this loop like this:

```
re = Regex("^\\s*(?:#|\$)")
for line = lines
  m = match(re, line)
  if m == nothing
    # non-comment
  else
    # comment
  end
end
```

Moreover, if the compiler could not determine that the regex object was constant over all loops, certain optimizations might not be possible, making this version still less efficient than the more convenient literal form above. Of course, there are still situations where the non-literal form is more convenient: if one needs to interpolate a variable into the regular expression, one must take this more verbose approach; in cases where the regular expression pattern itself is dynamic, potentially changing upon each loop iteration, a new regular expression object must be constructed on each iteration. In the vast majority of use cases, however, regular expressions are not constructed based on run-time data. In this majority of cases, the ability to write regular expressions as compile-time values is invaluable.

The mechanism for user-defined string literals is deeply, profoundly powerful. Not only are Julia's non-standard literals implemented using it, but also the command literal syntax ('echo "Hello, Sperson"') is implemented with the following innocuous-looking macro:

```
macro cmd(str)
    :(cmd_gen($shell_parse(str)))
end
```

Of course, a large amount of complexity is hidden in the functions used in this macro definition, but they are just functions, written entirely in Julia. You can read their source and see precisely what they do — and all they do is construct expression objects to be inserted into your program's syntax tree.

### 16.6 Generated functions

A very special macro is @generated, which allows you to define so-called *generated functions*. These have the capability to generate specialized code depending on the types of their arguments with more flexibility and/or less code than what can be achieved with multiple dispatch. While macros work with expressions at parsing-time and cannot access the types of their inputs, a generated function gets expanded at a time when the types of the arguments are known, but the function is not yet compiled.

Instead of performing some calculation or action, a generated function declaration returns a quoted expression which then forms the body for the method corresponding to the types of the arguments. When called, the body expression is compiled (or fetched from a cache, on subsequent calls) and only the returned expression - not the code that generated it - is evaluated. Thus, generated functions provide a flexible framework to move work from run-time to compile-time.

When defining generated functions, there are three main differences to ordinary functions:

- 1. You annotate the function declaration with the @generated macro. This adds some information to the AST that lets the compiler know that this is a generated function.
- 2. In the body of the generated function you only have access to the *types* of the arguments, not their values.
- 3. Instead of calculating something or performing some action, you return a *quoted expression* which, when evaluated, does what you want.

It's easiest to illustrate this with an example. We can declare a generated function foo as

Note that the body returns a quoted expression, namely : (x\*x), rather than just the value of x\*x.

From the caller's perspective, they are very similar to regular functions; in fact, you don't have to know if you're calling a regular or generated function - the syntax and result of the call is just the same. Let's see how £00 behaves:

```
# note: output is from println() statement in the body
julia> x = foo(2);
Int64

julia> x  # now we print x

4

julia> y = foo("bar");
ASCIIString

julia> y
"barbar"
```

So, we see that in the body of the generated function, x is the *type* of the passed argument, and the value returned by the generated function, is the result of evaluating the quoted expression we returned from the definition, now with the *value* of x.

What happens if we evaluate foo again with a type that we have already used?

```
julia> foo(4)
16
```

Note that there is no printout of Int64. The body of the generated function is only executed *once* (not entirely true, see note below) when the method for that specific set of argument types is compiled. After that, the expression returned from the generated function on the first invocation is re-used as the method body.

The reason for the disclaimer above is that the number of times a generated function is generated is really an implementation detail; it *might* be only once, but it *might* also be more often. As a consequence, you should *never* write a generated function with side effects - when, and how often, the side effects occur is undefined. (This is true for macros too - and just like for macros, the use of eval () (page 325) in a generated function is a sign that you're doing something the wrong way.)

The example generated function  $f \circ \circ$  above did not do anything a normal function  $f \circ \circ (x) = x * x$  could not do, except printing the type on the first invocation (and incurring a higher compile-time cost). However, the power of a generated function lies in its ability to compute different quoted expression depending on the types passed to it:

(although of course this contrived example is easily implemented using multiple dispatch...)

We can, of course, abuse this to produce some interesting behavior:

```
julia> @generated function baz(x)
    if rand() < .9
        return : (x^2)
    else
        return : ("boo!")
    end
    end
baz (generic function with 1 method)</pre>
```

Since the body of the generated function is non-deterministic, its behavior is undefined; the expression returned on the *first* invocation will be used for *all* subsequent invocations with the same type (again, with the exception covered by the disclaimer above). When we call the generated function with x of a new type, rand() (page 383) will be called again to see which method body to use for the new type. In this case, for one *type* out of ten, baz(x) will return the string "boo!".

Don't copy these examples!

These examples are hopefully helpful to illustrate how generated functions work, both in the definition end and at the call site; however, *don't copy them*, for the following reasons:

- the foo function has side-effects, and it is undefined exactly when, how often or how many times these side-effects will occur
- the bar function solves a problem that is better solved with multiple dispatch defining bar (x) = x and bar  $(x::Integer) = x^2$  will do the same thing, but it is both simpler and faster.
- the baz function is pathologically insane

Instead, now that we have a better understanding for how generated functions work, let's use them to build some more advanced functionality...

### 16.6.1 An advanced example

Julia's base library has a sub2ind() (page 394) function to calculate a linear index into an n-dimensional array, based on a set of n multilinear indices - in other words, to calculate the index i that can be used to index into an array A using A[i], instead of A[x,y,z,...]. One possible implementation is the following:

```
function sub2ind_loop{N} (dims::NTuple{N}, I::Integer...)
   ind = I[N] - 1
   for i = N-1:-1:1
      ind = I[i]-1 + dims[i]*ind
   end
   return ind + 1
end
```

The same thing can be done using recursion:

```
sub2ind_rec(dims::Tuple{}) = 1
sub2ind_rec(dims::Tuple{},i1::Integer, I::Integer...) =
    i1==1 ? sub2ind_rec(dims,I...) : throw(BoundsError())
sub2ind_rec(dims::Tuple{Integer, Vararg{Integer}}, i1::Integer) = i1
sub2ind_rec(dims::Tuple{Integer, Vararg{Integer}}, i1::Integer, I::Integer...) =
    i1 + dims[1]*(sub2ind_rec(tail(dims),I...)-1)
```

Both these implementations, although different, do essentially the same thing: a runtime loop over the dimensions of the array, collecting the offset in each dimension into the final index.

However, all the information we need for the loop is embedded in the type information of the arguments. Thus, we can utilize generated functions to move the iteration to compile-time; in compiler parlance, we use generated functions to manually unroll the loop. The body becomes almost identical, but instead of calculating the linear index, we build up an *expression* that calculates the index:

```
@generated function sub2ind_gen{N}(dims::NTuple{N}, I::Integer...)
    ex = :(I[$N] - 1)
    for i = N-1:-1:1
        ex = :(I[$i] - 1 + dims[$i]*$ex)
    end
    return :($ex + 1)
```

#### What code will this generate?

An easy way to find out, is to extract the body into another (regular) function:

We can now execute sub2ind\_gen\_impl and examine the expression it returns:

```
julia> sub2ind_gen_impl(Tuple{Int,Int}, Int, Int)
:(((I[1] - 1) + dims[1] * (I[2] - 1)) + 1)
```

So, the method body that will be used here doesn't include a loop at all - just indexing into the two tuples, multiplication and addition/subtraction. All the looping is performed compile-time, and we avoid looping during execution entirely. Thus, we only loop *once per type*, in this case once per N (except in edge cases where the function is generated more than once - see disclaimer above).

# **Multi-dimensional Arrays**

Julia, like most technical computing languages, provides a first-class array implementation. Most technical computing languages pay a lot of attention to their array implementation at the expense of other containers. Julia does not treat arrays in any special way. The array library is implemented almost completely in Julia itself, and derives its performance from the compiler, just like any other code written in Julia. As such, it's also possible to define custom array types by inheriting from AbstractArray. See the manual section on the AbstractArray interface for more details on implementing a custom array type.

An array is a collection of objects stored in a multi-dimensional grid. In the most general case, an array may contain objects of type Any. For most computational purposes, arrays should contain objects of a more specific type, such as Float 64 or Int 32.

In general, unlike many other technical computing languages, Julia does not expect programs to be written in a vectorized style for performance. Julia's compiler uses type inference and generates optimized code for scalar array indexing, allowing programs to be written in a style that is convenient and readable, without sacrificing performance, and using less memory at times.

In Julia, all arguments to functions are passed by reference. Some technical computing languages pass arrays by value, and this is convenient in many cases. In Julia, modifications made to input arrays within a function will be visible in the parent function. The entire Julia array library ensures that inputs are not modified by library functions. User code, if it needs to exhibit similar behavior, should take care to create a copy of inputs that it may modify.

# 17.1 Arrays

#### 17.1.1 Basic Functions

Function	Description
eltype (A) (page 337)	the type of the elements contained in A
length (A) (page 385)	the number of elements in A
ndims (A) (page 393)	the number of dimensions of A
size (A) (page 393)	a tuple containing the dimensions of A
size (A, n) (page 393)	the size of A in a particular dimension
eachindex (A) (page 393)	an efficient iterator for visiting each position in A
stride(A, k) (page 394)	the stride (linear index distance between adjacent elements) along dimension k
strides (A) (page 394)	a tuple of the strides in each dimension

#### 17.1.2 Construction and Initialization

Many functions for constructing and initializing arrays are provided. In the following list of such functions, calls with a dims... argument can either take a single tuple of dimension sizes or a series of dimension sizes passed as a variable number of arguments.

Function	Description
Array(type,	an uninitialized dense array
dims) (page 395)	
cell(dims)	an uninitialized cell array (heterogeneous array)
(page 395)	
zeros(type,	an array of all zeros of specified type, defaults to Float 64 if type not specified
dims) (page 395)	
zeros (A) (page 395)	an array of all zeros of same element type and shape of A
ones(type, dims)	an array of all ones of specified type, defaults to Float 64 if type not specified
(page 395)	
ones (A) (page 395)	an array of all ones of same element type and shape of A
trues(dims)	a Bool array with all values true
(page 395)	
falses(dims)	a Bool array with all values false
(page 395)	
reshape(A, dims)	an array with the same data as the given array, but with different dimensions.
(page 395)	
copy (A) (page 321)	сору А
deepcopy (A) (page 321)	copy A, recursively copying its elements
similar(A,	an uninitialized array of the same type as the given array (dense, sparse, etc.), but
element_type,	with the specified element type and dimensions. The second and third arguments
dims) (page 395)	are both optional, defaulting to the element type and dimensions of A if omitted.
reinterpret(type,	an array with the same binary data as the given array, but with the specified
A) (page 396)	element type
rand(dims) (page 383)	Array (page 395) of Float 64s with random, iid[#]_ and uniformly distributed
	values in the half-open interval $[0,1)$
randn(dims) (page 383)	Array (page 395) of Float 64s with random, iid and standard normally
	distributed random values
eye(n) (page 396)	n-by-n identity matrix
eye (m, n) (page 396)	m-by-n identity matrix
linspace(start,	range of n linearly spaced elements from start to stop
stop, n) (page 396)	
fill! (A, x) (page 395)	fill the array A with the value x
fill(x, dims)	create an array filled with the value x
(page 395)	

The syntax  $[A, B, C, \ldots]$  constructs a 1-d array (vector) of its arguments.

#### 17.1.3 Concatenation

Arrays can be constructed and also concatenated using the following functions:

Function	Description
cat (k, A) (page 397)	concatenate input n-d arrays along the dimension k
vcat (A) (page 397)	shorthand for cat (1, A)
hcat (A) (page 397)	shorthand for cat (2, A)

Scalar values passed to these functions are treated as 1-element arrays.

The concatenation functions are used so often that they have special syntax:

Expression	Calls		
[A; B; C;]	vcat () (page 397)		
[A B C]	hcat () (page 397)		
[A B; C D;]	hvcat () (page 397)		

hycat () (page 397) concatenates in both dimension 1 (with semicolons) and dimension 2 (with spaces).

### 17.1.4 Typed array initializers

An array with a specific element type can be constructed using the syntax  $T[A, B, C, \ldots]$ . This will construct a 1-d array with element type T, initialized to contain elements A, B, C, etc. For example Any[x, y, z] constructs a heterogeneous array that can contain any values.

Concatenation syntax can similarly be prefixed with a type to specify the element type of the result.

```
julia> [[1 2] [3 4]]
1x4 Array{Int64,2}:
1 2 3 4

julia> Int8[[1 2] [3 4]]
1x4 Array{Int8,2}:
1 2 3 4
```

### 17.1.5 Comprehensions

Comprehensions provide a general and powerful way to construct arrays. Comprehension syntax is similar to set construction notation in mathematics:

```
A = [F(x,y,...) \text{ for } x=rx, y=ry, ...]
```

The meaning of this form is that F(x,y,...) is evaluated with the variables x, y, etc. taking on each value in their given list of values. Values can be specified as any iterable object, but will commonly be ranges like 1:n or 2:(n-1), or explicit arrays of values like [1.2, 3.4, 5.7]. The result is an N-d dense array with dimensions that are the concatenation of the dimensions of the variable ranges rx, ry, etc. and each F(x,y,...) evaluation returns a scalar.

The following example computes a weighted average of the current element and its left and right neighbor along a 1-d grid. :

```
julia> const x = rand(8)
8-element Array{Float64,1}:
    0.843025
    0.869052
    0.365105
    0.699456
    0.977653
    0.994953
    0.41084
    0.809411

julia> [ 0.25*x[i-1] + 0.5*x[i] + 0.25*x[i+1] for i=2:length(x)-1 ]
6-element Array{Float64,1}:
    0.736559
    0.57468
    0.685417
```

17.1. Arrays 159

```
0.912429
0.8446
0.656511
```

**Note:** In the above example, x is declared as constant because type inference in Julia does not work as well on non-constant global variables.

The resulting array type is inferred from the expression; in order to control the type explicitly, the type can be prepended to the comprehension. For example, in the above example we could have avoided declaring x as constant, and ensured that the result is of type Float 64 by writing:

```
Float64[ 0.25*x[i-1] + 0.5*x[i] + 0.25*x[i+1] for i=2:length(x)-1 ]
```

### 17.1.6 Indexing

The general syntax for indexing into an n-dimensional array A is:

```
X = A[I_1, I_2, ..., I_n]
```

where each I\_k may be:

- 1. A scalar integer
- 2. A Range of the form a:b, or a:b:c
- 3. A: or Colon() to select entire dimensions
- 4. An arbitrary integer vector, including the empty vector []
- 5. A boolean vector

The result X generally has dimensions (length( $I_1$ ), length( $I_2$ ), ..., length( $I_n$ )), with location (i\_1, i\_2, ..., i\_n) of X containing the value A[I\_1[i\_1], I\_2[i\_2], ..., I\_n[i\_n]]. All dimensions indexed with scalars are dropped. For example, the result of A[2, I, 3] will be a vector with size (length(I),). Boolean vectors are first transformed with find; the size of a dimension indexed by a boolean vector will be the number of true values in the vector. As a special part of this syntax, the end keyword may be used to represent the last index of each dimension within the indexing brackets, as determined by the size of the innermost array being indexed.

Alternatively, single elements of a multidimensional array can be indexed as

```
x = A[I]
```

where I is a Cartesian Index, effectively an n-tuple of integers. See Iteration (page 162) below.

Indexing syntax is equivalent to a call to getindex:

```
X = getindex(A, I_1, I_2, ..., I_n)
```

#### Example:

```
julia> x = reshape(1:16, 4, 4)
4x4 Array{Int64,2}:
1  5  9  13
2  6  10  14
3  7  11  15
4  8  12  16
```

```
julia> x[2:3, 2:end-1]
2x2 Array{Int64,2}:
6  10
7  11
```

Empty ranges of the form n: n-1 are sometimes used to indicate the inter-index location between n-1 and n. For example, the searchsorted() (page 467) function uses this convention to indicate the insertion point of a value not found in a sorted array:

```
julia> a = [1,2,5,6,7];

julia> searchsorted(a, 3)
3:2
```

### 17.1.7 Assignment

The general syntax for assigning values in an n-dimensional array A is:

```
A[I_1, I_2, ..., I_n] = X
```

where each I\_k may be:

- 1. A scalar integer
- 2. A Range of the form a:b, or a:b:c
- 3. A: or Colon() to select entire dimensions
- 4. An arbitrary integer vector, including the empty vector []
- 5. A boolean vector

If X is an array, its size must be  $(length(I_1), length(I_2), \ldots, length(I_n))$ , and the value in location i\_1, i\_2, ..., i\_n of A is overwritten with the value  $X[I_1[i_1], I_2[i_2], \ldots, I_n[i_n]$ . If X is not an array, its value is written to all referenced locations of A.

A boolean vector used as an index behaves as in getindex() (page 342) (it is first transformed with find() (page 398)).

Index assignment syntax is equivalent to a call to setindex! () (page 342):

```
setindex!(A, X, I_1, I_2, ..., I_n)
```

#### Example:

```
julia> x = reshape(1:9, 3, 3)
3x3 Array{Int64,2}:
    1    4    7
    2    5    8
    3    6    9

julia> x[1:2, 2:3] = -1
-1

julia> x
3x3 Array{Int64,2}:
    1    -1    -1
    2    -1    -1
    3    6    9
```

17.1. Arrays 161

#### 17.1.8 Iteration

The recommended ways to iterate over a whole array are

```
for a in A
    # Do something with the element a
end

for i in eachindex(A)
    # Do something with i and/or A[i]
end
```

The first construct is used when you need the value, but not index, of each element. In the second construct, i will be an Int if A is an array type with fast linear indexing; otherwise, it will be a CartesianIndex:

```
A = rand(4,3)
B = sub(A, 1:3, 2:3)
julia> for i in eachindex(B)
    @show i
end
i = Base.IteratorsMD.CartesianIndex_2(1,1)
i = Base.IteratorsMD.CartesianIndex_2(2,1)
i = Base.IteratorsMD.CartesianIndex_2(3,1)
i = Base.IteratorsMD.CartesianIndex_2(1,2)
i = Base.IteratorsMD.CartesianIndex_2(1,2)
i = Base.IteratorsMD.CartesianIndex_2(2,2)
i = Base.IteratorsMD.CartesianIndex_2(3,2)
```

In contrast with for i = 1: length (A), iterating with eachindex provides an efficient way to iterate over any array type.

### 17.1.9 Array traits

If you write a custom AbstractArray type, you can specify that it has fast linear indexing using

```
Base.linearindexing{T<:MyArray}(::Type{T}) = LinearFast()</pre>
```

This setting will cause eachindex iteration over a MyArray to use integers. If you don't specify this trait, the default value LinearSlow() is used.

### 17.1.10 Vectorized Operators and Functions

The following operators are supported for arrays. The dot version of a binary operator should be used for elementwise operations.

- 1. Unary arithmetic -, +, !
- 2. Binary arithmetic -+, -,  $\star$ ,  $\cdot$ , div, mod
- 3. Comparison .==, .!=, .<, .<=, .>, .>=
- 4. Unary Boolean or bitwise ~
- 5. Binary Boolean or bitwise &, |, \$

Some operators without dots operate elementwise anyway when one argument is a scalar. These operators are  $\star$ , +, -, and the bitwise operators. The operators / and \ operate elementwise when the denominator is a scalar.

Note that comparisons such as == operate on whole arrays, giving a single boolean answer. Use dot operators for elementwise comparisons.

The following built-in functions are also vectorized, whereby the functions act elementwise:

```
abs abs2 angle cbrt
airy airyai airyaiprime airybi airybiprime airyprime
acos acosh asin asinh atan atan2 atanh
acsc acsch asec asech acot acoth
cos cospi cosh sin sinpi sinh tan tanh sinc cosc
csc csch sec sech cot coth
acosd asind atand asecd acscd acotd
cosd sind tand secd cscd cotd
besselh besseli besselj besselj0 besselj1 besselk bessely bessely0 bessely1
exp erf erfc erfinv erfcinv exp2 expm1
beta dawson digamma erfcx erfi
exponent eta zeta gamma
hankelh1 hankelh2
ceil floor round trunc
isfinite isinf isnan
lbeta lfact lgamma
log log10 log1p log2
copysign max min significand
sqrt hypot
```

Note that there is a difference between min() (page 362) and max() (page 362), which operate elementwise over multiple array arguments, and minimum() (page 338) and maximum() (page 338), which find the smallest and largest values within an array.

Julia provides the @vectorize\_larg() and @vectorize\_2arg() macros to automatically vectorize any function of one or two arguments respectively. Each of these takes two arguments, namely the Type of argument (which is usually chosen to be the most general possible) and the name of the function to vectorize. Here is a simple example:

#### 17.1.11 Broadcasting

It is sometimes useful to perform element-by-element binary operations on arrays of different sizes, such as adding a vector to each column of a matrix. An inefficient way to do this would be to replicate the vector to the size of the matrix:

```
julia> a = rand(2,1); A = rand(2,3);

julia> repmat(a,1,3)+A
2x3 Array{Float64,2}:
```

17.1. Arrays 163

```
1.20813 1.82068 1.25387
1.56851 1.86401 1.67846
```

This is wasteful when dimensions get large, so Julia offers broadcast () (page 396), which expands singleton dimensions in array arguments to match the corresponding dimension in the other array without using extra memory, and applies the given function elementwise:

```
julia> broadcast(+, a, A)
2x3 Array{Float64,2}:
1.20813   1.82068   1.25387
1.56851   1.86401   1.67846

julia> b = rand(1,2)
1x2 Array{Float64,2}:
   0.867535   0.00457906

julia> broadcast(+, a, b)
2x2 Array{Float64,2}:
1.71056   0.847604
1.73659   0.873631
```

Elementwise operators such as .+ and .\* perform broadcasting if necessary. There is also a <code>broadcast!()</code> (page 396) function to specify an explicit destination, and <code>broadcast\_getindex()</code> (page 397) and <code>broadcast\_setindex!()</code> (page 397) that broadcast the indices before indexing.

### 17.1.12 Implementation

The base array type in Julia is the abstract type AbstractArray{T,N}. It is parametrized by the number of dimensions N and the element type T. AbstractVector and AbstractMatrix are aliases for the 1-d and 2-d cases. Operations on AbstractArray objects are defined using higher level operators and functions, in a way that is independent of the underlying storage. These operations generally work correctly as a fallback for any specific array implementation.

The AbstractArray type includes anything vaguely array-like, and implementations of it might be quite different from conventional arrays. For example, elements might be computed on request rather than stored. However, any concrete AbstractArray{T,N} type should generally implement at least size(A) (page 393) (returning an Int tuple), getindex(A,i) (page 342) and getindex(A,i1,...,iN) (page 342); mutable arrays should also implement setindex! () (page 342). It is recommended that these operations have nearly constant time complexity, or technically  $\tilde{O}(1)$  complexity, as otherwise some array functions may be unexpectedly slow. Concrete types should also typically provide a similar(A, T=eltype(A), dims=size(A)) (page 395) method, which is used to allocate a similar array for copy() (page 321) and other out-of-place operations. No matter how an AbstractArray{T,N} is represented internally, T is the type of object returned by integer indexing (A[1, ..., 1], when A is not empty) and N should be the length of the tuple returned by size() (page 393).

DenseArray is an abstract subtype of AbstractArray intended to include all arrays that are laid out at regular offsets in memory, and which can therefore be passed to external C and Fortran functions expecting this memory layout. Subtypes should provide a method stride(A, k) (page 394) that returns the "stride" of dimension k: increasing the index of dimension k by 1 should increase the index i of getindex(A, i) (page 342) by stride(A, k) (page 394). If a pointer conversion method  $Base.unsafe\_convert(Ptr{T}, A)$  (page 481) is provided, the memory layout should correspond in the same way to these strides.

The Array (page 395) type is a specific instance of DenseArray where elements are stored in column-major order (see additional notes in *Performance Tips* (page 265)). Vector and Matrix are aliases for the 1-d and 2-d cases. Specific operations such as scalar indexing, assignment, and a few other basic storage-specific operations are all that have to be implemented for *Array* (page 395), so that the rest of the array library can be implemented in a generic manner.

SubArray is a specialization of AbstractArray that performs indexing by reference rather than by copying. A SubArray is created with the sub() (page 397) function, which is called the same way as getindex() (page 342) (with an array and a series of index arguments). The result of sub() (page 397) looks the same as the result of getindex() (page 342), except the data is left in place. sub() (page 397) stores the input index vectors in a SubArray object, which can later be used to index the original array indirectly.

StridedVector and StridedMatrix are convenient aliases defined to make it possible for Julia to call a wider range of BLAS and LAPACK functions by passing them either Array (page 395) or SubArray objects, and thus saving inefficiencies from memory allocation and copying.

The following example computes the QR decomposition of a small section of a larger array, without creating any temporaries, and by calling the appropriate LAPACK function with the right leading dimension size and stride parameters.

```
iulia > a = rand(10.10)
10x10 Array{Float64,2}:
0.217964
0.718915 0.537192
                    0.556946 0.996234 0.666232 0.509423 0.660788
0.493501 0.0565622 0.118392 0.493498
                                        0.262048 0.940693
                                                           0.252965
0.0470779 0.736979 0.264822 0.228787
                                         0.161441 0.897023
                                                           0.567641
          0.32327
                    0.795673 0.452242
0.343935
                                         0.468819 0.628507
                                                           0.511528
                                      ... 0.84589 0.178834
0.935597
          0.991511
                    0.571297 0.74485
                                                            0.284413
                    0.133158 0.65554
                                       0.371826 0.770628
0.160706 0.672252
                                                           0.0531208
         0.836126
                    0.301198 0.0224702
                                         0.39344 0.0370205 0.536062
0.306617
        0.168877
                                         0.096078 0.172048
0.890947
                    0.32002 0.486136
                                                           0.77672
         0.573567 0.220124 0.165816
0.507762
                                         0.211049 0.433277
                                                           0.539476
julia> b = sub(a, 2:2:8, 2:2:4)
4x2 SubArray{Float64,2,Array{Float64,2},Tuple{StepRange{Int64,Int64},StepRange{Int64,Int64}},1):
0.537192 0.996234
0.736979 0.228787
0.991511 0.74485
0.836126 0.0224702
julia> (q,r) = qr(b);
julia> q
4x2 Array{Float64,2}:
-0.338809 0.78934
-0.464815 -0.230274
-0.625349 0.194538
-0.527347 -0.534856
julia> r
2x2 Array{Float64,2}:
-1.58553 -0.921517
          0.866567
```

# 17.2 Sparse Matrices

Sparse matrices are matrices that contain enough zeros that storing them in a special data structure leads to savings in space and execution time. Sparse matrices may be used when operations on the sparse representation of a matrix lead to considerable gains in either time or space when compared to performing the same operations on a dense matrix.

### 17.2.1 Compressed Sparse Column (CSC) Storage

In Julia, sparse matrices are stored in the Compressed Sparse Column (CSC) format. Julia sparse matrices have the type SparseMatrixCSC{Tv,Ti}, where Tv is the type of the nonzero values, and Ti is the integer type for storing column pointers and row indices.:

The compressed sparse column storage makes it easy and quick to access the elements in the column of a sparse matrix, whereas accessing the sparse matrix by rows is considerably slower. Operations such as insertion of nonzero values one at a time in the CSC structure tend to be slow. This is because all elements of the sparse matrix that are beyond the point of insertion have to be moved one place over.

All operations on sparse matrices are carefully implemented to exploit the CSC data structure for performance, and to avoid expensive operations.

If you have data in CSC format from a different application or library, and wish to import it in Julia, make sure that you use 1-based indexing. The row indices in every column need to be sorted. If your SparseMatrixCSC object contains unsorted row indices, one quick way to sort them is by doing a double transpose.

In some applications, it is convenient to store explicit zero values in a SparseMatrixCSC. These *are* accepted by functions in Base (but there is no guarantee that they will be preserved in mutating operations). Such explicitly stored zeros are treated as structural nonzeros by many routines. The nnz() (page 403) function returns the number of elements explicitly stored in the sparse data structure, including structural nonzeros. In order to count the exact number of actual values that are nonzero, use countnz() (page 394), which inspects every stored element of a sparse matrix.

#### 17.2.2 Sparse matrix constructors

The simplest way to create sparse matrices is to use functions equivalent to the zeros() (page 395) and eye() (page 396) functions that Julia provides for working with dense matrices. To produce sparse matrices instead, you can use the same names with an sp prefix:

```
julia> spzeros(3,5)
3x5 sparse matrix with 0 Float64 entries:

julia> speye(3,5)
3x5 sparse matrix with 3 Float64 entries:
    [1, 1] = 1.0
    [2, 2] = 1.0
    [3, 3] = 1.0
```

The sparse() (page 402) function is often a handy way to construct sparse matrices. It takes as its input a vector I of row indices, a vector J of column indices, and a vector V of nonzero values. sparse(I, J, V) constructs a sparse matrix such that S[I[k], J[k]] = V[k].

```
julia> I = [1, 4, 3, 5]; J = [4, 7, 18, 9]; V = [1, 2, -5, 3];

julia> S = sparse(I,J,V)
5x18 sparse matrix with 4 Int64 entries:
        [1, 4] = 1
        [4, 7] = 2
```

```
[5 , 9] = 3
[3 , 18] = -5
```

The inverse of the *sparse()* (page 402) function is *findn()* (page 398), which retrieves the inputs used to create the sparse matrix.

```
julia> findn(S)
([1,4,5,3],[4,7,9,18])

julia> findnz(S)
([1,4,5,3],[4,7,9,18],[1,2,3,-5])
```

Another way to create sparse matrices is to convert a dense matrix into a sparse matrix using the *sparse()* (page 402) function:

```
julia> sparse(eye(5))
5x5 sparse matrix with 5 Float64 entries:
    [1, 1] = 1.0
    [2, 2] = 1.0
    [3, 3] = 1.0
    [4, 4] = 1.0
    [5, 5] = 1.0
```

You can go in the other direction using the full() (page 413) function. The issparse() (page 402) function can be used to query if a matrix is sparse.

```
julia> issparse(speye(5))
true
```

## 17.2.3 Sparse matrix operations

Arithmetic operations on sparse matrices also work as they do on dense matrices. Indexing of, assignment into, and concatenation of sparse matrices work in the same way as dense matrices. Indexing operations, especially assignment, are expensive, when carried out one element at a time. In many cases it may be better to convert the sparse matrix into (I, J, V) format using findnz() (page 398), manipulate the non-zeroes or the structure in the dense vectors (I, J, V), and then reconstruct the sparse matrix.

## 17.2.4 Correspondence of dense and sparse methods

The following table gives a correspondence between built-in methods on sparse matrices and their corresponding methods on dense matrix types. In general, methods that generate sparse matrices differ from their dense counterparts in that the resulting matrix follows the same sparsity pattern as a given sparse matrix S, or that the resulting sparse matrix has density d, i.e. each matrix element has a probability d of being non-zero.

Details can be found in the Sparse Vectors and Matrices (page 402) section of the standard library reference.

Sparse	Dense	Description
spzeros (m, n) (page 403)	zeros (m, n) (page 395)	Creates a <i>m</i> -by- <i>n</i> matrix of zeros.
		(spzeros (m, n) (page 403) is
		empty.)
spones(S) (page 403)	ones (m, n) (page 395)	Creates a matrix filled with ones.
		Unlike the dense version, spones ()
		(page 403) has the same sparsity
		pattern as S.
speye(n) (page 403)	eye (n) (page 396)	Creates a <i>n</i> -by- <i>n</i> identity matrix.
full(S) (page 413)	sparse (A) (page 402)	Interconverts between dense and
		sparse formats.
sprand(m, n, d) (page 403)	rand(m, n) (page 383)	Creates a <i>m</i> -by- <i>n</i> random matrix (of
		density d) with iid non-zero elements
		distributed uniformly on the half-open
		interval $[0,1)$ .
sprandn(m, n, d) (page 403)	randn(m, n) (page 383)	Creates a <i>m</i> -by- <i>n</i> random matrix (of
		density d) with iid non-zero elements
		distributed according to the standard
		normal (Gaussian) distribution.
sprandn(m, n, d, X) (page 403)	randn(m, n, X) (page 383)	Creates a <i>m</i> -by- <i>n</i> random matrix (of
		density d) with iid non-zero elements
		distributed according to the <i>X</i>
		distribution. (Requires the
		Distributions package.)
sprandbool(m,n,d)(page 403)	rand(Bool, m, n) (page 383)	Creates a <i>m</i> -by- <i>n</i> random matrix (of
		density <i>d</i> ) with non-zero Bool
		elements with probability $d$ ( $d$ =0.5
		for rand (Bool) (page 383).)

# Linear algebra

### 18.1 Matrix factorizations

Matrix factorizations (a.k.a. matrix decompositions) compute the factorization of a matrix into a product of matrices, and are one of the central concepts in linear algebra.

The following table summarizes the types of matrix factorizations that have been implemented in Julia. Details of their associated methods can be found in the *Linear Algebra* (page 413) section of the standard library documentation.

Cholesky	Cholesky factorization
CholeskyPivoted	Pivoted Cholesky factorization
LU	LU factorization
LUTridiagonal	LU factorization for Tridiagonal matrices
UmfpackLU	LU factorization for sparse matrices (computed by UMFPack)
QR	QR factorization
QRCompactWY	Compact WY form of the QR factorization
QRPivoted	Pivoted QR factorization
Hessenberg	Hessenberg decomposition
Eigen	Spectral decomposition
SVD	Singular value decomposition
GeneralizedSVD	Generalized SVD

# 18.2 Special matrices

Matrices with special symmetries and structures arise often in linear algebra and are frequently associated with various matrix factorizations. Julia features a rich collection of special matrix types, which allow for fast computation with specialized routines that are specially developed for particular matrix types.

The following tables summarize the types of special matrices that have been implemented in Julia, as well as whether hooks to various optimized methods for them in LAPACK are available.

Hermitian	Hermitian matrix
UpperTriangular	Upper triangular matrix
LowerTriangular	Lower triangular matrix
Tridiagonal (page 422)	Tridiagonal matrix
SymTridiagonal (page 422)	Symmetric tridiagonal matrix
Bidiagonal (page 422)	Upper/lower bidiagonal matrix
Diagonal	Diagonal matrix
UniformScaling	Uniform scaling operator

# 18.2.1 Elementary operations

Matrix type	+	_	*	\	Other functions with optimized methods
Hermitian				MV	inv() (page 424), sqrtm() (page 425), expm()
					(page 424)
UpperTriangular			MV	MV	inv() (page 424), det() (page 423)
LowerTriangular			MV	MV	inv() (page 424), det() (page 423)
SymTridiagonal	M	M	MS	MV	eigmax() (page 418), eigmin() (page 418)
(page 422)					
Tridiagonal	M	M	MS	MV	
(page 422)					
Bidiagonal	M	M	MS	MV	
(page 422)					
Diagonal	M	M	MV	MV	inv() (page 424), det() (page 423), logdet()
					(page 424), / () (page 353)
UniformScaling	M	M	MVS	MVS	/() (page 353)

## Legend:

M (matrix)	An optimized method for matrix-matrix operations is available
V (vector)	An optimized method for matrix-vector operations is available
S (scalar)	An optimized method for matrix-scalar operations is available

### 18.2.2 Matrix factorizations

Matrix type	LA-	eig()	eigvals()	eigvecs()	svd()	svdvals()
	PACK	(page 418)	(page 418)	(page 419)	(page 421)	(page 421)
Hermitian	HE		ARI			
UpperTriangula	ırTR	A	A	A		
LowerTriangula	ırTR	A	A	A		
SymTridiagonal	ST	A	ARI	AV		
(page 422)						
Tridiagonal	GT					
(page 422)						
Bidiagonal	BD				A	A
(page 422)						
Diagonal	DI		A			

## Legend:

A (all)	An optimized method to find all the characteristic values and/or vectors is available	e.g.
		eigvals(M)
R	An optimized method to find the ilth through the ihth characteristic values are	eigvals(M,
(range)	available	il, ih)
I (in-	An optimized method to find the characteristic values in the interval [vl, vh] is	eigvals(M,
terval)	available	vl, vh)
V	An optimized method to find the characteristic vectors corresponding to the	eigvecs(M,
(vec-	characteristic values $x = [x1, x2,]$ is available	x)
tors)		

## 18.2.3 The uniform scaling operator

A UniformScaling operator represents a scalar times the identity operator,  $\lambda \star I$ . The identity operator I is defined as a constant and is an instance of UniformScaling. The size of these operators are generic and match the other matrix in the binary operations + (page 353), - (page 353), + (page 385) and + (page 356). For A+I and A-I this means that A must be square. Multiplication with the identity operator I is a noop (except for checking that the scaling factor is one) and therefore almost without overhead.

# **Networking and Streams**

Julia provides a rich interface to deal with streaming I/O objects such as terminals, pipes and TCP sockets. This interface, though asynchronous at the system level, is presented in a synchronous manner to the programmer and it is usually unnecessary to think about the underlying asynchronous operation. This is achieved by making heavy use of Julia cooperative threading (*coroutine* (page 77)) functionality.

#### 19.1 Basic Stream I/O

All Julia streams expose at least a read() (page 450) and a write() (page 450) method, taking the stream as their first argument, e.g.:

```
julia> write(STDOUT, "Hello World"); # suppress return value 11 with;
Hello World
julia> read(STDIN, Char)
'\n'
```

Note that write() (page 450) returns 11, the number of bytes (in "Hello World") written to STDOUT (page 449), but this return value is suppressed with the ;.

Here Enter was pressed again so that Julia would read the newline. Now, as you can see from this example, write() (page 450) takes the data to write as its second argument, while read() (page 450) takes the type of the data to be read as the second argument.

For example, to read a simple byte array, we could do:

However, since this is slightly cumbersome, there are several convenience methods provided. For example, we could have written the above as:

```
julia> read(STDIN,4)
abcd
4-element Array{UInt8,1}:
    0x61
    0x62
    0x63
    0x64
```

or if we had wanted to read the entire line instead:

```
julia> readline(STDIN)
abcd
"abcd\n"
```

Note that depending on your terminal settings, your TTY may be line buffered and might thus require an additional enter before the data is sent to Julia.

To read every line from STDIN (page 449) you can use eachline () (page 454):

```
for line in eachline(STDIN)
    print("Found $line")
end
```

or read () (page 450) if you wanted to read by character instead:

```
while !eof(STDIN)
    x = read(STDIN, Char)
    println("Found: $x")
end
```

### 19.2 Text I/O

Note that the write () (page 450) method mentioned above operates on binary streams. In particular, values do not get converted to any canonical text representation but are written out as is:

```
julia> write(STDOUT,0x61); # suppress return value 1 with;
```

Note that a is written to STDOUT (page 449) by the write() (page 450) function and that the returned value is 1 (since  $0 \times 61$  is one byte).

For text I/O, use the print () (page 453) or show() (page 453) methods, depending on your needs (see the standard library reference for a detailed discussion of the difference between the two):

```
julia> print(STDOUT,0x61)
97
```

# 19.3 IO Output Contextual Properties

Sometimes IO output can benefit from the ability to pass contextual information into show methods. The IOContext object provides this framework for associating arbitrary metadata with an IO object. For example, showlimited adds a hinting parameter to the IO object that the invoked show method should print a shorter output (if applicable).

## 19.4 Working with Files

Like many other environments, Julia has an open () (page 449) function, which takes a filename and returns an IOStream object that you can use to read and write things from the file. For example if we have a file, hello.txt, whose contents are Hello, World!:

```
julia> f = open("hello.txt")
IOStream(<file hello.txt>)

julia> readlines(f)
1-element Array{Union{ASCIIString,UTF8String},1}:
    "Hello, World!\n"
```

If you want to write to a file, you can open it with the write ("w") flag:

```
julia> f = open("hello.txt","w")
IOStream(<file hello.txt>)

julia> write(f,"Hello again.")
12
```

If you examine the contents of hello.txt at this point, you will notice that it is empty; nothing has actually been written to disk yet. This is because the IOStream must be closed before the write is actually flushed to disk:

```
julia> close(f)
```

Examining hello.txt again will show its contents have been changed.

Opening a file, doing something to its contents, and closing it again is a very common pattern. To make this easier, there exists another invocation of <code>open()</code> (page 449) which takes a function as its first argument and filename as its second, opens the file, calls the function with the file as an argument, and then closes it again. For example, given a function:

```
function read_and_capitalize(f::IOStream)
    return uppercase(readstring(f))
end
```

You can call:

```
julia> open(read_and_capitalize, "hello.txt")
"HELLO AGAIN."
```

to open hello.txt, call read\_and\_capitalize on it, close hello.txt and return the capitalized contents.

To avoid even having to define a named function, you can use the do syntax, which creates an anonymous function on the fly:

## 19.5 A simple TCP example

Let's jump right in with a simple example involving TCP sockets. Let's first create a simple server:

To those familiar with the Unix socket API, the method names will feel familiar, though their usage is somewhat simpler than the raw Unix socket API. The first call to <code>listen()</code> (page 459) will create a server waiting for incoming connections on the specified port (2000) in this case. The same function may also be used to create various other kinds of servers:

```
julia> listen(2000) # Listens on localhost:2000 (IPv4)
TCPServer(active)

julia> listen(ip"127.0.0.1",2000) # Equivalent to the first
TCPServer(active)

julia> listen(ip"::1",2000) # Listens on localhost:2000 (IPv6)
TCPServer(active)

julia> listen(IPv4(0),2001) # Listens on port 2001 on all IPv4 interfaces
TCPServer(active)

julia> listen(IPv6(0),2001) # Listens on port 2001 on all IPv6 interfaces
TCPServer(active)

julia> listen("testsocket") # Listens on a domain socket/named pipe
PipeServer(active)
```

Note that the return type of the last invocation is different. This is because this server does not listen on TCP, but rather on a named pipe (Windows) or domain socket (UNIX). The difference is subtle and has to do with the <code>accept()</code> (page 459) and <code>connect()</code> (page 459) methods. The <code>accept()</code> (page 459) method retrieves a connection to the client that is connecting on the server we just created, while the <code>connect()</code> (page 459) function connects to a server using the specified method. The <code>connect()</code> (page 459) function takes the same arguments as <code>listen()</code> (page 459), so, assuming the environment (i.e. host, cwd, etc.) is the same you should be able to pass the same arguments to <code>connect()</code> (page 459) as you did to listen to establish the connection. So let's try that out (after having created the server above):

```
julia> connect(2000)
TCPSocket(open, 0 bytes waiting)
julia> Hello World
```

As expected we saw "Hello World" printed. So, let's actually analyze what happened behind the scenes. When we called <code>connect()</code> (page 459), we connect to the server we had just created. Meanwhile, the accept function returns a server-side connection to the newly created socket and prints "Hello World" to indicate that the connection was successful.

A great strength of Julia is that since the API is exposed synchronously even though the I/O is actually happening asynchronously, we didn't have to worry callbacks or even making sure that the server gets to run. When we called <code>connect()</code> (page 459) the current task waited for the connection to be established and only continued executing after that was done. In this pause, the server task resumed execution (because a connection request was now available), accepted the connection, printed the message and waited for the next client. Reading and writing works in the same

way. To see this, consider the following simple echo server:

```
julia> @async begin
         server = listen(2001)
         while true
           sock = accept(server)
           @async while isopen(sock)
             write(sock, readline(sock))
         end
       end
Task
julia> clientside=connect(2001)
TCPSocket(open, 0 bytes waiting)
julia> @async while true
          write(STDOUT, readline(clientside))
       end
julia > println(clientside, "Hello World from the Echo Server")
julia> Hello World from the Echo Server
```

As with other streams, use close () (page 450) to disconnect the socket:

```
julia> close(clientside)
```

## 19.6 Resolving IP Addresses

One of the <code>connect()</code> (page 459) methods that does not follow the <code>listen()</code> (page 459) methods is <code>connect(host::ASCIIString,port)</code>, which will attempt to connect to the host given by the host parameter on the port given by the port parameter. It allows you to do things like:

```
julia> connect("google.com",80)
TCPSocket(open, 0 bytes waiting)
```

At the base of this functionality is getaddrinfo() (page 459), which will do the appropriate address resolution:

```
julia> getaddrinfo("google.com")
IPv4(74.125.226.225)
```

Julia Language Documentation, Release 0.5.0-dev	

## **Parallel Computing**

Most modern computers possess more than one CPU, and several computers can be combined together in a cluster. Harnessing the power of these multiple CPUs allows many computations to be completed more quickly. There are two major factors that influence performance: the speed of the CPUs themselves, and the speed of their access to memory. In a cluster, it's fairly obvious that a given CPU will have fastest access to the RAM within the same computer (node). Perhaps more surprisingly, similar issues are relevant on a typical multicore laptop, due to differences in the speed of main memory and the cache. Consequently, a good multiprocessing environment should allow control over the "ownership" of a chunk of memory by a particular CPU. Julia provides a multiprocessing environment based on message passing to allow programs to run on multiple processes in separate memory domains at once.

Julia's implementation of message passing is different from other environments such as MPI <sup>1</sup>. Communication in Julia is generally "one-sided", meaning that the programmer needs to explicitly manage only one process in a two-process operation. Furthermore, these operations typically do not look like "message send" and "message receive" but rather resemble higher-level operations like calls to user functions.

Parallel programming in Julia is built on two primitives: *remote references* and *remote calls*. A remote reference is an object that can be used from any process to refer to an object stored on a particular process. A remote call is a request by one process to call a certain function on certain arguments on another (possibly the same) process.

Remote references come in two flavors -Future and RemoteChannel.

A remote call returns a Future to its result. Remote calls return immediately; the process that made the call proceeds to its next operation while the remote call happens somewhere else. You can wait for a remote call to finish by calling wait() (page 408) on the returned *Future*, and you can obtain the full value of the result using fetch() (page 408).

On the other hand RemoteRefs are rewritable. For example, multiple processes can co-ordinate their processing by referencing the same remote Channel.

Let's try this out. Starting with julia -p n provides n worker processes on the local machine. Generally it makes sense for n to equal the number of CPU cores on the machine.

```
$ ./julia -p 2

julia> r = remotecall(rand, 2, 2, 2)
Future(2,1,3,Nullable{Any}())

julia> s = @spawnat 2 1 .+ fetch(r)
Future(2,1,6,Nullable{Any}())

julia> fetch(s)
2x2 Float64 Array:
```

<sup>&</sup>lt;sup>1</sup> In this context, MPI refers to the MPI-1 standard. Beginning with MPI-2, the MPI standards committee introduced a new set of communication mechanisms, collectively referred to as Remote Memory Access (RMA). The motivation for adding RMA to the MPI standard was to facilitate one-sided communication patterns. For additional information on the latest MPI standard, see http://www.mpi-forum.org/docs.

```
1.60401 1.50111
1.17457 1.15741
```

The first argument to remotecall() (page 408) is the function to call. Most parallel programming in Julia does not reference specific processes or the number of processes available, but remotecall() (page 408) is considered a low-level interface providing finer control. The second argument to remotecall() (page 408) is the index of the process that will do the work, and the remaining arguments will be passed to the function being called.

As you can see, in the first line we asked process 2 to construct a 2-by-2 random matrix, and in the second line we asked it to add 1 to it. The result of both calculations is available in the two futures, r and s. The @spawnat (page 410) macro evaluates the expression in the second argument on the process specified by the first argument.

Occasionally you might want a remotely-computed value immediately. This typically happens when you read from a remote object to obtain data needed by the next local operation. The function remotecall\_fetch() (page 409) exists for this purpose. It is equivalent to fetch (remotecall(...)) but is more efficient.

```
julia> remotecall_fetch(getindex, 2, r, 1, 1)
0.10824216411304866
```

Remember that getindex(r, 1, 1) (page 342) is equivalent (page 160) to r[1, 1], so this call fetches the first element of the future r.

The syntax of remotecal1 () (page 408) is not especially convenient. The macro @spawn (page 409) makes things easier. It operates on an expression rather than a function, and picks where to do the operation for you:

```
julia> r = @spawn rand(2,2)
Future(2,1,4,Nullable{Any}())

julia> s = @spawn 1 .+ fetch(r)
Future(3,1,5,Nullable{Any}())

julia> fetch(s)
1.10824216411304866 1.13798233877923116
1.12376292706355074 1.18750497916607167
```

Note that we used 1 .+ fetch (r) instead of 1 .+ r. This is because we do not know where the code will run, so in general a fetch () (page 408) might be required to move r to the process doing the addition. In this case, @spawn (page 409) is smart enough to perform the computation on the process that owns r, so the fetch () (page 408) will be a no-op.

(It is worth noting that @spawn (page 409) is not built-in but defined in Julia as a *macro* (page 146). It is possible to define your own such constructs.)

An important thing to remember is that, once fetched, a Future will cache its value locally. Further fetch calls do not entail a network hop. Once all referencing Futures have fetched, the remote stored value is deleted.

# 20.1 Code Availability and Loading Packages

Your code must be available on any process that runs it. For example, type the following into the Julia prompt:

```
julia> fetch(@spawn rand2(2,2))
ERROR: On worker 2:
function rand2 not defined on process 2
```

Process 1 knew about the function rand2, but process 2 did not.

Most commonly you'll be loading code from files or packages, and you have a considerable amount of flexibility in controlling which processes load code. Consider a file, "DummyModule.jl", containing the following code:

```
module DummyModule

export MyType, f

type MyType
   a::Int
end

f(x) = x^2+1

println("loaded")
end
```

Starting julia with julia -p 2, you can use this to verify the following:

- include ("DummyModule.jl") (page 319) loads the file on just a single process (whichever one executes the statement).
- using DummyModule causes the module to be loaded on all processes; however, the module is brought into scope only on the one executing the statement.
- As long as DummyModule is loaded on process 2, commands like

```
rr = RemoteChannel(2)
put!(rr, MyType(7))
```

allow you to store an object of type MyType on process 2 even if DummyModule is not in scope on process 2.

You can force a command to run on all processes using the @everywhere (page 410) macro. For example, @everywhere (page 410) can also be used to directly define a function on all processes:

```
julia> @everywhere id = myid()
julia> remotecall_fetch(()->id, 2)
2
```

A file can also be preloaded on multiple processes at startup, and a driver script can be used to drive the computation:

```
julia -p <n> -L file1.jl -L file2.jl driver.jl
```

Each process has an associated identifier. The process providing the interactive Julia prompt always has an id equal to 1, as would the Julia process running the driver script in the example above. The processes used by default for parallel operations are referred to as "workers". When there is only one process, process 1 is considered a worker. Otherwise, workers are considered to be all processes other than process 1.

The base Julia installation has in-built support for two types of clusters:

- A local cluster specified with the -p option as shown above.
- A cluster spanning machines using the --machinefile option. This uses a passwordless ssh login to start julia worker processes (from the same path as the current host) on the specified machines.

Functions addprocs () (page 406), rmprocs () (page 407), workers () (page 407), and others are available as a programmatic means of adding, removing and querying the processes in a cluster.

Note that workers do not run a .juliarc.jl startup script, nor do they synchronize their global state (such as global variables, new method definitions, and loaded modules) with any of the other running processes.

Other types of clusters can be supported by writing your own custom ClusterManager, as described below in the *ClusterManagers* (page 189) section.

#### 20.2 Data Movement

Sending messages and moving data constitute most of the overhead in a parallel program. Reducing the number of messages and the amount of data sent is critical to achieving performance and scalability. To this end, it is important to understand the data movement performed by Julia's various parallel programming constructs.

fetch () (page 408) can be considered an explicit data movement operation, since it directly asks that an object be moved to the local machine. @spawn (page 409) (and a few related constructs) also moves data, but this is not as obvious, hence it can be called an implicit data movement operation. Consider these two approaches to constructing and squaring a random matrix:

```
# method 1
A = rand(1000,1000)
Bref = @spawn A^2
...
fetch(Bref)

# method 2
Bref = @spawn rand(1000,1000)^2
...
fetch(Bref)
```

The difference seems trivial, but in fact is quite significant due to the behavior of @spawn (page 409). In the first method, a random matrix is constructed locally, then sent to another process where it is squared. In the second method, a random matrix is both constructed and squared on another process. Therefore the second method sends much less data than the first.

In this toy example, the two methods are easy to distinguish and choose from. However, in a real program designing data movement might require more thought and likely some measurement. For example, if the first process needs matrix A then the first method might be better. Or, if computing A is expensive and only the current process has it, then moving it to another process might be unavoidable. Or, if the current process has very little to do between the <code>@spawn</code> (page 409) and <code>fetch(Bref)</code> then it might be better to eliminate the parallelism altogether. Or imagine <code>rand(1000,1000)</code> is replaced with a more expensive operation. Then it might make sense to add another <code>@spawn</code> (page 409) statement just for this step.

## 20.3 Parallel Map and Loops

Fortunately, many useful parallel computations do not require data movement. A common example is a Monte Carlo simulation, where multiple processes can handle independent simulation trials simultaneously. We can use @spawn (page 409) to flip coins on two processes. First, write the following function in count\_heads.jl:

```
function count_heads(n)
    c::Int = 0
    for i=1:n
        c += rand(Bool)
```

```
end
c
end
```

The function count\_heads simply adds together n random bits. Here is how we can perform some trials on two machines, and add together the results:

```
require("count_heads")
a = @spawn count_heads(100000000)
b = @spawn count_heads(100000000)
fetch(a)+fetch(b)
```

This example demonstrates a powerful and often-used parallel programming pattern. Many iterations run independently over several processes, and then their results are combined using some function. The combination process is called a *reduction*, since it is generally tensor-rank-reducing: a vector of numbers is reduced to a single number, or a matrix is reduced to a single row or column, etc. In code, this typically looks like the pattern x = f(x, v[i]), where x is the accumulator, f is the reduction function, and the v[i] are the elements being reduced. It is desirable for f to be associative, so that it does not matter what order the operations are performed in.

Notice that our use of this pattern with count\_heads can be generalized. We used two explicit @spawn (page 409) statements, which limits the parallelism to two processes. To run on any number of processes, we can use a *parallel* for loop, which can be written in Julia like this:

```
nheads = @parallel (+) for i=1:200000000
    Int(rand(Bool))
end
```

This construct implements the pattern of assigning iterations to multiple processes, and combining them with a specified reduction (in this case (+)). The result of each iteration is taken as the value of the last expression inside the loop. The whole parallel loop expression itself evaluates to the final answer.

Note that although parallel for loops look like serial for loops, their behavior is dramatically different. In particular, the iterations do not happen in a specified order, and writes to variables or arrays will not be globally visible since iterations run on different processes. Any variables used inside the parallel loop will be copied and broadcast to each process.

For example, the following code will not work as intended:

```
a = zeros(100000)
@parallel for i=1:100000
a[i] = i
end
```

However, this code will not initialize all of a, since each process will have a separate copy of it. Parallel for loops like these must be avoided. Fortunately, *Shared Arrays* (page ??) can be used to get around this limitation:

```
a = SharedArray(Float64,10)
@parallel for i=1:10
a[i] = i
end
```

Using "outside" variables in parallel loops is perfectly reasonable if the variables are read-only:

```
a = randn(1000)
@parallel (+) for i=1:100000
  f(a[rand(1:end)])
end
```

Here each iteration applies f to a randomly-chosen sample from a vector a shared by all processes.

As you could see, the reduction operator can be omitted if it is not needed. In that case, the loop executes asynchronously, i.e. it spawns independent tasks on all available workers and returns an array of *Future* (page 408) immediately without waiting for completion. The caller can wait for the *Future* (page 408) completions at a later point by calling *fetch()* (page 408) on them, or wait for completion at the end of the loop by prefixing it with @sync (page 410), like @sync @parallel for.

In some cases no reduction operator is needed, and we merely wish to apply a function to all integers in some range (or, more generally, to all elements in some collection). This is another useful operation called *parallel map*, implemented in Julia as the pmap() (page 408) function. For example, we could compute the singular values of several large random matrices in parallel as follows:

```
M = Matrix{Float64}[rand(1000,1000) for i=1:10]
pmap(svd, M)
```

Julia's pmap () (page 408) is designed for the case where each function call does a large amount of work. In contrast, <code>@parallel for</code> can handle situations where each iteration is tiny, perhaps merely summing two numbers. Only worker processes are used by both <code>pmap()</code> (page 408) and <code>@parallel for</code> for the parallel computation. In case of <code>@parallel for</code>, the final reduction is done on the calling process.

#### 20.4 Synchronization With Remote References

#### 20.5 Scheduling

Julia's parallel programming platform uses *Tasks (aka Coroutines)* (page 77) to switch among multiple computations. Whenever code performs a communication operation like *fetch()* (page 408) or *wait()* (page 408), the current task is suspended and a scheduler picks another task to run. A task is restarted when the event it is waiting for completes.

For many problems, it is not necessary to think about tasks directly. However, they can be used to wait for multiple events at the same time, which provides for *dynamic scheduling*. In dynamic scheduling, a program decides what to compute or where to compute it based on when other jobs finish. This is needed for unpredictable or unbalanced workloads, where we want to assign more work to processes only when they finish their current tasks.

As an example, consider computing the singular values of matrices of different sizes:

```
M = Matrix{Float64}[rand(800,800), rand(600,600), rand(800,800), rand(600,600)]
pmap(svd, M)
```

If one process handles both 800x800 matrices and another handles both 600x600 matrices, we will not get as much scalability as we could. The solution is to make a local task to "feed" work to each process when it completes its current task. This can be seen in the implementation of pmap() (page 408):

```
function pmap(f, lst)
    np = nprocs()  # determine the number of processes available
    n = length(lst)
    results = cell(n)
    i = 1
    # function to produce the next work item from the queue.
    # in this case it's just an index.
    nextidx() = (idx=i; i+=1; idx)
    @sync begin
    for p=1:np
        if p != myid() || np == 1
            @async begin
            while true
            idx = nextidx()
```

@async (page 410) is similar to @spawn (page 409), but only runs tasks on the local process. We use it to create a "feeder" task for each process. Each task picks the next index that needs to be computed, then waits for its process to finish, then repeats until we run out of indexes. Note that the feeder tasks do not begin to execute until the main task reaches the end of the @sync (page 410) block, at which point it surrenders control and waits for all the local tasks to complete before returning from the function. The feeder tasks are able to share state via nextidx() because they all run on the same process. No locking is required, since the threads are scheduled cooperatively and not preemptively. This means context switches only occur at well-defined points: in this case, when remotecall\_fetch() (page 409) is called.

#### 20.6 Channels

Channels provide for a fast means of inter-task communication. A Channel {T} (n::Int) is a shared queue of maximum length n holding objects of type T. Multiple readers can read off the channel via fetch and take!. Multiple writers can add to the channel via put!.isready tests for the presence of any object in the channel, while wait waits for an object to become available. close closes a Channel. On a closed channel, put! will fail, while take! and fetch successfully return any existing values till it is emptied.

A Channel can be used as an iterable object in a for loop, in which case the loop runs as long as the channel has data or is open. The loop variable takes on all values added to the channel. An empty, closed channel causes the for loop to terminate.

#### 20.7 Remote references and AbstractChannels

Remote references always refer to an implementation of an AbstractChannel

A concrete implementation of an AbstractChannel (like Channel), is required to implement put!, take!, fetch, isready and wait. The remote object referred to by a Future is stored in a Channel {Any} (1), i.e., a channel of size 1 capable of holding objects of Any type.

RemoteChannel, which is rewritable, can point to any type and size of channels, or any other implementation of an AbstractChannel.

The constructor RemoteChannel(f::Function, pid) allows us to construct references to channels holding more than one value of a specific type. f() is a function executed on pid and it must return an AbstractChannel.

For example, RemoteChannel(()->Channel{Int}(10), pid), will return a reference to a channel of type Int and size 10. The channel exists on worker pid.

Methods put!, take!, fetch, isready and wait on a RemoteChannel are proxied onto the backing store on the remote process.

RemoteChannel can thus be used to refer to user implemented AbstractChannel objects. A simple example of this is provided in examples/dictchannel.jl which uses a dictionary as its remote store.

20.6. Channels 185

#### 20.8 Remote References and Distributed Garbage Collection

Objects referred to by remote references can be freed only when all held references in the cluster are deleted.

The node where the value is stored keeps track of which of the workers have a reference to it. Every time a RemoteChannel or a (unfetched) Future is serialized to a worker, the node pointed to by the reference is notified. And every time a RemoteChannel or a (unfetched) Future is garbage collected locally, the node owning the value is again notified.

The notifications are done via sending of "tracking" messages - an "add reference" message when a reference is serialized to a different process and a "delete reference" message when a reference is locally garbage collected.

Since Futures are write-once and cached locally, the act of fetching a Future also updates reference tracking information on the node owning the value.

The node which owns the value frees it once all references to it are cleared.

With Futures, serializing an already fetched Future to a different node also sends the value since the original remote store may have collected the value by this time.

It is important to note that *when* an object is locally garbage collected depends on the size of the object and the current memory pressure in the system.

In case of remote references, the size of the local reference object is quite small, while the value stored on the remote node may be quite large. Since the local object may not be collected immediately, it is a good practice to explicitly call finalize on local instances of a RemoteChannel, or on unfetched Futures. Since calling fetch on a Future also removes its reference from the remote store, this is not required on fetched Futures. Explicitly calling finalize results in an immediate message sent to the remote node to go ahead and remove its reference to the value.

## 20.9 Shared Arrays

Shared Arrays use system shared memory to map the same array across many processes. While there are some similarities to a DArray, the behavior of a *SharedArray* (page 411) is quite different. In a DArray, each process has local access to just a chunk of the data, and no two processes share the same chunk; in contrast, in a *SharedArray* (page 411) each "participating" process has access to the entire array. A *SharedArray* (page 411) is a good choice when you want to have a large amount of data jointly accessible to two or more processes on the same machine.

SharedArray (page 411) indexing (assignment and accessing values) works just as with regular arrays, and is efficient because the underlying memory is available to the local process. Therefore, most algorithms work naturally on SharedArray (page 411)s, albeit in single-process mode. In cases where an algorithm insists on an Array (page 395) input, the underlying array can be retrieved from a SharedArray (page 411) by calling sdata() (page 411). For other AbstractArray types, sdata just returns the object itself, so it's safe to use sdata() (page 411) on any Array-type object.

The constructor for a shared array is of the form:

```
SharedArray(T::Type, dims::NTuple; init=false, pids=Int[])
```

which creates a shared array of a bitstype T and size dims across the processes specified by pids. Unlike distributed arrays, a shared array is accessible only from those participating workers specified by the pids named argument (and the creating process too, if it is on the same host).

If an init function, of signature initfn(S::SharedArray), is specified, it is called on all the participating workers. You can arrange it so that each worker runs the init function on a distinct portion of the array, thereby parallelizing initialization.

Here's a brief example:

```
julia> addprocs(3)
3-element Array{Int64,1}:
2
3
4

julia> S = SharedArray(Int, (3,4), init = S -> S[Base.localindexes(S)] = myid())
3x4 SharedArray{Int64,2}:
2  2  3  4
2  3  3  4
2  3  4  4

julia> S[3,2] = 7

julia> S
3x4 SharedArray{Int64,2}:
2  2  3  4
2  3  3  4
2  7  4  4
```

Base.localindexes() (page 411) provides disjoint one-dimensional ranges of indexes, and is sometimes convenient for splitting up tasks among processes. You can, of course, divide the work any way you wish:

```
julia> S = SharedArray(Int, (3,4), init = S -> S[indexpids(S):length(procs(S)):length(S)] = myid())
3x4 SharedArray{Int64,2}:
2  2  2  2
3  3  3  3
4  4  4  4  4
```

Since all processes have access to the underlying data, you do have to be careful not to set up conflicts. For example:

```
@sync begin
    for p in procs(S)
      @async begin
         remotecall_wait(fill!, p, S, p)
      end
    end
end
```

would result in undefined behavior: because each process fills the *entire* array with its own pid, whichever process is the last to execute (for any particular element of S) will have its pid retained.

As a more extended and complex example, consider running the following "kernel" in parallel:

```
q[i,j,t+1] = q[i,j,t] + u[i,j,t]
```

In this case, if we try to split up the work using a one-dimensional index, we are likely to run into trouble: if q[i,j,t] is near the end of the block assigned to one worker and q[i,j,t+1] is near the beginning of the block assigned to another, it's very likely that q[i,j,t] will not be ready at the time it's needed for computing q[i,j,t+1]. In such cases, one is better off chunking the array manually. Let's split along the second dimension:

```
# This function retuns the (irange, jrange) indexes assigned to this worker
@everywhere function myrange(q::SharedArray)
   idx = indexpids(q)
   if idx == 0
        # This worker is not assigned a piece
        return 1:0, 1:0
   end
   nchunks = length(procs(q))
```

```
splits = [round(Int, s) for s in linspace(0,size(q,2),nchunks+1)]
    1:size(q,1), splits[idx]+1:splits[idx+1]
end

# Here's the kernel
@everywhere function advection_chunk!(q, u, irange, jrange, trange)
    @show (irange, jrange, trange) # display so we can see what's happening
    for t in trange, j in jrange, i in irange
        q[i,j,t+1] = q[i,j,t] + u[i,j,t]
    end
    q
end

# Here's a convenience wrapper for a SharedArray implementation
@everywhere advection_shared_chunk!(q, u) = advection_chunk!(q, u, myrange(q)..., 1:size(q,3)-1)
```

Now let's compare three different versions, one that runs in a single process:

```
advection_serial!(q, u) = advection_chunk!(q, u, 1:size(q,1), 1:size(q,2), 1:size(q,3)-1)
```

one that uses @parallel:

and one that delegates in chunks:

```
function advection_shared!(q, u)
    @sync begin
    for p in procs(q)
        @async remotecall_wait(advection_shared_chunk!, p, q, u)
    end
    end
    q
end
```

If we create SharedArrays and time these functions, we get the following results (with julia -p 4):

```
q = SharedArray(Float64, (500,500,500))
u = SharedArray(Float64, (500,500,500))

# Run once to JIT-compile
advection_serial!(q, u)
advection_parallel!(q, u)
advection_shared!(q, u)

# Now the real results:
julia> @time advection_serial!(q, u);
(irange,jrange,trange) = (1:500,1:500,1:499)
830.220 milliseconds (216 allocations: 13820 bytes)
julia> @time advection_parallel!(q, u);
```

```
2.495 seconds (3999 k allocations: 289 MB, 2.09% gc time)

julia> @time advection_shared!(q,u);
    From worker 2: (irange, jrange, trange) = (1:500,1:125,1:499)
    From worker 4: (irange, jrange, trange) = (1:500,251:375,1:499)
    From worker 3: (irange, jrange, trange) = (1:500,126:250,1:499)
    From worker 5: (irange, jrange, trange) = (1:500,376:500,1:499)

238.119 milliseconds (2264 allocations: 169 KB)
```

The biggest advantage of advection\_shared! is that it minimizes traffic among the workers, allowing each to compute for an extended time on the assigned piece.

#### 20.10 ClusterManagers

The launching, management and networking of julia processes into a logical cluster is done via cluster managers. A ClusterManager is responsible for

- launching worker processes in a cluster environment
- · managing events during the lifetime of each worker
- optionally, a cluster manager can also provide data transport

A julia cluster has the following characteristics: - The initial julia process, also called the master is special and has a julia id of 1. - Only the master process can add or remove worker processes. - All processes can directly communicate with each other.

Connections between workers (using the in-built TCP/IP transport) is established in the following manner:

- addprocs () (page 406) is called on the master process with a ClusterManager object
- addprocs () (page 406) calls the appropriate <code>launch()</code> (page 411) method which spawns required number of worker processes on appropriate machines
- Each worker starts listening on a free port and writes out its host, port information to STDOUT (page 449)
- The cluster manager captures the stdout's of each worker and makes it available to the master process
- The master process parses this information and sets up TCP/IP connections to each worker
- Every worker is also notified of other workers in the cluster
- Each worker connects to all workers whose julia id is less than its own id
- In this way a mesh network is established, wherein every worker is directly connected with every other worker

While the default transport layer uses plain TCP sockets, it is possible for a julia cluster to provide its own transport.

Julia provides two in-built cluster managers:

- LocalManager, used when addprocs() (page 406) or addprocs(np::Integer) (page 406) are called
- SSHManager, used when addprocs (hostnames::Array) (page 406) is called with a list of hostnames

LocalManager is used to launch additional workers on the same host, thereby leveraging multi-core and multi-processor hardware.

Thus, a minimal cluster manager would need to:

- be a subtype of the abstract ClusterManager
- implement launch () (page 411), a method responsible for launching new workers

• implement manage () (page 412), which is called at various events during a worker's lifetime

addprocs (manager::FooManager) (page 406) requires FooManager to implement:

```
function launch(manager::FooManager, params::Dict, launched::Array, c::Condition)
...
end

function manage(manager::FooManager, id::Integer, config::WorkerConfig, op::Symbol)
...
end
```

As an example let us see how the LocalManager, the manager responsible for starting workers on the same host, is implemented:

The launch () (page 411) method takes the following arguments:

- manager::ClusterManager the cluster manager addprocs() (page 406) is called with
- params::Dict all the keyword arguments passed to addprocs() (page 406)
- launched:: Array the array to append one or more WorkerConfig objects to
- c::Condition the condition variable to be notified as and when workers are launched

The <code>launch()</code> (page 411) method is called asynchronously in a separate task. The termination of this task signals that all requested workers have been launched. Hence the <code>launch()</code> (page 411) function MUST exit as soon as all the requested workers have been launched.

Newly launched workers are connected to each other, and the master process, in a all-to-all manner. Specifying command argument, --worker results in the launched processes initializing themselves as workers and connections being setup via TCP/IP sockets. Optionally --bind-to bind\_addr[:port] may also be specified to enable other workers to connect to it at the specified bind\_addr and port. This is useful for multi-homed hosts.

For non-TCP/IP transports, for example, an implementation may choose to use MPI as the transport, --worker must NOT be specified. Instead newly launched workers should call init\_worker() before using any of the parallel constructs

For every worker launched, the <code>launch()</code> (page 411) method must add a <code>WorkerConfig</code> object (with appropriate fields initialized) to <code>launched</code>

```
type WorkerConfig

# Common fields relevant to all cluster managers
io::Nullable{IO}
host::Nullable{AbstractString}
port::Nullable{Integer}

# Used when launching additional workers at a host
count::Nullable{Union{Int, Symbol}}
exename::Nullable{AbstractString}
```

```
exeflags::Nullable{Cmd}

# External cluster managers can use this to store information at a per-worker level
# Can be a dict if multiple fields need to be stored.
userdata::Nullable{Any}

# SSHManager / SSH tunnel connections to workers
tunnel::Nullable{Bool}
bind_addr::Nullable{AbstractString}
sshflags::Nullable{Cmd}
max_parallel::Nullable{Integer}

connect_at::Nullable{Any}
.....
end
```

Most of the fields in WorkerConfig are used by the inbuilt managers. Custom cluster managers would typically specify only io or host / port:

If io is specified, it is used to read host/port information. A Julia worker prints out its bind address and port at startup. This allows Julia workers to listen on any free port available instead of requiring worker ports to be configured manually.

If io is not specified, host and port are used to connect.

count, exename and exeflags are relevant for launching additional workers from a worker. For example, a cluster manager may launch a single worker per node, and use that to launch additional workers. count with an integer value n will launch a total of n workers, while a value of :auto will launch as many workers as cores on that machine. exename is the name of the julia executable including the full path. exeflags should be set to the required command line arguments for new workers.

tunnel, bind\_addr, sshflags and max\_parallel are used when a ssh tunnel is required to connect to the workers from the master process.

userdata is provided for custom cluster managers to store their own worker specific information.

manage (manager::FooManager, id::Integer, config::WorkerConfig, op::Symbol) is called at different times during the worker's lifetime with appropriate op values:

- with :register/:deregister when a worker is added / removed from the Julia worker pool.
- with :interrupt when interrupt (workers) is called. The ClusterManager should signal the appropriate worker with an interrupt signal.
- with : finalize for cleanup purposes.

## 20.11 Cluster Managers with custom transports

Replacing the default TCP/IP all-to-all socket connections with a custom transport layer is a little more involved. Each julia process has as many communication tasks as the workers it is connected to. For example, consider a julia cluster of 32 processes in a all-to-all mesh network:

- Each julia process thus has 31 communication tasks
- Each task handles all incoming messages from a single remote worker in a message processing loop
- The message processing loop waits on an AsyncStream object for example, a TCP socket in the default implementation, reads an entire message, processes it and waits for the next one

• Sending messages to a process is done directly from any julia task - not just communication tasks - again, via the appropriate AsyncStream object

Replacing the default transport involves the new implementation to setup connections to remote workers, and to provide appropriate AsyncStream objects that the message processing loops can wait on. The manager specific callbacks to be implemented are:

```
connect(manager::FooManager, pid::Integer, config::WorkerConfig)
kill(manager::FooManager, pid::Int, config::WorkerConfig)
```

The default implementation (which uses TCP/IP sockets) is implemented as connect (manager::ClusterManager, pid::Integer, config::WorkerConfig).

connect should return a pair of AsyncStream objects, one for reading data sent from worker pid, and the other to write data that needs to be sent to worker pid. Custom cluster managers can use an in-memory BufferStream as the plumbing to proxy data between the custom, possibly non-AsyncStream transport and julia's in-built parallel infrastructure.

A BufferStream is an in-memory IOBuffer which behaves like an AsyncStream.

Folder examples/clustermanager/0mq is an example of using ZeroMQ is connect julia workers in a star network with a 0MQ broker in the middle. Note: The julia processes are still all *logically* connected to each other - any worker can message any other worker directly without any awareness of 0MQ being used as the transport layer.

When using custom transports:

- julia workers must NOT be started with --worker. Starting with --worker will result in the newly launched workers defaulting to the TCP/IP socket transport implementation
- For every incoming logical connection with a worker, Base.process\_messages (rd::AsyncStream, wr::AsyncStream) must be called. This launches a new task that handles reading and writing of messages from/to the worker represented by the AsyncStream objects
- init\_worker (manager::FooManager) MUST be called as part of worker process initialization
- Field connect\_at:: Any in WorkerConfig can be set by the cluster manager when launch is called. The value of this field is passed in in all connect callbacks. Typically, it carries information on how to connect to a worker. For example, the TCP/IP socket transport uses this field to specify the (host, port) tuple at which to connect to a worker

kill (manager, pid, config) is called to remove a worker from the cluster. On the master process, the corresponding AsyncStream objects must be closed by the implementation to ensure proper cleanup. The default implementation simply executes an exit () call on the specified remote worker.

examples/clustermanager/simple is an example that shows a simple implementation using unix domain sockets for cluster setup

# 20.12 Specifying network topology (Experimental)

Keyword argument topology to addprocs is used to specify how the workers must be connected to each other:

- : all to all: is the default, where all workers are connected to each other.
- :master\_slave: only the driver process, i.e. pid 1 has connections to the workers.
- :custom: the launch method of the cluster manager specifes the connection topology. Fields ident and connect\_idents in WorkerConfig are used to specify the same. connect\_idents is a list of ClusterManager provided identifiers to workers that worker with identified by ident must connect to.

Currently sending a message between unconnected workers results in an error. This behaviour, as also the functionality and interface should be considered experimental in nature and may change in future releases.

#### **Date and DateTime**

The Dates module provides two types for working with dates: Date and DateTime, representing day and millisecond precision, respectively; both are subtypes of the abstract TimeType. The motivation for distinct types is simple: some operations are much simpler, both in terms of code and mental reasoning, when the complexities of greater precision don't have to be dealt with. For example, since the Date type only resolves to the precision of a single date (i.e. no hours, minutes, or seconds), normal considerations for time zones, daylight savings/summer time, and leap seconds are unnecessary and avoided.

Both Date and DateTime are basically immutable Int64 wrappers. The single instant field of either type is actually a UTInstant {P} type, which represents a continuously increasing machine timeline based on the UT second <sup>1</sup>. The DateTime type is *timezone-unaware* (in Python parlance) or is analogous to a *LocalDateTime* in Java 8. Additional time zone functionality can be added through the Timezones.jl package, which compiles the Olsen Time Zone Database. Both Date and DateTime are based on the ISO 8601 standard, which follows the proleptic Gregorian calendar. One note is that the ISO 8601 standard is particular about BC/BCE dates. In general, the last day of the BC/BCE era, 1-12-31 BC/BCE, was followed by 1-1-1 AD/CE, thus no year zero exists. The ISO standard, however, states that 1 BC/BCE is year zero, so 0000-12-31 is the day before 0001-01-01, and year -0001 (yes, negative one for the year) is 2 BC/BCE, year -0002 is 3 BC/BCE, etc.

#### 21.1 Constructors

Date and DateTime types can be constructed by integer or Period types, by parsing, or through adjusters (more on those later):

```
julia> DateTime(2013)
2013-01-01T00:00:00

julia> DateTime(2013,7)
2013-07-01T00:00:00

julia> DateTime(2013,7,1)
2013-07-01T00:00:00

julia> DateTime(2013,7,1,12)
2013-07-01T12:00:00
```

<sup>&</sup>lt;sup>1</sup> The notion of the UT second is actually quite fundamental. There are basically two different notions of time generally accepted, one based on the physical rotation of the earth (one full rotation = 1 day), the other based on the SI second (a fixed, constant value). These are radically different! Think about it, a "UT second", as defined relative to the rotation of the earth, may have a different absolute length depending on the day! Anyway, the fact that Date and DateTime are based on UT seconds is a simplifying, yet honest assumption so that things like leap seconds and all their complexity can be avoided. This basis of time is formally called UT or UT1. Basing types on the UT second basically means that every minute has 60 seconds and every day has 24 hours and leads to more natural calculations when working with calendar dates.

```
julia > DateTime (2013, 7, 1, 12, 30)
2013-07-01T12:30:00
julia > DateTime (2013, 7, 1, 12, 30, 59)
2013-07-01T12:30:59
julia > DateTime (2013, 7, 1, 12, 30, 59, 1)
2013-07-01T12:30:59.001
julia> Date(2013)
2013-01-01
julia > Date (2013,7)
2013-07-01
julia > Date (2013, 7, 1)
2013-07-01
julia> Date(Dates.Year(2013), Dates.Month(7), Dates.Day(1))
2013-07-01
julia> Date(Dates.Month(7), Dates.Year(2013))
2013-07-01
```

Date or DateTime parsing is accomplished by the use of format strings. Format strings work by the notion of defining *delimited* or *fixed-width* "slots" that contain a period to parse and passing the text to parse and format string to a Date or DateTime constructor, of the form Date("2015-01-01", "y-m-d") or DateTime("20150101", "yyyymmdd").

Delimited slots are marked by specifying the delimiter the parser should expect between two subsequent periods; so "y-m-d" lets the parser know that between the first and second slots in a date string like "2014-07-16", it should find the – character. The y, m, and d characters let the parser know which periods to parse in each slot.

Fixed-width slots are specified by repeating the period character the number of times corresponding to the width with no delimiter between characters. So "yyyymmdd" would correspond to a date string like "20140716". The parser distinguishes a fixed-width slot by the absence of a delimiter, noting the transition "yyyymm" from one period character to the next.

Support for text-form month parsing is also supported through the u and U characters, for abbreviated and full-length month names, respectively. By default, only English month names are supported, so u corresponds to "Jan", "Feb", "Mar", etc. And U corresponds to "January", "February", "March", etc. Similar to other name=>value mapping functions dayname() and monthname(), custom locales can be loaded by passing in the locale=>Dict{UTF8String, Int} mapping to the MONTHTOVALUEABBR and MONTHTOVALUE dicts for abbreviated and full-name month names, respectively.

One note on parsing performance: using the Date(date\_string, format\_string) function is fine if only called a few times. If there are many similarly formatted date strings to parse however, it is much more efficient to first create a Dates.DateFormat, and pass it instead of a raw format string.

```
julia> df = Dates.DateFormat("y-m-d");
julia> dt = Date("2015-01-01",df)
2015-01-01

julia> dt2 = Date("2015-01-02",df)
2015-01-02
```

A full suite of parsing and formatting tests and examples is available in tests/dates/io.jl.

# 21.2 Durations/Comparisons

Finding the length of time between two Date or DateTime is straightforward given their underlying representation as UTInstant {Day} and UTInstant {Millisecond}, respectively. The difference between Date is returned in the number of Day, and DateTime in the number of Millisecond. Similarly, comparing TimeType is a simple matter of comparing the underlying machine instants (which in turn compares the internal Int 64 values).

```
julia> dt = Date(2012, 2, 29)
2012-02-29
julia > dt2 = Date(2000, 2, 1)
2000-02-01
julia> dump(dt)
Date
 instant: UTInstant{Day}
   periods: Day
     value: Int64 734562
julia> dump(dt2)
Date
instant: UTInstant{Day}
 periods: Day
   value: Int64 730151
julia> dt > dt2
true
julia> dt != dt2
true
julia> dt + dt2
Operation not defined for TimeTypes
julia> dt * dt2
Operation not defined for TimeTypes
julia> dt / dt2
Operation not defined for TimeTypes
julia> dt - dt2
4411 days
julia> dt2 - dt
-4411 days
julia> dt = DateTime(2012, 2, 29)
2012-02-29T00:00:00
julia > dt2 = DateTime(2000, 2, 1)
2000-02-01T00:00:00
julia> dt - dt2
381110402000 milliseconds
```

#### 21.3 Accessor Functions

Because the Date and DateTime types are stored as single Int64 values, date parts or fields can be retrieved through accessor functions. The lowercase accessors return the field as an integer:

```
julia> t = Date(2014,1,31)
2014-01-31

julia> Dates.year(t)
2014

julia> Dates.month(t)
1

julia> Dates.week(t)
5

julia> Dates.day(t)
31
```

While propercase return the same value in the corresponding Period type:

```
julia> Dates.Year(t)
2014 years

julia> Dates.Day(t)
31 days
```

Compound methods are provided, as they provide a measure of efficiency if multiple fields are needed at the same time:

```
julia> Dates.yearmonth(t)
(2014,1)

julia> Dates.monthday(t)
(1,31)

julia> Dates.yearmonthday(t)
(2014,1,31)
```

One may also access the underlying UTInstant or integer value:

```
julia> dump(t)
Date
instant: UTInstant{Day}
  periods: Day
    value: Int64 735264

julia> t.instant
UTInstant{Day}(735264 days)

julia> Dates.value(t)
735264
```

# 21.4 Query Functions

Query functions provide calendrical information about a TimeType. They include information about the day of the week:

```
julia> t = Date(2014,1,31)
2014-01-31

julia> Dates.dayofweek(t)
5

julia> Dates.dayname(t)
"Friday"

julia> Dates.dayofweekofmonth(t)
5 # 5th Friday of January
```

#### Month of the year:

```
julia> Dates.monthname(t)
"January"

julia> Dates.daysinmonth(t)
31
```

As well as information about the TimeType's year and quarter:

```
julia> Dates.isleapyear(t)
false

julia> Dates.dayofyear(t)
31

julia> Dates.quarterofyear(t)
1

julia> Dates.dayofquarter(t)
31
```

The dayname () and monthname () methods can also take an optional locale keyword that can be used to return the name of the day or month of the year for other languages/locales:

Similarly for the monthname() function, a mapping of locale=>Dict{Int,UTF8String} should be loaded in VALUETOMONTH.

## 21.5 TimeType-Period Arithmetic

It's good practice when using any language/date framework to be familiar with how date-period arithmetic is handled as there are some tricky issues to deal with (though much less so for day-precision types).

The Dates module approach tries to follow the simple principle of trying to change as little as possible when doing Period arithmetic. This approach is also often known as *calendrical* arithmetic or what you would probably guess if someone were to ask you the same calculation in a conversation. Why all the fuss about this? Let's take a classic example: add 1 month to January 31st, 2014. What's the answer? Javascript will say March 3 (assumes 31 days). PHP says March 2 (assumes 30 days). The fact is, there is no right answer. In the Dates module, it gives the result of February 28th. How does it figure that out? I like to think of the classic 7-7-7 gambling game in casinos.

