5001 Mini-Project

ZHANG Juntao - 20908272

December 10, 2022

1 Problem 1: Ordinary differential equation

From the known information, we have the following:

$$\frac{dT(t)}{dt} = -T(t) \text{ and } T(0) = 1$$

Then,

$$\frac{dT(t)}{T(t)} = -d(t) \Rightarrow d\ln T(t) = -dt \Rightarrow \ln T(t) = -t + C_1$$

So we get,

$$T(t) = ce^{-t}$$

From $T(0) = 1$ we can get $c = 1$. Therefore, we have the theoretical solution of this ODE:

$$T(t) = e^{-t}$$

For numerical solution:

$$\begin{aligned}
&T(\Delta t) - T(0) = -\Delta t T(0) \Rightarrow T(\Delta t) = 1 - \Delta t \\
&T(2\Delta t) - T(\Delta t) = -\Delta t T(\Delta t) \Rightarrow T(2\Delta t) = (1 - \Delta t)^2 \\
&T(3\Delta t) - T(2\Delta t) = -\Delta t T(2\Delta t) \Rightarrow T(3\Delta t) = (1 - \Delta t)^3 \\
&\vdots \\
&T(n\Delta t) - T(n\Delta t - \Delta t) = -\Delta t T(n\Delta t - \Delta t) \Rightarrow T(n\Delta t) = (1 - \Delta t)^n
\end{aligned}$$

Setting $\Delta t = 0.01$, $t = 10$, so we discrete 10 seconds into 1000 points. Through the above numerical solution, we can get the numerical results of $T(t)$ from the simulation and compare the theoretical and numerical results of $T(t)$, as Figure 1 shows.

From Figure 1, we can see that the numerical result perfectly agrees with the theoretical result.

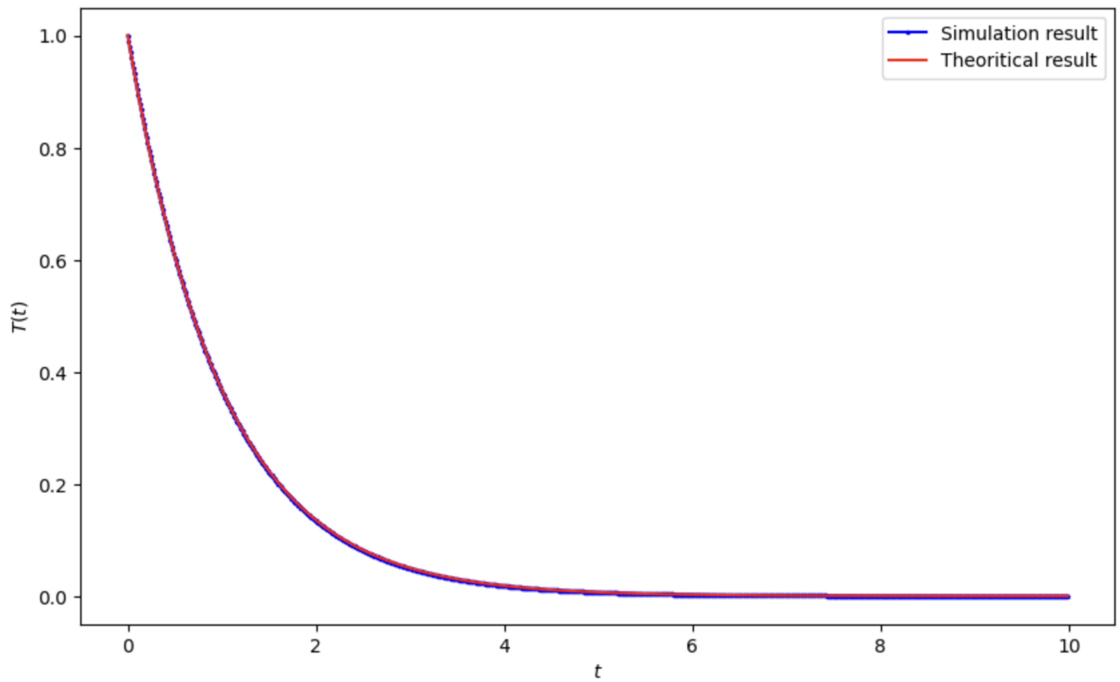

Figure 1: Numerical and theoretical results of $T(t)$

2 Problem 2: Partial differential equation

Two-dimensional heat conduction equation:

$$\frac{\partial T}{\partial t} = \kappa(\partial_x^2 T + \partial_y^2 T)$$

To numerical solve this PDE, we firstly set $\Delta x = \Delta y = \Delta h$, then:

$$\partial_x^2 T = \frac{T(x+\Delta x, y; t) + T(x-\Delta x, y; t) - 2T(x, y; t)}{\Delta x^2}$$
$$\partial_y^2 T = \frac{T(x, y+\Delta y; t) + T(x, y-\Delta y; t) - 2T(x, y; t)}{\Delta y^2}$$

$$\frac{\partial T}{\partial t} = \kappa \frac{T(x-\Delta h, y; t) + T(x+\Delta h, y; t) + T(x, y-\Delta h; t) + T(x, y+\Delta h; t) - 4T(x, y; t)}{\Delta h^2} = \alpha$$

Therefore,

$$T(x, y; t+\Delta t) = T(x, y; t) + \alpha \Delta t$$

In the simulation, we set $\Delta h = 0.5$, $\kappa = 0.5$, $\Delta t = 0.1$, the lengths of x and y are both 50, which means 100 steps in both x and y . And the length of t is 600, which means 6000 steps