Now just imagine that instead of 7-7-7, the slots are Year-Month-Day, or in our example, 2014-01-31. When you ask to add 1 month to this date, the month slot is incremented, so now we have 2014-02-31. Then the day number is checked if it is greater than the last valid day of the new month; if it is (as in the case above), the day number is adjusted down to the last valid day (28). What are the ramifications with this approach? Go ahead and add another month to our date, 2014-02-28 + Month(1) = 2014-03-28. What? Were you expecting the last day of March? Nope, sorry, remember the 7-7-7 slots. As few slots as possible are going to change, so we first increment the month slot by 1, 2014-03-28, and boom, we're done because that's a valid date. On the other hand, if we were to add 2 months to our original date, 2014-01-31, then we end up with 2014-03-31, as expected. The other ramification of this approach is a loss in associativity when a specific ordering is forced (i.e. adding things in different orders results in different outcomes). For example:

```
julia> (Date(2014,1,29)+Dates.Day(1)) + Dates.Month(1)
2014-02-28

julia> (Date(2014,1,29)+Dates.Month(1)) + Dates.Day(1)
2014-03-01
```

What's going on there? In the first line, we're adding 1 day to January 29th, which results in 2014-01-30; then we add 1 month, so we get 2014-02-30, which then adjusts down to 2014-02-28. In the second example, we add 1 month *first*, where we get 2014-02-29, which adjusts down to 2014-02-28, and *then* add 1 day, which results in 2014-03-01. One design principle that helps in this case is that, in the presence of multiple Periods, the operations will be ordered by the Periods' *types*, not their value or positional order; this means Year will always be added first, then Month, then Week, etc. Hence the following *does* result in associativity and Just Works:

```
julia> Date(2014,1,29) + Dates.Day(1) + Dates.Month(1)
2014-03-01

julia> Date(2014,1,29) + Dates.Month(1) + Dates.Day(1)
2014-03-01
```

Tricky? Perhaps. What is an innocent Dates user to do? The bottom line is to be aware that explicitly forcing a certain associativity, when dealing with months, may lead to some unexpected results, but otherwise, everything should work as expected. Thankfully, that's pretty much the extent of the odd cases in date-period arithmetic when dealing with time in UT (avoiding the "joys" of dealing with daylight savings, leap seconds, etc.).

# 21.6 Adjuster Functions

As convenient as date-period arithmetics are, often the kinds of calculations needed on dates take on a *calendrical* or *temporal* nature rather than a fixed number of periods. Holidays are a perfect example; most follow rules such as "Memorial Day = Last Monday of May", or "Thanksgiving = 4th Thursday of November". These kinds of temporal expressions deal with rules relative to the calendar, like first or last of the month, next Tuesday, or the first and third Wednesdays, etc.

The Dates module provides the *adjuster* API through several convenient methods that aid in simply and succinctly expressing temporal rules. The first group of adjuster methods deal with the first and last of weeks, months, quarters, and years. They each take a single TimeType as input and return or *adjust to* the first or last of the desired period relative to the input.

```
# Adjusts the input to the Monday of the input's week
julia> Dates.firstdayofweek(Date(2014,7,16))
2014-07-14

# Adjusts to the last day of the input's month
julia> Dates.lastdayofmonth(Date(2014,7,16))
2014-07-31

# Adjusts to the last day of the input's quarter
julia> Dates.lastdayofquarter(Date(2014,7,16))
2014-09-30
```

The next two higher-order methods, tonext(), and toprev(), generalize working with temporal expressions by taking a DateFunction as first argument, along with a starting TimeType. A DateFunction is just a function, usually anonymous, that takes a single TimeType as input and returns a Bool, true indicating a satisfied adjustment criterion. For example:

```
julia> istuesday = x->Dates.dayofweek(x) == Dates.Tuesday # Returns true if the day of
  (anonymous function)

julia> Dates.tonext(istuesday, Date(2014,7,13)) # 2014-07-13 is a Sunday
2014-07-15

# Convenience method provided for day of the week adjustments
julia> Dates.tonext(Date(2014,7,13), Dates.Tuesday)
2014-07-15
```

This is useful with the do-block syntax for more complex temporal expressions:

```
julia> Dates.tonext(Date(2014,7,13)) do x
    # Return true on the 4th Thursday of November (Thanksgiving)
    Dates.dayofweek(x) == Dates.Thursday &&
    Dates.dayofweekofmonth(x) == 4 &&
    Dates.month(x) == Dates.November
    end
2014-11-27
```

The final method in the adjuster API is the recur() function. recur() vectorizes the adjustment process by taking a start and stop date (optionally specificed by a StepRange), along with a DateFunction to specify all valid dates/moments to be returned in the specified range. In this case, the DateFunction is often referred to as the "inclusion" function because it specifies (by returning true) which dates/moments should be included in the returned vector of dates.

```
# Pittsburgh street cleaning; Every 2nd Tuesday from April to November
# Date range from January 1st, 2014 to January 1st, 2015
julia> dr = Dates.Date(2014):Dates.Date(2015);
julia> recur(dr) do x
          Dates.dayofweek(x) == Dates.Tue &&
          Dates.April <= Dates.month(x) <= Dates.Nov &&
          Dates.dayofweekofmonth(x) == 2
      end
8-element Array{Date, 1}:
 2014-04-08
 2014-05-13
 2014-06-10
 2014-07-08
 2014-08-12
 2014-09-09
 2014-10-14
```

```
2014-11-11
```

Additional examples and tests are available in test/dates/adjusters.jl.

# 21.7 Period Types

Periods are a human view of discrete, sometimes irregular durations of time. Consider 1 month; it could represent, in days, a value of 28, 29, 30, or 31 depending on the year and month context. Or a year could represent 365 or 366 days in the case of a leap year. Period types are simple Int64 wrappers and are constructed by wrapping any Int64 convertible type, i.e. Year(1) or Month(3.0). Arithmetic between Period of the same type behave like integers, and limited Period-Real arithmetic is available.

```
julia> y1 = Dates.Year(1)
1 year
julia> y2 = Dates.Year(2)
2 years
julia> y3 = Dates.Year(10)
10 years
julia> y1 + y2
3 years
julia> div(y3,y2)
5 years
julia> y3 - y2
8 years
julia> y3 * y2
20 years
julia> y3 % y2
0 years
julia> y1 + 20
21 years
julia > div(y3,3) # mirrors integer division
3 years
```

See the API reference for additional information on methods exported from the Dates module.

# **Interacting With Julia**

Julia comes with a full-featured interactive command-line REPL (read-eval-print loop) built into the julia executable. In addition to allowing quick and easy evaluation of Julia statements, it has a searchable history, tab-completion, many helpful keybindings, and dedicated help and shell modes. The REPL can be started by simply calling julia with no arguments or double-clicking on the executable:

To exit the interactive session, type ^D — the control key together with the d key on a blank line — or type quit () followed by the return or enter key. The REPL greets you with a banner and a julia> prompt.

## 22.1 The different prompt modes

#### 22.1.1 The Julian mode

The REPL has four main modes of operation. The first and most common is the Julian prompt. It is the default mode of operation; each new line initially starts with julia>. It is here that you can enter Julia expressions. Hitting return or enter after a complete expression has been entered will evaluate the entry and show the result of the last expression.

```
julia> string(1 + 2)
"3"
```

There are a number useful features unique to interactive work. In addition to showing the result, the REPL also binds the result to the variable ans. A trailing semicolon on the line can be used as a flag to suppress showing the result.

```
julia> string(3 * 4);
julia> ans
"12"
```

#### 22.1.2 Help mode

When the cursor is at the beginning of the line, the prompt can be changed to a help mode by typing?. Julia will attempt to print help or documentation for anything entered in help mode:

```
julia> ? # upon typing ?, the prompt changes (in place) to: help>
help> string
Base.string(xs...)

Create a string from any values using the "print" function.
```

In addition to function names, complete function calls may be entered to see which method is called for the given argument(s). Macros, types and variables can also be queried:

```
help> string(1)
string(x::Union{Int16,Int128,Int8,Int32,Int64}) at string.jl:1553
help> @printf
Base.@printf([io::IOStream], "%Fmt", args...)

Print arg(s) using C "printf()" style format specification
    string. Optionally, an IOStream may be passed as the first argument
    to redirect output.

help> AbstractString
DataType : AbstractString
    supertype: Any
    subtypes : Any [DirectIndexString,RepString,RevString{T<:AbstractString},RopeString,SubString{T<:AbstractString},RopeString,SubString{T<:AbstractString}</pre>
```

Help mode can be exited by pressing backspace at the beginning of the line.

#### 22.1.3 Shell mode

Just as help mode is useful for quick access to documentation, another common task is to use the system shell to execute system commands. Just as ? entered help mode when at the beginning of the line, a semicolon (;) will enter the shell mode. And it can be exited by pressing backspace at the beginning of the line.

```
julia> ; # upon typing ;, the prompt changes (in place) to: shell>
shell> echo hello
hello
```

#### 22.1.4 Search modes

In all of the above modes, the executed lines get saved to a history file, which can be searched. To initiate an incremental search through the previous history, type  $^R$ —the control key together with the r key. The prompt will change to (reverse-i-search) '':, and as you type the search query will appear in the quotes. The most recent result that matches the query will dynamically update to the right of the colon as more is typed. To find an older result using the same query, simply type  $^R$  again.

Just as ^R is a reverse search, ^S is a forward search, with the prompt (i-search) '':. The two may be used in conjunction with each other to move through the previous or next matching results, respectively.

# 22.2 Key bindings

The Julia REPL makes great use of key bindings. Several control-key bindings were already introduced above (^D to exit, ^R and ^S for searching), but there are many more. In addition to the control-key, there are also meta-key bindings. These vary more by platform, but most terminals default to using alt- or option- held down with a key to send the meta-key (or can be configured to do so).

Program control	
^D	Exit (when buffer is empty)
^C	Interrupt or cancel
^L	Clear console screen
Return/Enter, ^J	New line, executing if it is complete
meta-Return/Enter	Insert new line without executing it
? or ;	Enter help or shell mode (when at start of a line)
^R, ^S	Incremental history search, described above
<b>Cursor movement</b>	
Right arrow, ^F	Move right one character
Left arrow, ^B	Move left one character
Home, ^A	Move to beginning of line
End, ^E	Move to end of line
^P	Change to the previous or next history entry
^N	Change to the next history entry
Up arrow	Move up one line (or to the previous history entry)
Down arrow	Move down one line (or to the next history entry)
Page-up	Change to the previous history entry that matches the text before the cursor
Page-down	Change to the next history entry that matches the text before the cursor
meta-F	Move right one word
meta-B	Move left one word
Editing	
Backspace, ^H	Delete the previous character
Delete, ^D	Forward delete one character (when buffer has text)
meta-Backspace	Delete the previous word
meta-D	Forward delete the next word
^W	Delete previous text up to the nearest whitespace
^K	"Kill" to end of line, placing the text in a buffer
^Y	"Yank" insert the text from the kill buffer
^T	Transpose the characters about the cursor

#### 22.2.1 Customizing keybindings

Julia's REPL keybindings may be fully customized to a user's preferences by passing a dictionary to REPL.setup\_interface(). The keys of this dictionary may be characters or strings. The key '\*' refers to the default action. Control plus character x bindings are indicated with " $^x$ ". Meta plus x can be written " $^x$ ". The values of the custom keymap must be nothing (indicating that the input should be ignored) or functions that accept the signature (PromptState, AbstractREPL, Char). The REPL.setup\_interface() function must be called before the REPL is initialized, by registering the operation with atreplinit(). For example, to bind the up and down arrow keys to move through history without prefix search, one could put the following code in .juliarc.jl:

```
import Base: LineEdit, REPL
const mykeys = Dict{Any, Any}(
```

22.2. Key bindings 205

```
# Up Arrow
"\e[A" => (s,o...)->(LineEdit.edit_move_up(s) || LineEdit.history_prev(s, LineEdit.mode(s).hist)),
# Down Arrow
"\e[B" => (s,o...)->(LineEdit.edit_move_up(s) || LineEdit.history_next(s, LineEdit.mode(s).hist))
)

function customize_keys(repl)
   repl.interface = REPL.setup_interface(repl; extra_repl_keymap = mykeys)
end
atreplinit(customize_keys)
```

Users should refer to base/LineEdit.jl to discover the available actions on key input.

## 22.3 Tab completion

In both the Julian and help modes of the REPL, one can enter the first few characters of a function or type and then press the tab key to get a list all matches:

```
julia> stri
stride strides string stringmime strip

julia> Stri
StridedArray StridedVecOrMat AbstractString
StridedMatrix StridedVector
```

The tab key can also be used to substitute LaTeX math symbols with their Unicode equivalents, and get a list of LaTeX matches as well:

```
julia> \pi[TAB]
julia> \pi
\pi = 3.1415926535897...
julia > e \setminus 1[TAB] = [1,0]
julia > e1 = [1,0]
2-element Array{Int64,1}:
julia > e ^1[TAB] = [1 0]
julia> e^{1} = [1 \ 0]
1x2 Array{Int64,2}:
julia> \sqrt[TAB]2
                         \# \sqrt{} is equivalent to the sqrt() function
julia> √2
1.4142135623730951
julia > hbar[TAB](h) = h / 2 pi[TAB]
julia> \hbar(h) = h / 2\pi
\hbar (generic function with 1 method)
julia> \h[TAB]
\hat
                                                              \hookleftarrow
                                                                                   \hslash
                    \heartsuit
                                         \hksearow
\hbar
                   \hermitconjmatrix \hkswarow
                                                              \hookrightarrow
                                                                                   \hspace
```

A full list of tab-completions can be found in the *Unicode Input* (page 313) section of the manual.

#### 22.3.1 Customizing Colors

The colors used by Julia and the REPL can be customized, as well. To change the color of the Julia prompt you can add something like the following to your juliarc.jl file:

```
Base.active_repl.prompt_color = Base.text_colors[:cyan]
```

The available color keys in Base.text\_colors are :black, :red, :green, :yellow, :blue, :magenta, :cyan, :white, :normal, and :bold. Similarly, you can change the colors for the help and shell prompts and input and answer text by setting the appropriate member of Base.active\_repl (respectively, help\_color, shell\_color, input\_color, and answer\_color). For the latter two, be sure that the envcolors member is also set to false.

You can also customize the color used to render warning and informational messages by setting the appropriate environment variable. For instance, to render warning messages in yellow and informational messages in cyan you can add the following to your juliarc.jl file:

```
ENV["JULIA_WARN_COLOR"] = :yellow
ENV["JULIA_INFO_COLOR"] = :cyan
```

Julia Language Documentation, Release 0.5.0-dev			

# **Running External Programs**

Julia borrows backtick notation for commands from the shell, Perl, and Ruby. However, in Julia, writing

```
julia> `echo hello`
  `echo hello`
```

differs in several aspects from the behavior in various shells, Perl, or Ruby:

- Instead of immediately running the command, backticks create a Cmd object to represent the command. You can use this object to connect the command to others via pipes, run it, and read or write to it.
- When the command is run, Julia does not capture its output unless you specifically arrange for it to. Instead, the output of the command by default goes to STDOUT (page 449) as it would using libc's system call.
- The command is never run with a shell. Instead, Julia parses the command syntax directly, appropriately interpolating variables and splitting on words as the shell would, respecting shell quoting syntax. The command is run as julia's immediate child process, using fork and exec calls.

Here's a simple example of running an external program:

```
julia> run(`echo hello`)
hello
```

The hello is the output of the echo command, sent to STDOUT (page 449). The run method itself returns nothing, and throws an ErrorException (page 330) if the external command fails to run successfully.

If you want to read the output of the external command, readstring () (page 454) can be used instead:

```
julia> a=readstring(`echo hello`)
"hello\n"

julia> (chomp(a)) == "hello"
true
```

More generally, you can use open () (page 449) to read from or write to an external command.

## 23.1 Interpolation

Suppose you want to do something a bit more complicated and use the name of a file in the variable file as an argument to a command. You can use \$ for interpolation much as you would in a string literal (see *Strings* (page 41)):

```
julia> file = "/etc/passwd"
  "/etc/passwd"

julia> `sort $file`
  `sort /etc/passwd`
```

A common pitfall when running external programs via a shell is that if a file name contains characters that are special to the shell, they may cause undesirable behavior. Suppose, for example, rather than /etc/passwd, we wanted to sort the contents of the file /Volumes/External HD/data.csv. Let's try it:

```
julia> file = "/Volumes/External HD/data.csv"
   "/Volumes/External HD/data.csv"

julia> `sort $file`
   `sort '/Volumes/External HD/data.csv'`
```

How did the file name get quoted? Julia knows that file is meant to be interpolated as a single argument, so it quotes the word for you. Actually, that is not quite accurate: the value of file is never interpreted by a shell, so there's no need for actual quoting; the quotes are inserted only for presentation to the user. This will even work if you interpolate a value as part of a shell word:

```
julia> path = "/Volumes/External HD"
  "/Volumes/External HD"

julia> name = "data"
  "data"

julia> ext = "csv"
  "csv"

julia> `sort $path/$name.$ext`
  `sort '/Volumes/External HD/data.csv'`
```

As you can see, the space in the path variable is appropriately escaped. But what if you *want* to interpolate multiple words? In that case, just use an array (or any other iterable container):

```
julia> files = ["/etc/passwd","/Volumes/External HD/data.csv"]
2-element Array{ASCIIString,1}:
   "/etc/passwd"
   "/Volumes/External HD/data.csv"

julia> `grep foo $files`
   `grep foo /etc/passwd '/Volumes/External HD/data.csv'`
```

If you interpolate an array as part of a shell word, Julia emulates the shell's {a,b,c} argument generation:

```
julia> names = ["foo","bar","baz"]
3-element Array{ASCIIString,1}:
    "foo"
    "bar"
    "baz"
```

```
julia> `grep xylophone $names.txt`
`grep xylophone foo.txt bar.txt baz.txt`
```

Moreover, if you interpolate multiple arrays into the same word, the shell's Cartesian product generation behavior is emulated:

```
julia> names = ["foo","bar","baz"]
3-element Array{ASCIIString,1}:
    "foo"
    "bar"
    "baz"

julia> exts = ["aux","log"]
2-element Array{ASCIIString,1}:
    "aux"
    "log"

julia> `rm -f $names.$exts`
    `rm -f foo.aux foo.log bar.aux bar.log baz.aux baz.log`
```

Since you can interpolate literal arrays, you can use this generative functionality without needing to create temporary array objects first:

```
julia> `rm -rf $["foo","bar","baz","qux"].$["aux","log","pdf"]`
`rm -rf foo.aux foo.log foo.pdf bar.aux bar.log bar.pdf baz.aux baz.log baz.pdf qux.aux qux.log qux.]
```

## 23.2 Quoting

Inevitably, one wants to write commands that aren't quite so simple, and it becomes necessary to use quotes. Here's a simple example of a Perl one-liner at a shell prompt:

```
sh$ perl -le '$|=1; for (0..3) { print }'
0
1
2
3
```

The Perl expression needs to be in single quotes for two reasons: so that spaces don't break the expression into multiple shell words, and so that uses of Perl variables like \$ | (yes, that's the name of a variable in Perl), don't cause interpolation. In other instances, you may want to use double quotes so that interpolation *does* occur:

```
sh$ first="A"
sh$ second="B"
sh$ perl -le '$|=1; print for @ARGV' "1: $first" "2: $second"
1: A
2: B
```

In general, the Julia backtick syntax is carefully designed so that you can just cut-and-paste shell commands as is into backticks and they will work: the escaping, quoting, and interpolation behaviors are the same as the shell's. The only difference is that the interpolation is integrated and aware of Julia's notion of what is a single string value, and what is a container for multiple values. Let's try the above two examples in Julia:

23.2. Quoting 211

```
0
1
2
3

julia> first = "A"; second = "B";

julia> `perl -le 'print for @ARGV' "1: $first" "2: $second"`
  `perl -le 'print for @ARGV' '1: A' '2: B'`

julia> run(ans)
1: A
2: B
```

The results are identical, and Julia's interpolation behavior mimics the shell's with some improvements due to the fact that Julia supports first-class iterable objects while most shells use strings split on spaces for this, which introduces ambiguities. When trying to port shell commands to Julia, try cut and pasting first. Since Julia shows commands to you before running them, you can easily and safely just examine its interpretation without doing any damage.

## 23.3 Pipelines

Shell metacharacters, such as |, &, and >, are not special inside of Julia's backticks: unlike in the shell, inside of Julia's backticks, a pipe is always just a pipe:

```
julia> run(`echo hello | sort`)
hello | sort
```

This expression invokes the echo command with three words as arguments: "hello", "|", and "sort". The result is that a single line is printed: "hello | sort". Inside of backticks, a "|" is just a literal pipe character. How, then, does one construct a pipeline? Instead of using "|" inside of backticks, one uses pipeline() (page 328):

```
julia> run(pipeline(`echo hello`, `sort`))
hello
```

This pipes the output of the echo command to the sort command. Of course, this isn't terribly interesting since there's only one line to sort, but we can certainly do much more interesting things:

```
julia> run(pipeline(`cut -d: -f3 /etc/passwd`, `sort -n`, `tail -n5`))
210
211
212
213
214
```

This prints the highest five user IDs on a UNIX system. The cut, sort and tail commands are all spawned as immediate children of the current julia process, with no intervening shell process. Julia itself does the work to setup pipes and connect file descriptors that is normally done by the shell. Since Julia does this itself, it retains better control and can do some things that shells cannot.

Julia can run multiple commands in parallel:

```
julia> run(`echo hello` & `echo world`)
world
hello
```

The order of the output here is non-deterministic because the two echo processes are started nearly simultaneously, and race to make the first write to the STDOUT (page 449) descriptor they share with each other and the julia parent

process. Julia lets you pipe the output from both of these processes to another program:

```
julia> run(pipeline(`echo world` & `echo hello`, `sort`))
hello
world
```

In terms of UNIX plumbing, what's happening here is that a single UNIX pipe object is created and written to by both echo processes, and the other end of the pipe is read from by the sort command.

IO redirection can be accomplished by passing keyword arguments stdin, stdout, and stderr to the pipeline function:

```
pipeline(`do_work`, stdout=pipeline(`sort`, "out.txt"), stderr="errs.txt")
```

### 23.3.1 Avoiding Deadlock in Pipelines

When reading and writing to both ends of a pipeline from a single process, it is important to avoid forcing the kernel to buffer all of the data.

For example, when reading all of the output from a command, call read(out), not wait (process), since the former will actively consume all of the data written by the process, whereas the latter will attempt to store the data in the kernel's buffers while waiting for a reader to be connected.

Another common solution is to separate the reader and writer of the pipeline into separate Tasks:

```
writer = @async writeall(process, "data")
reader = @async do_compute(read(process))
wait(process)
fetch(reader)
```

### 23.3.2 Complex Example

B 9

The combination of a high-level programming language, a first-class command abstraction, and automatic setup of pipes between processes is a powerful one. To give some sense of the complex pipelines that can be created easily, here are some more sophisticated examples, with apologies for the excessive use of Perl one-liners:

```
julia> prefixer(prefix, sleep) = `perl -nle '$|=1; print "'$prefix' ", $_; sleep '$sleep';'`;

julia> run(pipeline(`perl -le '$|=1; for(0..9){ print; sleep 1 }'`, prefixer("A",2) & prefixer("B",2
A 0
B 1
A 2
B 3
A 4
B 5
A 6
B 7
A 8
```

This is a classic example of a single producer feeding two concurrent consumers: one perl process generates lines with the numbers 0 through 9 on them, while two parallel processes consume that output, one prefixing lines with the letter "A", the other with the letter "B". Which consumer gets the first line is non-deterministic, but once that race has been won, the lines are consumed alternately by one process and then the other. (Setting  $\$ \mid = 1$  in Perl causes each print statement to flush the STDOUT (page 449) handle, which is necessary for this example to work. Otherwise all the output is buffered and printed to the pipe at once, to be read by just one consumer process.)

Here is an even more complex multi-stage producer-consumer example:

23.3. Pipelines 213

This example is similar to the previous one, except there are two stages of consumers, and the stages have different latency so they use a different number of parallel workers, to maintain saturated throughput.

We strongly encourage you to try all these examples to see how they work.

## **Calling C and Fortran Code**

Though most code can be written in Julia, there are many high-quality, mature libraries for numerical computing already written in C and Fortran. To allow easy use of this existing code, Julia makes it simple and efficient to call C and Fortran functions. Julia has a "no boilerplate" philosophy: functions can be called directly from Julia without any "glue" code, code generation, or compilation — even from the interactive prompt. This is accomplished just by making an appropriate call with ccall syntax, which looks like an ordinary function call.

The code to be called must be available as a shared library. Most C and Fortran libraries ship compiled as shared libraries already, but if you are compiling the code yourself using GCC (or Clang), you will need to use the <code>-shared</code> and <code>-fPIC</code> options. The machine instructions generated by Julia's JIT are the same as a native C call would be, so the resulting overhead is the same as calling a library function from C code. (Non-library function calls in both C and Julia can be inlined and thus may have even less overhead than calls to shared library functions. When both libraries and executables are generated by LLVM, it is possible to perform whole-program optimizations that can even optimize across this boundary, but Julia does not yet support that. In the future, however, it may do so, yielding even greater performance gains.)

Shared libraries and functions are referenced by a tuple of the form (:function, "library") or ("function", "library") where function is the C-exported function name. library refers to the shared library name: shared libraries available in the (platform-specific) load path will be resolved by name, and if necessary a direct path may be specified.

A function name may be used alone in place of the tuple (just:function or "function"). In this case the name is resolved within the current process. This form can be used to call C library functions, functions in the Julia runtime, or functions in an application linked to Julia.

By default, Fortran compilers generate mangled names (for example, converting function names to lowercase or uppercase, often appending an underscore), and so to call a Fortran function via ccall you must pass the mangled identifier corresponding to the rule followed by your Fortran compiler. Also, when calling a Fortran function, all inputs must be passed by reference.

Finally, you can use ccall to actually generate a call to the library function. Arguments to ccall are as follows:

- 1. (:function, "library") pair (must be a constant, but see below).
- 2. Return type (see below for mapping the declared C type to Julia)
  - This argument will be evaluated at compile-time.
- 3. A tuple of input types. The input types must be written as a literal tuple, not a tuple-valued variable or expression.
  - This argument will be evaluated at compile-time.
- 4. The following arguments, if any, are the actual argument values passed to the function.

As a complete but simple example, the following calls the clock function from the standard C library:

```
julia> t = ccall( (:clock, "libe"), Int32, ())
2292761

julia> t
2292761

julia> typeof(ans)
Int32
```

clock takes no arguments and returns an Int32. One common gotcha is that a 1-tuple must be written with a trailing comma. For example, to call the getenv function to get a pointer to the value of an environment variable, one makes a call like this:

```
julia> path = ccall((:getenv, "libc"), Cstring, (Cstring,), "SHELL")
Cstring(@0x00007fff5fbffc45)

julia> bytestring(path)
"/bin/bash"
```

Note that the argument type tuple must be written as (Cstring,), rather than (Cstring). This is because (Cstring) is just the expression Cstring surrounded by parentheses, rather than a 1-tuple containing Cstring:

```
julia> (Cstring)
Cstring

julia> (Cstring,)
(Cstring,)
```

In practice, especially when providing reusable functionality, one generally wraps ccall uses in Julia functions that set up arguments and then check for errors in whatever manner the C or Fortran function indicates them, propagating to the Julia caller as exceptions. This is especially important since C and Fortran APIs are notoriously inconsistent about how they indicate error conditions. For example, the getenv C library function is wrapped in the following Julia function, which is a simplified version of the actual definition from env.jl:

The C getenv function indicates an error by returning NULL, but other standard C functions indicate errors in various different ways, including by returning -1, 0, 1 and other special values. This wrapper throws an exception clearly indicating the problem if the caller tries to get a non-existent environment variable:

```
julia> getenv("SHELL")
  "/bin/bash"

julia> getenv("FOOBAR")
getenv: undefined variable: FOOBAR
```

Here is a slightly more complex example that discovers the local machine's hostname:

```
hostname, sizeof(hostname))
hostname[end] = 0; # ensure null-termination
return bytestring(pointer(hostname))
end
```

This example first allocates an array of bytes, then calls the C library function <code>gethostname</code> to fill the array in with the hostname, takes a pointer to the hostname buffer, and converts the pointer to a Julia string, assuming that it is a NUL-terminated C string. It is common for C libraries to use this pattern of requiring the caller to allocate memory to be passed to the callee and filled in. Allocation of memory from Julia like this is generally accomplished by creating an uninitialized array and passing a pointer to its data to the C function. This is why we don't use the <code>Cstring</code> type here: as the array is uninitialized, it could contain NUL bytes. Converting to a <code>Cstring</code> as part of the <code>ccall</code> checks for contained NUL bytes and could therefore throw a conversion error.

# 24.1 Creating C-Compatible Julia Function Pointers

It is possible to pass Julia functions to native c-functions that accept function pointer arguments. For example, to match c-prototypes of the form:

```
typedef returntype (*functiontype)(argumenttype,...)
```

The function cfunction generates the c-compatible function pointer for a call to a Julia library function. Arguments to cfunction are as follows:

- 1. A Julia Function
- 2. Return type
- 3. A tuple of input types

A classic example is the standard C library qsort function, declared as:

The base argument is a pointer to an array of length nmemb, with elements of size bytes each. compare is a callback function which takes pointers to two elements a and b and returns an integer less/greater than zero if a should appear before/after b (or zero if any order is permitted). Now, suppose that we have a 1d array A of values in Julia that we want to sort using the qsort function (rather than Julia's built-in sort function). Before we worry about calling qsort and passing arguments, we need to write a comparison function that works for some arbitrary type T:

```
function mycompare{T}(a::T, b::T)
   return convert(Cint, a < b ? -1 : a > b ? +1 : 0)::Cint
end
```

Notice that we have to be careful about the return type: qsort expects a function returning a C int, so we must be sure to return Cint via a call to convert and a typeassert.

In order to pass this function to C, we obtain its address using the function cfunction:

```
const mycompare_c = cfunction(mycompare, Cint, (Ref{Cdouble}, Ref{Cdouble}))
```

cfunction accepts three arguments: the Julia function (mycompare), the return type (Cint), and a tuple of the argument types, in this case to sort an array of Cdouble (Float64) elements.

The final call to qsort looks like this:

After this executes, A is changed to the sorted array [-2.7, 1.3, 3.1, 4.4]. Note that Julia knows how to convert an array into a Ptr{Cdouble}, how to compute the size of a type in bytes (identical to C's sizeof operator), and so on. For fun, try inserting a println("mycompare(\$a,\$b)") line into mycompare, which will allow you to see the comparisons that qsort is performing (and to verify that it is really calling the Julia function that you passed to it).

# 24.2 Mapping C Types to Julia

It is critical to exactly match the declared C type with its declaration in Julia. Inconsistencies can cause code that works correctly on one system to fail or produce indeterminate results on a different system.

Note that no C header files are used anywhere in the process of calling C functions: you are responsible for making sure that your Julia types and call signatures accurately reflect those in the C header file. (The Clang package can be used to auto-generate Julia code from a C header file.)

#### 24.2.1 Auto-conversion:

Julia automatically inserts calls to the convert function to convert each argument to the specified type. For example, the following call:

```
ccall((:foo, "libfoo"), Void, (Int32, Float64), x, y)
```

will behave as if the following were written:

convert normally just calls convert, but can be defined to return an arbitrary new object more appropriate for passing to C. For example, this is used to convert an Array of objects (e.g. strings) to an array of pointers.

unsafe\_convert handles conversion to Ptr types. It is considered unsafe because converting an object to a native pointer can hide the object from the garbage collector, causing it to be freed prematurely.

### 24.2.2 Type Correspondences:

First, a review of some relevant Julia type terminology:

Syntax / Keyword	Example	Description
type	ASCIIString	"Leaf Type" :: A group of related data that includes a type-tag, is managed by the Julia GC, and is defined by object-identity. The type parameters of a leaf type must be fully defined (no TypeVars are allowed) in order for the instance to be constructed.
abstract	<pre>Any, AbstractArray{T,N}, Complex{T}</pre>	"Super Type" :: A super-type (not a leaf-type) that cannot be instantiated, but can be used to describe a group of types.
{T}	Vector{Int}	"Type Parameter" :: A specialization of a type (typically used for dispatch or storage optimization). "TypeVar" :: The T in the type parameter declaration is referred to as a TypeVar (short for type variable).
bitstype	Int,Float64	"Bits Type" :: A type with no fields, but a size. It is stored and defined by-value.
immutable	Pair{Int,Int} Complex128(isbits)	"Immutable" :: A type with all fields defined to be constant. It is defined by-value. And may be stored with a type-tag.  "Is-Bits" :: A bitstype, or an immutable type where all fields are other isbits types. It is defined by-value, and is stored without a type-tag.
type; end	nothing	"Singleton" :: a Leaf Type or Immutable with no fields.
() or tuple() '	(1,2,3)	"Tuple" :: an immutable data- structure similar to an anonymous immutable type, or a constant array. Represented as either an array or a struct.
typealias	Not applicable here	Type aliases, and other similar mechanisms of doing type indirection, are resolved to their base type (this includes assigning a type to another name, or getting the type out of a function call).

## 24.2.3 Bits Types:

There are several special types to be aware of, as no other type can be defined to behave the same:

**Float32** Exactly corresponds to the float type in C (or REAL\*4 in Fortran).

**Float 64** Exactly corresponds to the double type in C (or REAL \*8 in Fortran).

 $\textbf{Complex64} \ \ \textbf{Exactly corresponds to the complex float type in } C \ \ (or \ \ \texttt{COMPLEX*8 in Fortran}).$ 

**Complex128** Exactly corresponds to the complex double type in C (or COMPLEX\*16 in Fortran).

**Signed** Exactly corresponds to the signed type annotation in C (or any INTEGER type in Fortran). Any Julia type that is not a subtype of Signed is assumed to be unsigned.

**Ref{T}** Behaves like a  $Ptr\{T\}$  that owns its memory.

**Array** $\{T, N\}$  When an array is passed to C as a Ptr $\{T\}$  argument, it is not reinterpret-cast: Julia requires that the element type of the array matches T, and the address of the first element is passed.

Therefore, if an Array contains data in the wrong format, it will have to be explicitly converted using a call such as trunc(Int32, a).

To pass an array A as a pointer of a different type *without* converting the data beforehand (for example, to pass a Float64 array to a function that operates on uninterpreted bytes), you can declare the argument as Ptr{Void}.

If an array of eltype  $Ptr\{T\}$  is passed as a  $Ptr\{Ptr\{T\}\}$  argument, the Julia base library convert function will attempt to first make a null-terminated copy of the array with each element replaced by its convert version. This allows, for example, passing an argv pointer array of type  $Vector\{ByteString\}$  to an argument of type  $Ptr\{Cchar\}\}$ .

On all systems we currently support, basic C/C++ value types may be translated to Julia types as follows. Every C type also has a corresponding Julia type with the same name, prefixed by C. This can help for writing portable code (and remembering that an int in C is not the same as an Int in Julia).

#### **System Independent:**

C name	Fortran name	Standard Julia Alias	Julia Base Type
unsigned char	CHARACTER	Cuchar	UInt8
bool (C++)			
short	INTEGER*2	Cshort	Int16
	LOGICAL*2		
unsigned short		Cushort	UInt16
int	INTEGER*4	Cint	Int32
BOOL (C, typical)	LOGICAL*4		
unsigned int		Cuint	UInt32
long long	INTEGER*8	Clonglong	Int64
	LOGICAL*8		
unsigned long long		Culonglong	UInt64
intmax_t		Cintmax_t	Int64
uintmax_t		Cuintmax_t	UInt64
float	REAL*4i	Cfloat	Float32
double	REAL*8	Cdouble	Float64
complex float	COMPLEX*8	Complex64	Complex{Float32}
complex double	COMPLEX*16	Complex128	Complex{Float64}
ptrdiff_t		Cptrdiff_t	Int
ssize_t		Cssize_t	Int
size_t		Csize_t	UInt
void			Void
void*			Ptr{Void}
T* (where T represents an			Ref{T}
appropriately defined type)			
char* (or char[], e.g. a	CHARACTER*N		Cstring if NUL-
string)			terminated, or
			Ptr{UInt8} if not
char**(or*char[])			Ptr{Ptr{UInt8}}
jl_value_t* (any Julia			Any
Type)			
jl_value_t** (a refer-			Ref{Any}
ence to a Julia Type)			
va_arg			Not supported
(variadic function			T (where T is one
specification)			of the above types, vari-
			adic functions of different
			argument types are not sup-
			ported)

The <code>Cstring</code> type is essentially a synonym for <code>Ptr{UInt8}</code>, except the conversion to <code>Cstring</code> throws an error if the Julia string contains any embedded NUL characters (which would cause the string to be silently truncated if the C routine treats NUL as the terminator). If you are passing a <code>char\*</code> to a C routine that does not assume NUL termination (e.g. because you pass an explicit string length), or if you know for certain that your Julia string does not contain NUL and want to skip the check, you can use <code>Ptr{UInt8}</code> as the argument type. <code>Cstring</code> can also be used as the <code>ccall</code> return type, but in that case it obviously does not introduce any extra checks and is only meant to improve readability of the call.

#### **System-dependent:**

C name	Standard Julia Alias	Julia Base Type
char	Cchar	Int8 (x86, x86_64)
		UInt8 (powerpc, arm)
long	Clong	Int (UNIX)
		Int32 (Windows)
unsigned long	Culong	UInt (UNIX)
		UInt32 (Windows)
wchar_t	Cwchar_t	Int32 (UNIX)
		UInt16 (Windows)

**Note:** When calling a Fortran function, all inputs must be passed by reference, so all type correspondences above should contain an additional Ptr{..} or Ref{..} wrapper around their type specification.

**Warning:** For string arguments (char\*) the Julia type should be Cstring (if NUL- terminated data is expected) or either Ptr{Cchar} or Ptr{UInt8} otherwise (these two pointer types have the same effect), as described above, not ASCIIString. Similarly, for array arguments (T[] or T\*), the Julia type should again be Ptr{T}, not Vector{T}.

Warning: Julia's Char type is 32 bits, which is not the same as the wide character type (wchar\_t or wint\_t) on all platforms.

Note: For wchar\_t\* arguments, the Julia type should be Cwstring (if the C routine expects a NUL-terminated string) or Ptr{Cwchar\_t} otherwise, and data can be converted to/from ordinary Julia strings by the wstring(s) function (equivalent to either utf16(s) or utf32(s) depending upon the width of Cwchar\_t); this conversion will be called automatically for Cwstring arguments. Note also that ASCII, UTF-8, UTF-16, and UTF-32 string data in Julia is internally NUL-terminated, so it can be passed to C functions expecting NUL-terminated data without making a copy (but using the Cwstring type will cause an error to be thrown if the string itself contains NUL characters).

**Note:** C functions that take an argument of the type char\*\* can be called by using a Ptr{Ptr{UInt8}} type within Julia. For example, C functions of the form:

```
int main(int argc, char **argv);
```

can be called via the following Julia code:

```
argv = [ "a.out", "arg1", "arg2" ]
ccall(:main, Int32, (Int32, Ptr{Ptr{UInt8}}), length(argv), argv)
```

**Note:** A C function declared to return Void will return the value nothing in Julia.

## 24.2.4 Struct Type correspondences

Composite types, aka struct in C or TYPE in Fortran90 (or STRUCTURE / RECORD in some variants of F77), can be mirrored in Julia by creating a type or immutable definition with the same field layout.

When used recursively, isbits types are stored inline. All other types are stored as a pointer to the data. When mirroring a struct used by-value inside another struct in C, it is imperative that you do not attempt to manually copy

the fields over, as this will not preserve the correct field alignment. Instead, declare an immutable isbits type and use that instead. Unnamed structs are not possible in the translation to Julia.

Packed structs and union declarations are not supported by Julia.

You can get a near approximation of a union if you know, a priori, the field that will have the greatest size (potentially including padding). When translating your fields to Julia, declare the Julia field to be only of that type.

Arrays of parameters must be expanded manually, currently (either inline, or in an immutable helper-type). For example:

```
in C:
struct B {
    int A[3];
};
b_a_2 = B.A[2];

in Julia:
immutable B_A
    A_1::Cint
    A_2::Cint
    A_3::Cint
end
type B
    A::B_A
end
b_a_2 = B.A.(2)
```

Arrays of unknown size are not supported.

In the future, some of these restrictions may be reduced or eliminated.

### 24.2.5 Memory Ownership:

#### malloc/free

Memory allocation and deallocation of such objects must be handled by calls to the appropriate cleanup routines in the libraries being used, just like in any C program. Do not try to free an object received from a C library with Libc.free in Julia, as this may result in the free function being called via the wrong libc library and cause Julia to crash. The reverse (passing an object allocated in Julia to be freed by an external library) is equally invalid.

```
Ptr{T} vs. Array{T} vs. Ref{T} vs. T
```

The choice of type-wrapper declaration strongly depends on who allocated the memory, and the declared type. In general, use T if the memory is intended to be allocated in (and managed by) Julia (with type-tag). Use  $Ptr\{T\}$  if the memory is expected to be populated by C (without type-tag). Use  $Ref\{T\}$  if you have an isbits type, but you want to turn it into a pointer to a struct in another struct definition.

See issue #2818 for some work that needs to be done to simplify this so that Julia types can be used to recursively mirror c-style structs, without requiring as much manual management of the Ptr conversions. After #2818 is implemented, it will be true that an  $Vector\{T\}$  will be equivalent to an  $Ptr\{Ptr\{T\}\}$ . That is currently not true, and the conversion must be explicitly.

## 24.3 Mapping C Functions to Julia

### 24.3.1 ccall/cfunction argument translation guide

For translating a c argument list to Julia:

- T, where T is one of the primitive types: char, int, long, short, float, double, complex, enum or any of their typedef equivalents
  - T, where T is an equivalent Julia Bits Type (per the table above)
  - if T is an enum, the argument type should be equivalent to Cint or Cuint
  - argument value will be copied (passed by-value)
- struct T (including typedef to a struct)
  - T, where T is a Julia Leaf Type
  - argument value will be copied (passed by-value)
- void\*
  - depends on how this parameter is used, first translate this to the intended pointer type, then determine the Julia equivalent using the remaining rules in this list
  - this argument may be declared as Ptr{Void}, if it really is just an unknown pointer
- jl\_value\_t\*
  - Any
  - argument value must be a valid Julia object
  - currently unsupported by cfunction
- jl\_value\_t\*\*
  - Ref{Any}
  - argument value must be a valid Julia object (or C\_NULL)
  - currently unsupported by cfunction
- T\*
  - Ref { T }, where T is the Julia type corresponding to T
  - argument value will be copied if it is an isbits type otherwise, the value must be a valid Julia object
- (T\*) (...) (e.g. a pointer to a function)
  - Ptr{Void} (you may need to use cfunction explicitly to create this pointer)
- ... (e.g. a vararg)
  - T..., where T is the Julia type
- va\_arg
  - not supported

### 24.3.2 ccall/cfunction return type translation guide

For translating a c return type to Julia:

- void
  - Void (this will return the singleton instance nothing::Void)
- T, where T is one of the primitive types: char, int, long, short, float, double, complex, enum or any of their typedef equivalents
  - T, where T is an equivalent Julia Bits Type (per the table above)
  - if T is an enum, the argument type should be equivalent to Cint or Cuint
  - argument value will be copied (returned by-value)
- struct T (including typedef to a struct)
  - T, where T is a Julia Leaf Type
  - argument value will be copied (returned by-value)
- void\*
  - depends on how this parameter is used, first translate this to the intended pointer type, then determine the Julia equivalent using the remaining rules in this list
  - this argument may be declared as Ptr{Void}, if it really is just an unknown pointer
- jl\_value\_t\*
  - Any
  - argument value must be a valid Julia object
- jl\_value\_t\*\*
  - Ref{Any}
  - argument value must be a valid Julia object (or C\_NULL)
- T\*
  - If the memory is already owned by Julia, or is an isbits type, and is known to be non-null:
    - \* Ref { T }, where T is the Julia type corresponding to T
    - \* a return type of Ref{Any} is invalid, it should either be Any (corresponding to jl\_value\_t\*) or Ptr{Any} (corresponding to Ptr{Any})
    - \* currently partially unsupported by cfunction due to #2818
    - \* C MUST NOT modify the memory returned via Ref { T } if T is an isbits type
  - If the memory is owned by C:
    - \*  $Ptr\{T\}$ , where T is the Julia type corresponding to T
- (T\*) (...) (e.g. a pointer to a function)
  - Ptr{Void} (you may need to use cfunction explicitly to create this pointer)

### 24.3.3 Passing Pointers for Modifying Inputs

Because C doesn't support multiple return values, often C functions will take pointers to data that the function will modify. To accomplish this within a ccall, you need to first encapsulate the value inside an  $Ref\{T\}$  of the appropriate type. When you pass this Ref object as an argument, julia will automatically pass a C pointer to the encapsulated data:

```
width = Ref{Cint}(0)
range = Ref{Cfloat}(0)
ccall(:foo, Void, (Ref{Cint}, Ref{Cfloat}), width, range)
```

Upon return, the contents of width and range can be retrieved (if they were changed by foo) by width[] and range[]; that is, they act like zero-dimensional arrays.

### 24.3.4 Special Reference Syntax for ccall (deprecated):

The & syntax is deprecated, use the  $Ref\{T\}$  argument type instead.

A prefix & is used on an argument to ccall to indicate that a pointer to a scalar argument should be passed instead of the scalar value itself (required for all Fortran function arguments, as noted above). The following example computes a dot product using a BLAS function.

The meaning of prefix & is not quite the same as in C. In particular, any changes to the referenced variables will not be visible in Julia unless the type is mutable (declared via type). However, even for immutable types it will not cause any harm for called functions to attempt such modifications (that is, writing through the passed pointers). Moreover, & may be used with any expression, such as &0 or &f(x).

When a scalar value is passed with & as an argument of type Ptr{T}, the value will first be converted to type T.

# 24.4 Garbage Collection Safety

When passing data to a ccall, it is best to avoid using the pointer() function. Instead define a convert method and pass the variables directly to the ccall. ccall automatically arranges that all of its arguments will be preserved from garbage collection until the call returns. If a C API will store a reference to memory allocated by Julia, after the ccall returns, you must arrange that the object remains visible to the garbage collector. The suggested way to handle this is to make a global variable of type Array{Ref,1} to hold these values, until the C library notifies you that it is finished with them.

Whenever you have created a pointer to Julia data, you must ensure the original data exists until you are done with using the pointer. Many methods in Julia such as unsafe\_load() and bytestring() make copies of data instead of taking ownership of the buffer, so that it is safe to free (or alter) the original data without affecting Julia. A notable exception is pointer\_to\_array() which, for performance reasons, shares (or can be told to take ownership of) the underlying buffer.

The garbage collector does not guarantee any order of finalization. That is, if a contained a reference to b and both a and b are due for garbage collection, there is no guarantee that b would be finalized after a. If proper finalization of a depends on b being valid, it must be handled in other ways.

## 24.5 Non-constant Function Specifications

A (name, library) function specification must be a constant expression. However, it is possible to use computed values as function names by staging through eval as follows:

```
@eval ccall(($(string("a","b")),"lib"), ...
```

This expression constructs a name using string, then substitutes this name into a new ccall expression, which is then evaluated. Keep in mind that eval only operates at the top level, so within this expression local variables will not be available (unless their values are substituted with \$). For this reason, eval is typically only used to form top-level definitions, for example when wrapping libraries that contain many similar functions.

If your usage is more dynamic, use indirect calls as described in the next section.

### 24.6 Indirect Calls

The first argument to ccall can also be an expression evaluated at run time. In this case, the expression must evaluate to a Ptr, which will be used as the address of the native function to call. This behavior occurs when the first ccall argument contains references to non-constants, such as local variables, function arguments, or non-constant globals.

For example, you might lookup the function via dlsym, then cache it in a global variable for that session. For example:

```
macro dlsym(func, lib)
    z, zlocal = gensym(string(func)), gensym()
    eval(current_module(),:(global $z = C_NULL))
    z = esc(z)
    quote
        let $zlocal::Ptr{Void} = $z::Ptr{Void}
            if $zlocal == C_NULL
               $zlocal = dlsym($(esc(lib))::Ptr{Void}, $(esc(func)))
               global $z = $zlocal
            end
            $zlocal
        end
    end
end
mylibvar = dlopen("mylib")
ccall(@dlsym("myfunc", mylibvar), Void, ())
```

## 24.7 Calling Convention

The second argument to ccall can optionally be a calling convention specifier (immediately preceding return type). Without any specifier, the platform-default C calling convention is used. Other supported conventions are: stdcall, cdecl, fastcall, and thiscall. For example (from base/libc.jl) we see the same gethostname ccall as above, but with the correct signature for Windows:

```
hn = Array(UInt8, 256)
err = ccall(:gethostname, stdcall, Int32, (Ptr{UInt8}, UInt32), hn, length(hn))
```

For more information, please see the LLVM Language Reference.

## 24.8 Accessing Global Variables

Global variables exported by native libraries can be accessed by name using the cglobal function. The arguments to cglobal are a symbol specification identical to that used by ccall, and a type describing the value stored in the variable:

```
julia> cglobal((:errno,:libc), Int32)
Ptr{Int32} @0x00007f418d0816b8
```

The result is a pointer giving the address of the value. The value can be manipulated through this pointer using unsafe\_load and unsafe\_store.

## 24.9 Accessing Data through a Pointer

The following methods are described as "unsafe" because a bad pointer or type declaration can cause Julia to terminate abruptly (although, that's quite alike with ccall).

Given a  $Ptr{T}$ , the contents of type T can generally be copied from the referenced memory into a Julia object using unsafe\_load(ptr, [index]). The index argument is optional (default is 1), and follows the Julia-convention of 1-based indexing. This function is intentionally similar to the behavior of getindex() and setindex!() (e.g. [] access syntax).

The return value will be a new object initialized to contain a copy of the contents of the referenced memory. The referenced memory can safely be freed or released.

If T is Any, then the memory is assumed to contain a reference to a Julia object (a jl\_value\_t\*), the result will be a reference to this object, and the object will not be copied. You must be careful in this case to ensure that the object was always visible to the garbage collector (pointers do not count, but the new reference does) to ensure the memory is not prematurely freed. Note that if the object was not originally allocated by Julia, the new object will never be finalized by Julia's garbage collector. If the Ptr itself is actually a jl\_value\_t\*, it can be converted back to a Julia object reference by unsafe\_pointer\_to\_objref(ptr). (Julia values v can be converted to jl\_value\_t\* pointers, as Ptr{Void}, by calling pointer\_from\_objref(v).)

The reverse operation (writing data to a  $Ptr\{T\}$ ), can be performed using unsafe\_store! (ptr, value, [index]). Currently, this is only supported for bitstypes or other pointer-free (isbits) immutable types.

Any operation that throws an error is probably currently unimplemented and should be posted as a bug so that it can be resolved.

If the pointer of interest is a plain-data array (bitstype or immutable), the function pointer\_to\_array(ptr,dims,[own]) may be more useful. The final parameter should be true if Julia should "take ownership" of the underlying buffer and call free(ptr) when the returned Array object is finalized. If the own parameter is omitted or false, the caller must ensure the buffer remains in existence until all access is complete.

Arithmetic on the Ptr type in Julia (e.g. using +) does not behave the same as C's pointer arithmetic. Adding an integer to a Ptr in Julia always moves the pointer by some number of *bytes*, not elements. This way, the address values obtained from pointer arithmetic do not depend on the element types of pointers.

## 24.10 Thread-safety

Some C libraries execute their callbacks from a different thread, and since Julia isn't thread-safe you'll need to take some extra precautions. In particular, you'll need to set up a two-layered system: the C callback should only *schedule* (via Julia's event loop) the execution of your "real" callback. To do this, you pass a function of one argument (the AsyncWork object for which the event was triggered, which you'll probably just ignore) to SingleAsyncWork:

```
cb = Base.SingleAsyncWork(data -> my_real_callback(args))
```

The callback you pass to C should only execute a ccall to :uv\_async\_send, passing cb.handle as the argument.

### 24.11 More About Callbacks

For more details on how to pass callbacks to C libraries, see this blog post.

### 24.12 C++

Limited support for C++ is provided by the Cpp, Clang, and Cxx packages.

# 24.13 Handling Operating System Variation

When dealing with platform libraries, it is often necessary to provide special cases for various platforms. The variable OS\_NAME can be used to write these special cases. Additionally, there are several macros intended to make this easier: @windows, @unix, @linux, and @osx. Note that linux and osx are mutually exclusive subsets of unix. Their usage takes the form of a ternary conditional operator, as demonstrated in the following examples.

Simple blocks:

```
ccall((@windows?:_fopen::fopen), ...)
```

Complex blocks:

```
@linux? (
    begin
        some_complicated_thing(a)
    end
    : begin
        some_different_thing(a)
    end
)
```

Chaining (parentheses optional, but recommended for readability):

```
@windows? :a : (@osx? :b : :c)
```

Julia Language Documentation, Release 0	.5.0-dev	

## **Embedding Julia**

As we have seen in *Calling C and Fortran Code* (page 213), Julia has a simple and efficient way to call functions written in C. But there are situations where the opposite is needed: calling Julia function from C code. This can be used to integrate Julia code into a larger C/C++ project, without the need to rewrite everything in C/C++. Julia has a C API to make this possible. As almost all programming languages have some way to call C functions, the Julia C API can also be used to build further language bridges (e.g. calling Julia from Python or C#).

## 25.1 High-Level Embedding

We start with a simple C program that initializes Julia and calls some Julia code:

```
#include <julia.h>
int main(int argc, char *argv[])
{
    /* required: setup the julia context */
    jl_init(NULL);

    /* run julia commands */
    jl_eval_string("print(sqrt(2.0))");

    /* strongly recommended: notify julia that the
        program is about to terminate. this allows
        julia time to cleanup pending write requests
        and run all finalizers
    */
    jl_atexit_hook(0);
    return 0;
}
```

In order to build this program you have to put the path to the Julia header into the include path and link against libjulia. For instance, when Julia is installed to \$JULIA\_DIR, one can compile the above test program test.c with gcc using:

```
gcc -o test -fPIC -I$JULIA_DIR/include/julia -L$JULIA_DIR/usr/lib test.c -ljulia
```

Alternatively, look at the embedding.c program in the Julia source tree in the examples/ folder. The file ui/repl.c program is another simple example of how to set jl\_options options while linking against libjulia.

The first thing that has to be done before calling any other Julia C function is to initialize Julia. This is done by calling jl\_init, which takes as argument a C string (const char\*) to the location where Julia is installed. When the

argument is NULL, Julia tries to determine the install location automatically.

The second statement in the test program evaluates a Julia statement using a call to jl eval string.

Before the program terminates, it is strongly recommended to call <code>jl\_atexit\_hook</code>. The above example program calls this before returning from main.

**Note:** Currently, dynamically linking with the libjulia shared library requires passing the RTLD\_GLOBAL option. In Python, this looks like:

```
>>> julia=CDLL('./libjulia.dylib',RTLD_GLOBAL)
>>> julia.jl_init.argtypes = [c_char_p]
>>> julia.jl_init('.')
250593296
```

### 25.1.1 Using julia-config to automatically determine build parameters

The script *julia-config.jl* was created to aid in determining what build parameters are required by a program that uses embedded Julia. This script uses the build parameters and system configuration of the particular Julia distribution it is invoked by to export the necessary compiler flags for an embedding program to interact with that distribution. This script is located in the Julia shared data directory.

#### **Example**

Below is essentially the same as above with one small change; the argument to jl\_init is now **JULIA\_INIT\_DIR** which is defined by *julia-config.jl*.:

```
#include <julia.h>
int main(int argc, char *argv[])
{
    jl_init(JULIA_INIT_DIR);
    (void) jl_eval_string("println(sqrt(2.0))");
    jl_atexit_hook(0);
    return 0;
}
```

#### On the command line

A simple use of this script is from the command line. Assuming that *julia-config.jl* is located in /usr/local/julia/share/julia, it can be invoked on the command line directly and takes any combination of 3 flags:

```
/usr/local/julia/share/julia/julia-config.jl
Usage: julia-config [--cflags|--ldflags|--ldlibs]
```

If the above example source is saved in the file *embed\_example.c*, then the following command will compile it into a running program on Linux and Windows (MSYS2 environment), or if on OS/X, then substitute clang for gcc.:

```
/usr/local/julia/share/julia/julia-config.jl --cflags --ldflags --ldlibs | xargs gcc embed_example.c
```

#### Use in Makefiles

But in general, embedding projects will be more complicated than the above, and so the following allows general makefile support as well – assuming GNU make because of the use of the **shell** macro expansions. Additionally, though many times *julia-config.jl* may be found in the directory */usr/local*, this is not necessarily the case, but Julia can be used to locate *julia-config.jl* too, and the makefile can be used to take advantage of that. The above example is extended to use a Makefile:

```
JL_SHARE = $(shell julia -e 'print(joinpath(JULIA_HOME, Base.DATAROOTDIR, "julia"))')
CFLAGS += $(shell $(JL_SHARE)/julia-config.jl --cflags)
CXXFLAGS += $(shell $(JL_SHARE)/julia-config.jl --cflags)
LDFLAGS += $(shell $(JL_SHARE)/julia-config.jl --ldflags)
LDLIBS += $(shell $(JL_SHARE)/julia-config.jl --ldlibs)
all: embed_example
```

Now the build command is simply make.

## 25.2 Converting Types

Real applications will not just need to execute expressions, but also return their values to the host program. jl\_eval\_string returns a jl\_value\_t\*, which is a pointer to a heap-allocated Julia object. Storing simple data types like Float 64 in this way is called boxing, and extracting the stored primitive data is called unboxing. Our improved sample program that calculates the square root of 2 in Julia and reads back the result in C looks as follows:

```
jl_value_t *ret = jl_eval_string("sqrt(2.0)");

if (jl_is_float64(ret)) {
    double ret_unboxed = jl_unbox_float64(ret);
    printf("sqrt(2.0) in C: %e \n", ret_unboxed);
}
```

In order to check whether ret is of a specific Julia type, we can use the <code>jl\_is\_...</code> functions. By typing typeof(sqrt(2.0)) into the Julia shell we can see that the return type is Float64 (double in C). To convert the boxed Julia value into a C double the <code>jl\_unbox\_float64</code> function is used in the above code snippet.

Corresponding jl\_box\_... functions are used to convert the other way:

```
jl_value_t *a = jl_box_float64(3.0);
jl_value_t *b = jl_box_float32(3.0f);
jl_value_t *c = jl_box_int32(3);
```

As we will see next, boxing is required to call Julia functions with specific arguments.

# 25.3 Calling Julia Functions

While <code>jl\_eval\_string</code> allows C to obtain the result of a Julia expression, it does not allow passing arguments computed in C to Julia. For this you will need to invoke Julia functions directly, using <code>jl\_call</code>:

```
jl_function_t *func = jl_get_function(jl_base_module, "sqrt");
jl_value_t *argument = jl_box_float64(2.0);
jl_value_t *ret = jl_call1(func, argument);
```

In the first step, a handle to the Julia function sqrt is retrieved by calling jl\_get\_function. The first argument passed to jl\_get\_function is a pointer to the Base module in which sqrt is defined. Then, the double value is boxed using jl\_box\_float64. Finally, in the last step, the function is called using jl\_call1. jl\_call0, jl\_call2, and jl\_call3 functions also exist, to conveniently handle different numbers of arguments. To pass more arguments, use jl\_call:

```
jl_value_t *jl_call(jl_function_t *f, jl_value_t **args, int32_t nargs)
```

Its second argument args is an array of jl\_value\_t \* arguments and nargs is the number of arguments.

## 25.4 Memory Management

As we have seen, Julia objects are represented in C as pointers. This raises the question of who is responsible for freeing these objects.

Typically, Julia objects are freed by a garbage collector (GC), but the GC does not automatically know that we are holding a reference to a Julia value from C. This means the GC can free objects out from under you, rendering pointers invalid.

The GC can only run when Julia objects are allocated. Calls like  $jl\_box\_float64$  perform allocation, and allocation might also happen at any point in running Julia code. However, it is generally safe to use pointers in between  $jl\_...$  calls. But in order to make sure that values can survive  $jl\_...$  calls, we have to tell Julia that we hold a reference to a Julia value. This can be done using the  $Jl\_GC\_PUSH$  macros:

```
jl_value_t *ret = jl_eval_string("sqrt(2.0)");

JL_GC_PUSH1(&ret);

// Do something with ret

JL_GC_POP();
```

The  $JL\_GC\_POP$  call releases the references established by the previous  $JL\_GC\_PUSH$ . Note that  $JL\_GC\_PUSH$  is working on the stack, so it must be exactly paired with a  $JL\_GC\_POP$  before the stack frame is destroyed.

Several Julia values can be pushed at once using the JL\_GC\_PUSH2, JL\_GC\_PUSH3, and JL\_GC\_PUSH4 macros. To push an array of Julia values one can use the JL\_GC\_PUSHARGS macro, which can be used as follows:

```
jl_value_t **args;

JL_GC_PUSHARGS(args, 2); // args can now hold 2 `jl_value_t*` objects
args[0] = some_value;
args[1] = some_other_value;
// Do something with args (e.g. call jl_... functions)
JL_GC_POP();
```

The garbage collector also operates under the assumption that it is aware of every old-generation object pointing to a young-generation one. Any time a pointer is updated breaking that assumption, it must be signaled to the collector with the <code>jl\_gc\_wb</code> (write barrier) function like so:

```
jl_value_t *parent = some_old_value, *child = some_young_value;
((some_specific_type*)parent)->field = child;
jl_gc_wb(parent, child);
```

It is in general impossible to predict which values will be old at runtime, so the write barrier must be inserted after all explicit stores. One notable exception is if the parent object was just allocated and garbage collection was not run since then. Remember that most jl\_... functions can sometimes invoke garbage collection.

The write barrier is also necessary for arrays of pointers when updating their data directly. For example:

```
jl_array_t *some_array = ...; // e.g. a Vector{Any}
void **data = (void**)jl_array_data(some_array);
jl_value_t *some_value = ...;
data[0] = some_value;
jl_gc_wb(some_array, some_value);
```

### 25.4.1 Manipulating the Garbage Collector

There are some functions to control the GC. In normal use cases, these should not be necessary.

jl_gc_collect()	Force a GC run
jl_gc_enable(0)	Disable the GC, return previous state as int
jl_gc_enable(1)	Enable the GC, return previous state as int
<pre>jl_gc_is_enabled()</pre>	Return current state as int

## 25.5 Working with Arrays

Julia and C can share array data without copying. The next example will show how this works.

Julia arrays are represented in C by the datatype <code>jl\_array\_t\*</code>. Basically, <code>jl\_array\_t</code> is a struct that contains:

- Information about the datatype
- A pointer to the data block
- Information about the sizes of the array

To keep things simple, we start with a 1D array. Creating an array containing Float64 elements of length 10 is done by:

```
jl_value_t* array_type = jl_apply_array_type(jl_float64_type, 1);
jl_array_t* x = jl_alloc_array_ld(array_type, 10);
```

Alternatively, if you have already allocated the array you can generate a thin wrapper around its data:

```
double *existingArray = (double*)malloc(sizeof(double)*10);
jl_array_t *x = jl_ptr_to_array_1d(array_type, existingArray, 10, 0);
```

The last argument is a boolean indicating whether Julia should take ownership of the data. If this argument is non-zero, the GC will call free on the data pointer when the array is no longer referenced.

In order to access the data of x, we can use jl\_array\_data:

```
double *xData = (double*)jl_array_data(x);
```

Now we can fill the array:

```
for(size_t i=0; i<jl_array_len(x); i++)
    xData[i] = i;</pre>
```

Now let us call a Julia function that performs an in-place operation on x:

```
jl_function_t *func = jl_get_function(jl_base_module, "reverse!");
jl_call1(func, (jl_value_t*)x);
```

By printing the array, one can verify that the elements of x are now reversed.

### 25.5.1 Accessing Returned Arrays

If a Julia function returns an array, the return value of <code>jl\_eval\_string</code> and <code>jl\_call</code> can be cast to a <code>jl\_array\_t\*</code>:

```
jl_function_t *func = jl_get_function(jl_base_module, "reverse");
jl_array_t *y = (jl_array_t*) jl_call1(func, (jl_value_t*)x);
```

Now the content of y can be accessed as before using jl\_array\_data. As always, be sure to keep a reference to the array while it is in use.

### 25.5.2 Multidimensional Arrays

Julia's multidimensional arrays are stored in memory in column-major order. Here is some code that creates a 2D array and accesses its properties:

```
// Create 2D array of float64 type
jl_value_t *array_type = jl_apply_array_type(jl_float64_type, 2);
jl_array_t *x = jl_alloc_array_2d(array_type, 10, 5);

// Get array pointer
double *p = (double*) jl_array_data(x);

// Get number of dimensions
int ndims = jl_array_ndims(x);

// Get the size of the i-th dim
size_t size0 = jl_array_dim(x,0);
size_t size1 = jl_array_dim(x,1);

// Fill array with data
for(size_t i=0; i<size1; i++)
    for(size_t j=0; j<size0; j++)
    p[j + size0*i] = i + j;</pre>
```

Notice that while Julia arrays use 1-based indexing, the C API uses 0-based indexing (for example in calling jl array dim) in order to read as idiomatic C code.

# 25.6 Exceptions

Julia code can throw exceptions. For example, consider:

```
jl_eval_string("this_function_does_not_exist()");
```

This call will appear to do nothing. However, it is possible to check whether an exception was thrown:

```
if (jl_exception_occurred())
    printf("%s \n", jl_typeof_str(jl_exception_occurred()));
```

If you are using the Julia C API from a language that supports exceptions (e.g. Python, C#, C++), it makes sense to wrap each call into libjulia with a function that checks whether an exception was thrown, and then rethrows the exception in the host language.

### 25.6.1 Throwing Julia Exceptions

When writing Julia callable functions, it might be necessary to validate arguments and throw exceptions to indicate errors. A typical type check looks like:

```
if (!jl_is_float64(val)) {
    jl_type_error(function_name, (jl_value_t*)jl_float64_type, val);
}
```

General exceptions can be raised using the functions:

```
void jl_error(const char *str);
void jl_errorf(const char *fmt, ...);
```

jl\_error takes a C string, and jl\_errorf is called like printf:

```
jl_errorf("argument x = %d is too large", x);
```

where in this example  $\boldsymbol{x}$  is assumed to be an integer.

25.6. Exceptions 237

## **Packages**

Julia has a built-in package manager for installing add-on functionality written in Julia. It can also install external libraries using your operating system's standard system for doing so, or by compiling from source. The list of registered Julia packages can be found at http://pkg.julialang.org. All package manager commands are found in the Pkg (page 469) module, included in Julia's Base install.

First we'll go over the mechanics of the Pkg family of commands and then we'll provide some guidance on how to get your package registered. Be sure to read the section below on package naming conventions, tagging versions and the importance of a REQUIRE file for when you're ready to add your code to the curated METADATA repository.

## 26.1 Package Status

The Pkg. status () (page 472) function prints out a summary of the state of packages you have installed. Initially, you'll have no packages installed:

```
julia> Pkg.status()
INFO: Initializing package repository /Users/stefan/.julia/v0.4
INFO: Cloning METADATA from git://github.com/JuliaLang/METADATA.jl
No packages installed.
```

Your package directory is automatically initialized the first time you run a *Pkg* (page 469) command that expects it to exist – which includes *Pkg.status* () (page 472). Here's an example non-trivial set of required and additional packages:

These packages are all on registered versions, managed by Pkg (page 469). Packages can be in more complicated states, indicated by annotations to the right of the installed package version; we will explain these states and annotations as we encounter them. For programmatic usage, Pkg.installed() (page 472) returns a dictionary, mapping installed package names to the version of that package which is installed:

```
julia> Pkg.installed()
Dict{ASCIIString, VersionNumber} with 4 entries:
"Distributions" => v"0.2.8"
"Stats" => v"0.2.6"
```

```
"UTF16" => v"0.2.0"
"NumericExtensions" => v"0.2.17"
```

## 26.2 Adding and Removing Packages

Julia's package manager is a little unusual in that it is declarative rather than imperative. This means that you tell it what you want and it figures out what versions to install (or remove) to satisfy those requirements optimally – and minimally. So rather than installing a package, you just add it to the list of requirements and then "resolve" what needs to be installed. In particular, this means that if some package had been installed because it was needed by a previous version of something you wanted, and a newer version doesn't have that requirement anymore, updating will actually remove that package.

Your package requirements are in the file  $\sim$ /.julia/v0.4/REQUIRE. You can edit this file by hand and then call Pkg.resolve() (page 471) to install, upgrade or remove packages to optimally satisfy the requirements, or you can do Pkg.edit() (page 471), which will open REQUIRE in your editor (configured via the EDITOR or VISUAL environment variables), and then automatically call Pkg.resolve() (page 471) afterwards if necessary. If you only want to add or remove the requirement for a single package, you can also use the non-interactive Pkg.add() (page 471) and Pkg.rm() (page 471) commands, which add or remove a single requirement to REQUIRE and then call Pkg.resolve() (page 471).

You can add a package to the list of requirements with the Pkg.add() (page 471) function, and the package and all the packages that it depends on will be installed:

```
julia> Pkg.status()
No packages installed.
julia> Pkg.add("Distributions")
INFO: Cloning cache of Distributions from git://github.com/JuliaStats/Distributions.jl.git
INFO: Cloning cache of NumericExtensions from git://github.com/lindahua/NumericExtensions.jl.git
INFO: Cloning cache of Stats from git://github.com/JuliaStats/Stats.jl.git
INFO: Installing Distributions v0.2.7
INFO: Installing NumericExtensions v0.2.17
INFO: Installing Stats v0.2.6
INFO: REQUIRE updated.
julia> Pkg.status()
Required packages:
- Distributions
                                 0.2.7
Additional packages:
- NumericExtensions
                                 0.2.17
- Stats
                                 0.2.6
```

What this is doing is first adding Distributions to your ~/.julia/v0.4/REQUIRE file:

```
$ cat ~/.julia/v0.4/REQUIRE
Distributions
```

It then runs Pkg.resolve() (page 471) using these new requirements, which leads to the conclusion that the Distributions package should be installed since it is required but not installed. As stated before, you can accomplish the same thing by editing your  $\sim$ /. julia/v0.4/REQUIRE file by hand and then running Pkg.resolve() (page 471) yourself:

```
$ echo UTF16 >> ~/.julia/v0.4/REQUIRE

julia> Pkg.resolve()

INFO: Cloning cache of UTF16 from git://github.com/nolta/UTF16.jl.git
```

This is functionally equivalent to calling *Pkg.add("UTF16")* (page 471), except that *Pkg.add()* (page 471) doesn't change REQUIRE until *after* installation has completed, so if there are problems, REQUIRE will be left as it was before calling *Pkg.add()* (page 471). The format of the REQUIRE file is described in *Requirements Specification* (page 254); it allows, among other things, requiring specific ranges of versions of packages.

When you decide that you don't want to have a package around any more, you can use Pkg.rm() (page 471) to remove the requirement for it from the REQUIRE file:

Once again, this is equivalent to editing the REQUIRE file to remove the line with each package name on it then running <code>Pkg.resolve()</code> (page 471) to update the set of installed packages to match. While <code>Pkg.add()</code> (page 471) and <code>Pkg.rm()</code> (page 471) are convenient for adding and removing requirements for a single package, when you want to add or remove multiple packages, you can call <code>Pkg.edit()</code> (page 471) to manually change the contents of <code>REQUIRE</code> and then update your packages accordingly. <code>Pkg.edit()</code> (page 471) does not roll back the contents of <code>REQUIRE</code> if <code>Pkg.resolve()</code> (page 471) fails – rather, you have to run <code>Pkg.edit()</code> (page 471) again to fix the files contents yourself.

Because the package manager uses git internally to manage the package git repositories, users may run into protocol issues (if behind a firewall, for example), when running Pkg.add() (page 471). By default, all GitHub-hosted packages wil be accessed via 'https'; this default can be modified by calling Pkg.setprotocol!() (page 472). The following command can be run from the command line in order to tell git to use 'https' instead of the 'git' protocol when cloning all repositories, wherever they are hosted:

```
git config --global url."https://".insteadOf git://
```

However, this change will be system-wide and thus the use of Pkq.setprotocol! () (page 472) is preferable.

## 26.3 Offline Installation of Packages

For machines with no Internet connection, packages may be installed by copying the package root directory (given by *Pkg.dir()* (page 471)) from a machine with the same operating system and environment.

Pkg. add () (page 471) does the following within the package root directory:

- 1. Adds the name of the package to REQUIRE.
- 2. Downloads the package to .cache, then copies the package to the package root directory.
- 3. Recursively performs step 2 against all the packages listed in the package's REQUIRE file.
- 4. Runs Pkg.build() (page 472)

**Warning:** Copying installed packages from a different machine is brittle for packages requiring binary external dependencies. Such packages may break due to differences in operating system versions, build environments, and/or absolute path dependencies.

# 26.4 Installing Unregistered Packages

Julia packages are simply git repositories, clonable via any of the protocols that git supports, and containing Julia code that follows certain layout conventions. Official Julia packages are registered in the METADATA.jl repository, available at a well-known location <sup>1</sup>. The *Pkg.add()* (page 471) and *Pkg.rm()* (page 471) commands in the previous section interact with registered packages, but the package manager can install and work with unregistered packages too. To install an unregistered package, use *Pkg.clone(url)* (page 471), where url is a git URL from which the package can be cloned:

```
julia> Pkg.clone("git://example.com/path/to/Package.jl.git")
INFO: Cloning Package from git://example.com/path/to/Package.jl.git
Cloning into 'Package'...
remote: Counting objects: 22, done.
remote: Compressing objects: 100% (10/10), done.
remote: Total 22 (delta 8), reused 22 (delta 8)
Receiving objects: 100% (22/22), 2.64 KiB, done.
Resolving deltas: 100% (8/8), done.
```

By convention, Julia repository names end with .jl (the additional .git indicates a "bare" git repository), which keeps them from colliding with repositories for other languages, and also makes Julia packages easy to find in search engines. When packages are installed in your .julia/v0.4 directory, however, the extension is redundant so we leave it off.

If unregistered packages contain a REQUIRE file at the top of their source tree, that file will be used to determine which registered packages the unregistered package depends on, and they will automatically be installed. Unregistered packages participate in the same version resolution logic as registered packages, so installed package versions will be adjusted as necessary to satisfy the requirements of both registered and unregistered packages.

## 26.5 Updating Packages

When package developers publish new registered versions of packages that you're using, you will, of course, want the new shiny versions. To get the latest and greatest versions of all your packages, just do Pkg.update() (page 472):

<sup>&</sup>lt;sup>1</sup> The official set of packages is at https://github.com/JuliaLang/METADATA.jl, but individuals and organizations can easily use a different metadata repository. This allows control which packages are available for automatic installation. One can allow only audited and approved package versions, and make private packages or forks available. See *Custom METADATA* (page 243) for details.

```
julia> Pkg.update()
INFO: Updating METADATA...
INFO: Computing changes...
INFO: Upgrading Distributions: v0.2.8 => v0.2.10
INFO: Upgrading Stats: v0.2.7 => v0.2.8
```

The first step of updating packages is to pull new changes to ~/.julia/v0.4/METADATA and see if any new registered package versions have been published. After this, Pkg.update() (page 472) attempts to update packages that are checked out on a branch and not dirty (i.e. no changes have been made to files tracked by git) by pulling changes from the package's upstream repository. Upstream changes will only be applied if no merging or rebasing is necessary – i.e. if the branch can be "fast-forwarded". If the branch cannot be fast-forwarded, it is assumed that you're working on it and will update the repository yourself.

Finally, the update process recomputes an optimal set of package versions to have installed to satisfy your top-level requirements and the requirements of "fixed" packages. A package is considered fixed if it is one of the following:

- 1. **Unregistered:** the package is not in METADATA you installed it with *Pkg.clone()* (page 471).
- 2. Checked out: the package repo is on a development branch.
- 3. **Dirty:** changes have been made to files in the repo.

If any of these are the case, the package manager cannot freely change the installed version of the package, so its requirements must be satisfied by whatever other package versions it picks. The combination of top-level requirements in  $\sim$ /.julia/v0.4/REQUIRE and the requirement of fixed packages are used to determine what should be installed.

## 26.6 Checkout, Pin and Free

You may want to use the master version of a package rather than one of its registered versions. There might be fixes or functionality on master that you need that aren't yet published in any registered versions, or you may be a developer of the package and need to make changes on master or some other development branch. In such cases, you can do <code>Pkg.checkout(pkg)</code> (page 472) to checkout the master branch of pkg or <code>Pkg.checkout(pkg,branch)</code> (page 472) to checkout some other branch:

```
julia> Pkg.add("Distributions")
INFO: Installing Distributions v0.2.9
INFO: Installing NumericExtensions v0.2.17
INFO: Installing Stats v0.2.7
INFO: REQUIRE updated.
julia > Pkg.status()
Required packages:
- Distributions
                                  0.2.9
Additional packages:
- NumericExtensions
                                  0.2.17
                                  0.2.7
- Stats
julia> Pkg.checkout("Distributions")
INFO: Checking out Distributions master...
INFO: No packages to install, update or remove.
julia> Pkg.status()
Required packages:
                                  0.2.9 +
- Distributions
                                                     master
Additional packages:
```

- NumericExtensions	0.2.17
- Stats	0.2.7

Immediately after installing Distributions with Pkg.add() (page 471) it is on the current most recent registered version -0.2.9 at the time of writing this. Then after running Pkg.checkout("Distributions") (page 472), you can see from the output of Pkg.status() (page 472) that Distributions is on an unregistered version greater than 0.2.9, indicated by the "pseudo-version" number 0.2.9+.

When you checkout an unregistered version of a package, the copy of the REQUIRE file in the package repo takes precedence over any requirements registered in METADATA, so it is important that developers keep this file accurate and up-to-date, reflecting the actual requirements of the current version of the package. If the REQUIRE file in the package repo is incorrect or missing, dependencies may be removed when the package is checked out. This file is also used to populate newly published versions of the package if you use the API that Pkg (page 469) provides for this (described below).

When you decide that you no longer want to have a package checked out on a branch, you can "free" it back to the control of the package manager with Pkg.free(pkg) (page 472):

After this, since the package is on a registered version and not on a branch, its version will be updated as new registered versions of the package are published.

If you want to pin a package at a specific version so that calling Pkg.update() (page 472) won't change the version the package is on, you can use the Pkg.pin() (page 472) function:

After this, the Stats package will remain pinned at version 0.2.7 – or more specifically, at commit 47c198b1, but since versions are permanently associated a given git hash, this is the same thing. Pkg.pin() (page 472) works by creating a throw-away branch for the commit you want to pin the package at and then checking that branch out. By default, it pins a package at the current commit, but you can choose a different version by passing a second argument:

- NumericExtensions	0.2.17	
- Stats	0.2.5	pinned.1fd0983b.tmp

Now the Stats package is pinned at commit 1fd0983b, which corresponds to version 0.2.5. When you decide to "unpin" a package and let the package manager update it again, you can use Pkg.free() (page 472) like you would to move off of any branch:

After this, the Stats package is managed by the package manager again, and future calls to Pkg.update() (page 472) will upgrade it to newer versions when they are published. The throw-away pinned.lfd0983b.tmp branch remains in your local Stats repo, but since git branches are extremely lightweight, this doesn't really matter; if you feel like cleaning them up, you can go into the repo and delete those branches <sup>2</sup>.

## 26.7 Custom METADATA Repository

By default, Julia assumes you will be using the official METADATA.jl repository for downloading and installing packages. You can also provide a different metadata repository location. A common approach is to keep your metadata-v2 branch up to date with the Julia official branch and add another branch with your custom packages. You can initialize your local metadata repository using that custom location and branch and then periodically rebase your custom branch with the official metadata-v2 branch. In order to use a custom repository and branch, issue the following command:

```
julia> Pkg.init("https://me.example.com/METADATA.jl.git", "branch")
```

The branch argument is optional and defaults to metadata-v2. Once initialized, a file named META\_BRANCH in your ~/.julia/vX.Y/ path will track the branch that your METADATA repository was initialized with. If you want to change branches, you will need to either modify the META\_BRANCH file directly (be careful!) or remove the vX.Y directory and re-initialize your METADATA repository using the Pkg.init command.

<sup>&</sup>lt;sup>2</sup> Packages that aren't on branches will also be marked as dirty if you make changes in the repo, but that's a less common thing to do.

# **Package Development**

Julia's package manager is designed so that when you have a package installed, you are already in a position to look at its source code and full development history. You are also able to make changes to packages, commit them using git, and easily contribute fixes and enhancements upstream. Similarly, the system is designed so that if you want to create a new package, the simplest way to do so is within the infrastructure provided by the package manager.

## 27.1 Initial Setup

Since packages are git repositories, before doing any package development you should setup the following standard global git configuration settings:

```
$ git config --global user.name "FULL NAME"
$ git config --global user.email "EMAIL"
```

where FULL NAME is your actual full name (spaces are allowed between the double quotes) and EMAIL is your actual email address. Although it isn't necessary to use GitHub to create or publish Julia packages, most Julia packages as of writing this are hosted on GitHub and the package manager knows how to format origin URLs correctly and otherwise work with the service smoothly. We recommend that you create a free account on GitHub and then do:

```
$ git config --global github.user "USERNAME"
```

where USERNAME is your actual GitHub user name. Once you do this, the package manager knows your GitHub user name and can configure things accordingly. You should also upload your public SSH key to GitHub and set up an SSH agent on your development machine so that you can push changes with minimal hassle. In the future, we will make this system extensible and support other common git hosting options like BitBucket and allow developers to choose their favorite. Since the package development functions has been moved to the PkgDev package, you need to run Pkg.add("PkgDev"); import PkgDev to access the functions starting with PkgDev. in the document below.

# 27.2 Making changes to an existing package

## 27.2.1 Documentation changes

If you want to improve the online documentation of a package, the easiest approach (at least for small changes) is to use GitHub's online editing functionality. First, navigate to the repository's GitHub "home page," find the file (e.g., README.md) within the repository's folder structure, and click on it. You'll see the contents displayed, along with a small "pencil" icon in the upper right hand corner. Clicking that icon opens the file in edit mode. Make your changes,

write a brief summary describing the changes you want to make (this is your *commit message*), and then hit "Propose file change." Your changes will be submitted for consideration by the package owner(s) and collaborators.

For larger documentation changes—and especially ones that you expect to have to update in response to feedback—you might find it easier to use the procedure for code changes described below.

#### 27.2.2 Code changes

#### **Executive summary**

Here we assume you've already set up git on your local machine and have a GitHub account (see above). Let's imagine you're fixing a bug in the Images package:

```
Pkg.checkout("Images")  # check out the master branch
<here, make sure your bug is still a bug and hasn't been fixed already>
cd(Pkg.dir("Images"))
;git checkout -b myfixes  # create a branch for your changes
<edit code>  # be sure to add a test for your bug
Pkg.test("Images")  # make sure everything works now
;git commit -a -m "Fix foo by calling bar"  # write a descriptive message
Pkg.submit("Images")
```

The last line will present you with a link to submit a pull request to incorporate your changes.

#### **Detailed description**

If you want to fix a bug or add new functionality, you want to be able to test your changes before you submit them for consideration. You also need to have an easy way to update your proposal in response to the package owner's feedback. Consequently, in this case the strategy is to work locally on your own machine; once you are satisfied with your changes, you submit them for consideration. This process is called a *pull request* because you are asking to "pull" your changes into the project's main repository. Because the online repository can't see the code on your private machine, you first *push* your changes to a publicly-visible location, your own online *fork* of the package (hosted on your own personal GitHub account).

Let's assume you already have the Foo package installed. In the description below, anything starting with Pkg. or PkgDev. is meant to be typed at the Julia prompt; anything starting with git is meant to be typed in *julia's shell mode* (page 202) (or using the shell that comes with your operating system). Within Julia, you can combine these two modes:

```
julia> cd(Pkg.dir("Foo"))  # go to Foo's folder
shell> git command arguments... # command will apply to Foo
```

Now suppose you're ready to make some changes to Foo. While there are several possible approaches, here is one that is widely used:

- From the Julia prompt, type <code>Pkg.checkout("Foo")</code> (page 472). This ensures you're running the latest code (the master branch), rather than just whatever "official release" version you have installed. (If you're planning to fix a bug, at this point it's a good idea to check again whether the bug has already been fixed by someone else. If it has, you can request that a new official release be tagged so that the fix gets distributed to the rest of the community.) If you receive an error <code>Foo is dirty</code>, bailing, see <code>Dirty packages</code> (page 247) below.
- Create a branch for your changes: navigate to the package folder (the one that Julia reports from *Pkg.dir("Foo")* (page 471)) and (in shell mode) create a new branch using git checkout -b <newbranch>, where <newbranch> might be some descriptive name (e.g., fixbar). By creating a branch,

you ensure that you can easily go back and forth between your new work and the current master branch (see http://git-scm.com/book/en/v2/Git-Branching-Branches-in-a-Nutshell).

If you forget to do this step until after you've already made some changes, don't worry: see *more detail about branching* (page 247) below.

- Make your changes. Whether it's fixing a bug or adding new functionality, in most cases your change should include updates to both the src/ and test/ folders. If you're fixing a bug, add your minimal example demonstrating the bug (on the current code) to the test suite; by contributing a test for the bug, you ensure that the bug won't accidentally reappear at some later time due to other changes. If you're adding new functionality, creating tests demonstrates to the package owner that you've made sure your code works as intended.
- Run the package's tests and make sure they pass. There are several ways to run the tests:
  - From Julia, run Pkg. test ("Foo") (page 472): this will run your tests in a separate (new) julia process.
  - From Julia, include ("runtests.jl") from the package's test/ folder (it's possible the file has a different name, look for one that runs all the tests): this allows you to run the tests repeatedly in the same session without reloading all the package code; for packages that take a while to load, this can be much faster. With this approach, you do have to do some extra work to make changes in the package code.
  - From the shell, run julia ../test/runtests.jl from within the package's src/ folder.
- Commit your changes: see http://git-scm.com/book/en/v2/Git-Basics-Recording-Changes-to-the-Repository.
- Submit your changes: From the Julia prompt, type PkgDev.submit ("Foo"). This will push your changes to your GitHub fork, creating it if it doesn't already exist. (If you encounter an error, *make sure you've set up your SSH keys* (page 245).) Julia will then give you a hyperlink; open that link, edit the message, and then click "submit." At that point, the package owner will be notified of your changes and may initiate discussion.
- The package owner may suggest additional improvements. To respond to those suggestions, you can easily update the pull request (this only works for changes that have not already been merged; for merged pull requests, make new changes by starting a new branch):
  - If you've changed branches in the meantime, make sure you go back to the same branch with git checkout fixbar (from shell mode) or Pkg.checkout ("Foo", "fixbar") (page 472) (from the Julia prompt).
  - As above, make your changes, run the tests, and commit your changes.
  - From the shell, type git push. This will add your new commit(s) to the same pull request; you should see them appear automatically on the page holding the discussion of your pull request.

One potential type of change the owner may request is that you squash your commits. See *Squashing* (page 248) below.

#### 27.2.3 Dirty packages

If you can't change branches because the package manager complains that your package is dirty, it means you have some changes that have not been committed. From the shell, use git diff to see what these changes are; you can either discard them (git checkout changedfile.jl) or commit them before switching branches. If you can't easily resolve the problems manually, as a last resort you can delete the entire "Foo" folder and reinstall a fresh copy with Pkg.add("Foo") (page 471). Naturally, this deletes any changes you've made.

#### 27.2.4 Making a branch post hoc

Especially for newcomers to git, one often forgets to create a new branch until after some changes have already been made. If you haven't yet staged or committed your changes, you can create a new branch with git checkout -b

<newbranch> just as usual—git will kindly show you that some files have been modified and create the new branch
for you. Your changes have not yet been committed to this new branch, so the normal work rules still apply.

However, if you've already made a commit to master but wish to go back to the official master (called origin/master), use the following procedure:

- Create a new branch. This branch will hold your changes.
- Make sure everything is committed to this branch.
- git checkout master. If this fails, *do not* proceed further until you have resolved the problems, or you may lose your changes.
- *Reset* master (your current branch) back to an earlier state with git reset --hard origin/master (see http://git-scm.com/blog/2011/07/11/reset.html).

This requires a bit more familiarity with git, so it's much better to get in the habit of creating a branch at the outset.

#### 27.2.5 Squashing and rebasing

Depending on the tastes of the package owner (s)he may ask you to "squash" your commits. This is especially likely if your change is quite simple but your commit history looks like this:

```
WIP: add new 1-line whizbang function (currently breaks package)
Finish whizbang function
Fix typo in variable name
Oops, don't forget to supply default argument
Split into two 1-line functions
Rats, forgot to export the second function
...
```

This gets into the territory of more advanced git usage, and you're encouraged to do some reading (http://git-scm.com/book/en/v2/Git-Branching-Rebasing). However, a brief summary of the procedure is as follows:

- To protect yourself from error, start from your fixbar branch and create a new branch with git checkout -b fixbar\_backup. Since you started from fixbar, this will be a copy. Now go back to the one you intend to modify with git checkout fixbar.
- From the shell, type git rebase -i origin/master.
- To combine commits, change pick to squash (for additional options, consult other sources). Save the file and close the editor window.
- Edit the combined commit message.

If the rebase goes badly, you can go back to the beginning to try again like this:

```
git checkout fixbar
git reset --hard fixbar_backup
```

Now let's assume you've rebased successfully. Since your fixbar repository has now diverged from the one in your GitHub fork, you're going to have to do a *force push*:

- To make it easy to refer to your GitHub fork, create a "handle" for it with git remote add myfork https://github.com/myaccount/Foo.jl.git, where the URL comes from the "clone URL" on your GitHub fork's page.
- Force-push to your fork with git push myfork +fixbar. The + indicates that this should replace the fixbar branch found at myfork.

# 27.3 Creating a new Package

#### 27.3.1 REQUIRE speaks for itself

You should have a REQUIRE file in your package repository, with a bare minimum directive of what Julia version you expect your users to be running for the package to work. Putting a floor on what Julia version your package supports is done by simply adding julia 0.x in this file. While this line is partly informational, it also has the consequence of whether Pkg.update() will update code found in .julia version directories. It will not update code found in version directories beneath the floor of what's specified in your REQUIRE.

As the development version 0. y matures, you may find yourself using it more frequently, and wanting your package to support it. Be warned, the development branch of Julia is the land of breakage, and you can expect things to break. When you go about fixing whatever broke your package in the development 0. y branch, you will likely find that you just broke your package on the stable version.

There is a mechanism found in the Compat package that will enable you to support both the stable version and breaking changes found in the development version. Should you decide to use this solution, you will need to add Compat to your REQUIRE file. In this case, you will still have  $julia\ 0.x$  in your REQUIRE. The x is the floor version of what your package supports.

You might also have no interest in supporting the development version of Julia. Just as you can add a floor to the version you expect your users to be on, you can set an upper bound. In this case, you would put julia 0.x 0.y-in your REQUIRE file. The – at the end of the version number means pre-release versions of that specific version from the very first commit. By setting it as the ceiling, you mean the code supports everything up to but not including the ceiling version.

Another scenario is that you are writing the bulk of the code for your package with Julia 0.y and do not want to support the current stable version of Julia. If you choose to do this, simply add julia 0.y- to your REQUIRE. Just remember to change the julia 0.y- to julia 0.y in your REQUIRE file once 0.y is officially released. If you don't edit the dash cruft you are suggesting that you support both the development and stable versions of the same version number! That would be madness. See the *Requirements Specification* (page 254) for the full format of REQUIRE.

#### 27.3.2 Guidelines for naming a package

Package names should be sensible to most Julia users, even to those who are not domain experts. When you submit your package to METADATA, you can expect a little back and forth about the package name with collaborators, especially if it's ambiguous or can be confused with something other than what it is. During this bike-shedding, it's not uncommon to get a range of different name suggestions. These are only suggestions though, with the intent being to keep a tidy namespace in the curated METADATA repository. Since this repository belongs to the entire community, there will likely be a few collaborators who care your package name. Here are some guidelines to follow in naming your package:

- 1. Avoid jargon. In particular, avoid acronyms unless there is minimal possibility of confusion.
  - It's ok to say USA if you're talking about the USA.
  - It's not ok to say PMA, even if you're talking about positive mental attitude.
- 2. Avoid using Julia in your package name.
  - It is usually clear from context and to your users that the package is a Julia package.
  - Having Julia in the name can imply that the package is connected to, or endorsed by, contributors to the Julia language itself.
- 3. Packages that provide most of their functionality in association with a new type should have pluralized names.

- DataFrames provides the DataFrame type.
- BloomFilters provides the BloomFilter type.
- In contrast, JuliaParser provides no new type, but instead new functionality in the JuliaParser.parse() function.
- 4. Err on the side of clarity, even if clarity seems long-winded to you.
  - RandomMatrices is a less ambiguous name than RndMat or RMT, even though the latter are shorter.
- 5. A less systematic name may suit a package that implements one of several possible approaches to its domain.
  - Julia does not have a single comprehensive plotting package. Instead, Gadfly, PyPlot, Winston and other packages each implement a unique approach based on a particular design philosophy.
  - In contrast, SortingAlgorithms provides a consistent interface to use many well-established sorting algorithms.
- 6. Packages that wrap external libraries or programs should be named after those libraries or programs.
  - CPLEX.jl wraps the CPLEX library, which can be identified easily in a web search.
  - MATLAB. jl provides an interface to call the MATLAB engine from within Julia.

#### 27.3.3 Generating the package

Suppose you want to create a new Julia package called FooBar. To get started, do PkgDev.generate(pkg,license) where pkg is the new package name and license is the name of a license that the package generator knows about:

```
julia> PkgDev.generate("FooBar","MIT")
INFO: Initializing FooBar repo: /Users/stefan/.julia/v0.4/FooBar
INFO: Origin: git://github.com/StefanKarpinski/FooBar.jl.git
INFO: Generating LICENSE.md
INFO: Generating README.md
INFO: Generating src/FooBar.jl
INFO: Generating test/runtests.jl
INFO: Generating REQUIRE
INFO: Generating .travis.yml
INFO: Generating appveyor.yml
INFO: Generating .gitignore
INFO: Committing FooBar generated files
```

This creates the directory ~/.julia/v0.4/FooBar, initializes it as a git repository, generates a bunch of files that all packages should have, and commits them to the repository:

```
$ cd ~/.julia/v0.4/FooBar && git show --stat

commit 84b8e266dae6de30ab9703150b3bf771ec7b6285

Author: Stefan Karpinski <stefan@karpinski.org>
Date: Wed Oct 16 17:57:58 2013 -0400

FooBar.jl generated files.

license: MIT
authors: Stefan Karpinski
years: 2013
user: StefanKarpinski

Julia Version 0.3.0-prerelease+3217 [5fcfb13*]
```

At the moment, the package manager knows about the MIT "Expat" License, indicated by "MIT", the Simplified BSD License, indicated by "BSD", and version 2.0 of the Apache Software License, indicated by "ASL". If you want to use a different license, you can ask us to add it to the package generator, or just pick one of these three and then modify the ~/.julia/v0.4/PACKAGE/LICENSE.md file after it has been generated.

If you created a GitHub account and configured git to know about it, PkgDev.generate() will set an appropriate origin URL for you. It will also automatically generate a .travis.yml file for using the Travis automated testing service, and an appveyor.yml file for using AppVeyor. You will have to enable testing on the Travis and AppVeyor websites for your package repository, but once you've done that, it will already have working tests. Of course, all the default testing does is verify that using FooBar in Julia works.

#### 27.3.4 Making Your Package Available

Once you've made some commits and you're happy with how FooBar is working, you may want to get some other people to try it out. First you'll need to create the remote repository and push your code to it; we don't yet automatically do this for you, but we will in the future and it's not too hard to figure out <sup>3</sup>. Once you've done this, letting people try out your code is as simple as sending them the URL of the published repo – in this case:

```
git://github.com/StefanKarpinski/FooBar.jl.git
```

For your package, it will be your GitHub user name and the name of your package, but you get the idea. People you send this URL to can use *Pkg.clone()* (page 471) to install the package and try it out:

```
julia> Pkg.clone("git://github.com/StefanKarpinski/FooBar.jl.git")
INFO: Cloning FooBar from git@github.com:StefanKarpinski/FooBar.jl.git
```

## 27.3.5 Tagging and Publishing Your Package

Once you've decided that FooBar is ready to be registered as an official package, you can add it to your local copy of METADATA using PkgDev.register():

```
julia> PkgDev.register("FooBar")
INFO: Registering FooBar at git://github.com/StefanKarpinski/FooBar.jl.git
INFO: Committing METADATA for FooBar
```

This creates a commit in the ~/.julia/v0.4/METADATA repo:

```
$ cd ~/.julia/v0.4/METADATA && git show

commit 9f71f4becb05cadacb983c54a72eed744e5c019d
Author: Stefan Karpinski <stefan@karpinski.org>
Date: Wed Oct 16 18:46:02 2013 -0400
```

<sup>&</sup>lt;sup>3</sup> Installing and using GitHub's "hub" tool is highly recommended. It allows you to do things like run hub create in the package repo and have it automatically created via GitHub's API.

```
Register FooBar

diff --git a/FooBar/url b/FooBar/url

new file mode 100644

index 0000000..30e525e
--- /dev/null
+++ b/FooBar/url
@@ -0,0 +1 @@
+git://github.com/StefanKarpinski/FooBar.jl.git
```

This commit is only locally visible, however. To make it visible to the Julia community, you need to merge your local METADATA upstream into the official repo. The PkgDev.publish() command will fork the METADATA repository on GitHub, push your changes to your fork, and open a pull request:

```
julia> PkgDev.publish()
INFO: Validating METADATA
INFO: No new package versions to publish
INFO: Submitting METADATA changes
INFO: Forking JuliaLang/METADATA.jl to StefanKarpinski
INFO: Pushing changes as branch pull-request/ef45f54b
INFO: To create a pull-request open:
    https://github.com/StefanKarpinski/METADATA.jl/compare/pull-request/ef45f54b
```

#### Tip: If PkgDev.publish() fails with error:

```
ERROR: key not found: "token"
```

then you may have encountered an issue from using the GitHub API on multiple systems. The solution is to delete the "Julia Package Manager" personal access token from your Github account and try again.

Other failures may require you to circumvent PkgDev.publish() by creating a pull request on GitHub. See: *Publishing METADATA manually* (page 253) below.

Once the package URL for FooBar is registered in the official METADATA repo, people know where to clone the package from, but there still aren't any registered versions available. You can tag and register it with the PkgDev.tag() command:

```
julia> PkgDev.tag("FooBar")
INFO: Tagging FooBar v0.0.1
INFO: Committing METADATA for FooBar
```

#### This tags v0.0.1 in the FooBar repo:

```
$ cd ~/.julia/v0.4/FooBar && git tag v0.0.1
```

It also creates a new version entry in your local METADATA repo for FooBar:

```
$ cd ~/.julia/v0.4/FooBar && git show commit de77ee4dc0689b12c5e8b574aef7f70e8b311b0e Author: Stefan Karpinski <stefan@karpinski.org> Date: Wed Oct 16 23:06:18 2013 -0400

Tag FooBar v0.0.1

diff --git a/FooBar/versions/0.0.1/sha1 b/FooBar/versions/0.0.1/sha1 new file mode 100644
```

```
index 0000000..c1cb1c1
--- /dev/null
+++ b/FooBar/versions/0.0.1/sha1
@@ -0,0 +1 @@
+84b8e266dae6de30ab9703150b3bf771ec7b6285
```

The PkgDev.tag() command takes an optional second argument that is either an explicit version number object like v"0.0.1" or one of the symbols :patch, :minor or :major. These increment the patch, minor or major version number of your package intelligently.

Adding a tagged version of your package will expedite the official registration into METADATA.jl by collaborators. It is strongly recommended that you complete this process, regardless if your package is completely ready for an official release.

As a general rule, packages should be tagged 0.0.1 first. Since Julia itself hasn't achieved 1.0 status, it's best to be conservative in your package's tagged versions.

As with PkgDev.register(), these changes to METADATA aren't available to anyone else until they've been included upstream. Again, use the PkgDev.publish() command, which first makes sure that individual package repos have been tagged, pushes them if they haven't already been, and then opens a pull request to METADATA:

```
julia> PkgDev.publish()
INFO: Validating METADATA
INFO: Pushing FooBar permanent tags: v0.0.1
INFO: Submitting METADATA changes
INFO: Forking JuliaLang/METADATA.jl to StefanKarpinski
INFO: Pushing changes as branch pull-request/3ef4f5c4
INFO: To create a pull-request open:
    https://github.com/StefanKarpinski/METADATA.jl/compare/pull-request/3ef4f5c4
```

#### **Publishing METADATA manually**

If PkgDev.publish () fails you can follow these instructions to manually publish your package.

By "forking" the main METADATA repository, you can create a personal copy (of METADATA.jl) under your GitHub account. Once that copy exists, you can push your local changes to your copy (just like any other GitHub project).

- 1. go to https://github.com/JuliaLang/METADATA.jl/fork and create your own fork.
- 2. add your fork as a remote repository for the METADATA repository on your local computer (in the terminal where USERNAME is your github username):

```
cd ~/.julia/v0.4/METADATA
git remote add USERNAME https://github.com/USERNAME/METADATA.jl.git
```

3. push your changes to your fork:

```
git push USERNAME metadata-v2
```

4. If all of that works, then go back to the GitHub page for your fork, and click the "pull request" link.

# 27.4 Fixing Package Requirements

If you need to fix the registered requirements of an already-published package version, you can do so just by editing the metadata for that version, which will still have the same commit hash – the hash associated with a version is permanent:

```
$ cd ~/.julia/v0.4/METADATA/FooBar/versions/0.0.1 && cat requires
julia 0.3-
$ vi requires
```

Since the commit hash stays the same, the contents of the REQUIRE file that will be checked out in the repo will **not** match the requirements in METADATA after such a change; this is unavoidable. When you fix the requirements in METADATA for a previous version of a package, however, you should also fix the REQUIRE file in the current version of the package.

# 27.5 Requirements Specification

The ~/.julia/v0.4/REQUIRE file, the REQUIRE file inside packages, and the METADATA package requires files use a simple line-based format to express the ranges of package versions which need to be installed. Package REQUIRE and METADATA requires files should also include the range of versions of julia the package is expected to work with.

Here's how these files are parsed and interpreted.

- Everything after a # mark is stripped from each line as a comment.
- If nothing but whitespace is left, the line is ignored.
- If there are non-whitespace characters remaining, the line is a requirement and the is split on whitespace into words.

The simplest possible requirement is just the name of a package name on a line by itself:

```
Distributions
```

This requirement is satisfied by any version of the Distributions package. The package name can be followed by zero or more version numbers in ascending order, indicating acceptable intervals of versions of that package. One version opens an interval, while the next closes it, and the next opens a new interval, and so on; if an odd number of version numbers are given, then arbitrarily large versions will satisfy; if an even number of version numbers are given, the last one is an upper limit on acceptable version numbers. For example, the line:

```
Distributions 0.1
```

is satisfied by any version of Distributions greater than or equal to 0.1.0. Suffixing a version with — allows any pre-release versions as well. For example:

```
Distributions 0.1-
```

is satisfied by pre-release versions such as 0.1-dev or 0.1-rc1, or by any version greater than or equal to 0.1.0.

This requirement entry:

```
Distributions 0.1 0.2.5
```

is satisfied by versions from 0.1.0 up to, but not including 0.2.5. If you want to indicate that any 0.1.x version will do, you will want to write:

```
Distributions 0.1 0.2-
```

If you want to start accepting versions after 0.2.7, you can write:

```
Distributions 0.1 0.2- 0.2.7
```

If a requirement line has leading words that begin with @, it is a system-dependent requirement. If your system matches these system conditionals, the requirement is included, if not, the requirement is ignored. For example:

```
@osx Homebrew
```

will require the Homebrew package only on systems where the operating system is OS X. The system conditions that are currently supported are:

```
@windows
@unix
@osx
@linux
```

The @unix condition is satisfied on all UNIX systems, including OS X, Linux and FreeBSD. Negated system conditionals are also supported by adding a! after the leading @. Examples:

```
@!windows
@unix @!osx
```

The first condition applies to any system but Windows and the second condition applies to any UNIX system besides OS X.

Runtime checks for the current version of Julia can be made using the built-in VERSION variable, which is of type VersionNumber. Such code is occasionally necessary to keep track of new or deprecated functionality between various releases of Julia. Examples of runtime checks:

```
VERSION < v"0.3-" #exclude all pre-release versions of 0.3
v"0.2-" <= VERSION < v"0.3-" #get all 0.2 versions, including pre-releases, up to the above v"0.2" <= VERSION < v"0.3-" #To get only stable 0.2 versions (Note v"0.2" == v"0.2.0")
VERSION >= v"0.2.1" #get at least version 0.2.1
```

See the section on *version number literals* (page 53) for a more complete description.

Julia Language Documentation, Release 0.5.0-dev				

# **Profiling**

The *Profile* (page 490) module provides tools to help developers improve the performance of their code. When used, it takes measurements on running code, and produces output that helps you understand how much time is spent on individual line(s). The most common usage is to identify "bottlenecks" as targets for optimization.

Profile (page 490) implements what is known as a "sampling" or statistical profiler. It works by periodically taking a backtrace during the execution of any task. Each backtrace captures the currently-running function and line number, plus the complete chain of function calls that led to this line, and hence is a "snapshot" of the current state of execution.

If much of your run time is spent executing a particular line of code, this line will show up frequently in the set of all backtraces. In other words, the "cost" of a given line—or really, the cost of the sequence of function calls up to and including this line—is proportional to how often it appears in the set of all backtraces.

A sampling profiler does not provide complete line-by-line coverage, because the backtraces occur at intervals (by default, 1 ms on Unix systems and 10 ms on Windows, although the actual scheduling is subject to operating system load). Moreover, as discussed further below, because samples are collected at a sparse subset of all execution points, the data collected by a sampling profiler is subject to statistical noise.

Despite these limitations, sampling profilers have substantial strengths:

- You do not have to make any modifications to your code to take timing measurements (in contrast to the alternative instrumenting profiler).
- It can profile into Julia's core code and even (optionally) into C and Fortran libraries.
- By running "infrequently" there is very little performance overhead; while profiling, your code can run at nearly native speed.

For these reasons, it's recommended that you try using the built-in sampling profiler before considering any alternatives.

# 28.1 Basic usage

Let's work with a simple test case:

```
function myfunc()
    A = rand(100, 100, 200)
    maximum(A)
end
```

It's a good idea to first run the code you intend to profile at least once (unless you want to profile Julia's JIT-compiler):

```
julia> myfunc() # run once to force compilation
```

Now we're ready to profile this function:

```
julia> @profile myfunc()
```

To see the profiling results, there is a graphical browser available, but here we'll use the text-based display that comes with the standard library:

```
julia> Profile.print()
23 client.jl; _start; line: 373
23 client.jl; run_repl; line: 166
23 client.jl; eval_user_input; line: 91
23 profile.jl; anonymous; line: 14
8 none; myfunc; line: 2
8 dSFMT.jl; dsfmt_gv_fill_array_close_open!; line: 128
15 none; myfunc; line: 3
2 reduce.jl; max; line: 35
2 reduce.jl; max; line: 36
11 reduce.jl; max; line: 37
```

Each line of this display represents a particular spot (line number) in the code. Indentation is used to indicate the nested sequence of function calls, with more-indented lines being deeper in the sequence of calls. In each line, the first "field" indicates the number of backtraces (samples) taken *at this line or in any functions executed by this line*. The second field is the file name, followed by a semicolon; the third is the function name followed by a semicolon, and the fourth is the line number. Note that the specific line numbers may change as Julia's code changes; if you want to follow along, it's best to run this example yourself.

In this example, we can see that the top level is client.jl's \_start function. This is the first Julia function that gets called when you launch Julia. If you examine line 373 of client.jl, you'll see that (at the time of this writing) it calls run\_repl(), mentioned on the second line. This in turn calls eval\_user\_input(). These are the functions in client.jl that interpret what you type at the REPL, and since we're working interactively these functions were invoked when we entered @profile myfunc(). The next line reflects actions taken in the @profile (page 491) macro.

The first line shows that 23 backtraces were taken at line 373 of client.jl, but it's not that this line was "expensive" on its own: the second line reveals that all 23 of these backtraces were actually triggered inside its call to run\_repl, and so on. To find out which operations are actually taking the time, we need to look deeper in the call chain.

The first "important" line in this output is this one:

```
8 none; myfunc; line: 2
```

none refers to the fact that we defined myfunc in the REPL, rather than putting it in a file; if we had used a file, this would show the file name. Line 2 of myfunc() contains the call to rand, and there were 8 (out of 23) backtraces that occurred at this line. Below that, you can see a call to dsfmt\_gv\_fill\_array\_close\_open!() inside dSFMT.jl. You might be surprised not to see the rand function listed explicitly: that's because rand is *inlined*, and hence doesn't appear in the backtraces.

A little further down, you see:

```
15 none; myfunc; line: 3
```

Line 3 of myfunc contains the call to max, and there were 15 (out of 23) backtraces taken here. Below that, you can see the specific places in base/reduce.jl that carry out the time-consuming operations in the max function for this type of input data.

Overall, we can tentatively conclude that finding the maximum element is approximately twice as expensive as generating the random numbers. We could increase our confidence in this result by collecting more samples:

```
julia> @profile (for i = 1:100; myfunc(); end)
```

```
julia> Profile.print()
    3121 client.jl; _start; line: 373
    3121 client.jl; run_repl; line: 166
    3121 client.jl; eval_user_input; line: 91
        3121 profile.jl; anonymous; line: 1
        848 none; myfunc; line: 2
        842 dSFMT.jl; dsfmt_gv_fill_array_close_open!; line: 128
        1510 none; myfunc; line: 3
        74 reduce.jl; max; line: 35
        122 reduce.jl; max; line: 36
        1314 reduce.jl; max; line: 37
```

In general, if you have N samples collected at a line, you can expect an uncertainty on the order of sqrt (N) (barring other sources of noise, like how busy the computer is with other tasks). The major exception to this rule is garbage collection, which runs infrequently but tends to be quite expensive. (Since Julia's garbage collector is written in C, such events can be detected using the C=true output mode described below, or by using ProfileView.jl.)

This illustrates the default "tree" dump; an alternative is the "flat" dump, which accumulates counts independent of their nesting:

```
julia> Profile.print(format=:flat)
Count File
                  Function
                                                    Line
 3121 client.jl
                                                     373
                  _start
 3121 client.jl eval_user_input
                                                      91
 3121 client.jl
                                                     166
                  run_repl
  842 dSFMT.jl
                  dsfmt_gv_fill_array_close_open!
                                                     128
  848 none
                   myfunc
 1510 none
                   myfunc
 3121 profile.jl anonymous
                                                       1
   74 reduce.jl
                                                      35
                   max
  122 reduce.jl
                                                      36
                   max
 1314 reduce.jl
                   max
                                                      37
```

If your code has recursion, one potentially-confusing point is that a line in a "child" function can accumulate more counts than there are total backtraces. Consider the following function definitions:

If you were to profile dumbsum3, and a backtrace was taken while it was executing dumbsum(1), the backtrace would look like this:

```
dumbsum3
dumbsum(3)
dumbsum(2)
dumbsum(1)
```

Consequently, this child function gets 3 counts, even though the parent only gets one. The "tree" representation makes this much clearer, and for this reason (among others) is probably the most useful way to view the results.

# 28.2 Accumulation and clearing

Results from <code>@profile</code> (page 491) accumulate in a buffer; if you run multiple pieces of code under <code>@profile</code> (page 491), then <code>Profile.print()</code> (page 491) will show you the combined results. This can be very useful, but sometimes you want to start fresh; you can do so with <code>Profile.clear()</code> (page 491).

## 28.3 Options for controlling the display of profile results

Profile.print () (page 491) has more options than we've described so far. Let's see the full declaration:

```
function print(io::IO = STDOUT, data = fetch(); format = :tree, C = false, combine = true, cols = tt
```

Let's discuss these arguments in order:

- The first argument allows you to save the results to a file, but the default is to print to STDOUT (the console).
- The second argument contains the data you want to analyze; by default that is obtained from *Profile.fetch()* (page 491), which pulls out the backtraces from a pre-allocated buffer. For example, if you want to profile the profiler, you could say:

```
data = copy(Profile.fetch())
Profile.clear()
@profile Profile.print(STDOUT, data) # Prints the previous results
Profile.print() # Prints results from Profile.print()
```

- The first keyword argument, format, was introduced above. The possible choices are :tree and :flat.
- C, if set to true, allows you to see even the calls to C code. Try running the introductory example with Profile.print(C = true). This can be extremely helpful in deciding whether it's Julia code or C code that is causing a bottleneck; setting C=true also improves the interpretability of the nesting, at the cost of longer profile dumps.
- Some lines of code contain multiple operations; for example, s += A[i] contains both an array reference (A[i]) and a sum operation. These correspond to different lines in the generated machine code, and hence there may be two or more different addresses captured during backtraces on this line. combine=true lumps them together, and is probably what you typically want, but you can generate an output separately for each unique instruction pointer with combine=false.
- cols allows you to control the number of columns that you are willing to use for display. When the text would be wider than the display, you might see output like this:

```
33 inference.jl; abstract_call; line: 645
33 inference.jl; abstract_call; line: 645
33 ...rence.jl; abstract_call_gf; line: 567
33 ...nce.jl; typeinf; line: 1201
+1 5 ...nce.jl; ...t_interpret; line: 900
+3 5 ...ence.jl; abstract_eval; line: 758
+4 5 ...ence.jl; ...ct_eval_call; line: 733
+6 5 ...ence.jl; abstract_call; line: 645
```

File/function names are sometimes truncated (with . . . ), and indentation is truncated with a +n at the beginning, where n is the number of extra spaces that would have been inserted, had there been room. If you want a complete profile of deeply-nested code, often a good idea is to save to a file and use a very wide cols setting:

```
s = open("/tmp/prof.txt","w")
Profile.print(s,cols = 500)
close(s)
```

- maxdepth can be used to limit the size of the output in :tree format (it nests only up to level maxdepth)
- sortedby = :count sorts the : flat format in order of increasing counts

# 28.4 Configuration

*@profile* (page 491) just accumulates backtraces, and the analysis happens when you call *Profile.print* () (page 491). For a long-running computation, it's entirely possible that the pre-allocated buffer for storing backtraces will be filled. If that happens, the backtraces stop but your computation continues. As a consequence, you may miss some important profiling data (you will get a warning when that happens).

You can obtain and configure the relevant parameters this way:

```
Profile.init() # returns the current settings
Profile.init(n, delay)
Profile.init(delay = 0.01)
```

n is the total number of instruction pointers you can store, with a default value of 10<sup>6</sup>. If your typical backtrace is 20 instruction pointers, then you can collect 50000 backtraces, which suggests a statistical uncertainty of less than 1%. This may be good enough for most applications.

Consequently, you are more likely to need to modify delay, expressed in seconds, which sets the amount of time that Julia gets between snapshots to perform the requested computations. A very long-running job might not need frequent backtraces. The default setting is delay = 0.001. Of course, you can decrease the delay as well as increase it; however, the overhead of profiling grows once the delay becomes similar to the amount of time needed to take a backtrace (~30 microseconds on the author's laptop).

28.4. Configuration 263

# Memory allocation analysis

One of the most common techniques to improve performance is to reduce memory allocation. The total amount of allocation can be measured with <code>@time</code> (page 328) and <code>@allocated</code> (page 329), and specific lines triggering allocation can often be inferred from profiling via the cost of garbage collection that these lines incur. However, sometimes it is more efficient to directly measure the amount of memory allocated by each line of code.

To measure allocation line-by-line, start Julia with the <code>--track-allocation=<setting></code> command-line option, for which you can choose <code>none</code> (the default, do not measure allocation), <code>user</code> (measure memory allocation everywhere except Julia's core code), or <code>all</code> (measure memory allocation at each line of Julia code). Allocation gets measured for each line of compiled code. When you quit Julia, the cumulative results are written to text files with <code>.mem</code> appended after the file name, residing in the same directory as the source file. Each line lists the total number of bytes allocated. The <code>Coverage</code> package contains some elementary analysis tools, for example to sort the lines in order of number of bytes allocated.

In interpreting the results, there are a few important details. Under the user setting, the first line of any function directly called from the REPL will exhibit allocation due to events that happen in the REPL code itself. More significantly, JIT-compilation also adds to allocation counts, because much of Julia's compiler is written in Julia (and compilation usually requires memory allocation). The recommended procedure is to force compilation by executing all the commands you want to analyze, then call <code>Profile.clear\_malloc\_data()</code> (page 492) to reset all allocation counters. Finally, execute the desired commands and quit Julia to trigger the generation of the .mem files.

Julia Language Documentation, Release 0.5.0-dev				

# **Performance Tips**

In the following sections, we briefly go through a few techniques that can help make your Julia code run as fast as possible.

# 30.1 Avoid global variables

A global variable might have its value, and therefore its type, change at any point. This makes it difficult for the compiler to optimize code using global variables. Variables should be local, or passed as arguments to functions, whenever possible.

Any code that is performance critical or being benchmarked should be inside a function.

We find that global names are frequently constants, and declaring them as such greatly improves performance:

```
const DEFAULT_VAL = 0
```

Uses of non-constant globals can be optimized by annotating their types at the point of use:

```
global x
y = f(x::Int + 1)
```

Writing functions is better style. It leads to more reusable code and clarifies what steps are being done, and what their inputs and outputs are.

**NOTE:** All code in the REPL is evaluated in global scope, so a variable defined and assigned at toplevel will be a **global** variable.

In the following REPL session:

```
julia> x = 1.0
```

is equivalent to:

```
julia> global x = 1.0
```

so all the performance issues discussed previously apply.

# 30.2 Measure performance with @time and pay attention to memory allocation

The most useful tool for measuring performance is the @time (page 328) macro. The following example illustrates good working style:

On the first call (@time f(1)), f gets compiled. (If you've not yet used @time (page 328) in this session, it will also compile functions needed for timing.) You should not take the results of this run seriously. For the second run, note that in addition to reporting the time, it also indicated that a large amount of memory was allocated. This is the single biggest advantage of @time (page 328) vs. functions like tic() (page 328) and toc() (page 328), which only report time.

Unexpected memory allocation is almost always a sign of some problem with your code, usually a problem with type-stability. Consequently, in addition to the allocation itself, it's very likely that the code generated for your function is far from optimal. Take such indications seriously and follow the advice below.

As a teaser, note that an improved version of this function allocates no memory (except to pass back the result back to the REPL) and has an order of magnitude faster execution after the first call:

```
julia> @time f_improved(1) # first call
elapsed time: 0.003702172 seconds (78944 bytes allocated)
0.5

julia> @time f_improved(10^6)
elapsed time: 0.004313644 seconds (112 bytes allocated)
2.5000025e11
```

Below you'll learn how to spot the problem with f and how to fix it.

In some situations, your function may need to allocate memory as part of its operation, and this can complicate the simple picture above. In such cases, consider using one of the *tools* (page 266) below to diagnose problems, or write a version of your function that separates allocation from its algorithmic aspects (see *Pre-allocating outputs* (page 275)).

#### **30.3 Tools**

Julia and its package ecosystem includes tools that may help you diagnose problems and improve the performance of your code:

- *Profiling* (page 491) allows you to measure the performance of your running code and identify lines that serve as bottlenecks. For complex projects, the ProfileView package can help you visualize your profiling results.
- Unexpectedly-large memory allocations—as reported by <code>@time</code> (page 328), <code>@allocated</code> (page 329), or the profiler (through calls to the garbage-collection routines)—hint that there might be issues with your code. If you don't see another reason for the allocations, suspect a type problem. You can also start Julia with the <code>--track-allocation=user</code> option and examine the resulting \*.mem files to see information about where those allocations occur. See <code>Memory allocation analysis</code> (page 263).
- @code\_warntype generates a representation of your code that can be helpful in finding expressions that result in type uncertainty. See @code\_warntype (page 281) below.
- The Lint and TypeCheck packages can also warn you of certain types of programming errors.

# 30.4 Avoid containers with abstract type parameters

When working with parameterized types, including arrays, it is best to avoid parameterizing with abstract types where possible.

Consider the following:

```
a = Real[] # typeof(a) = Array{Real,1}
if (f = rand()) < .8
    push!(a, f)
end</pre>
```

Because a is a an array of abstract type Real, it must be able to hold any Real value. Since Real objects can be of arbitrary size and structure, a must be represented as an array of pointers to individually allocated Real objects. Because f will always be a Float 64 (page 380), we should instead, use:

```
a = Float64[] # typeof(a) = Array{Float64,1}
```

which will create a contiguous block of 64-bit floating-point values that can be manipulated efficiently.

See also the discussion under *Parametric Types* (page 94).

# 30.5 Type declarations

In many languages with optional type declarations, adding declarations is the principal way to make code run faster. This is *not* the case in Julia. In Julia, the compiler generally knows the types of all function arguments, local variables, and expressions. However, there are a few specific instances where declarations are helpful.

#### 30.5.1 Avoid fields with abstract type

Types can be declared without specifying the types of their fields:

```
julia> type MyAmbiguousType

a
end
```

This allows a to be of any type. This can often be useful, but it does have a downside: for objects of type MyAmbiguousType, the compiler will not be able to generate high-performance code. The reason is that the compiler uses the types of objects, not their values, to determine how to build code. Unfortunately, very little can be inferred about an object of type MyAmbiguousType:

```
julia> b = MyAmbiguousType("Hello")
MyAmbiguousType("Hello")

julia> c = MyAmbiguousType(17)
MyAmbiguousType(17)

julia> typeof(b)
MyAmbiguousType

julia> typeof(c)
MyAmbiguousType
```

b and c have the same type, yet their underlying representation of data in memory is very different. Even if you stored just numeric values in field a, the fact that the memory representation of a UInt8 differs from a Float64 also means that the CPU needs to handle them using two different kinds of instructions. Since the required information is not available in the type, such decisions have to be made at run-time. This slows performance.

You can do better by declaring the type of a. Here, we are focused on the case where a might be any one of several types, in which case the natural solution is to use parameters. For example:

```
julia> type MyType{T<:AbstractFloat}
    a::T
    end</pre>
```

This is a better choice than

```
julia> type MyStillAmbiguousType
    a::AbstractFloat
    end
```

because the first version specifies the type of a from the type of the wrapper object. For example:

```
julia> m = MyType(3.2)
MyType{Float64}(3.2)

julia> t = MyStillAmbiguousType(3.2)
MyStillAmbiguousType(3.2)

julia> typeof(m)
MyType{Float64}

julia> typeof(t)
MyStillAmbiguousType
```

The type of field a can be readily determined from the type of m, but not from the type of t. Indeed, in t it's possible to change the type of field a:

```
julia> typeof(t.a)
Float64

julia> t.a = 4.5f0
4.5f0

julia> typeof(t.a)
Float32
```

In contrast, once m is constructed, the type of m.a cannot change:

```
julia> m.a = 4.5f0
4.5
```

```
julia> typeof(m.a)
Float64
```

The fact that the type of m. a is known from m's type—coupled with the fact that its type cannot change mid-function—allows the compiler to generate highly-optimized code for objects like m but not for objects like t.

Of course, all of this is true only if we construct m with a concrete type. We can break this by explicitly constructing it with an abstract type:

```
julia> m = MyType{AbstractFloat}(3.2)
MyType{AbstractFloat}(3.2)

julia> typeof(m.a)
Float64

julia> m.a = 4.5f0
4.5f0

julia> typeof(m.a)
Float32
```

For all practical purposes, such objects behave identically to those of MyStillAmbiguousType.

It's quite instructive to compare the sheer amount code generated for a simple function

```
func(m::MyType) = m.a+1
```

#### using

```
code_llvm(func, (MyType{Float64},))
code_llvm(func, (MyType{AbstractFloat},))
code_llvm(func, (MyType,))
```

For reasons of length the results are not shown here, but you may wish to try this yourself. Because the type is fully-specified in the first case, the compiler doesn't need to generate any code to resolve the type at run-time. This results in shorter and faster code.

#### 30.5.2 Avoid fields with abstract containers

The same best practices also work for container types:

#### For example:

```
julia> c = MySimpleContainer(1:3);
julia> typeof(c)
MySimpleContainer{UnitRange{Int64}}
julia> c = MySimpleContainer([1:3;]);
julia> typeof(c)
```

```
MySimpleContainer{Array{Int64,1}}

julia> b = MyAmbiguousContainer(1:3);

julia> typeof(b)
MyAmbiguousContainer{Int64}

julia> b = MyAmbiguousContainer([1:3;]);

julia> typeof(b)
MyAmbiguousContainer{Int64}
```

For MySimpleContainer, the object is fully-specified by its type and parameters, so the compiler can generate optimized functions. In most instances, this will probably suffice.

While the compiler can now do its job perfectly well, there are cases where *you* might wish that your code could do different things depending on the *element type* of a. Usually the best way to achieve this is to wrap your specific operation (here,  $f \circ 0$ ) in a separate function:

```
function sumfoo(c::MySimpleContainer)
    s = 0
for x in c.a
    s += foo(x)
end
s
end
foo(x::Integer) = x
foo(x::AbstractFloat) = round(x)
```

This keeps things simple, while allowing the compiler to generate optimized code in all cases.

However, there are cases where you may need to declare different versions of the outer function for different element types of a. You could do it like this:

```
function myfun{T<:AbstractFloat}(c::MySimpleContainer{Vector{T}})
    ...
end
function myfun{T<:Integer}(c::MySimpleContainer{Vector{T}})
    ...
end</pre>
```

This works fine for Vector{T}, but we'd also have to write explicit versions for UnitRange{T} or other abstract types. To prevent such tedium, you can use two parameters in the declaration of MyContainer:

```
type MyContainer{T, A<:AbstractVector}
    a::A
end
MyContainer(v::AbstractVector) = MyContainer{eltype(v), typeof(v)}(v)

julia> b = MyContainer(1.3:5);

julia> typeof(b)
MyContainer{Float64, UnitRange{Float64}}
```

Note the somewhat surprising fact that T doesn't appear in the declaration of field a, a point that we'll return to in a moment. With this approach, one can write functions such as:

```
function myfunc{T<:Integer, A<:AbstractArray}(c::MyContainer{T,A})</pre>
    return c.a[1]+1
end
# Note: because we can only define MyContainer for
# A<:AbstractArray, and any unspecified parameters are arbitrary,
# the previous could have been written more succinctly as
      function myfunc{T<:Integer} (c::MyContainer{T})</pre>
function myfunc{T<:AbstractFloat}(c::MyContainer{T})</pre>
    return c.a[1]+2
end
function myfunc{T<:Integer}(c::MyContainer{T, Vector{T}})</pre>
    return c.a[1]+3
end
julia> myfunc(MyContainer(1:3))
julia> myfunc(MyContainer(1.0:3))
3.0
julia> myfunc(MyContainer([1:3]))
```

As you can see, with this approach it's possible to specialize on both the element type T and the array type A.

However, there's one remaining hole: we haven't enforced that A has element type T, so it's perfectly possible to construct an object like this:

```
julia> b = MyContainer{Int64, UnitRange{Float64}} (1.3:5);

julia> typeof(b)
MyContainer{Int64,UnitRange{Float64}}
```

To prevent this, we can add an inner constructor:

```
type MyBetterContainer{T<:Real, A<:AbstractVector}
    a::A

    MyBetterContainer(v::AbstractVector{T}) = new(v)
end
MyBetterContainer(v::AbstractVector) = MyBetterContainer{eltype(v),typeof(v)}(v)

julia> b = MyBetterContainer(1.3:5);

julia> typeof(b)
MyBetterContainer{Float64,UnitRange{Float64}}

julia> b = MyBetterContainer{Int64, UnitRange{Float64}} (1.3:5);
ERROR: no method MyBetterContainer(UnitRange{Float64},)
```

The inner constructor requires that the element type of A be T.

#### 30.5.3 Annotate values taken from untyped locations

It is often convenient to work with data structures that may contain values of any type, such as cell arrays (arrays of type Array {Any}). But, if you're using one of these structures and happen to know the type of an element, it helps to share this knowledge with the compiler:

```
function foo(a::Array{Any,1})
    x = a[1]::Int32
    b = x+1
    ...
end
```

Here, we happened to know that the first element of a would be an Int32. Making an annotation like this has the added benefit that it will raise a run-time error if the value is not of the expected type, potentially catching certain bugs earlier.

#### 30.5.4 Declare types of keyword arguments

Keyword arguments can have declared types:

```
function with_keyword(x; name::Int = 1)
...
end
```

Functions are specialized on the types of keyword arguments, so these declarations will not affect performance of code inside the function. However, they will reduce the overhead of calls to the function that include keyword arguments.

Functions with keyword arguments have near-zero overhead for call sites that pass only positional arguments.

Passing dynamic lists of keyword arguments, as in f(x; keywords...), can be slow and should be avoided in performance-sensitive code.

# 30.6 Break functions into multiple definitions

Writing a function as many small definitions allows the compiler to directly call the most applicable code, or even inline it.

Here is an example of a "compound function" that should really be written as multiple definitions:

```
function norm(A)
   if isa(A, Vector)
        return sqrt(real(dot(A,A)))
   elseif isa(A, Matrix)
        return max(svd(A)[2])
   else
        error("norm: invalid argument")
   end
end
```

This can be written more concisely and efficiently as:

```
norm(x::Vector) = sqrt(real(dot(x,x)))
norm(A::Matrix) = max(svd(A)[2])
```

# 30.7 Write "type-stable" functions

When possible, it helps to ensure that a function always returns a value of the same type. Consider the following definition:

```
pos(x) = x < 0 ? 0 : x
```

Although this seems innocent enough, the problem is that 0 is an integer (of type Int) and x might be of any type. Thus, depending on the value of x, this function might return a value of either of two types. This behavior is allowed, and may be desirable in some cases. But it can easily be fixed as follows:

```
pos(x) = x < 0 ? zero(x) : x
```

There is also a one () (page 378) function, and a more general of type (x, y) (page 322) function, which returns y converted to the type of x.

# 30.8 Avoid changing the type of a variable

An analogous "type-stability" problem exists for variables used repeatedly within a function:

```
function foo()
    x = 1
    for i = 1:10
        x = x/bar()
    end
    return x
end
```

Local variable x starts as an integer, and after one loop iteration becomes a floating-point number (the result of / (page 353) operator). This makes it more difficult for the compiler to optimize the body of the loop. There are several possible fixes:

- Initialize x with x = 1.0
- Declare the type of x: x::Float64 = 1
- Use an explicit conversion: x = one(T)

# 30.9 Separate kernel functions

Many functions follow a pattern of performing some set-up work, and then running many iterations to perform a core computation. Where possible, it is a good idea to put these core computations in separate functions. For example, the following contrived function returns an array of a randomly-chosen type:

```
function strange_twos(n)
    a = Array(rand(Bool) ? Int64 : Float64, n)
    for i = 1:n
        a[i] = 2
    end
    return a
end
```

This should be written as:

```
function fill_twos!(a)
    for i=1:length(a)
        a[i] = 2
    end
end

function strange_twos(n)
    a = Array(rand(Bool) ? Int64 : Float64, n)
    fill_twos!(a)
    return a
end
```

Julia's compiler specializes code for argument types at function boundaries, so in the original implementation it does not know the type of a during the loop (since it is chosen randomly). Therefore the second version is generally faster since the inner loop can be recompiled as part of fill\_twos! for different types of a.

The second form is also often better style and can lead to more code reuse.

This pattern is used in several places in the standard library. For example, see hvcat\_fill in abstractarray.jl, or the fill! function, which we could have used instead of writing our own fill twos!.

Functions like strange\_twos occur when dealing with data of uncertain type, for example data loaded from an input file that might contain either integers, floats, strings, or something else.

# 30.10 Access arrays in memory order, along columns

Multidimensional arrays in Julia are stored in column-major order. This means that arrays are stacked one column at a time. This can be verified using the vec function or the syntax [:] as shown below (notice that the array is ordered [1 3 2 4], not [1 2 3 4]):

```
julia> x = [1 2; 3 4]
2x2 Array{Int64,2}:
    1    2
    3    4

julia> x[:]
4-element Array{Int64,1}:
    1
    3
    2
    4
```

This convention for ordering arrays is common in many languages like Fortran, Matlab, and R (to name a few). The alternative to column-major ordering is row-major ordering, which is the convention adopted by C and Python (numpy) among other languages. Remembering the ordering of arrays can have significant performance effects when looping over arrays. A rule of thumb to keep in mind is that with column-major arrays, the first index changes most rapidly. Essentially this means that looping will be faster if the inner-most loop index is the first to appear in a slice expression.

Consider the following contrived example. Imagine we wanted to write a function that accepts a Vector and returns a square Matrix with either the rows or the columns filled with copies of the input vector. Assume that it is not important whether rows or columns are filled with these copies (perhaps the rest of the code can be easily adapted accordingly). We could conceivably do this in at least four ways (in addition to the recommended call to the built-in repmat () (page 424)):

```
function copy_cols{T}(x::Vector{T})
n = size(x, 1)
```

```
out = Array(eltype(x), n, n)
    for i=1:n
        out[:, i] = x
    end
    out
end
function copy_rows{T} (x::Vector{T})
   n = size(x, 1)
   out = Array(eltype(x), n, n)
    for i=1:n
        out[i, :] = x
    end
    out
end
function copy_col_row{T}(x::Vector{T})
   n = size(x, 1)
   out = Array(T, n, n)
    for col=1:n, row=1:n
        out[row, col] = x[row]
    out
end
function copy_row_col{T}(x::Vector{T})
   n = size(x, 1)
   out = Array(T, n, n)
   for row=1:n, col=1:n
        out[row, col] = x[col]
    end
    out
```

Now we will time each of these functions using the same random 10000 by 1 input vector:

Notice that copy\_cols is much faster than copy\_rows. This is expected because copy\_cols respects the column-based memory layout of the Matrix and fills it one column at a time. Additionally, copy\_col\_row is much faster than copy\_row\_col because it follows our rule of thumb that the first element to appear in a slice expression should be coupled with the inner-most loop.

# 30.11 Pre-allocating outputs

If your function returns an Array or some other complex type, it may have to allocate memory. Unfortunately, oftentimes allocation and its converse, garbage collection, are substantial bottlenecks. Sometimes you can circumvent the need to allocate memory on each function call by preallocating the output. As a trivial example, compare

```
function xinc(x)
    return [x, x+1, x+2]
end

function loopinc()
    y = 0
    for i = 1:10^7
        ret = xinc(i)
        y += ret[2]
    end
    y
end
```

with

```
function xinc!{T}(ret::AbstractVector{T}, x::T)
    ret[1] = x
    ret[2] = x+1
    ret[3] = x+2
    nothing
end

function loopinc_prealloc()
    ret = Array(Int, 3)
    y = 0
    for i = 1:10^7
        xinc!(ret, i)
        y += ret[2]
    end
    y
end
```

#### Timing results:

```
julia> @time loopinc()
elapsed time: 1.955026528 seconds (1279975584 bytes allocated)
50000015000000

julia> @time loopinc_prealloc()
elapsed time: 0.078639163 seconds (144 bytes allocated)
50000015000000
```

Preallocation has other advantages, for example by allowing the caller to control the "output" type from an algorithm. In the example above, we could have passed a SubArray rather than an Array (page 395), had we so desired.

Taken to its extreme, pre-allocation can make your code uglier, so performance measurements and some judgment may be required.

# 30.12 Avoid string interpolation for I/O

When writing data to a file (or other I/O device), forming extra intermediate strings is a source of overhead. Instead of:

```
println(file, "$a $b")
```

use:

```
println(file, a, " ", b)
```

The first version of the code forms a string, then writes it to the file, while the second version writes values directly to the file. Also notice that in some cases string interpolation can be harder to read. Consider:

```
println(file, "$(f(a))$(f(b))")
```

versus:

```
println(file, f(a), f(b))
```

# 30.13 Optimize network I/O during parallel execution

When executing a remote function in parallel:

```
responses = cell(nworkers())
@sync begin
    for (idx, pid) in enumerate(workers())
        @async responses[idx] = remotecall_fetch(pid, foo, args...)
    end
end
```

is faster than:

```
refs = cell(nworkers())
for (idx, pid) in enumerate(workers())
    refs[idx] = @spawnat pid foo(args...)
end
responses = [fetch(r) for r in refs]
```

The former results in a single network round-trip to every worker, while the latter results in two network calls - first by the @spawnat and the second due to the fetch (or even a wait). The fetch/wait is also being executed serially resulting in an overall poorer performance.

# 30.14 Fix deprecation warnings

A deprecated function internally performs a lookup in order to print a relevant warning only once. This extra lookup can cause a significant slowdown, so all uses of deprecated functions should be modified as suggested by the warnings.

#### **30.15 Tweaks**

These are some minor points that might help in tight inner loops.

- Avoid unnecessary arrays. For example, instead of sum([x,y,z]) (page 339) use x+y+z.
- Use abs2(z) (page 363) instead of abs(z)^2 (page 363) for complex z. In general, try to rewrite code to use abs2() (page 363) instead of abs() (page 363) for complex arguments.
- Use div(x, y) (page 354) for truncating division of integers instead of trunc(x/y) (page 362), fld(x, y) (page 354) instead of floor(x/y) (page 362), and cld(x, y) (page 354) instead of ceil(x/y) (page 362).

#### 30.16 Performance Annotations

Sometimes you can enable better optimization by promising certain program properties.

- Use @inbounds to eliminate array bounds checking within expressions. Be certain before doing this. If the subscripts are ever out of bounds, you may suffer crashes or silent corruption.
- Use @fastmath to allow floating point optimizations that are correct for real numbers, but lead to differences for IEEE numbers. Be careful when doing this, as this may change numerical results. This corresponds to the -ffast-math option of clang.
- Write @simd in front of for loops that are amenable to vectorization. This feature is experimental and could change or disappear in future versions of Julia.

Note: While @simd needs to be placed directly in front of a loop, both @inbounds and @fastmath can be applied to several statements at once, e.g. using begin ... end, or even to a whole function.

Here is an example with both @inbounds and @simd markup:

```
function inner( x, y )
    s = zero(eltype(x))
    for i=1:length(x)
        Qinbounds s += x[i] * y[i]
    end
end
function innersimd( x, y )
    s = zero(eltype(x))
    @simd for i=1:length(x)
        Qinbounds s += x[i] * y[i]
    end
end
function timeit ( n, reps )
   x = rand(Float32, n)
   y = rand(Float32, n)
    s = zero(Float64)
   time = @elapsed for j in 1:reps
        s + = inner(x, y)
    println("GFlop
                          = ",2.0*n*reps/time*1E-9)
    time = @elapsed for j in 1:reps
        s + = innersimd(x, y)
    println("GFlop (SIMD) = ",2.0*n*reps/time*1E-9)
end
timeit(1000,1000)
```

On a computer with a 2.4GHz Intel Core i5 processor, this produces:

```
GFlop = 1.9467069505224963
GFlop (SIMD) = 17.578554163920018
```

The range for a @simd for loop should be a one-dimensional range. A variable used for accumulating, such as s in the example, is called a *reduction variable*. By using @simd, you are asserting several properties of the loop:

• It is safe to execute iterations in arbitrary or overlapping order, with special consideration for reduction variables.

- Floating-point operations on reduction variables can be reordered, possibly causing different results than without @simd.
- No iteration ever waits on another iteration to make forward progress.

A loop containing break, continue, or @goto will cause a compile-time error.

Using @simd merely gives the compiler license to vectorize. Whether it actually does so depends on the compiler. To actually benefit from the current implementation, your loop should have the following additional properties:

- The loop must be an innermost loop.
- The loop body must be straight-line code. This is why @inbounds is currently needed for all array accesses. The compiler can sometimes turn short &&, ||, and ?: expressions into straight-line code, if it is safe to evaluate all operands unconditionally. Consider using ifelse() (page 320) instead of ?: in the loop if it is safe to do so.
- Accesses must have a stride pattern and cannot be "gathers" (random-index reads) or "scatters" (random-index writes).
- The stride should be unit stride.
- In some simple cases, for example with 2-3 arrays accessed in a loop, the LLVM auto-vectorization may kick in automatically, leading to no further speedup with @simd.

Here is an example with all three kinds of markup. This program first calculates the finite difference of a onedimensional array, and then evaluates the L2-norm of the result:

```
function init!(u)
   n = length(u)
    dx = 1.0 / (n-1)
    @fastmath @inbounds @simd for i in 1:n
        u[i] = sin(2pi*dx*i)
    end
end
function deriv! (u, du)
   n = length(u)
   dx = 1.0 / (n-1)
    @fastmath @inbounds du[1] = (u[2] - u[1]) / dx
    @fastmath @inbounds @simd for i in 2:n-1
        du[i] = (u[i+1] - u[i-1]) / (2*dx)
    end
    @fastmath @inbounds du[n] = (u[n] - u[n-1]) / dx
end
function norm(u)
   n = length(u)
   T = eltype(u)
    s = zero(T)
    @fastmath @inbounds @simd for i in 1:n
        s += u[i]^2
    end
    @fastmath @inbounds return sqrt(s/n)
end
function main()
   n = 2000
   u = Array(Float64, n)
    init!(u)
    du = similar(u)
```

On a computer with a 2.7 GHz Intel Core i7 processor, this produces:

```
$ julia wave.jl
elapsed time: 1.207814709 seconds (0 bytes allocated)
4.443986180758243

$ julia --math-mode=ieee wave.jl
elapsed time: 4.487083643 seconds (0 bytes allocated)
4.443986180758243
```

Here, the option --math-mode=ieee disables the @fastmath macro, so that we can compare results.

In this case, the speedup due to @fastmath is a factor of about 3.7. This is unusually large – in general, the speedup will be smaller. (In this particular example, the working set of the benchmark is small enough to fit into the L1 cache of the processor, so that memory access latency does not play a role, and computing time is dominated by CPU usage. In many real world programs this is not the case.) Also, in this case this optimization does not change the result – in general, the result will be slightly different. In some cases, especially for numerically unstable algorithms, the result can be very different.

The annotation <code>@fastmath</code> re-arranges floating point expressions, e.g. changing the order of evaluation, or assuming that certain special cases (inf, nan) cannot occur. In this case (and on this particular computer), the main difference is that the expression 1 / (2\*dx) in the function <code>deriv</code> is hoisted out of the loop (i.e. calculated outside the loop), as if one had written idx = 1 / (2\*dx). In the loop, the expression . . . / (2\*dx) then becomes . . . \* idx, which is much faster to evaluate. Of course, both the actual optimization that is applied by the compiler as well as the resulting speedup depend very much on the hardware. You can examine the change in generated code by using Julia's <code>code\_native()</code> (page 333) function.

#### 30.17 Treat Subnormal Numbers as Zeros

Subnormal numbers, formerly called denormal numbers, are useful in many contexts, but incur a performance penalty on some hardware. A call <code>set\_zero\_subnormals(true)</code> (page 381) grants permission for floating-point operations to treat subnormal inputs or outputs as zeros, which may improve performance on some hardware. A call <code>set\_zero\_subnormals(false)</code> (page 381) enforces strict IEEE behavior for subnormal numbers.

Below is an example where subnormals noticeably impact performance on some hardware:

```
b[n] = 0
                                         # Boundary condition
end
function heatflow{T}( a::Vector{T}, nstep::Integer )
   b = similar(a)
   for t=1:div(nstep, 2)
                                         # Assume nstep is even
        timestep(b,a,T(0.1))
        timestep(a,b,T(0.1))
    end
end
heatflow(zeros(Float32,10),2)
                                       # Force compilation
for trial=1:6
   a = zeros(Float32, 1000)
    set_zero_subnormals(iseven(trial)) # Odd trials use strict IEEE arithmetic
    @time heatflow(a, 1000)
end
```

This example generates many subnormal numbers because the values in a become an exponentially decreasing curve, which slowly flattens out over time.

Treating subnormals as zeros should be used with caution, because doing so breaks some identities, such as x-y==0 implies x==y:

```
julia> x=3f-38; y=2f-38;

julia> set_zero_subnormals(false); (x-y, x==y)
(1.0000001f-38, false)

julia> set_zero_subnormals(true); (x-y, x==y)
(0.0f0, false)
```

In some applications, an alternative to zeroing subnormal numbers is to inject a tiny bit of noise. For example, instead of initializing a with zeros, initialize it with:

```
a = rand(Float32,1000) * 1.f-9
```

# 30.18 @code\_warntype

The macro @code\_warntype (page 333) (or its function variant code\_warntype () (page 332)) can sometimes be helpful in diagnosing type-related problems. Here's an example:

```
pos(x) = x < 0 ? 0 : x

function f(x)
    y = pos(x)
    sin(y*x+1)
end

julia> @code_warntype f(3.2)
Variables:
    x::Float64
    y::UNION(INT64,FLOAT64)
    _var0::Float64
    _var3::Tuple{Int64}
    _var4::UNION(INT64,FLOAT64)
    _var1::Float64
```

Interpreting the output of @code\_warntype (page 333), like that of its cousins @code\_lowered (page 332), @code\_typed (page 332), @code\_llvm (page 333), and @code\_native (page 333), takes a little practice. Your code is being presented in form that has been partially digested on its way to generating compiled machine code. Most of the expressions are annotated by a type, indicated by the ::T (where T might be Float64 (page 380), for example). The most important characteristic of @code\_warntype (page 333) is that non-concrete types are displayed in red; in the above example, such output is shown in all-caps.

The top part of the output summarizes the type information for the different variables internal to the function. You can see that y, one of the variables you created, is a Union{Int64, Float64}, due to the type-instability of pos. There is another variable, \_var4, which you can see also has the same type.

The next lines represent the body of f. The lines starting with a number followed by a colon (1:, 2:) are labels, and represent targets for jumps (via goto) in your code. Looking at the body, you can see that pos has been *inlined* into f—everything before 2: comes from code defined in pos.

Starting at 2:, the variable y is defined, and again annotated as a Union type. Next, we see that the compiler created the temporary variable \_var1 to hold the result of y\*x. Because a Float64 (page 380) times either an Int64 or Float64 (page 380) yields a Float64 (page 380), all type-instability ends here. The net result is that f(x::Float64) will not be type-unstable in its output, even if some of the intermediate computations are type-unstable

How you use this information is up to you. Obviously, it would be far and away best to fix pos to be type-stable: if you did so, all of the variables in f would be concrete, and its performance would be optimal. However, there are circumstances where this kind of *ephemeral* type instability might not matter too much: for example, if pos is never used in isolation, the fact that f's output is type-stable (for Float64 (page 380) inputs) will shield later code from the propagating effects of type instability. This is particularly relevant in cases where fixing the type instability is difficult or impossible: for example, currently it's not possible to infer the return type of an anonymous function. In such cases, the tips above (e.g., adding type annotations and/or breaking up functions) are your best tools to contain the "damage" from type instability.

The following examples may help you interpret expressions marked as containing non-leaf types:

- Function body ending in end::Union{T1,T2})
  - Interpretation: function with unstable return type
  - Suggestion: make the return value type-stable, even if you have to annotate it
- f(x::T)::Union{T1,T2}
  - Interpretation: call to a type-unstable function
  - Suggestion: fix the function, or if necessary annotate the return value

- (top(arrayref))(A::Array{Any,1},1)::Any
  - Interpretation: accessing elements of poorly-typed arrays
  - Suggestion: use arrays with better-defined types, or if necessary annotate the type of individual element accesses
- (top(getfield))(A::ArrayContainer{Float64},:data)::Array{Float64,N}
  - Interpretation: getting a field that is of non-leaf type. In this case, ArrayContainer had a field data::Array{T}. But Array needs the dimension N, too, to be a concrete type.
  - Suggestion: use concrete types like  $Array\{T,3\}$  or  $Array\{T,N\}$ , where N is now a parameter of ArrayContainer

# **Style Guide**

The following sections explain a few aspects of idiomatic Julia coding style. None of these rules are absolute; they are only suggestions to help familiarize you with the language and to help you choose among alternative designs.

# 31.1 Write functions, not just scripts

Writing code as a series of steps at the top level is a quick way to get started solving a problem, but you should try to divide a program into functions as soon as possible. Functions are more reusable and testable, and clarify what steps are being done and what their inputs and outputs are. Furthermore, code inside functions tends to run much faster than top level code, due to how Julia's compiler works.

It is also worth emphasizing that functions should take arguments, instead of operating directly on global variables (aside from constants like pi (page 379)).

# 31.2 Avoid writing overly-specific types

Code should be as generic as possible. Instead of writing:

```
convert(Complex{Float64}, x)
```

it's better to use available generic functions:

```
complex(float(x))
```

The second version will convert x to an appropriate type, instead of always the same type.

This style point is especially relevant to function arguments. For example, don't declare an argument to be of type Int or Int32 if it really could be any integer, expressed with the abstract type Integer. In fact, in many cases you can omit the argument type altogether, unless it is needed to disambiguate from other method definitions, since a MethodError (page 330) will be thrown anyway if a type is passed that does not support any of the requisite operations. (This is known as duck typing.)

For example, consider the following definitions of a function addone that returns one plus its argument:

```
addone(x::Int) = x + 1  # works only for Int
addone(x::Integer) = x + one(x)  # any integer type
addone(x::Number) = x + one(x)  # any numeric type
addone(x) = x + one(x)  # any type supporting + and one
```

The last definition of addone handles any type supporting one() (page 378) (which returns 1 in the same type as x, which avoids unwanted type promotion) and the + (page 353) function with those arguments. The key thing to realize is that there is no performance penalty to defining only the general addone(x) = x + one(x), because Julia will automatically compile specialized versions as needed. For example, the first time you call addone(12), Julia will automatically compile a specialized addone function for x::Int arguments, with the call to one() (page 378) replaced by its inlined value 1. Therefore, the first three definitions of addone above are completely redundant.

# 31.3 Handle excess argument diversity in the caller

Instead of:

```
function foo(x, y)
    x = Int(x); y = Int(y)
    ...
end
foo(x, y)
```

use:

```
function foo(x::Int, y::Int)
    ...
end
foo(Int(x), Int(y))
```

This is better style because foo does not really accept numbers of all types; it really needs Int s.

One issue here is that if a function inherently requires integers, it might be better to force the caller to decide how non-integers should be converted (e.g. floor or ceiling). Another issue is that declaring more specific types leaves more "space" for future method definitions.

# 31.4 Append! to names of functions that modify their arguments

Instead of:

```
function double{T<:Number} (a::AbstractArray{T})
   for i = 1:endof(a); a[i] *= 2; end
   a
end</pre>
```

use:

```
function double!{T<:Number}(a::AbstractArray{T})
    for i = 1:endof(a); a[i] *= 2; end
    a
end</pre>
```

The Julia standard library uses this convention throughout and contains examples of functions with both copying and modifying forms (e.g., <code>sort()</code> (page 466) and <code>sort!()</code> (page 466)), and others which are just modifying (e.g., <code>push!()</code> (page 346), <code>pop!()</code> (page 344), <code>splice!()</code> (page 348)). It is typical for such functions to also return the modified array for convenience.

# 31.5 Avoid strange type Unions

Types such as Union {Function, AbstractString} are often a sign that some design could be cleaner.

# 31.6 Avoid type Unions in fields

When creating a type such as:

```
type MyType
    ...
    x::Union{Void, T}
end
```

ask whether the option for x to be nothing (of type Void) is really necessary. Here are some alternatives to consider:

- Find a safe default value to initialize x with
- Introduce another type that lacks x
- If there are many fields like x, store them in a dictionary
- Determine whether there is a simple rule for when x is nothing. For example, often the field will start as nothing but get initialized at some well-defined point. In that case, consider leaving it undefined at first.
- If x really needs to hold no value at some times, define it as ::Nullable {T} instead, as this guarantees type-stability in the code accessing this field (see *Nullable types* (page 103))

# 31.7 Avoid elaborate container types

It is usually not much help to construct arrays like the following:

```
a = Array(Union{Int, AbstractString, Tuple, Array}, n)
```

In this case cell(n) (page 395) is better. It is also more helpful to the compiler to annotate specific uses (e.g. a[i]::Int) than to try to pack many alternatives into one type.

# 31.8 Use naming conventions consistent with Julia's base/

- modules and type names use capitalization and camel case: module SparseMatrix, immutable UnitRange.
- functions are lowercase (maximum() (page 338), convert() (page 321)) and, when readable, with multiple words squashed together (isequal() (page 320), haskey() (page 343)). When necessary, use underscores as word separators. Underscores are also used to indicate a combination of concepts (remotecall\_fetch() (page 409) as a more efficient implementation of fetch (remotecall(...))) or as modifiers (sum\_kbn() (page 401)).
- conciseness is valued, but avoid abbreviation (indexin() (page 337) rather than indxin()) as it becomes difficult to remember whether and how particular words are abbreviated.

If a function name requires multiple words, consider whether it might represent more than one concept and might be better split into pieces.

# 31.9 Don't overuse try-catch

It is better to avoid errors than to rely on catching them.

# 31.10 Don't parenthesize conditions

Julia doesn't require parens around conditions in if and while. Write:

```
if a == b
```

instead of:

```
if (a == b)
```

#### 31.11 Don't overuse ...

Splicing function arguments can be addictive. Instead of [a..., b...], use simply [a; b], which already concatenates arrays. collect(a) (page 341) is better than [a...], but since a is already iterable it is often even better to leave it alone, and not convert it to an array.

# 31.12 Don't use unnecessary static parameters

A function signature:

```
foo{T<:Real}(x::T) = ...
```

should be written as:

```
foo(x::Real) = ...
```

instead, especially if T is not used in the function body. Even if T is used, it can be replaced with typeof(x) (page 321) if convenient. There is no performance difference. Note that this is not a general caution against static parameters, just against uses where they are not needed.

Note also that container types, specifically may need type parameters in function calls. See the FAQ *Avoid fields with abstract containers* (page 269) for more information.

# 31.13 Avoid confusion about whether something is an instance or a type

Sets of definitions like the following are confusing:

```
foo(::Type{MyType}) = ...
foo(::MyType) = foo(MyType)
```

Decide whether the concept in question will be written as MyType or MyType (), and stick to it.

The preferred style is to use instances by default, and only add methods involving Type {MyType} later if they become necessary to solve some problem.

If a type is effectively an enumeration, it should be defined as a single (ideally immutable) type, with the enumeration values being instances of it. Constructors and conversions can check whether values are valid. This design is preferred over making the enumeration an abstract type, with the "values" as subtypes.

#### 31.14 Don't overuse macros

Be aware of when a macro could really be a function instead.

Calling eval () (page 325) inside a macro is a particularly dangerous warning sign; it means the macro will only work when called at the top level. If such a macro is written as a function instead, it will naturally have access to the run-time values it needs.

## 31.15 Don't expose unsafe operations at the interface level

If you have a type that uses a native pointer:

```
type NativeType
    p::Ptr{UInt8}
    ...
end
```

don't write definitions like the following:

```
getindex(x::NativeType, i) = unsafe_load(x.p, i)
```

The problem is that users of this type can write x[i] without realizing that the operation is unsafe, and then be susceptible to memory bugs.

Such a function should either check the operation to ensure it is safe, or have unsafe somewhere in its name to alert callers.

# 31.16 Don't overload methods of base container types

It is possible to write definitions like the following:

```
show(io::IO, v::Vector{MyType}) = ...
```

This would provide custom showing of vectors with a specific new element type. While tempting, this should be avoided. The trouble is that users will expect a well-known type like Vector() to behave in a certain way, and overly customizing its behavior can make it harder to work with.

# 31.17 Be careful with type equality

You generally want to use isa() (page 320) and <: (issubtype() (page 322)) for testing types, not ==. Checking types for exact equality typically only makes sense when comparing to a known concrete type (e.g. T = Float64), or if you *really*, *really* know what you're doing.

#### 31.18 Do not write x->f(x)

Since higher-order functions are often called with anonymous functions, it is easy to conclude that this is desirable or even necessary. But any function can be passed directly, without being "wrapped" in an anonymous function. Instead of writing map (x-)f(x), a), write map(f, a) (page 340).

# 31.19 Avoid using floats for numeric literals in generic code when possible

If you write generic code which handles numbers, and which can be expected to run with many different numeric type arguments, try using literals of a numeric type that will affect the arguments as little as possible through promotion.

For example,

```
julia> f(x) = 2.0 * x
f (generic function with 1 method)

julia> f(1//2)
1.0

julia> f(1/2)
1.0

julia> f(1)
2.0
```

#### while

```
julia> g(x) = 2 * x
g (generic function with 1 method)

julia> g(1//2)
1//1

julia> g(1/2)
1.0

julia> g(2)
4
```

As you can see, the second version, where we used an Int literal, preserved the type of the input argument, while the first didn't. This is because e.g. promote\_type(Int, Float64) == Float64, and promotion happens with the multiplication. Similarly, Rational literals are less type disruptive than Float64 (page 380) literals, but more disruptive than Ints:

```
julia> h(x) = 2//1 * x

h (generic function with 1 method)

julia> h(1//2)

1//1

julia> h(1/2)

1.0

julia> h(1)

2//1
```

Thus, use Int literals when possible, with Rational {Int} for literal non-integer numbers, in order to make it easier to use your code.

# **Frequently Asked Questions**

#### 32.1 Sessions and the REPL

#### 32.1.1 How do I delete an object in memory?

Julia does not have an analog of MATLAB's clear function; once a name is defined in a Julia session (technically, in module Main), it is always present.

If memory usage is your concern, you can always replace objects with ones that consume less memory. For example, if A is a gigabyte-sized array that you no longer need, you can free the memory with A = 0. The memory will be released the next time the garbage collector runs; you can force this to happen with gc() (page 332).

#### 32.1.2 How can I modify the declaration of a type/immutable in my session?

Perhaps you've defined a type and then realize you need to add a new field. If you try this at the REPL, you get the error:

```
ERROR: invalid redefinition of constant MyType
```

Types in module Main cannot be redefined.

While this can be inconvenient when you are developing new code, there's an excellent workaround. Modules can be replaced by redefining them, and so if you wrap all your new code inside a module you can redefine types and constants. You can't import the type names into Main and then expect to be able to redefine them there, but you can use the module name to resolve the scope. In other words, while developing you might use a workflow something like this:

```
include("mynewcode.jl")  # this defines a module MyModule
obj1 = MyModule.ObjConstructor(a, b)
obj2 = MyModule.somefunction(obj1)
# Got an error. Change something in "mynewcode.jl"
include("mynewcode.jl")  # reload the module
obj1 = MyModule.ObjConstructor(a, b) # old objects are no longer valid, must reconstruct
obj2 = MyModule.somefunction(obj1)  # this time it worked!
obj3 = MyModule.someotherfunction(obj2, c)
...
```

#### 32.2 Functions

# 32.2.1 I passed an argument x to a function, modified it inside that function, but on the outside, the variable x is still unchanged. Why?

Suppose you call a function like this:

In Julia, any function (including change\_value! ()) can't change the binding of a local variable. If x (in the calling scope) is bound to a immutable object (like a real number), you can't modify the object; likewise, if x is bound in the calling scope to a Dict, you can't change it to be bound to an ASCIIString.

But here is a thing you should pay attention to: suppose x is bound to an Array (or any other mutable type). You cannot "unbind" x from this Array. But, since an Array is a *mutable* type, you can change its content. For example:

Here we created a function change\_array! (), that assigns 5 to the first element of the Array. We passed x (which was previously bound to an Array) to the function. Notice that, after the function call, x is still bound to the same Array, but the content of that Array changed.

#### 32.2.2 Can I use using or import inside a function?

No, you are not allowed to have a using or import statement inside a function. If you want to import a module but only use its symbols inside a specific function or set of functions, you have two options:

1. Use import:

```
import Foo
function bar(...)
    ... refer to Foo symbols via Foo.baz ...
end
```

This loads the module Foo and defines a variable Foo that refers to the module, but does not import any of the other symbols from the module into the current namespace. You refer to the Foo symbols by their qualified names Foo.bar etc.

#### 2. Wrap your function in a module:

```
module Bar
export bar
using Foo
function bar(...)
    ... refer to Foo.baz as simply baz ....
end
end
using Bar
```

This imports all the symbols from Foo, but only inside the module Bar.

#### 32.2.3 What does the . . . operator do?

#### 32.2.4 The two uses of the . . . operator: slurping and splatting

Many newcomers to Julia find the use of . . . operator confusing. Part of what makes the . . . operator confusing is that it means two different things depending on context.

#### 32.2.5 . . . combines many arguments into one argument in function definitions

In the context of function definitions, the . . . operator is used to combine many different arguments into a single argument. This use of . . . for combining many different arguments into a single argument is called slurping:

If Julia were a language that made more liberal use of ASCII characters, the slurping operator might have been written as <-... instead of ....

#### 32.2.6 . . . splits one argument into many different arguments in function calls

In contrast to the use of the ... operator to denote slurping many different arguments into one argument when defining a function, the ... operator is also used to cause a single function argument to be split apart into many different arguments when used in the context of a function call. This use of ... is called splatting:

```
julia> function threeargs(a, b, c)
          @printf("a = %s::%s\n", a, typeof(a))
          @printf("b = %s::%s\n", b, typeof(b))
          @printf("c = %s::%s\n", c, typeof(c))
          end
threeargs (generic function with 1 method)
```

32.2. Functions 297

```
julia> vec = [1, 2, 3]
3-element Array{Int64,1}:
    1
    2
    3

julia> threeargs(vec...)
a = 1::Int64
b = 2::Int64
c = 3::Int64
```

If Julia were a language that made more liberal use of ASCII characters, the splatting operator might have been written as ...-> instead of ....

# 32.3 Types, type declarations, and constructors

#### 32.3.1 What does "type-stable" mean?

It means that the type of the output is predictable from the types of the inputs. In particular, it means that the type of the output cannot vary depending on the *values* of the inputs. The following code is *not* type-stable:

```
function unstable(flag::Bool)
   if flag
      return 1
   else
      return 1.0
   end
end
```

It returns either an Int or a Float 64 depending on the value of its argument. Since Julia can't predict the return type of this function at compile-time, any computation that uses it will have to guard against both types possibly occurring, making generation of fast machine code difficult.

# 32.3.2 Why does Julia give a DomainError for certain seemingly-sensible operations?

Certain operations make mathematical sense but result in errors:

```
julia> sqrt(-2.0)
ERROR: DomainError
in sqrt at math.jl:128

julia> 2^-5
ERROR: DomainError
in power_by_squaring at intfuncs.jl:70
in ^ at intfuncs.jl:84
```

This behavior is an inconvenient consequence of the requirement for type-stability. In the case of sqrt() (page 364), most users want sqrt(2.0) to give a real number, and would be unhappy if it produced the complex number 1.4142135623730951 + 0.0im. One could write the sqrt() (page 364) function to switch to a complex-valued output only when passed a negative number (which is what sqrt() (page 364) does in some other languages), but then the result would not be type-stable (page 296) and the sqrt() (page 364) function would have poor performance.

In these and other cases, you can get the result you want by choosing an *input type* that conveys your willingness to accept an *output type* in which the result can be represented:

```
julia> sqrt(-2.0+0im)
0.0 + 1.4142135623730951im

julia> 2.0^-5
0.03125
```

#### 32.3.3 Why does Julia use native machine integer arithmetic?

Julia uses machine arithmetic for integer computations. This means that the range of Int values is bounded and wraps around at either end so that adding, subtracting and multiplying integers can overflow or underflow, leading to some results that can be unsettling at first:

```
julia> typemax(Int)
9223372036854775807

julia> ans+1
-9223372036854775808

julia> -ans
-9223372036854775808

julia> 2*ans
0
```

Clearly, this is far from the way mathematical integers behave, and you might think it less than ideal for a high-level programming language to expose this to the user. For numerical work where efficiency and transparency are at a premium, however, the alternatives are worse.

One alternative to consider would be to check each integer operation for overflow and promote results to bigger integer types such as Int128 or BigInt (page 380) in the case of overflow. Unfortunately, this introduces major overhead on every integer operation (think incrementing a loop counter) – it requires emitting code to perform run-time overflow checks after arithmetic instructions and branches to handle potential overflows. Worse still, this would cause every computation involving integers to be type-unstable. As we mentioned above, type-stability is crucial (page 296) for effective generation of efficient code. If you can't count on the results of integer operations being integers, it's impossible to generate fast, simple code the way C and Fortran compilers do.

A variation on this approach, which avoids the appearance of type instability is to merge the Int and <code>BigInt</code> (page 380) types into a single hybrid integer type, that internally changes representation when a result no longer fits into the size of a machine integer. While this superficially avoids type-instability at the level of Julia code, it just sweeps the problem under the rug by foisting all of the same difficulties onto the C code implementing this hybrid integer type. This approach <code>can</code> be made to work and can even be made quite fast in many cases, but has several drawbacks. One problem is that the in-memory representation of integers and arrays of integers no longer match the natural representation used by C, Fortran and other languages with native machine integers. Thus, to interoperate with those languages, we would ultimately need to introduce native integer types anyway. Any unbounded representation of integers cannot have a fixed number of bits, and thus cannot be stored inline in an array with fixed-size slots – large integer values will always require separate heap-allocated storage. And of course, no matter how clever a hybrid integer implementation one uses, there are always performance traps – situations where performance degrades unexpectedly. Complex representation, lack of interoperability with C and Fortran, the inability to represent integer arrays without additional heap storage, and unpredictable performance characteristics make even the cleverest hybrid integer implementations a poor choice for high-performance numerical work.

An alternative to using hybrid integers or promoting to BigInts is to use saturating integer arithmetic, where adding to the largest integer value leaves it unchanged and likewise for subtracting from the smallest integer value. This is precisely what Matlab<sup>TM</sup> does:

```
>> int64(9223372036854775807)
ans =
    9223372036854775807
>> int64(9223372036854775807) + 1
ans =
    9223372036854775807
>> int64(-9223372036854775808)
ans =
    -9223372036854775808
>> int64(-9223372036854775808) - 1
ans =
    -9223372036854775808
```

At first blush, this seems reasonable enough since 9223372036854775807 is much closer to 9223372036854775808 than -9223372036854775808 is and integers are still represented with a fixed size in a natural way that is compatible with C and Fortran. Saturated integer arithmetic, however, is deeply problematic. The first and most obvious issue is that this is not the way machine integer arithmetic works, so implementing saturated operations requires emitting instructions after each machine integer operation to check for underflow or overflow and replace the result with typemin (Int) (page 322) or typemax (Int) (page 323) as appropriate. This alone expands each integer operation from a single, fast instruction into half a dozen instructions, probably including branches. Ouch. But it gets worse – saturating integer arithmetic isn't associative. Consider this Matlab computation:

```
>> n = int64(2)^62
4611686018427387904
>> n + (n - 1)
9223372036854775807
>> (n + n) - 1
9223372036854775806
```

This makes it hard to write many basic integer algorithms since a lot of common techniques depend on the fact that machine addition with overflow is associative. Consider finding the midpoint between integer values lo and hi in Julia using the expression (lo + hi) >>> 1:

```
julia> n = 2^62
4611686018427387904

julia> (n + 2n) >>> 1
6917529027641081856
```

See? No problem. That's the correct midpoint between  $2^62$  and  $2^63$ , despite the fact that n + 2n is -4611686018427387904. Now try it in Matlab:

```
>> (n + 2*n)/2
ans =
```

```
4611686018427387904
```

Oops. Adding a >>> operator to Matlab wouldn't help, because saturation that occurs when adding n and 2n has already destroyed the information necessary to compute the correct midpoint.

Not only is lack of associativity unfortunate for programmers who cannot rely it for techniques like this, but it also defeats almost anything compilers might want to do to optimize integer arithmetic. For example, since Julia integers use normal machine integer arithmetic, LLVM is free to aggressively optimize simple little functions like f(k) = 5k-1. The machine code for this function is just this:

```
julia> code_native(f,(Int,))
    .section __TEXT,__text,regular,pure_instructions
Filename: none
Source line: 1
    push    RBP
    mov RBP, RSP
Source line: 1
    lea RAX, QWORD PTR [RDI + 4*RDI - 1]
    pop RBP
    ret
```

The actual body of the function is a single lea instruction, which computes the integer multiply and add at once. This is even more beneficial when f gets inlined into another function:

```
julia> function g(k,n)
         for i = 1:n
           k = f(k)
         end
         return k
g (generic function with 2 methods)
julia> code_native(q,(Int,Int))
              ___TEXT,__text,regular,pure_instructions
    .section
Filename: none
Source line: 3
   push
   mov RBP, RSP
   test RSI, RSI
   jle 22
   mov EAX, 1
Source line: 3
   lea RDI, QWORD PTR [RDI + 4*RDI - 1]
   inc RAX
   cmp RAX, RSI
Source line: 2
   jle -17
Source line: 5
   mov RAX, RDI
   pop RBP
```

Since the call to f gets inlined, the loop body ends up being just a single lea instruction. Next, consider what happens if we make the number of loop iterations fixed:

```
return k
       end
g (generic function with 2 methods)
julia> code_native(g,(Int,))
               ___TEXT,__text,regular,pure_instructions
    .section
Filename: none
Source line: 3
          RBP
   push
   mov RBP, RSP
Source line: 3
           RAX, RDI, 9765625
   imul
   add RAX, -2441406
Source line: 5
   pop RBP
    ret
```

Because the compiler knows that integer addition and multiplication are associative and that multiplication distributes over addition – neither of which is true of saturating arithmetic – it can optimize the entire loop down to just a multiply and an add. Saturated arithmetic completely defeats this kind of optimization since associativity and distributivity can fail at each loop iteration, causing different outcomes depending on which iteration the failure occurs in. The compiler can unroll the loop, but it cannot algebraically reduce multiple operations into fewer equivalent operations.

The most reasonable alternative to having integer arithmetic silently overflow is to do checked arithmetic everywhere, raising errors when adds, subtracts, and multiplies overflow, producing values that are not value-correct. In this blog post, Dan Luu analyzes this and finds that rather than the trivial cost that this approach should in theory have, it ends up having a substantial cost due to compilers (LLVM and GCC) not gracefully optimizing around the added overflow checks. If this improves in the future, we could consider defaulting to checked integer arithmetic in Julia, but for now, we have to live with the possibility of overflow.

# 32.4 Packages and Modules

#### 32.4.1 What is the difference between "using" and "importall"?

There is only one difference, and on the surface (syntax-wise) it may seem very minor. The difference between using and importall is that with using you need to say function Foo.bar(... to extend module Foo's function bar with a new method, but with importall or import Foo.bar, you only need to say function bar(... and it automatically extends module Foo's function bar.

If you use importall, then function Foo.bar(... and function bar(... become equivalent. If you use using, then they are different.

The reason this is important enough to have been given separate syntax is that you don't want to accidentally extend a function that you didn't know existed, because that could easily cause a bug. This is most likely to happen with a method that takes a common type like a string or integer, because both you and the other module could define a method to handle such a common type. If you use importall, then you'll replace the other module's implementation of bar(s::AbstractString) with your new implementation, which could easily do something completely different (and break all/many future usages of the other functions in module Foo that depend on calling bar).

# 32.5 Nothingness and missing values

#### 32.5.1 How does "null" or "nothingness" work in Julia?

Unlike many languages (for example, C and Java), Julia does not have a "null" value. When a reference (variable, object field, or array element) is uninitialized, accessing it will immediately throw an error. This situation can be detected using the isdefined function.

Some functions are used only for their side effects, and do not need to return a value. In these cases, the convention is to return the value nothing, which is just a singleton object of type Void. This is an ordinary type with no fields; there is nothing special about it except for this convention, and that the REPL does not print anything for it. Some language constructs that would not otherwise have a value also yield nothing, for example if false; end.

For situations where a value exists only sometimes (for example, missing statistical data), it is best to use the Nullable {T} type, which allows specifying the type of a missing value.

The empty tuple (()) is another form of nothingness. But, it should not really be thought of as nothing but rather a tuple of zero values.

In code written for Julia prior to version 0.4 you may occasionally see None, which is quite different. It is the empty (or "bottom") type, a type with no values and no subtypes (except itself). This is now written as Union{} (an empty union type). You will generally not need to use this type.

# 32.6 Memory

#### 32.6.1 Why does x += y allocate memory when x and y are arrays?

In julia, x += y gets replaced during parsing by x = x + y. For arrays, this has the consequence that, rather than storing the result in the same location in memory as x, it allocates a new array to store the result.

While this behavior might surprise some, the choice is deliberate. The main reason is the presence of immutable objects within julia, which cannot change their value once created. Indeed, a number is an immutable object; the statements x = 5; x += 1 do not modify the meaning of 5, they modify the value bound to x. For an immutable, the only way to change the value is to reassign it.

To amplify a bit further, consider the following function:

```
function power_by_squaring(x, n::Int)
  ispow2(n) || error("This implementation only works for powers of 2")
  while n >= 2
        x *= x
        n >>= 1
  end
        x
end
```

After a call like x = 5;  $y = power_by_squaring(x, 4)$ , you would get the expected result: x == 5 && y == 625. However, now suppose that \*=, when used with matrices, instead mutated the left hand side. There would be two problems:

- For general square matrices, A = A\*B cannot be implemented without temporary storage: A[1,1] gets computed and stored on the left hand side before you're done using it on the right hand side.
- Suppose you were willing to allocate a temporary for the computation (which would eliminate most of the point of making \*= work in-place); if you took advantage of the mutability of x, then this function would behave

differently for mutable vs. immutable inputs. In particular, for immutable x, after the call you'd have (in general) y = x, but for mutable x you'd have y = x.

Because supporting generic programming is deemed more important than potential performance optimizations that can be achieved by other means (e.g., using explicit loops), operators like += and \*= work by rebinding new values.

# 32.7 Asynchronous IO and concurrent synchronous writes

#### 32.7.1 Why do concurrent writes to the same stream result in inter-mixed output?

While the streaming I/O API is synchronous, the underlying implementation is fully asynchronous.

The following:

```
@sync for i in 1:3
    @async write(STDOUT, string(i), " Foo ", " Bar ")
end
```

results in:

```
123 Foo Foo Bar Bar Bar
```

This is happening because, while the write call is synchronous, the writing of each argument yields to other tasks while waiting for that part of the I/O to complete.

print and println "lock" the stream during a call. Consequently changing write to println in the above example results in:

```
1 Foo Bar
2 Foo Bar
3 Foo Bar
```

You can lock your writes with a ReentrantLock like this:

```
l = ReentrantLock()
@sync for i in 1:3
    @async begin
    lock(1)
    try
        write(STDOUT, string(i), " Foo ", " Bar ")
    finally
        unlock(1)
    end
end
end
```

#### 32.8 Julia Releases

## 32.8.1 Do I want to use a release, beta, or nightly version of Julia?

You may prefer the release version of Julia if you are looking for a stable code base. Releases generally occur every 6 months, giving you a stable platform for writing code.

You may prefer the beta version of Julia if you don't mind being slightly behind the latest bugfixes and changes, but find the slightly faster rate of changes more appealing. Additionally, these binaries are tested before they are published to ensure they are fully functional.

You may prefer the nightly version of Julia if you want to take advantage of the latest updates to the language, and don't mind if the version available today occasionally doesn't actually work.

Finally, you may also consider building Julia from source for yourself. This option is mainly for those individuals who are comfortable at the command line, or interested in learning. If this describes you, you may also be interested in reading our guidelines for contributing.

Links to each of these download types can be found on the download page at http://julialang.org/downloads/. Note that not all versions of Julia are available for all platforms.

#### 32.8.2 When are deprecated functions removed?

Deprecated functions are removed after the subsequent release. For example, functions marked as deprecated in the 0.1 release will not be available starting with the 0.2 release.

32.8. Julia Releases 305

Julia Language Documentation, Release 0.5.0-dev						

# **Noteworthy Differences from other Languages**

# 33.1 Noteworthy differences from MATLAB

Although MATLAB users may find Julia's syntax familiar, Julia is not a MATLAB clone. There are major syntactic and functional differences. The following are some noteworthy differences that may trip up Julia users accustomed to MATLAB:

- Julia arrays are indexed with square brackets, A[i, j].
- Julia arrays are assigned by reference. After A=B, changing elements of B will modify A as well.
- Julia values are passed and assigned by reference. If a function modifies an array, the changes will be visible in the caller.
- Julia does not automatically grow arrays in an assignment statement. Whereas in MATLAB a (4) = 3.2 can create the array a = [0 0 0 3.2] and a (5) = 7 can grow it into a = [0 0 0 3.2 7], the corresponding Julia statement a [5] = 7 throws an error if the length of a is less than 5 or if this statement is the first use of the identifier a. Julia has push! () (page 346) and append! () (page 349), which grow Vectors much more efficiently than MATLAB's a (end+1) = val.
- The imaginary unit sqrt (-1) is represented in Julia as im (page 379), not i or j as in MATLAB.
- In Julia, literal numbers without a decimal point (such as 42) create integers instead of floating point numbers. Arbitrarily large integer literals are supported. As a result, some operations such as 2^-1 will throw a domain error as the result is not an integer (see *the FAQ entry on domain errors* (page 296) for details).
- In Julia, multiple values are returned and assigned as tuples, e.g. (a, b) = (1, 2) or a, b = 1, 2. MATLAB's nargout, which is often used in MATLAB to do optional work based on the number of returned values, does not exist in Julia. Instead, users can use optional and keyword arguments to achieve similar capabilities.
- Julia has true one-dimensional arrays. Column vectors are of size N, not Nx1. For example, rand (N) (page 383) makes a 1-dimensional array.
- In Julia v0.3, concatenating scalars and arrays with the syntax [x, y, z] concatenates in the first dimension ("vertically"). For concatenation in the second dimension ("horizontally"), use spaces as in [x y z]. To construct block matrices (concatenating in the first two dimensions), the syntax [a b; c d] is used to avoid confusion. In Julia v0.4, the concatenation syntax [x, [y, z]] is deprecated in favor of [x; [y, z]].
- In Julia, a:b and a:b:c construct Range objects. To construct a full vector like in MATLAB, use collect (a:b) (page 341). Generally, there is no need to call collect though. Range will act like a normal array in most cases but is more efficient because it lazily computes its values. This pattern of creating specialized objects instead of full arrays is used frequently, and is also seen in functions such as linspace

(page 396), or with iterators such as *enumerate* (page 335), and *zip* (page 335). The special objects can mostly be used as if they were normal arrays.

- Functions in Julia return values from their last expression or the return keyword instead of listing the names of variables to return in the function definition (see *The return Keyword* (page 56) for details).
- A Julia script may contain any number of functions, and all definitions will be externally visible when the file is loaded. Function definitions can be loaded from files outside the current working directory.
- In Julia, reductions such as <code>sum()</code> (page 339), <code>prod()</code> (page 340), and <code>max()</code> (page 362) are performed over every element of an array when called with a single argument, as in <code>sum(A)</code>, even if A has more than one dimension.
- In Julia, functions such as <code>sort()</code> (page 466) that operate column-wise by default (<code>sort(A)</code> is equivalent to <code>sort(A, 1)</code>) do not have special behavior for <code>1xN</code> arrays; the argument is returned unmodified since it still performs <code>sort(A, 1)</code>. To sort a <code>1xN</code> matrix like a vector, use <code>sort(A, 2)</code>.
- In Julia, if A is a 2-dimensional array, fft (A) computes a 2D FFT. In particular, it is not equivalent to fft (A, 1), which computes a 1D FFT acting column-wise.
- In Julia, parentheses must be used to call a function with zero arguments, like in tic() (page 328) and toc() (page 328).
- Julia discourages the used of semicolons to end statements. The results of statements are not automatically printed (except at the interactive prompt), and lines of code do not need to end with semicolons. println() (page 454) or @printf() (page 454) can be used to print specific output.
- In Julia, if A and B are arrays, logical comparison operations like A == B do not return an array of booleans. Instead, use A .== B, and similarly for the other boolean operators like < (page 356), > (page 356) and =.
- In Julia, the operators & (page 356), / (page 356), and \$ (page 356) perform the bitwise operations equivalent to and, or, and xor respectively in MATLAB, and have precedence similar to Python's bitwise operators (unlike C). They can operate on scalars or element-wise across arrays and can be used to combine logical arrays, but note the difference in order of operations: parentheses may be required (e.g., to select elements of A equal to 1 or 2 use (A .== 1) | (A .== 2)).
- In Julia, the elements of a collection can be passed as arguments to a function using the splat operator . . ., as in xs=[1,2]; f(xs...).
- Julia's svd () (page 421) returns singular values as a vector instead of as a dense diagonal matrix.
- In Julia, . . . is not used to continue lines of code. Instead, incomplete expressions automatically continue onto the next line.
- In both Julia and MATLAB, the variable ans is set to the value of the last expression issued in an interactive session. In Julia, unlike MATLAB, ans is not set when Julia code is run in non-interactive mode.
- Julia's types do not support dynamically adding fields at runtime, unlike MATLAB's classes. Instead, use a Dict (page 343).

# 33.2 Noteworthy differences from R

One of Julia's goals is to provide an effective language for data analysis and statistical programming. For users coming to Julia from R, these are some noteworthy differences:

- Julia's single quotes enclose characters, not strings.
- Julia can create substrings by indexing into strings. In R, strings must be converted into character vectors before creating substrings.

- In Julia, like Python but unlike R, strings can be created with triple quotes """ ... """. This syntax is convenient for constructing strings that contain line breaks.
- In Julia, varargs are specified using the splat operator . . . , which always follows the name of a specific variable, unlike R, for which . . . can occur in isolation.
- In Julia, modulus, is % (page 356), not %%.
- In Julia, not all data structures support logical indexing. Furthermore, logical indexing in Julia is supported only with vectors of length equal to the object being indexed. For example: In R, c(1, 2, 3, 4) [c(TRUE, FALSE)] is equivalent to c(1,3). In R, c(1, 2, 3, 4) [c(TRUE, FALSE, TRUE, FALSE)] is equivalent to c(1,3). In Julia, [1, 2, 3, 4] [[true, false]] throws a BoundsError (page 330). In Julia, [1, 2, 3, 4] [[true, false, true, false]] produces [1, 3].
- Like many languages, Julia does not always allow operations on vectors of different lengths, unlike R where the vectors only need to share a common index range. For example, c(1,2,3,4) + c(1,2) is valid R but the equivalent [1:4] + [1:2] will throw an error in Julia.
- Julia's apply () takes the function first, then its arguments, unlike lapply (<structure>, function, arg2, ...) in R.
- Julia uses end to denote the end of conditional blocks, like if, loop blocks, like while/for, and functions. In lieu of the one-line if (cond) statement, Julia allows statements of the form if cond; statement; end, cond && statement and !cond || statement. Assignment statements in the latter two syntaxes must be explicitly wrapped in parentheses, e.g. cond && (x = value).
- In Julia, <-, <<- and -> are not assignment operators.
- Julia's -> creates an anonymous function, like Python.
- Julia constructs vectors using brackets. Julia's [1, 2, 3] is the equivalent of R's c (1, 2, 3).
- Julia's \* (page 385) operator can perform matrix multiplication, unlike in R. If A and B are matrices, then A \* B denotes a matrix multiplication in Julia, equivalent to R's A % \* B. In R, this same notation would perform an element-wise (Hadamard) product. To get the element-wise multiplication operation, you need to write A . \* B in Julia.
- Julia performs matrix transposition using the ' operator and conjugated transposition using the ' operator. Julia's A.' is therefore equivalent to R's t (A).
- Julia does not require parentheses when writing if statements or for/while loops: use for i in [1, 2, 3] instead of for (i in c(1, 2, 3)) and if i == 1 instead of if (i == 1).
- Julia does not treat the numbers 0 and 1 as Booleans. You cannot write if (1) in Julia, because if statements accept only booleans. Instead, you can write if true, if Bool(1), or if 1==1.
- Julia does not provide nrow and ncol. Instead, use size (M, 1) for nrow (M) and size (M, 2) for ncol (M).
- Julia is careful to distinguish scalars, vectors and matrices. In R, 1 and c(1) are the same. In Julia, they can not be used interchangeably. One potentially confusing result of this is that x' \* y for vectors x and y is a 1-element vector, not a scalar. To get a scalar, use dot(x, y) (page 413).
- Julia's diag () (page 422) and diagm () (page 422) are not like R's.
- Julia cannot assign to the results of function calls on the left hand side of an assignment operation: you cannot write diag (M) = ones (n).
- Julia discourages populating the main namespace with functions. Most statistical functionality for Julia is found in packages under the JuliaStats organization. For example:
  - Functions pertaining to probability distributions are provided by the Distributions package.

- The DataFrames package provides data frames.
- Generalized linear models are provided by the GLM package.
- Julia provides tuples and real hash tables, but not R-style lists. When returning multiple items, you should typically use a tuple: instead of list (a = 1, b = 2), use (1, 2).
- Julia encourages users to write their own types, which are easier to use than S3 or S4 objects in R. Julia's multiple dispatch system means that table (x::TypeA) and table (x::TypeB) act like R's table.TypeA(x) and table.TypeB(x).
- In Julia, values are passed and assigned by reference. If a function modifies an array, the changes will be visible in the caller. This is very different from R and allows new functions to operate on large data structures much more efficiently.
- In Julia, vectors and matrices are concatenated using hcat () (page 397), vcat () (page 397) and hvcat () (page 397), not c, rbind and cbind like in R.
- In Julia, a range like a:b is not shorthand for a vector like in R, but is a specialized Range that is used for iteration without high memory overhead. To convert a range into a vector, use collect (a:b) (page 341).
- Julia's max() (page 362) and min() (page 362) are the equivalent of pmax and pmin respectively in R, but both arguments need to have the same dimensions. While maximum() (page 338) and minimum() (page 338) replace max and min in R, there are important differences.
- Julia's sum() (page 339), prod() (page 340), maximum() (page 338), and minimum() (page 338) are different from their counterparts in R. They all accept one or two arguments. The first argument is an iterable collection such as an array. If there is a second argument, then this argument indicates the dimensions, over which the operation is carried out. For instance, let  $A = [[1 \ 2], [3 \ 4]]$  in Julia and B = rbind(c(1,2), c(3,4)) be the same matrix in R. Then sum(A) gives the same result as sum(B), but sum(A, 1) is a row vector containing the sum over each column and sum(A, 2) is a column vector containing the sum over each row. This contrasts to the behavior of R, where sum(B, 1) = 11 and sum(B, 2) = 12. If the second argument is a vector, then it specifies all the dimensions over which the sum is performed, e.g., sum(A, [1, 2]) = 10. It should be noted that there is no error checking regarding the second argument.
- Julia has several functions that can mutate their arguments. For example, it has both *sort()* (page 466) and *sort()* (page 466).
- In R, performance requires vectorization. In Julia, almost the opposite is true: the best performing code is often achieved by using devectorized loops.
- Julia is eagerly evaluated and does not support R-style lazy evaluation. For most users, this means that there are very few unquoted expressions or column names.
- Julia does not support the NULL type.
- Julia lacks the equivalent of R's assign or get.
- In Julia, return does not require parentheses.

# 33.3 Noteworthy differences from Python

- In Julia, a vector of vectors can automatically concatenate into a one-dimensional vector *if* no explicit element type is specified. For example:
  - In Julia, [1, [2, 3]] concatenates into [1, 2, 3], like in R.
  - In Julia, Int[1, Int[2, 3]] will *not* concatenate, but instead throw an error.
  - In Julia, Any [1, [2, 3]] will not concatenate.

- In Julia, Vector{Int}[[1, 2], [3, 4]] will *not* concatenate, but produces an object similar to Python's list of lists. This object is *different* from a two-dimensional Array (page 395) of Ints.
- Julia requires end to end a block. Unlike Python, Julia has no pass keyword.
- In Julia, indexing of arrays, strings, etc. is 1-based not 0-based.
- Julia's slice indexing includes the last element, unlike in Python. a [2:3] in Julia is a [1:3] in Python.
- Julia does not support negative indexes. In particular, the last element of a list or array is indexed with end in Julia, not -1 as in Python.
- Julia's list comprehensions do not support the optional if clause that Python has.
- Julia's for, if, while, etc. blocks are terminated by the end keyword. Indentation level is not significant as it is in Python.
- Julia has no line continuation syntax: if, at the end of a line, the input so far is a complete expression, it is considered done; otherwise the input continues. One way to force an expression to continue is to wrap it in parentheses.
- Julia arrays are column major (Fortran ordered) whereas NumPy arrays are row major (C-ordered) by default. To get optimal performance when looping over arrays, the order of the loops should be reversed in Julia relative to NumPy (see relevant section of *Performance Tips* (page 265)).
- Julia's updating operators (e.g. +=, -=, ...) are *not in-place* whereas NumPy's are. This means A = ones (4); B = A; B += 3 doesn't change values in A, it rather rebinds the name B to the result of the right-hand side B = B + 3, which is a new array. Use B[:] += 3, explicit loops, or InplaceOps.jl.
- Julia evaluates default values of function arguments every time the method is invoked, unlike in Python where the default values are evaluated only once when the function is defined. For example, the function f (x=rand()) = x returns a new random number every time it is invoked without argument. On the other hand, the function g(x=[1,2]) = push! (x,3) returns [1,2,3] every time it is called as g().

# 33.4 Noteworthy differences from C/C++

- Julia arrays are indexed with square brackets, and can have more than one dimension A[i,j]. This syntax is not just syntactic sugar for a reference to a pointer or address as in C/C++. See the Julia documentation for the syntax for array construction (it has changed between versions).
- In Julia, indexing of arrays, strings, etc. is 1-based not 0-based.
- Julia arrays are assigned by reference. After A=B, changing elements of B will modify A as well. Updating operators like += do not operate in-place, they are equivalent to A = A + B which rebinds the left-hand side to the result of the right-hand side expression.
- Julia arrays are column major (Fortran ordered) whereas C/C++ arrays are row major ordered by default. To get optimal performance when looping over arrays, the order of the loops should be reversed in Julia relative to C/C++ (see relevant section of *Performance Tips* (page 265)).
- Julia values are passed and assigned by reference. If a function modifies an array, the changes will be visible in the caller.
- In Julia, whitespace is significant, unlike C/C++, so care must be taken when adding/removing whitespace from a Julia program.
- In Julia, literal numbers without a decimal point (such as 42) create signed integers, of type Int, but literals too large to fit in the machine word size will automatically be promoted to a larger size type, such as Int64 (if Int is Int32), Int128, or the arbitrarily large BigInt type. There are no numeric literal suffixes, such as L, LL, U, ULL to indicate unsigned and/or signed vs. unsigned. Decimal literals are always signed,

and hexadecimal literals (which start with 0x like C/C++), are unsigned. Hexadecimal literals also, unlike C/C++/Java and unlike decimal literals in Julia, have a type based on the *length* of the literal, including leading 0s. For example, 0x0 and 0x00 have type UInt8, 0x000 and 0x0000 have type UInt16, then literals with 5 to 8 hex digits have type UInt32, 9 to 16 hex digits type UInt64 and 17 to 32 hex digits type UInt128. This needs to be taken into account when defining hexadecimal masks, for example  $\sim 0xf = 0xf0$  is very different from  $\sim 0x000f = 0xfff0$ . 64 bit Float64 and 32 bit Float32 bit literals are expressed as 1.0 and 1.0f0 respectively. Floating point literals are rounded (and not promoted to the BigFloat type) if they can not be exactly represented. Floating point literals are closer in behavior to C/C++. Octal (prefixed with 0o) and binary (prefixed with 0b) literals are also treated as unsigned.

- String literals can be delimited with either " or """, """ delimited literals can contain " characters without quoting it like "\"" String literals can have values of other variables or expressions interpolated into them, indicated by \$variablename or \$(expression), which evaluates the variable name or the expression in the context of the function.
- // indicates a Rational number, and not a single-line comment (which is # in Julia)
- #= indicates the start of a multiline comment, and =# ends it.
- Functions in Julia return values from their last expression(s) or the return keyword. Multiple values can be returned from functions and assigned as tuples, e.g. (a, b) = myfunction() or a, b = myfunction(), instead of having to pass pointers to values as one would have to do in C/C++ (i.e. a = myfunction(&b).
- Julia does not require the use of semicolons to end statements. The results of expressions are not automatically printed (except at the interactive prompt, i.e. the REPL), and lines of code do not need to end with semicolons. <code>println()</code> (page 454) or <code>@printf()</code> (page 454) can be used to print specific output. In the REPL, ; can be used to suppress output. ; also has a different meaning within [ ], something to watch out for. ; can be used to separate expressions on a single line, but are not strictly necessary in many cases, and are more an aid to readability.
- In Julia, the operator \$\( \phi \) (page 356) performs the bitwise XOR operation, i.e. ^\( (page 385) \) in C/C++. Also, the bitwise operators do not have the same precedence as C/++, so parenthesis may be required.
- Julia's ^ (page 385) is exponentiation (pow), not bitwise XOR as in C/C++ (use \$ (page 356) in Julia)
- Julia has two right-shift operators, >> and >>>. >>> performs an arithmetic shift, >> always performs a logical shift, unlike C/C++, where the meaning of >> depends on the type of the value being shifted.
- Julia's -> creates an anonymous function, it does not access a member via a pointer.
- Julia does not require parentheses when writing if statements or for/while loops: use for i in [1, 2, 3] instead of for (int i=1; i <= 3; i++) and if i == 1 instead of if (i == 1).
- Julia does not treat the numbers 0 and 1 as Booleans. You cannot write if (1) in Julia, because if statements accept only booleans. Instead, you can write if true, if Bool (1), or if 1==1.
- Julia uses end to denote the end of conditional blocks, like if, loop blocks, like while/for, and functions. In lieu of the one-line if (cond) statement, Julia allows statements of the form if cond; statement; end, cond && statement and !cond || statement. Assignment statements in the latter two syntaxes must be explicitly wrapped in parentheses, e.g. cond && (x = value), because of the operator precedence.
- Julia has no line continuation syntax: if, at the end of a line, the input so far is a complete expression, it is considered done; otherwise the input continues. One way to force an expression to continue is to wrap it in parentheses.
- Julia macros operate on parsed expressions, rather than the text of the program, which allows them to perform sophisticated transformations of Julia code. Macro names start with the @ character, and have both a function-like syntax, @mymacro(arg1, arg2, arg3), and a statement-like syntax, @mymacro arg1 arg2 arg3. The forms are interchangable; the function-like form is particularly useful if the macro appears

within another expression, and is often clearest. The statement-like form is often used to annotate blocks, as in the parallel for construct: @parallel for i in 1:n; #= body =#; end. Where the end of the macro construct may be unclear, use the function-like form.

- Julia now has an enumeration type, expressed using the macro @enum(name, value1, value2, ...)
  For example: @enum(Fruit, Banana=1, Apple, Pear)
- By convention, functions that modify their arguments have a! at the end of the name, for example push!.
- In C++, by default, you have static dispatch, i.e. you need to annotate a function as virtual, in order to have dynamic dispatch. On the other hand, in Julia every method is "virtual" (although it's more general than that since methods are dispatched on every argument type, not only this, using the most-specific-declaration rule).



CHAPTER 3	34
-----------	----

	_
Unicode	Innut
Ollioudo	put

Please see the online documentation.

# Part II The Julia Standard Library

### **Essentials**

### 35.1 Introduction

The Julia standard library contains a range of functions and macros appropriate for performing scientific and numerical computing, but is also as broad as those of many general purpose programming languages. Additional functionality is available from a growing collection of available packages. Functions are grouped by topic below.

Some general notes:

- Except for functions in built-in modules (*Pkg* (page 469), *Collections* (page 350), *Test* (page 475) and *Profile*), all functions documented here are directly available for use in programs.
- To use module functions, use import Module to import the module, and Module.fn(x) to use the functions.
- Alternatively, using Module will import all exported Module functions into the current namespace.
- By convention, function names ending with an exclamation point (!) modify their arguments. Some functions have both modifying (e.g., sort!) and non-modifying (sort) versions.

# 35.2 Getting Around

### exit([code])

Quit (or control-D at the prompt). The default exit code is zero, indicating that the processes completed successfully.

### quit()

Quit the program indicating that the processes completed successfully. This function calls exit(0) (see exit() (page 317)).

### $\mathtt{atexit}(f)$

Register a zero-argument function f() to be called at process exit. atexit() hooks are called in last in first out (LIFO) order and run before object finalizers.

### atreplinit(f)

Register a one-argument function to be called before the REPL interface is initialized in interactive sessions; this is useful to customize the interface. The argument of f is the REPL object. This function should be called from within the <code>.juliarc.jl</code> initialization file.

### $\textbf{isinteractive}\,(\,)\,\to Bool$

Determine whether Julia is running an interactive session.

### whos ([io,] [Module,] [pattern::Regex])

Print information about exported global variables in a module, optionally restricted to those matching pattern.

The memory consumption estimate is an approximate lower bound on the size of the internal structure of the object.

### Base.summarysize(obj; $exclude=Union\{Module, Function, DataType, TypeName\}$ ) $\rightarrow$ Int

Compute the amount of memory used by all unique objects reachable from the argument. Keyword argument exclude specifies a type of objects to exclude from the traversal.

### edit (path::AbstractString[, line])

Edit a file or directory optionally providing a line number to edit the file at. Returns to the julia prompt when you quit the editor.

### edit (function[, types])

Edit the definition of a function, optionally specifying a tuple of types to indicate which method to edit.

### @edit()

Evaluates the arguments to the function call, determines their types, and calls the edit function on the resulting expression.

### less(file::AbstractString[, line])

Show a file using the default pager, optionally providing a starting line number. Returns to the julia prompt when you quit the pager.

### less(function[, types])

Show the definition of a function using the default pager, optionally specifying a tuple of types to indicate which method to see.

#### @less()

Evaluates the arguments to the function call, determines their types, and calls the less function on the resulting expression.

### ${\tt clipboard}\,(x)$

Send a printed form of x to the operating system clipboard ("copy").

### $clipboard() \rightarrow AbstractString$

Return a string with the contents of the operating system clipboard ("paste").

### reload(name::AbstractString)

Force reloading of a package, even if it has been loaded before. This is intended for use during package development as code is modified.

### require (module::Symbol)

This function is part of the implementation of using / import, if a module is not already defined in Main. It can also be called directly to force reloading a module, regardless of whether it has been loaded before (for example, when interactively developing libraries).

Loads a source files, in the context of the Main module, on every active node, searching standard locations for files. require is considered a top-level operation, so it sets the current include path but does not use it to search for files (see help for include). This function is typically used to load library code, and is implicitly called by using to load packages.

When searching for files, require first looks in the current working directory, then looks for package code under Pkq.dir(), then tries paths in the global array LOAD\_PATH.

### Base.compilecache(module::ByteString)

Creates a precompiled cache file for module (see help for require) and all of its dependencies. This can be used to reduce package load times. Cache files are stored in LOAD\_CACHE\_PATH[1], which defaults to ~/.julia/lib/VERSION. See *Module initialization and precompilation* (page 136) for important notes.

### \_\_precompile\_\_(isprecompilable::Bool=true)

Specify whether the file calling this function is precompilable. If isprecompilable is true, then \_\_precompile\_\_ throws an exception when the file is loaded by using/import/require unless the file is being precompiled, and in a module file it causes the module to be automatically precompiled when it is imported. Typically, \_\_precompile\_\_() should occur before the module declaration in the file, or better yet VERSION >= v"0.4" && \_\_precompile\_\_() in order to be backward-compatible with Julia 0.3.

If a module or file is *not* safely precompilable, it should call \_\_precompile\_\_(false) in order to throw an error if Julia attempts to precompile it.

\_\_precompile\_\_() should *not* be used in a module unless all of its dependencies are also using \_\_precompile\_\_(). Failure to do so can result in a runtime error when loading the module.

### include (path::AbstractString)

Evaluate the contents of a source file in the current context. During including, a task-local include path is set to the directory containing the file. Nested calls to include will search relative to that path. All paths refer to files on node 1 when running in parallel, and files will be fetched from node 1. This function is typically used to load source interactively, or to combine files in packages that are broken into multiple source files.

### include\_string(code::AbstractString[, filename])

Like include, except reads code from the given string rather than from a file. Since there is no file path involved, no path processing or fetching from node 1 is done.

### include\_dependency (path::AbstractString)

In a module, declare that the file specified by path (relative or absolute) is a dependency for precompilation; that is, the module will need to be recompiled if this file changes.

This is only needed if your module depends on a file that is not used via include. It has no effect outside of compilation.

### apropos (string)

Search through all documentation for a string, ignoring case.

### which (f, types)

Returns the method of f (a Method object) that would be called for arguments of the given types.

If types is an abstract type, then the method that would be called by invoke is returned.

### which (symbol)

Return the module in which the binding for the variable referenced by symbol was created.

### @which()

Applied to a function call, it evaluates the arguments to the specified function call, and returns the Method object for the method that would be called for those arguments. Applied to a variable, it returns the module in which the variable was bound. It calls out to the which function.

### methods(f, types)

Returns the method table for f.

If types is specified, returns an array of methods whose types match.

### methodswith (typ[, module or function][, showparents])

Return an array of methods with an argument of type typ. If optional showparents is true, also return arguments with a parent type of typ, excluding type Any.

The optional second argument restricts the search to a particular module or function.

### @show()

Show an expression and result, returning the result.

### versioninfo([verbose::Bool])

Print information about the version of Julia in use. If the verbose argument is true, detailed system information is shown as well.

#### workspace()

Replace the top-level module (Main) with a new one, providing a clean workspace. The previous Main module is made available as LastMain. A previously-loaded package can be accessed using a statement such as using LastMain.Package.

This function should only be used interactively.

#### ans

A variable referring to the last computed value, automatically set at the interactive prompt.

# 35.3 All Objects

```
is (x, y) \rightarrow \text{Bool}

=== (x, y) \rightarrow \text{Bool}

\equiv (x, y) \rightarrow \text{Bool}
```

Determine whether x and y are identical, in the sense that no program could distinguish them. Compares mutable objects by address in memory, and compares immutable objects (such as numbers) by contents at the bit level. This function is sometimes called egal.

#### **isa** $(x, type) \rightarrow Bool$

Determine whether x is of the given type.

### isequal(x, y)

Similar to ==, except treats all floating-point NaN values as equal to each other, and treats -0.0 as unequal to 0.0. The default implementation of isequal calls ==, so if you have a type that doesn't have these floating-point subtleties then you probably only need to define ==.

is equal is the comparison function used by hash tables (Dict). is equal (x, y) must imply that hash (x) == hash (y).

This typically means that if you define your own == function then you must define a corresponding hash (and vice versa). Collections typically implement isequal by calling isequal recursively on all contents.

Scalar types generally do not need to implement isequal separate from ==, unless they represent floating-point numbers amenable to a more efficient implementation than that provided as a generic fallback (based on isnan, signbit, and ==).

#### isless(x, y)

Test whether x is less than y, according to a canonical total order. Values that are normally unordered, such as NaN, are ordered in an arbitrary but consistent fashion. This is the default comparison used by sort. Non-numeric types with a canonical total order should implement this function. Numeric types only need to implement it if they have special values such as NaN.

#### ifelse(condition::Bool, x, y)

Return x if condition is true, otherwise return y. This differs from ? or if in that it is an ordinary function, so all the arguments are evaluated first. In some cases, using ifelse instead of an if statement can eliminate the branch in generated code and provide higher performance in tight loops.

### lexcmp(x, y)

Compare x and y lexicographically and return -1, 0, or 1 depending on whether x is less than, equal to, or greater than y, respectively. This function should be defined for lexicographically comparable types, and lexless will call lexcmp by default.

### lexless(x, y)

Determine whether x is lexicographically less than y.

### typeof(x)

Get the concrete type of x.

### **tuple** (*xs...*)

Construct a tuple of the given objects.

### ntuple (f::Function, n)

Create a tuple of length n, computing each element as f(i), where i is the index of the element.

### $object_id(x)$

Get a unique integer id for x. object\_id(x) == object\_id(y) if and only if is (x, y).

### $\mathbf{hash}(x[,h::UInt])$

Compute an integer hash code such that isequal(x, y) implies hash(x) == hash(y). The optional second argument h is a hash code to be mixed with the result.

New types should implement the 2-argument form, typically by calling the 2-argument hash method recursively in order to mix hashes of the contents with each other (and with h). Typically, any type that implements hash should also implement its own == (hence isequal) to guarantee the property mentioned above.

### finalizer(x, function)

Register a function f(x) to be called when there are no program-accessible references to x. The behavior of this function is unpredictable if x is of a bits type.

#### finalize(x)

Immediately run finalizers registered for object x.

### copy(x)

Create a shallow copy of x: the outer structure is copied, but not all internal values. For example, copying an array produces a new array with identically-same elements as the original.

### deepcopy(x)

Create a deep copy of x: everything is copied recursively, resulting in a fully independent object. For example, deep-copying an array produces a new array whose elements are deep copies of the original elements. Calling deepcopy on an object should generally have the same effect as serializing and then deserializing it.

As a special case, functions can only be actually deep-copied if they are anonymous, otherwise they are just copied. The difference is only relevant in the case of closures, i.e. functions which may contain hidden internal references.

While it isn't normally necessary, user-defined types can override the default deepcopy behavior by defining a specialized version of the function deepcopy\_internal(x::T, dict::ObjectIdDict) (which shouldn't otherwise be used), where T is the type to be specialized for, and dict keeps track of objects copied so far within the recursion. Within the definition, deepcopy\_internal should be used in place of deepcopy, and the dict variable should be updated as appropriate before returning.

### isdefined([object], index | symbol)

Tests whether an assignable location is defined. The arguments can be an array and index, a composite object and field name (as a symbol), or a module and a symbol. With a single symbol argument, tests whether a global variable with that name is defined in current\_module().

#### convert(T, x)

Convert x to a value of type T.

If T is an Integer type, an InexactError (page 330) will be raised if x is not representable by T, for example if x is not integer-valued, or is outside the range supported by T.

35.3. All Objects 323

```
julia> convert(Int, 3.0)
3

julia> convert(Int, 3.5)
ERROR: InexactError()
in convert at int.jl:209
```

If T is a AbstractFloat or Rational type, then it will return the closest value to x representable by T.

```
julia> x = 1/3
0.333333333333333333

julia> convert(Float32, x)
0.333333334f0

julia> convert(Rational{Int32}, x)
1//3

julia> convert(Rational{Int64}, x)
6004799503160661//18014398509481984
```

### promote (xs...)

Convert all arguments to their common promotion type (if any), and return them all (as a tuple).

### oftype (x, y)

Convert y to the type of x (convert (typeof (x), y)).

### widen $(type \mid x)$

If the argument is a type, return a "larger" type (for numeric types, this will be a type with at least as much range and precision as the argument, and usually more). Otherwise the argument x is converted to widen (typeof(x)).

```
julia> widen(Int32)
Int64

julia> widen(1.5f0)
1.5
```

### identity(x)

The identity function. Returns its argument.

# **35.4 Types**

### supertype (T::DataType)

Return the supertype of DataType T.

### issubtype (type1, type2)

Return true if and only if all values of type1 are also of type2. Can also be written using the <: infix operator as type1 <: type2.

### <: (*T1*, *T2*)

Subtype operator, equivalent to issubtype (T1, T2).

### subtypes (T::DataType)

Return a list of immediate subtypes of DataType T. Note that all currently loaded subtypes are included, including those not visible in the current module.

### typemin(T)

The lowest value representable by the given (real) numeric DataType T.

### typemax(T)

The highest value representable by the given (real) numeric DataType.

### realmin(T)

The smallest in absolute value non-subnormal value representable by the given floating-point DataType T.

#### realmax (T

The highest finite value representable by the given floating-point DataType T.

### maxintfloat(T)

The largest integer losslessly representable by the given floating-point DataType T.

#### sizeof(T)

Size, in bytes, of the canonical binary representation of the given DataType T, if any.

### eps(T)

The distance between 1.0 and the next larger representable floating-point value of DataType T. Only floating-point types are sensible arguments.

#### eps()

The distance between 1.0 and the next larger representable floating-point value of Float 64.

#### eps(x)

The distance between x and the next larger representable floating-point value of the same DataType as x.

### promote\_type (type1, type2)

Determine a type big enough to hold values of each argument type without loss, whenever possible. In some cases, where no type exists to which both types can be promoted losslessly, some loss is tolerated; for example, promote\_type (Int64,Float64) returns Float64 even though strictly, not all Int64 values can be represented exactly as Float64 values.

### promote\_rule (type1, type2)

Specifies what type should be used by promote when given values of types type1 and type2. This function should not be called directly, but should have definitions added to it for new types as appropriate.

### getfield(value, name::Symbol)

Extract a named field from a value of composite type. The syntax a.b calls getfield(a, :b), and the syntax a.(b) calls getfield(a, b).

### setfield! (value, name::Symbol, x)

Assign x to a named field in value of composite type. The syntax a.b = c calls setfield! (a, :b, c), and the syntax a.(b) = c calls setfield! (a, b, c).

### fieldoffset (type, i)

The byte offset of field i of a type relative to the data start. For example, we could use it in the following manner to summarize information about a struct type:

```
julia> structinfo(T) = [(fieldoffset(T,i), fieldname(T,i), fieldtype(T,i)) for i =

julia> structinfo(StatStruct)

12-element Array{Tuple{Int64, Symbol, DataType}, 1}:

(0,:device, UInt64)
(8,:inode, UInt64)
(16,:mode, UInt64)
(24,:nlink, Int64)
(32,:uid, UInt64)
(40,:gid, UInt64)
(48,:rdev, UInt64)
(56,:size, Int64)
```

35.4. Types 325

```
(64,:blksize,Int64)
(72,:blocks,Int64)
(80,:mtime,Float64)
(88,:ctime,Float64)
```

### fieldtype (T, name::Symbol | index::Int)

Determine the declared type of a field (specified by name or index) in a composite DataType T.

#### isimmutable(v)

Return true iff value v is immutable. See *Immutable Composite Types* (page 92) for a discussion of immutability. Note that this function works on values, so if you give it a type, it will tell you that a value of DataType is mutable.

### isbits(T)

Return true if T is a "plain data" type, meaning it is immutable and contains no references to other values. Typical examples are numeric types such as UInt8, Float64, and Complex  $\{Float64\}$ .

```
julia> isbits(Complex{Float64})
  true

julia> isbits(Complex)
  false
```

### isleaftype(T)

Determine whether T is a concrete type that can have instances, meaning its only subtypes are itself and None (but T itself is not None).

### typejoin(T, S)

Compute a type that contains both  ${\mathbb T}$  and  ${\mathbb S}$ .

### typeintersect(T, S)

Compute a type that contains the intersection of  $\mathbb{T}$  and  $\mathbb{S}$ . Usually this will be the smallest such type or one close to it.

### **Val{c}**()

Create a "value type" out of c, which must be an isbits value. The intent of this construct is to be able to dispatch on constants, e.g.,  $f(Val\{false\})$  allows you to dispatch directly (at compile-time) to an implementation  $f(::Type\{Val\{false\}\})$ , without having to test the boolean value at runtime.

### @enum EnumName EnumValue1[=x] EnumValue2[=y]

Create an Enum type with name EnumName and enum member values of EnumValue1 and EnumValue2 with optional assigned values of x and y, respectively. EnumName can be used just like other types and enum member values as regular values, such as

```
julia> @enum FRUIT apple=1 orange=2 kiwi=3

julia> f(x::FRUIT) = "I'm a FRUIT with value: $(Int(x))"
   f (generic function with 1 method)

julia> f(apple)
"I'm a FRUIT with value: 1"
```

### instances(T::Type)

Return a collection of all instances of the given type, if applicable. Mostly used for enumerated types (see @enum).

### 35.5 Generic Functions

### $method_exists(f, Tuple\ type) \rightarrow Bool$

Determine whether the given generic function has a method matching the given Tuple of argument types.

```
julia> method_exists(length, Tuple{Array})
true
```

### **applicable** $(f, args...) \rightarrow Bool$

Determine whether the given generic function has a method applicable to the given arguments.

### **invoke** (*f*, (*types*...), *args*...)

Invoke a method for the given generic function matching the specified types (as a tuple), on the specified arguments. The arguments must be compatible with the specified types. This allows invoking a method other than the most specific matching method, which is useful when the behavior of a more general definition is explicitly needed (often as part of the implementation of a more specific method of the same function).

### | > (x, f)

Applies a function to the preceding argument. This allows for easy function chaining.

```
julia> [1:5;] |> x->x.^2 |> sum |> inv
0.018181818181818
```

### **call** (*x*, *args*...)

If x is not a Function, then x (args...) is equivalent to call (x, args...). This means that function-like behavior can be added to any type by defining new call methods.

# 35.6 Syntax

### eval([m::Module], expr::Expr)

Evaluate an expression in the given module and return the result. Every Module (except those defined with baremodule) has its own 1-argument definition of eval, which evaluates expressions in that module.

### @eval()

Evaluate an expression and return the value.

### evalfile (path::AbstractString)

Load the file using include, evaluate all expressions, and return the value of the last one.

```
esc (e::ANY)
```

Only valid in the context of an Expr returned from a macro. Prevents the macro hygiene pass from turning embedded variables into gensym variables. See the *Macros* (page 146) section of the Metaprogramming chapter of the manual for more details and examples.

### gensym([tag])

Generates a symbol which will not conflict with other variable names.

### @gensym()

Generates a gensym symbol for a variable. For example, gensym x y is transformed into x = gensym("x"); y = gensym("y").

### parse (str, start; greedy=true, raise=true)

Parse the expression string and return an expression (which could later be passed to eval for execution). start is the index of the first character to start parsing. If greedy is true (default), parse will try to consume as much input as it can; otherwise, it will stop as soon as it has parsed a valid expression. Incomplete but otherwise syntactically valid expressions will return Expr(:incomplete, "(error message)"). If raise is true (default), syntax errors other than incomplete expressions will raise an error. If raise is false, parse will return an expression that will raise an error upon evaluation.

#### parse (str; raise=true)

Parse the expression string greedily, returning a single expression. An error is thrown if there are additional characters after the first expression. If raise is true (default), syntax errors will raise an error; otherwise, parse will return an expression that will raise an error upon evaluation.

### 35.7 Nullables

### Nullable (x)

Wrap value x in an object of type Nullable, which indicates whether a value is present. Nullable (x) yields a non-empty wrapper, and Nullable  $\{T\}$  () yields an empty instance of a wrapper that might contain a value of type T.

#### qet(x)

Attempt to access the value of the Nullable object, x. Returns the value if it is present; otherwise, throws a NullException.

#### qet(x, y)

Attempt to access the value of the  $Nullable\{T\}$  object, x. Returns the value if it is present; otherwise, returns convert (T, y).

### isnull(x)

Is the Nullable object x null, i.e. missing a value?

# 35.8 System

#### run (command)

Run a command object, constructed with backticks. Throws an error if anything goes wrong, including the process exiting with a non-zero status.

### spawn (command)

Run a command object asynchronously, returning the resulting Process object.

### DevNull

Used in a stream redirect to discard all data written to it. Essentially equivalent to /dev/null on Unix or NUL on Windows. Usage: run('cat test.txt' |> DevNull)

### success (command)

Run a command object, constructed with backticks, and tell whether it was successful (exited with a code of 0). An exception is raised if the process cannot be started.

### process\_running(p::Process)

Determine whether a process is currently running.

#### process exited(p::Process)

Determine whether a process has exited.

### **kill** (p::Process, signum=SIGTERM)

Send a signal to a process. The default is to terminate the process.

### Sys.set process title (title::AbstractString)

Set the process title. No-op on some operating systems. (not exported)

### Sys.get process title()

Get the process title. On some systems, will always return empty string. (not exported)

#### readandwrite(command)

Starts running a command asynchronously, and returns a tuple (stdout,stdin,process) of the output stream and input stream of the process, and the process object itself.

#### ignorestatus(command)

Mark a command object so that running it will not throw an error if the result code is non-zero.

### detach (command)

Mark a command object so that it will be run in a new process group, allowing it to outlive the julia process, and not have Ctrl-C interrupts passed to it.

# Cmd(cmd::Cmd; ignorestatus, detach, windows\_verbatim, windows\_hide, env, dir)

Construct a new Cmd object, representing an external program and arguments, from cmd, while changing the settings of the optional keyword arguments:

- •ignorestatus::Bool: If true (defaults to false), then the Cmd will not throw an error if the return code is nonzero.
- •detach::Bool: If true (defaults to false), then the Cmd will be run in a new process group, allowing it to outlive the julia process and not have Ctrl-C passed to it.
- •windows\_verbatim::Bool: If true (defaults to false), then on Windows the Cmd will send a command-line string to the process with no quoting or escaping of arguments, even arguments containing spaces. (On Windows, arguments are sent to a program as a single "command-line" string, and programs are responsible for parsing it into arguments. By default, empty arguments and arguments with spaces or tabs are quoted with double quotes " in the command line, and \ or " are preceded by backslashes. windows\_verbatim=true is useful for launching programs that parse their command line in non-standard ways.) Has no effect on non-Windows systems.
- •windows\_hide::Bool: If true (defaults to false), then on Windows no new console window is displayed when the Cmd is executed. This has no effect if a console is already open or on non-Windows systems.
- •env: Set environment variables to use when running the Cmd. env is either a dictionary mapping strings to strings, an array of strings of the form "var=val", an array or tuple of "var"=>val pairs, or nothing. In order to modify (rather than replace) the existing environment, create env by copy (ENV) and then set env ["var"]=val as desired.
- •dir::AbstractString: Specify a working directory for the command (instead of the current directory).

For any keywords that are not specified, the current settings from cmd are used. Normally, to create a Cmd object in the first place, one uses backticks, e.g.

```
Cmd(`echo "Hello world"`, ignorestatus=true, detach=false)
```

### setenv (command, env; dir=working\_dir)

Set environment variables to use when running the given command. env is either a dictionary mapping strings

35.8. System 329

to strings, an array of strings of the form "var=val", or zero or more "var"=>val pair arguments. In order to modify (rather than replace) the existing environment, create env by copy (ENV) and then setting env ["var"]=val as desired, or use withenv.

The dir keyword argument can be used to specify a working directory for the command.

### withenv (f::Function, kv::Pair...)

Execute f () in an environment that is temporarily modified (not replaced as in setenv) by zero or more "var"=>val arguments kv. withenv is generally used via the withenv(kv...) do ... end syntax. A value of nothing can be used to temporarily unset an environment variable (if it is set). When withenv returns, the original environment has been restored.

### pipeline (from, to, ...)

Create a pipeline from a data source to a destination. The source and destination can be commands, I/O streams, strings, or results of other pipeline calls. At least one argument must be a command. Strings refer to filenames. When called with more than two arguments, they are chained together from left to right. For example pipeline (a, b, c) is equivalent to pipeline (pipeline (a, b), c). This provides a more concise way to specify multi-stage pipelines.

### **Examples:**

```
run(pipeline(`ls`, `grep xyz`))
run(pipeline(`ls`, "out.txt"))
run(pipeline("out.txt", `grep xyz`))
```

### pipeline (command; stdin, stdout, stderr, append=false)

Redirect I/O to or from the given command. Keyword arguments specify which of the command's streams should be redirected. append controls whether file output appends to the file. This is a more general version of the 2-argument pipeline function. pipeline (from, to) is equivalent to pipeline (from, stdout=to) when from is a command, and to pipe (to, stdin=from) when from is another kind of data source.

### **Examples**:

```
run(pipeline(`dothings`, stdout="out.txt", stderr="errs.txt"))
run(pipeline(`update`, stdout="log.txt", append=true))
```

### **gethostname**() → AbstractString

Get the local machine's host name.

### $\texttt{getipaddr}() \rightarrow IPAddr$

Get the IP address of the local machine.

### $getpid() \rightarrow Int32$

Get julia's process ID.

### time()

Get the system time in seconds since the epoch, with fairly high (typically, microsecond) resolution.

### time\_ns()

Get the time in nanoseconds. The time corresponding to 0 is undefined, and wraps every 5.8 years.

### tic()

Set a timer to be read by the next call to toc() (page 328) or toq() (page 328). The macro call @time expr can also be used to time evaluation.

#### toc()

Print and return the time elapsed since the last tic() (page 328).

### toq()

Return, but do not print, the time elapsed since the last tic() (page 328).

#### @time()

A macro to execute an expression, printing the time it took to execute, the number of allocations, and the total number of bytes its execution caused to be allocated, before returning the value of the expression.

#### @timev()

This is a verbose version of the <code>@time</code> macro. It first prints the same information as <code>@time</code>, then any non-zero memory allocation counters, and then returns the value of the expression.

#### @timed()

A macro to execute an expression, and return the value of the expression, elapsed time, total bytes allocated, garbage collection time, and an object with various memory allocation counters.

#### @elapsed()

A macro to evaluate an expression, discarding the resulting value, instead returning the number of seconds it took to execute as a floating-point number.

### @allocated()

A macro to evaluate an expression, discarding the resulting value, instead returning the total number of bytes allocated during evaluation of the expression. Note: the expression is evaluated inside a local function, instead of the current context, in order to eliminate the effects of compilation, however, there still may be some allocations due to JIT compilation. This also makes the results inconsistent with the <code>@time</code> macros, which do not try to adjust for the effects of compilation.

### **EnvHash**() $\rightarrow$ EnvHash

A singleton of this type provides a hash table interface to environment variables.

#### **ENV**

Reference to the singleton EnvHash, providing a dictionary interface to system environment variables.

### @unix()

Given @unix? a: b, do a on Unix systems (including Linux and OS X) and b elsewhere. See documentation for Handling Platform Variations in the Calling C and Fortran Code section of the manual.

### @osx()

Given @osx? a: b, do a on OS X and b elsewhere. See documentation for Handling Platform Variations in the Calling C and Fortran Code section of the manual.

### @linux()

Given @linux? a : b, do a on Linux and b elsewhere. See documentation for Handling Platform Variations in the Calling C and Fortran Code section of the manual.

### @windows()

Given @windows? a: b, do a on Windows and b elsewhere. See documentation for Handling Platform Variations in the Calling C and Fortran Code section of the manual.

### 35.9 Errors

### error (message::AbstractString)

Raise an ErrorException with the given message.

### throw(e)

Throw an object as an exception.

### rethrow([e])

Throw an object without changing the current exception backtrace. The default argument is the current exception (if called within a catch block).

### backtrace()

Get a backtrace object for the current program point.

35.9. Errors 331

#### catch backtrace()

Get the backtrace of the current exception, for use within catch blocks.

### assert (cond)

Throw an AssertionError if cond is false. Also available as the macro @assert expr.

### @assert cond [text]

Throw an AssertionError if cond is false. Preferred syntax for writing assertions. Message text is optionally displayed upon assertion failure.

### ArgumentError (msg)

The parameters to a function call do not match a valid signature. Argument msq is a descriptive error string.

### AssertionError ([msg])

The asserted condition did not evaluate to true. Optional argument msq is a descriptive error string.

### BoundsError ([a][,i])

An indexing operation into an array, a, tried to access an out-of-bounds element, i.

### DimensionMismatch ([msg])

The objects called do not have matching dimensionality. Optional argument msq is a descriptive error string.

#### DivideError()

Integer division was attempted with a denominator value of 0.

### DomainError()

The arguments to a function or constructor are outside the valid domain.

#### EOFError()

No more data was available to read from a file or stream.

### ErrorException (msg)

Generic error type. The error message, in the .msg field, may provide more specific details.

### InexactError()

Type conversion cannot be done exactly.

### InterruptException()

The process was stopped by a terminal interrupt (CTRL+C).

### KeyError (key)

An indexing operation into an Associative (Dict) or Set like object tried to access or delete a non-existent element.

### LoadError (file::AbstractString, line::Int, error)

An error occurred while includeing, requireing, or using a file. The error specifics should be available in the .error field.

### MethodError (f, args)

A method with the required type signature does not exist in the given generic function.

### NullException()

An attempted access to a Nullable with no defined value.

### OutOfMemoryError()

An operation allocated too much memory for either the system or the garbage collector to handle properly.

### ReadOnlyMemoryError()

An operation tried to write to memory that is read-only.

### OverflowError()

The result of an expression is too large for the specified type and will cause a wraparound.

### ParseError (msg)

The expression passed to the parse function could not be interpreted as a valid Julia expression.

### ProcessExitedException()

After a client Julia process has exited, further attempts to reference the dead child will throw this exception.

### StackOverflowError()

The function call grew beyond the size of the call stack. This usually happens when a call recurses infinitely.

```
SystemError (prefix::AbstractString[, errno::Int32])
```

A system call failed with an error code (in the errno global variable).

### **TypeError** (func::Symbol, context::AbstractString, expected::Type, got)

A type assertion failure, or calling an intrinsic function with an incorrect argument type.

#### UndefRefError()

The item or field is not defined for the given object.

### UndefVarError (var::Symbol)

A symbol in the current scope is not defined.

### InitError (mod::Symbol, error)

An error occurred when running a module's \_\_init\_\_ function. The actual error thrown is available in the .error field.

### **35.10 Events**

### Timer (callback::Function, delay, repeat=0)

Create a timer to call the given callback function. The callback is passed one argument, the timer object itself. The callback will be invoked after the specified initial delay, and then repeating with the given repeat interval. If repeat is 0, the timer is only triggered once. Times are in seconds. A timer is stopped and has its resources freed by calling close on it.

### **Timer** (*delay*, *repeat=0*)

Create a timer that wakes up tasks waiting for it (by calling wait on the timer object) at a specified interval. Times are in seconds. Waiting tasks are woken with an error when the timer is closed (by close). Use isopen to check whether a timer is still active.

### 35.11 Reflection

### $module\_name(m::Module) \rightarrow Symbol$

Get the name of a Module as a Symbol.

### ${\tt module\_parent}\;(m{::}Module)\;\to {\rm Module}$

Get a module's enclosing Module. Main is its own parent, as is LastMain after workspace().

### $current\_module() \rightarrow Module$

Get the *dynamically* current Module, which is the Module code is currently being read from. In general, this is not the same as the module containing the call to this function.

### fullname (m::Module)

Get the fully-qualified name of a module as a tuple of symbols. For example, fullname (Base.Pkg) gives (:Base,:Pkg), and fullname (Main) gives ().

### names (x::Module[, all=false[, imported=false]])

Get an array of the names exported by a Module, with optionally more Module globals according to the additional parameters.

35.10. Events 333

#### **nfields** (x::DataType) $\rightarrow$ Int

Get the number of fields of a DataType.

### fieldnames (x::DataType)

Get an array of the fields of a DataType.

### fieldname (x::DataType, i)

Get the name of field i of a DataType.

### $isconst([m::Module], s::Symbol) \rightarrow Bool$

Determine whether a global is declared const in a given Module. The default Module argument is current\_module().

### $isgeneric (f::Function) \rightarrow Bool$

Determine whether a Function is generic.

### **function\_name** (f::Function) $\rightarrow$ Symbol

Get the name of a generic Function as a symbol, or : anonymous.

### **function\_module** (*f::Function*, *types*) → Module

Determine the module containing a given definition of a generic function.

### functionloc (f::Function, types)

Returns a tuple (filename, line) giving the location of a generic Function definition.

### functionloc(m::Method)

Returns a tuple (filename, line) giving the location of a Method definition.

### 35.12 Internals

#### qc()

Perform garbage collection. This should not generally be used.

### gc\_enable (on::Bool)

Control whether garbage collection is enabled using a boolean argument (true for enabled, false for disabled). Returns previous GC state. Disabling garbage collection should be used only with extreme caution, as it can cause memory use to grow without bound.

### macroexpand(x)

Takes the expression x and returns an equivalent expression with all macros removed (expanded).

#### expand(x)

Takes the expression x and returns an equivalent expression in lowered form.

### ${\tt code\_lowered}\,(f,\,types)$

Returns an array of lowered ASTs for the methods matching the given generic function and type signature.

### @code lowered()

Evaluates the arguments to the function call, determines their types, and calls <code>code\_lowered()</code> (page 332) on the resulting expression.

### code\_typed (f, types; optimize=true)

Returns an array of lowered and type-inferred ASTs for the methods matching the given generic function and type signature. The keyword argument optimize controls whether additional optimizations, such as inlining, are also applied.

### @code\_typed()

Evaluates the arguments to the function call, determines their types, and calls <code>code\_typed()</code> (page 332) on the resulting expression.

### code\_warntype (f, types)

Displays lowered and type-inferred ASTs for the methods matching the given generic function and type signature. The ASTs are annotated in such a way as to cause "non-leaf" types to be emphasized (if color is available, displayed in red). This serves as a warning of potential type instability. Not all non-leaf types are particularly problematic for performance, so the results need to be used judiciously. See @code\_warntype (page 281) for more information.

### @code\_warntype()

Evaluates the arguments to the function call, determines their types, and calls <code>code\_warntype()</code> (page 332) on the resulting expression.

### $code_1lvm(f, types)$

Prints the LLVM bitcodes generated for running the method matching the given generic function and type signature to STDOUT (page 449).

All metadata and dbg.\* calls are removed from the printed bitcode. Use code\_llvm\_raw for the full IR.

### @code\_llvm()

Evaluates the arguments to the function call, determines their types, and calls <code>code\_llvm()</code> (page 333) on the resulting expression.

### code\_native (f, types)

Prints the native assembly instructions generated for running the method matching the given generic function and type signature to STDOUT.

### @code\_native()

Evaluates the arguments to the function call, determines their types, and calls <code>code\_native()</code> (page 333) on the resulting expression.

### precompile (f, args::Tuple{Vararg{Any}})

Compile the given function f for the argument tuple (of types) args, but do not execute it.

35.12. Internals 335

### **Collections and Data Structures**

### 36.1 Iteration

Sequential iteration is implemented by the methods start() (page 335), done() (page 335), and next() (page 335). The general for loop:

is translated into:

```
state = start(I)
while !done(I, state)
    (i, state) = next(I, state)
    # body
end
```

The state object may be anything, and should be chosen appropriately for each iterable type. See the manual section on the iteration interface for more details about defining a custom iterable type.

```
start(iter) \rightarrow state
```

Get initial iteration state for an iterable object.

```
done (iter, state) \rightarrow Bool
```

Test whether we are done iterating.

```
next (iter, state) \rightarrow item, state
```

For a given iterable object and iteration state, return the current item and the next iteration state.

```
zip (iters...)
```

For a set of iterable objects, returns an iterable of tuples, where the ith tuple contains the ith component of each input iterable.

```
Note that zip() (page 335) is its own inverse: collect (zip(zip(a...)...)) == collect(a).
```

### enumerate(iter)

An iterator that yields (i, x) where i is an index starting at 1, and x is the ith value from the given iterator. It's useful when you need not only the values x over which you are iterating, but also the index i of the iterations.

```
1 a
2 b
3 c
```

### rest (iter, state)

An iterator that yields the same elements as iter, but starting at the given state.

### countfrom (start=1, step=1)

An iterator that counts forever, starting at start and incrementing by step.

#### take(iter, n)

An iterator that generates at most the first n elements of iter.

### drop(iter, n)

An iterator that generates all but the first n elements of iter.

### cycle (iter)

An iterator that cycles through iter forever.

### repeated (x[, n::Int])

An iterator that generates the value x forever. If n is specified, generates x that many times (equivalent to take (repeated (x), n)).

### Fully implemented by:

- Range
- UnitRange
- NDRange
- Tuple
- Number
- AbstractArray
- IntSet (page 345)
- ObjectIdDict
- Dict (page 343)
- WeakKeyDict
- EachLine
- AbstractString
- Set (page 345)
- *Task* (page 405)

### 36.2 General Collections

### $isempty(collection) \rightarrow Bool$

Determine whether a collection is empty (has no elements).

```
julia> isempty([])
  true

julia> isempty([1 2 3])
  false
```

```
empty! (collection) \rightarrow collection
```

Remove all elements from a collection.

### $length(collection) \rightarrow Integer$

For ordered, indexable collections, the maximum index i for which getindex (collection, i) is valid. For unordered collections, the number of elements.

### **endof** (collection) $\rightarrow$ Integer

Returns the last index of the collection.

```
julia> endof([1,2,4])
3
```

### Fully implemented by:

- Range
- UnitRange
- Tuple
- Number
- AbstractArray
- IntSet (page 345)
- *Dict* (page 343)
- WeakKeyDict
- AbstractString
- Set (page 345)

### 36.3 Iterable Collections

```
in (item, collection) → Bool

∈ (item, collection) → Bool

∋ (collection, item) → Bool

∉ (item, collection) → Bool

\not\ni (collection, item) → Bool
```

Determine whether an item is in the given collection, in the sense that it is == to one of the values generated by iterating over the collection. Some collections need a slightly different definition; for example Set (page 345)s check whether the item isequal() (page 320) to one of the elements. Dict (page 343)s look for (key, value) pairs, and the key is compared using isequal() (page 320). To test for the presence of a key in a dictionary, use haskey() (page 343) or k in keys (dict).

### eltype(type)

Determine the type of the elements generated by iterating a collection of the given type. For associative collection types, this will be a  $Pair\{KeyType, ValType\}$ . The definition eltype(x) = eltype(typeof(x)) is provided for convenience so that instances can be passed instead of types. However the form that accepts a type argument should be defined for new types.

#### $indexin(a \ b)$

Returns a vector containing the highest index in b for each value in a that is a member of b. The output vector contains 0 wherever a is not a member of b.

#### findin(a, b)

Returns the indices of elements in collection a that appear in collection b.

### unique (itr[, dim])

Returns an array containing only the unique elements of the iterable itr, in the order that the first of each set of equivalent elements originally appears. If dim is specified, returns unique regions of the array itr along dim.

### unique(f, itr)

Returns an array containing one value from itr for each unique value produced by f applied to elements of itr.

### reduce(op, v0, itr)

Reduce the given collection itr with the given binary operator op. v0 must be a neutral element for op that will be returned for empty collections. It is unspecified whether v0 is used for non-empty collections.

Reductions for certain commonly-used operators have special implementations which should be used instead: maximum(itr), minimum(itr), sum(itr), prod(itr), any(itr), all(itr).

The associativity of the reduction is implementation dependent. This means that you can't use non-associative operations like – because it is undefined whether reduce(-, [1, 2, 3]) should be evaluated as (1-2)-3 or 1-(2-3). Use fold1 or foldr instead for guaranteed left or right associativity.

Some operations accumulate error, and parallelism will also be easier if the reduction can be executed in groups. Future versions of Julia might change the algorithm. Note that the elements are not reordered if you use an ordered collection.

### reduce (op, itr)

Like reduce (op, v0, itr). This cannot be used with empty collections, except for some special cases (e.g. when op is one of +, \*, max, min, &, |) when Julia can determine the neutral element of op.

### foldl(op, v0, itr)

Like reduce () (page 338), but with guaranteed left associativity. v0 will be used exactly once.

### foldl (op, itr)

Like foldl (op, v0, itr), but using the first element of itr as v0. In general, this cannot be used with empty collections (see reduce (op, itr)).

### foldr(op, v0, itr)

Like reduce () (page 338), but with guaranteed right associativity. v0 will be used exactly once.

#### foldr (op, itr)

Like foldr (op, v0, itr), but using the last element of itr as v0. In general, this cannot be used with empty collections (see reduce (op, itr)).

### maximum(itr)

Returns the largest element in a collection.

### maximum(A, dims)

Compute the maximum value of an array over the given dimensions.

#### maximum! (r, A)

Compute the maximum value of A over the singleton dimensions of r, and write results to r.

### minimum(itr)

Returns the smallest element in a collection.

#### minimum (A, dims)

Compute the minimum value of an array over the given dimensions.

### minimum!(r, A)

Compute the minimum value of A over the singleton dimensions of r, and write results to r.

### extrema(itr)

Compute both the minimum and maximum element in a single pass, and return them as a 2-tuple.

#### $indmax(itr) \rightarrow Integer$

Returns the index of the maximum element in a collection.

### $indmin(itr) \rightarrow Integer$

Returns the index of the minimum element in a collection.

### $findmax(itr) \rightarrow (x, index)$

Returns the maximum element and its index.

#### $findmax(A, dims) \rightarrow (maxval, index)$

For an array input, returns the value and index of the maximum over the given dimensions.

### **findmin** (itr) -> (x, index)

Returns the minimum element and its index.

### $findmin(A, dims) \rightarrow (minval, index)$

For an array input, returns the value and index of the minimum over the given dimensions.

### findmax! (rval, rind, A, [init=true]) -> (maxval, index)

Find the maximum of A and the corresponding linear index along singleton dimensions of rval and rind, and store the results in rval and rind.

### findmin! (rval, rind, A, [init=true]) -> (minval, index)

Find the minimum of A and the corresponding linear index along singleton dimensions of rval and rind, and store the results in rval and rind.

#### maxabs(itr)

Compute the maximum absolute value of a collection of values.

#### maxabs(A.dims)

Compute the maximum absolute values over given dimensions.

### maxabs!(r, A)

Compute the maximum absolute values over the singleton dimensions of r, and write values to r.

### minabs(itr)

Compute the minimum absolute value of a collection of values.

### minabs(A, dims)

Compute the minimum absolute values over given dimensions.

### minabs!(r, A)

Compute the minimum absolute values over the singleton dimensions of r, and write values to r.

#### sum(itr)

Returns the sum of all elements in a collection.

#### sum(A, dims)

Sum elements of an array over the given dimensions.

### sum!(r,A)

Sum elements of A over the singleton dimensions of r, and write results to r.

### $\mathbf{sum}\left(f,\,itr\right)$

Sum the results of calling function f on each element of itr.

#### sumabs(itr)

Sum absolute values of all elements in a collection. This is equivalent to sum (abs (itr)) but faster.

### sumabs(A, dims)

Sum absolute values of elements of an array over the given dimensions.

#### sumabs!(r,A)

Sum absolute values of elements of A over the singleton dimensions of r, and write results to r.

#### sumabs2 (itr)

Sum squared absolute values of all elements in a collection. This is equivalent to sum (abs2 (itr)) but faster.

### sumabs2(A, dims)

Sum squared absolute values of elements of an array over the given dimensions.

#### sumabs2!(r, A)

Sum squared absolute values of elements of A over the singleton dimensions of r, and write results to r.

#### prod (itr)

Returns the product of all elements of a collection.

### prod(A, dims)

Multiply elements of an array over the given dimensions.

### prod!(r, A)

Multiply elements of A over the singleton dimensions of r, and write results to r.

### **any** $(itr) \rightarrow Bool$

Test whether any elements of a boolean collection are true.

#### any(A, dims)

Test whether any values along the given dimensions of an array are true.

### any! (r, A)

Test whether any values in A along the singleton dimensions of r are true, and write results to r.

### **all** (itr) $\rightarrow$ Bool

Test whether all elements of a boolean collection are true.

#### **all** (*A. dims*)

Test whether all values along the given dimensions of an array are true.

### all!(r, A)

Test whether all values in A along the singleton dimensions of r are true, and write results to r.

### **count** $(p, itr) \rightarrow$ Integer

Count the number of elements in itr for which predicate p returns true.

### $any(p, itr) \rightarrow Bool$

Determine whether predicate p returns true for any elements of itr.

### **all** $(p, itr) \rightarrow Bool$

Determine whether predicate p returns true for all elements of itr.

```
julia> all(i->(4<=i<=6), [4,5,6])
true
```

### foreach $(f, c...) \rightarrow Void$

Call function f on each element of iterable c. For multiple iterable arguments, f is called element-wise. foreach should be used instead of map when the results of f are not needed, for example in foreach (println, array).

### $map(f, c...) \rightarrow collection$

Transform collection c by applying f to each element. For multiple collection arguments, apply f elementwise.

```
julia> map((x) -> x * 2, [1, 2, 3])
3-element Array{Int64,1}:
2
4
6

julia> map(+, [1, 2, 3], [10, 20, 30])
```

### map! (function, collection)

In-place version of map () (page 340).

### map! (function, destination, collection...)

Like map () (page 340), but stores the result in destination rather than a new collection. destination must be at least as large as the first collection.

### mapreduce(f, op, v0, itr)

Apply function f to each element in itr, and then reduce the result using the binary function op. v0 must be a neutral element for op that will be returned for empty collections. It is unspecified whether v0 is used for non-empty collections.

mapreduce() (page 341) is functionally equivalent to calling reduce(op, v0, map(f, itr)), but will in general execute faster since no intermediate collection needs to be created. See documentation for reduce() (page 338) and map() (page 340).

```
julia> mapreduce(x->x^2, +, [1:3;]) # == 1 + 4 + 9
14
```

The associativity of the reduction is implementation-dependent. Additionally, some implementations may reuse the return value of f for elements that appear multiple times in itr. Use <code>mapfold1()</code> (page 341) or <code>mapfoldr()</code> (page 341) instead for guaranteed left or right associativity and invocation of f for every value.

### mapreduce(f, op, itr)

Like mapreduce (f, op, v0, itr). In general, this cannot be used with empty collections (see reduce (op, itr)).

### mapfoldl(f, op, v0, itr)

Like mapreduce () (page 341), but with guaranteed left associativity. v0 will be used exactly once.

### mapfoldl(f, op, itr)

Like mapfoldl(f, op, v0, itr), but using the first element of itr as v0. In general, this cannot be used with empty collections (see reduce (op, itr)).

### **mapfoldr** (f, op, v0, itr)

Like mapreduce () (page 341), but with guaranteed right associativity. v0 will be used exactly once.

### mapfoldr(f, op, itr)

Like mapfoldr(f, op, v0, itr), but using the first element of itr as v0. In general, this cannot be used with empty collections (see reduce (op, itr)).

### first (coll)

Get the first element of an iterable collection. Returns the start point of a Range even if it is empty.

#### last (coll

Get the last element of an ordered collection, if it can be computed in O(1) time. This is accomplished by calling endof() (page 337) to get the last index. Returns the end point of a Range even if it is empty.

#### step(r)

Get the step size of a Range object.

### collect (collection)

Return an array of all items in a collection. For associative collections, returns Pair{KeyType, ValType}.

#### **collect** (*element type*, *collection*)

Return an array of type Array {element\_type, 1} of all items in a collection.

### issubset(a, b)

 $\subseteq (a, b) \rightarrow Bool$ 

 $\nsubseteq (a, b) \rightarrow Bool$ 

 $\subseteq (a, b) \rightarrow Bool$ 

Determine whether every element of a is also in b, using in () (page 337).

### filter (function, collection)

Return a copy of collection, removing elements for which function is false. For associative collections, the function is passed two arguments (key and value).

### filter! (function, collection)

Update collection, removing elements for which function is false. For associative collections, the function is passed two arguments (key and value).

### 36.4 Indexable Collections

### getindex (collection, key...)

Retrieve the value(s) stored at the given key or index within a collection. The syntax a[i, j, ...] is converted by the compiler to getindex (a, i, j, ...).

### setindex! (collection, value, key...)

Store the given value at the given key or index within a collection. The syntax a[i,j,...] = x is converted by the compiler to (setindex! (a, x, i, j, ...); x).

Fully implemented by:

- Array (page 395)
- BitArray
- AbstractArray
- SubArray
- ObjectIdDict
- Dict (page 343)
- WeakKeyDict
- AbstractString

### Partially implemented by:

- Range
- UnitRange
- Tuple

### 36.5 Associative Collections

Dict (page 343) is the standard associative collection. Its implementation uses hash() (page 321) as the hashing function for the key, and isequal() (page 320) to determine equality. Define these two functions for custom types to override how they are stored in a hash table.

ObjectIdDict is a special hash table where the keys are always object identities.

WeakKeyDict is a hash table implementation where the keys are weak references to objects, and thus may be garbage collected even when referenced in a hash table.

Dict (page 343)s can be created by passing pair objects constructed with =>() to a Dict (page 343) constructor: Dict("A"=>1, "B"=>2). This call will attempt to infer type information from the keys and values (i.e. this example creates a Dict{ASCIIString, Int64}). To explicitly specify types use the syntax Dict{KeyType, ValueType}(...). For example, Dict{ASCIIString, Int32}("A"=>1, "B"=>2).

As with Array (page 395)s, Dict (page 343)s may be created with comprehensions. For example, [i => f(i)] for i = 1:10.

Given a dictionary D, the syntax D[x] returns the value of key x (if it exists) or throws an error, and D[x] = y stores the key-value pair x = y in D (replacing any existing value for the key x). Multiple arguments to D[x] are converted to tuples; for example, the syntax D[x, y] is equivalent to D[(x, y)], i.e. it refers to the value keyed by the tuple (x, y).

### Dict([itr])

 $Dict\{K, V\}$  () constructs a hash table with keys of type K and values of type V.

Given a single iterable argument, constructs a <code>Dict</code> (page 343) whose key-value pairs are taken from 2-tuples (key, value) generated by the argument.

```
julia> Dict([("A", 1), ("B", 2)])
Dict{ASCIIString, Int64} with 2 entries:
    "B" => 2
    "A" => 1
```

Alternatively, a sequence of pair arguments may be passed.

```
julia> Dict("A"=>1, "B"=>2)
Dict{ASCIIString, Int64} with 2 entries:
    "B" => 2
    "A" => 1
```

### **haskey** (collection, key) $\rightarrow$ Bool

Determine whether a collection has a mapping for a given key.

### get (collection, key, default)

Return the value stored for the given key, or the given default value if no mapping for the key is present.

#### get (f::Function, collection, key)

Return the value stored for the given key, or if no mapping for the key is present, return f(). Use get!() (page 343) to also store the default value in the dictionary.

This is intended to be called using do block syntax

```
get(dict, key) do
    # default value calculated here
    time()
end
```

#### **get!** (collection, key, default)

Return the value stored for the given key, or if no mapping for the key is present, store key => default, and return default.

### get! (f::Function, collection, key)

Return the value stored for the given key, or if no mapping for the key is present, store key = f(), and return f().

This is intended to be called using do block syntax:

```
get!(dict, key) do
    # default value calculated here
    time()
end
```

### getkey (collection, key, default)

Return the key matching argument key if one exists in collection, otherwise return default.

### delete! (collection, key)

Delete the mapping for the given key in a collection, and return the collection.

```
pop! (collection, key[, default])
```

Delete and return the mapping for key if it exists in collection, otherwise return default, or throw an error if default is not specified.

### keys (collection)

Return an iterator over all keys in a collection. collect (keys (d)) returns an array of keys.

#### values (collection)

Return an iterator over all values in a collection. collect (values (d)) returns an array of values.

### merge (collection, others...)

Construct a merged collection from the given collections. If necessary, the types of the resulting collection will be promoted to accommodate the types of the merged collections. If the same key is present in another collection, the value for that key will be the value it has in the last collection listed.

```
julia> a = Dict("foo" => 0.0, "bar" => 42.0)
Dict{ASCIIString,Float64} with 2 entries:
  "bar" => 42.0
  "foo" => 0.0
julia> b = Dict(utf8("baz") => 17, utf8("bar") => 4711)
Dict{UTF8String, Int64} with 2 entries:
  "bar" => 4711
  "baz" => 17
julia> merge(a, b)
Dict{UTF8String,Float64} with 3 entries:
  "bar" \Rightarrow 4711.0
  "baz" => 17.0
  "foo" => 0.0
julia> merge(b, a)
Dict{UTF8String,Float64} with 3 entries:
  "bar" => 42.0
  "baz" => 17.0
  "foo" => 0.0
```

### merge! (collection, others...)

Update collection with pairs from the other collections.

### sizehint!(s, n)

Suggest that collection s reserve capacity for at least n elements. This can improve performance.

### keytype (type)

Get the key type of an associative collection type. Behaves similarly to eltype.

### valtype (type)

Get the value type of an associative collection type. Behaves similarly to eltype.

Fully implemented by:

- ObjectIdDict
- Dict (page 343)
- WeakKeyDict

### Partially implemented by:

- IntSet (page 345)
- Set (page 345)
- EnvHash (page 329)
- Array (page 395)
- BitArray

### 36.6 Set-Like Collections

### Set([itr])

Construct a Set (page 345) of the values generated by the given iterable object, or an empty set. Should be used instead of IntSet (page 345) for sparse integer sets, or for sets of arbitrary objects.

### IntSet ([itr])

Construct a sorted set of positive Ints generated by the given iterable object, or an empty set. Implemented as a bit string, and therefore designed for dense integer sets. Only Ints greater than 0 can be stored. If the set will be sparse (for example holding a few very large integers), use Set (page 345) instead.

### union (s1, s2...)

 $\cup$  (s1, s2...)

Construct the union of two or more sets. Maintains order with arrays.

### union! (s, iterable)

Union each element of iterable into set s in-place.

### **intersect** (*s1*, *s2*...)

 $\cap$  (s1, s2)

Construct the intersection of two or more sets. Maintains order and multiplicity of the first argument for arrays and ranges.

### setdiff(s1, s2)

Construct the set of elements in s1 but not s2. Maintains order with arrays. Note that both arguments must be collections, and both will be iterated over. In particular, setdiff(set,element) where element is a potential member of set, will not work in general.

### setdiff! (s, iterable)

Remove each element of iterable from set s in-place.

### symdiff(s1, s2...)

Construct the symmetric difference of elements in the passed in sets or arrays. Maintains order with arrays.

### symdiff!(s, n)

The set s is destructively modified to toggle the inclusion of integer n.

### symdiff! (s, itr)

For each element in itr, destructively toggle its inclusion in set s.

### symdiff!(s1, s2)

Construct the symmetric difference of sets \$1 and \$2, storing the result in \$1.

### complement (s)

Returns the set-complement of IntSet (page 345) s.

### complement! (s)

Mutates IntSet (page 345) s into its set-complement.

### intersect! (s1, s2)

Intersects sets s1 and s2 and overwrites the set s1 with the result. If needed, s1 will be expanded to the size of s2.

### $issubset(A, S) \rightarrow Bool$

 $\subseteq$  (A, S)  $\rightarrow$  Bool

Return true if A is a subset of or equal to S.

Fully implemented by:

- IntSet (page 345)
- Set (page 345)

Partially implemented by:

• Array (page 395)

# 36.7 Dequeues

**push!** (*collection*, *items*...)  $\rightarrow$  collection

Insert one or more items at the end of collection.

```
julia> push!([1, 2, 3], 4, 5, 6)
6-element Array{Int64,1}:
    1
    2
    3
    4
    5
    6
```

Use append! () (page 349) to add all the elements of another collection to collection. The result of the preceding example is equivalent to append! ([1, 2, 3], [4, 5, 6]).

### **pop!** (collection) $\rightarrow$ item

Remove the last item in collection and return it.

```
julia> A=[1, 2, 3, 4, 5, 6]
6-element Array{Int64,1}:
     1
     2
     3
     4
     5
     6

julia> pop!(A)
6

julia> A
5-element Array{Int64,1}:
     1
```

```
2
3
4
5
```

### **unshift!** (*collection*, *items*...) $\rightarrow$ collection

Insert one or more items at the beginning of collection.

```
julia> unshift!([1, 2, 3, 4], 5, 6)
6-element Array{Int64,1}:
5
6
1
2
3
4
```

### **shift!** (collection) $\rightarrow$ item

Remove the first item from collection.

```
julia> A = [1, 2, 3, 4, 5, 6]
6-element Array{Int64,1}:
    1
    2
    3
    4
    5
    6

julia> shift!(A)
    1

julia> A
5-element Array{Int64,1}:
    2
    3
    4
    5
    6
```

### insert! (collection, index, item)

Insert an item into collection at the given index. index is the index of item in the resulting collection.

```
julia> insert!([6, 5, 4, 2, 1], 4, 3)
6-element Array{Int64,1}:
6
5
4
3
2
1
```

### deleteat! (collection, index)

Remove the item at the given index and return the modified collection. Subsequent items are shifted to fill the resulting gap.

```
julia> deleteat!([6, 5, 4, 3, 2, 1], 2)
5-element Array{Int64,1}:
```

36.7. Dequeues 349

```
6
4
3
2
1
```

### deleteat! (collection, itr)

Remove the items at the indices given by itr, and return the modified collection. Subsequent items are shifted to fill the resulting gap. itr must be sorted and unique.

```
julia> deleteat!([6, 5, 4, 3, 2, 1], 1:2:5)
3-element Array{Int64,1}:
5
3
1

julia> deleteat!([6, 5, 4, 3, 2, 1], (2, 2))
ERROR: ArgumentError: indices must be unique and sorted
in deleteat! at array.j1:543
```

### splice! (collection, index[, replacement]) $\rightarrow$ item

Remove the item at the given index, and return the removed item. Subsequent items are shifted down to fill the resulting gap. If specified, replacement values from an ordered collection will be spliced in place of the removed item.

```
julia> A = [6, 5, 4, 3, 2, 1]; splice!(A, 5)
julia> A
5-element Array{Int64,1}:
5
4
 3
1
julia> splice!(A, 5, −1)
julia> A
5-element Array{Int64,1}:
  5
 4
 3
julia> splice!(A, 1, [-1, -2, -3])
julia> A
7-element Array{Int64,1}:
 -1
 -2
 -3
  5
  4
  3
```

```
-1
```

To insert replacement before an index n without removing any items, use splice! (collection, n:n-1, replacement).

### **splice!** (collection, range[, replacement]) $\rightarrow$ items

Remove items in the specified index range, and return a collection containing the removed items. Subsequent items are shifted down to fill the resulting gap. If specified, replacement values from an ordered collection will be spliced in place of the removed items.

To insert replacement before an index n without removing any items, use splice! (collection, n:n-1, replacement).

```
julia> splice!(A, 4:3, 2)
0-element Array{Int64,1}

julia> A
8-element Array{Int64,1}:
-1
-2
-3
2
5
4
3
-1
```

### **resize!** (*collection*, n) $\rightarrow$ collection

Resize collection to contain n elements. If n is smaller than the current collection length, the first n elements will be retained. If n is larger, the new elements are not guaranteed to be initialized.

```
julia> resize!([6, 5, 4, 3, 2, 1], 3)
3-element Array{Int64,1}:
6
5
4
```

```
julia> resize!([6, 5, 4, 3, 2, 1], 8)
8-element Array{Int64,1}:
6
5
4
3
2
1
0
0
```

### **append!** (*collection*, *collection*2) $\rightarrow$ collection.

Add the elements of collection2 to the end of collection.

```
julia> append!([1],[2,3])
3-element Array{Int64,1}:
    1
    2
    3
```

```
julia> append!([1, 2, 3], [4, 5, 6])
6-element Array{Int64,1}:
1
```

36.7. Dequeues 351

```
2
3
4
5
6
```

Use *push!* () (page 346) to add individual items to collection which are not already themselves in another collection. The result is of the preceding example is equivalent to push! ([1, 2, 3], 4, 5, 6).

### **prepend!** (*collection*, *items*) $\rightarrow$ collection

Insert the elements of items to the beginning of collection.

```
julia> prepend!([3],[1,2])
3-element Array{Int64,1}:
    1
    2
    3
```

### Fully implemented by:

- Vector (a.k.a. 1-dimensional Array (page 395))
- BitVector (a.k.a. 1-dimensional BitArray)

# 36.8 PriorityQueue

The *PriorityQueue* (page 350) type is available from the Collections module. It provides a basic priority queue implementation allowing for arbitrary key and priority types. Multiple identical keys are not permitted, but the priority of existing keys can be changed efficiently.

### PriorityQueue (K, V[, ord])

Construct a new *PriorityQueue* (page 350), with keys of type K and values/priorites of type V. If an order is not given, the priority queue is min-ordered using the default comparison for V.

A PriorityQueue acts like a Dict, mapping values to their priorities, with the addition of a dequeue! function to remove the lowest priority element.

### enqueue! (pq, k, v)

Insert the a key k into a priority queue pg with priority v.

#### dequeue! (pq)

Remove and return the lowest priority key from a priority queue.

### peek(pq)

Return the lowest priority key from a priority queue without removing that key from the queue.

PriorityQueue (page 350) also behaves similarly to a Dict in that keys can be inserted and priorities accessed or changed using indexing notation.

# 36.9 Heap Functions

Along with the *PriorityQueue* (page 350) type, the Collections module provides lower level functions for performing binary heap operations on arrays. Each function takes an optional ordering argument. If not given, default ordering is used, so that elements popped from the heap are given in ascending order.

```
heapify (v[,ord])
```

Returns a new vector in binary heap order, optionally using the given ordering.

```
heapify! (v[, ord])
In-place heapify() (page 351).
```

```
isheap(v[,ord])
```

Return true if an array is heap-ordered according to the given order.

```
heappush! (v, x[, ord])
```

Given a binary heap-ordered array, push a new element x, preserving the heap property. For efficiency, this function does not check that the array is indeed heap-ordered.

```
heappop! (v[,ord])
```

Given a binary heap-ordered array, remove and return the lowest ordered element. For efficiency, this function does not check that the array is indeed heap-ordered.

Julia Language Documentation, Release 0.5.0-dev			

## **Mathematics**

# **37.1 Mathematical Operators**

```
-(x)
      Unary minus operator.
+(x, y...)
      Addition operator. x+y+z+... calls this function with all arguments, i.e. +(x, y, z, ...).
-(x, y)
      Subtraction operator.
\star (x, y...)
      Multiplication operator. x * y * z * \dots calls this function with all arguments, i.e. * (x, y, z, \dots).
/(x, y)
      Right division operator: multiplication of x by the inverse of y on the right. Gives floating-point results for
      integer arguments.
(x, y)
      Left division operator: multiplication of y by the inverse of x on the left. Gives floating-point results for integer
      arguments.
^{\wedge}(x, y)
      Exponentiation operator.
. + (x, y)
      Element-wise addition operator.
-(x, y)
      Element-wise subtraction operator.
\star (x, y)
      Element-wise multiplication operator.
(x, y)
      Element-wise right division operator.
(x, y)
      Element-wise left division operator.
. ^{\wedge}(x, y)
      Element-wise exponentiation operator.
```

```
fma (x, y, z)
```

Computes x\*y+z without rounding the intermediate result x\*y. On some systems this is significantly more expensive than x\*y+z. fma is used to improve accuracy in certain algorithms. See muladd.

#### $\mathbf{muladd}(x, y, z)$

Combined multiply-add, computes x \* y + z in an efficient manner. This may on some systems be equivalent to x \* y + z, or to fma (x, y, z). muladd is used to improve performance. See fma.

#### $\mathbf{div}(x, y)$

 $\div (x, y)$ 

The quotient from Euclidean division. Computes x/y, truncated to an integer.

#### fld (x, y)

Largest integer less than or equal to x/y.

#### cld(x, y)

Smallest integer larger than or equal to x/y.

### mod(x, y)

Modulus after flooring division, returning in the range [0, "y"], if y is positive, or (y,0] if y is negative.

```
x == fld(x,y) *y + mod(x,y)
```

#### mod2pi(x)

Modulus after division by 2pi, returning in the range [0,2pi).

This function computes a floating point representation of the modulus after division by numerically exact 2pi, and is therefore not exactly the same as mod(x,2pi), which would compute the modulus of x relative to division by the floating-point number 2pi.

## $\mathbf{rem}(x, y)$

% (x, y)

Remainder from Euclidean division, returning a value of the same sign as x, and smaller in magnitude than y. This value is always exact.

```
x == div(x,y)*y + rem(x,y)
```

## divrem(x, y)

The quotient and remainder from Euclidean division. Equivalent to (div(x,y), rem(x,y)) or  $(x \div y, x \div y)$ .

#### fldmod(x, y)

The floored quotient and modulus after division. Equivalent to (fld(x,y), mod(x,y)).

## fld1(x, y)

Flooring division, returning a value consistent with mod1 (x, y)

```
x == fld(x,y) *y + mod(x,y)

x == (fld1(x,y)-1) *y + mod1(x,y)
```

#### mod1(x, y)

Modulus after flooring division, returning a value in the range (0, y).

#### fldmod1(x, y)

Return (fld1(x,y), mod1(x,y)).

## rem1(x, y)

(Deprecated.) Remainder after division, returning in the range (0, y].

#### **//** (num, den)

Divide two integers or rational numbers, giving a Rational result.

```
rationalize ([Type=Int], x; tol=eps(x))
```

Approximate floating point number x as a Rational number with components of the given integer type. The result will differ from x by no more than tol.

#### num(x)

Numerator of the rational representation of x.

#### den(x)

Denominator of the rational representation of x.

### <<(x, n)

Left bit shift operator.

>> (x, n)

Right bit shift operator, preserving the sign of x.

#### >> (x, n)

Unsigned right bit shift operator.

```
: (start, step, stop)
```

Range operator. a:b constructs a range from a to b with a step size of 1, and a:s:b is similar but uses a step size of s. These syntaxes call the function colon. The colon is also used in indexing to select whole dimensions.

```
colon (start[, step], stop)
```

Called by: syntax for constructing ranges.

```
range (start, step, length)
```

Construct a range by length, given a starting value and optional step (defaults to 1).

```
==(x, y)
```

Generic equality operator, giving a single Bool result. Falls back to ===. Should be implemented for all types with a notion of equality, based on the abstract value that an instance represents. For example, all numeric types are compared by numeric value, ignoring type. Strings are compared as sequences of characters, ignoring encoding.

Follows IEEE semantics for floating-point numbers.

Collections should generally implement == by calling == recursively on all contents.

New numeric types should implement this function for two arguments of the new type, and handle comparison to other types via promotion rules where possible.

```
! = (x, y)
\neq (x, y)
```

Not-equals comparison operator. Always gives the opposite answer as ==. New types should generally not implement this, and rely on the fallback definition !=(x,y)=!(x==y) instead.

```
=== (x, y)

\equiv (x, y)

See the is() (page 320) operator.

!== (x, y)

\not\equiv (x, y)

Equivalent to !is(x, y).
```

Less-than comparison operator. New numeric types should implement this function for two arguments of the new type. Because of the behavior of floating-point NaN values, < implements a partial order. Types with a canonical partial order should implement <, and types with a canonical total order should implement isless.

```
\leftarrow = (x, y)
```

 $\langle (x, y) \rangle$ 

```
\leq (x, y)
      Less-than-or-equals comparison operator.
>(x, y)
      Greater-than comparison operator. Generally, new types should implement < instead of this function, and rely
      on the fallback definition > (x, y) = y < x.
>= (x, y)
\geq (x, y)
      Greater-than-or-equals comparison operator.
      Element-wise equality comparison operator.
.! = (x, y)
\cdot \neq (x, y)
      Element-wise not-equals comparison operator.
. < (x, y)
      Element-wise less-than comparison operator.
. <= (x, y)
\leq (x, y)
      Element-wise less-than-or-equals comparison operator.
. > (x, y)
      Element-wise greater-than comparison operator.
.>=(x, y)
. \ge (x, y)
      Element-wise greater-than-or-equals comparison operator.
cmp(x, y)
      Return -1, 0, or 1 depending on whether x is less than, equal to, or greater than y, respectively. Uses the
      total order implemented by isless. For floating-point numbers, uses < but throws an error for unordered
      arguments.
\sim (x)
      Bitwise not.
\& (x, y)
      Bitwise and.
|(x, y)|
      Bitwise or.
(x, y)
      Bitwise exclusive or.
!(x)
      Boolean not.
ж && у
      Short-circuiting boolean AND.
x \mid \mid y
      Short-circuiting boolean OR.
A_ldiv_Bc(A, B)
      For matrices or vectors A and B, calculates A \setminus B^{H}.
```

 $A\_ldiv\_Bt(A, B)$ 

For matrices or vectors A and B, calculates  $A \setminus B^{T}$ .

### A mul B! $(Y, A, B) \rightarrow Y$

Calculates the matrix-matrix or matrix-vector product  $A \cdot B$  and stores the result in Y, overwriting the existing value of Y. Note that Y must not be aliased with either A or B.

```
julia> A=[1.0 2.0; 3.0 4.0]; B=[1.0 1.0; 1.0 1.0]; Y = similar(B); A_mul_B!(Y, A, B);
julia> Y
2x2 Array{Float64,2}:
3.0 3.0
7.0 7.0
```

## $A_{\underline{}}$ mul $\underline{}$ Bc (A, B)

For matrices or vectors A and B, calculates  $A \cdot B^{H}$ .

### $A_{mul}Bt(A, B)$

For matrices or vectors A and B, calculates  $A \cdot B^{T}$ .

## $A_rdiv_Bc(A, B)$

For matrices or vectors A and B, calculates  $A/B^{H}$ .

## $A_rdiv_Bt(A, B)$

For matrices or vectors A and B, calculates  $A/B^{T}$ .

### $Ac_ldiv_B(A, B)$

For matrices or vectors A and B, calculates  $A^{H} \setminus B$ .

## $Ac\_ldiv\_Bc(A, B)$

For matrices or vectors A and B, calculates  $A^{H} \setminus B^{H}$ .

#### Ac mul B(A, B)

For matrices or vectors A and B, calculates  $A^{H} \cdot B$ .

## $Ac_{mul}Bc(A, B)$

For matrices or vectors A and B, calculates  $A^{H}B^{H}$ .

## $Ac\_rdiv\_B(A, B)$

For matrices or vectors A and B, calculates  $A^{\rm H}/B$ .

## $Ac\_rdiv\_Bc(A, B)$

For matrices or vectors A and B, calculates  $A^{\rm H}/B^{\rm H}$ .

## $At_ldiv_B(A, B)$

For matrices or vectors A and B, calculates  $A^{T} \setminus B$ .

## $At\_ldiv\_Bt(A, B)$

For matrices or vectors A and B, calculates  $A^{T} \setminus B^{T}$ .

## $At_mul_B(A, B)$

For matrices or vectors A and B, calculates  $A^{T} \cdot B$ .

#### At mul Bt (A, B)

For matrices or vectors A and B, calculates  $A^{T} \cdot B^{T}$ .

## $At\_rdiv\_B(A, B)$

For matrices or vectors A and B, calculates  $A^{T}/B$ .

### At rdiv Bt (A, B)

For matrices or vectors A and B, calculates  $A^{T}/B^{T}$ .

## 37.2 Mathematical Functions

```
isapprox (x, y; rtol::Real=sqrt(eps), atol::Real=0)
     Inexact equality comparison: true if norm(x-y) \le atol + rtol*max(norm(x), norm(y)).
     The default atol is zero and the default rtol depends on the types of x and y.
     For real or complex floating-point values, rtol defaults to sqrt (eps (typeof (real (x-y)))). This
     corresponds to requiring equality of about half of the significand digits. For other types, rtol defaults to zero.
     x and y may also be arrays of numbers, in which case norm defaults to vecnorm but may be changed by
     passing a norm:: Function keyword argument. (For numbers, norm is the same thing as abs.)
     The binary operator \approx is equivalent to isapprox with the default arguments, and x \not\approx y is equivalent to
      !isapprox(x, y).
sin(x)
     Compute sine of x, where x is in radians.
\cos(x)
     Compute cosine of x, where x is in radians.
tan(x)
     Compute tangent of x, where x is in radians.
sind(x)
     Compute sine of x, where x is in degrees.
cosd(x)
     Compute cosine of x, where x is in degrees.
tand(x)
     Compute tangent of x, where x is in degrees.
sinpi(x)
     Compute \sin(\pi x) more accurately than \sin(\text{pi}*x), especially for large x.
cospi(x)
     Compute \cos(\pi x) more accurately than \cos(\text{pi}*x), especially for large x.
sinh(x)
     Compute hyperbolic sine of x.
cosh(x)
     Compute hyperbolic cosine of x.
tanh(x)
     Compute hyperbolic tangent of x.
asin(x)
     Compute the inverse sine of x, where the output is in radians.
acos(x)
     Compute the inverse cosine of x, where the output is in radians
atan(x)
     Compute the inverse tangent of x, where the output is in radians.
atan2(y, x)
     Compute the inverse tangent of y/x, using the signs of both x and y to determine the quadrant of the return
```

asind(x)

Compute the inverse sine of x, where the output is in degrees.

#### acosd(x)

Compute the inverse cosine of x, where the output is in degrees.

## atand(x)

Compute the inverse tangent of x, where the output is in degrees.

#### sec(x)

Compute the secant of x, where x is in radians.

#### csc(x)

Compute the cosecant of x, where x is in radians.

#### cot(x)

Compute the cotangent of x, where x is in radians.

#### secd(x)

Compute the secant of x, where x is in degrees.

#### $\operatorname{cscd}(x)$

Compute the cosecant of x, where x is in degrees.

#### cotd(x)

Compute the cotangent of x, where x is in degrees.

#### asec(x)

Compute the inverse secant of x, where the output is in radians.

#### acsc(x)

Compute the inverse cosecant of x, where the output is in radians

#### acot (x)

Compute the inverse cotangent of x, where the output is in radians.

## $\mathbf{asecd}(x)$

Compute the inverse secant of x, where the output is in degrees.

#### acscd(x)

Compute the inverse cosecant of  $\boldsymbol{x}$ , where the output is in degrees.

## acotd(x)

Compute the inverse cotangent of x, where the output is in degrees.

## $\mathbf{sech}(x)$

Compute the hyperbolic secant of x

## $\mathtt{csch}\,(x)$

Compute the hyperbolic cosecant of  $\boldsymbol{x}$ .

#### $\coth(x)$

Compute the hyperbolic cotangent of x.

## asinh(x)

Compute the inverse hyperbolic sine of x.

## $\mathbf{acosh}\left( x\right)$

Compute the inverse hyperbolic cosine of  $\boldsymbol{x}$ .

#### atanh(x)

Compute the inverse hyperbolic tangent of x.

## $\mathbf{asech}(x)$

Compute the inverse hyperbolic secant of x.

#### acsch(x)

Compute the inverse hyperbolic cosecant of x.

### acoth(x)

Compute the inverse hyperbolic cotangent of x.

#### sinc(x)

Compute  $\sin(\pi x)/(\pi x)$  if  $x \neq 0$ , and 1 if x = 0.

#### cosc(x)

Compute  $\cos(\pi x)/x - \sin(\pi x)/(\pi x^2)$  if  $x \neq 0$ , and 0 if x = 0. This is the derivative of sinc (x).

#### deg2rad(x)

Convert x from degrees to radians.

### rad2deg(x)

Convert x from radians to degrees.

#### hypot(x, y)

Compute the  $\sqrt{x^2 + y^2}$  avoiding overflow and underflow.

#### log(x)

Compute the natural logarithm of x. Throws DomainError for negative Real arguments. Use complex negative arguments to obtain complex results.

There is an experimental variant in the Base. Math. JuliaLibm module, which is typically faster and more accurate.

#### log(b, x)

Compute the base b logarithm of x. Throws DomainError for negative Real arguments.

#### log2(x)

Compute the logarithm of x to base 2. Throws DomainError for negative Real arguments.

#### log10(x)

Compute the logarithm of x to base 10. Throws DomainError for negative Real arguments.

## log1p(x)

Accurate natural logarithm of 1+x. Throws DomainError for Real arguments less than -1.

There is an experimental variant in the Base. Math. JuliaLibm module, which is typically faster and more accurate.

## frexp(val)

Return (x, exp) such that x has a magnitude in the interval [1/2, 1) or 0, and val =  $x \times 2^{exp}$ .

## exp(x)

Compute  $e^x$ .

#### exp2(x)

Compute  $2^x$ .

## exp10(x)

Compute  $10^x$ .

## $\mathtt{ldexp}\,(x,n)$

Compute  $x \times 2^n$ .

## modf(x)

Return a tuple (fpart,ipart) of the fractional and integral parts of a number. Both parts have the same sign as the argument.

## expm1(x)

Accurately compute  $e^x - 1$ .

```
round([T], x[, digits[, base]][, r::RoundingMode])
```

round (x) rounds x to an integer value according to the default rounding mode (see *rounding()* (page 380)), returning a value of the same type as x. By default (*RoundNearest* (page 361)), this will round to the nearest integer, with ties (fractional values of 0.5) being rounded to the even integer.

```
julia> round(1.7)
2.0

julia> round(1.5)
2.0

julia> round(2.5)
2.0
```

The optional RoundingMode (page 361) argument will change how the number gets rounded.

round (T, x, [r::RoundingMode]) converts the result to type T, throwing an InexactError (page 330) if the value is not representable.

round (x, digits) rounds to the specified number of digits after the decimal place (or before if negative). round (x, digits, base) rounds using a base other than 10.

```
julia> round(pi, 2)
3.14

julia> round(pi, 3, 2)
3.125
```

#### note

Rounding to specified digits in bases other than 2 can be inexact when operating on binary floating point numbers. For example, the Float 64 value represented by 1.15 is actually *less* than 1.15, yet will be rounded to 1.2.

```
julia> x = 1.15
1.15

julia> @sprintf "%.20f" x
"1.149999999999991118"

julia> x < 115//100
true

julia> round(x, 1)
1.2
```

## RoundingMode

A type which controls rounding behavior. Currently supported rounding modes are:

- •RoundNearest (page 361) (default)
- •RoundNearestTiesAway (page 362)
- •RoundNearestTiesUp (page 362)
- •RoundToZero (page 362)
- •RoundUp (page 362)
- •RoundDown (page 362)

#### RoundNearest

The default rounding mode. Rounds to the nearest integer, with ties (fractional values of 0.5) being rounded to the nearest even integer.

## RoundNearestTiesAway

Rounds to nearest integer, with ties rounded away from zero (C/C++ round() (page 360) behaviour).

### RoundNearestTiesUp

Rounds to nearest integer, with ties rounded toward positive infinity (Java/JavaScript round() (page 360) behaviour).

#### RoundToZero

round () (page 360) using this rounding mode is an alias for trunc () (page 362).

## RoundUp

round () (page 360) using this rounding mode is an alias for ceil () (page 362).

#### RoundDown

round () (page 360) using this rounding mode is an alias for floor () (page 362).

#### round (z, RoundingModeReal, RoundingModeImaginary)

Returns the nearest integral value of the same type as the complex-valued z to z, breaking ties using the specified RoundingMode (page 361)s. The first RoundingMode (page 361) is used for rounding the real components while the second is used for rounding the imaginary components.

## ceil([T], x[, digits[, base]])

ceil(x) returns the nearest integral value of the same type as x that is greater than or equal to x.

ceil (T, x) converts the result to type T, throwing an InexactError if the value is not representable.

digits and base work as for round() (page 360).

## floor ([T], x[, digits[, base]])

floor (x) returns the nearest integral value of the same type as x that is less than or equal to x.

floor(T, x) converts the result to type T, throwing an InexactError if the value is not representable.

digits and base work as for round() (page 360).

## trunc([T], x[, digits[, base]])

trunc(x) returns the nearest integral value of the same type as x whose absolute value is less than or equal to x.

trunc(T, x) converts the result to type T, throwing an InexactError if the value is not representable.

digits and base work as for round() (page 360).

## $unsafe\_trunc(T, x)$

unsafe\_trunc(T, x) returns the nearest integral value of type T whose absolute value is less than or equal to x. If the value is not representable by T, an arbitrary value will be returned.

## signif(x, digits[, base])

Rounds (in the sense of round) x so that there are digits significant digits, under a base base representation, default 10. E.g., signif (123.456, 2) is 120.0, and signif (357.913, 4, 2) is 352.0.

## min(x, y, ...)

Return the minimum of the arguments. Operates elementwise over arrays.

#### max(x, y, ...)

Return the maximum of the arguments. Operates elementwise over arrays.

#### minmax(x, y)

Return  $(\min(x,y), \max(x,y))$ . See also: extrema() (page 338) that returns  $(\min\max(x), \max\min(x))$ .

### clamp(x, lo, hi)

Return x if  $10 \le x \le hi$ . If  $x \le 10$ , return 10. If x > hi, return hi. Arguments are promoted to a common type. Operates elementwise over x if it is an array.

#### abs(x)

The absolute value of x.

When abs is applied to signed integers, overflow may occur, resulting in the return of a negative value. This overflow occurs only when abs is applied to the minimum representable value of a signed integer. That is, when x == typemin(typeof(x)), abs (x) == x < 0, not -x as might be expected.

#### Base.checked\_abs(x)

Calculates abs(x), checking for overflow errors where applicable. For example, standard two's complement signed integers (e.g. Int) cannot represent abs(typemin(Int)), thus leading to an overflow.

The overflow protection may impose a perceptible performance penalty.

#### Base.checked\_neg(x)

Calculates -x, checking for overflow errors where applicable. For example, standard two's complement signed integers (e.g. Int) cannot represent -typemin(Int), thus leading to an overflow.

The overflow protection may impose a perceptible performance penalty.

#### Base.checked add (x, y)

Calculates x+y, checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

## Base.checked\_sub (x, y)

Calculates x-y, checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

## Base.checked\_mul (x, y)

Calculates x \* y, checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

## Base.checked\_div (x, y)

Calculates div(x, y), checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

## Base.checked rem(x, y)

Calculates x %y, checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

#### Base.checked fld(x, y)

Calculates fld (x, y), checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

## Base.checked\_mod(x, y)

Calculates mod(x, y), checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

#### Base.checked\_cld(x, y)

Calculates cld (x, y), checking for overflow errors where applicable.

The overflow protection may impose a perceptible performance penalty.

### abs2(x)

Squared absolute value of x.

```
copysign(x, y)
      Return x such that it has the same sign as y
      Return zero if x==0 and x/|x| otherwise (i.e., \pm 1 for real x).
signbit(x)
      Returns true if the value of the sign of x is negative, otherwise false.
flipsign (x, y)
      Return x with its sign flipped if y is negative. For example abs (x) = flipsign(x, x).
\mathbf{sqrt}(x)
      Return \sqrt{x}. Throws DomainError for negative Real arguments. Use complex negative arguments instead.
      The prefix operator \sqrt{} is equivalent to sqrt.
isqrt(n)
      Integer square root: the largest integer m such that m*m \le n.
cbrt (x)
      Return x^{1/3}. The prefix operator \sqrt[3]{} is equivalent to cbrt.
erf(x)
      Compute the error function of x, defined by \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt for arbitrary complex x.
      Compute the complementary error function of x, defined by 1 - \operatorname{erf}(x).
erfcx(x)
      Compute the scaled complementary error function of x, defined by e^{x^2}\operatorname{erfc}(x). Note also that \operatorname{erfcx}(-ix)
      computes the Faddeeva function w(x).
erfi(x)
      Compute the imaginary error function of x, defined by -i \operatorname{erf}(ix).
dawson(x)
      Compute the Dawson function (scaled imaginary error function) of x, defined by \frac{\sqrt{\pi}}{2}e^{-x^2}\operatorname{erfi}(x).
      Compute the inverse error function of a real x, defined by erf(erfinv(x)) = x.
erfcinv(x)
      Compute the inverse error complementary function of a real x, defined by \operatorname{erfc}(\operatorname{erfcinv}(x)) = x.
real(z)
      Return the real part of the complex number z.
      Return the imaginary part of the complex number z.
reim(z)
      Return both the real and imaginary parts of the complex number z.
      Compute the complex conjugate of a complex number z.
angle (z)
      Compute the phase angle in radians of a complex number z.
cis(z)
      Return \exp(iz).
binomial(n, k)
```

Number of ways to choose k out of n items.

## factorial(n)

Factorial of n. If n is an Integer, the factorial is computed as an integer (promoted to at least 64 bits). Note that this may overflow if n is not small, but you can use factorial (big (n)) to compute the result exactly in arbitrary precision. If n is not an Integer, factorial (n) is equivalent to gamma(n+1) (page 366).

## factorial(n, k)

Compute factorial (n) / factorial (k).

#### **factor** $(n) \rightarrow \text{Dict}$

Compute the prime factorization of an integer n. Returns a dictionary. The keys of the dictionary correspond to the factors, and hence are of the same type as n. The value associated with each key indicates the number of times the factor appears in the factorization.

```
julia> factor(100) # == 2*2*5*5
Dict{Int64, Int64} with 2 entries:
   2 => 2
   5 => 2
```

#### gcd(x, y)

Greatest common (positive) divisor (or zero if x and y are both zero).

#### lcm(x, y)

Least common (non-negative) multiple.

#### gcdx(x, y)

Computes the greatest common (positive) divisor of x and y and their Bézout coefficients, i.e. the integer coefficients u and v that satisfy ux + vy = d = gcd(x, y).

```
julia> gcdx(12, 42)
(6,-3,1)
```

```
julia> gcdx(240, 46)
(2,-9,47)
```

### note

Bézout coefficients are *not* uniquely defined. gcdx returns the minimal Bézout coefficients that are computed by the extended Euclid algorithm. (Ref: D. Knuth, TAoCP, 2/e, p. 325, Algorithm X.) These coefficients u and v are minimal in the sense that  $|u| < |\frac{y}{d}$  and  $|v| < |\frac{x}{d}$ . Furthermore, the signs of u and v are chosen so that d is positive.

## $ispow2(n) \rightarrow Bool$

Test whether n is a power of two.

#### nextpow2 (n)

The smallest power of two not less than n. Returns 0 for n==0, and returns -nextpow2 (-n) for negative arguments.

## prevpow2 (n)

The largest power of two not greater than n. Returns 0 for n==0, and returns -prevpow2 (-n) for negative arguments.

#### nextpow(a, x)

The smallest  $a^n$  not less than x, where n is a non-negative integer. a must be greater than 1, and x must be greater than 0.

## prevpow(a, x)

The largest  $a^n$  not greater than x, where n is a non-negative integer. a must be greater than 1, and x must not be less than 1.

```
nextprod ([k_1, k_2, ...], n)
      Next integer not less than n that can be written as \prod k_i^{p_i} for integers p_1, p_2, etc.
      For a list of integers i1, i2, i3, find the smallest
      i1^n1 * i2^n2 * i3^n3 >= x
      for integer n1, n2, n3
invmod(x, m)
      Take the inverse of x modulo m: y such that xy = 1 \pmod{m}.
powermod(x, p, m)
      Compute x^p \pmod{m}.
gamma(x)
      Compute the gamma function of x.
lgamma(x)
      Compute the logarithm of the absolute value of gamma () (page 366) for Real x, while for Complex x it
      computes the logarithm of gamma (x).
lfact(x)
      Compute the logarithmic factorial of x
digamma(x)
      Compute the digamma function of x (the logarithmic derivative of gamma (x))
invdigamma(x)
      Compute the inverse digamma function of x.
trigamma(x)
      Compute the trigamma function of x (the logarithmic second derivative of gamma (x)).
polygamma(m, x)
      Compute the polygamma function of order m of argument x (the (m+1) th derivative of the logarithm of
      gamma(x))
airy(k, x)
      The kth derivative of the Airy function Ai(x).
airyai(x)
      Airy function Ai(x).
airyprime(x)
      Airy function derivative Ai'(x).
airyaiprime(x)
      Airy function derivative Ai'(x).
airybi(x)
      Airy function Bi(x).
airybiprime(x)
      Airy function derivative Bi'(x).
airyx(k, x)
      scaled kth derivative of the Airy function, return \operatorname{Ai}(x)e^{\frac{2}{3}x\sqrt{x}} for k == 0 \mid \mid k == 1, and
      \operatorname{Ai}(x)e^{-\left|\operatorname{Re}\left(\frac{2}{3}x\sqrt{x}\right)\right|} for k == 2 | | k == 3.
```

besselj0(x)

Bessel function of the first kind of order 0,  $J_0(x)$ .

besselj1(x)

Bessel function of the first kind of order 1,  $J_1(x)$ .

besselj(nu, x)

Bessel function of the first kind of order nu,  $J_{\nu}(x)$ .

besseljx(nu, x)

Scaled Bessel function of the first kind of order nu,  $J_{\nu}(x)e^{-|\operatorname{Im}(x)|}$ .

bessely0 (x)

Bessel function of the second kind of order 0,  $Y_0(x)$ .

bessely1(x)

Bessel function of the second kind of order 1,  $Y_1(x)$ .

 $\verb"bessely"\,(nu,x)$ 

Bessel function of the second kind of order nu,  $Y_{\nu}(x)$ .

besselyx(nu, x)

Scaled Bessel function of the second kind of order nu,  $Y_{\nu}(x)e^{-|\operatorname{Im}(x)|}$ .

hankelh1(nu, x)

Bessel function of the third kind of order nu,  $H_{\nu}^{(1)}(x)$ .

hankelh1x(nu, x)

Scaled Bessel function of the third kind of order nu,  $H_{\nu}^{(1)}(x)e^{-xi}$ .

hankelh2(nu, x)

Bessel function of the third kind of order nu,  $H_{\nu}^{(2)}(x)$ .

hankelh2x(nu,x)

Scaled Bessel function of the third kind of order nu,  $H_{\nu}^{(2)}(x)e^{xi}$ .

besselh(nu, k, x)

Bessel function of the third kind of order nu (Hankel function). k is either 1 or 2, selecting hankelh1 or hankelh2, respectively.

besseli(nu, x)

Modified Bessel function of the first kind of order nu,  $I_{\nu}(x)$ .

besselix(nu,x)

Scaled modified Bessel function of the first kind of order nu,  $I_{\nu}(x)e^{-|\operatorname{Re}(x)|}$ .

besselk(nu, x)

Modified Bessel function of the second kind of order nu,  $K_{\nu}(x)$ .

 $\verb|besselkx| (nu, x)$ 

Scaled modified Bessel function of the second kind of order nu,  $K_{\nu}(x)e^{x}$ .

beta(x, y)

Euler integral of the first kind  $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$ .

lbeta(x, y)

Natural logarithm of the absolute value of the beta function log(|B(x, y)|).

eta(x)

Dirichlet eta function  $\eta(s) = \sum_{n=1}^{\infty} (-)^{n-1}/n^s$ .

zeta(s)

Riemann zeta function  $\zeta(s)$ .

zeta (s. z

Hurwitz zeta function  $\zeta(s,z)$ . (This is equivalent to the Riemann zeta function  $\zeta(s)$  for the case of z=1.)

#### ndigits(n, b)

Compute the number of digits in number n written in base b.

## widemul(x, y)

Multiply x and y, giving the result as a larger type.

## @evalpoly(z, c...)

Evaluate the polynomial  $\sum_k c[k]z^{k-1}$  for the coefficients c[1], c[2], ...; that is, the coefficients are given in ascending order by power of z. This macro expands to efficient inline code that uses either Horner's method or, for complex z, a more efficient Goertzel-like algorithm.

## 37.3 Statistics

## mean(v[, region])

Compute the mean of whole array v, or optionally along the dimensions in region. Note: Julia does not ignore NaN values in the computation. For applications requiring the handling of missing data, the DataArray package is recommended.

### mean!(r, v)

Compute the mean of v over the singleton dimensions of r, and write results to r.

## std(v[, region])

Compute the sample standard deviation of a vector or array v, optionally along dimensions in region. The algorithm returns an estimator of the generative distribution's standard deviation under the assumption that each entry of v is an IID drawn from that generative distribution. This computation is equivalent to calculating sqrt (sum((v - mean(v)).^2) / (length(v) - 1)). Note: Julia does not ignore NaN values in the computation. For applications requiring the handling of missing data, the DataArray package is recommended.

#### stdm(v, m)

Compute the sample standard deviation of a vector v with known mean m. Note: Julia does not ignore NaN values in the computation.

## var(v[, region])

Compute the sample variance of a vector or array v, optionally along dimensions in region. The algorithm will return an estimator of the generative distribution's variance under the assumption that each entry of v is an IID drawn from that generative distribution. This computation is equivalent to calculating sumabs2 (v - mean(v)) / (length(v) - 1). Note: Julia does not ignore NaN values in the computation. For applications requiring the handling of missing data, the DataArray package is recommended.

## varm(v, m)

Compute the sample variance of a vector v with known mean m. Note: Julia does not ignore NaN values in the computation.

#### middle(x)

Compute the middle of a scalar value, which is equivalent to x itself, but of the type of middle (x, x) for consistency.

### middle(x, y)

Compute the middle of two reals x and y, which is equivalent in both value and type to computing their mean ((x + y) / 2).

#### middle(range)

Compute the middle of a range, which consists in computing the mean of its extrema. Since a range is sorted, the mean is performed with the first and last element.

## middle(array)

Compute the middle of an array, which consists in finding its extrema and then computing their mean.

## median(v[, region])

Compute the median of whole array v, or optionally along the dimensions in region. For even number of elements no exact median element exists, so the result is equivalent to calculating mean of two median elements. NaN is returned if the data contains any NaN values. For applications requiring the handling of missing data, the DataArrays package is recommended.

### median!(v)

Like median, but may overwrite the input vector.

## **hist** $(v[, n]) \rightarrow e$ , counts

Compute the histogram of v, optionally using approximately n bins. The return values are a range e, which correspond to the edges of the bins, and counts containing the number of elements of v in each bin. Note: Julia does not ignore NaN values in the computation.

### **hist** $(v, e) \rightarrow e$ , counts

Compute the histogram of v using a vector/range e as the edges for the bins. The result will be a vector of length length (e) -1, such that the element at location i satisfies  $sum(e[i] \cdot v \cdot = e[i+1])$ . Note: Julia does not ignore NaN values in the computation.

#### **hist!** (*counts*, v, e) $\rightarrow$ e, counts

Compute the histogram of v, using a vector/range e as the edges for the bins. This function writes the resultant counts to a pre-allocated array counts.

## $hist2d(M, e1, e2) \rightarrow (edge1, edge2, counts)$

Compute a "2d histogram" of a set of N points specified by N-by-2 matrix M. Arguments e1 and e2 are bins for each dimension, specified either as integer bin counts or vectors of bin edges. The result is a tuple of edge1 (the bin edges used in the first dimension), edge2 (the bin edges used in the second dimension), and counts, a histogram matrix of size (length (edge1) -1, length (edge2) -1). Note: Julia does not ignore NaN values in the computation.

## **hist2d!** (*counts*, M, e1, e2) -> (e1, e2, *counts*)

Compute a "2d histogram" with respect to the bins delimited by the edges given in e1 and e2. This function writes the results to a pre-allocated array counts.

### histrange(v, n)

Compute *nice* bin ranges for the edges of a histogram of v, using approximately n bins. The resulting step sizes will be 1, 2 or 5 multiplied by a power of 10. Note: Julia does not ignore NaN values in the computation.

#### midpoints(e)

Compute the midpoints of the bins with edges e. The result is a vector/range of length length (e) - 1. Note: Julia does not ignore NaN values in the computation.

## quantile (v, p; sorted=false)

Compute the quantile(s) of a vector v at a specified probability or vector p. The keyword argument sorted indicates whether v can be assumed to be sorted.

The p should be on the interval [0,1], and v should not have any NaN values.

Quantiles are computed via linear interpolation between the points ((k-1)/(n-1), v[k]), for k=1:n where n=length(v). This corresponds to Definition 7 of Hyndman and Fan (1996), and is the same as the R default.

•Hyndman, R.J and Fan, Y. (1996) "Sample Quantiles in Statistical Packages", *The American Statistician*, Vol. 50, No. 4, pp. 361-365

# quantile! ([q], v, p; sorted=false)

Compute the quantile(s) of a vector v at the probabilities p, with optional output into array q (if not provided, a new output array is created). The keyword argument sorted indicates whether v can be assumed to be sorted; if false (the default), then the elements of v may be partially sorted.

The elements of p should be on the interval [0,1], and v should not have any NaN values.

37.3. Statistics 371

Quantiles are computed via linear interpolation between the points ((k-1)/(n-1), v[k]), for k = 1:n where n = length(v). This corresponds to Definition 7 of Hyndman and Fan (1996), and is the same as the R default.

•Hyndman, R.J and Fan, Y. (1996) "Sample Quantiles in Statistical Packages", *The American Statistician*, Vol. 50, No. 4, pp. 361-365

cov(x[, corrected=true])

Compute the variance of the vector x. If corrected is true (the default) then the sum is scaled with n-1 wheares the sum is scaled with n if corrected is false where n = length(x).

cov(X[, vardim=1, corrected=true])

Compute the covariance matrix of the matrix X along the dimension vardim. If corrected is true (the default) then the sum is scaled with n-1 wheares the sum is scaled with n if corrected is false where n = size(X, vardim).

cov(x, y[, corrected=true])

Compute the covariance between the vectors x and y. If corrected is true (the default) then the sum is scaled with n-1 wheares the sum is scaled with n if corrected is false where n = length(x) = length(y).

cov(X, Y[, vardim=1, corrected=true])

Compute the covariance between the vectors or matrices X and Y along the dimension vardim. If corrected is true (the default) then the sum is scaled with n-1 wheares the sum is scaled with n if corrected is false where n = size(X, vardim) = size(Y, vardim).

cor(x)

Return the number one.

cor(X[, vardim=1])

Compute the Pearson correlation matrix of the matrix X along the dimension vardim.

cor(x, y)

Compute the Pearson correlation between the vectors x and y.

cor(X, Y[, vardim=1])

Compute the Pearson correlation between the vectors or matrices X and Y along the dimension vardim.

# 37.4 Signal Processing

Fast Fourier transform (FFT) functions in Julia are implemented by calling functions from FFTW. By default, Julia does not use multi-threaded FFTW. Higher performance may be obtained by experimenting with multi-threading. Use FFTW.set\_num\_threads(np) to use np threads.

**fft** (A[, dims])

Performs a multidimensional FFT of the array A. The optional dims argument specifies an iterable subset of dimensions (e.g. an integer, range, tuple, or array) to transform along. Most efficient if the size of A along the transformed dimensions is a product of small primes; see <code>nextprod()</code>. See also <code>plan\_ffft()</code> for even greater efficiency.

A one-dimensional FFT computes the one-dimensional discrete Fourier transform (DFT) as defined by

$$\mathrm{DFT}(A)[k] = \sum_{n=1}^{\mathrm{length}(A)} \exp\left(-i\frac{2\pi(n-1)(k-1)}{\mathrm{length}(A)}\right) A[n].$$

A multidimensional FFT simply performs this operation along each transformed dimension of A.

Higher performance is usually possible with multi-threading. Use FFTW.set\_num\_threads(np) to use np threads, if you have np processors.

fft! (A[, dims])

Same as fft () (page 370), but operates in-place on A, which must be an array of complex floating-point numbers.

ifft(A[, dims])

Multidimensional inverse FFT.

A one-dimensional inverse FFT computes

$$IDFT(A)[k] = \frac{1}{\operatorname{length}(A)} \sum_{n=1}^{\operatorname{length}(A)} \exp\left(+i\frac{2\pi(n-1)(k-1)}{\operatorname{length}(A)}\right) A[n].$$

A multidimensional inverse FFT simply performs this operation along each transformed dimension of A.

ifft! (A[, dims])

Same as ifft () (page 371), but operates in-place on A.

bfft(A[,dims])

Similar to *ifft()* (page 371), but computes an unnormalized inverse (backward) transform, which must be divided by the product of the sizes of the transformed dimensions in order to obtain the inverse. (This is slightly more efficient than *ifft()* (page 371) because it omits a scaling step, which in some applications can be combined with other computational steps elsewhere.)

$$BDFT(A)[k] = length(A) IDFT(A)[k]$$

bfft! (A[, dims])

Same as bfft () (page 371), but operates in-place on A.

plan\_fft (A [, dims]; flags=FFTW.ESTIMATE; timelimit=Inf)

Pre-plan an optimized FFT along given dimensions (dims) of arrays matching the shape and type of A. (The first two arguments have the same meaning as for fft () (page 370).) Returns an object P which represents the linear operator computed by the FFT, and which contains all of the information needed to compute fft (A, dims) quickly.

To apply P to an array A, use P  $\star$  A; in general, the syntax for applying plans is much like that of matrices. (A plan can only be applied to arrays of the same size as the A for which the plan was created.) You can also apply a plan with a preallocated output array  $\hat{A}$  by calling A\_mul\_B! ( $\hat{A}$ , plan, A). You can compute the inverse-transform plan by inv (P) and apply the inverse plan with P \  $\hat{A}$  (the inverse plan is cached and reused for subsequent calls to inv or \), and apply the inverse plan to a pre-allocated output array A with A ldiv B! (A, P,  $\hat{A}$ ).

The flags argument is a bitwise-or of FFTW planner flags, defaulting to FFTW.ESTIMATE. e.g. passing FFTW.MEASURE or FFTW.PATIENT will instead spend several seconds (or more) benchmarking different possible FFT algorithms and picking the fastest one; see the FFTW manual for more information on planner flags. The optional timelimit argument specifies a rough upper bound on the allowed planning time, in seconds. Passing FFTW.MEASURE or FFTW.PATIENT may cause the input array A to be overwritten with zeros during plan creation.

plan\_fft!() (page 372) is the same as plan\_fft() (page 371) but creates a plan that operates in-place on its argument (which must be an array of complex floating-point numbers). plan\_ifft() (page 371) and so on are similar but produce plans that perform the equivalent of the inverse transforms ifft() (page 371) and so on.

plan\_ifft (A [, dims]; flags=FFTW.ESTIMATE; timelimit=Inf)

Same as plan\_fft () (page 371), but produces a plan that performs inverse transforms ifft () (page 371).

plan\_bfft (A [, dims]; flags=FFTW.ESTIMATE; timelimit=Inf)

Same as  $plan_{fft}()$  (page 371), but produces a plan that performs an unnormalized backwards transform bfft() (page 371).

## rfft(A[, dims])

Multidimensional FFT of a real array A, exploiting the fact that the transform has conjugate symmetry in order to save roughly half the computational time and storage costs compared with fft () (page 370). If A has size  $(n_1, \ldots, n_d)$ , the result has size  $(div(n_1, 2) + 1, \ldots, n_d)$ .

The optional dims argument specifies an iterable subset of one or more dimensions of A to transform, similar to fft () (page 370). Instead of (roughly) halving the first dimension of A in the result, the dims [1] dimension is (roughly) halved in the same way.

## irfft(A, d[, dims])

Inverse of rfft() (page 372): for a complex array A, gives the corresponding real array whose FFT yields A in the first half. As for rfft() (page 372), dims is an optional subset of dimensions to transform, defaulting to 1:ndims (A).

d is the length of the transformed real array along the dims[1] dimension, which must satisfy div(d,2)+1 == size(A,dims[1]). (This parameter cannot be inferred from size(A) since both 2\*size(A,dims[1])-2 as well as 2\*size(A,dims[1])-1 are valid sizes for the transformed real array.)

## brfft(A, d[, dims])

Similar to *irfft()* (page 372) but computes an unnormalized inverse transform (similar to *bfft()* (page 371)), which must be divided by the product of the sizes of the transformed dimensions (of the real output array) in order to obtain the inverse transform.

## plan\_rfft (A [, dims]; flags=FFTW.ESTIMATE; timelimit=Inf)

Pre-plan an optimized real-input FFT, similar to  $plan_ffft()$  (page 371) except for rfft() (page 372) instead of fft() (page 370). The first two arguments, and the size of the transformed result, are the same as for rfft() (page 372).

## plan\_brfft (A, d [, dims]; flags=FFTW.ESTIMATE; timelimit=Inf)

Pre-plan an optimized real-input unnormalized transform, similar to plan\_rfft() (page 372) except for brfft() (page 372) instead of rfft() (page 372). The first two arguments and the size of the transformed result, are the same as for brfft() (page 372).

#### plan irfft(A, d[, dims]; flags=FFTW.ESTIMATE; timelimit=Inf)

Pre-plan an optimized inverse real-input FFT, similar to <code>plan\_rfft()</code> (page 372) except for <code>irfft()</code> (page 372) and <code>brfft()</code> (page 372), respectively. The first three arguments have the same meaning as for <code>irfft()</code> (page 372).

## dct(A[,dims])

Performs a multidimensional type-II discrete cosine transform (DCT) of the array A, using the unitary normalization of the DCT. The optional dims argument specifies an iterable subset of dimensions (e.g. an integer, range, tuple, or array) to transform along. Most efficient if the size of A along the transformed dimensions is a product of small primes; see <code>nextprod()</code> (page 365). See also <code>plan\_dct()</code> (page 373) for even greater efficiency.

## dct!(A[,dims])

Same as dct! () (page 372), except that it operates in-place on A, which must be an array of real or complex floating-point values.

```
idct(A[, dims])
```

Computes the multidimensional inverse discrete cosine transform (DCT) of the array A (technically, a type-III DCT with the unitary normalization). The optional dims argument specifies an iterable subset of dimensions (e.g. an integer, range, tuple, or array) to transform along. Most efficient if the size of A along the transformed dimensions is a product of small primes; see <code>nextprod()</code> (page 365). See also <code>plan\_idct()</code> (page 373) for even greater efficiency.

## idct!(A[,dims])

Same as idct! () (page 373), but operates in-place on A.

## $plan_dct(A[, dims[, flags[, timelimit]]])$

Pre-plan an optimized discrete cosine transform (DCT), similar to plan\_fft() (page 371) except producing a function that computes dct() (page 372). The first two arguments have the same meaning as for dct() (page 372).

## plan\_dct! (A[, dims[, flags[, timelimit]]])

Same as plan\_dct () (page 373), but operates in-place on A.

```
plan_idct (A[, dims[, flags[, timelimit]]])
```

Pre-plan an optimized inverse discrete cosine transform (DCT), similar to plan\_fft() (page 371) except producing a function that computes idct() (page 372). The first two arguments have the same meaning as for idct() (page 372).

## plan\_idct! (A[, dims[, flags[, timelimit]]])

Same as plan\_idct() (page 373), but operates in-place on A.

#### fftshift(x)

Swap the first and second halves of each dimension of x.

#### **fftshift** (x, dim)

Swap the first and second halves of the given dimension of array x.

## ifftshift(x[,dim])

Undoes the effect of fft.shift.

## **filt** (b, a, x[, si])

Apply filter described by vectors a and b to vector x, with an optional initial filter state vector si (defaults to zeros).

## filt! (out, b, a, x[, si])

Same as filt () (page 373) but writes the result into the out argument, which may alias the input x to modify it in-place.

### deconv(b, a)

Construct vector c such that b = conv(a, c) + r. Equivalent to polynomial division.

#### conv(u, v)

Convolution of two vectors. Uses FFT algorithm.

## conv2(u, v, A)

2-D convolution of the matrix  $\mathbb A$  with the 2-D separable kernel generated by the vectors  $\mathbb u$  and  $\mathbb v$ . Uses 2-D FFT algorithm.

#### conv2(B, A)

2-D convolution of the matrix B with the matrix A. Uses 2-D FFT algorithm.

#### xcorr(u, v)

Compute the cross-correlation of two vectors.

The following functions are defined within the  ${\tt Base.FFTW}$  module.

```
r2r(A, kind[, dims])
```

Performs a multidimensional real-input/real-output (r2r) transform of type kind of the array A, as defined in the FFTW manual. kind specifies either a discrete cosine transform of various types (FFTW.REDFT00, FFTW.REDFT01, FFTW.REDFT11), a discrete sine transform of various types (FFTW.RODFT01, FFTW.RODFT01, FFTW.RODFT11), a real-input DFT with halfcomplex-format output (FFTW.R2HC and its inverse FFTW.HC2R), or a discrete Hartley transform (FFTW.DHT). The kind argument may be an array or tuple in order to specify different transform types along the different dimensions of A; kind[end] is used for any unspecified dimensions. See the FFTW manual for precise definitions of these transform types, at http://www.fftw.org/doc.

The optional dims argument specifies an iterable subset of dimensions (e.g. an integer, range, tuple, or array) to transform along. kind[i] is then the transform type for dims[i], with kind[end] being used for i > length(kind).

See also plan\_r2r() (page 374) to pre-plan optimized r2r transforms.

```
r2r! (A, kind, dims)
```

Same as r2r() (page 373), but operates in-place on A, which must be an array of real or complex floating-point numbers.

```
plan_r2r (A, kind[, dims[, flags[, timelimit]]])
```

Pre-plan an optimized r2r transform, similar to  $Base.plan_fft()$  (page 371) except that the transforms (and the first three arguments) correspond to r2r() (page 373) and r2r() (page 374), respectively.

```
plan_r2r! (A, kind[, dims[, flags[, timelimit]]])
Similar to Base.plan_fft() (page 371), but corresponds to r2r! () (page 374).
```

# 37.5 Numerical Integration

Although several external packages are available for numeric integration and solution of ordinary differential equations, we also provide some built-in integration support in Julia.

```
quadqk (f, a, b, c...; reltol=sqrt(eps), abstol=0, maxevals=10^7, order=7, norm=vecnorm)
```

Numerically integrate the function f(x) from a to b, and optionally over additional intervals b to c and so on. Keyword options include a relative error tolerance reltol (defaults to sqrt (eps) in the precision of the endpoints), an absolute error tolerance abstol (defaults to 0), a maximum number of function evaluations maxevals (defaults to  $10^7$ ), and the order of the integration rule (defaults to 7).

Returns a pair (I,E) of the estimated integral I and an estimated upper bound on the absolute error E. If maxevals is not exceeded then  $E \le \max(abstol, reltol*norm(I))$  will hold. (Note that it is useful to specify a positive abstol in cases where norm(I) may be zero.)

The endpoints a etcetera can also be complex (in which case the integral is performed over straight-line segments in the complex plane). If the endpoints are <code>BigFloat</code>, then the integration will be performed in <code>BigFloat</code> precision as well (note: it is advisable to increase the integration <code>order</code> in rough proportion to the precision, for smooth integrands). More generally, the precision is set by the precision of the integration endpoints (promoted to floating-point types).

The integrand f (x) can return any numeric scalar, vector, or matrix type, or in fact any type supporting +, -, multiplication by real values, and a norm (i.e., any normed vector space). Alternatively, a different norm can be specified by passing a norm-like function as the norm keyword argument (which defaults to vecnorm).

[Only one-dimensional integrals are provided by this function. For multi-dimensional integration (cubature), there are many different algorithms (often much better than simple nested 1d integrals) and the optimal choice tends to be very problem-dependent. See the Julia external-package listing for available algorithms for multidimensional integration or other specialized tasks (such as integrals of highly oscillatory or singular functions).]

The algorithm is an adaptive Gauss-Kronrod integration technique: the integral in each interval is estimated using a Kronrod rule (2\*order+1 points) and the error is estimated using an embedded Gauss rule (order points). The interval with the largest error is then subdivided into two intervals and the process is repeated until the desired error tolerance is achieved.

These quadrature rules work best for smooth functions within each interval, so if your function has a known discontinuity or other singularity, it is best to subdivide your interval to put the singularity at an endpoint. For example, if f has a discontinuity at x=0.7 and you want to integrate from 0 to 1, you should use quadgk (f, 0, 0.7, 1) to subdivide the interval at the point of discontinuity. The integrand is never evaluated exactly at the endpoints of the intervals, so it is possible to integrate functions that diverge at the endpoints as long as the singularity is integrable (for example, a log(x) or 1/sgrt(x) singularity).

For real-valued endpoints, the starting and/or ending points may be infinite. (A coordinate transformation is performed internally to map the infinite interval to a finite one.)

## **Numbers**

# 38.1 Standard Numeric Types

Bool Int8 UInt8 Int16 UInt16 Int32 UInt32 Int64 UInt64 Int128 UInt128 Float16 Float32 Float64 Complex64 Complex128

## 38.2 Data Formats

 $\mathbf{bin}\,(n\big[,pad\,\big])$ 

Convert an integer to a binary string, optionally specifying a number of digits to pad to.

 $\mathbf{hex}(n[,pad])$ 

Convert an integer to a hexadecimal string, optionally specifying a number of digits to pad to.

dec(n[,pad])

Convert an integer to a decimal string, optionally specifying a number of digits to pad to.

oct (n[, pad])

Convert an integer to an octal string, optionally specifying a number of digits to pad to.

**base** (base, n[, pad])

Convert an integer to a string in the given base, optionally specifying a number of digits to pad to. The base can be specified as either an integer, or as a UInt8 array of character values to use as digit symbols.

digits([T], n[, base][, pad])

Returns an array with element type T (default Int) of the digits of n in the given base, optionally padded with zeros to a specified size. More significant digits are at higher indexes, such that  $n = sum([digits[k]*base^(k-1) for k=1:length(digits)])$ .

digits! (array, n[, base])

Fills an array of the digits of n in the given base. More significant digits are at higher indexes. If the array length is insufficient, the least significant digits are filled up to the array length. If the array length is excessive, the excess portion is filled with zeros.

 $\mathtt{bits}\,(n)$ 

A string giving the literal bit representation of a number.

parse (type, str |, base |)

Parse a string as a number. If the type is an integer type, then a base can be specified (the default is 10). If the type is a floating point type, the string is parsed as a decimal floating point number. If the string does not contain a valid number, an error is raised.

## tryparse (type, str[, base])

Like parse, but returns a Nullable of the requested type. The result will be null if the string does not contain a valid number.

#### $\mathbf{big}(x)$

Convert a number to a maximum precision representation (typically BigInt or BigFloat). See BigFloat for information about some pitfalls with floating-point numbers.

#### signed(x)

Convert a number to a signed integer. If the argument is unsigned, it is reinterpreted as signed without checking for overflow.

## **unsigned** $(x) \rightarrow \text{Unsigned}$

Convert a number to an unsigned integer. If the argument is signed, it is reinterpreted as unsigned without checking for negative values.

#### float(x)

Convert a number, array, or string to a AbstractFloat data type. For numeric data, the smallest suitable AbstractFloat type is used. Converts strings to Float64.

#### significand(x)

Extract the significand(s) (a.k.a. mantissa), in binary representation, of a floating-point number or array. If x is a non-zero finite number, than the result will be a number of the same type on the interval [1, 2). Otherwise x is returned.

```
julia> significand(15.2)/15.2
0.125

julia> significand(15.2)*8
15.2
```

## **exponent** $(x) \rightarrow Int$

Get the exponent of a normalized floating-point number.

## complex(r|,i|)

Convert real numbers or arrays to complex. i defaults to zero.

### bswap(n)

Byte-swap an integer.

## num2hex(f)

Get a hexadecimal string of the binary representation of a floating point number.

#### hex2num(str)

Convert a hexadecimal string to the floating point number it represents.

#### hex2bytes (s::ASCIIString)

Convert an arbitrarily long hexadecimal string to its binary representation. Returns an  $Array{UInt8,1}$ , i.e. an array of bytes.

## bytes2hex (bin\_arr::Array{UInt8, 1})

Convert an array of bytes to its hexadecimal representation. All characters are in lower-case. Returns an ASCIIString.

## 38.3 General Number Functions and Constants

### one (x)

Get the multiplicative identity element for the type of x (x can also specify the type itself). For matrices, returns an identity matrix of the appropriate size and type.

```
zero(x)
      Get the additive identity element for the type of x (x can also specify the type itself).
рi
      The constant pi
im
      The imaginary unit
е
      The constant e
catalan
      Catalan's constant
eulergamma
      Euler's constant
golden
      The golden ratio
Inf
      Positive infinity of type Float 64
Inf32
      Positive infinity of type Float 32
Inf16
      Positive infinity of type Float16
NaN
      A not-a-number value of type Float 64
NaN32
      A not-a-number value of type Float 32
NaN16
      A not-a-number value of type Float16
issubnormal(f) \rightarrow Bool
      Test whether a floating point number is subnormal.
isfinite(f) \rightarrow Bool
      Test whether a number is finite
isinf(f) \rightarrow Bool
      Test whether a number is infinite.
isnan(f) \rightarrow Bool
      Test whether a floating point number is not a number (NaN).
inf(f)
      Returns positive infinity of the floating point type f or of the same floating point type as f.
nan(f)
      Returns NaN (not-a-number) of the floating point type f or of the same floating point type as f
nextfloat(f)
      Get the next floating point number in lexicographic order.
```

```
prevfloat(f) \rightarrow AbstractFloat
```

Get the previous floating point number in lexicographic order.

```
isinteger(x) \rightarrow Bool
```

Test whether x or all its elements are numerically equal to some integer

```
isreal(x) \rightarrow Bool
```

Test whether x or all its elements are numerically equal to some real number.

```
Float32 (x[, mode::RoundingMode])
```

Create a Float32 from x. If x is not exactly representable then mode determines how x is rounded.

```
julia> Float32(1/3, RoundDown)
0.333333336

julia> Float32(1/3, RoundUp)
0.333333334f0
```

See rounding for available rounding modes.

## Float 64 (x, mode::Rounding Mode)

Create a Float64 from x. If x is not exactly representable then mode determines how x is rounded.

```
julia> Float64(pi, RoundDown)
3.141592653589793

julia> Float64(pi, RoundUp)
3.1415926535897936
```

See rounding for available rounding modes.

### BigInt(x)

Create an arbitrary precision integer. x may be an Int (or anything that can be converted to an Int). The usual mathematical operators are defined for this type, and results are promoted to a BigInt.

Instances can be constructed from strings via parse () (page 326), or using the big string literal.

#### BigFloat (x)

Create an arbitrary precision floating point number. x may be an Integer, a Float64 or a BigInt. The usual mathematical operators are defined for this type, and results are promoted to a BigFloat.

Note that because decimal literals are converted to floating point numbers when parsed, BigFloat (2.1) may not yield what you expect. You may instead prefer to initialize constants from strings via parse () (page 326), or using the big string literal.

#### rounding (T)

Get the current floating point rounding mode for type T, controlling the rounding of basic arithmetic functions (+()) (page 353), -()) (page 353), +()) (page 353) and sqrt()) (page 364)) and type conversion.

Valid modes are RoundNearest, RoundToZero, RoundUp, RoundDown, and RoundFromZero (BigFloat only).

## setrounding(T, mode)

Set the rounding mode of floating point type T, controlling the rounding of basic arithmetic functions (+() (page 353), -() (page 353), \*() (page 385), /() (page 353) and sqrt() (page 364)) and type conversion.

Note that this may affect other types, for instance changing the rounding mode of Float 64 will change the rounding mode of Float 32. See rounding for available modes

## setrounding (f::Function, T, mode)

Change the rounding mode of floating point type T for the duration of f. It is logically equivalent to:

```
old = rounding(T)
setrounding(T, mode)
f()
setrounding(T, old)
```

See rounding for available rounding modes.

## $\texttt{get}\_\texttt{zero}\_\texttt{subnormals}\,(\,) \, \to Bool$

Returns false if operations on subnormal floating-point values ("denormals") obey rules for IEEE arithmetic, and true if they might be converted to zeros.

## $set\_zero\_subnormals(yes::Bool) \rightarrow Bool$

If yes is false, subsequent floating-point operations follow rules for IEEE arithmetic on subnormal values ("denormals"). Otherwise, floating-point operations are permitted (but not required) to convert subnormal inputs or outputs to zero. Returns true unless yes==true but the hardware does not support zeroing of subnormal numbers.

set\_zero\_subnormals(true) can speed up some computations on some hardware. However, it can break identities such as (x-y==0) == (x==y).

## 38.3.1 Integers

### $count\_ones(x::Integer) \rightarrow Integer$

Number of ones in the binary representation of x.

```
julia> count_ones(7)
3
```

#### $count\_zeros(x::Integer) \rightarrow Integer$

Number of zeros in the binary representation of x.

```
julia> count_zeros(Int32(2 ^ 16 - 1))
16
```

#### **leading\_zeros** (x::Integer) $\rightarrow$ Integer

Number of zeros leading the binary representation of x.

```
julia> leading_zeros(Int32(1))
31
```

## $leading\_ones(x::Integer) \rightarrow Integer$

Number of ones leading the binary representation of x.

```
julia> leading_ones(UInt32(2 ^ 32 - 2))
31
```

## $trailing\_zeros(x::Integer) \rightarrow Integer$

Number of zeros trailing the binary representation of x.

```
julia> trailing_zeros(2)
1
```

## $trailing\_ones(x::Integer) \rightarrow Integer$

Number of ones trailing the binary representation of x.

```
julia> trailing_ones(3)
2
```

## $isprime(x::Integer) \rightarrow Bool$

Returns true if x is prime, and false otherwise.

```
julia> isprime(3)
true
```

```
isprime(x::BigInt[, reps = 25]) \rightarrow Bool
```

Probabilistic primality test. Returns true if x is prime; and false if x is not prime with high probability. The false positive rate is about  $0.25^{\text{reps}}$ . reps = 25 is considered safe for cryptographic applications (Knuth, Seminumerical Algorithms).

```
julia> isprime(big(3))
true
```

## primes([lo], hi)

Returns a collection of the prime numbers (from 10, if specified) up to hi.

```
primesmask([lo], hi)
```

Returns a prime sieve, as a BitArray, of the positive integers (from lo, if specified) up to hi. Useful when working with either primes or composite numbers.

## $isodd(x::Integer) \rightarrow Bool$

Returns true if x is odd (that is, not divisible by 2), and false otherwise.

```
julia> isodd(9)
true

julia> isodd(10)
false
```

#### **iseven** (x::Integer) $\rightarrow$ Bool

Returns true is x is even (that is, divisible by 2), and false otherwise.

```
julia> iseven(9)
false

julia> iseven(10)
true
```

# 38.4 BigFloats

The BigFloat type implements arbitrary-precision floating-point arithmetic using the GNU MPFR library.

```
precision (num::AbstractFloat)
```

Get the precision of a floating point number, as defined by the effective number of bits in the mantissa.

### precision (BigFloat)

Get the precision (in bits) currently used for BigFloat arithmetic.

```
setprecision([T=BigFloat], precision::Int)
```

Set the precision (in bits) to be used for T arithmetic.

```
setprecision (f::Function[, T=BigFloat], precision::Integer)
```

Change the T arithmetic precision (in bits) for the duration of f. It is logically equivalent to:

```
old = precision(BigFloat)
setprecision(BigFloat, precision)
f()
setprecision(BigFloat, old)
```

Often used as setprecision (T, precision) do ... end

## 38.5 Random Numbers

Random number generation in Julia uses the Mersenne Twister library via MersenneTwister objects. Julia has a global RNG, which is used by default. Other RNG types can be plugged in by inheriting the AbstractRNG type; they can then be used to have multiple streams of random numbers. Besides MersenneTwister, Julia also provides the RandomDevice RNG type, which is a wrapper over the OS provided entropy.

Most functions related to random generation accept an optional AbstractRNG as the first argument, rng, which defaults to the global one if not provided. Morever, some of them accept optionally dimension specifications dims... (which can be given as a tuple) to generate arrays of random values.

A MersenneTwister or RandomDevice RNG can generate random numbers of the following types: Float16, Float32, Float64, Bool, Int8, UInt8, Int16, UInt16, Int32, UInt32, Int64, UInt64, Int128, UInt128, BigInt (or complex numbers of those types). Random floating point numbers are generated uniformly in [0,1). As BigInt represents unbounded integers, the interval must be specified (e.g. rand (big (1:6))).

```
srand([rng][, seed])
```

Reseed the random number generator. If a seed is provided, the RNG will give a reproducible sequence of numbers, otherwise Julia will get entropy from the system. For MersenneTwister, the seed may be a non-negative integer, a vector of UInt32 integers or a filename, in which case the seed is read from a file. RandomDevice does not support seeding.

# ${\tt MersenneTwister} ( [seed ] )$

Create a MersenneTwister RNG object. Different RNG objects can have their own seeds, which may be useful for generating different streams of random numbers.

#### RandomDevice()

Create a RandomDevice RNG object. Two such objects will always generate different streams of random numbers.

```
rand([rng][,S][,dims...])
```

Pick a random element or array of random elements from the set of values specified by S; S can be

```
•an indexable collection (for example 1:n or ['x','y','z']), or
```

•a type: the set of values to pick from is then equivalent to typemin(S): typemax(S) for integers (this is not applicable to BigInt), and to [0,1) for floating point numbers;

S defaults to Float 64.

```
rand! ([rng], A[, coll])
```

Populate the array A with random values. If the indexable collection coll is specified, the values are picked randomly from coll. This is equivalent to copy! (A, rand(rng, coll, size(A))) or copy! (A, rand(rng, eltype(A), size(A))) but without allocating a new array.

```
\mathtt{bitrand}\,(\big[\mathit{rng}\,\big]\big[,\mathit{dims}...\,\big])
```

Generate a BitArray of random boolean values.

```
\mathtt{randn}\,(\big[\mathit{rng}\,\big]\big[,\mathit{dims}...\,\big])
```

Generate a normally-distributed random number with mean 0 and standard deviation 1. Optionally generate an array of normally-distributed random numbers.

38.5. Random Numbers 385

randn! ([rng], A::Array{Float64, N})

Fill the array A with normally-distributed (mean 0, standard deviation 1) random numbers. Also see the rand function.

randexp([rng][, dims...])

Generate a random number according to the exponential distribution with scale 1. Optionally generate an array of such random numbers.

randexp! ([rng], A::Array{Float64, N})

Fill the array A with random numbers following the exponential distribution (with scale 1).

 $randjump(r::MersenneTwister, jumps[, jumppoly]) \rightarrow Vector\{MersenneTwister\}$ 

Create an array of the size jumps of initialized MersenneTwister RNG objects where the first RNG object given as a parameter and following MersenneTwister RNGs in the array initialized such that a state of the RNG object in the array would be moved forward (without generating numbers) from a previous RNG object array element on a particular number of steps encoded by the jump polynomial jumppoly.

Default jump polynomial moves forward MersenneTwister RNG state by 10^20 steps.

# **Strings**

## length(s)

The number of characters in string s.

## sizeof(s::AbstractString)

The number of bytes in string s.

#### $\star (s, t)$

Concatenate strings. The \* operator is an alias to this function.

```
julia> "Hello " * "world"
"Hello world"
```

#### $^{\wedge}(s,n)$

Repeat n times the string s. The repeat function is an alias to this operator.

```
julia> "Test "^3
"Test Test Test "
```

## string(xs...)

Create a string from any values using the print function.

### repr(x)

Create a string from any value using the showall function.

## bytestring (::Ptr{UInt8}[, length])

Create a string from the address of a C (0-terminated) string encoded in ASCII or UTF-8. A copy is made; the ptr can be safely freed. If length is specified, the string does not have to be 0-terminated.

## bytestring(s)

Convert a string to a contiguous byte array representation appropriate for passing it to C functions. The string will be encoded as either ASCII or UTF-8.

#### **ascii** (::Array{UInt8, 1})

Create an ASCII string from a byte array.

## ascii(s)

Convert a string to a contiguous ASCII string (all characters must be valid ASCII characters).

## ascii (::Ptr{UInt8}, length])

Create an ASCII string from the address of a C (0-terminated) string encoded in ASCII. A copy is made; the ptr can be safely freed. If length is specified, the string does not have to be 0-terminated.

## **utf8** (::Array{UInt8, 1})

Create a UTF-8 string from a byte array.

## utf8 (::Ptr{UInt8}[, length])

Create a UTF-8 string from the address of a C (0-terminated) string encoded in UTF-8. A copy is made; the ptr can be safely freed. If length is specified, the string does not have to be 0-terminated.

## utf8(s)

Convert a string to a contiguous UTF-8 string (all characters must be valid UTF-8 characters).

#### @r str -> Regex

Construct a regex, such as  $r"^[a-z] *$ . The regex also accepts one or more flags, listed after the ending quote, to change its behaviour:

- •i enables case-insensitive matching
- •m treats the ^ and \$ tokens as matching the start and end of individual lines, as opposed to the whole string.
- •s allows the . modifier to match newlines.
- •x enables "comment mode": whitespace is enabled except when escaped with \, and # is treated as starting a comment.

For example, this regex has all three flags enabled:

```
julia> match(r"a+.*b+.*?d$"ism, "Goodbye,\nOh, angry,\nBad world\n")
RegexMatch("angry,\nBad world")
```

#### @html\_str -> Docs.HTML

Create an HTML object from a literal string.

#### @text\_str -> Docs.Text

Create a Text object from a literal string.

#### normalize\_string(s, normalform::Symbol)

Normalize the string s according to one of the four "normal forms" of the Unicode standard: normalform can be :NFC, :NFKC, or :NFKC. Normal forms C (canonical composition) and D (canonical decomposition) convert different visually identical representations of the same abstract string into a single canonical form, with form C being more compact. Normal forms KC and KD additionally canonicalize "compatibility equivalents": they convert characters that are abstractly similar but visually distinct into a single canonical choice (e.g. they expand ligatures into the individual characters), with form KC being more compact.

Alternatively, finer control and additional transformations may be be obtained by calling normalize\_string(s; keywords...), where any number of the following boolean keywords options (which all default to false except for compose) are specified:

- compose=false: do not perform canonical composition
- •decompose=true: do canonical decomposition instead of canonical composition (compose=true is ignored if present)
- •compat=true: compatibility equivalents are canonicalized
- •casefold=true: perform Unicode case folding, e.g. for case-insensitive string comparison
- •newline2lf=true, newline2ls=true, or newline2ps=true: convert various newline sequences (LF, CRLF, CR, NEL) into a linefeed (LF), line-separation (LS), or paragraph-separation (PS) character, respectively
- •stripmark=true: strip diacritical marks (e.g. accents)
- •stripignore=true: strip Unicode's "default ignorable" characters (e.g. the soft hyphen or the left-to-right marker)
- •stripcc=true: strip control characters; horizontal tabs and form feeds are converted to spaces; new-lines are also converted to spaces unless a newline-conversion flag was specified

- •rejectna=true: throw an error if unassigned code points are found
- •stable=true: enforce Unicode Versioning Stability

For example, NFKC corresponds to the options compose=true, compat=true, stable=true.

## **graphemes** $(s) \rightarrow$ iterator over substrings of s

Returns an iterator over substrings of s that correspond to the extended graphemes in the string, as defined by Unicode UAX #29. (Roughly, these are what users would perceive as single characters, even though they may contain more than one codepoint; for example a letter combined with an accent mark is a single grapheme.)

### $isvalid(value) \rightarrow Bool$

Returns true if the given value is valid for its type, which currently can be one of Char, ASCIIString, UTF8String, UTF16String, or UTF32String.

### $isvalid(T, value) \rightarrow Bool$

Returns true if the given value is valid for that type. Types currently can be Char, ASCIIString, UTF8String, UTF16String, or UTF32String Values for Char can be of type Char or UInt32 Values for ASCIIString and UTF8String can be of that type, or Vector{UInt8} Values for UTF16String can be UTF16String or Vector{UInt16} Values for UTF32String can be UTF32String, Vector{Char} or Vector{UInt32}

#### isvalid(str, i)

Tells whether index i is valid for the given string.

## $is_assigned_char(c) \rightarrow Bool$

Returns true if the given char or integer is an assigned Unicode code point.

### $ismatch(r::Regex, s::AbstractString) \rightarrow Bool$

Test whether a string contains a match of the given regular expression.

## match (r::Regex, s::AbstractString[, idx::Integer[, addopts]])

Search for the first match of the regular expression r in s and return a RegexMatch object containing the match, or nothing if the match failed. The matching substring can be retrieved by accessing m.match and the captured sequences can be retrieved by accessing m.captures The optional idx argument specifies an index at which to start the search.

# $\verb| eachmatch| (r::Regex, s::AbstractString[, overlap::Bool=false])|$

Search for all matches of a the regular expression r in s and return a iterator over the matches. If overlap is true, the matching sequences are allowed to overlap indices in the original string, otherwise they must be from distinct character ranges.

**matchall** (r::Regex, s::AbstractString[, overlap::Bool=false[)  $\rightarrow$  Vector{AbstractString} Return a vector of the matching substrings from eachmatch.

### lpad(string, n, p)

Make a string at least n columns wide when printed, by padding on the left with copies of p.

#### rpad(string, n, p)

Make a string at least n columns wide when printed, by padding on the right with copies of p.

## search (string, chars, start)

Search for the first occurrence of the given characters within the given string. The second argument may be a single character, a vector or a set of characters, a string, or a regular expression (though regular expressions are only allowed on contiguous strings, such as ASCII or UTF-8 strings). The third argument optionally specifies a starting index. The return value is a range of indexes where the matching sequence is found, such that s[search(s,x)] = x:

```
search(string, "substring") = start:end such that string[start:end] ==
"substring", or 0:-1 if unmatched.
```

search (string, 'c') = index such that string[index] == 'c', or 0 if unmatched.

## rsearch (string, chars[, start])

Similar to search, but returning the last occurrence of the given characters within the given string, searching in reverse from start.

## searchindex (string, substring[, start])

Similar to search, but return only the start index at which the substring is found, or 0 if it is not.

## rsearchindex(string, substring[, start])

Similar to rsearch, but return only the start index at which the substring is found, or 0 if it is not.

## contains (haystack, needle)

Determine whether the second argument is a substring of the first.

### **reverse** (s::AbstractString) $\rightarrow$ AbstractString

Reverses a string.

## replace(string, pat, r[, n])

Search for the given pattern pat, and replace each occurrence with r. If n is provided, replace at most n occurrences. As with search, the second argument may be a single character, a vector or a set of characters, a string, or a regular expression. If r is a function, each occurrence is replaced with r(s) where s is the matched substring. If pat is a regular expression and r is a SubstitutionString, then capture group references in r are replaced with the corresponding matched text.

## split (string, [chars]; limit=0, keep=true)

Return an array of substrings by splitting the given string on occurrences of the given character delimiters, which may be specified in any of the formats allowed by search's second argument (i.e. a single character, collection of characters, string, or regular expression). If chars is omitted, it defaults to the set of all space characters, and keep is taken to be false. The two keyword arguments are optional: they are are a maximum size for the result and a flag determining whether empty fields should be kept in the result.

### rsplit (string, [chars]; limit=0, keep=true)

Similar to split, but starting from the end of the string.

## strip(string[, chars])

Return string with any leading and trailing whitespace removed. If chars (a character, or vector or set of characters) is provided, instead remove characters contained in it.

# lstrip(string[, chars])

Return string with any leading whitespace removed. If chars (a character, or vector or set of characters) is provided, instead remove characters contained in it.

# rstrip(string[, chars])

Return string with any trailing whitespace removed. If chars (a character, or vector or set of characters) is provided, instead remove characters contained in it.

### startswith (string, prefix | chars)

Returns true if string starts with prefix. If the second argument is a vector or set of characters, tests whether the first character of string belongs to that set.

## endswith (string, suffix | chars)

Returns true if string ends with suffix. If the second argument is a vector or set of characters, tests whether the last character of string belongs to that set.

#### uppercase (string)

Returns string with all characters converted to uppercase.

### lowercase(string)

Returns string with all characters converted to lowercase.

## ucfirst (string)

Returns string with the first character converted to uppercase.

#### lcfirst (string)

Returns string with the first character converted to lowercase.

## join (strings, delim | , last | )

Join an array of strings into a single string, inserting the given delimiter between adjacent strings. If last is given, it will be used instead of delim between the last two strings. For example

```
join(["apples", "bananas", "pineapples"], ", ", " and ") == "apples, bananas and pineapples"
```

strings can be any iterable over elements x which are convertible to strings via print (io::IOBuffer, x).

### chop (string)

Remove the last character from a string.

#### chomp (string)

Remove a trailing newline from a string.

#### ind2chr (string, i)

Convert a byte index to a character index.

#### chr2ind(string, i)

Convert a character index to a byte index.

#### nextind (str, i)

Get the next valid string index after i. Returns a value greater than endof (str) at or after the end of the string.

### prevind(str, i)

Get the previous valid string index before i. Returns a value less than 1 at the beginning of the string.

# randstring([rng], len=8)

Create a random ASCII string of length len, consisting of upper- and lower-case letters and the digits 0-9. The optional rng argument specifies a random number generator, see *Random Numbers* (page 383).

### charwidth(c)

Gives the number of columns needed to print a character.

### strwidth(s)

Gives the number of columns needed to print a string.

### $isalnum(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is alphanumeric, or whether this is true for all elements of a string. A character is classified as alphabetic if it belongs to the Unicode general category Letter or Number, i.e. a character whose category code begins with 'L' or 'N'.

### $isalpha(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is alphabetic, or whether this is true for all elements of a string. A character is classified as alphabetic if it belongs to the Unicode general category Letter, i.e. a character whose category code begins with 'L'.

### $isascii(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character belongs to the ASCII character set, or whether this is true for all elements of a string.

### $iscntrl(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is a control character, or whether this is true for all elements of a string. Control characters are the non-printing characters of the Latin-1 subset of Unicode.

### $isdigit(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is a numeric digit (0-9), or whether this is true for all elements of a string.

### $isgraph(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is printable, and not a space, or whether this is true for all elements of a string. Any character that would cause a printer to use ink should be classified with isgraph (c) ==true.

### $islower(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is a lowercase letter, or whether this is true for all elements of a string. A character is classified as lowercase if it belongs to Unicode category Ll, Letter: Lowercase.

### $isnumber(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is numeric, or whether this is true for all elements of a string. A character is classified as numeric if it belongs to the Unicode general category Number, i.e. a character whose category code begins with 'N'.

### $isprint(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is printable, including spaces, but not a control character. For strings, tests whether this is true for all elements of the string.

### $ispunct(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character belongs to the Unicode general category Punctuation, i.e. a character whose category code begins with 'P'. For strings, tests whether this is true for all elements of the string.

### $isspace(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is any whitespace character. Includes ASCII characters '\t', '\n', '\v', '\f', '\r', and '', Latin-1 character U+0085, and characters in Unicode category Zs. For strings, tests whether this is true for all elements of the string.

### $isupper(c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is an uppercase letter, or whether this is true for all elements of a string. A character is classified as uppercase if it belongs to Unicode category Lu, Letter: Uppercase, or Lt, Letter: Titlecase.

### $isxdigit (c::Union\{Char, AbstractString\}) \rightarrow Bool$

Tests whether a character is a valid hexadecimal digit, or whether this is true for all elements of a string.

### $symbol(x...) \rightarrow Symbol$

Create a Symbol by concatenating the string representations of the arguments together.

### **escape\_string**(*str::AbstractString*) → AbstractString

General escaping of traditional C and Unicode escape sequences. See print\_escaped() (page 452) for more general escaping.

### unescape\_string (s::AbstractString) → AbstractString

General unescaping of traditional C and Unicode escape sequences. Reverse of escape\_string() (page 390). See also print\_unescaped() (page 452).

#### utf16(s)

Create a UTF-16 string from a byte array, array of UInt16, or any other string type. (Data must be valid UTF-16. Conversions of byte arrays check for a byte-order marker in the first two bytes, and do not include it in the resulting string.)

Note that the resulting UTF16String data is terminated by the NUL codepoint (16-bit zero), which is not treated as a character in the string (so that it is mostly invisible in Julia); this allows the string to be passed directly to external functions requiring NUL-terminated data. This NUL is appended automatically by the utf16(s) conversion function. If you have a UInt16 array A that is already NUL-terminated valid UTF-16 data, then you can instead use UTF16String(A) to construct the string without making a copy of the data and treating the NUL as a terminator rather than as part of the string.

# **utf16** (::Union{Ptr{UInt16}, Ptr{Int16}}[, length])

Create a string from the address of a NUL-terminated UTF-16 string. A copy is made; the pointer can be safely freed. If length is specified, the string does not have to be NUL-terminated.

### utf32(s)

Create a UTF-32 string from a byte array, array of Char or UInt32, or any other string type. (Conversions of byte arrays check for a byte-order marker in the first four bytes, and do not include it in the resulting string.)

Note that the resulting UTF32String data is terminated by the NUL codepoint (32-bit zero), which is not treated as a character in the string (so that it is mostly invisible in Julia); this allows the string to be passed directly to external functions requiring NUL-terminated data. This NUL is appended automatically by the utf32(s) conversion function. If you have a Char or UInt32 array A that is already NUL-terminated UTF-32 data, then you can instead use UTF32String(A) to construct the string without making a copy of the data and treating the NUL as a terminator rather than as part of the string.

# utf32 (::Union{Ptr{Char}, Ptr{UInt32}, Ptr{Int32}}[, length])

Create a string from the address of a NUL-terminated UTF-32 string. A copy is made; the pointer can be safely freed. If length is specified, the string does not have to be NUL-terminated.

#### wstring(s)

This is a synonym for either utf32(s) or utf16(s), depending on whether Cwchar\_t is 32 or 16 bits, respectively. The synonym WString for UTF32String or UTF16String is also provided.

394 Chapter 39. Strings

# **Arrays**

# 40.1 Basic functions

```
ndims(A) \rightarrow Integer
```

Returns the number of dimensions of A.

```
size(A[,dim...])
```

Returns a tuple containing the dimensions of A. Optionally you can specify the dimension(s) you want the length of, and get the length of that dimension, or a tuple of the lengths of dimensions you asked for.:

```
julia> A = rand(2,3,4);

julia> size(A, 2)
3

julia> size(A,3,2)
(4,3)
```

### iseltype(A, T)

Tests whether A or its elements are of type T.

### **length** $(A) \rightarrow$ Integer

Returns the number of elements in A.

#### eachindex(A...)

Creates an iterable object for visiting each index of an AbstractArray A in an efficient manner. For array types that have opted into fast linear indexing (like Array), this is simply the range 1:length (A). For other array types, this returns a specialized Cartesian range to efficiently index into the array with indices specified for every dimension. For other iterables, including strings and dictionaries, this returns an iterator object supporting arbitrary index types (e.g. unevenly spaced or non-integer indices).

Example for a sparse 2-d array:

```
A[iter] = 1
  (iter.I[1],iter.I[2]) = (2,1)
A[iter] = -5
  (iter.I[1],iter.I[2]) = (1,2)
A[iter] = 0
  (iter.I[1],iter.I[2]) = (2,2)
A[iter] = 0
  (iter.I[1],iter.I[2]) = (1,3)
A[iter] = 2
  (iter.I[1],iter.I[2]) = (2,3)
A[iter] = 0
```

If you supply more than one AbstractArray argument, eachindex will create an iterable object that is fast for all arguments (a UnitRange if all inputs have fast linear indexing, a CartesianRange otherwise). If the arrays have different sizes and/or dimensionalities, eachindex returns an iterable that spans the largest range along each dimension.

#### Base.linearindexing (A)

linearindexing defines how an AbstractArray most efficiently accesses its elements. If Base.linearindexing(A) returns Base.LinearFast(), this means that linear indexing with only one index is an efficient operation. If it instead returns Base.LinearSlow() (by default), this means that the array intrinsically accesses its elements with indices specified for every dimension. Since converting a linear index to multiple indexing subscripts is typically very expensive, this provides a traits-based mechanism to enable efficient generic code for all array types.

An abstract array subtype MyArray that wishes to opt into fast linear indexing behaviors should define linearindexing in the type-domain:

```
Base.linearindexing{T<:MyArray}(::Type{T}) = Base.LinearFast()</pre>
```

#### countnz (A)

Counts the number of nonzero values in array A (dense or sparse). Note that this is not a constant-time operation. For sparse matrices, one should usually use nnz, which returns the number of stored values.

#### conj!(A)

Convert an array to its complex conjugate in-place.

### stride(A, k)

Returns the distance in memory (in number of elements) between adjacent elements in dimension k.

### strides(A)

Returns a tuple of the memory strides in each dimension.

#### **ind2sub** (dims, index) $\rightarrow$ subscripts

Returns a tuple of subscripts into an array with dimensions dims, corresponding to the linear index index.

**Example**: i, j, ... = ind2sub(size(A), indmax(A)) provides the indices of the maximum element

### **ind2sub** (a, index) $\rightarrow$ subscripts

Returns a tuple of subscripts into array a corresponding to the linear index index.

### $sub2ind(dims, i, j, k...) \rightarrow index$

The inverse of ind2 sub, returns the linear index corresponding to the provided subscripts.

### LinAlg.checksquare(A)

Check that a matrix is square, then return its common dimension. For multiple arguments, return a vector.

396 Chapter 40. Arrays

# 40.2 Constructors

### Array (dims)

Array (T) (dims) constructs an uninitialized dense array with element type T. dims may be a tuple or a series of integer arguments. The syntax Array (T, dims) is also available, but deprecated.

### getindex (type[, elements...])

Construct a 1-d array of the specified type. This is usually called with the syntax Type[]. Element values can be specified using Type[a,b,c,...].

### cell (dims)

Construct an uninitialized cell array (heterogeneous array). dims can be either a tuple or a series of integer arguments.

### zeros (type, dims)

Create an array of all zeros of specified type. The type defaults to Float64 if not specified.

#### zeros(A)

Create an array of all zeros with the same element type and shape as A.

#### ones (type, dims)

Create an array of all ones of specified type. The type defaults to Float 64 if not specified.

### ones(A)

Create an array of all ones with the same element type and shape as A.

#### trues (dims)

Create a BitArray with all values set to true.

#### falses (dims)

Create a BitArray with all values set to false.

#### **fill** (*x*, *dims*)

Create an array filled with the value x. For example, fill(1.0, (10,10)) returns a 10x10 array of floats, with each element initialized to 1.0.

If x is an object reference, all elements will refer to the same object. fill (Foo(), dims) will return an array filled with the result of evaluating Foo() once.

### fill! (A, x)

Fill array A with the value x. If x is an object reference, all elements will refer to the same object. fill! (A, Foo()) will return A filled with the result of evaluating Foo() once.

#### reshape (A, dims)

Create an array with the same data as the given array, but with different dimensions. An implementation for a particular type of array may choose whether the data is copied or shared.

# similar (array[, element\_type=eltype(array)][, dims=size(array)])

Create an uninitialized mutable array with the given element type and size, based upon the given source array. The second and third arguments are both optional, defaulting to the given array's eltype and size. The dimensions may be specified either as a single tuple argument or as a series of integer arguments.

Custom AbstractArray subtypes may choose which specific array type is best-suited to return for the given element type and dimensionality. If they do not specialize this method, the default is an Array (element\_type, dims...).

For example, similar (1:10, 1, 4) returns an uninitialized Array {Int, 2} since ranges are neither mutable nor support 2 dimensions:

40.2. Constructors 397

```
julia> similar(1:10, 1, 4)
1x4 Array{Int64,2}:
   4419743872  4419743888  0
```

Conversely, similar (trues (10, 10), 2) returns an uninitialized BitVector with two elements since BitArrays are both mutable and can support 1-dimensional arrays:

```
julia> similar(trues(10,10), 2)
2-element BitArray{1}:
   false
   false
```

Since BitArrays can only store elements of type Bool, however, if you request a different element type it will create a regular Array instead:

#### reinterpret (type, A)

Change the type-interpretation of a block of memory. For example, reinterpret (Float32, UInt32(7)) interprets the 4 bytes corresponding to UInt32(7) as a Float32. For arrays, this constructs an array with the same binary data as the given array, but with the specified element type.

#### eye(n)

n-by-n identity matrix.

## eye(m, n)

m-by-n identity matrix.

#### eye(A)

Constructs an identity matrix of the same dimensions and type as A.

### linspace (start, stop, n=50)

Construct a range of n linearly spaced elements from start to stop.

### logspace (start, stop, n=50)

Construct a vector of n logarithmically spaced numbers from 10^start to 10^stop.

# 40.3 Mathematical operators and functions

All mathematical operations and functions are supported for arrays

```
broadcast (f, As...)
```

Broadcasts the arrays As to a common size by expanding singleton dimensions, and returns an array of the results f(as...) for each position.

```
broadcast! (f, dest, As...)
```

Like broadcast, but store the result of broadcast (f, As...) in the dest array. Note that dest is only used to store the result, and does not supply arguments to f unless it is also listed in the As, as in broadcast! (f, A, A, B) to perform A[:] = broadcast(f, A, B).

### bitbroadcast(f, As...)

Like broadcast, but allocates a BitArray to store the result, rather then an Array.

## ${\tt broadcast\_function}\,(f)$

```
Returns a function broadcast_f such that broadcast_function(f)(As...)
```

```
=== broadcast(f, As...). Most useful in the form const broadcast_f = broadcast_function(f).
```

### $broadcast!_function(f)$

Like broadcast\_function, but for broadcast!.

# 40.4 Indexing, Assignment, and Concatenation

### getindex (A, inds...)

Returns a subset of array A as specified by inds, where each ind may be an Int, a Range, or a Vector. See the manual section on *array indexing* (page 160) for details.

#### **sub** (*A*, *inds...*)

Like <code>getindex()</code> (page 342), but returns a view into the parent array A with the given indices instead of making a copy. Calling <code>getindex()</code> (page 342) or <code>setindex!()</code> (page 342) on the returned <code>SubArray</code> computes the indices to the parent array on the fly without checking bounds.

#### parent(A)

Returns the "parent array" of an array view type (e.g., SubArray), or the array itself if it is not a view.

### parentindexes (A)

From an array view A, returns the corresponding indexes in the parent.

#### slicedim(A, d, i)

Return all the data of A where the index for dimension d equals i. Equivalent to A[:,:,...,i,:,:,...] where i is in position d.

#### slice(A, inds...)

Returns a view of array A with the given indices like sub() (page 397), but drops all dimensions indexed with scalars.

#### setindex! (A, X, inds...)

Store values from array X within some subset of A as specified by inds.

#### broadcast\_getindex (A, inds...)

Broadcasts the inds arrays to a common size like broadcast, and returns an array of the results A[ks...], where ks goes over the positions in the broadcast.

### broadcast setindex! (A, X, inds...)

Broadcasts the X and inds arrays to a common size and stores the value from each position in X at the indices given by the same positions in inds.

#### cat(dims, A...)

Concatenate the input arrays along the specified dimensions in the iterable dims. For dimensions not in dims, all input arrays should have the same size, which will also be the size of the output array along that dimension. For dimensions in dims, the size of the output array is the sum of the sizes of the input arrays along that dimension. If dims is a single number, the different arrays are tightly stacked along that dimension. If dims is an iterable containing several dimensions, this allows one to construct block diagonal matrices and their higher-dimensional analogues by simultaneously increasing several dimensions for every new input array and putting zero blocks elsewhere. For example, cat([1,2], matrices...) builds a block diagonal matrix, i.e. a block matrix with matrices[1], matrices[2], ... as diagonal blocks and matching zero blocks away from the diagonal.

### vcat(A...)

Concatenate along dimension 1.

#### hcat(A...)

Concatenate along dimension 2.

### hvcat (rows::Tuple{Vararg{Int}}, values...)

Horizontal and vertical concatenation in one call. This function is called for block matrix syntax. The first argument specifies the number of arguments to concatenate in each block row.

```
julia> a, b, c, d, e, f = 1, 2, 3, 4, 5, 6
(1, 2, 3, 4, 5, 6)
julia> [a b c; d e f]
2x3 Array{Int64,2}:
1 2 3
4 5 6
julia > hvcat((3,3), a,b,c,d,e,f)
2x3 Array{Int64,2}:
1 2 3
4 5 6
julia> [a b;c d; e f]
3x2 Array{Int64,2}:
3 4
5 6
julia > hvcat((2,2,2), a,b,c,d,e,f)
3x2 Array{Int64,2}:
1 2
3 4
 5
```

If the first argument is a single integer n, then all block rows are assumed to have n block columns.

### flipdim(A, d)

Reverse A in dimension d.

### circshift (A, shifts)

Circularly shift the data in an array. The second argument is a vector giving the amount to shift in each dimension.

#### find(A)

Return a vector of the linear indexes of the non-zeros in  $\mathbb{A}$  (determined by  $\mathbb{A}[i]!=0$ ). A common use of this is to convert a boolean array to an array of indexes of the true elements.

#### find(f, A)

Return a vector of the linear indexes of A where f returns true.

### findn(A)

Return a vector of indexes for each dimension giving the locations of the non-zeros in A (determined by A[i]!=0).

#### findnz(A)

Return a tuple (I, J, V) where I and J are the row and column indexes of the non-zero values in matrix A, and V is a vector of the non-zero values.

### findfirst(A)

Return the index of the first non-zero value in A (determined by A [i]!=0).

## $\mathbf{findfirst} \; (A, \, v)$

Return the index of the first element equal to v in A.

#### findfirst (predicate, A)

Return the index of the first element of A for which predicate returns true.

#### findlast (A)

Return the index of the last non-zero value in A (determined by A [i]!=0).

### findlast(A, v)

Return the index of the last element equal to v in A.

### findlast (predicate, A)

Return the index of the last element of A for which predicate returns true.

#### findnext(A, i)

Find the next index  $\geq$  i of a non-zero element of A, or 0 if not found.

#### **findnext** (predicate, A, i)

Find the next index >= i of an element of A for which predicate returns true, or 0 if not found.

### findnext(A, v, i)

Find the next index  $\ge 1$  of an element of A equal to v (using ==), or 0 if not found.

### findprev(A, i)

Find the previous index  $\leq$  i of a non-zero element of A, or 0 if not found.

### **findprev** (predicate, A, i)

Find the previous index <= i of an element of A for which predicate returns true, or 0 if not found.

#### findprev(A, v, i)

Find the previous index  $\leq$  i of an element of A equal to v (using ==), or 0 if not found.

#### permutedims (A, perm)

Permute the dimensions of array A. perm is a vector specifying a permutation of length ndims (A). This is a generalization of transpose for multi-dimensional arrays. Transpose is equivalent to permutedims (A, [2,1]).

### ipermutedims (A, perm)

Like permutedims () (page 399), except the inverse of the given permutation is applied.

### permutedims! (dest, src, perm)

Permute the dimensions of array src and store the result in the array dest. perm is a vector specifying a permutation of length ndims(src). The preallocated array dest should have size(dest) == size(src)[perm] and is completely overwritten. No in-place permutation is supported and unexpected results will happen if src and dest have overlapping memory regions.

### squeeze(A, dims)

Remove the dimensions specified by dims from array A. Elements of dims must be unique and within the range 1: ndims (A).

#### **vec** $(Array) \rightarrow Vector$

Vectorize an array using column-major convention.

#### $promote\_shape(s1, s2)$

Check two array shapes for compatibility, allowing trailing singleton dimensions, and return whichever shape has more dimensions.

## checkbounds (array, indexes...)

Throw an error if the specified indexes are not in bounds for the given array. Subtypes of AbstractArray should specialize this method if they need to provide custom bounds checking behaviors.

### checkbounds (::Type{Bool}, dimlength::Integer, index)

Return a Bool describing if the given index is within the bounds of the given dimension length. Custom types that would like to behave as indices for all arrays can extend this method in order to provide a specialized bounds checking implementation.

### randsubseq $(A, p) \rightarrow \text{Vector}$

Return a vector consisting of a random subsequence of the given array A, where each element of A is included (in order) with independent probability p. (Complexity is linear in p\*length(A), so this function is efficient even if p is small and A is large.) Technically, this process is known as "Bernoulli sampling" of A.

### randsubseq! (S, A, p)

Like randsubseq, but the results are stored in S (which is resized as needed).

# 40.5 Array functions

# $\mathtt{cumprod}(A[,dim])$

Cumulative product along a dimension dim (defaults to 1). See also *cumprod!* () (page 400) to use a preal-located output array, both for performance and to control the precision of the output (e.g. to avoid overflow).

# cumprod! (B, A[, dim])

Cumulative product of A along a dimension, storing the result in B. The dimension defaults to 1.

# $\operatorname{cumsum}(A[,dim])$

Cumulative sum along a dimension dim (defaults to 1). See also <code>cumsum!()</code> (page 400) to use a preallocated output array, both for performance and to control the precision of the output (e.g. to avoid overflow).

# cumsum! (B, A[, dim])

Cumulative sum of A along a dimension, storing the result in B. The dimension defaults to 1.

# $\operatorname{cumsum\_kbn}(A[,dim])$

Cumulative sum along a dimension, using the Kahan-Babuska-Neumaier compensated summation algorithm for additional accuracy. The dimension defaults to 1.

# $\operatorname{cummin}(A[,dim])$

Cumulative minimum along a dimension. The dimension defaults to 1.

### $\operatorname{cummax}(A, dim)$

Cumulative maximum along a dimension. The dimension defaults to 1.

## diff(A[,dim])

Finite difference operator of matrix or vector.

### gradient (F[,h])

Compute differences along vector F, using h as the spacing between points. The default spacing is one.

#### rot180(A)

Rotate matrix A 180 degrees.

### rot180(A, k)

Rotate matrix A 180 degrees an integer k number of times. If k is even, this is equivalent to a copy.

### $\mathtt{rot190}\,(A)$

Rotate matrix A left 90 degrees.

#### rot190(A, k)

Rotate matrix A left 90 degrees an integer k number of times. If k is zero or a multiple of four, this is equivalent to a copy.

### rotr90(A)

Rotate matrix A right 90 degrees.

#### rotr90(A, k)

Rotate matrix A right 90 degrees an integer k number of times. If k is zero or a multiple of four, this is equivalent to a copy.

402 Chapter 40. Arrays

```
reducedim(f, A, dims[, initial])
```

Reduce 2-argument function f along dimensions of A. dims is a vector specifying the dimensions to reduce, and initial is the initial value to use in the reductions. For +,  $\star$ , max and min the initial argument is optional.

The associativity of the reduction is implementation-dependent; if you need a particular associativity, e.g. left-to-right, you should write your own loop. See documentation for reduce.

# mapreducedim(f, op, A, dims[, initial])

Evaluates to the same as reducedim(op, map(f, A), dims, f(initial)), but is generally faster because the intermediate array is avoided.

### mapslices (f, A, dims)

Transform the given dimensions of array A using function f. f is called on each slice of A of the form  $A[\ldots,:,\ldots,:,\ldots]$ . dims is an integer vector specifying where the colons go in this expression. The results are concatenated along the remaining dimensions. For example, if dims is [1,2] and A is 4-dimensional, f is called on A[:,:,i,j] for all i and j.

### $sum_kbn(A)$

Returns the sum of all array elements, using the Kahan-Babuska-Neumaier compensated summation algorithm for additional accuracy.

# 40.6 Combinatorics

# randperm([rng], n)

Construct a random permutation of length n. The optional rng argument specifies a random number generator, see *Random Numbers* (page 383).

### invperm(v)

Return the inverse permutation of v

### $isperm(v) \rightarrow Bool$

Returns true if v is a valid permutation.

### permute!(v, p)

Permute vector v in-place, according to permutation p. No checking is done to verify that p is a permutation.

To return a new permutation, use v[p]. Note that this is generally faster than permute! (v,p) for large vectors.

### ipermute!(v, p)

Like permute!, but the inverse of the given permutation is applied.

# randcycle([rng], n)

Construct a random cyclic permutation of length n. The optional rng argument specifies a random number generator, see *Random Numbers* (page 383).

```
shuffle([rng], v)
```

Return a randomly permuted copy of v. The optional rng argument specifies a random number generator, see *Random Numbers* (page 383).

# **shuffle!** ([rng], v)

In-place version of shuffle () (page 401).

# reverse (v[, start=1[, stop=length(v)]])

Return a copy of v reversed from start to stop.

40.6. Combinatorics 403

```
reverseind(v, i)
```

Given an index i in reverse (v), return the corresponding index in v so that v[reverseind(v, i)] = reverse(v)[i]. (This can be nontrivial in the case where v is a Unicode string.)

**reverse!**  $(v[, start=1[, stop=length(v)]]) \rightarrow v$  In-place version of reverse () (page 388).

# 40.7 BitArrays

```
bitpack (A::AbstractArray\{T, N\}) \rightarrow BitArray
```

Converts a numeric array to a packed boolean array.

```
bitunpack(B::BitArray\{N\}) \rightarrow Array\{Bool,N\}
```

Converts a packed boolean array to an array of booleans.

```
\textbf{flipbits!} \; (\textit{B::BitArray\{N\}}) \; \rightarrow BitArray\{N\}
```

Performs a bitwise not operation on B. See ~ operator (page 356).

rol! (dest::BitArray{1}, src::BitArray{1}, i::Integer)  $\rightarrow$  BitArray{1}

Performs a left rotation operation on src and put the result into dest.

 $\verb"rol!" (B::BitArray \{1\}, i::Integer") \rightarrow BitArray \{1\}$ 

Performs a left rotation operation on B.

 $rol(B::BitArray\{1\}, i::Integer) \rightarrow BitArray\{1\}$ 

Performs a left rotation operation.

 $ror! (dest::BitArray{1}, src::BitArray{1}, i::Integer) \rightarrow BitArray{1}$ 

Performs a right rotation operation on src and put the result into dest.

 $ror! (B::BitArray\{1\}, i::Integer) \rightarrow BitArray\{1\}$ 

Performs a right rotation operation on B.

 $ror(B::BitArray\{1\}, i::Integer) \rightarrow BitArray\{1\}$ 

Performs a right rotation operation.

# 40.8 Sparse Vectors and Matrices

Sparse vectors and matrices largely support the same set of operations as their dense counterparts. The following functions are specific to sparse arrays.

```
sparse(I, J, V[, m, n, combine])
```

Create a sparse matrix S of dimensions m x n such that S[I[k], J[k]] = V[k]. The combine function is used to combine duplicates. If m and n are not specified, they are set to maximum(I) and maximum(J) respectively. If the combine function is not supplied, duplicates are added by default. All elements of I must satisfy  $1 \le I[k] \le m$ , and all elements of J must satisfy  $1 \le J[k] \le m$ .

```
sparsevec(I, V[, m, combine])
```

Create a sparse vector S of length m such that S[I[k]] = V[k]. Duplicates are combined using the combine function, which defaults to + if it is not provided.

```
sparsevec(D::Dict[,m])
```

Create a sparse vector of length m where the nonzero indices are keys from the dictionary, and the nonzero values are the values from the dictionary.

issparse(S)

Returns true if S is sparse, and false otherwise.

#### sparse(A)

Convert an AbstractMatrix A into a sparse matrix.

### sparsevec(A)

Convert a vector A into a sparse vector of length m.

#### full(S)

Convert a sparse matrix or vector S into a dense matrix or vector.

#### nnz(A)

Returns the number of stored (filled) elements in a sparse array.

# spzeros(m[,n])

Create a sparse vector of length m or sparse matrix of size  $m \times n$ . This sparse array will not contain any nonzero values. No storage will be allocated for nonzero values during construction.

#### spones(S)

Create a sparse array with the same structure as that of S, but with every nonzero element having the value 1.0.

# speye (type, m[, n])

Create a sparse identity matrix of specified type of size  $m \times m$ . In case n is supplied, create a sparse identity matrix of size  $m \times n$ .

# spdiagm(B, d[, m, n])

Construct a sparse diagonal matrix. B is a tuple of vectors containing the diagonals and d is a tuple containing the positions of the diagonals. In the case the input contains only one diagonal, B can be a vector (instead of a tuple) and d can be the diagonal position (instead of a tuple), defaulting to D (diagonal). Optionally, D and D specify the size of the resulting sparse matrix.

# sprand([rng], m[, n], p::AbstractFloat[, rfn])

Create a random length m sparse vector or m by n sparse matrix, in which the probability of any element being nonzero is independently given by p (and hence the mean density of nonzeros is also exactly p). Nonzero values are sampled from the distribution specified by rfn. The uniform distribution is used in case rfn is not specified. The optional rng argument specifies a random number generator, see *Random Numbers* (page 383).

# sprandn(m[, n], p::AbstractFloat)

Create a random sparse vector of length m or sparse matrix of size m by n with the specified (independent) probability p of any entry being nonzero, where nonzero values are sampled from the normal distribution.

# sprandbool(m[,n],p)

Create a random m by n sparse boolean matrix or length m sparse boolean vector with the specified (independent) probability p of any entry being true.

## etree(A[,post])

Compute the elimination tree of a symmetric sparse matrix A from triu (A) and, optionally, its post-ordering permutation.

#### symperm(A, p)

Return the symmetric permutation of A, which is A[p,p]. A should be symmetric, sparse, and only contain nonzeros in the upper triangular part of the matrix is stored. This algorithm ignores the lower triangular part of the matrix. Only the upper triangular part of the result is returned.

### nonzeros(A)

Return a vector of the structural nonzero values in sparse array A. This includes zeros that are explicitly stored in the sparse array. The returned vector points directly to the internal nonzero storage of A, and any modifications to the returned vector will mutate A as well. See rowvals (A) and nzrange (A, col).

### rowvals (A::SparseMatrixCSC)

Return a vector of the row indices of A. Any modifications to the returned vector will mutate A as well. Providing access to how the row indices are stored internally can be useful in conjunction with iterating over structural nonzero values. See also nonzeros (A) and nzrange (A, col).

### nzrange (A::SparseMatrixCSC, col)

Return the range of indices to the structural nonzero values of a sparse matrix column. In conjunction with nonzeros (A) and rowvals (A), this allows for convenient iterating over a sparse matrix:

```
A = sparse(I,J,V)
rows = rowvals(A)
vals = nonzeros(A)
m, n = size(A)
for i = 1:n
    for j in nzrange(A, i)
        row = rows[j]
        val = vals[j]
        # perform sparse wizardry...
    end
end
```

406 Chapter 40. Arrays

# **Tasks and Parallel Computing**

## 41.1 Tasks

#### Task (func)

Create a Task (i.e. thread, or coroutine) to execute the given function (which must be callable with no arguments). The task exits when this function returns.

## yieldto (task, arg = nothing)

Switch to the given task. The first time a task is switched to, the task's function is called with no arguments. On subsequent switches, arg is returned from the task's last call to yieldto. This is a low-level call that only switches tasks, not considering states or scheduling in any way. Its use is discouraged.

### current\_task()

Get the currently running Task.

### $istaskdone(task) \rightarrow Bool$

Tell whether a task has exited.

#### $istaskstarted(task) \rightarrow Bool$

Tell whether a task has started executing.

### consume (task, values...)

Receive the next value passed to produce by the specified task. Additional arguments may be passed, to be returned from the last produce call in the producer.

### produce (value)

Send the given value to the last consume call, switching to the consumer task. If the next consume call passes any values, they are returned by produce.

#### yield()

Switch to the scheduler to allow another scheduled task to run. A task that calls this function is still runnable, and will be restarted immediately if there are no other runnable tasks.

### task\_local\_storage (symbol)

Look up the value of a symbol in the current task's task-local storage.

### task\_local\_storage (symbol, value)

Assign a value to a symbol in the current task's task-local storage.

### task\_local\_storage (body, symbol, value)

Call the function body with a modified task-local storage, in which value is assigned to symbol; the previous value of symbol, or lack thereof, is restored afterwards. Useful for emulating dynamic scoping.

### Condition()

Create an edge-triggered event source that tasks can wait for. Tasks that call wait on a Condition are

suspended and queued. Tasks are woken up when notify is later called on the Condition. Edge triggering means that only tasks waiting at the time notify is called can be woken up. For level-triggered notifications, you must keep extra state to keep track of whether a notification has happened. The Channel type does this, and so can be used for level-triggered events.

### notify (condition, val=nothing; all=true, error=false)

Wake up tasks waiting for a condition, passing them val. If all is true (the default), all waiting tasks are woken, otherwise only one is. If error is true, the passed value is raised as an exception in the woken tasks.

### schedule (t::Task, [val]; error=false)

Add a task to the scheduler's queue. This causes the task to run constantly when the system is otherwise idle, unless the task performs a blocking operation such as wait.

If a second argument is provided, it will be passed to the task (via the return value of yieldto) when it runs again. If error is true, the value is raised as an exception in the woken task.

### @schedule()

Wrap an expression in a Task and add it to the local machine's scheduler queue.

### @task()

Wrap an expression in a Task without executing it, and return the Task. This only creates a task, and does not run it.

### sleep (seconds)

Block the current task for a specified number of seconds. The minimum sleep time is 1 millisecond or input of 0.001.

#### ReentrantLock()

Creates a reentrant lock. The same task can acquire the lock as many times as required. Each lock must be matched with an unlock

### lock (l::ReentrantLock)

Associates 1 with the current task. If 1 is already locked by a different task, waits for it to become available. The same task can acquire the lock multiple times. Each "lock" must be matched by an "unlock"

#### unlock (l::ReentrantLock)

Releases ownership of the lock by the current task. If the lock had been acquired before, it just decrements an internal counter and returns immediately.

### Channel {T} (sz::Int)

Constructs a Channel that can hold a maximum of sz objects of type T. put! calls on a full channel block till an object is removed with take!.

### Other constructors:

- •Channel () equivalent to Channel (Any) (32)
- •Channel (sz::Int) equivalent to Channel (Any) (sz)

# 41.2 General Parallel Computing Support

### **addprocs** (n::Integer; exeflags=""") $\rightarrow$ List of process identifiers

Launches workers using the in-built LocalManager which only launches workers on the local host. This can be used to take advantage of multiple cores. addprocs (4) will add 4 processes on the local machine.

### **addprocs** () $\rightarrow$ List of process identifiers

Equivalent to addprocs (CPU CORES)

Note that workers do not run a .juliarc.jl startup script, nor do they synchronize their global state (such as global variables, new method definitions, and loaded modules) with any of the other running processes.

**addprocs** (machines; tunnel=false, sshflags=", max\_parallel=10, exeflags=") → List of process identifiers Add processes on remote machines via SSH. Requires julia to be installed in the same location on each node, or to be available via a shared file system.

machines is a vector of machine specifications. Worker are started for each specification.

A machine specification is either a string machine\_spec or a tuple - (machine\_spec, count).

machine\_spec is a string of the form [user@]host[:port] [bind\_addr[:port]]. user defaults to current user, port to the standard ssh port. If [bind\_addr[:port]] is specified, other workers will connect to this worker at the specified bind\_addr and port.

count is the number of workers to be launched on the specified host. If specified as : auto it will launch as many workers as the number of cores on the specific host.

### Keyword arguments:

tunnel: if true then SSH tunneling will be used to connect to the worker from the master process.

sshflags: specifies additional ssh options, e.g.

```
sshflags=`-i /home/foo/bar.pem`
```

max\_parallel: specifies the maximum number of workers connected to in parallel at a host. Defaults to 10.

dir: specifies the working directory on the workers. Defaults to the host's current directory (as found by pwd())

exename: name of the julia executable. Defaults to "\$JULIA\_HOME/julia" or "\$JULIA\_HOME/julia-debug" as the case may be.

exeflags: additional flags passed to the worker processes.

### Environment variables:

If the master process fails to establish a connection with a newly launched worker within 60.0 seconds, the worker treats it a fatal situation and terminates. This timeout can be controlled via environment variable <code>JULIA\_WORKER\_TIMEOUT</code>. The value of <code>JULIA\_WORKER\_TIMEOUT</code> on the master process, specifies the number of seconds a newly launched worker waits for connection establishment.

## addprocs (manager::ClusterManager; kwargs...) $\rightarrow$ List of process identifiers

Launches worker processes via the specified cluster manager.

For example Beowulf clusters are supported via a custom cluster manager implemented in package ClusterManagers.

The number of seconds a newly launched worker waits for connection establishment from the master can be specified via variable <code>JULIA\_WORKER\_TIMEOUT</code> in the worker process's environment. Relevant only when using TCP/IP as transport.

### nprocs()

Get the number of available processes.

#### nworkers()

Get the number of available worker processes. This is one less than nprocs(). Equal to nprocs() if nprocs() == 1.

#### procs()

Returns a list of all process identifiers.

## workers()

Returns a list of all worker process identifiers.

#### rmprocs (pids...)

Removes the specified workers.

## interrupt(|pids...|)

Interrupt the current executing task on the specified workers. This is equivalent to pressing Ctrl-C on the local machine. If no arguments are given, all workers are interrupted.

#### myid()

Get the id of the current process.

### pmap (f, lsts...; err\_retry=true, err\_stop=false, pids=workers())

Transform collections lsts by applying f to each element in parallel. (Note that f must be made available to all worker processes; see *Code Availability and Loading Packages* (page 180) for details.) If nprocs () > 1, the calling process will be dedicated to assigning tasks. All other available processes will be used as parallel workers, or on the processes specified by pids.

If err\_retry is true, it retries a failed application of f on a different worker. If err\_stop is true, it takes precedence over the value of err\_retry and pmap stops execution on the first error.

### remotecall (func, id, args...)

Call a function asynchronously on the given arguments on the specified process. Returns a Future.

#### Future()

Create a Future on the local machine.

#### Future (n)

Create a Future on process n.

#### RemoteChannel()

Make an reference to a Channel {Any} (1) on the local machine.

### RemoteChannel(n)

Make an reference to a Channel {Any} (1) on process n.

## RemoteChannel (f::Function, pid)

Create references to remote channels of a specific size and type. f() is a function that when executed on pid must return an implementation of an AbstractChannel.

For example, RemoteChannel(()->Channel{Int}(10), pid), will return a reference to a channel of type Int and size 10 on pid.

## wait([x])

Block the current task until some event occurs, depending on the type of the argument:

- •RemoteChannel: Wait for a value to become available on the specified remote channel.
- •Future: Wait for a value to become available for the specified future.
- •Channel: Wait for a value to be appended to the channel.
- •Condition: Wait for notify on a condition.
- •Process: Wait for a process or process chain to exit. The exitcode field of a process can be used to determine success or failure.
- •Task: Wait for a Task to finish, returning its result value. If the task fails with an exception, the exception is propagated (re-thrown in the task that called wait).
- •RawFD: Wait for changes on a file descriptor (see poll\_fd for keyword arguments and return code)

If no argument is passed, the task blocks for an undefined period. A task can only be restarted by an explicit call to schedule or yieldto.

Often wait is called within a while loop to ensure a waited-for condition is met before proceeding.

#### fetch(x)

Waits and fetches a value from x depending on the type of x. Does not remove the item fetched:

- •Future: Wait for and get the value of a Future. The fetched value is cached locally. Further calls to fetch on the same reference return the cached value. If the remote value is an exception, throws a RemoteException which captures the remote exception and backtrace.
- •RemoteChannel: Wait for and get the value of a remote reference. Exceptions raised are same as for a Future.
- •Channel: Wait for and get the first available item from the channel.

### remotecall\_wait (func, id, args...)

Perform wait (remotecall (...)) in one message.

### remotecall\_fetch (func, id, args...)

Perform fetch (remotecall (...)) in one message. Any remote exceptions are captured in a RemoteException and thrown.

### put! (RemoteChannel, value)

Store a value to the remote channel. If the channel is full, blocks until space is available. Returns its first argument.

### put! (Future, value)

Store a value to a future. Future's are write-once remote references. A put! on an already set Future throws an Exception. All asynchronous remote calls return Futures and set the value to the return value of the call upon completion.

#### put! (Channel, value)

Appends an item to the channel. Blocks if the channel is full.

#### take! (RemoteChannel)

Fetch a value from a remote channel, also removing it in the processs.

### take! (Channel)

Removes and returns a value from a Channel. Blocks till data is available.

### isready (r::RemoteChannel)

Determine whether a RemoteChannel has a value stored to it. Note that this function can cause race conditions, since by the time you receive its result it may no longer be true. However, it can be safely used on a Future since they are assigned only once.

### isready (r::Future)

Determine whether a Future has a value stored to it.

If the argument Future is owned by a different node, this call will block to wait for the answer. It is recommended to wait for r in a separate task instead, or to use a local Channel as a proxy:

```
c = Channel(1)
@async put!(c, remotecall_fetch(long_computation, p))
isready(c) # will not block
```

#### close(Channel)

Closes a channel. An exception is thrown by:

- •put! on a closed channel.
- •take! and fetch on an empty, closed channel.

### timedwait (testcb::Function, secs::Float64; pollint::Float64=0.1)

Waits till testeb returns true or for secs seconds, whichever is earlier. testeb is polled every pollint seconds.

### @spawn()

Creates a closure around an expression and runs it on an automatically-chosen process, returning a Future to the result.

#### @spawnat()

Accepts two arguments, p and an expression. A closure is created around the expression and run asynchronously on process p. Returns a Future to the result.

#### @fetch()

Equivalent to fetch (@spawn expr).

#### @fetchfrom()

Equivalent to fetch (@spawnat p expr).

### @async()

Like @schedule, @async wraps an expression in a Task and adds it to the local machine's scheduler queue. Additionally it adds the task to the set of items that the nearest enclosing @sync waits for. @async also wraps the expression in a let x=x, y=y, ... block to create a new scope with copies of all variables referenced in the expression.

#### @sync()

Wait until all dynamically-enclosed uses of @async, @spawn, @spawnat and @parallel are complete. All exceptions thrown by enclosed async operations are collected and thrown as a CompositeException.

#### @parallel()

A parallel for loop of the form:

```
@parallel [reducer] for var = range
     body
end
```

The specified range is partitioned and locally executed across all workers. In case an optional reducer function is specified, <code>@parallel</code> performs local reductions on each worker with a final reduction on the calling process.

Note that without a reducer function, <code>@parallel</code> executes asynchronously, i.e. it spawns independent tasks on all available workers and returns immediately without waiting for completion. To wait for completion, prefix the call with <code>@sync</code>, like:

### @everywhere()

Execute an expression on all processes. Errors on any of the processes are collected into a CompositeException and thrown.

### Base.remoteref\_id(r::AbstractRemoteRef) -> (whence, id)

A low-level API which returns the unique identifying tuple for a remote reference. A reference id is a tuple of two elements - pid where the reference was created from and a one-up number from that node.

### Base.channel\_from\_id(refid) $\rightarrow$ c

A low-level API which returns the backing AbstractChannel for an id returned by remoteref\_id. The call is valid only on the node where the backing channel exists.

### Base.worker id from socket $(s::IO) \rightarrow pid$

A low-level API which given a IO connection, returns the pid of the worker it is connected to. This is useful when writing custom serialize methods for a type, which optimizes the data written out depending on the receiving process id.

# 41.3 Shared Arrays

### SharedArray (T::Type, dims::NTuple; init=false, pids=Int[])

Construct a SharedArray of a bitstype T and size dims across the processes specified by pids - all of which have to be on the same host.

If pids is left unspecified, the shared array will be mapped across all processes on the current host, including the master. But, localindexes and indexpids will only refer to worker processes. This facilitates work distribution code to use workers for actual computation with the master process acting as a driver.

If an init function of the type initfn(S::SharedArray) is specified, it is called on all the participating workers

**SharedArray** (filename::AbstractString, T::Type, dims::NTuple, [offset=0]; mode=nothing, init=false, pids=Int[])

Construct a SharedArray backed by the file filename, with element type T (must be a bitstype) and size dims, across the processes specified by pids - all of which have to be on the same host. This file is mmapped into the host memory, with the following consequences:

- •The array data must be represented in binary format (e.g., an ASCII format like CSV cannot be supported)
- •Any changes you make to the array values (e.g., A[3] = 0) will also change the values on disk

If pids is left unspecified, the shared array will be mapped across all processes on the current host, including the master. But, localindexes and indexpids will only refer to worker processes. This facilitates work distribution code to use workers for actual computation with the master process acting as a driver.

mode must be one of "r", "r+", "w+", or "a+", and defaults to "r+" if the file specified by filename already exists, or "w+" if not. If an init function of the type initfn (S::SharedArray) is specified, it is called on all the participating workers. You cannot specify an init function if the file is not writable.

offset allows you to skip the specified number of bytes at the beginning of the file.

### procs (S::SharedArray)

Get the vector of processes that have mapped the shared array.

#### sdata(S::SharedArray)

Returns the actual Array object backing S.

### indexpids (S::SharedArray)

Returns the index of the current worker into the pids vector, i.e., the list of workers mapping the SharedArray

#### localindexes (S::SharedArray)

Returns a range describing the "default" indexes to be handled by the current process. This range should be interpreted in the sense of linear indexing, i.e., as a sub-range of 1:length(S). In multi-process contexts, returns an empty range in the parent process (or any process for which indexpids returns 0).

It's worth emphasizing that localindexes exists purely as a convenience, and you can partition work on the array among workers any way you wish. For a SharedArray, all indexes should be equally fast for each worker process.

# 41.4 Cluster Manager Interface

This interface provides a mechanism to launch and manage Julia workers on different cluster environments. Local-Manager, for launching additional workers on the same host and SSHManager, for launching on remote hosts via ssh are present in Base. TCP/IP sockets are used to connect and transport messages between processes. It is possible for Cluster Managers to provide a different transport.

launch (manager::FooManager, params::Dict, launched::Vector{WorkerConfig}, launch\_ntfy::Condition)

Implemented by cluster managers. For every Julia worker launched by this function, it should append a WorkerConfig entry to launched and notify launch\_ntfy. The function MUST exit once all workers, requested by manager have been launched. params is a dictionary of all keyword arguments addprocs was called with.

manage (manager::FooManager, pid::Int, config::WorkerConfig. op::Symbol)

Implemented by cluster managers. It is called on the master process, during a worker's lifetime, with appropriate op values:

- •with :register/:deregister when a worker is added / removed from the Julia worker pool.
- •with :interrupt when interrupt (workers) is called. The ClusterManager should signal the appropriate worker with an interrupt signal.
- •with : finalize for cleanup purposes.

### **kill** (manager::FooManager, pid::Int, config::WorkerConfig)

Implemented by cluster managers. It is called on the master process, by rmprocs. It should cause the remote worker specified by pid to exit. Base.kill(manager::ClusterManager....) executes a remote exit() on pid

#### init\_worker (manager::FooManager)

Called by cluster managers implementing custom transports. It initializes a newly launched process as a worker. Command line argument ——worker has the effect of initializing a process as a worker using TCP/IP sockets for transport.

connect (manager::FooManager, pid::Int, config::WorkerConfig) -> (instrm::AsyncStream, outstrm::AsyncStream)

Implemented by cluster managers using custom transports. It should establish a logical connection to worker with id pid, specified by config and return a pair of AsyncStream objects. Messages from pid to current process will be read off instrm, while messages to be sent to pid will be written to outstrm. The custom transport implementation must ensure that messages are delivered and received completely and in order. Base.connect (manager::ClusterManager....) sets up TCP/IP socket connections in-between workers.

#### Base.process messages (instrm::AsyncStream, outstrm::AsyncStream)

Called by cluster managers using custom transports. It should be called when the custom transport implementation receives the first message from a remote worker. The custom transport must manage a logical connection to the remote worker and provide two AsyncStream objects, one for incoming messages and the other for messages addressed to the remote worker.

# Linear Algebra

# 42.1 Standard Functions

Linear algebra functions in Julia are largely implemented by calling functions from LAPACK. Sparse factorizations call functions from SuiteSparse.

 $\star (A, B)$ 

Matrix multiplication.

 $\setminus (A, B)$ 

Matrix division using a polyalgorithm. For input matrices A and B, the result X is such that  $A \star X == B$  when A is square. The solver that is used depends upon the structure of A. A direct solver is used for upper or lower triangular A. For Hermitian A (equivalent to symmetric A for non-complex A) the BunchKaufman factorization is used. Otherwise an LU factorization is used. For rectangular A the result is the minimum-norm least squares solution computed by a pivoted QR factorization of A and a rank estimate of A based on the R factor.

When A is sparse, a similar polyalgorithm is used. For indefinite matrices, the LDLt factorization does not use pivoting during the numerical factorization and therefore the procedure can fail even for invertible matrices.

```
\mathbf{dot}(x, y) \\ \cdot (x, y)
```

Compute the dot product. For complex vectors, the first vector is conjugated.

### vecdot(x, y)

For any iterable containers x and y (including arrays of any dimension) of numbers (or any element type for which dot is defined), compute the Euclidean dot product (the sum of dot (x[i], y[i])) as if they were vectors.

```
{\tt cross}\,(x,y)
```

 $\mathbf{x}(x, y)$ 

Compute the cross product of two 3-vectors.

### factorize(A)

Compute a convenient factorization (including LU, Cholesky, Bunch-Kaufman, LowerTriangular, UpperTriangular) of A, based upon the type of the input matrix. The return value can then be reused for efficient solving of multiple systems. For example: A=factorize(A);  $x=A \ \ y=A \ \ C$ .

#### full(F)

Reconstruct the matrix A from the factorization F=factorize(A).

### $\mathbf{1u}(A) \rightarrow L, U, p$

Compute the LU factorization of A, such that A[p, :] = L\*U.

# $lufact(A[, pivot=Val\{true\}]) \rightarrow F$

Compute the LU factorization of A. The return type of F depends on the type of A. In most cases, if A is a subtype S of AbstractMatrix with an element type T supporting +, -,  $\star$  and / the return type is LU{T,S{T}}. If pivoting is chosen (default) the element type should also support abs and <. When A is sparse and have element of type Float32, Float64, Complex{Float32}, or Complex{Float64} the return type is UmfpackLU. Some examples are shown in the table below.

Type of input A	Type of output F Relationship between F and A		
Matrix()	LU	F[:L]*F[:U] == A[F[:p], :]	
Tridiagonal() (page 422)	LU{T,Tridiagonal{	F}[:L]*F[:U] == A[F[:p], :]	
SparseMatrixCSC()	UmfpackLU	F[:L]*F[:U] == (F[:Rs] .* A)[F[:p], F[:q]]	

The individual components of the factorization F can be accessed by indexing:

Component	Description	LU	LU{T,Tridiagonal{T}}	UmfpackLU
F[:L]	L (lower triangular) part of LU	X	X	X
F[:U]	U (upper triangular) part of LU	X	X	X
F[:p]	(right) permutation Vector	X	X	X
F[:P]	(right) permutation Matrix	X	X	
F[:q]	left permutation Vector			X
F[:Rs]	Vector of scaling factors			X
F[:(:)]	(L, U, p, q, Rs) components			X

Supported function	LU	LU{T,Tridiagonal{T}}	UmfpackLU
/	X		
\	X	X	X
cond	X		X
det	X	X	X
logdet	X	X	
logabsdet	X	X	
size	X	X	

### lufact! $(A) \rightarrow LU$

lufact! is the same as lufact() (page 413), but saves space by overwriting the input A, instead of creating a copy. For sparse A the nzval field is not overwritten but the index fields, colptr and rowval are decremented in place, converting from 1-based indices to 0-based indices.

### **chol** (A::AbstractMatrix) $\rightarrow$ U

Compute the Cholesky factorization of a positive definite matrix A and return the UpperTriangular matrix U such that A = U'U.

### **chol** (x::Number) $\rightarrow$ y

Compute the square root of a non-negative number x.

# $\textbf{cholfact} \ (A :: Strided Matrix, uplo :: Symbol, Val \{false\}) \ \rightarrow \textbf{Cholesky}$

Compute the Cholesky factorization of a dense symmetric positive definite matrix A and return a Cholesky factorization. The uplo argument may be :L for using the lower part or :U for the upper part of A. The default is to use :U. The triangular Cholesky factor can be obtained from the factorization F with: F[:L] and F[:U]. The following functions are available for Cholesky objects: size, , 'inv, det. A PosDefException exception is thrown in case the matrix is not positive definite.

## $\textbf{cholfact} \ (A::StridedMatrix, uplo::Symbol, Val\{true\}; \ tol = 0.0) \ \rightarrow \textbf{CholeskyPivoted}$

Compute the pivoted Cholesky factorization of a dense symmetric positive semi-definite matrix A and return a CholeskyPivoted factorization. The uplo argument may be :L for using the lower part or :U for the upper part of A. The default is to use :U. The triangular Cholesky factor can be obtained from the factorization F with: F[:L] and F[:U]. The following functions are available for PivotedCholesky objects: size, ,

'inv, det, and rank. The argument tol determines the tolerance for determining the rank. For negative values, the tolerance is the machine precision.

},Hermitian{Complex{Float64},SparseMatrixCSC{Complex{Float64},() (in module Base)

```
cholfact(::Union{SparseMatrixCSC,Symmetric{Float64,SparseMatrixCSC{Flaot64,
SuiteSparse_long}},Hermitian{Complex{Float64},SparseMatrixCSC{Complex{Float64},()
SuiteSparse_long}}}; shift = 0.0, perm=Int[]) -> CHOLMOD.Factor
```

Compute the Cholesky factorization of a sparse positive definite matrix A. A fill-reducing permutation is used. F = cholfact (A) is most frequently used to solve systems of equations with F, but also the methods diag, det, logdet are defined for F. You can also extract individual factors from F, using F[:L]. However, since pivoting is on by default, the factorization is internally represented as A = P' \* L \* L' \* P with a permutation matrix P; using just L without accounting for P will give incorrect answers. To include the effects of permutation, it's typically preferable to extact "combined" factors like PtL = F[:PtL] (the equivalent of P' \* L) and LtP = F[:UP] (the equivalent of L' \* P).

Setting optional shift keyword argument computes the factorization of A+shift\*I instead of A. If the perm argument is nonempty, it should be a permutation of 1:size(A, 1) giving the ordering to use (instead of CHOLMOD's default AMD ordering).

The function calls the C library CHOLMOD and many other functions from the library are wrapped but not exported.

!() (in module Base)

```
cholfact!(F::Factor, A::Union{SparseMatrixCSC,
Symmetric{Float64, SparseMatrixCSC{Float64, SuiteSparse_long}}, ()
Hermitian{Complex{Float64}, SparseMatrixCSC{Complex{Float64}, SuiteSparse_long}}}; ()
shift = 0.0) -> CHOLMOD.Factor
```

Compute the LDLt factorization of A, reusing the symbolic factorization F.

```
cholfact! (A::StridedMatrix, uplo::Symbol, Val\{false\}) \rightarrow Cholesky
```

The same as cholfact, but saves space by overwriting the input A, instead of creating a copy.

```
cholfact! (A::StridedMatrix, uplo::Symbol, Val\{true\}) \rightarrow PivotedCholesky
```

The same as cholfact, but saves space by overwriting the input A, instead of creating a copy.

```
lowrankupdate (C::Cholesky, v::StridedVector) \rightarrow CC::Cholesky
```

```
Update a Cholesky factorization C with the vector v. If A = C[:U]'C[:U] then CC = \text{cholfact}(C[:U]'C[:U] + v*v') but the computation of CC only uses O(n^2) operations.
```

```
\textbf{lowrankdowndate} (\textit{C::Cholesky}, \textit{v::StridedVector}) \rightarrow \text{CC::Cholesky}
```

```
Downdate a Cholesky factorization C with the vector v. If A = C[:U]'C[:U] then CC = \text{cholfact}(C[:U]'C[:U] - v*v') but the computation of CC only uses O(n^2) operations.
```

```
lowrankupdate! (C::Cholesky, v::StridedVector) \rightarrow CC::Cholesky
```

Update a Cholesky factorization C with the vector v. If A = C[:U]'C[:U] then CC = cholfact(C[:U]'C[:U] + v\*v') but the computation of CC only uses  $O(n^2)$  operations. The input factorization C is updated in place such that on exit C = CC. The vector v is destroyed during the computation.

```
lowrankdowndate! (C::Cholesky, v::StridedVector) \rightarrow CC::Cholesky
```

Downdate a Cholesky factorization C with the vector v. If A = C[:U]'C[:U] then CC = Cholfact(C[:U]'C[:U] - v\*v') but the computation of CC only uses  $O(n^2)$  operations. The input factorization C is updated in place such that on exit C = CC. The vector v is destroyed during the computation.

```
ldltfact(::SymTridiagonal) \rightarrow LDLt
```

Compute an LDLt factorization of a real symmetric tridiagonal matrix such that A = L\*Diagonal(d)\*L' where L is a unit lower triangular matrix and d is a vector. The main use of an LDLt factorization F = ldltfact(A) is to solve the linear system of equations Ax = b with  $F \setminus b$ .

!() (in module Base)

```
ldltfact(::Union{SparseMatrixCSC,
Symmetric{Float64, SparseMatrixCSC{Flaot64, SuiteSparse_long}}, ()
Hermitian{Complex{Float64}, SparseMatrixCSC{Complex{Float64}, SuiteSparse long}}}; ()
shift = 0.0, perm=Int[]) -> CHOLMOD.Factor
     Compute the LDLt factorization of a sparse symmetric or Hermitian matrix. A fill-reducing permutation is used.
     F = 1dltfact(A) is most frequently used to solve systems of equations A * x = b with F. The returned
     factorization object F also supports the methods diag, det, and logdet. You can extract individual factors
     from F using F[:L]. However, since pivoting is on by default, the factorization is internally represented as
     A = P' \star L \star D \star L' \star P with a permutation matrix P; using just L without accounting for P will give incorrect
     answers. To include the effects of permutation, it's typically preferable to extact "combined" factors like PtL
     = F[:PtL] (the equivalent of P' \starL) and LtP = F[:UP] (the equivalent of L' \starP). The complete list of
     supported factors is :L, :PtL, :D, :UP, :U, :LD, :DU, :PtLD, :DUP.
     Setting optional shift keyword argument computes the factorization of A+shift*I instead of A. If the
     perm argument is nonempty, it should be a permutation of 1:size (A, 1) giving the ordering to use (instead
     of CHOLMOD's default AMD ordering).
     The function calls the C library CHOLMOD and many other functions from the library are wrapped but not
     exported.
!() (in module Base)
ldltfact!(F::Factor, A::Union{SparseMatrixCSC,
Symmetric{Float64,SparseMatrixCSC{Float64,SuiteSparse long}},()
Hermitian{Complex{Float64}, SparseMatrixCSC{Complex{Float64}, SuiteSparse_long}}}; ()
shift = 0.0) -> CHOLMOD.Factor
     Compute the LDLt factorization of A, reusing the symbolic factorization F.
ldltfact! (::SymTridiagonal) \rightarrow LDLt
     Same as ldltfact, but saves space by overwriting the input A, instead of creating a copy.
qr (v::AbstractVector)
     Computes the polar decomposition of a vector.
     Input:
         •v::AbstractVector - vector to normalize
     Outputs:
         •w - A unit vector in the direction of v
         •r - The norm of v
     See also:
     normalize, normalize!, LinAlq.qr!
LinAlq.qr! (v::AbstractVector)
     Computes the polar decomposition of a vector.
                                                              Instead of returning a new vector as
     gr (v:: AbstractVector), this function mutates the input vector v in place.
     Input:
         •v::AbstractVector - vector to normalize
         •w - A unit vector in the direction of v (This is a mutation of v).
         •r - The norm of v
     See also:
```

normalize, normalize!, gr

$$qr(A[, pivot=Val\{false\}][;thin=true]) \rightarrow Q, R, [p]$$

Compute the (pivoted) QR factorization of A such that either A = Q \*R or A[:,p] = Q \*R. Also see qrfact. The default is to compute a thin factorization. Note that R is not extended with zeros when the full Q is requested.

$$qrfact(A[, pivot=Val\{false\}]) \rightarrow F$$

Computes the QR factorization of A. The return type of F depends on the element type of A and whether pivoting is specified (with pivot==Val{true}).

Return type	eltype(A)	pivot	Relationship between F and A
QR	not BlasFloat	either	A==F[:Q]*F[:R]
QRCompactWY	BlasFloat	Val{false}	A==F[:Q]*F[:R]
QRPivoted	BlasFloat	Val{true}	A[:,F[:p]]==F[:Q]*F[:R]

BlasFloat refers to any of: Float32, Float64, Complex64 or Complex128.

The individual components of the factorization F can be accessed by indexing:

Compo-	Description	QR	QRCompactWY	QRPivoted
nent				
F[:Q]	Q (orthogonal/unitary) part of	X	X	X
	QR	(QRPackedQ)	(QRCompactWYQ)	(QRPackedQ)
F[:R]	R (upper right triangular) part of	X	X	X
	QR			
F[:p]	pivot Vector			X
F[:P]	(pivot) permutation Matrix			X

The following functions are available for the QR objects: size, \. When A is rectangular, \ will return a least squares solution and if the solution is not unique, the one with smallest norm is returned.

Multiplication with respect to either thin or full Q is allowed, i.e. both  $F[:Q] \star F[:R]$  and  $F[:Q] \star A$  are supported. A Q matrix can be converted into a regular matrix with full() (page 413) which has a named argument thin.

#### note

qrfact returns multiple types because LAPACK uses several representations that minimize the memory storage requirements of products of Householder elementary reflectors, so that the Q and R matrices can be stored compactly rather as two separate dense matrices.

The data contained in QR or QRPivoted can be used to construct the QRPackedQ type, which is a compact representation of the rotation matrix:

$$Q = \prod_{i=1}^{\min(m,n)} (I - \tau_i v_i v_i^T)$$

where  $\tau_i$  is the scale factor and  $v_i$  is the projection vector associated with the  $i^{th}$  Householder elementary reflector.

The data contained in QRCompactWY can be used to construct the QRCompactWYQ type, which is a compact representation of the rotation matrix

$$Q = I + YTY^T$$

where Y is  $m \times r$  lower trapezoidal and T is  $r \times r$  upper triangular. The *compact WY* representation [Schreiber1989] (page 493) is not to be confused with the older, WY representation [Bischof1987] (page 493). (The LAPACK documentation uses V in lieu of Y.)

#### $qrfact(A) \rightarrow SPQR$ . Factorization

Compute the QR factorization of a sparse matrix A. A fill-reducing permutation is used. The main application of this type is to solve least squares problems with \. The function calls the C library SPQR and a few additional functions from the library are wrapped but not exported.

# $qrfact! (A[, pivot=Val\{false\}])$

qrfact! is the same as qrfact() (page 417) when A is a subtype of StridedMatrix, but saves space by overwriting the input A, instead of creating a copy.

## **full** ( $QRCompactWYQ[, thin=true]) \rightarrow Matrix$

Converts an orthogonal or unitary matrix stored as a QRCompactWYQ object, i.e. in the compact WY format [Bischof1987] (page 493), to a dense matrix.

Optionally takes a thin Boolean argument, which if true omits the columns that span the rows of R in the QR factorization that are zero. The resulting matrix is the Q in a thin QR factorization (sometimes called the reduced QR factorization). If false, returns a Q that spans all rows of R in its corresponding QR factorization.

### $bkfact(A) \rightarrow BunchKaufman$

Compute the Bunch-Kaufman [Bunch1977] (page 493) factorization of a real symmetric or complex Hermitian matrix A and return a BunchKaufman object. The following functions are available for BunchKaufman objects: size, \, inv, issym, ishermitian.

#### **bkfact!** $(A) \rightarrow BunchKaufman$

bkfact! is the same as bkfact() (page 418), but saves space by overwriting the input A, instead of creating a copy.

## $\texttt{eig}(A,[irange,][vl,][vu,][permute=true,][scale=true]) \rightarrow D, V$

Computes eigenvalues and eigenvectors of A. See eigfact() (page 419) for details on the balance keyword argument.

```
julia> eig([1.0 0.0 0.0; 0.0 3.0 0.0; 0.0 0.0 18.0])
([1.0,3.0,18.0],
3x3 Array{Float64,2}:
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0)
```

eig is a wrapper around eigfact () (page 419), extracting all parts of the factorization to a tuple; where possible, using eigfact () (page 419) is recommended.

### $eig(A, B) \rightarrow D, V$

Computes generalized eigenvalues and vectors of  ${\tt A}$  with respect to  ${\tt B}$ .

eig is a wrapper around eigfact () (page 419), extracting all parts of the factorization to a tuple; where possible, using eigfact () (page 419) is recommended.

### eigvals $(A,[irange,][vl,][vu]) \rightarrow values$

Returns the eigenvalues of A. If A is Symmetric, Hermitian or SymTridiagonal, it is possible to calculate only a subset of the eigenvalues by specifying either a UnitRange irange covering indices of the sorted eigenvalues, or a pair vl and vu for the lower and upper boundaries of the eigenvalues.

For general non-symmetric matrices it is possible to specify how the matrix is balanced before the eigenvector calculation. The option permute=true permutes the matrix to become closer to upper triangular, and scale=true scales the matrix by its diagonal elements to make rows and columns moreequal in norm. The default is true for both options.

### **eigvals!** $(A,[irange,][vl,][vu]) \rightarrow \text{values}$

Same as eigvals, but saves space by overwriting the input A (and B), instead of creating a copy.

### eigmax(A)

Returns the largest eigenvalue of A.

#### eigmin(A)

Returns the smallest eigenvalue of A.

### eigvecs (A, [eigvals,][permute=true,][scale=true]) $\rightarrow$ Matrix

Returns a matrix M whose columns are the eigenvectors of A. (The kth eigenvector can be obtained from the slice M[:, k].) The permute and scale keywords are the same as for eigfact () (page 419).

For SymTridiagonal (page 422) matrices, if the optional vector of eigenvalues eigvals is specified, returns the specific corresponding eigenvectors.

### $eigfact (A,[irange,][vl,][vu,][permute=true,][scale=true]) \rightarrow Eigen$

Computes the eigenvalue decomposition of A, returning an Eigen factorization object F which contains the eigenvalues in F[:values] and the eigenvectors in the columns of the matrix F[:vectors]. (The kth eigenvector can be obtained from the slice F[:vectors][:, k].)

The following functions are available for Eigen objects: inv, det.

If A is Symmetric, Hermitian or *SymTridiagonal* (page 422), it is possible to calculate only a subset of the eigenvalues by specifying either a UnitRange irange covering indices of the sorted eigenvalues or a pair v1 and vu for the lower and upper boundaries of the eigenvalues.

For general nonsymmetric matrices it is possible to specify how the matrix is balanced before the eigenvector calculation. The option permute=true permutes the matrix to become closer to upper triangular, and scale=true scales the matrix by its diagonal elements to make rows and columns more equal in norm. The default is true for both options.

### eigfact $(A, B) \rightarrow$ GeneralizedEigen

Computes the generalized eigenvalue decomposition of A and B, returning a GeneralizedEigen factorization object F which contains the generalized eigenvalues in F[:values] and the generalized eigenvectors in the columns of the matrix F[:vectors]. (The kth generalized eigenvector can be obtained from the slice F[:vectors][:, k].)

## eigfact! (A[,B])

Same as eigfact () (page 419), but saves space by overwriting the input A (and B), instead of creating a copy.

#### hessfact (A)

Compute the Hessenberg decomposition of A and return a Hessenberg object. If F is the factorization object, the unitary matrix can be accessed with F [:Q] and the Hessenberg matrix with F [:H]. When Q is extracted, the resulting type is the HessenbergQ object, and may be converted to a regular matrix with full() (page 413).

### hessfact!(A)

hessfact! is the same as hessfact() (page 419), but saves space by overwriting the input A, instead of creating a copy.

#### $schurfact (A::StridedMatrix) \rightarrow F::Schur$

Computes the Schur factorization of the matrix A. The (quasi) triangular Schur factor can be obtained from the Schur object F with either F[:Schur] or F[:T] and the orthogonal/unitary Schur vectors can be obtained with F[:vectors] or F[:Z] such that A = F[:vectors] \*F[:Schur] \*F[:vectors]'. The eigenvalues of A can be obtained with F[:values].

### **schurfact!** (A::StridedMatrix) $\rightarrow$ F::Schur

Same as schurfact but uses the input argument as workspace.

### **schur** (A::StridedMatrix) $\rightarrow$ T::Matrix, Z::Matrix, $\lambda$ ::Vector

Computes the Schur factorization of the matrix A. The methods return the (quasi) triangular Schur factor T and the orthogonal/unitary Schur vectors Z such that A = Z \* T \* Z'. The eigenvalues of A are returned in the vector  $\lambda$ .

See schurfact.

- $ordschur(F::Schur, select::Union\{Vector\{Bool\}, BitVector\}) \rightarrow F::Schur$ 
  - Reorders the Schur factorization F of a matrix A = Z\*T\*Z' according to the logical array select returning the reordered factorization F object. The selected eigenvalues appear in the leading diagonal of F[:Schur] and the corresponding leading columns of F[:vectors] form an orthogonal/unitary basis of the corresponding right invariant subspace. In the real case, a complex conjugate pair of eigenvalues must be either both included or both excluded via select.
- ordschur! (F::Schur, select::Union{Vector{Bool}, BitVector})  $\rightarrow$  F::Schur Same as ordschur but overwrites the factorization F.
- ordschur (T::StridedMatrix, Z::StridedMatrix,  $select::Union\{Vector\{Bool\}, BitVector\}$ )  $\rightarrow$  T::StridedMatrix, Z::StridedMatrix,  $\lambda::Vector$

Reorders the Schur factorization of a real matrix A = Z \* T \* Z' according to the logical array select returning the reordered matrices T and Z as well as the vector of eigenvalues  $\lambda$ . The selected eigenvalues appear in the leading diagonal of T and the the corresponding leading columns of Z form an orthogonal/unitary basis of the corresponding right invariant subspace. In the real case, a complex conjugate pair of eigenvalues must be either both included or both excluded via select.

- ordschur! (T::StridedMatrix, Z::StridedMatrix, select::Union{Vector{Bool}, BitVector}) → T::StridedMatrix, Z::StridedMatrix, λ::Vector Same as ordschur but overwrites the input arguments.
- $schurfact (A::StridedMatrix, B::StridedMatrix) \rightarrow F::GeneralizedSchur$

Computes the Generalized Schur (or QZ) factorization of the matrices A and B. The (quasi) triangular Schur factors can be obtained from the Schur object F with F[:S] and F[:T], the left unitary/orthogonal Schur vectors can be obtained with F[:left] or F[:Q] and the right unitary/orthogonal Schur vectors can be obtained with F[:right] or F[:Z] such that A=F[:left]\*F[:S]\*F[:right]' and B=F[:left]\*F[:T]\*F[:right]'. The generalized eigenvalues of A and B can be obtained with F[:alpha]./F[:beta].

- **schurfact!** (A::StridedMatrix, B::StridedMatrix)  $\rightarrow$  F::GeneralizedSchur Same as schurfact but uses the input matrices A and B as workspace.
- ordschur (F::GeneralizedSchur, select::Union{Vector{Bool}, BitVector})  $\rightarrow$  F::GeneralizedSchur Reorders the Generalized Schur factorization F of a matrix pair (A, B) = ( $Q \times S \times Z'$ ,  $Q \times T \times Z'$ ) according to the logical array select and returns a GeneralizedSchur object F. The selected eigenvalues appear in the leading diagonal of both F[:S] and F[:T], and the left and right orthogonal/unitary Schur vectors are also reordered such that (A, B) = F[:Q] \* (F[:S], F[:T]) \* F[:Z]' still holds and the generalized eigenvalues of A and B can still be obtained with F[:alpha]./F[:beta].
- **ordschur!** (F::GeneralizedSchur, select:: $Union\{Vector\{Bool\}, BitVector\}$ )  $\rightarrow$  F::GeneralizedSchur Same as ordschur but overwrites the factorization F.
- ordschur (S::StridedMatrix, T::StridedMatrix, Q::StridedMatrix, Z::StridedMatrix, select)  $\rightarrow$  S::StridedMatrix, T::StridedMatrix, Q::StridedMatrix, Z::StridedMatrix,  $\alpha$ ::Vector,  $\beta$ ::Vector Reorders the Generalized Schur factorization of a matrix pair (A, B) = (Q\*S\*Z', Q\*T\*Z') according to the logical array select and returns the matrices S, T, Q, Z and vectors  $\alpha$  and  $\beta$ . The selected eigenvalues appear in the leading diagonal of both S and T, and the left and right unitary/orthogonal Schur vectors are also reordered such that (A, B) = Q\*(S, T)\*Z' still holds and the generalized eigenvalues of A and B can still be obtained with  $\alpha$ .  $/\beta$ .
- ordschur! (S::StridedMatrix, T::StridedMatrix, Q::StridedMatrix, Z::StridedMatrix, select)  $\rightarrow$  S::StridedMatrix, T::StridedMatrix, Q::StridedMatrix, Z::StridedMatrix,  $\alpha$ ::Vector,  $\beta$ ::Vector Same as ordschur but overwrites the factorization the input arguments.
- $svdfact(A[, thin=true]) \rightarrow SVD$

Compute the Singular Value Decomposition (SVD) of A and return an SVD object. U, S, V and Vt can be obtained from the factorization F with F[:U], F[:S], F[:V] and F[:Vt], such that A = U\*diagm(S)\*Vt. If thin is true, an economy mode decomposition is returned. The algorithm produces Vt and hence Vt is more efficient to extract than V. The default is to produce a thin decomposition.

# **svdfact!** $(A[, thin=true]) \rightarrow SVD$

svdfact! is the same as svdfact() (page 420), but saves space by overwriting the input A, instead of creating a copy. If thin is true, an economy mode decomposition is returned. The default is to produce a thin decomposition.

# $svd(A[, thin=true]) \rightarrow U, S, V$

Wrapper around svdfact extracting all parts the factorization to a tuple. Direct use of svdfact is therefore generally more efficient. Computes the SVD of A, returning U, vector S, and V such that A == U\*diagm(S)\*V'. If thin is true, an economy mode decomposition is returned. The default is to produce a thin decomposition.

#### svdvals(A)

Returns the singular values of A.

### svdvals!(A)

Returns the singular values of A, while saving space by overwriting the input.

#### $svdfact(A, B) \rightarrow GeneralizedSVD$

Compute the generalized SVD of A and B, returning a Generalized SVD Factorization object F, such that A = F[:U] \*F[:D] \*F[:R0] \*F[:Q]' and B = F[:V] \*F[:D2] \*F[:R0] \*F[:Q]'.

### $svd(A, B) \rightarrow U, V, O, D1, D2, R0$

Wrapper around svdfact extracting all parts the factorization to a tuple. Direct use of svdfact is therefore generally more efficient. The function returns the generalized SVD of A and B, returning U, V, Q, D1, D2, and R0 such that A = U\*D1\*R0\*Q' and B = V\*D2\*R0\*Q'.

### svdvals(A, B)

Return only the singular values from the generalized singular value decomposition of A and B.

### $qivens{T}$ (::T, ::T, ::Integer, ::Integer) $\rightarrow$ {Givens, T}

Computes the tuple (G, r) = givens (f, g, i1, i2) where G is a Givens rotation and r is a scalar such that G\*x=y with x[i1]=f, x[i2]=g, y[i1]=r, and y[i2]=0. The cosine and sine of the rotation angle can be extracted from the Givens type with G.c and G.s respectively. The arguments f and g can be either Float32, Float64, Complex{Float32}, or Complex{Float64}. The Givens type supports left multiplication G\*A and conjugated transpose right multiplication A\*G'. The type doesn't have a size and can therefore be multiplied with matrices of arbitrary size as long as i2 <= size(A, 2) for G\*A or i2 <= size(A, 1) for A\*G'.

### $givens\{T\}$ (::AbstractArray{T}, ::Integer, ::Integer, ::Integer) $\rightarrow$ {Givens, T}

Computes the tuple (G, r) = givens (A, i1, i2, col) where G is Givens rotation and r is a scalar such that G\*A[:,col]=y with y[i1]=r, and y[i2]=0. The cosine and sine of the rotation angle can be extracted from the Givens type with G.c and G.s respectively. The element type of A can be either Float 32, Float 64, Complex {Float 32}, or Complex {Float 64}. The Givens type supports left multiplication G\*A and conjugated transpose right multiplication A\*G'. The type doesn't have a size and can therefore be multiplied with matrices of arbitrary size as long as i2 <= size(A, 2) for G\*A or i2 <= size(A, 1) for A\*G'.

#### triu(M)

Upper triangle of a matrix.

#### triu(M, k)

Returns the upper triangle of M starting from the kth superdiagonal.

#### **triu!** (*M*)

Upper triangle of a matrix, overwriting M in the process.

#### triu!(M, k)

Returns the upper triangle of M starting from the kth superdiagonal, overwriting M in the process.

#### tril(M)

Lower triangle of a matrix.

#### tril(M, k)

Returns the lower triangle of M starting from the kth superdiagonal.

#### **tril!** (*M*)

Lower triangle of a matrix, overwriting M in the process.

#### tril!(M, k)

Returns the lower triangle of M starting from the kth superdiagonal, overwriting M in the process.

# diagind(M[,k])

A Range giving the indices of the kth diagonal of the matrix M.

## diag(M[,k])

The kth diagonal of a matrix, as a vector. Use diagm to construct a diagonal matrix.

## diagm(v[,k])

Construct a diagonal matrix and place v on the kth diagonal.

### scale(A, b)

#### scale(b, A)

Scale an array A by a scalar b, returning a new array.

If A is a matrix and b is a vector, then scale(A,b) scales each column i of A by b[i] (similar to A\*diagm(b)), while scale(b,A) scales each row i of A by b[i] (similar to diagm(b)\*A), returning a new array.

Note: for large A, scale can be much faster than A .\* b or b .\* A, due to the use of BLAS.

### scale!(A, b)

## scale!(b, A)

Scale an array A by a scalar b, similar to scale () (page 422) but overwriting A in-place.

If A is a matrix and b is a vector, then scale!(A,b) scales each column i of A by b[i] (similar to A\*diagm(b)), while scale!(b,A) scales each row i of A by b[i] (similar to diagm(b)\*A), again operating in-place on A.

### Tridiagonal (dl, d, du)

Construct a tridiagonal matrix from the lower diagonal, diagonal, and upper diagonal, respectively. The result is of type Tridiagonal and provides efficient specialized linear solvers, but may be converted into a regular matrix with full() (page 413).

#### Bidiagonal (*dv*, *ev*, *isupper*)

Constructs an upper (isupper=true) or lower (isupper=false) bidiagonal matrix using the given diagonal (dv) and off-diagonal (ev) vectors. The result is of type Bidiagonal and provides efficient specialized linear solvers, but may be converted into a regular matrix with full() (page 413).

### SymTridiagonal(d, du)

Construct a real symmetric tridiagonal matrix from the diagonal and upper diagonal, respectively. The result is of type SymTridiagonal and provides efficient specialized eigensolvers, but may be converted into a regular matrix with full() (page 413).

### ${\tt rank}\,(M)$

Compute the rank of a matrix.

### norm(A, p)

Compute the p-norm of a vector or the operator norm of a matrix A, defaulting to the p=2-norm.

For vectors, p can assume any numeric value (even though not all values produce a mathematically valid vector norm). In particular, norm (A, Inf) returns the largest value in abs (A), whereas norm (A, -Inf) returns the smallest.

For matrices, the matrix norm induced by the vector p-norm is used, where valid values of p are 1, 2, or Inf. (Note that for sparse matrices, p=2 is currently not implemented.) Use vecnorm() (page 423) to compute the Frobenius norm.

# vecnorm(A[,p])

For any iterable container A (including arrays of any dimension) of numbers (or any element type for which norm is defined), compute the p-norm (defaulting to p=2) as if A were a vector of the corresponding length.

For example, if A is a matrix and p=2, then this is equivalent to the Frobenius norm.

# normalize! (v[, p=2])

Normalize the vector v in-place with respect to the p-norm.

Inputs:

•v::AbstractVector - vector to be normalized

•p::Real - The p-norm to normalize with respect to. Default: 2

Output:

•v - A unit vector being the input vector, rescaled to have norm 1. The input vector is modified in-place.

See also:

normalize, qr

# normalize (v[, p=2])

Normalize the vector v with respect to the p-norm.

Inputs:

•v::AbstractVector - vector to be normalized

•p::Real - The p-norm to normalize with respect to. Default: 2

Output:

•v - A unit vector being a copy of the input vector, scaled to have norm 1

See also:

normalize!, qr

# cond(M[,p])

Condition number of the matrix M, computed using the operator p-norm. Valid values for p are 1, 2 (default), or Inf.

 $\verb|condskeel| (M[,x,p])$ 

$$\kappa_S(M, p) = \left\| |M| \left| M^{-1} \right| \right\|_p$$
  
$$\kappa_S(M, x, p) = \left\| |M| \left| M^{-1} \right| |x| \right\|_p$$

Skeel condition number  $\kappa_S$  of the matrix M, optionally with respect to the vector x, as computed using the operator p-norm. p is Inf by default, if not provided. Valid values for p are 1, 2, or Inf.

This quantity is also known in the literature as the Bauer condition number, relative condition number, or componentwise relative condition number.

trace(M)

Matrix trace.

#### det(M)

Matrix determinant.

### logdet(M)

Log of matrix determinant. Equivalent to log (det (M)), but may provide increased accuracy and/or speed.

### logabsdet (M)

Log of absolute value of determinant of real matrix. Equivalent to  $(\log(abs(det(M))))$ , sign(det(M))), but may provide increased accuracy and/or speed.

#### inv(M)

Matrix inverse.

# pinv(M[,tol])

Computes the Moore-Penrose pseudoinverse.

For matrices M with floating point elements, it is convenient to compute the pseudoinverse by inverting only singular values above a given threshold, tol.

The optimal choice of tol varies both with the value of M and the intended application of the pseudoinverse. The default value of tol is eps(real(float(one(eltype(M)))))\*maximum(size(A)), which is essentially machine epsilon for the real part of a matrix element multiplied by the larger matrix dimension. For inverting dense ill-conditioned matrices in a least-squares sense, tol = sqrt(eps(real(float(one(eltype(M)))))) is recommended.

For more information, see [issue8859] (page 493), [B96] (page 493), [S84] (page 493), [KY88] (page 493).

#### nullspace (M)

Basis for nullspace of M.

### repmat(A, n, m)

Construct a matrix by repeating the given matrix n times in dimension 1 and m times in dimension 2.

```
repeat (A, inner = Int[], outer = Int[])
```

Construct an array by repeating the entries of A. The i-th element of inner specifies the number of times that the individual entries of the i-th dimension of A should be repeated. The i-th element of outer specifies the number of times that a slice along the i-th dimension of A should be repeated.

#### kron(A, B)

Kronecker tensor product of two vectors or two matrices.

### blkdiag(A...)

Concatenate matrices block-diagonally. Currently only implemented for sparse matrices.

### $linreg(x, y) \rightarrow a, b$

Perform linear regression. Returns a and b such that a + b \*x is the closest straight line to the given points (x, y), i.e., such that the squared error between y and a + b \*x is minimized.

#### **Example:**

```
using PyPlot
x = [1.0:12.0;]
y = [5.5, 6.3, 7.6, 8.8, 10.9, 11.79, 13.48, 15.02, 17.77, 20.81, 22.0, 22.99]
a, b = linreg(x, y)  # Linear regression
plot(x, y, "o")  # Plot (x, y) points
plot(x, [a+b*i for i in x]) # Plot line determined by linear regression
```

### linreg(x, y, w)

Weighted least-squares linear regression.

#### expm(A)

Compute the matrix exponential of A, defined by

$$e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!}.$$

For symmetric or Hermitian A, an eigendecomposition (eigfact () (page 419)) is used, otherwise the scaling and squaring algorithm (see [H05] (page 493)) is chosen.

#### logm (A::StridedMatrix)

If A has no negative real eigenvalue, compute the principal matrix logarithm of A, i.e. the unique matrix X such that  $e^X = A$  and  $-\pi < Im(\lambda) < \pi$  for all the eigenvalues  $\lambda$  of X. If A has nonpositive eigenvalues, a nonprincipal matrix function is returned whenever possible.

If A is symmetric or Hermitian, its eigendecomposition (eigfact() (page 419)) is used, if A is triangular an improved version of the inverse scaling and squaring method is employed (see [AH12] (page 493) and [AHR13] (page 493)). For general matrices, the complex Schur form (schur() (page 419)) is computed and the triangular algorithm is used on the triangular factor.

#### sqrtm(A)

If A has no negative real eigenvalues, compute the principal matrix square root of A, that is the unique matrix X with eigenvalues having positive real part such that  $X^2 = A$ . Otherwise, a nonprincipal square root is returned.

If A is symmetric or Hermitian, its eigendecomposition (eigfact () (page 419)) is used to compute the square root. Otherwise, the square root is determined by means of the Björck-Hammarling method, which computes the complex Schur form (schur() (page 419)) and then the complex square root of the triangular factor.

### lyap(A, C)

Computes the solution X to the continuous Lyapunov equation AX + XA' + C = 0, where no eigenvalue of A has a zero real part and no two eigenvalues are negative complex conjugates of each other.

### sylvester(A, B, C)

Computes the solution X to the Sylvester equation AX + XB + C = 0, where A, B and C have compatible dimensions and A and B have no eigenvalues with equal real part.

#### $issym(A) \rightarrow Bool$

Test whether a matrix is symmetric.

#### $isposdef(A) \rightarrow Bool$

Test whether a matrix is positive definite.

### $isposdef!(A) \rightarrow Bool$

Test whether a matrix is positive definite, overwriting A in the processes.

#### $istril(A) \rightarrow Bool$

Test whether a matrix is lower triangular.

#### $istriu(A) \rightarrow Bool$

Test whether a matrix is upper triangular.

### $isdiag(A) \rightarrow Bool$

Test whether a matrix is diagonal.

#### $ishermitian(A) \rightarrow Bool$

Test whether a matrix is Hermitian.

#### transpose(A)

The transposition operator (.').

### transpose! (dest, src)

Transpose array src and store the result in the preallocated array dest, which should have a size corresponding

to (size(src, 2), size(src, 1)). No in-place transposition is supported and unexpected results will happen if src and dest have overlapping memory regions.

#### ctranspose(A)

The conjugate transposition operator (').

#### ctranspose! (dest, src)

Conjugate transpose array src and store the result in the preallocated array dest, which should have a size corresponding to (size(src, 2), size(src, 1)). No in-place transposition is supported and unexpected results will happen if src and dest have overlapping memory regions.

eigs (A; nev=6, nev=max(20, 2\*nev+1), which="LM", tol=0.0, maxiter=300, sigma=nothing, ritzvec=true, v0=zeros((0, ))) -> (d[, v], nconv, niter, nmult, resid)

Computes eigenvalues d of A using Lanczos or Arnoldi iterations for real symmetric or general nonsymmetric matrices respectively.

The following keyword arguments are supported:

- •nev: Number of eigenvalues
- •ncv: Number of Krylov vectors used in the computation; should satisfy  $nev+1 \le ncv \le n$  for real symmetric problems and  $nev+2 \le ncv \le n$  for other problems, where n is the size of the input matrix A. The default is ncv = max(20, 2\*nev+1). Note that these restrictions limit the input matrix A to be of dimension at least 2.
- •which: type of eigenvalues to compute. See the note below.

which	type of eigenvalues
:LM	eigenvalues of largest magnitude (default)
:SM	eigenvalues of smallest magnitude
:LR	eigenvalues of largest real part
:SR	eigenvalues of smallest real part
:LI	eigenvalues of largest imaginary part (nonsymmetric or complex A only)
:SI	eigenvalues of smallest imaginary part (nonsymmetric or complex A only)
:BE	compute half of the eigenvalues from each end of the spectrum, biased in favor of the high end.
	(real symmetric A only)

- •tol: tolerance ( $tol \le 0.0$  defaults to DLAMCH ('EPS'))
- •maxiter: Maximum number of iterations (default = 300)
- •sigma: Specifies the level shift used in inverse iteration. If nothing (default), defaults to ordinary (forward) iterations. Otherwise, find eigenvalues close to sigma using shift and invert iterations.
- •ritzvec: Returns the Ritz vectors v (eigenvectors) if true
- •v0: starting vector from which to start the iterations

eigs returns the new requested eigenvalues in d, the corresponding Ritz vectors v (only if ritzvec=true), the number of converged eigenvalues nconv, the number of iterations niter and the number of matrix vector multiplications nmult, as well as the final residual vector resid.

#### note

The sigma and which keywords interact: the description of eigenvalues searched for by which do \_not\_ necessarily refer to the eigenvalues of A, but rather the linear operator constructed by the specification of the iteration mode implied by sigma.

sigma	iteration mode	which refers to eigenvalues of
nothing	ordinary (forward)	A
real or complex	inverse with level shift sigma	$(A - \sigma I)^{-1}$

eigs  $(A, B; nev=6, ncv=max(20, 2*nev+1), which="LM", tol=0.0, maxiter=300, sigma=nothing, ritzvec=true, <math>v0=zeros((0, ))) \rightarrow (d[, v], nconv, niter, nmult, resid)$ 

Computes generalized eigenvalues d of A and B using Lanczos or Arnoldi iterations for real symmetric or general nonsymmetric matrices respectively.

The following keyword arguments are supported:

- •nev: Number of eigenvalues
- •ncv: Number of Krylov vectors used in the computation; should satisfy  $nev+1 \le ncv \le n$  for real symmetric problems and  $nev+2 \le ncv \le n$  for other problems, where n is the size of the input matrices A and B. The default is ncv = max(20, 2\*nev+1). Note that these restrictions limit the input matrix A to be of dimension at least 2.
- •which: type of eigenvalues to compute. See the note below.

which	type of eigenvalues
:LM	eigenvalues of largest magnitude (default)
:SM	eigenvalues of smallest magnitude
:LR	eigenvalues of largest real part
:SR	eigenvalues of smallest real part
:LI	eigenvalues of largest imaginary part (nonsymmetric or complex A only)
:SI	eigenvalues of smallest imaginary part (nonsymmetric or complex A only)
:BE	compute half of the eigenvalues from each end of the spectrum, biased in favor of the high end.
	(real symmetric A only)

- •tol: tolerance (tol < 0.0 defaults to DLAMCH ('EPS'))
- •maxiter: Maximum number of iterations (default = 300)
- •sigma: Specifies the level shift used in inverse iteration. If nothing (default), defaults to ordinary (forward) iterations. Otherwise, find eigenvalues close to sigma using shift and invert iterations.
- •ritzvec: Returns the Ritz vectors v (eigenvectors) if true
- •v0: starting vector from which to start the iterations

eigs returns the new requested eigenvalues in d, the corresponding Ritz vectors v (only if ritzvec=true), the number of converged eigenvalues nconv, the number of iterations niter and the number of matrix vector multiplications nmult, as well as the final residual vector resid.

### note

The sigma and which keywords interact: the description of eigenvalues searched for by which do \_not\_ necessarily refer to the eigenvalue problem  $Av = Bv\lambda$ , but rather the linear operator constructed by the specification of the iteration mode implied by sigma.

sigma	iteration mode	which refers to the problem
nothing	ordinary (forward)	$Av = Bv\lambda$
real or complex	inverse with level shift sigma	$(A - \sigma B)^{-1}B = v\nu$

**svds** (A; nsv=6, ritzvec=true, tol=0.0, maxiter=1000) -> (left\_sv, s, right\_sv, nconv, niter, nmult, resid) svds computes largest singular values s of A using Lanczos or Arnoldi iterations. Uses eigs() (page 426) underneath.

### Inputs are:

- •A: Linear operator. It can either subtype of AbstractArray (e.g., sparse matrix) or duck typed. For duck typing A has to support size (A), eltype (A), A \* vector and A' \* vector.
- •nsv: Number of singular values.

- •ritzvec: Whether to return the left and right singular vectors left\_sv and right\_sv, default is true. If false the singular vectors are omitted from the output.
- •tol: tolerance, see eigs () (page 426).
- •maxiter: Maximum number of iterations, see eigs () (page 426).

#### **Example**

```
X = sprand(10, 5, 0.2)

svds(X, nsv = 2)
```

### peakflops (n; parallel=false)

peakflops computes the peak flop rate of the computer by using double precision Base.LinAlg.BLAS.gemm! () (page 429). By default, if no arguments are specified, it multiplies a matrix of size  $n \times n$ , where n = 2000. If the underlying BLAS is using multiple threads, higher flop rates are realized. The number of BLAS threads can be set with blas\_set\_num\_threads(n).

If the keyword argument parallel is set to true, peakflops is run in parallel on all the worker processors. The flop rate of the entire parallel computer is returned. When running in parallel, only 1 BLAS thread is used. The argument n still refers to the size of the problem that is solved on each processor.

### 42.2 BLAS Functions

Base. LinAlg. BLAS (page 428) provides wrappers for some of the BLAS functions for linear algebra. Those BLAS functions that overwrite one of the input arrays have names ending in '!'.

Usually a function has 4 methods defined, one each for Float64, Float32, Complex128 and Complex64 arrays.

```
dot(n, X, incx, Y, incy)
```

Dot product of two vectors consisting of n elements of array X with stride incx and n elements of array Y with stride incy.

```
dotu(n, X, incx, Y, incy)
```

Dot function for two complex vectors.

```
dotc(n, X, incx, U, incy)
```

Dot function for two complex vectors conjugating the first vector.

```
blascopy! (n, X, incx, Y, incy)
```

Copy n elements of array X with stride incx to array Y with stride incy. Returns Y.

```
nrm2 (n, X, incx)
```

2-norm of a vector consisting of n elements of array X with stride incx.

```
asum(n, X, incx)
```

Sum of the absolute values of the first n elements of array X with stride incx.

```
axpv!(a, X, Y)
```

Overwrite Y with a \* X + Y. Returns Y.

#### scal!(n, a, X, incx)

Overwrite X with a \* X. Returns X.

### scal(n, a, X, incx)

Returns a \* X.

#### ger! (alpha, x, y, A)

Rank-1 update of the matrix A with vectors x and y as alpha\*x\*y' + A.

#### syr! (uplo, alpha, x, A)

Rank-1 update of the symmetric matrix A with vector x as alpha $\star$ x $\star$ x.' + A. When uplo is 'U' the upper triangle of A is updated ('L' for lower triangle). Returns A.

#### syrk! (uplo, trans, alpha, A, beta, C)

Rank-k update of the symmetric matrix C as alpha\*A\*A.' + beta\*C or alpha\*A.'\*A + beta\*C according to whether trans is 'N' or 'T'. When uplo is 'U' the upper triangle of C is updated ('L' for lower triangle). Returns C.

### syrk (uplo, trans, alpha, A)

Returns either the upper triangle or the lower triangle, according to uplo ('U' or 'L'), of alpha\*A\*A.' or alpha\*A.'\*A, according to trans ('N' or 'T').

#### **her!** (uplo, alpha, x, A)

Methods for complex arrays only. Rank-1 update of the Hermitian matrix A with vector x as alpha\*x\*x' + A. When uplo is 'U' the upper triangle of A is updated ('L' for lower triangle). Returns A.

#### herk! (uplo, trans, alpha, A, beta, C)

Methods for complex arrays only. Rank-k update of the Hermitian matrix C as alpha\*A\*A' + beta\*C or alpha\*A'\*A + beta\*C according to whether trans is 'N' or 'T'. When uplo is 'U' the upper triangle of C is updated ('L' for lower triangle). Returns C.

#### herk (uplo, trans, alpha, A)

Methods for complex arrays only. Returns either the upper triangle or the lower triangle, according to uplo ('U' or 'L'), of alpha\*A\*A' or alpha\*A'\*A, according to trans ('N' or 'T').

#### gbmv! (trans, m, kl, ku, alpha, A, x, beta, y)

Update vector y as alpha\*A\*x + beta\*y or alpha\*A'\*x + beta\*y according to trans ('N' or 'T'). The matrix A is a general band matrix of dimension m by size(A, 2) with kl sub-diagonals and ku super-diagonals. Returns the updated y.

### gbmv (trans, m, kl, ku, alpha, A, x, beta, y)

Returns alpha\*A\*x or alpha\*A' \*x according to trans ('N' or 'T'). The matrix A is a general band matrix of dimension m by size (A, 2) with kl sub-diagonals and ku super-diagonals.

#### **sbmv!** (uplo, k, alpha, A, x, beta, y)

Update vector y as alpha\*A\*x + beta\*y where A is a a symmetric band matrix of order size (A, 2) with k super-diagonals stored in the argument A. The storage layout for A is described the reference BLAS module, level-2 BLAS at <http://www.netlib.org/lapack/explore-html/>.

Returns the updated y.

#### sbmv(uplo, k, alpha, A, x)

Returns alpha\*A\*x where A is a symmetric band matrix of order size (A, 2) with k super-diagonals stored in the argument A.

#### sbmv(uplo, k, A, x)

Returns A \* x where A is a symmetric band matrix of order size(A, 2) with k super-diagonals stored in the argument A.

### gemm! (tA, tB, alpha, A, B, beta, C)

Update C as alpha\*A\*B + beta\*C or the other three variants according to tA (transpose A) and tB. Returns the updated C.

#### gemm(tA, tB, alpha, A, B)

Returns alpha\*A\*B or the other three variants according to tA (transpose A) and tB.

### gemm(tA, tB, A, B)

Returns A\*B or the other three variants according to tA (transpose A) and tB.

42.2. BLAS Functions 431

#### gemv! (tA, alpha, A, x, beta, y)

Update the vector y as alpha\*A\*x + beta\*y or alpha\*A'x + beta\*y according to tA (transpose A). Returns the updated y.

### gemv(tA, alpha, A, x)

Returns alpha\*A\*x or alpha\*A'x according to tA (transpose A).

#### gemv(tA, A, x)

Returns  $A \times x$  or  $A' \times according to <math>tA$  (transpose A).

#### symm! (side, ul, alpha, A, B, beta, C)

Update C as alpha\*A\*B + beta\*C or alpha\*B\*A + beta\*C according to side. A is assumed to be symmetric. Only the ul triangle of A is used. Returns the updated C.

#### symm(side, ul, alpha, A, B)

Returns alpha\*A\*B or alpha\*B\*A according to side. A is assumed to be symmetric. Only the ul triangle of A is used.

### symm(side, ul, A, B)

Returns A\*B or B\*A according to side. A is assumed to be symmetric. Only the ul triangle of A is used.

#### symm(tA, tB, alpha, A, B)

Returns alpha\*A\*B or the other three variants according to tA (transpose A) and tB.

#### **symv!** (ul, alpha, A, x, beta, y)

Update the vector y as alpha\*A\*x + beta\*y. A is assumed to be symmetric. Only the ul triangle of A is used. Returns the updated y.

#### symv(ul, alpha, A, x)

Returns alpha\*A\*x. A is assumed to be symmetric. Only the ul triangle of A is used.

#### symv(ul, A, x)

Returns A\*x. A is assumed to be symmetric. Only the ul triangle of A is used.

### trmm! (side, ul, tA, dA, alpha, A, B)

Update B as alpha\*A\*B or one of the other three variants determined by side (A on left or right) and tA (transpose A). Only the ul triangle of A is used. dA indicates if A is unit-triangular (the diagonal is assumed to be all ones). Returns the updated B.

#### trmm (side, ul, tA, dA, alpha, A, B)

Returns alpha\*A\*B or one of the other three variants determined by side (A on left or right) and tA (transpose A). Only the ul triangle of A is used. dA indicates if A is unit-triangular (the diagonal is assumed to be all ones).

#### trsm! (side, ul, tA, dA, alpha, A, B)

Overwrite B with the solution to A\*X = alpha\*B or one of the other three variants determined by side (A on left or right of X) and tA (transpose A). Only the ul triangle of A is used. dA indicates if A is unit-triangular (the diagonal is assumed to be all ones). Returns the updated B.

### trsm(side, ul, tA, dA, alpha, A, B)

Returns the solution to  $A \star X = alpha \star B$  or one of the other three variants determined by side (A on left or right of X) and tA (transpose A). Only the ul triangle of A is used. dA indicates if A is unit-triangular (the diagonal is assumed to be all ones).

#### trmv! (side, ul, tA, dA, alpha, A, b)

Update b as alpha\*A\*b or one of the other three variants determined by side (A on left or right) and tA (transpose A). Only the ul triangle of A is used. dA indicates if A is unit-triangular (the diagonal is assumed to be all ones). Returns the updated b.

#### trmv (side, ul, tA, dA, alpha, A, b)

Returns alpha\*A\*b or one of the other three variants determined by side (A on left or right) and tA (trans-

pose A). Only the ul triangle of A is used. dA indicates if A is unit-triangular (the diagonal is assumed to be all ones).

#### **trsv!** (*ul*, *tA*, *dA*, *A*, *b*)

Overwrite b with the solution to A\*x = b or one of the other two variants determined by tA (transpose A) and ul (triangle of A used). dA indicates if A is unit-triangular (the diagonal is assumed to be all ones). Returns the updated b.

#### trsv(ul, tA, dA, A, b)

Returns the solution to A\*x = b or one of the other two variants determined by tA (transpose A) and ul (triangle of A is used.) dA indicates if A is unit-triangular (the diagonal is assumed to be all ones).

#### blas\_set\_num\_threads(n)

Set the number of threads the BLAS library should use.

I

An object of type UniformScaling, representing an identity matrix of any size.

# 42.3 LAPACK Functions

Base. LinAlg. LAPACK (page 431) provides wrappers for some of the LAPACK functions for linear algebra. Those functions that overwrite one of the input arrays have names ending in '!'.

Usually a function has 4 methods defined, one each for Float64, Float32, Complex128 and Complex64 arrays.

Note that the LAPACK API provided by Julia can and will change in the future. Since this API is not user-facing, there is no commitment to support/deprecate this specific set of functions in future releases.

#### $gbtrf!(kl, ku, m, AB) \rightarrow (AB, ipiv)$

Compute the LU factorization of a banded matrix AB. kl is the first subdiagonal containing a nonzero band, ku is the last superdiagonal containing one, and m is the first dimension of the matrix AB. Returns the LU factorization in-place and ipiv, the vector of pivots used.

### gbtrs! (trans, kl, ku, m, AB, ipiv, B)

Solve the equation AB \* X = B. trans determines the orientation of AB. It may be N (no transpose), T (transpose), or C (conjugate transpose). kl is the first subdiagonal containing a nonzero band, ku is the last superdiagonal containing one, and m is the first dimension of the matrix AB. ipiv is the vector of pivots returned from gbtrf!. Returns the vector or matrix X, overwriting B in-place.

#### $gebal!(job, A) \rightarrow (ilo, ihi, scale)$

Balance the matrix A before computing its eigensystem or Schur factorization. job can be one of N (A will not be permuted or scaled), P (A will only be permuted), S (A will only be scaled), or B (A will be both permuted and scaled). Modifies A in-place and returns ilo, ihi, and scale. If permuting was turned on, A[i,j] = 0 if j > i and 1 < j < ilo or j > ihi. scale contains information about the scaling/permutations performed.

#### **gebak!** (*job*, *side*, *ilo*, *ihi*, *scale*, *V*)

Transform the eigenvectors V of a matrix balanced using gebal! to the unscaled/unpermuted eigenvectors of the original matrix. Modifies V in-place. side can be L (left eigenvectors are transformed) or R (right eigenvectors are transformed).

### **gebrd!** (*A*) -> (*A*, *d*, *e*, tauq, taup)

Reduce A in-place to bidiagonal form A = QBP'. Returns A, containing the bidiagonal matrix B; d, containing the diagonal elements of B; e, containing the off-diagonal elements of B; tauq, containing the elementary reflectors representing Q; and taup, containing the elementary reflectors representing P.

#### gelqf!(A, tau)

Compute the LQ factorization of A, A = LQ. tau contains scalars which parameterize the elementary reflectors of the factorization. tau must have length greater than or equal to the smallest dimension of A.

Returns A and tau modified in-place.

### $gelqf!(A) \rightarrow (A, tau)$

Compute the LQ factorization of A, A = LQ.

Returns A, modified in-place, and tau, which contains scalars which parameterize the elementary reflectors of the factorization.

### geqlf! (A, tau)

Compute the QL factorization of A, A = QL. tau contains scalars which parameterize the elementary reflectors of the factorization. tau must have length greater than or equal to the smallest dimension of A.

Returns A and tau modified in-place.

#### $geqlf!(A) \rightarrow (A, tau)$

Compute the QL factorization of A, A = QL.

Returns A, modified in-place, and tau, which contains scalars which parameterize the elementary reflectors of the factorization.

#### geqrf! (A, tau)

Compute the QR factorization of A, A = QR. tau contains scalars which parameterize the elementary reflectors of the factorization. tau must have length greater than or equal to the smallest dimension of A.

Returns A and tau modified in-place.

#### $qeqrf!(A) \rightarrow (A, tau)$

Compute the QR factorization of A, A = QR.

Returns A, modified in-place, and tau, which contains scalars which parameterize the elementary reflectors of the factorization.

#### geqp3! (A, jpvt, tau)

Compute the pivoted QR factorization of A, AP = QR using BLAS level 3. P is a pivoting matrix, represented by jpvt. tau stores the elementary reflectors. jpvt must have length length greater than or equal to n if A is an  $(m \times n)$  matrix. tau must have length greater than or equal to the smallest dimension of A.

A, jpvt, and tau are modified in-place.

### $qeqp3! (A, ipvt) \rightarrow (A, ipvt, tau)$

Compute the pivoted QR factorization of A, AP = QR using BLAS level 3. P is a pivoting matrix, represented by jpvt. jpvt must have length greater than or equal to n if A is an  $(m \times n)$  matrix.

Returns A and jpvt, modified in-place, and tau, which stores the elementary reflectors.

#### geqp3! $(A) \rightarrow (A, jpvt, tau)$

Compute the pivoted QR factorization of A, AP = QR using BLAS level 3.

Returns A, modified in-place, jpvt, which represents the pivoting matrix P, and tau, which stores the elementary reflectors.

#### gerqf! (A, tau)

Compute the RQ factorization of A, A = RQ. tau contains scalars which parameterize the elementary reflectors of the factorization. tau must have length greater than or equal to the smallest dimension of A.

Returns A and tau modified in-place.

### $gerqf!(A) \rightarrow (A, tau)$

Compute the RQ factorization of A, A = RQ.

Returns A, modified in-place, and tau, which contains scalars which parameterize the elementary reflectors of the factorization.

#### geqrt!(A, T)

Compute the blocked QR factorization of A, A = QR. T contains upper triangular block reflectors which parameterize the elementary reflectors of the factorization. The first dimension of T sets the block size and it must be between 1 and n. The second dimension of T must equal the smallest dimension of A.

Returns A and T modified in-place.

#### **geqrt!** (A, nb) -> (A, T)

Compute the blocked QR factorization of A, A = QR. nb sets the block size and it must be between 1 and n, the second dimension of A.

Returns A, modified in-place, and T, which contains upper triangular block reflectors which parameterize the elementary reflectors of the factorization.

#### geqrt3!(A, T)

Recursively computes the blocked QR factorization of A, A = QR. T contains upper triangular block reflectors which parameterize the elementary reflectors of the factorization. The first dimension of T sets the block size and it must be between 1 and n. The second dimension of T must equal the smallest dimension of A.

Returns A and T modified in-place.

#### **geqrt3!** (A) -> (A, T)

Recursively computes the blocked QR factorization of A, A = QR.

Returns A, modified in-place, and T, which contains upper triangular block reflectors which parameterize the elementary reflectors of the factorization.

#### $getrf!(A) \rightarrow (A, ipiv, info)$

Compute the pivoted LU factorization of A, A = LU.

Returns A, modified in-place, ipiv, the pivoting information, and an info code which indicates success (info = 0), a singular value in U (info = i, in which case U[i,i] is singular), or an error code (info < 0).

#### $tzrzf!(A) \rightarrow (A, tau)$

Transforms the upper trapezoidal matrix A to upper triangular form in-place. Returns A and tau, the scalar parameters for the elementary reflectors of the transformation.

#### ormrz! (side, trans, A, tau, C)

Multiplies the matrix C by Q from the transformation supplied by tzrzf!. Depending on side or trans the multiplication can be left-sided (side = L, Q\*C) or right-sided (side = R, C\*Q) and Q can be unmodified (trans = R), transposed (trans = R), or conjugate transposed (trans = R). Returns matrix R0 which is modified in-place with the result of the multiplication.

#### $qels!(trans, A, B) \rightarrow (F, B, ssr)$

Solves the linear equation  $A \star X = B$ ,  $A \cdot ' \star X = B$ , or  $A' \star X = B$  using a QR or LQ factorization. Modifies the matrix/vector B in place with the solution. A is overwritten with its QR or LQ factorization. trans may be one of N (no modification), T (transpose), or C (conjugate transpose). gels! searches for the minimum norm/least squares solution. A may be under or over determined. The solution is returned in B.

#### **gesv!** (A, B) -> (B, A, ipiv)

Solves the linear equation  $A \star X = B$  where A is a square matrix using the LU factorization of A. A is overwritten with its LU factorization and B is overwritten with the solution X. ipiv contains the pivoting information for the LU factorization of A.

### getrs! (trans, A, ipiv, B)

Solves the linear equation  $A \star X = B$ ,  $A \cdot ' \star X = B$ , or  $A' \star X = B$  for square A. Modifies the matrix/vector B in place with the solution. A is the LU factorization from getrf!, with ipiv the pivoting information. trans may be one of N (no modification), T (transpose), or C (conjugate transpose).

#### getri! (A, ipiv)

Computes the inverse of A, using its LU factorization found by getrf!. ipiv is the pivot information output and A contains the LU factorization of getrf!. A is overwritten with its inverse.

#### gesvx! (fact, trans, A, AF, ipiv, equed, R, C, B) -> (X, equed, R, C, B, rcond, ferr, berr, work)

Solves the linear equation A \* X = B (trans = N), A.' \* X = B (trans = T), or A' \* X = B (trans = C) using the LU factorization of A. fact may be E, in which case A will be equilibrated and copied to AF; F, in which case AF and ipiv from a previous LU factorization are inputs; or N, in which case A will be copied to AF and then factored. If fact = F, equed may be N, meaning A has not been equilibrated; R, meaning A was multiplied by diagm(R) from the left; C, meaning A was multiplied by diagm(C) from the right; or B, meaning A was multiplied by diagm(R) from the left and diagm(C) from the right. If fact = F and equed = R or B the elements of R must all be positive. If fact = F and equed = C or B the elements of C must all be positive.

Returns the solution X; equed, which is an output if fact is not N, and describes the equilibration that was performed; R, the row equilibration diagonal; C, the column equilibration diagonal; B, which may be overwritten with its equilibrated form diagm(R) \*B(iftrans = N and equed = R, B) or diagm(C) \*B(iftrans = T, C and equed = C, B); rcond, the reciprocal condition number of A after equilibrating; ferr, the forward error bound for each solution vector in X; berr, the forward error bound for each solution vector in X; and work, the reciprocal pivot growth factor.

#### qesvx!(A, B)

The no-equilibration, no-transpose simplification of gesvx!.

#### $gelsd! (A, B, rcond) \rightarrow (B, rnk)$

Computes the least norm solution of  $A \star X = B$  by finding the SVD factorization of A, then dividing-and-conquering the problem. B is overwritten with the solution X. Singular values below roond will be treated as zero. Returns the solution in B and the effective rank of A in rnk.

#### $gelsy! (A, B, rcond) \rightarrow (B, rnk)$

Computes the least norm solution of A \* X = B by finding the full QR factorization of A, then dividing-and-conquering the problem. B is overwritten with the solution X. Singular values below roond will be treated as zero. Returns the solution in B and the effective rank of A in rnk.

#### **gglse!** $(A, c, B, d) \rightarrow (X, res)$

Solves the equation  $A \star x = c$  where x is subject to the equality constraint  $B \star x = d$ . Uses the formula  $| c - A \star x | ^2 = 0$  to solve. Returns X and the residual sum-of-squares.

### geev! $(jobvl, jobvr, A) \rightarrow (W, VL, VR)$

Finds the eigensystem of A. If jobvl = N, the left eigenvectors of A aren't computed. If jobvr = N, the right eigenvectors of A aren't computed. If jobvl = V or jobvr = V, the corresponding eigenvectors are computed. Returns the eigenvalues in W, the right eigenvectors in VR, and the left eigenvectors in VL.

#### gesdd! $(job, A) \rightarrow (U, S, VT)$

Finds the singular value decomposition of A, A = U \* S \* V', using a divide and conquer approach. If  $j \circ b = A$ , all the columns of U and the rows of V' are computed. If  $j \circ b = N$ , no columns of U or rows of V' are computed. If  $j \circ b = O$ , A is overwritten with the columns of (thin) U and the rows of (thin) V'. If  $j \circ b = S$ , the columns of (thin) U and the rows of (thin) V' are computed and returned separately.

### gesvd! $(jobu, jobvt, A) \rightarrow (U, S, VT)$

Finds the singular value decomposition of A, A = U \* S \* V'. If jobu = A, all the columns of U are computed. If jobvt = A all the rows of V' are computed. If jobu = N, no columns of U are computed. If jobvt = N no rows of V' are computed. If jobu = O, A is overwritten with the columns of (thin) U. If jobvt = O, A is overwritten with the rows of (thin) V'. If jobu = S, the columns of (thin) U are computed and returned separately. If jobvt = S the rows of (thin) V' are computed and returned separately. jobvt and jobvt can't both be O.

Returns U, S, and Vt, where S are the singular values of A.

### ggsvd! (jobu, joby, jobq, A, B) -> (U, V, Q, alpha, beta, k, l, R)

Finds the generalized singular value decomposition of A and B, U' \*A\*Q = D1\*R and V' \*B\*Q = D2\*R. D1 has alpha on its diagonal and D2 has beta on its diagonal. If jobu = U, the orthogonal/unitary matrix U is computed. If jobv = V the orthogonal/unitary matrix V is computed. If jobq = Q, the orthogonal/unitary matrix Q is computed. If jobu, jobv or jobq is N, that matrix is not computed. This function is only available in LAPACK versions prior to 3.6.0.

### ggsvd3! $(jobu, jobv, jobq, A, B) \rightarrow (U, V, Q, alpha, beta, k, l, R)$

Finds the generalized singular value decomposition of A and B, U' \*A\*Q = D1\*R and V' \*B\*Q = D2\*R. D1 has alpha on its diagonal and D2 has beta on its diagonal. If jobu = U, the orthogonal/unitary matrix U is computed. If jobv = V the orthogonal/unitary matrix V is computed. If jobq = Q, the orthogonal/unitary matrix Q is computed. If jobu, jobv, or jobq is N, that matrix is not computed. This function requires LAPACK 3.6.0.

#### **geevx!** (balanc, jobvl, jobvr, sense, A) -> (A, w, VL, VR, ilo, ihi, scale, abnrm, rconde, rcondv)

Finds the eigensystem of A with matrix balancing. If jobvl = N, the left eigenvectors of A aren't computed. If jobvl = N of the right eigenvectors of A aren't computed. If jobvl = V or jobvr = V, the corresponding eigenvectors are computed. If balanc = N, no balancing is performed. If balanc = P, A is permuted but not scaled. If balanc = S, A is scaled but not permuted. If balanc = B, A is permuted and scaled. If sense = N, no reciprocal condition numbers are computed. If sense = E, reciprocal condition numbers are computed for the eigenvalues only. If sense = V, reciprocal condition numbers are computed for the right eigenvectors only. If sense = B, reciprocal condition numbers are computed for the right eigenvectors. If sense = E, B, the right and left eigenvectors must be computed.

### **ggev!** $(jobvl, jobvr, A, B) \rightarrow (alpha, beta, vl, vr)$

Finds the generalized eigendecomposition of A and B. If jobvl = N, the left eigenvectors aren't computed. If jobvl = N or jobvl = V, the corresponding eigenvectors are computed.

### gtsv!(dl, d, du, B)

Solves the equation  $A \star X = B$  where A is a tridiagonal matrix with d1 on the subdiagonal, d on the diagonal, and du on the superdiagonal.

Overwrites B with the solution X and returns it.

#### gttrf! $(dl, d, du) \rightarrow (dl, d, du, du2, ipiv)$

Finds the LU factorization of a tridiagonal matrix with dl on the subdiagonal, d on the diagonal, and du on the superdiagonal.

Modifies d1, d, and du in-place and returns them and the second superdiagonal du2 and the pivoting vector ipiv.

#### gttrs! (trans, dl, d, du, du2, ipiv, B)

Solves the equation  $A \star X = B$  (trans = N),  $A \cdot ' \star X = B$  (trans = T), or  $A' \star X = B$  (trans = C) using the LU factorization computed by gttrf!. B is overwritten with the solution X.

#### orglq! (A, tau, k = length(tau))

Explicitly finds the matrix Q of a LQ factorization after calling gelqf! on A. Uses the output of gelqf!. A is overwritten by Q.

### orgqr! (A, tau, k = length(tau))

Explicitly finds the matrix Q of a QR factorization after calling geqrf! on A. Uses the output of geqrf!. A is overwritten by Q.

### **orgql!** (A, tau, k = length(tau))

Explicitly finds the matrix Q of a QL factorization after calling geqlf! on A. Uses the output of geqlf!. A is overwritten by Q.

### orgrq! (A, tau, k = length(tau))

Explicitly finds the matrix Q of a RQ factorization after calling gerqf! on A. Uses the output of gerqf!. A is

overwritten by Q.

#### ormlq! (side, trans, A, tau, C)

Computes Q \* C (trans = N), Q.' \* C (trans = T), Q' \* C (trans = C) for side = L or the equivalent right-sided multiplication for side = R using Q from a LQ factorization of A computed using gelqf!. C is overwritten.

#### ormgr! (side, trans, A, tau, C)

Computes Q \* C (trans = N), Q.' \* C (trans = T), Q' \* C (trans = C) for side = L or the equivalent right-sided multiplication for side = R using Q from a QR factorization of A computed using geqrf!. C is overwritten.

### ormql! (side, trans, A, tau, C)

Computes Q \* C (trans = N), Q.' \* C (trans = T), Q' \* C (trans = C) for side = L or the equivalent right-sided multiplication for side = R using Q from a QL factorization of A computed using geqlf!. C is overwritten.

#### ormrq! (side, trans, A, tau, C)

Computes Q \* C (trans = N), Q.' \* C (trans = T), Q' \* C (trans = C) for side = L or the equivalent right-sided multiplication for side = R using Q from a RQ factorization of A computed using gerqf!. C is overwritten.

#### gemgrt! (side, trans, V, T, C)

Computes  $Q \star C$  (trans = N),  $Q \cdot ' \star C$  (trans = T),  $Q' \star C$  (trans = C) for side = L or the equivalent right-sided multiplication for side = R using Q from a QR factorization of A computed using qeqrt!. C is overwritten.

### posv! $(uplo, A, B) \rightarrow (A, B)$

Finds the solution to  $A \star X = B$  where A is a symmetric or Hermitian positive definite matrix. If uplo = U the upper Cholesky decomposition of A is computed. If uplo = L the lower Cholesky decomposition of A is computed. A is overwritten by its Cholesky decomposition. B is overwritten with the solution X.

### potrf! (uplo, A)

Computes the Cholesky (upper if uplo = U, lower if uplo = L) decomposition of positive-definite matrix A. A is overwritten and returned with an info code.

#### potri! (uplo, A)

Computes the inverse of positive-definite matrix A after calling potrf! to find its (upper if uplo = U, lower if uplo = L) Cholesky decomposition.

A is overwritten by its inverse and returned.

### potrs! (uplo, A, B)

Finds the solution to A \* X = B where A is a symmetric or Hermitian positive definite matrix whose Cholesky decomposition was computed by potrf!. If uplo = U the upper Cholesky decomposition of A was computed. If uplo = L the lower Cholesky decomposition of A was computed. B is overwritten with the solution X.

### pstrf! (uplo, A, tol) -> (A, piv, rank, info)

Computes the (upper if uplo = U, lower if uplo = L) pivoted Cholesky decomposition of positive-definite matrix A with a user-set tolerance tol. A is overwritten by its Cholesky decomposition.

Returns A, the pivots piv, the rank of A, and an info code. If info = 0, the factorization succeeded. If info = i > 0 'A is indefinite or rank-deficient.

### **ptsv!** (*D*, *E*, *B*)

Solves  $A \star X = B$  for positive-definite tridiagonal A. D is the diagonal of A and E is the off-diagonal. B is overwritten with the solution X and returned.

#### pttrf!(D, E)

Computes the LDLt factorization of a positive-definite tridiagonal matrix with D as diagonal and E as off-diagonal. D and E are overwritten and returned.

#### pttrs! (D, E, B)

Solves  $A \star X = B$  for positive-definite tridiagonal A with diagonal D and off-diagonal E after computing A's LDLt factorization using pttrf!. B is overwritten with the solution X.

### trtri! (uplo, diag, A)

Finds the inverse of (upper if uplo = U, lower if uplo = L) triangular matrix A. If diag = N, A has non-unit diagonal elements. If diag = U, all diagonal elements of A are one. A is overwritten with its inverse.

#### trtrs! (uplo, trans, diag, A, B)

Solves A  $\star$  X = B (trans = N), A.'  $\star$  X = B (trans = T), or A'  $\star$  X = B (trans = C) for (upper if uplo = U, lower if uplo = L) triangular matrix A. If diag = N, A has non-unit diagonal elements. If diag = U, all diagonal elements of A are one. B is overwritten with the solution X.

### trcon! (norm, uplo, diag, A)

Finds the reciprocal condition number of (upper if uplo = U, lower if uplo = L) triangular matrix A. If diag = N, A has non-unit diagonal elements. If diag = U, all diagonal elements of A are one. If norm = I, the condition number is found in the infinity norm. If norm = O or 1, the condition number is found in the one norm.

#### **trevc!** (side, howmny, select, T, VL = similar(T), VR = similar(T))

Finds the eigensystem of an upper triangular matrix T. If side = R, the right eigenvectors are computed. If side = L, the left eigenvectors are computed. If side = B, both sets are computed. If howmny = A, all eigenvectors are found. If howmny = B, all eigenvectors are found and backtransformed using VL and VR. If howmny = S, only the eigenvectors corresponding to the values in select are computed.

#### trrfs! (uplo, trans, diag, A, B, X, Ferr, Berr) -> (Ferr, Berr)

Estimates the error in the solution to A  $\star$  X = B (trans = N), A.'  $\star$  X = B (trans = T), A'  $\star$  X = B (trans = C) for side = L, or the equivalent equations a right-handed side = R X  $\star$  A after computing X using trtrs!. If uplo = U, A is upper triangular. If uplo = L, A is lower triangular. If diag = N, A has non-unit diagonal elements. If diag = U, all diagonal elements of A are one. Ferr and Berr are optional inputs. Ferr is the forward error and Berr is the backward error, each component-wise.

#### **stev!** $(job, dv, ev) \rightarrow (dv, Zmat)$

Computes the eigensystem for a symmetric tridiagonal matrix with dv as diagonal and ev as off-diagonal. If job = N only the eigenvalues are found and returned in dv. If job = V then the eigenvectors are also found and returned in Zmat.

### **stebz!** (range, order, vl, vu, il, iu, abstol, dv, ev) -> (dv, iblock, isplit)

Computes the eigenvalues for a symmetric tridiagonal matrix with dv as diagonal and ev as off-diagonal. If range = A, all the eigenvalues are found. If range = V, the eigenvalues in the half-open interval (vl, vu] are found. If range = I, the eigenvalues with indices between il and iu are found. If order = B, eigvalues are ordered within a block. If order = E, they are ordered across all the blocks. abstol can be set as a tolerance for convergence.

### **stegr!** (jobz, range, dv, ev, vl, vu, il, iu) -> (w, Z)

Computes the eigenvalues (jobz = N) or eigenvalues and eigenvectors (jobz = V) for a symmetric tridiagonal matrix with dv as diagonal and ev as off-diagonal. If range = A, all the eigenvalues are found. If range = V, the eigenvalues in the half-open interval (vl, vu] are found. If range = I, the eigenvalues with indices between il and iu are found. The eigenvalues are returned in w and the eigenvectors in Z.

#### stein! (dv, ev\_in, w\_in, iblock\_in, isplit\_in)

Computes the eigenvectors for a symmetric tridiagonal matrix with dv as diagonal and ev\_in as off-diagonal. w\_in specifies the input eigenvalues for which to find corresponding eigenvectors. iblock\_in specifies the submatrices corresponding to the eigenvalues in w\_in. isplit\_in specifies the splitting points between the submatrix blocks.

#### syconv! $(uplo, A, ipiv) \rightarrow (A, work)$

Converts a symmetric matrix A (which has been factorized into a triangular matrix) into two matrices L and D. If uplo = U, A is upper triangular. If uplo = L, it is lower triangular. ipiv is the pivot vector from the triangular factorization. A is overwritten by L and D.

#### **sysv!** $(uplo, A, B) \rightarrow (B, A, ipiv)$

Finds the solution to A \* X = B for symmetric matrix A. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored. B is overwritten by the solution X. A is overwritten by its Bunch-Kaufman factorization. ipiv contains pivoting information about the factorization.

#### $sytrf! (uplo, A) \rightarrow (A, ipiv)$

Computes the Bunch-Kaufman factorization of a symmetric matrix A. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored.

Returns A, overwritten by the factorization, and a pivot vector ipiv.

#### sytri! (uplo, A, ipiv)

Computes the inverse of a symmetric matrix A using the results of sytrf!. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored. A is overwritten by its inverse.

### sytrs! (uplo, A, ipiv, B)

Solves the equation A \* X = B for a symmetric matrix A using the results of sytrf!. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored. B is overwritten by the solution X.

#### **hesv!** $(uplo, A, B) \rightarrow (B, A, ipiv)$

Finds the solution to  $A \star X = B$  for Hermitian matrix A. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored. B is overwritten by the solution X. A is overwritten by its Bunch-Kaufman factorization. ipiv contains pivoting information about the factorization.

### $hetrf! (uplo, A) \rightarrow (A, ipiv)$

Computes the Bunch-Kaufman factorization of a Hermitian matrix A. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored.

Returns A, overwritten by the factorization, and a pivot vector.

#### hetri! (uplo, A, ipiv)

Computes the inverse of a Hermitian matrix A using the results of sytrf!. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored. A is overwritten by its inverse.

#### hetrs! (uplo, A, ipiv, B)

Solves the equation A \* X = B for a Hermitian matrix A using the results of sytrf!. If uplo = U, the upper half of A is stored. If uplo = L, the lower half is stored. B is overwritten by the solution X.

### syev! (jobz, uplo, A)

Finds the eigenvalues (jobz = N) or eigenvalues and eigenvectors (jobz = V) of a symmetric matrix A. If uplo = U, the upper triangle of A is used. If uplo = L, the lower triangle of A is used.

### **syevr!** (jobz, range, uplo, A, vl, vu, il, iu, abstol) -> (W, Z)

Finds the eigenvalues (jobz = N) or eigenvalues and eigenvectors (jobz = V) of a symmetric matrix A. If uplo = U, the upper triangle of A is used. If uplo = L, the lower triangle of A is used. If range = A, all the eigenvalues are found. If range = V, the eigenvalues in the half-open interval (vl, vu] are found. If range = I, the eigenvalues with indices between il and iu are found. abstol can be set as a tolerance for convergence.

The eigenvalues are returned in W and the eigenvectors in Z.

### **sygvd!** (jobz, range, uplo, A, vl, vu, il, iu, abstol) -> (w, A, B)

Finds the generalized eigenvalues (jobz = N) or eigenvalues and eigenvectors (jobz = V) of a symmetric matrix A and symmetric positive-definite matrix B. If uplo = U, the upper triangles of A and B are used. If uplo = L, the lower triangles of A and B are used. If itype = 1, the problem to solve is A  $\star$  x =

lambda \* B \* x. If itype = 2, the problem to solve is A \* B \* x = lambda \* x. If itype = 3, the problem to solve is B \* A \* x = lambda \* x.

#### **bdsqr!** (*uplo*, *d*, $e_{-}$ , Vt, U, C) -> (d, Vt, U, C)

Computes the singular value decomposition of a bidiagonal matrix with d on the diagonal and e\_ on the off-diagonal. If uplo = U, e\_ is the superdiagonal. If uplo = L, e\_ is the subdiagonal. Can optionally also compute the product  $Q' \star C$ .

Returns the singular values in d, and the matrix C overwritten with  $Q' \star C$ .

### **bdsdc!** (uplo, compq, d, $e_{-}$ ) -> (d, e, u, vt, q, iq)

Computes the singular value decomposition of a bidiagonal matrix with d on the diagonal and e\_ on the off-diagonal using a divide and conqueq method. If uplo = U, e\_ is the superdiagonal. If uplo = L, e\_ is the subdiagonal. If compq = N, only the singular values are found. If compq = I, the singular values and vectors are found. If compq = P, the singular values and vectors are found in compact form. Only works for real types.

Returns the singular values in d, and if compq = P, the compact singular vectors in iq.

#### gecon! (normtype, A, anorm)

Finds the reciprocal condition number of matrix A. If normtype = I, the condition number is found in the infinity norm. If normtype = O or 1, the condition number is found in the one norm. A must be the result of getrf! and anorm is the norm of A in the relevant norm.

#### gehrd! (ilo, ihi, A) -> (A, tau)

Converts a matrix A to Hessenberg form. If A is balanced with gebal! then ilo and ihi are the outputs of gebal!. Otherwise they should be ilo = 1 and ihi = size(A, 2). tau contains the elementary reflectors of the factorization.

#### orghr! (ilo, ihi, A, tau)

Explicitly finds Q, the orthogonal/unitary matrix from gehrd!. ilo, ihi, A, and tau must correspond to the input/output to gehrd!.

### gees! $(jobvs, A) \rightarrow (A, vs, w)$

Computes the eigenvalues (jobvs = N) or the eigenvalues and Schur vectors (jobvs = V) of matrix A. A is overwritten by its Schur form.

Returns A, vs containing the Schur vectors, and w, containing the eigenvalues.

#### gges! $(jobvsl, jobvsr, A, B) \rightarrow (A, B, alpha, beta, vsl, vsr)$

Computes the generalized eigenvalues, generalized Schur form, left Schur vectors (jobsvl = V), or right Schur vectors (jobvsr = V) of A and B.

The generalized eigenvalues are returned in alpha and beta. The left Schur vectors are returned in vsl and the right Schur vectors are returned in vsr.

#### trexc! $(compq, ifst, ilst, T, Q) \rightarrow (T, Q)$

Reorder the Schur factorization of a matrix. If compq = V, the Schur vectors Q are reordered. If compq = N they are not modified. if st and ilst specify the reordering of the vectors.

### trsen! $(compq, job, select, T, Q) \rightarrow (T, Q, w)$

Reorder the Schur factorization of a matrix and optionally finds reciprocal condition numbers. If job = N, no condition numbers are found. If job = E, only the condition number for this cluster of eigenvalues is found. If job = V, only the condition number for the invariant subspace is found. If job = B then the condition numbers for the cluster and subspace are found. If compq = V the Schur vectors Q are updated. If compq = N the Schur vectors are not modified. select determines which eigenvalues are in the cluster.

Returns T, Q, and reordered eigenvalues in w.

#### tgsen! (select, S, T, Q, Z) $\rightarrow$ (S, T, alpha, beta, Q, Z)

Reorders the vectors of a generalized Schur decomposition. select specifices the eigenvalues in each cluster.

### **trsyl!** $(transa, transb, A, B, C, isgn=1) \rightarrow (C, scale)$

Solves the Sylvester matrix equation A \* X +/- X \* B = scale\*C where A and B are both quasi-upper triangular. If transa = N, A is not modified. If transa = T, A is transposed. If transa = C, A is conjugate transposed. Similarly for transb and B. If isgn = 1, the equation A \* X + X \* B = scale \* C is solved. If isgn = -1, the equation A \* X - X \* B = scale \* C is solved.

Returns X (overwriting C) and scale.

# **Constants**

#### nothing

The singleton instance of type Void, used by convention when there is no value to return (as in a C void function). Can be converted to an empty Nullable value.

#### OS NAME

A symbol representing the name of the operating system. Possible values are : Linux, : Darwin (OS X), or : Windows.

#### **ARGS**

An array of the command line arguments passed to Julia, as strings.

#### C NULL

The C null pointer constant, sometimes used when calling external code.

#### CPU\_CORES

The number of CPU cores in the system.

#### WORD\_SIZE

Standard word size on the current machine, in bits.

#### VERSION

An object describing which version of Julia is in use.

#### LOAD PATH

An array of paths (as strings) where the require function looks for code.

### JULIA\_HOME

A string containing the full path to the directory containing the julia executable.

#### ANY

Equivalent to Any for dispatch purposes, but signals the compiler to skip code generation specialization for that field

### See also:

STDIN (page 449) STDOUT (page 449) STDERR (page 449) ENV (page 329) ENDIAN\_BOM (page 461) MS\_ASYNC MS\_INVALIDATE MS\_SYNC DL\_LOAD\_PATH RTLD\_DEEPBIND RTLD\_LOCAL RTLD\_NOLOAD RTLD\_LAZY RTLD\_NOW RTLD\_GLOBAL RTLD\_NODELETE RTLD\_FIRST

# **Filesystem**

```
\textbf{pwd} \ (\ ) \ \rightarrow AbstractString
```

Get the current working directory.

```
cd ([dir::AbstractString=homedir()])
Set the current working directory.
```

cd(f[, dir=homedir()])

Temporarily changes the current working directory and applies function f before returning.

```
readdir([dir]) \rightarrow Vector\{ByteString\}
```

Returns the files and directories in the directory dir (or the current working directory if not given).

```
walkdir (dir; topdown=true, follow_symlinks=false, onerror=throw)
```

The walkdir method return an iterator that walks the directory tree of a directory. The iterator returns a tuple containing (rootpath, dirs, files). The directory tree can be traversed top-down or bottom-up. If walkdir encounters a SystemError it will raise the error. A custom error handling function can be provided through onerror keyword argument, the function is called with a SystemError as argument.

```
for (root, dirs, files) in walkdir(".")
    println("Directories in $root")
    for dir in dirs
        println(joinpath(root, dir)) # path to directories
    end
    println("Files in $root")
    for file in files
        println(joinpath(root, file)) # path to files
    end
end
```

### mkdir (path[, mode])

Make a new directory with name path and permissions mode. mode defaults to 0o777, modified by the current file creation mask.

### mkpath (path, mode)

Create all directories in the given path, with permissions mode. mode defaults to 0o777, modified by the current file creation mask.

#### symlink (target, link)

Creates a symbolic link to target with the name link.

#### note

This function raises an error under operating systems that do not support soft symbolic links, such as Windows XP.

#### **readlink** (*path*) → AbstractString

Returns the value of a symbolic link path.

#### chmod (path, mode)

Change the permissions mode of path to mode. Only integer modes (e.g. 00777) are currently supported.

#### stat (file)

Returns a structure whose fields contain information about the file. The fields of the structure are:

Name	Description
size	The size (in bytes) of the file
device	ID of the device that contains the file
inode	The inode number of the file
mode	The protection mode of the file
nlink	The number of hard links to the file
uid	The user id of the owner of the file
gid	The group id of the file owner
rdev	If this file refers to a device, the ID of the device it refers to
blksize	The file-system preferred block size for the file
blocks	The number of such blocks allocated
mtime	Unix timestamp of when the file was last modified
ctime	Unix timestamp of when the file was created

#### lstat (file)

Like stat, but for symbolic links gets the info for the link itself rather than the file it refers to. This function must be called on a file path rather than a file object or a file descriptor.

### ctime(file)

Equivalent to stat (file) .ctime

### mtime (file)

Equivalent to stat (file) .mtime.

### filemode(file)

Equivalent to stat (file) .mode

#### filesize (path...)

Equivalent to stat (file) .size.

### $\mathtt{uperm}\,(\mathit{file}\,)$

Gets the permissions of the owner of the file as a bitfield of

Value	Description
01	Execute Permission
02	Write Permission
04	Read Permission

For allowed arguments, see stat.

#### gperm (file)

Like uperm but gets the permissions of the group owning the file.

#### operm (file)

Like uperm but gets the permissions for people who neither own the file nor are a member of the group owning the file

cp (src::AbstractString, dst::AbstractString; remove\_destination::Bool=false, follow\_symlinks::Bool=false)

Copy the file, link, or directory from src to dest. remove\_destination=true will first remove an existing dst.

If follow\_symlinks=false, and src is a symbolic link, dst will be created as a symbolic link. If follow\_symlinks=true and src is a symbolic link, dst will be a copy of the file or directory src refers to

# download(url[, localfile])

Download a file from the given url, optionally renaming it to the given local file name. Note that this function relies on the availability of external tools such as curl, wget or fetch to download the file and is provided for convenience. For production use or situations in which more options are needed, please use a package that provides the desired functionality instead.

### **mv** (src::AbstractString, dst::AbstractString; remove\_destination::Bool=false)

Move the file, link, or directory from src to dst. remove\_destination=true will first remove an existing dst.

### **rm** (path::AbstractString; force=false, recursive=false)

Delete the file, link, or empty directory at the given path. If force=true is passed, a non-existing path is not treated as error. If recursive=true is passed and the path is a directory, then all contents are removed recursively.

### touch (path::AbstractString)

Update the last-modified timestamp on a file to the current time.

### tempname()

Generate a unique temporary file path.

#### tempdir()

Obtain the path of a temporary directory (possibly shared with other processes).

### mktemp ([parent=tempdir()])

Returns (path, io), where path is the path of a new temporary file in parent and io is an open file object for this path.

# mktemp (f::Function[, parent=tempdir()])

Apply the function f to the result of mktemp (parent) and remove the temporary file upon completion.

### mktempdir([parent=tempdir()])

Create a temporary directory in the parent directory and return its path.

### mktempdir (f::Function[, parent=tempdir()])

Apply the function f to the result of mktempdir (parent) and remove the temporary directory upon completion.

#### $isblockdev(path) \rightarrow Bool$

Returns true if path is a block device, false otherwise.

#### $ischardev(path) \rightarrow Bool$

Returns true if path is a character device, false otherwise.

#### $isdir(path) \rightarrow Bool$

Returns true if path is a directory, false otherwise.

#### **isfifo** $(path) \rightarrow Bool$

Returns true if path is a FIFO, false otherwise.

#### $isfile(path) \rightarrow Bool$

Returns true if path is a regular file, false otherwise.

### $islink(path) \rightarrow Bool$

Returns true if path is a symbolic link, false otherwise.

### $ismount(path) \rightarrow Bool$

Returns true if path is a mount point, false otherwise.

#### $ispath(path) \rightarrow Bool$

Returns true if path is a valid filesystem path, false otherwise.

#### $issetgid(path) \rightarrow Bool$

Returns true if path has the setgid flag set, false otherwise.

### $issetuid(path) \rightarrow Bool$

Returns true if path has the setuid flag set, false otherwise.

#### $issocket(path) \rightarrow Bool$

Returns true if path is a socket, false otherwise.

### $issticky(path) \rightarrow Bool$

Returns true if path has the sticky bit set, false otherwise.

#### **homedir**() $\rightarrow$ AbstractString

Return the current user's home directory.

#### **dirname** (path::AbstractString) → AbstractString

Get the directory part of a path.

#### $basename(path::AbstractString) \rightarrow AbstractString$

Get the file name part of a path.

### @\_\_FILE\_\_ -> AbstractString

@\_\_FILE\_\_ expands to a string with the absolute path and file name of the script being run. Returns nothing if run from a REPL or an empty string if evaluated by julia -e <expr>.

#### @\_\_LINE\_\_ -> Int

@ LINE expands to the line number of the call-site.

### **isabspath** (*path::AbstractString*) → Bool

Determines whether a path is absolute (begins at the root directory).

### **isdirpath** (*path::AbstractString*) → Bool

Determines whether a path refers to a directory (for example, ends with a path separator).

### joinpath (*parts*...) → AbstractString

Join path components into a full path. If some argument is an absolute path, then prior components are dropped.

#### **abspath** (*path::AbstractString*) → AbstractString

Convert a path to an absolute path by adding the current directory if necessary.

### **normpath** (*path::AbstractString*) → AbstractString

Normalize a path, removing "." and ".." entries.

#### realpath (path::AbstractString) → AbstractString

Canonicalize a path by expanding symbolic links and removing "." and ".." entries.

### relpath (path::AbstractString, startpath::AbstractString = ".") → AbstractString

Return a relative filepath to path either from the current directory or from an optional start directory. This is a path computation: the filesystem is not accessed to confirm the existence or nature of path or startpath.

### $expanduser(path::AbstractString) \rightarrow AbstractString$

On Unix systems, replace a tilde character at the start of a path with the current user's home directory.

#### splitdir(path::AbstractString) -> (AbstractString, AbstractString)

Split a path into a tuple of the directory name and file name.

### splitdrive (path::AbstractString) -> (AbstractString, AbstractString)

On Windows, split a path into the drive letter part and the path part. On Unix systems, the first component is always the empty string.

splitext(path::AbstractString) -> (AbstractString, AbstractString)

If the last component of a path contains a dot, split the path into everything before the dot and everything including and after the dot. Otherwise, return a tuple of the argument unmodified and the empty string.

# I/O and Network

# 45.1 General I/O

#### STDOUT

Global variable referring to the standard out stream.

#### **STDERR**

Global variable referring to the standard error stream.

#### STDIN

Global variable referring to the standard input stream.

 $open(file\_name[, read, write, create, truncate, append]) \rightarrow IOStream$ 

Open a file in a mode specified by five boolean arguments. The default is to open files for reading only. Returns a stream for accessing the file.

$$\mathtt{open}\ (\mathit{file\_name}\big[,\mathit{mode}\ \big])\ \to \mathrm{IOStream}$$

Alternate syntax for open, where a string-based mode specifier is used instead of the five booleans. The values of mode correspond to those from fopen (3) or Perl open, and are equivalent to setting the following boolean groups:

Mode	Description
r	read
r+	read, write
W	write, create, truncate
w+	read, write, create, truncate
a	write, create, append
a+	read, write, create, append

open (command, mode::AbstractString="r", stdio=DevNull)

Start running command asynchronously, and return a tuple (stream, process). If mode is "r", then stream reads from the process's standard output and stdio optionally specifies the process's standard input stream. If mode is "w", then stream writes to the process's standard input and stdio optionally specifies the process's standard output stream.

open (f::Function, command, mode::AbstractString="r", stdio=DevNull)

Similar to open (command, mode, stdio), but calls f (stream) on the resulting read or write stream, then closes the stream and waits for the process to complete. Returns the value returned by f.

open (f::Function, args...)

Apply the function f to the result of open (args...) and close the resulting file descriptor upon completion.

Example: open (readstring, "file.txt")

#### $IOBuffer() \rightarrow IOBuffer$

Create an in-memory I/O stream.

#### IOBuffer (size::Int)

Create a fixed size IOBuffer. The buffer will not grow dynamically.

#### IOBuffer (string)

Create a read-only IOBuffer on the data underlying the given string.

```
IOBuffer ([data][, readable, writable[, maxsize]])
```

Create an IOBuffer, which may optionally operate on a pre-existing array. If the readable/writable arguments are given, they restrict whether or not the buffer may be read from or written to respectively. By default the buffer is readable but not writable. The last argument optionally specifies a size beyond which the buffer may not be grown.

### takebuf\_array (b::IOBuffer)

Obtain the contents of an IOBuffer as an array, without copying. Afterwards, the IOBuffer is reset to its initial state.

### takebuf\_string(b::IOBuffer)

Obtain the contents of an IOBuffer as a string, without copying. Afterwards, the IOBuffer is reset to its initial state.

# fdio([name::AbstractString], fd::Integer[, own::Bool]) $\rightarrow$ IOStream

Create an IOStream object from an integer file descriptor. If own is true, closing this object will close the underlying descriptor. By default, an IOStream is closed when it is garbage collected. name allows you to associate the descriptor with a named file.

#### flush(stream)

Commit all currently buffered writes to the given stream.

#### close (stream)

Close an I/O stream. Performs a flush first.

### write (stream or filename, x)

Write the canonical binary representation of a value to the given stream or file. Returns the number of bytes written into the stream.

You can write multiple values with the same :func:write call. i.e. the following are equivalent:

```
write(stream, x, y...)
write(stream, x) + write(stream, y...)
```

#### read (stream, type)

Read a value of the given type from a stream, in canonical binary representation.

### read (stream, type, dims)

Read a series of values of the given type from a stream, in canonical binary representation. dims is either a tuple or a series of integer arguments specifying the size of Array to return.

#### read! (stream or filename, array::Array)

Read binary data from a stream or file, filling in the argument array.

### readbytes! (stream, b::Vector{UInt8}, nb=length(b); all=true)

Read at most nb bytes from the stream into b, returning the number of bytes read (increasing the size of b as needed).

See read for a description of the all option.

### read (stream, nb=typemax(Int); all=true)

Read at most nb bytes from the stream, returning a Vector{UInt8} of the bytes read.

If all is true (the default), this function will block repeatedly trying to read all requested bytes, until an error or end-of-file occurs. If all is false, at most one read call is performed, and the amount of data returned is device-dependent. Note that not all stream types support the all option.

### unsafe\_read (io, ref, nbytes)

Copy nbytes from the IO stream object into ref (converted to a pointer).

It is recommended that IO subtypes override the exact method signature below to provide more efficient implementations: unsafe\_read(s::IO, p::Ptr{UInt8}, n::UInt)

#### unsafe\_write(io, ref, nbytes)

Copy nbytes from ref (converted to a pointer) into the IO stream object.

It is recommended that IO subtypes override the exact method signature below to provide more efficient implementations: unsafe\_write(s::IO, p::Ptr{UInt8}, n::UInt)

#### position(s)

Get the current position of a stream.

#### seek(s, pos)

Seek a stream to the given position.

#### seekstart(s)

Seek a stream to its beginning.

#### seekend(s)

Seek a stream to its end.

### skip(s, offset)

Seek a stream relative to the current position.

#### mark(s)

Add a mark at the current position of stream s. Returns the marked position.

```
See also unmark () (page 451), reset () (page 451), ismarked () (page 451).
```

#### unmark(s)

Remove a mark from stream s. Returns true if the stream was marked, false otherwise.

```
See also mark () (page 451), reset () (page 451), ismarked () (page 451).
```

#### reset(s)

Reset a stream s to a previously marked position, and remove the mark. Returns the previously marked position. Throws an error if the stream is not marked.

```
See also mark () (page 451), unmark () (page 451), ismarked () (page 451).
```

#### ismarked(s)

Returns true if stream s is marked.

```
See also mark () (page 451), unmark () (page 451), reset () (page 451).
```

### eof (stream) → Bool

Tests whether an I/O stream is at end-of-file. If the stream is not yet exhausted, this function will block to wait for more data if necessary, and then return false. Therefore it is always safe to read one byte after seeing eof return false. eof will return false as long as buffered data is still available, even if the remote end of a connection is closed.

#### $isreadonly(stream) \rightarrow Bool$

Determine whether a stream is read-only.

#### iswritable (io) $\rightarrow$ Bool

Returns true if the specified IO object is writable (if that can be determined).

45.1. General I/O 453

#### $isreadable(io) \rightarrow Bool$

Returns true if the specified IO object is readable (if that can be determined).

#### $isopen(object) \rightarrow Bool$

Determine whether an object - such as a stream, timer, or mmap – is not yet closed. Once an object is closed, it will never produce a new event. However, a closed stream may still have data to read in its buffer, use eof to check for the ability to read data. Use poll\_fd to be notified when a stream might be writable or readable.

#### serialize(stream, value)

Write an arbitrary value to a stream in an opaque format, such that it can be read back by deserialize. The read-back value will be as identical as possible to the original. In general, this process will not work if the reading and writing are done by different versions of Julia, or an instance of Julia with a different system image. Ptr values are serialized as all-zero bit patterns (NULL).

### deserialize(stream)

Read a value written by serialize.

#### print\_escaped (io, str::AbstractString, esc::AbstractString)

General escaping of traditional C and Unicode escape sequences, plus any characters in esc are also escaped (with a backslash).

### print\_unescaped(io, s::AbstractString)

General unescaping of traditional C and Unicode escape sequences. Reverse of print\_escaped() (page 452).

# print\_joined(io, items, delim[, last])

Print elements of items to io with delim between them. If last is specified, it is used as the final delimiter instead of delim.

#### print\_shortest (io, x)

Print the shortest possible representation, with the minimum number of consecutive non-zero digits, of number x, ensuring that it would parse to the exact same number.

### **fd** (stream)

Returns the file descriptor backing the stream or file. Note that this function only applies to synchronous File's and IOStream's not to any of the asynchronous streams.

#### redirect\_stdout()

Create a pipe to which all C and Julia level STDOUT output will be redirected. Returns a tuple (rd, wr) representing the pipe ends. Data written to STDOUT may now be read from the rd end of the pipe. The wr end is given for convenience in case the old STDOUT object was cached by the user and needs to be replaced elsewhere.

#### redirect stdout(stream)

Replace STDOUT by stream for all C and julia level output to STDOUT. Note that stream must be a TTY, a Pipe or a TCPSocket.

# redirect\_stderr([stream])

Like redirect\_stdout, but for STDERR.

### redirect\_stdin([stream])

Like redirect\_stdout, but for STDIN. Note that the order of the return tuple is still (rd,wr), i.e. data to be read from STDIN, may be written to wr.

#### readchomp(x)

Read the entirety of x as a string but remove trailing newlines. Equivalent to chomp (readstring (x)).

### truncate (file, n)

Resize the file or buffer given by the first argument to exactly n bytes, filling previously unallocated space with '\0' if the file or buffer is grown.

### skipchars (stream, predicate; linecomment::Char)

Advance the stream until before the first character for which predicate returns false. For example skipchars (stream, isspace) will skip all whitespace. If keyword argument linecomment is specified, characters from that character through the end of a line will also be skipped.

# countlines (io[, eol::Char])

Read io until the end of the stream/file and count the number of lines. To specify a file pass the filename as the first argument. EOL markers other than '\n' are supported by passing them as the second argument.

#### PipeBuffer()

An IOBuffer that allows reading and performs writes by appending. Seeking and truncating are not supported. See IOBuffer for the available constructors.

# PipeBuffer (data::Vector{UInt8}[, maxsize])

Create a PipeBuffer to operate on a data vector, optionally specifying a size beyond which the underlying Array may not be grown.

### readavailable (stream)

Read all available data on the stream, blocking the task only if no data is available. The result is a Vector{UInt8,1}.

#### IOContext{<:IO} <: IO</pre>

IOContext provides a mechanism for passing output-configuration keyword arguments through arbitrary show methods.

In short, it is an immutable Dictionary that is a subclass of IO.

```
IOContext(io::IO, KV::Pair)
```

Create a new entry in the IO Dictionary for the key => value pair

•use (key => value) in dict to see if this particular combination is in the properties set

•use get (dict, key, default) to retrieve the most recent value for a particular key

```
IOContext(io::IO, context::IOContext)
```

Create a IOContext that wraps an alternate IO but inherits the keyword arguments from the context

### 45.2 Text I/O

#### show(x)

Write an informative text representation of a value to the current output stream. New types should overload show(io, x) where the first argument is a stream. The representation used by show generally includes Julia-specific formatting and type information.

#### showcompact (x)

Show a more compact representation of a value. This is used for printing array elements. If a new type has a different compact representation, it should test Base.limit\_output (io) in its normal show method.

### showall(x)

Similar to show, except shows all elements of arrays.

#### summary(x)

Return a string giving a brief description of a value. By default returns string (typeof (x)), e.g. Int 64.

For arrays, returns a string of size and type info, e.g. 10-element Array {Int 64, 1}.

#### print(x)

Write (to the default output stream) a canonical (un-decorated) text representation of a value if there is one,

45.2. Text I/O 455

otherwise call show. The representation used by print includes minimal formatting and tries to avoid Julia-specific details.

#### println(x)

Print (using print () (page 453)) x followed by a newline.

# print\_with\_color(color::Symbol[, io], strings...)

Print strings in a color specified as a symbol, for example : red or :blue.

#### info (msg)

Display an informational message. Argument msg is a string describing the information to be displayed.

#### warn (msg)

Display a warning. Argument msg is a string describing the warning to be displayed.

```
@printf([io::IOStream], "%Fmt", args...)
```

Print args using C printf() style format specification string. Optionally, an IOStream may be passed as the first argument to redirect output.

```
@sprintf("%Fmt", args...)
```

Return @printf formatted output as string.

```
julia> s = @sprintf "this is a %s %15.1f" "test" 34.567;

julia> println(s)
this is a test 34.6
```

#### sprint (f::Function, args...)

Call the given function with an I/O stream and the supplied extra arguments. Everything written to this I/O stream is returned as a string.

#### showerror(io, e)

Show a descriptive representation of an exception object.

#### $\mathbf{dump}(x)$

Show all user-visible structure of a value.

### xdump(x)

Show all structure of a value, including all fields of objects.

#### readstring(stream or filename)

Read the entire contents of an I/O stream or a file as a string.

### readline (stream=STDIN or filename)

Read a single line of text, including a trailing newline character (if one is reached before the end of the input), from the given stream or file (defaults to STDIN),

#### readuntil (stream or filename, delim)

Read a string, up to and including the given delimiter byte.

### readlines (stream or filename)

Read all lines as an array.

#### eachline (stream or filename)

Create an iterable object that will yield each line.

Read a matrix from the source where each line (separated by eol) gives one row, with elements separated by the given delimeter. The source can be a text file, stream or byte array. Memory mapped files can be used by passing the byte array representation of the mapped segment as source.

If T is a numeric type, the result is an array of that type, with any non-numeric elements as NaN for floating-point types, or zero. Other useful values of T include ASCIIString, AbstractString, and Any.

If header is true, the first row of data will be read as header and the tuple (data\_cells, header\_cells) is returned instead of only data\_cells.

Specifying skipstart will ignore the corresponding number of initial lines from the input.

If skipblanks is true, blank lines in the input will be ignored.

If use\_mmap is true, the file specified by source is memory mapped for potential speedups. Default is true except on Windows. On Windows, you may want to specify true if the file is large, and is only read once and not written to.

If ignore\_invalid\_chars is true, bytes in source with invalid character encoding will be ignored. Otherwise an error is thrown indicating the offending character position.

If quotes is true, column enclosed within double-quote (") characters are allowed to contain new lines and column delimiters. Double-quote characters within a quoted field must be escaped with another double-quote. Specifying dims as a tuple of the expected rows and columns (including header, if any) may speed up reading of large files. If comments is true, lines beginning with comment\_char and text following comment\_char in any line are ignored.

#### readdlm(source, delim::Char, eol::Char; options...)

If all data is numeric, the result will be a numeric array. If some elements cannot be parsed as numbers, a cell array of numbers and strings is returned.

#### readdlm(source, delim::Char, T::Type; options...)

The end of line delimiter is taken as n.

#### readdlm(source, delim::Char; options...)

The end of line delimiter is taken as n. If all data is numeric, the result will be a numeric array. If some elements cannot be parsed as numbers, a cell array of numbers and strings is returned.

### readdlm(source, T::Type; options...)

The columns are assumed to be separated by one or more whitespaces. The end of line delimiter is taken as n.

#### readdlm(source; options...)

The columns are assumed to be separated by one or more whitespaces. The end of line delimiter is taken as n. If all data is numeric, the result will be a numeric array. If some elements cannot be parsed as numbers, a cell array of numbers and strings is returned.

### writedlm(f, A, delim='\t')

Write A (a vector, matrix or an iterable collection of iterable rows) as text to f (either a filename string or an IO stream) using the given delimeter delim (which defaults to tab, but can be any printable Julia object, typically a Char or AbstractString).

For example, two vectors x and y of the same length can be written as two columns of tab-delimited text to f by either writedlm(f,  $[x \ y]$ ) or by writedlm(f, zip(x, y)).

### readcsv (source, [T::Type]; options...)

Equivalent to readdlm with delim set to comma.

#### writecsv (filename, A)

Equivalent to writedlm with delim set to comma.

#### Base64EncodePipe (ostream)

Returns a new write-only I/O stream, which converts any bytes written to it into base64-encoded ASCII bytes written to ostream. Calling close on the Base64Pipe stream is necessary to complete the encoding (but does not close ostream).

45.2. Text I/O 457

#### Base64DecodePipe (istream)

Returns a new read-only I/O stream, which decodes base64-encoded data read from istream.

```
base64encode (writefunc, args...)
base64encode (args...)
```

Given a write-like function writefunc, which takes an I/O stream as its first argument, base64 (writefunc, args...) calls writefunc to write args... to a base64-encoded string, and returns the string. base64 (args...) is equivalent to base64 (write, args...): it converts its arguments into bytes using the standard write functions and returns the base64-encoded string.

### base64decode (string)

Decodes the base64-encoded string and returns a Vector {UInt8} of the decoded bytes.

```
displaysize (io) -> (lines, columns)
```

Return the nominal size of the screen that may be used for rendering output to this io object

```
limit\_output(io) \rightarrow Bool
```

Output hinting for identifying contexts where the user requested a compact output

### 45.3 Multimedia I/O

Just as text output is performed by print and user-defined types can indicate their textual representation by over-loading show, Julia provides a standardized mechanism for rich multimedia output (such as images, formatted text, or even audio and video), consisting of three parts:

- A function display (x) to request the richest available multimedia display of a Julia object x (with a plaintext fallback).
- Overloading writemime allows one to indicate arbitrary multimedia representations (keyed by standard MIME types) of user-defined types.
- Multimedia-capable display backends may be registered by subclassing a generic Display type and pushing them onto a stack of display backends via pushdisplay.

The base Julia runtime provides only plain-text display, but richer displays may be enabled by loading external modules or by using graphical Julia environments (such as the IPython-based IJulia notebook).

```
display (x)
display (d::Display, x)
display (mime, x)
display (d::Display, mime, x)
```

Display x using the topmost applicable display in the display stack, typically using the richest supported multimedia output for x, with plain-text STDOUT output as a fallback. The display (d, x) variant attempts to display x on the given display d only, throwing a MethodError if d cannot display objects of this type.

There are also two variants with a mime argument (a MIME type string, such as "image/png"), which attempt to display x using the requested MIME type only, throwing a MethodError if this type is not supported by either the display(s) or by x. With these variants, one can also supply the "raw" data in the requested MIME type by passing x::AbstractString (for MIME types with text-based storage, such as text/html or application/postscript) or x::Vector{UInt8} (for binary MIME types).

```
redisplay (x)
redisplay (d::Display, x)
redisplay (mime, x)
redisplay (d::Display, mime, x)
```

By default, the redisplay functions simply call display. However, some display backends may override redisplay to modify an existing display of x (if any). Using redisplay is also a hint to the backend that

x may be redisplayed several times, and the backend may choose to defer the display until (for example) the next interactive prompt.

### displayable $(mime) \rightarrow Bool$ displayable $(d::Display, mime) \rightarrow Bool$

Returns a boolean value indicating whether the given mime type (string) is displayable by any of the displays in the current display stack, or specifically by the display d in the second variant.

#### writemime (stream, mime, x)

The display functions ultimately call writemime in order to write an object x as a given mime type to a given I/O stream (usually a memory buffer), if possible. In order to provide a rich multimedia representation of a user-defined type T, it is only necessary to define a new writemime method for T, via: writemime (stream, ::MIME"mime", x::T) = ..., where mime is a MIME-type string and the function body calls write (or similar) to write that representation of x to stream. (Note that the MIME"" notation only supports literal strings; to construct MIME types in a more flexible manner use MIME  $\{\text{symbol}("")\}$ .)

For example, if you define a MyImage type and know how to write it to a PNG file, you could define a function writemime (stream, ::MIME"image/png", x::MyImage) = ... to allow your images to be displayed on any PNG-capable Display (such as IJulia). As usual, be sure to import Base.writemime in order to add new methods to the built-in Julia function writemime.

Technically, the MIME "mime" macro defines a singleton type for the given mime string, which allows us to exploit Julia's dispatch mechanisms in determining how to display objects of any given type.

#### mimewritable (mime, x)

Returns a boolean value indicating whether or not the object x can be written as the given mime type. (By default, this is determined automatically by the existence of the corresponding writemime function for typeof(x).)

#### reprmime(mime, x)

Returns an AbstractString or Vector {UInt8} containing the representation of x in the requested mime type, as written by writemime (throwing a MethodError if no appropriate writemime is available). An AbstractString is returned for MIME types with textual representations (such as "text/html" or "application/postscript"), whereas binary data is returned as Vector{UInt8}. (The function istext (mime) returns whether or not Julia treats a given mime type as text.)

As a special case, if x is an AbstractString (for textual MIME types) or a Vector{UInt8} (for binary MIME types), the reprmime function assumes that x is already in the requested mime format and simply returns x.

#### stringmime(mime, x)

Returns an AbstractString containing the representation of x in the requested mime type. This is similar to reprime except that binary data is base64-encoded as an ASCII string.

As mentioned above, one can also define new display backends. For example, a module that can display PNG images in a window can register this capability with Julia, so that calling display (x) on types with PNG representations will automatically display the image using the module's window.

In order to define a new display backend, one should first create a subtype D of the abstract class Display. Then, for each MIME type (mime string) that can be displayed on D, one should define a function display(d::D, ::MIME"mime", x) = ... that displays x as that MIME type, usually by calling reprmime (mime, x). A MethodError should be thrown if x cannot be displayed as that MIME type; this is automatic if one calls reprmime. Finally, one should define a function display(d::D, x) that queries mimewritable (mime, x) for the mime types supported by D and displays the "best" one; a MethodError should be thrown if no supported MIME types are found for x. Similarly, some subtypes may wish to override redisplay (d::D, ...). (Again, one should import Base.display to add new methods to display.) The return values of these functions are up to the implementation (since in some cases it may be useful to return a display "handle" of some type). The display

45.3. Multimedia I/O 459

functions for D can then be called directly, but they can also be invoked automatically from display (x) simply by pushing a new display onto the display-backend stack with:

#### pushdisplay (d::Display)

Pushes a new display d on top of the global display-backend stack. Calling display(x) or display(mime, x) will display x on the topmost compatible backend in the stack (i.e., the topmost backend that does not throw a MethodError).

```
popdisplay()
popdisplay(d::Display)
```

Pop the topmost backend off of the display-backend stack, or the topmost copy of d in the second variant.

#### TextDisplay (stream)

Returns a TextDisplay <: Display, which can display any object as the text/plain MIME type (only), writing the text representation to the given I/O stream. (The text representation is the same as the way an object is printed in the Julia REPL.)

```
istext (m::MIME)
```

Determine whether a MIME type is text data.

# 45.4 Memory-mapped I/O

```
Mmap.Anonymous (name, readonly, create)
```

Create an IO-like object for creating zeroed-out mmapped-memory that is not tied to a file for use in Mmap.mmap. Used by SharedArray for creating shared memory arrays.

```
\label{loss-mmap} \begin{tabular}{l} $\mathsf{Mmap.mmap}(io::Union\{IOStream,AbstractString,Mmap.AnonymousMmap\}[,\ type::Type\{Array\{T,N\}\},\ dims,\ offset];\ grow::Bool=true,\ shared::Bool=true) \\ \begin{tabular}{l} $\mathsf{Mmap.mmap}(type::Type\{Array\{T,N\}\},\ dims) \end{tabular}
```

Create an Array whose values are linked to a file, using memory-mapping. This provides a convenient way of working with data too large to fit in the computer's memory.

The type is an Array{T,N} with a bits-type element of T and dimension N that determines how the bytes of the array are interpreted. Note that the file must be stored in binary format, and no format conversions are possible (this is a limitation of operating systems, not Julia).

dims is a tuple or single Integer specifying the size or length of the array.

The file is passed via the stream argument, either as an open IOStream or filename string. When you initialize the stream, use "r" for a "read-only" array, and "w+" to create a new array used to write values to disk.

If no type argument is specified, the default is Vector {UInt8}.

Optionally, you can specify an offset (in bytes) if, for example, you want to skip over a header in the file. The default value for the offset is the current stream position for an IOStream.

The grow keyword argument specifies whether the disk file should be grown to accommodate the requested size of array (if the total file size is < requested array size). Write privileges are required to grow the file.

The shared keyword argument specifies whether the resulting Array and changes made to it will be visible to other processes mapping the same file.

For example, the following code

```
# Create a file for mmapping
# (you could alternatively use mmap to do this step, too)
A = rand(1:20, 5, 30)
s = open("/tmp/mmap.bin", "w+")
# We'll write the dimensions of the array as the first two Ints in the file
write(s, size(A,1))
```

```
write(s, size(A,2))
# Now write the data
write(s, A)
close(s)

# Test by reading it back in
s = open("/tmp/mmap.bin") # default is read-only
m = read(s, Int)
n = read(s, Int)
A2 = Mmap.mmap(s, Matrix{Int}, (m,n))
```

creates a m-by-n Matrix{Int}, linked to the file associated with stream s.

A more portable file would need to encode the word size – 32 bit or 64 bit – and endianness information in the header. In practice, consider encoding binary data using standard formats like HDF5 (which can be used with memory-mapping).

```
Mmap.mmap(io, BitArray[, dims, offset])
```

Create a BitArray whose values are linked to a file, using memory-mapping; it has the same purpose, works in the same way, and has the same arguments, as mmap(), but the byte representation is different.

```
Example: B = Mmap.mmap(s, BitArray, (25, 30000))
```

This would create a 25-by-30000 BitArray, linked to the file associated with stream s.

```
Mmap.sync! (array)
```

Forces synchronization between the in-memory version of a memory-mapped Array or BitArray and the on-disk version.

# 45.5 Network I/O

```
connect ([host], port) \rightarrow TCPSocket
Connect to the host on port port.
```

```
connect (path) → PipeEndpoint
```

Connect to the Named Pipe / Domain Socket at path.

```
listen([addr], port) \rightarrow TCPServer
```

Listen on port on the address specified by addr. By default this listens on localhost only. To listen on all interfaces pass IPv4(0) or IPv6(0) as appropriate.

```
listen(path) \rightarrow PipeServer
```

Create and listen on a Named Pipe / Domain Socket.

```
getaddrinfo(host)
```

Gets the IP address of the host (may have to do a DNS lookup)

```
getsockname (sock::Union{TCPServer, TCPSocket}) -> (IPAddr, UInt16)
```

Get the IP address and the port that the given TCP socket is connected to (or bound to, in the case of TCPServer).

```
\textbf{IPv4} \; (\textit{host::Integer}) \; \rightarrow \text{IPv4}
```

Returns IPv4 object from ip address formatted as Integer.

```
IPv6 (host::Integer) \rightarrow IPv6
```

Returns IPv6 object from ip address formatted as Integer

```
nb_available(stream)
```

Returns the number of bytes available for reading before a read from this stream or buffer will block.

45.5. Network I/O 461

### accept (server[, client])

Accepts a connection on the given server and returns a connection to the client. An uninitialized client stream may be provided, in which case it will be used instead of creating a new stream.

#### listenany (port\_hint) -> (UInt16, TCPServer)

Create a TCPServer on any port, using hint as a starting point. Returns a tuple of the actual port that the server was created on and the server itself.

#### poll fd(fd, timeout s::Real; readable=false, writable=false)

Monitor a file descriptor fd for changes in the read or write availability, and with a timeout given by timeout\_s seconds.

The keyword arguments determine which of read and/or write status should be monitored; at least one of them must be set to true.

The returned value is an object with boolean fields readable, writable, and timedout, giving the result of the polling.

### poll\_file (path, interval\_s::Real, timeout\_s::Real) -> (previous::StatStruct, current::StatStruct)

Monitor a file for changes by polling every interval\_s seconds until a change occurs or timeout\_s seconds have elapsed. The interval\_s should be a long period; the default is 5.007 seconds.

Returns a pair of StatStruct objects (previous, current) when a change is detected.

To determine when a file was modified, compare mtime (prev) != mtime (current) to detect notification of changes. However, using watch\_file for this operation is preferred, since it is more reliable and efficient, although in some situations it may not be available.

#### watch\_file (path, timeout\_s::Real)

Watch file or directory path for changes until a change occurs or timeout\_s seconds have elapsed.

The returned value is an object with boolean fields changed, renamed, and timedout, giving the result of watching the file.

This behavior of this function varies slightly across platforms. See <a href="https://nodejs.org/api/fs.html#fs\_caveats">https://nodejs.org/api/fs.html#fs\_caveats</a> for more detailed information.

### bind (socket::Union{UDPSocket, TCPSocket}, host::IPAddr, port::Integer; ipv6only=false)

Bind socket to the given host:port. Note that 0.0.0.0 will listen on all devices. ipv6only parameter disables dual stack mode. If it's true, only IPv6 stack is created.

### send (socket::UDPSocket, host::IPv4, port::Integer, msg)

Send msg over socket to host:port.

### $\verb"recv" (socket:: UDPSocket)$

Read a UDP packet from the specified socket, and return the bytes received. This call blocks.

#### recvfrom (socket::UDPSocket) -> (address, data)

Read a UDP packet from the specified socket, returning a tuple of (address, data), where address will be either IPv4 or IPv6 as appropriate.

# **setopt** (sock::UDPSocket; multicast\_loop = nothing, multicast\_ttl=nothing, enable\_broadcast=nothing, ttl=nothing)

Set UDP socket options. multicast\_loop: loopback for multicast packets (default: true). multicast\_ttl: TTL for multicast packets. enable\_broadcast: flag must be set to true if socket will be used for broadcast messages, or else the UDP system will return an access error (default: false). ttl: Time-to-live of packets sent on the socket.

#### ntoh(x)

Converts the endianness of a value from Network byte order (big-endian) to that used by the Host.

### hton(x)

Converts the endianness of a value from that used by the Host to Network byte order (big-endian).

## ltoh(x)

Converts the endianness of a value from Little-endian to that used by the Host.

## htol(x)

Converts the endianness of a value from that used by the Host to Little-endian.

## ENDIAN\_BOM

The 32-bit byte-order-mark indicates the native byte order of the host machine. Little-endian machines will contain the value 0x04030201. Big-endian machines will contain the value 0x01020304.

45.5. Network I/O 463

Julia Language Documentation, Release 0.5.0-dev				

# **Punctuation**

Extended documentation for mathematical symbols & functions is *here* (page 353).

symbol	meaning
@m	invoke macro m; followed by space-separated expressions
!	prefix "not" operator
a!()	at the end of a function name, ! indicates that a function modifies its argument(s)
#	begin single line comment
#=	begin multi-line comment (these are nestable)
=#	end multi-line comment
\$	bitwise xor operator, string and expression interpolation
%	remainder operator
^	exponent operator
&	bitwise and
& &	short-circuiting boolean and
	bitwise or
	short-circuiting boolean or
*	multiply, or matrix multiply
()	the empty tuple
~	bitwise not operator
\	backslash operator
,	complex transpose operator A <sup>H</sup>
a[]	array indexing
[,]	vertical concatenation
[;]	also vertical concatenation
[ ]	with space-separated expressions, horizontal concatenation
T { }	parametric type instantiation
{ }	construct a cell array (deprecated in 0.4 in favor of Any [])
;	statement separator
,	separate function arguments or tuple components
?	3-argument conditional operator (conditional ? if_true : if_false)
" "	delimit string literals
11	delimit character literals
<b>'</b>	delimit external process (command) specifications
	splice arguments into a function call or declare a varargs function or type
•	access named fields in objects or names inside modules, also prefixes elementwise operators
a:b	range a, a+1, a+2,, b
a:s:b	range a, a+s, a+2s,, b
<u> </u>	Continued on next page

Table 46.1 – continued from previous page

symbol	meaning
:	index an entire dimension (1:end)
::	type annotation, depending on context
:()	quoted expression
:a	symbol a

# **Sorting and Related Functions**

Julia has an extensive, flexible API for sorting and interacting with already-sorted arrays of values. By default, Julia picks reasonable algorithms and sorts in standard ascending order:

```
julia> sort([2,3,1])
3-element Array{Int64,1}:
    1
    2
    3
```

You can easily sort in reverse order as well:

```
julia> sort([2,3,1], rev=true)
3-element Array{Int64,1}:
3
2
1
```

To sort an array in-place, use the "bang" version of the sort function:

```
julia> a = [2,3,1];

julia> sort!(a);

julia> a
3-element Array{Int64,1}:
    1
    2
    3
```

Instead of directly sorting an array, you can compute a permutation of the array's indices that puts the array into sorted order:

```
julia> v = randn(5)
5-element Array{Float64,1}:
    0.297288
    0.382396
    -0.597634
    -0.0104452
    -0.839027

julia> p = sortperm(v)
5-element Array{Int64,1}:
    5
    3
```

```
4
1
2

julia> v[p]
5-element Array{Float64,1}:
-0.839027
-0.597634
-0.0104452
0.297288
0.382396
```

Arrays can easily be sorted according to an arbitrary transformation of their values:

```
julia> sort(v, by=abs)
5-element Array{Float64,1}:
    -0.0104452
    0.297288
    0.382396
    -0.597634
    -0.839027
```

Or in reverse order by a transformation:

```
julia> sort(v, by=abs, rev=true)
5-element Array{Float64,1}:
    -0.839027
    -0.597634
    0.382396
    0.297288
    -0.0104452
```

If needed, the sorting algorithm can be chosen:

```
julia> sort(v, alg=InsertionSort)
5-element Array{Float64,1}:
   -0.839027
   -0.597634
   -0.0104452
   0.297288
   0.382396
```

All the sorting and order related functions rely on a "less than" relation defining a total order on the values to be manipulated. The isless function is invoked by default, but the relation can be specified via the lt keyword.

# **47.1 Sorting Functions**

```
sort! (v, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false])
```

Sort the vector v in place. QuickSort is used by default for numeric arrays while MergeSort is used for other arrays. You can specify an algorithm to use via the alg keyword (see Sorting Algorithms for available algorithms). The by keyword lets you provide a function that will be applied to each element before comparison; the lt keyword allows providing a custom "less than" function; use rev=true to reverse the sorting order. These options are independent and can be used together in all possible combinations: if both by and lt are specified, the lt function is applied to the result of the by function; rev=true reverses whatever ordering specified via the by and lt keywords.

- sort (v, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false])
  Variant of sort! that returns a sorted copy of v leaving v itself unmodified.
- **sort** (*A*, *dim*, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false]) Sort a multidimensional array A along the given dimension.
- sortperm(v, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false])

Return a permutation vector of indices of v that puts it in sorted order. Specify alg to choose a particular sorting algorithm (see Sorting Algorithms). MergeSort is used by default, and since it is stable, the resulting permutation will be the lexicographically first one that puts the input array into sorted order – i.e. indices of equal elements appear in ascending order. If you choose a non-stable sorting algorithm such as QuickSort, a different permutation that puts the array into order may be returned. The order is specified using the same keywords as sort!.

See also sortperm! () (page 467).

sortperm! (ix, v, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false,] [initialized=false])

Like sortperm, but accepts a preallocated index vector ix. If initialized is false (the default), ix is initialized to contain the values 1:length(v).

See also sortperm() (page 467).

- **sortrows** (*A*, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false]) Sort the rows of matrix A lexicographically.
- **sortcols** (*A*, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false]) Sort the columns of matrix A lexicographically.

## 47.2 Order-Related Functions

issorted(v, [by=<transform>,] [lt=<comparison>,] [rev=false])

Test whether a vector is in sorted order. The by, lt and rev keywords modify what order is considered to be sorted just as they do for sort.

searchsorted(a, x, [by=<transform>,] [lt=<comparison>,] [rev=false])

Returns the range of indices of a which compare as equal to x according to the order specified by the by, lt and rev keywords, assuming that a is already sorted in that order. Returns an empty range located at the insertion point if a does not contain values equal to x.

searchsortedfirst (a, x, [by=<transform>,] [lt=<comparison>,] [rev=false])

Returns the index of the first value in a greater than or equal to x, according to the specified order. Returns length (a) +1 if x is greater than all values in a.

searchsortedlast (a, x, [by=<transform>,] [lt=<comparison>,] [rev=false])

Returns the index of the last value in a less than or equal to x, according to the specified order. Returns 0 if x is less than all values in a.

select! (v, k, [by=<transform>,] [lt=<comparison>,] [rev=false])

Partially sort the vector v in place, according to the order specified by by, lt and rev so that the value at index k (or range of adjacent values if k is a range) occurs at the position where it would appear if the array were fully sorted via a non-stable algorithm. If k is a single index, that value is returned; if k is a range, an array of values at those indices is returned. Note that select! does not fully sort the input array.

select (v, k, [by=<transform>,] [lt=<comparison>,] [rev=false])

Variant of select! which copies v before partially sorting it, thereby returning the same thing as select! but leaving v unmodified.

```
selectperm(v, k, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false])
```

Return a partial permutation of the the vector v, according to the order specified by by, lt and rev, so that v[output] returns the first k (or range of adjacent values if k is a range) values of a fully sorted version of v. If k is a single index (Integer), an array of the first k indices is returned; if k is a range, an array of those indices is returned. Note that the handling of integer values for k is different from select in that it returns a vector of k elements instead of just the k th element. Also note that this is equivalent to, but more efficient than, calling sortperm(...) [k]

```
selectperm! (ix, v, k, [alg=<algorithm>,] [by=<transform>,] [lt=<comparison>,] [rev=false,] [initial-ized=false])
```

Like selectperm, but accepts a preallocated index vector ix. If initialized is false (the default), ix is initialized to contain the values 1:length(ix).

# 47.3 Sorting Algorithms

There are currently four sorting algorithms available in base Julia:

- InsertionSort
- OuickSort
- PartialQuickSort(k)
- MergeSort

InsertionSort is an  $O(n^2)$  stable sorting algorithm. It is efficient for very small n, and is used internally by QuickSort.

QuickSort is an O(n log n) sorting algorithm which is in-place, very fast, but not stable – i.e. elements which are considered equal will not remain in the same order in which they originally appeared in the array to be sorted. QuickSort is the default algorithm for numeric values, including integers and floats.

PartialQuickSort (k) is similar to QuickSort, but the output array is only sorted up to index k if k is an integer, or in the range of k if k is an OrdinalRange. For example:

```
x = rand(1:500, 100)
k = 50
k2 = 50:100
s = sort(x; alg=QuickSort)
ps = sort(x; alg=PartialQuickSort(k))
qs = sort(x; alg=PartialQuickSort(k2))
map(issorted, (s, ps, qs))
                                        # => (true, false, false)
map(x->issorted(x[1:k]), (s, ps, qs))
                                       # => (true, true, false)
map(x->issorted(x[k2]), (s, ps, qs))
                                        # => (true, false, true)
                                        # => true
s[1:k] == ps[1:k]
s[k2] == qs[k2]
                                        # => true
```

MergeSort is an O(n log n) stable sorting algorithm but is not in-place – it requires a temporary array of half the size of the input array – and is typically not quite as fast as QuickSort. It is the default algorithm for non-numeric data.

The default sorting algorithms are chosen on the basis that they are fast and stable, or *appear* to be so. For numeric types indeed, QuickSort is selected as it is faster and indistinguishable in this case from a stable sort (unless the array records its mutations in some way). The stability property comes at a non-negligible cost, so if you don't need it, you may want to explicitly specify your preferred algorithm, e.g. sort! (v, alg=QuickSort).

The mechanism by which Julia picks default sorting algorithms is implemented via the Base. Sort.defalg function. It allows a particular algorithm to be registered as the default in all sorting functions for specific arrays. For example, here are the two default methods from sort.jl:

```
defalg(v::AbstractArray) = MergeSort
defalg(T<:Number) (v::AbstractArray{T}) = QuickSort</pre>
```

As for numeric arrays, choosing a non-stable default algorithm for array types for which the notion of a stable sort is meaningless (i.e. when two values comparing equal can not be distinguished) may make sense.

Julia Language Documentation, Release 0.5.0-dev					

# **Package Manager Functions**

All package manager functions are defined in the Pkg module. None of the Pkg module's functions are exported; to use them, you'll need to prefix each function call with an explicit Pkg., e.g. Pkg. status() or Pkg.dir().

Functions for package development (e.g. tag, publish, etc.) have been moved to the PkgDev package. See PkgDev README for the documentation of those functions.

## $dir() \rightarrow AbstractString$

absolute package This defaults Returns the path of the directory. to joinpath(homedir(),".julia","v\$(VERSION.major).\$(VERSION.minor)") ~/.julia/v0.5 in UNIX shell syntax). on all platforms (i.e. If the JULIA PKGDIR is set, environment variable then that path is used returned joinpath(ENV["JULIA\_PKGDIR"], "v\$(VERSION.major).\$(VERSION.minor)"). JULIA\_PKGDIR is a relative path, it is interpreted relative to whatever the current working directory

#### $dir(names...) \rightarrow AbstractString$

Equivalent to normpath (Pkg.dir(), names...) – i.e. it appends path components to the package directory and normalizes the resulting path. In particular, Pkg.dir(pkg) returns the path to the package pkg.

## init (meta::AbstractString=DEFAULT\_META, branch::AbstractString=META\_BRANCH)

Initialize Pkg.dir() as a package directory. This will be done automatically when the JULIA\_PKGDIR is not set and Pkg.dir() uses its default value. As part of this process, clones a local METADATA git repository from the site and branch specified by its arguments, which are typically not provided. Explicit (non-default) arguments can be used to support a custom METADATA setup.

#### resolve()

Determines an optimal, consistent set of package versions to install or upgrade to. The optimal set of package versions is based on the contents of Pkg.dir("REQUIRE") and the state of installed packages in Pkg.dir(), Packages that are no longer required are moved into Pkg.dir(".trash").

#### edit()

Opens Pkg.dir("REQUIRE") in the editor specified by the VISUAL or EDITOR environment variables; when the editor command returns, it runs Pkg.resolve() to determine and install a new optimal set of installed package versions.

## **add** (*pkg*, *vers...*)

Add a requirement entry for pkg to Pkg.dir ("REQUIRE") and call Pkg.resolve(). If vers are given, they must be VersionNumber objects and they specify acceptable version intervals for pkg.

#### rm (pkg)

Remove all requirement entries for pkg from Pkg.dir ("REQUIRE") and call Pkg.resolve().

## clone(url[,pkg])

Clone a package directly from the git URL url. The package does not need to be registered in

Pkg.dir("METADATA"). The package repo is cloned by the name pkg if provided; if not provided, pkg is determined automatically from url.

## clone(pkg)

If pkg has a URL registered in Pkg.dir("METADATA"), clone it from that URL on the default branch. The package does not need to have any registered versions.

#### setprotocol! (proto)

Set the protocol used to access GitHub-hosted packages. Defaults to 'https', with a blank proto delegating the choice to the package developer.

## $available() \rightarrow Vector\{ASCIIString\}$

Returns the names of available packages.

## **available** $(pkg) \rightarrow \text{Vector}\{\text{VersionNumber}\}$

Returns the version numbers available for package pkg.

## installed() → Dict{ASCIIString,VersionNumber}

Returns a dictionary mapping installed package names to the installed version number of each package.

## $installed(pkg) \rightarrow Void \mid VersionNumber$

If pkg is installed, return the installed version number, otherwise return nothing.

#### status()

Prints out a summary of what packages are installed and what version and state they're in.

## update()

Update the metadata repo – kept in Pkg.dir ("METADATA") – then update any fixed packages that can safely be pulled from their origin; then call Pkg.resolve() to determine a new optimal set of packages versions.

## checkout (pkg, [branch="master"]; merge=true, pull=true)

Checkout the Pkg.dir(pkg) repo to the branch branch. Defaults to checking out the "master" branch. To go back to using the newest compatible released version, use Pkg.free(pkg). Changes are merged (fast-forward only) if the keyword argument merge == true, and the latest version is pulled from the upstream repo if pull == true.

## pin(pkg)

Pin pkg at the current version. To go back to using the newest compatible released version, use Pkg.free(pkg)

## pin (pkg, version)

Pin pkg at registered version version.

#### free(pkg)

Free the package pkg to be managed by the package manager again. It calls Pkg.resolve() to determine optimal package versions after. This is an inverse for both Pkg.checkout and Pkg.pin.

You can also supply an iterable collection of package names, e.g., Pkg.free (("Pkg1", "Pkg2")) to free multiple packages at once.

## build()

Run the build scripts for all installed packages in depth-first recursive order.

#### build(pkgs...)

Run the build script in deps/build.jl for each package in pkgs and all of their dependencies in depth-first recursive order. This is called automatically by Pkg.resolve() on all installed or updated packages.

#### test()

Run the tests for all installed packages ensuring that each package's test dependencies are installed for the duration of the test. A package is tested by running its test/runtests.jl file and test dependencies are specified in test/REQUIRE.

## test (pkgs...)

Run the tests for each package in pkgs ensuring that each package's test dependencies are installed for the duration of the test. A package is tested by running its test/runtests.jl file and test dependencies are specified in test/REQUIRE.

Julia Language Documentation, Release 0.5.0-dev					

# **Unit Testing**

# 49.1 Testing Base Julia

Julia is under rapid development and has an extensive test suite to verify functionality across multiple platforms. If you build Julia from source, you can run this test suite with make test. In a binary install, you can run the test suite using Base.runtests().

```
runtests([tests=["all"][, numcores=iceil(CPU_CORES/2)]])
```

Run the Julia unit tests listed in tests, which can be either a string or an array of strings, using numcores processors. (not exported)

## 49.2 Basic Unit Tests

The Base. Test module provides simple *unit testing* functionality. Unit testing is a way to see if your code is correct by checking that the results are what you expect. It can be helpful to ensure your code still works after you make changes, and can be used when developing as a way of specifying the behaviors your code should have when complete.

Simple unit testing can be performed with the @test() and @test\_throws() macros:

## @test ex

Tests that the expression ex evaluates to true. Returns a Pass Result if it does, a Fail Result if it is false, and an Error Result if it could not be evaluated.

#### @test\_throws extype ex

Tests that the expression ex throws an exception of type extype.

For example, suppose we want to check our new function foo(x) works as expected:

```
julia> using Base.Test

julia> foo(x) = length(x)^2
foo (generic function with 1 method)
```

If the condition is true, a Pass is returned:

```
julia> @test foo("bar") == 9
Test Passed
  Expression: foo("bar") == 9
   Evaluated: 9 == 9
julia> @test foo("fizz") >= 10
```

```
Test Passed
  Expression: foo("fizz") >= 10
  Evaluated: 16 >= 10
```

If the condition is false, then a Fail is returned and an exception is thrown:

```
julia> @test foo("f") == 20
Test Failed
  Expression: foo("f") == 20
    Evaluated: 1 == 20
ERROR: There was an error during testing
  in record at test.jl:268
  in do_test at test.jl:191
```

If the condition could not be evaluated because an exception was thrown, which occurs in this case because length () is not defined for symbols, an Error object is returned and an exception is thrown:

```
julia> @test foo(:cat) == 1
Error During Test
  Test threw an exception of type MethodError
  Expression: foo(:cat) == 1
  MethodError: `length` has no method matching length(::Symbol)
  in foo at none:1
  in anonymous at test.jl:159
  in do_test at test.jl:180
ERROR: There was an error during testing
  in record at test.jl:268
  in do_test at test.jl:191
```

If we expect that evaluating an expression *should* throw an exception, then we can use @test\_throws() to check this occurs:

```
julia> @test_throws MethodError foo(:cat)
Test Passed
  Expression: foo(:cat)
  Evaluated: MethodError
```

# 49.3 Working with Test Sets

Typically a large of number of tests are used to make sure functions work correctly over a range of inputs. In the event a test fails, the default behavior is to throw an exception immediately. However, it is normally preferable to run the rest of the tests first to get a better picture of how many errors there are in the code being tested.

The @testset() macro can be used to group tests into *sets*. All the tests in a test set will be run, and at the end of the test set a summary will be printed. If any of the tests failed, or could not be evaluated due to an error, the test set will then throw a TestSetException.

```
@testset [CustomTestSet] [option=val ...] ["description"] begin ... end
@testset [CustomTestSet] [option=val ...] ["description $v"] for v in (...) ... end
@testset [CustomTestSet] [option=val ...] ["description $v, $w"] for v in (...), w in (...
Starts a new test set, or multiple test sets if a for loop is provided.
```

If no custom testset type is given it defaults to creating a DefaultTestSet. DefaultTestSet records all the results and, and if there are any Fails or Errors, throws an exception at the end of the top-level (non-nested) test set, along with a summary of the test results.

Any custom testset type (subtype of AbstractTestSet) can be given and it will also be used for any nested @testset invocations. The given options are only applied to the test set where they are given. The default test set type does not take any options.

The description string accepts interpolation from the loop indices. If no description is provided, one is constructed based on the variables.

By default the @testset macro will return the testset object itself, though this behavior can be customized in other testset types. If a for loop is used then the macro collects and returns a list of the return values of the finish method, which by default will return a list of the testset objects used in each iteration.

We can put our tests for the foo(x) function in a test set:

Test sets can all also be nested:

In the event that a nested test set has no failures, as happened here, it will be hidden in the summary. If we do have a test failure, only the details for the failed test sets will be shown:

```
julia> @testset "Foo Tests" begin
           @testset "Animals" begin
               @testset "Felines" begin
                   @test foo("cat") == 9
               end
               @testset "Canines" begin
                   @test foo("dog") == 9
               end
           end
           @testset "Arrays" begin
               @test foo(zeros(2)) == 4
               @test foo(ones(4)) == 15
           end
       end
Arrays: Test Failed
 Expression: foo(ones(4)) == 15
  Evaluated: 16 == 15
in record at test.jl:297
in do_test at test.jl:191
Test Summary: | Pass Fail
                            Total
```

```
Foo Tests | 3 1 4
Animals | 2 2
Arrays | 1 1 2
ERROR: Some tests did not pass: 3 passed, 1 failed, 0 errored.
in finish at test.jl:362
```

## 49.4 Other Test Macros

As calculations on floating-point values can be imprecise, you can perform approximate equality checks using either @test a  $\approx$  b (where  $\approx$ , typed via tab completion of \approx, is the isapprox() function) or use isapprox() directly.

An alternative is the <code>@test\_approx\_eq</code> macro (which differs from <code>isapprox</code> in that it treats NaN values as equal and has a smaller default tolerance) or <code>@test\_approx\_eq\_eps</code> (which takes an extra argument indicating the relative tolerance):

```
julia> @test 1 \approx 0.999999
ERROR: test failed: 1 isapprox 0.999999
in expression: 1 \approx 0.999999
in error at error.jl:21
in default_handler at test.jl:30
in do_test at test.jl:53
julia > @test_approx_eq 1. 0.9999999999
ERROR: assertion failed: |1.0 - 0.999999999| < 2.220446049250313e-12
 1.0 = 1.0
 in test_approx_eq at test.jl:75
in test_approx_eq at test.jl:80
julia > @test_approx_eq_eps 1. 0.999 1e-2
julia> @test_approx_eq_eps 1. 0.999 1e-3
ERROR: assertion failed: |1.0 - 0.999| <= 0.001
 1.0 = 1.0
 0.999 = 0.999
 in error at error.jl:22
in test_approx_eq at test.jl:68
```

Note that these macros will fail immediately, and are not compatible with <code>@testset()</code>, so using <code>@testisapprox</code> is encouraged when writing new tests.

## $@test_approx_eq(a, b)$

Test two floating point numbers a and b for equality taking in account small numerical errors.

#### @test\_approx\_eq\_eps (a, b, tol)

Test two floating point numbers a and b for equality taking in account a margin of tolerance given by tol.

# 49.5 Creating Custom AbstractTestSet Types

Packages can create their own AbstractTestSet subtypes by implementing the record and finish methods. The subtype should have a one-argument constructor taking a description string, with any options passed in as keyword arguments.

```
record (ts::AbstractTestSet, res::Result)
```

Record a result to a testset. This function is called by the @testset infrastructure each time a contained @test macro completes, and is given the test result (which could be an Error). This will also be called with an Error if an exception is thrown inside the test block but outside of a @test context.

#### **finish** (ts::AbstractTestSet)

Do any final processing necessary for the given testset. This is called by the @testset infrastructure after a test block executes. One common use for this function is to record the testset to the parent's results list, using get\_testset.

Base. Test takes responsibility for maintaining a stack of nested testsets as they are executed, but any result accumulation is the responsibility of the AbstractTestSet subtype. You can access this stack with the get\_testset and get\_testset\_depth methods. Note that these functions are not exported.

```
get_testset()
```

Retrieve the active test set from the task's local storage. If no test set is active, use the fallback default test set.

#### get\_testset\_depth()

Returns the number of active test sets, not including the defaut test set

Base. Test also makes sure that nested @testset invocations use the same AbstractTestSet subtype as their parent unless it is set explicitly. It does not propagate any properties of the testset option inheritance behavior can be implemented by packages using the stack infrastructure that Base. Test provides.

Defining a basic AbstractTestSet subtype might look like:

```
import Base. Test: record, finish
using Base.Test: AbstractTestSet, Result, Pass, Fail, Error
using Base.Test: get_testset_depth, get_testset
immutable CustomTestSet <: Base.Test.AbstractTestSet</pre>
   description::AbstractString
    foo::Int
    results::Vector
    # constructor takes a description string and options keyword arguments
    CustomTestSet(desc; foo=1) = new(desc, foo, [])
end
record(ts::CustomTestSet, child::AbstractTestSet) = push!(ts.results, child)
record(ts::CustomTestSet, res::Result) = push!(ts.results, res)
function finish(ts::CustomTestSet)
    # just record if we're not the top-level parent
    if get_testset_depth() > 0
        record(get_testset(), ts)
    end
    ts
end
```

And using that testset looks like:

```
@testset CustomTestSet foo=4 "custom testset inner 2" begin
# this testset should inherit the type, but not the argument.
@testset "custom testset inner" begin
@test true
```

end end

## **C** Interface

```
ccall ((symbol, library) or function_pointer, ReturnType, (ArgumentType1, ...), ArgumentValue1, ...)
```

Call function in C-exported shared library, specified by (function name, library) tuple, where each component is a string or symbol.

Note that the argument type tuple must be a literal tuple, and not a tuple-valued variable or expression. Alternatively, ccall may also be used to call a function pointer, such as one returned by dlsym.

Each ArgumentValue to the ccall will be converted to the corresponding ArgumentType, by automatic insertion of calls to unsafe\_convert(ArgumentType, cconvert(ArgumentType, ArgumentValue)). (See also the documentation for each of these functions for further details.) In most cases, this simply results in a call to convert (ArgumentType, ArgumentValue).

```
cglobal ((symbol, library)[, type=Void])
```

Obtain a pointer to a global variable in a C-exported shared library, specified exactly as in ccall. Returns a Ptr{Type}, defaulting to Ptr{Void} if no Type argument is supplied. The values can be read or written by unsafe load or unsafe store!, respectively.

```
cfunction (function::Function, ReturnType::Type, (ArgumentTypes...))
```

Generate C-callable function pointer from Julia function. Type annotation of the return value in the callback function is a must for situations where Julia cannot infer the return type automatically.

For example:

```
function foo()
    # body

retval::Float64
end

bar = cfunction(foo, Float64, ())
```

#### $unsafe\_convert(T, x)$

Convert x to a value of type T

In cases where convert would need to take a Julia object and turn it into a Ptr, this function should be used to define and perform that conversion.

Be careful to ensure that a julia reference to x exists as long as the result of this function will be used. Accordingly, the argument x to this function should never be an expression, only a variable name or field reference. For example, x=a.b.c is acceptable, but x=[a,b,c] is not.

The unsafe prefix on this function indicates that using the result of this function after the x argument to this function is no longer accessible to the program may cause undefined behavior, including program corruption or segfaults, at any later time.

#### cconvert(T, x)

Convert x to a value of type T, typically by calling convert (T, x)

In cases where x cannot be safely converted to T, unlike convert, cconvert may return an object of a type different from T, which however is suitable for unsafe\_convert to handle.

Neither convert nor convert should take a Julia object and turn it into a Ptr.

#### unsafe load (p::Ptr{T}, i::Integer)

Load a value of type T from the address of the ith element (1-indexed) starting at p. This is equivalent to the C expression p[i-1].

The unsafe prefix on this function indicates that no validation is performed on the pointer p to ensure that it is valid. Incorrect usage may segfault your program or return garbage answers, in the same manner as C.

## unsafe\_store! (p::Ptr{T}, x, i::Integer)

Store a value of type T to the address of the ith element (1-indexed) starting at p. This is equivalent to the C expression p[i-1] = x.

The unsafe prefix on this function indicates that no validation is performed on the pointer p to ensure that it is valid. Incorrect usage may corrupt or segfault your program, in the same manner as C.

## unsafe\_copy! (dest::Ptr{T}, src::Ptr{T}, N)

Copy N elements from a source pointer to a destination, with no checking. The size of an element is determined by the type of the pointers.

The unsafe prefix on this function indicates that no validation is performed on the pointers dest and src to ensure that they are valid. Incorrect usage may corrupt or segfault your program, in the same manner as C.

## unsafe\_copy! (dest::Array, do, src::Array, so, N)

Copy N elements from a source array to a destination, starting at offset so in the source and do in the destination (1-indexed).

The unsafe prefix on this function indicates that no validation is performed to ensure that N is inbounds on either array. Incorrect usage may corrupt or segfault your program, in the same manner as C.

#### copy! (dest, src)

Copy all elements from collection src to array dest. Returns dest.

## copy! (dest, do, src, so, N)

Copy N elements from collection src starting at offset so, to array dest starting at offset do. Returns dest.

## pointer(array[, index])

Get the native address of an array or string element. Be careful to ensure that a julia reference to a exists as long as this pointer will be used. This function is "unsafe" like unsafe\_convert.

Calling Ref (array[, index]) is generally preferable to this function.

## pointer\_to\_array (pointer, dims[, take\_ownership::Bool])

Wrap a native pointer as a Julia Array object. The pointer element type determines the array element type. own optionally specifies whether Julia should take ownership of the memory, calling free on the pointer when the array is no longer referenced.

## pointer\_from\_objref(object\_instance)

Get the memory address of a Julia object as a Ptr. The existence of the resulting Ptr will not protect the object from garbage collection, so you must ensure that the object remains referenced for the whole time that the Ptr will be used.

## unsafe\_pointer\_to\_objref(p::Ptr)

Convert a Ptr to an object reference. Assumes the pointer refers to a valid heap-allocated Julia object. If this is not the case, undefined behavior results, hence this function is considered "unsafe" and should be used with care.

## disable\_sigint (f::Function)

Disable Ctrl-C handler during execution of a function, for calling external code that is not interrupt safe. Intended to be called using do block syntax as follows:

```
disable_sigint() do
    # interrupt-unsafe code
    ...
end
```

## reenable\_sigint (f::Function)

Re-enable Ctrl-C handler during execution of a function. Temporarily reverses the effect of disable\_sigint.

## systemerror (sysfunc, iftrue)

Raises a SystemError for errno with the descriptive string sysfunc if iftrue is true

## Ptr{T}

A memory address referring to data of type T. However, there is no guarantee that the memory is actually valid, or that it actually represents data of the specified type.

#### Ref{T}

An object that safely references data of type T. This type is guaranteed to point to valid, Julia-allocated memory of the correct type. The underlying data is protected from freeing by the garbage collector as long as the Ref itself is referenced.

When passed as a ccall argument (either as a Ptr or Ref type), a Ref object will be converted to a native pointer to the data it references.

There is no invalid (NULL) Ref.

#### Cchar

Equivalent to the native char c-type

## Cuchar

Equivalent to the native unsigned char c-type (UInt8)

#### Cshort

Equivalent to the native signed short c-type (Int16)

#### Cushort

Equivalent to the native unsigned short c-type (UInt16)

## Cint

Equivalent to the native signed int c-type (Int32)

#### Cuint

Equivalent to the native unsigned int c-type (UInt32)

#### Clong

Equivalent to the native signed long c-type

## Culong

Equivalent to the native unsigned long c-type

## Clonglong

Equivalent to the native signed long long c-type (Int64)

#### Culonglong

Equivalent to the native unsigned long long c-type (UInt64)

#### Cintmax\_t

Equivalent to the native intmax\_t c-type (Int64)

## Cuintmax\_t

Equivalent to the native uintmax\_t c-type (UInt64)

## Csize\_t

Equivalent to the native size\_t c-type (UInt)

## Cssize\_t

Equivalent to the native ssize\_t c-type

## Cptrdiff\_t

Equivalent to the native ptrdiff\_t c-type (Int)

## Cwchar\_t

Equivalent to the native wchar\_t c-type (Int32)

## Cfloat

Equivalent to the native float c-type (Float32)

## Cdouble

Equivalent to the native double c-type (Float64)

## **LLVM Interface**

11vmcall (IR::String, ReturnType, (ArgumentType1, ...), ArgumentValue1, ...)

 $\textbf{11vmcal1} \ ((declarations::String, IR::String), ReturnType, (ArgumentType1, ...), ArgumentValue1, ...)$ 

Call LLVM IR string in the first argument. Similar to an LLVM function define block, arguments are available as consecutive unnamed SSA variables (%0, %1, etc.).

The optional declarations string contains external functions declarations that are necessary for llvm to compile the IR string. Multiple declarations can be passed in by separating them with line breaks.

Note that the argument type tuple must be a literal tuple, and not a tuple-valued variable or expression.

Each ArgumentValue to llvmcall will be converted to the corresponding ArgumentType, by automatic insertion of calls to unsafe\_convert(ArgumentType, cconvert(ArgumentType, ArgumentValue)). (see also the documentation for each of these functions for further details). In most cases, this simply results in a call to convert (ArgumentType, ArgumentValue).

See test/llvmcall.jl for usage examples.

# C Standard Library

## $malloc(size::Integer) \rightarrow Ptr{Void}$

Call malloc from the C standard library.

## $\texttt{calloc} (\textit{num}::Integer, \textit{size}::Integer) \rightarrow Ptr\{Void\}$

Call calloc from the C standard library.

## $realloc(addr::Ptr, size::Integer) \rightarrow Ptr{Void}$

Call realloc from the C standard library.

See warning in the documentation for free regarding only using this on memory originally obtained from malloc.

## free (addr::Ptr)

Call free from the C standard library. Only use this on memory obtained from malloc, not on pointers retrieved from other C libraries. Ptr objects obtained from C libraries should be freed by the free functions defined in that library, to avoid assertion failures if multiple libc libraries exist on the system.

## errno(|code|)

Get the value of the C library's errno. If an argument is specified, it is used to set the value of errno.

The value of errno is only valid immediately after a ccall to a C library routine that sets it. Specifically, you cannot call errno at the next prompt in a REPL, because lots of code is executed between prompts.

#### strerror(n=errno())

Convert a system call error code to a descriptive string

## GetLastError()

Call the Win32 GetLastError function [only available on Windows].

#### FormatMessage (n=GetLastError())

Convert a Win32 system call error code to a descriptive string [only available on Windows].

#### time (t::TmStruct)

Converts a TmStruct struct to a number of seconds since the epoch.

## strftime([format], time)

Convert time, given as a number of seconds since the epoch or a TmStruct, to a formatted string using the given format. Supported formats are the same as those in the standard C library.

## strptime([format], timestr)

Parse a formatted time string into a TmStruct giving the seconds, minute, hour, date, etc. Supported formats are the same as those in the standard C library. On some platforms, timezones will not be parsed correctly. If the result of this function will be passed to time to convert it to seconds since the epoch, the isdst field should be filled in manually. Setting it to -1 will tell the C library to use the current system settings to determine the timezone.

## TmStruct([seconds])

Convert a number of seconds since the epoch to broken-down format, with fields sec, min, hour, mday, month, year, wday, yday, and isdst.

#### flush\_cstdio()

Flushes the C stdout and stderr streams (which may have been written to by external C code).

## msync(ptr, len[, flags])

Forces synchronization of the mmap () ped memory region from ptr to ptr+len. Flags defaults to MS\_SYNC, but can be a combination of MS\_ASYNC, MS\_SYNC, or MS\_INVALIDATE. See your platform man page for specifics. The flags argument is not valid on Windows.

You may not need to call msync, because synchronization is performed at intervals automatically by the operating system. However, you can call this directly if, for example, you are concerned about losing the result of a long-running calculation.

## MS\_ASYNC

Enum constant for msync () (page 488). See your platform man page for details. (not available on Windows).

## MS\_SYNC

Enum constant for msync() (page 488). See your platform man page for details. (not available on Windows).

#### MS\_INVALIDATE

Enum constant for msync() (page 488). See your platform man page for details. (not available on Windows).

# **Dynamic Linker**

## dlopen(libfile::AbstractString[, flags::Integer])

Load a shared library, returning an opaque handle.

The optional flags argument is a bitwise-or of zero or more of RTLD\_LOCAL, RTLD\_GLOBAL, RTLD\_LAZY, RTLD\_NOW, RTLD\_NODELETE, RTLD\_NOLOAD, RTLD\_DEEPBIND, and RTLD\_FIRST. These are converted to the corresponding flags of the POSIX (and/or GNU libc and/or MacOS) dlopen command, if possible, or are ignored if the specified functionality is not available on the current platform. The default flags are platform specific. On MacOS the default dlopen flags are RTLD\_LAZY|RTLD\_DEEPBIND|RTLD\_GLOBAL while on other platforms the defaults are RTLD\_LAZY|RTLD\_DEEPBIND|RTLD\_LOCAL. An important usage of these flags is to specify non default behavior for when the dynamic library loader binds library references to exported symbols and if the bound references are put into process local or global scope. For instance RTLD\_LAZY|RTLD\_DEEPBIND|RTLD\_GLOBAL allows the library's symbols to be available for usage in other shared libraries, addressing situations where there are dependencies between shared libraries.

## dlopen\_e (libfile::AbstractString[, flags::Integer])

Similar to <code>dlopen()</code> (page 489), except returns a <code>NULL</code> pointer instead of raising errors.

## RTLD DEEPBIND

Enum constant for <code>dlopen()</code> (page 489). See your platform man page for details, if applicable.

## RTLD\_FIRST

Enum constant for dlopen () (page 489). See your platform man page for details, if applicable.

#### RTLD GLOBAL

Enum constant for <code>dlopen()</code> (page 489). See your platform man page for details, if applicable.

## RTLD LAZY

Enum constant for <code>dlopen()</code> (page 489). See your platform man page for details, if applicable.

#### RTLD LOCAL

Enum constant for <code>dlopen()</code> (page 489). See your platform man page for details, if applicable.

#### RTLD NODELETE

Enum constant for dlopen () (page 489). See your platform man page for details, if applicable.

#### RTLD NOLOAD

Enum constant for <code>dlopen()</code> (page 489). See your platform man page for details, if applicable.

#### RTLD NOW

Enum constant for <code>dlopen()</code> (page 489). See your platform man page for details, if applicable.

#### **dlsym** (handle, sym)

Look up a symbol from a shared library handle, return callable function pointer on success.

### dlsym\_e (handle, sym)

Look up a symbol from a shared library handle, silently return NULL pointer on lookup failure.

## dlclose(handle)

Close shared library referenced by handle.

## dlext

File extension for dynamic libraries (e.g. dll, dylib, so) on the current platform.

### find\_library (names, locations)

Searches for the first library in names in the paths in the locations list, DL\_LOAD\_PATH, or system library paths (in that order) which can successfully be dlopen'd. On success, the return value will be one of the names (potentially prefixed by one of the paths in locations). This string can be assigned to a global const and used as the library name in future ccall's. On failure, it returns the empty string.

## DL\_LOAD\_PATH

When calling dlopen, the paths in this list will be searched first, in order, before searching the system locations for a valid library handle.

# **Profiling**

## @profile()

<code>@profile <expression> runs your expression while taking periodic backtraces. These are appended to an internal buffer of backtraces.</code>

The methods in Base.Profile are not exported and need to be called e.g. as Profile.print().

#### clear()

Clear any existing backtraces from the internal buffer.

print ([io::IO = STDOUT], [data::Vector]; format = :tree, C = false, combine = true, maxdepth = typemax(Int), sortedby = :filefuncline)

Prints profiling results to io (by default, STDOUT). If you do not supply a data vector, the internal buffer of accumulated backtraces will be used. format can be :tree or :flat. If C==true, backtraces from C and Fortran code are shown. combine==true merges instruction pointers that correspond to the same line of code. maxdepth can be used to limit the depth of printing in :tree format, while sortedby can be used to control the order in :flat format (:filefuncline sorts by the source line, whereas :count sorts in order of number of collected samples).

print ([io::IO = STDOUT], data::Vector, lidict::Dict; kwargs)

Prints profiling results to io. This variant is used to examine results exported by a previous call to retrieve() (page 491). Supply the vector data of backtraces and a dictionary lidict of line information.

See Profile.print([io], data) for an explanation of the valid keyword arguments.

#### init (; n::Integer, delay::Float64)

Configure the delay between backtraces (measured in seconds), and the number n of instruction pointers that may be stored. Each instruction pointer corresponds to a single line of code; backtraces generally consist of a long list of instruction pointers. Default settings can be obtained by calling this function with no arguments, and each can be set independently using keywords or in the order (n, delay).

#### **fetch**() $\rightarrow$ data

Returns a reference to the internal buffer of backtraces. Note that subsequent operations, like clear() (page 491), can affect data unless you first make a copy. Note that the values in data have meaning only on this machine in the current session, because it depends on the exact memory addresses used in JIT-compiling. This function is primarily for internal use; retrieve() (page 491) may be a better choice for most users.

## retrieve() → data, lidict

"Exports" profiling results in a portable format, returning the set of all backtraces (data) and a dictionary that maps the (session-specific) instruction pointers in data to LineInfo values that store the file name, function name, and line number. This function allows you to save profiling results for future analysis.

 $\textbf{callers} (funcname[, data, lidict][, filename = < filename > ][, linerange = < start: stop > ]) \rightarrow Vector{Tuple{count, linfo}}$ 

Given a previous profiling run, determine who called a particular function. Supplying the filename (and op-

tionally, range of line numbers over which the function is defined) allows you to disambiguate an overloaded method. The returned value is a vector containing a count of the number of calls and line information about the caller. One can optionally supply backtrace data obtained from retrieve() (page 491); otherwise, the current internal profile buffer is used.

## clear\_malloc\_data()

Clears any stored memory allocation data when running julia with --track-allocation. Execute the command(s) you want to test (to force JIT-compilation), then call <code>clear\_malloc\_data()</code> (page 492). Then execute your command(s) again, quit Julia, and examine the resulting \*.mem files.

- [Clarke61] Arthur C. Clarke, Profiles of the Future (1961): Clarke's Third Law.
- [Bischof1987] C Bischof and C Van Loan, "The WY representation for products of Householder matrices", SIAM J Sci Stat Comput 8 (1987), s2-s13. doi:10.1137/0908009
- [Schreiber1989] R Schreiber and C Van Loan, "A storage-efficient WY representation for products of Householder transformations", SIAM J Sci Stat Comput 10 (1989), 53-57. doi:10.1137/0910005
- [Bunch1977] J R Bunch and L Kaufman, Some stable methods for calculating inertia and solving symmetric linear systems, Mathematics of Computation 31:137 (1977), 163-179. url.
- [issue8859] Issue 8859, "Fix least squares", https://github.com/JuliaLang/julia/pull/8859
- [B96] Åke Björck, "Numerical Methods for Least Squares Problems", SIAM Press, Philadelphia, 1996, "Other Titles in Applied Mathematics", Vol. 51. doi:10.1137/1.9781611971484
- [S84] 7. (a) Stewart, "Rank Degeneracy", SIAM Journal on Scientific and Statistical Computing, 5(2), 1984, 403-413, doi:10.1137/0905030
- [KY88] Konstantinos Konstantinides and Kung Yao, "Statistical analysis of effective singular values in matrix rank determination", IEEE Transactions on Acoustics, Speech and Signal Processing, 36(5), 1988, 757-763. doi:10.1109/29.1585
- [H05] Nicholas J. Higham, "The squaring and scaling method for the matrix exponential revisited", SIAM Journal on Matrix Analysis and Applications, 26(4), 2005, 1179-1193. doi:10.1137/090768539
- [AH12] Awad H. Al-Mohy and Nicholas J. Higham, "Improved inverse scaling and squaring algorithms for the matrix logarithm", SIAM Journal on Scientific Computing, 34(4), 2012, C153-C169. doi:10.1137/110852553
- [AHR13] Awad H. Al-Mohy, Nicholas J. Higham and Samuel D. Relton, "Computing the Fréchet derivative of the matrix logarithm and estimating the condition number", SIAM Journal on Scientific Computing, 35(4), 2013, C394-C410. doi:10.1137/120885991
- [BH83] Åke Björck and Sven Hammarling, "A Schur method for the square root of a matrix", Linear Algebra and its Applications, 52-53, 1983, 127-140. doi:10.1016/0024-3795(83)80010-X

496 Bibliography

Symbols	≢() (in module Base), 355
*() (in module Base), 353, 385, 413	∌() (in module Base), 337
+() (in module Base), 353	∉() (in module Base), 337
-() (in module Base), 353	$\nsubseteq$ () (in module Base), 341
	$\subseteq$ () (in module Base), 341, 346
=() (in module Base), 356	$\subsetneq$ () (in module Base), 341
.*() (in module Base), 353	÷() (in module Base), 354
.+() (in module Base), 353	$\times$ () (in module Base), 413
() (in module Base), 353	precompile() (in module Base), 318
./() (in module Base), 353	^() (in module Base), 353, 385
.==() (in module Base), 356	~() (in module Base), 356
.≥() (in module Base), 356	\() (in module Base), 353, 413
.≤() (in module Base), 356	l() (in module Base), 356
$\neq$ (in module Base), 356	>() (in module Base), 325
.^() (in module Base), 353	>() (in module Base), 356
.\() (in module Base), 353	>=() (in module Base), 356
.>() (in module Base), 356	>>() (in module Base), 355
.>=() (in module Base), 356	>>>() (in module Base), 355
.<() (in module Base), 356	<() (in module Base), 355
.<=() (in module Base), 356	<:() (in module Base), 322
/() (in module Base), 353	<=() (in module Base), 355
//() (in module Base), 354	<<() (in module Base), 355
:() (in module Base), 355	Λ
==() (in module Base), 355	Α
===() (in module Base), 320, 355	A_ldiv_Bc() (in module Base), 356
\$() (in module Base), 356	A_ldiv_Bt() (in module Base), 356
%() (in module Base), 354	A_mul_B
&() (in module Base), 356	() (in module Base), 356
$\gamma$ (in module Base), 379	A_mul_Bc() (in module Base), 357
$\phi$ (in module Base), 379	A_mul_Bt() (in module Base), 357
$\pi$ (in module Base), 379	A_rdiv_Bc() (in module Base), 357
$\cap$ () (in module Base), 345	A_rdiv_Bt() (in module Base), 357
·() (in module Base), 413	abs() (in module Base), 363
∪() (in module Base), 345	abs2() (in module Base), 363
$\equiv$ () (in module Base), 320, 355	abspath() (in module Base), 446
≥() (in module Base), 356	Ac_ldiv_B() (in module Base), 357
∈() (in module Base), 337	Ac_ldiv_Bc() (in module Base), 357
≤() (in module Base), 355	Ac_mul_B() (in module Base), 357
$\ni$ () (in module Base), 337	Ac_mul_Bc() (in module Base), 357
$\neq$ () (in module Base), 355	Ac_rdiv_B() (in module Base), 357
, , , , , , , , , , , , , , , , , , , ,	Ac_rdiv_Bc() (in module Base), 357

accept() (in module Base), 459	atanh() (in module Base), 359
acos() (in module Base), 358	atexit() (in module Base), 317
acosd() (in module Base), 358	atreplinit() (in module Base), 317
acosh() (in module Base), 359	available() (in module Base.Pkg), 472
acot() (in module Base), 359	axpy
acotd() (in module Base), 359	() (in module Base.LinAlg.BLAS), 428
acoth() (in module Base), 360	_
acsc() (in module Base), 359	В
acscd() (in module Base), 359	backtrace() (in module Base), 329
acsch() (in module Base), 359	baremodule, 133
add() (in module Base.Pkg), 471	base() (in module Base), 377
addprocs() (in module Base), 406, 407	Base.channel_from_id() (in module Base), 410
airy() (in module Base), 366	Base.checked_abs() (in module Base), 363
airyai() (in module Base), 366	Base.checked_add() (in module Base), 363
airyaiprime() (in module Base), 366	Base.checked_cld() (in module Base), 363
airybi() (in module Base), 366	Base.checked_div() (in module Base), 363
airybiprime() (in module Base), 366	Base.checked_fld() (in module Base), 363
airyprime() (in module Base), 366	Base.checked_mod() (in module Base), 363
airyx() (in module Base), 366	Base.checked_mul() (in module Base), 363
all	Base.checked_neg() (in module Base), 363
() (in module Base), 340	Base.checked_rem() (in module Base), 363
all() (in module Base), 340	Base.checked_sub() (in module Base), 363
angle() (in module Base), 364	Base.Collections (module), 350
ans (in module Base), 320	Base.compilecache() (in module Base), 318
any	Base.LinAlg (module), 413
() (in module Base), 340	Base.LinAlg.BLAS (module), 428
ANY (in module Base), 441	Base.LinAlg.LAPACK (module), 431
any() (in module Base), 340	Base.linearindexing() (in module Base), 394
append	Base.Pkg (module), 469
() (in module Base), 349	Base.process_messages() (in module Base), 412
applicable() (in module Base), 325	Base.remoteref_id() (in module Base), 410
apropos() (in module Base), 319	Base.summarysize() (in module Base), 318
ARGS (in module Base), 441	Base.Test (module), 475
ArgumentError() (in module Base), 330	Base.worker_id_from_socket() (in module Base), 410
Array() (in module Base), 395	base64decode() (in module Base), 456
ascii() (in module Base), 385	Base64DecodePipe() (in module Base), 455
asec() (in module Base), 359	base64encode() (in module Base), 456
asecd() (in module Base), 359	Base64EncodePipe() (in module Base), 455
asech() (in module Base), 359	basename() (in module Base), 446
asin() (in module Base), 358	bdsdc
asind() (in module Base), 358	() (in module Base.LinAlg.LAPACK), 439
asinh() (in module Base), 359	bdsqr
assert() (in module Base), 330	() (in module Base.LinAlg.LAPACK), 439
AssertionError() (in module Base), 330	besselh() (in module Base), 367
asum() (in module Base.LinAlg.BLAS), 428	besseli() (in module Base), 367
At_ldiv_B() (in module Base), 357	besselix() (in module Base), 367
At_ldiv_Bt() (in module Base), 357	besselj() (in module Base), 367
At_mul_B() (in module Base), 357	besselj0() (in module Base), 366
At_mul_Bt() (in module Base), 357	besselj1() (in module Base), 366
At_rdiv_B() (in module Base), 357	besseljx() (in module Base), 367
At_rdiv_Bt() (in module Base), 357	besselk() (in module Base), 367
atan() (in module Base), 358	besselkx() (in module Base), 367
atan2() (in module Base), 358	bessely() (in module Base), 367
atand() (in module Base), 359	bessely0() (in module Base), 367

498 Index

bessely1() (in module Base), 367	Cdouble (in module Base), 484
besselyx() (in module Base), 367	ceil() (in module Base), 362
beta() (in module Base), 367	cell() (in module Base), 395
bfft	Cfloat (in module Base), 484
() (in module Base), 371	cfunction() (in module Base), 481
bfft() (in module Base), 371	cglobal() (in module Base), 481
Bidiagonal() (in module Base), 422	Channel{T}() (in module Base), 406
big() (in module Base), 378	charwidth() (in module Base), 389
BigFloat() (in module Base), 380	checkbounds() (in module Base), 399
BigInt() (in module Base), 380	checkout() (in module Base.Pkg), 472
bin() (in module Base), 377	chmod() (in module Base), 444
bind() (in module Base), 460	chol() (in module Base), 414
binomial() (in module Base), 364	cholfact
bitbroadcast() (in module Base), 396	() (in module Base), 415
bitpack() (in module Base), 402	cholfact() (in module Base), 414
bitrand() (in module Base), 383	chomp() (in module Base), 389
bits() (in module Base), 377	chop() (in module Base), 389
bitunpack() (in module Base), 402	chr2ind() (in module Base), 389
bkfact	Cint (in module Base), 483
() (in module Base), 418	Cintmax_t (in module Base), 483
bkfact() (in module Base), 418	circshift() (in module Base), 398
blas_set_num_threads() (in module Base.LinAlg.BLAS),	cis() (in module Base), 364
431	clamp() (in module Base), 362
blascopy	cld() (in module Base), 354
() (in module Base.LinAlg.BLAS), 428	clear() (in module Base.Profile), 491
blkdiag() (in module Base), 424	clear_malloc_data() (in module Base.Profile), 492
BoundsError() (in module Base), 330	clipboard() (in module Base), 318
brfft() (in module Base), 372	clone() (in module Base.Pkg), 471, 472
broadcast	Clong (in module Base), 483
() (in module Base), 396	Clonglong (in module Base), 483
_function() (in module Base), 397	close() (in module Base), 409, 450
broadcast() (in module Base), 396	cmp() (in module Base), 356
broadcast_function() (in module Base), 396	code_llvm() (in module Base), 333
broadcast_getindex() (in module Base), 397	code_lowered() (in module Base), 332
broadcast_setindex	code_native() (in module Base), 333
() (in module Base), 397	code_typed() (in module Base), 332
bswap() (in module Base), 378	code_warntype() (in module Base), 332
build() (in module Base.Pkg), 472	collect() (in module Base), 341
bytes2hex() (in module Base), 378	colon() (in module Base), 355
bytestring() (in module Base), 385	complement
	() (in module Base), 345
C	complement() (in module Base), 345
C_NULL (in module Base), 441	complex() (in module Base), 378
call() (in module Base), 325	cond() (in module Base), 423
callers() (in module Base.Profile), 491	Condition() (in module Base), 405
calloc() (in module Libc), 487	condskeel() (in module Base), 423
cat() (in module Base), 397	conj
catalan (in module Base), 379	() (in module Base), 394
catch_backtrace() (in module Base), 329	conj() (in module Base), 364
cbrt() (in module Base), 364	connect() (in module Base), 412, 459
ccall() (in module Base), 481	consume() (in module Base), 405
Cchar (in module Base), 483	contains() (in module Base), 388
cconvert() (in module Base), 481	conv() (in module Base), 373
cd() (in module Base), 443	conv2() (in module Base), 373

convert() (in module Base), 321	D
сору	dawson() (in module Base), 364
() (in module Base), 482	dct
copy() (in module Base), 321	() (in module Base), 372
copysign() (in module Base), 363	dct() (in module Base), 372
cor() (in module Base), 370	dec() (in module Base), 377
cos() (in module Base), 358	deconv() (in module Base), 373
cosc() (in module Base), 360	deepcopy() (in module Base), 321
cosd() (in module Base), 358	deg2rad() (in module Base), 360
cosh() (in module Base), 358	delete
cospi() (in module Base), 358	() (in module Base), 344
cot() (in module Base), 359	deleteat
cotd() (in module Base), 359	() (in module Base), 347
coth() (in module Base), 359	den() (in module Base), 355
count() (in module Base), 340	dequeue
count_ones() (in module Base), 381	() (in module Base.Collections), 350
count_zeros() (in module Base), 381	deserialize() (in module Base), 452
countfrom() (in module Base), 336	det() (in module Base), 423
countlines() (in module Base), 453	detach() (in module Base), 327
countnz() (in module Base), 394	DevNull (in module Base), 326
cov() (in module Base), 370	diag() (in module Base), 422
cp() (in module Base), 444	diagind() (in module Base), 422
Cptrdiff_t (in module Base), 484	diagm() (in module Base), 422
CPU_CORES (in module Base), 441	Dict() (in module Base), 343
cross() (in module Base), 413	diff() (in module Base), 400
csc() (in module Base), 359	digamma() (in module Base), 366
cscd() (in module Base), 359	digits
csch() (in module Base), 359	() (in module Base), 377
Cshort (in module Base), 483	digits() (in module Base), 377
Csize_t (in module Base), 484	DimensionMismatch() (in module Base), 330
Cssize_t (in module Base), 484	dir() (in module Base.Pkg), 471
ctime() (in module Base), 444	dir() (in module Base), 446
ctranspose	disable_sigint() (in module Base), 482
() (in module Base), 426	display() (in module Base), 456
ctranspose() (in module Base), 426	displayable() (in module Base), 457
Cuchar (in module Base), 483	displaysize() (in module Base), 456
Cuint (in module Base), 483	div() (in module Base), 354
Cuintmax_t (in module Base), 483	DivideError() (in module Base), 330
Culong (in module Base), 483	divrem() (in module Base), 354
Culonglong (in module Base), 483	DL_LOAD_PATH (in module Libdl), 490
cummax() (in module Base), 400	dlclose() (in module Libdl), 490
cummin() (in module Base), 400	dlext (in module Libdl), 490
cumprod	dlopen() (in module Libdl), 489
() (in module Base), 400	dlopen_e() (in module Libdl), 489
cumprod() (in module Base), 400	dlsym() (in module Libdl), 489
cumsum	dlsym_e() (in module Libdl), 489
() (in module Base), 400	DomainError() (in module Base), 330
cumsum() (in module Base), 400	done() (in module Base), 335
cumsum_kbn() (in module Base), 400	dot() (in module Base), 413
current_module() (in module Base), 331	dot() (in module Base.), 413 dot() (in module Base.LinAlg.BLAS), 428
current_task() (in module Base), 405	dotc() (in module Base.LinAlg.BLAS), 428
Cushort (in module Base), 483	dotu() (in module Base.LinAlg.BLAS), 428
Cwchar_t (in module Base), 484	download() (in module Base), 445
cycle() (in module Base), 336	drop() (in module Base), 336
	r () (

dump() (in module Base), 454	exp2() (in module Base), 360
E	expand() (in module Base), 332 expanduser() (in module Base), 446
e (in module Base), 379	expm() (in module Base), 424
eachindex() (in module Base), 393	expm1() (in module Base), 360
	exponent() (in module Base), 378
eachline() (in module Base), 454	export, 133
eachmatch() (in module Base), 387	extrema() (in module Base), 338
edit() (in module Base), 318	eye() (in module Base), 396
edit() (in module Base.Pkg), 471	cyc() (III module base), 390
eig() (in module Base), 418	F
eigfact	•
() (in module Base), 419	factor() (in module Base), 365
eigfact() (in module Base), 419	factorial() (in module Base), 364, 365
eigmax() (in module Base), 418	factorize() (in module Base), 413
eigmin() (in module Base), 418	falses() (in module Base), 395
eigs() (in module Base), 426	fd() (in module Base), 452
eigvals	fdio() (in module Base), 450
() (in module Base), 418	fetch() (in module Base), 408
eigvals() (in module Base), 418	fetch() (in module Base.Profile), 491
eigvecs() (in module Base), 419	fft
eltype() (in module Base), 337	() (in module Base), 371
empty	fft() (in module Base), 370
() (in module Base), 336	fftshift() (in module Base), 373
ENDIAN_BOM (in module Base), 461	fieldname() (in module Base), 332
endof() (in module Base), 337	fieldnames() (in module Base), 332
endswith() (in module Base), 388	fieldoffset() (in module Base), 323
enqueue	fieldtype() (in module Base), 324
() (in module Base.Collections), 350	filemode() (in module Base), 444
enumerate() (in module Base), 335	filesize() (in module Base), 444
ENV (in module Base), 329	fill
EnvHash() (in module Base), 329	() (in module Base), 395
eof() (in module Base), 451	fill() (in module Base), 395
EOFError() (in module Base), 330	filt
eps() (in module Base), 323	() (in module Base), 373
erf() (in module Base), 364	filt() (in module Base), 373
erfc() (in module Base), 364	filter
erfcinv() (in module Base), 364	() (in module Base), 342
erfcx() (in module Base), 364	filter() (in module Base), 342
erfi() (in module Base), 364	finalize() (in module Base), 321
erfiny() (in module Base), 364	finalize() (in module Base), 321
errno() (in module Libc), 487	find() (in module Base), 398
error() (in module Base), 329	find_library() (in module Libdl), 490
ErrorException() (in module Base), 330	findfirst() (in module Base), 398
esc() (in module Base), 325	findin() (in module Base), 337
escape_string() (in module Base), 390	findlast() (in module Base), 398, 399
eta() (in module Base), 367	findmax
etree() (in module Base), 403	() (in module Base), 339
eu (in module Base), 379	findmax() (in module Base), 339
eulergamma (in module Base), 379	findmin
eval() (in module Base), 325	() (in module Base), 339
evalfile() (in module Base), 325	findmin() (in module Base), 339
exit() (in module Base), 317	findn() (in module Base), 398
exp() (in module Base), 360	findnext() (in module Base), 399
exp10() (in module Base) 360	findnz() (in module Base) 398

findprev() (in module Base), 399	() (in module Base.LinAlg.LAPACK), 439
finish() (in module Base.Test), 479	geev
first() (in module Base), 341	() (in module Base.LinAlg.LAPACK), 434
fld() (in module Base), 354	geevx
fld1() (in module Base), 354	() (in module Base.LinAlg.LAPACK), 435
fldmod() (in module Base), 354	gehrd
fldmod1() (in module Base), 354	() (in module Base.LinAlg.LAPACK), 439
flipbits	gelqf
() (in module Base), 402	() (in module Base.LinAlg.LAPACK), 431, 432
flipdim() (in module Base), 398	gels
flipsign() (in module Base), 364	() (in module Base.LinAlg.LAPACK), 433
float() (in module Base), 378	gelsd
Float32() (in module Base), 380	() (in module Base.LinAlg.LAPACK), 434
Float64() (in module Base), 380	gelsy
floor() (in module Base), 362	() (in module Base.LinAlg.LAPACK), 434
flush() (in module Base), 450	gemm
flush_cstdio() (in module Libc), 488	() (in module Base.LinAlg.BLAS), 429
fma() (in module Base), 353	gemm() (in module Base.LinAlg.BLAS), 429
foldl() (in module Base), 338	gemqrt
foldr() (in module Base), 338	() (in module Base.LinAlg.LAPACK), 436
foreach() (in module Base), 340	gemv
FormatMessage() (in module Libc), 487	() (in module Base.LinAlg.BLAS), 429
free() (in module Base.Pkg), 472	gemv() (in module Base.LinAlg.BLAS), 430
free() (in module Libc), 487	gensym() (in module Base), 325
frexp() (in module Base), 360	geqlf
full() (in module Base), 403, 413, 418	() (in module Base.LinAlg.LAPACK), 432
fullname() (in module Base), 331	geqp3
function_module() (in module Base), 332	() (in module Base.LinAlg.LAPACK), 432
function_name() (in module Base), 332	geqrf
functionloc() (in module Base), 332	() (in module Base.LinAlg.LAPACK), 432
Future() (in module Base), 408	geqrt
<u> </u>	() (in module Base.LinAlg.LAPACK), 433
G	geqrt3
gamma() (in module Base), 366	() (in module Base.LinAlg.LAPACK), 433
gbmv	ger
() (in module Base.LinAlg.BLAS), 429	() (in module Base.LinAlg.BLAS), 428
gbmv() (in module Base.LinAlg.BLAS), 429	gerqf
gbtrf	() (in module Base.LinAlg.LAPACK), 432
() (in module Base.LinAlg.LAPACK), 431	gesdd
gbtrs	() (in module Base.LinAlg.LAPACK), 434
() (in module Base.LinAlg.LAPACK), 431	gesv
gc() (in module Base), 332	() (in module Base.LinAlg.LAPACK), 433
gc_enable() (in module Base), 332	gesvd
gcd() (in module Base), 365	() (in module Base.LinAlg.LAPACK), 434
gcdx() (in module Base), 365	gesvx
gebak	() (in module Base.LinAlg.LAPACK), 434
() (in module Base.LinAlg.LAPACK), 431	get
gebal	() (in module Base), 343
() (in module Base.LinAlg.LAPACK), 431	get() (in module Base), 326, 343
gebrd	get_testset() (in module Base.Test), 479
() (in module Base.LinAlg.LAPACK), 431	get_testset_depth() (in module Base.Test), 479
gecon	
•	get_zero_subnormals() (in module Base), 381
() (in module Base.LinAlg.LAPACK), 439 gees	

gethostname() (in module Base), 328 getindex() (in module Base), 342, 395, 397	herk() (in module Base.LinAlg.BLAS), 429 Hermitian{Complex{Float64},SparseMatrixCSC{Complex{Float64},Suite
getipaddr() (in module Base), 328	415, 416
getkey() (in module Base), 343	hessfact
GetLastError() (in module Libc), 487	() (in module Base), 419
getpid() (in module Base), 328	hessfact() (in module Base), 419
getrf	hesv
() (in module Base.LinAlg.LAPACK), 433	() (in module Base.LinAlg.LAPACK), 438
getri	hetrf
() (in module Base.LinAlg.LAPACK), 433	() (in module Base.LinAlg.LAPACK), 438
getrs	hetri
() (in module Base.LinAlg.LAPACK), 433	() (in module Base.LinAlg.LAPACK), 438
getsockname() (in module Base), 459	hetrs
	() (in module Base.LinAlg.LAPACK), 438
() (in module Passa Lin Alg LAPACK) 430	hex() (in module Base), 377
() (in module Base.LinAlg.LAPACK), 439	
ggev	hex2bytes() (in module Base), 378
() (in module Base.LinAlg.LAPACK), 435	hex2num() (in module Base), 378
gglse	hist
() (in module Base.LinAlg.LAPACK), 434	() (in module Base), 369
ggsvd	hist() (in module Base), 369
() (in module Base.LinAlg.LAPACK), 434	hist2d
ggsvd3	() (in module Base), 369
() (in module Base.LinAlg.LAPACK), 435	hist2d() (in module Base), 369
givens{T}() (in module Base), 421	histrange() (in module Base), 369
golden (in module Base), 379	homedir() (in module Base), 446
gperm() (in module Base), 444	htol() (in module Base), 461
gradient() (in module Base), 400	hton() (in module Base), 460
graphemes() (in module Base), 387	hvcat() (in module Base), 397
gtsv	hypot() (in module Base), 360
() (in module Base.LinAlg.LAPACK), 435	
gttrf	
() (in module Base.LinAlg.LAPACK), 435	L (in modula Pasa Lin Ala DLAS) 421
gttrs	I (in module Base.LinAlg.BLAS), 431
() (in module Base.LinAlg.LAPACK), 435	idet
() (III IIIodule Base, EliiAig, EAI ACR), 433	() (in module Base), 373
H	idct() (in module Base), 372
	identity() (in module Base), 322
hankelh1() (in module Base), 367	ifelse() (in module Base), 320
hankelh1x() (in module Base), 367	ifft
hankelh2() (in module Base), 367	() (in module Base), 371
hankelh2x() (in module Base), 367	ifft() (in module Base), 371
hash() (in module Base), 321	ifftshift() (in module Base), 373
haskey() (in module Base), 343	ignorestatus() (in module Base), 327
hcat() (in module Base), 397	im (in module Base), 379
heapify	imag() (in module Base), 364
() (in module Base.Collections), 351	import, 133
heapify() (in module Base.Collections), 351	importall, 133
heappop	in() (in module Base), 337
() (in module Base.Collections), 351	include() (in module Base), 319
heappush	include_dependency() (in module Base), 319
() (in module Base.Collections), 351	include_string() (in module Base), 319
her  () (in module Pass Lin Alg PL AS), 420	ind2chr() (in module Base), 389
() (in module Base.LinAlg.BLAS), 429	ind2sub() (in module Base), 394
herk	indexin() (in module Base), 337
() (in module Base, LinAlg, BLAS), 429	indexpids() (in module Base), 411

indmax() (in module Base), 338	isequal() (in module Base), 320
indmin() (in module Base), 339	iseven() (in module Base), 382
InexactError() (in module Base), 330	isfifo() (in module Base), 445
Inf (in module Base), 379	isfile() (in module Base), 445
inf() (in module Base), 379	isfinite() (in module Base), 379
Inf16 (in module Base), 379	isgeneric() (in module Base), 332
Inf32 (in module Base), 379	isgraph() (in module Base), 389
info() (in module Base), 454	isheap() (in module Base.Collections), 351
init() (in module Base.Pkg), 471	ishermitian() (in module Base), 425
init() (in module Base.Profile), 491	isimmutable() (in module Base), 324
init_worker() (in module Base), 412	isinf() (in module Base), 379
InitError() (in module Base), 331	isinteger() (in module Base), 380
insert	isinteractive() (in module Base), 317
() (in module Base), 347	isleaftype() (in module Base), 324
installed() (in module Base.Pkg), 472	isless() (in module Base), 320
instances() (in module Base), 324	islink() (in module Base), 445
interrupt() (in module Base), 408	islower() (in module Base), 390
InterruptException() (in module Base), 330	ismarked() (in module Base), 451
intersect	ismatch() (in module Base), 387
() (in module Base), 345	ismount() (in module Base), 445
intersect() (in module Base), 345	isnan() (in module Base), 379
IntSet() (in module Base), 345	isnull() (in module Base), 326
inv() (in module Base), 424	isnumber() (in module Base), 390
invdigamma() (in module Base), 366	isodd() (in module Base), 382
invmod() (in module Base), 366	isopen() (in module Base), 452
invoke() (in module Base), 325	ispath() (in module Base), 445
invperm() (in module Base), 401	isperm() (in module Base), 401
IOBuffer() (in module Base), 449, 450	isposdef
ipermute	() (in module Base), 425
() (in module Base), 401	isposdef() (in module Base), 425
ipermutedims() (in module Base), 399	ispow2() (in module Base), 365
IPv4() (in module Base), 459	isprime() (in module Base), 382
IPv6() (in module Base), 459	isprint() (in module Base), 390
irfft() (in module Base), 372	ispunct() (in module Base), 390
is() (in module Base), 320	isqrt() (in module Base), 364
is_assigned_char() (in module Base), 387	isreadable() (in module Base), 451
isa() (in module Base), 320	isreadonly() (in module Base), 451
isabspath() (in module Base), 446	isready() (in module Base), 409
isalnum() (in module Base), 389	isreal() (in module Base), 380
isalpha() (in module Base), 389	issetgid() (in module Base), 446
isapprox() (in module Base), 358	issetuid() (in module Base), 446
isascii() (in module Base), 389	issocket() (in module Base), 446
isbits() (in module Base), 324	issorted() (in module Base), 467
isblockdev() (in module Base), 445	isspace() (in module Base), 390
ischardev() (in module Base), 445	issparse() (in module Base), 402
iscntrl() (in module Base), 389	issticky() (in module Base), 446
isconst() (in module Base), 332	issubnormal() (in module Base), 379
isdefined() (in module Base), 321	issubset() (in module Base), 341, 346
isdiag() (in module Base), 425	issubtype() (in module Base), 322
isdigit() (in module Base), 389	issym() (in module Base), 425
isdir() (in module Base), 445	istaskdone() (in module Base), 405
isdirpath() (in module Base), 446	istaskstarted() (in module Base), 405
iseltype() (in module Base), 393	istext() (in module Base), 458
isempty() (in module Base), 336	istril() (in module Base), 425

istriu() (in module Base), 425 isupper() (in module Base), 390	log2() (in module Base), 360 logabsdet() (in module Base), 424
isvalid() (in module Base), 387	logdet() (in module Base), 424
iswritable() (in module Base), 451	logm() (in module Base), 425
isxdigit() (in module Base), 390	logspace() (in module Base), 396
isheight() (iii iiiodale Base), 350	lowercase() (in module Base), 388
J	lowrankdowndate
join() (in module Base), 389	() (in module Base.LinAlg), 415
joinpath() (in module Base), 446	lowrankdowndate() (in module Base.LinAlg), 415
JULIA_HOME (in module Base), 441	lowrankupdate
	() (in module Base.LinAlg), 415
K	lowrankupdate() (in module Base.LinAlg), 415
KeyError() (in module Base), 330	lpad() (in module Base), 387
keys() (in module Base), 344	lstat() (in module Base), 444
keytype() (in module Base), 344	lstrip() (in module Base), 388
kill() (in module Base), 327, 412	ltoh() (in module Base), 461
kron() (in module Base), 424	lu() (in module Base), 413
	lufact
L	() (in module Base), 414
last() (in module Base), 341	lufact() (in module Base), 413
launch() (in module Base), 411	lyap() (in module Base), 425
lbeta() (in module Base), 367	
lcfirst() (in module Base), 388	M
lcm() (in module Base), 365	macroexpand() (in module Base), 332
ldexp() (in module Base), 360	malloc() (in module Libc), 487
ldltfact	manage() (in module Base), 412
	map
() (in module Base), 416	() (in module Base), 341
Idltfact() (in module Base), 415	map() (in module Base), 340
leading_ones() (in module Base), 381	mapfoldl() (in module Base), 341
leading_zeros() (in module Base), 381	mapfoldr() (in module Base), 341
length() (in module Base), 337, 385, 393	mapreduce() (in module Base), 341
less() (in module Base), 318	mapreducedim() (in module Base), 401
lexcmp() (in module Base), 320	mapslices() (in module Base), 401
lexless() (in module Base), 320	mark() (in module Base), 451
lfact() (in module Base), 366	match() (in module Base), 431
lgamma() (in module Base), 366	matchall() (in module Base), 387
Libc (module), 485	max() (in module Base), 362
Libdl (module), 488	maxabs
limit_output() (in module Base), 456	() (in module Base), 339
LinAlg.checksquare() (in module Base), 394	maxabs() (in module Base), 339
LinAlg.qr	· · · · · · · · · · · · · · · · · · ·
() (in module Base), 416	maximum
linreg() (in module Base), 424	() (in module Base), 338
linspace() (in module Base), 396	maximum() (in module Base), 338
listen() (in module Base), 459	maxintfloat() (in module Base), 323
listenany() (in module Base), 460	mean
llvmcall() (in module Base), 485	() (in module Base), 368
LOAD_PATH (in module Base), 441	mean() (in module Base), 368
LoadError() (in module Base), 330	median
localindexes() (in module Base), 411	() (in module Base), 369
lock() (in module Base), 406	median() (in module Base), 369
log() (in module Base), 360	merge
log10() (in module Base), 360	() (in module Base), 344
log1p() (in module Base), 360	merge() (in module Base), 344

MersenneTwister() (in module Base), 383	nextprod() (in module Base), 365
method_exists() (in module Base), 325	nfields() (in module Base), 332
MethodError() (in module Base), 330	nnz() (in module Base), 403
methods() (in module Base), 319	nonzeros() (in module Base), 403
methodswith() (in module Base), 319	norm() (in module Base), 422
middle() (in module Base), 368	normalize
midpoints() (in module Base), 369	() (in module Base), 423
mimewritable() (in module Base), 457	normalize() (in module Base), 423
min() (in module Base), 362	normalize_string() (in module Base), 386
minabs	normpath() (in module Base), 446
() (in module Base), 339	nothing (in module Base), 441
minabs() (in module Base), 339	notify() (in module Base), 406
minimum	nprocs() (in module Base), 407
() (in module Base), 338	nrm2() (in module Base.LinAlg.BLAS), 428
minimum() (in module Base), 338	ntoh() (in module Base), 460
minmax() (in module Base), 362	ntuple() (in module Base), 321
mkdir() (in module Base), 443	Nullable() (in module Base), 326
mkpath() (in module Base), 443	NullException() (in module Base), 330
mktemp() (in module Base), 445	nullspace() (in module Base), 424
mktempdir() (in module Base), 445	num() (in module Base), 355
Mmap.Anonymous() (in module Base), 458	num2hex() (in module Base), 378
Mmap.mmap() (in module Base), 458, 459	nworkers() (in module Base), 407
• • • •	nzrange() (in module Base), 403
Mmap.sync	nzrange() (in module base), 403
() (in module Base), 459	0
mod() (in module Base), 354	
mod1() (in module Base), 354	object_id() (in module Base), 321
mod2pi() (in module Base), 354	oct() (in module Base), 377
modf() (in module Base), 360	oftype() (in module Base), 322
module, 133	one() (in module Base), 378
module_name() (in module Base), 331	ones() (in module Base), 395
module_parent() (in module Base), 331	open() (in module Base), 449
MS_ASYNC (in module Libc), 488	operm() (in module Base), 444
MS_INVALIDATE (in module Libc), 488	ordschur
MS_SYNC (in module Libc), 488	() (in module Base), 420
msync() (in module Libc), 488	ordschur() (in module Base), 419, 420
mtime() (in module Base), 444	orghr
muladd() (in module Base), 354	() (in module Base.LinAlg.LAPACK), 439
mv() (in module Base), 445	orglq
myid() (in module Base), 408	() (in module Base.LinAlg.LAPACK), 435
N.I.	orgql
N	() (in module Base.LinAlg.LAPACK), 435
names() (in module Base), 331	orgqr
NaN (in module Base), 379	() (in module Base.LinAlg.LAPACK), 435
nan() (in module Base), 379	orgrq
NaN16 (in module Base), 379	() (in module Base.LinAlg.LAPACK), 435
NaN32 (in module Base), 379	ormlq
nb_available() (in module Base), 459	() (in module Base.LinAlg.LAPACK), 436
ndigits() (in module Base), 367	ormql
ndims() (in module Base), 393	() (in module Base.LinAlg.LAPACK), 436
next() (in module Base), 335	ormqr
nextfloat() (in module Base), 379	() (in module Base.LinAlg.LAPACK), 436
nextind() (in module Base), 389	ormrq
nextpow() (in module Base), 365	() (in module Base.LinAlg.LAPACK), 436
nextpow2() (in module Base), 365	ormrz
1	********

() (in module Base.LinAlg.LAPACK), 433	posv
OS_NAME (in module Base), 441	() (in module Base.LinAlg.LAPACK), 436
OutOfMemoryError() (in module Base), 330	potrf
OverflowError() (in module Base), 330	() (in module Base.LinAlg.LAPACK), 436
D	potri
P	() (in module Base.LinAlg.LAPACK), 436
parent() (in module Base), 397	potrs
parentindexes() (in module Base), 397	() (in module Base.LinAlg.LAPACK), 436
parse() (in module Base), 326, 377	powermod() (in module Base), 366
ParseError() (in module Base), 330	precision() (in module Base), 382
peakflops() (in module Base), 428	precompile() (in module Base), 333
peek() (in module Base.Collections), 350	prepend
permute	() (in module Base), 350
() (in module Base), 401	prevfloat() (in module Base), 379
permutedims	prevind() (in module Base), 389
() (in module Base), 399	prevpow() (in module Base), 365
permutedims() (in module Base), 399	prevpow2() (in module Base), 365
pi (in module Base), 379	primes() (in module Base), 382
pin() (in module Base.Pkg), 472	primesmask() (in module Base), 382
pinv() (in module Base), 424	print() (in module Base), 453
PipeBuffer() (in module Base), 453	print() (in module Base.Profile), 491
pipeline() (in module Base), 328	print_escaped() (in module Base), 452
plan_bfft	print_joined() (in module Base), 452
() (in module Base), 372	print_shortest() (in module Base), 452
plan_bfft() (in module Base), 371	print_unescaped() (in module Base), 452
plan_brfft() (in module Base), 372	print_with_color() (in module Base), 454
plan_dct	println() (in module Base), 454
() (in module Base), 373	PriorityQueue() (in module Base.Collections), 350
plan_dct() (in module Base), 373	process_exited() (in module Base), 326
•	process_running() (in module Base), 326
plan_fft () (in module Pass) 272	ProcessExitedException() (in module Base), 331
() (in module Base), 372	procs() (in module Base), 407, 411
plan_fft() (in module Base), 371	prod
plan_idet	() (in module Base), 340
() (in module Base), 373	prod() (in module Base), 340
plan_idct() (in module Base), 373	produce() (in module Base), 405
plan_ifft (in module Page) 272	Profile (module), 490
() (in module Base), 372	promote() (in module Base), 322
plan_ifft() (in module Base), 371	promote_rule() (in module Base), 323
plan_irfft() (in module Base), 372	promote_shape() (in module Base), 329
plan_r2r	promote_type() (in module Base), 323
() (in module Base.FFTW), 374	pstrf
plan_r2r() (in module Base.FFTW), 374	() (in module Base.LinAlg.LAPACK), 436
plan_rfft() (in module Base), 372	Ptr{T} (in module Base), 483
pmap() (in module Base), 408	
pointer() (in module Base), 482	ptsv () (in modulo Poso Lin Ala LADACK) 436
pointer_from_objref() (in module Base), 482	() (in module Base.LinAlg.LAPACK), 436
pointer_to_array() (in module Base), 482	pttrf () (in module Pess Lin Ala LA DACK) 426
poll_fd() (in module Base), 460	() (in module Base.LinAlg.LAPACK), 436
poll_file() (in module Base), 460	pttrs () (in madula Pasa Lin Ala LAPACK) 427
polygamma() (in module Base), 366	() (in module Base.LinAlg.LAPACK), 437
pop	push
() (in module Base), 344, 346	() (in module Base), 346
popdisplay() (in module Base), 458	pushdisplay() (in module Base), 458
position() (in module Base), 451	put

() (in module Base), 409 pwd() (in module Base), 443	readstring() (in module Base), 454 readuntil() (in module Base), 454
Q	real() (in module Base), 364
	realloc() (in module Libc), 487
qr() (in module Base), 416, 417	realmax() (in module Base), 323
qrfact	realmin() (in module Base), 323
() (in module Base), 418	realpath() (in module Base), 446
qrfact() (in module Base), 417	record() (in module Base.Test), 479
quadgk() (in module Base), 374	recv() (in module Base), 460
quantile	recvfrom() (in module Base), 460
() (in module Base), 369	redirect_stderr() (in module Base), 452
quantile() (in module Base), 369	redirect_stdin() (in module Base), 452
quit() (in module Base), 317	redirect_stdout() (in module Base), 452
Б	redisplay() (in module Base), 456
R	reduce() (in module Base), 338
r2r	reducedim() (in module Base), 400
() (in module Base.FFTW), 374	reenable_sigint() (in module Base), 483
r2r() (in module Base.FFTW), 373	ReentrantLock() (in module Base), 406
rad2deg() (in module Base), 360	Ref{T} (in module Base), 483
rand	reim() (in module Base), 364
() (in module Base), 383	reinterpret() (in module Base), 396
rand() (in module Base), 383	reload() (in module Base), 318
randcycle() (in module Base), 401	relpath() (in module Base), 446
randexp	rem() (in module Base), 354
() (in module Base), 384	rem1() (in module Base), 354
randexp() (in module Base), 384	remotecall() (in module Base), 408
randjump() (in module Base), 384	remotecall_fetch() (in module Base), 40
randn	remotecall_wait() (in module Base), 409
() (in module Base), 383	RemoteChannel() (in module Base), 408
randn() (in module Base), 383	repeat() (in module Base), 424
RandomDevice() (in module Base), 383	repeated() (in module Base), 336
randperm() (in module Base), 401	replace() (in module Base), 388
randstring() (in module Base), 389	repmat() (in module Base), 424
randsubseq	repr() (in module Base), 385
() (in module Base), 400	reprmime() (in module Base), 457
randsubseq() (in module Base), 399	require() (in module Base), 318
range() (in module Base), 355	reset() (in module Base), 451
rank() (in module Base), 422	reshape() (in module Base), 395
rationalize() (in module Base), 354	resize
read	() (in module Base), 349
() (in module Base), 450	resolve() (in module Base.Pkg), 471
read() (in module Base), 450	rest() (in module Base), 336
readandwrite() (in module Base), 327	rethrow() (in module Base), 329
readavailable() (in module Base), 453	retrieve() (in module Base.Profile), 491
	reverse
readbytes	() (in module Base), 402
() (in module Base), 450 readchomp() (in module Base), 452	reverse() (in module Base), 388, 401
readcsv() (in module Base), 455	reverseind() (in module Base), 401
V .	rfft() (in module Base), 372
readdir() (in module Base), 443	rm() (in module Base), 445
readdlm() (in module Base), 454, 455	rm() (in module Base.Pkg), 471
readline() (in module Base), 454	rmprocs() (in module Base), 407
readlines() (in module Base), 454	rol
readlink() (in module Base), 443	() (in module Base), 402
ReadOnlyMemoryError() (in module Base), 330	V \

rol() (in module Base), 402	sec() (in module Base), 359
ror	secd() (in module Base), 359
() (in module Base), 402	sech() (in module Base), 359
ror() (in module Base), 402	seek() (in module Base), 451
rot180() (in module Base), 400	seekend() (in module Base), 451
rotl90() (in module Base), 400	seekstart() (in module Base), 451
rotr90() (in module Base), 400	select
round() (in module Base), 360, 362	() (in module Base), 467
RoundDown (in module Base), 362	select() (in module Base), 467
rounding() (in module Base), 380	selectperm
RoundingMode (in module Base), 361	() (in module Base), 468
RoundNearest (in module Base), 361	selectperm() (in module Base), 467
RoundNearestTiesAway (in module Base), 362	send() (in module Base), 460
RoundNearestTiesUp (in module Base), 362	serialize() (in module Base), 452
RoundToZero (in module Base), 362	Set() (in module Base), 345
RoundUp (in module Base), 362	set_zero_subnormals() (in module Base), 38
rowvals() (in module Base), 403	setdiff
rpad() (in module Base), 387	() (in module Base), 345
rsearch() (in module Base), 387	setdiff() (in module Base), 345
rsearchindex() (in module Base), 388	setenv() (in module Base), 327
rsplit() (in module Base), 388	setfield
rstrip() (in module Base), 388	() (in module Base), 323
RTLD_DEEPBIND (in module Libdl), 489	setindex
RTLD_FIRST (in module Libdl), 489	() (in module Base), 342, 397
RTLD_GLOBAL (in module Libdl), 489	setopt() (in module Base), 460
RTLD_LAZY (in module Libdl), 489	setprecision() (in module Base), 382
RTLD_LOCAL (in module Libdl), 489	setprotocol
RTLD_NODELETE (in module Libdl), 489	() (in module Base.Pkg), 472
RTLD_NOLOAD (in module Libdl), 489	setrounding() (in module Base), 380, 381
RTLD_NOW (in module Libdl), 489	SharedArray() (in module Base), 411
run() (in module Base), 326	shift
runtests() (in module Base), 475	() (in module Base), 347
Tunests() (in module buse), 173	show() (in module Base), 453
S	showall() (in module Base), 453
	showcompact() (in module Base), 453
sbmv	showerror() (in module Base), 454
() (in module Base.LinAlg.BLAS), 429	shuffle
sbmv() (in module Base.LinAlg.BLAS), 429	() (in module Base), 401
scal	shuffle() (in module Base), 401
() (in module Base.LinAlg.BLAS), 428	sign() (in module Base), 364
scal() (in module Base.LinAlg.BLAS), 428	signbit() (in module Base), 364
scale	signed() (in module Base), 378
() (in module Base), 422	signif() (in module Base), 378 signif() (in module Base), 362
scale() (in module Base), 422	significand() (in module Base), 378
schedule() (in module Base), 406	
schur() (in module Base), 419, 420	similar() (in module Base), 395
schurfact	sin() (in module Base), 358
() (in module Base), 419, 420	sinc() (in module Base), 360
schurfact() (in module Base), 419, 420	sind() (in module Base), 358
sdata() (in module Base), 411	sinh() (in module Base), 358
search() (in module Base), 387	sinpi() (in module Base), 358
searchindex() (in module Base), 388	size() (in module Base), 393
searchsorted() (in module Base), 467	sizehint
searchsortedfirst() (in module Base), 467	() (in module Base), 344
searchsortedlast() (in module Base), 467	sizeof() (in module Base), 323, 385

skip() (in module Base), 451	strftime() (in module Libc), 487
skipchars() (in module Base), 452	stride() (in module Base), 394
sleep() (in module Base), 406	strides() (in module Base), 394
slice() (in module Base), 397	string() (in module Base), 385
slicedim() (in module Base), 397	stringmime() (in module Base), 457
sort	strip() (in module Base), 388
() (in module Base), 466	strptime() (in module Libc), 487
sort() (in module Base), 466, 467	strwidth() (in module Base), 389
sortcols() (in module Base), 467	sub() (in module Base), 397
sortperm	sub2ind() (in module Base), 394
() (in module Base), 467	subtypes() (in module Base), 322
sortperm() (in module Base), 467	success() (in module Base), 326
sortrows() (in module Base), 467	SuiteSparse_long 415
sparse() (in module Base), 402	sum
sparsevec() (in module Base), 402, 403	() (in module Base), 339
spawn() (in module Base), 326	sum() (in module Base), 339
spdiagm() (in module Base), 403	sum_kbn() (in module Base), 401
speye() (in module Base), 403	sumabs
splice	() (in module Base), 339
() (in module Base), 348	sumabs() (in module Base), 339
split() (in module Base), 388	sumabs2
splitdir() (in module Base), 446	() (in module Base), 340
splitdrive() (in module Base), 446	sumabs2() (in module Base), 339, 340
splitext() (in module Base), 446	summary() (in module Base), 453
spones() (in module Base), 403	supertype() (in module Base), 322
sprand() (in module Base), 403	svd() (in module Base), 421
sprandbool() (in module Base), 403	sydfact
sprandn() (in module Base), 403	() (in module Base), 421
sprint() (in module Base), 454	svdfact() (in module Base), 420, 421
spzeros() (in module Base), 403	svds() (in module Base), 427
sqrt() (in module Base), 364	svdvals
sqrtm() (in module Base), 425	() (in module Base), 421
squeeze() (in module Base), 399	svdvals() (in module Base), 421
srand() (in module Base), 383	syconv
StackOverflowError() (in module Base), 331	() (in module Base.LinAlg.LAPACK), 437
start() (in module Base), 335	syev
startswith() (in module Base), 388	() (in module Base.LinAlg.LAPACK), 438
stat() (in module Base), 444	syevr
status() (in module Base.Pkg), 472	() (in module Base.LinAlg.LAPACK), 438
std() (in module Base), 368	sygvd
STDERR (in module Base), 449	() (in module Base.LinAlg.LAPACK), 438
STDIN (in module Base), 449	sylvester() (in module Base), 425
stdm() (in module Base), 368	symbol() (in module Base), 390
STDOUT (in module Base), 449	symdiff
stebz	() (in module Base), 345
() (in module Base.LinAlg.LAPACK), 437	symdiff() (in module Base), 345
stegr	symlink() (in module Base), 443
() (in module Base.LinAlg.LAPACK), 437	symm
stein	() (in module Base.LinAlg.BLAS), 430
() (in module Base.LinAlg.LAPACK), 437	symm() (in module Base.LinAlg.BLAS), 430
step() (in module Base), 341	Symmetric{Float64,SparseMatrixCSC{Flaot64,SuiteSparse_long}},(
stev	(in module Base), 415
() (in module Base.LinAlg.LAPACK), 437	Symmetric{Float64,SparseMatrixCSC{Float64,SuiteSparse_long}},(
strerror() (in module Libc), 487	(in module Base), 415, 416

symperm() (in module Base), 403	transpose
SymTridiagonal() (in module Base), 422	() (in module Base), 425
symv	transpose() (in module Base), 425
() (in module Base.LinAlg.BLAS), 430	trcon
symv() (in module Base.LinAlg.BLAS), 430	() (in module Base.LinAlg.LAPACK), 437
syr	trevc
() (in module Base.LinAlg.BLAS), 428	() (in module Base.LinAlg.LAPACK), 437
syrk	trexc
() (in module Base.LinAlg.BLAS), 429	() (in module Base.LinAlg.LAPACK), 439
syrk() (in module Base.LinAlg.BLAS), 429	Tridiagonal() (in module Base), 422
Sys.get_process_title() (in module Base), 327	trigamma() (in module Base), 366
Sys.set_process_title() (in module Base), 327	tril
SystemError() (in module Base), 331	() (in module Base), 422
systemerror() (in module Base), 483	tril() (in module Base), 422
sysv	triu
() (in module Base.LinAlg.LAPACK), 438	() (in module Base), 421
sytrf	triu() (in module Base), 421
() (in module Base.LinAlg.LAPACK), 438	trmm
sytri	() (in module Base.LinAlg.BLAS), 430
() (in module Base.LinAlg.LAPACK), 438	trmm() (in module Base.LinAlg.BLAS), 430
sytrs	trmv
() (in module Base.LinAlg.LAPACK), 438	() (in module Base.LinAlg.BLAS), 430
() (III IIIOddic Basc.LiliAig.LAi ACR), 436	trmv() (in module Base.LinAlg.BLAS), 430
T	trrfs
take	() (in module Base.LinAlg.LAPACK), 437
() (in module Base), 409	trsen () (in modulo Poso Lin Alg I A DACK) 420
take() (in module Base), 336	() (in module Base.LinAlg.LAPACK), 439
takebuf_array() (in module Base), 450	trsm
takebuf_string() (in module Base), 450	() (in module Base.LinAlg.BLAS), 430
tan() (in module Base), 358	trsm() (in module Base.LinAlg.BLAS), 430
tand() (in module Base), 358	trsv
tanh() (in module Base), 358	() (in module Base.LinAlg.BLAS), 431
Task() (in module Base), 405	trsv() (in module Base.LinAlg.BLAS), 431
task_local_storage() (in module Base), 405	trsyl
tempdir() (in module Base), 445	() (in module Base.LinAlg.LAPACK), 439
tempname() (in module Base), 445	trtri
test() (in module Base.Pkg), 472	() (in module Base.LinAlg.LAPACK), 437
TextDisplay() (in module Base), 458	trtrs
tgsen	() (in module Base.LinAlg.LAPACK), 437
() (in module Base.LinAlg.LAPACK), 439	trues() (in module Base), 395
throw() (in module Base), 329	trunc() (in module Base), 362
tic() (in module Base), 328	truncate() (in module Base), 452
time() (in module Base), 328	tryparse() (in module Base), 377
time() (in module Libc), 487	tuple() (in module Base), 321
time_ns() (in module Base), 328	TypeError() (in module Base), 331
timedwait() (in module Base), 409	typeintersect() (in module Base), 324
Timer() (in module Base), 331	typejoin() (in module Base), 324
TmStruct() (in module Libc), 487	typemax() (in module Base), 323
toc() (in module Base), 328	typemin() (in module Base), 322
toq() (in module Base), 328	typeof() (in module Base), 321
touch() (in module Base), 445	tzrzf
trace() (in module Base), 423	() (in module Base.LinAlg.LAPACK), 433
trailing_ones() (in module Base), 381	
trailing_zeros() (in module Base), 381	

U
ucfirst() (in module Base), 388
UndefRefError() (in module Base), 331
UndefVarError() (in module Base), 331
unescape_string() (in module Base), 390
union
() (in module Base), 345
union() (in module Base), 345
unique() (in module Base), 337, 338
unlock() (in module Base), 406
unmark() (in module Base), 451
unsafe_convert() (in module Base), 481
unsafe_copy
() (in module Base), 482
unsafe_load() (in module Base), 482
unsafe_pointer_to_objref() (in module Base), 482
unsafe_read() (in module Base), 451
unsafe_store
() (in module Base), 482
unsafe_trunc() (in module Base), 362
unsafe_write() (in module Base), 451
unshift
() (in module Base), 346
unsigned() (in module Base), 378
update() (in module Base.Pkg), 472
uperm() (in module Base), 444
uppercase() (in module Base), 388
using, 133
utf16() (in module Base), 390
utf32() (in module Base), 390, 391
utf8() (in module Base), 385, 386
V
Val{c}() (in module Base), 324
valtype() (in module Base), 344
values() (in module Base), 344
var() (in module Base), 368
varm() (in module Base), 368
vcat() (in module Base), 397
vec() (in module Base), 399
vecdot() (in module Base), 413
vecnorm() (in module Base), 423
VERSION (in module Base), 441
versioninfo() (in module Base), 319
10/
W
wait() (in module Base), 408
walkdir() (in module Base), 443
warn() (in module Base), 454
watch_file() (in module Base), 460
which() (in module Base), 319
whos() (in module Base), 317
V \

widemul() (in module Base), 368

widen() (in module Base), 322 withenv() (in module Base), 328 WORD\_SIZE (in module Base), 441 workers() (in module Base), 407 workspace() (in module Base), 320 write() (in module Base), 450 writecsv() (in module Base), 455 writedlm() (in module Base), 455 writemime() (in module Base), 457 wstring() (in module Base), 391

## X

xcorr() (in module Base), 373 xdump() (in module Base), 454

## Y

yield() (in module Base), 405 yieldto() (in module Base), 405

## Ζ

zero() (in module Base), 378 zeros() (in module Base), 395 zeta() (in module Base), 367 zip() (in module Base), 335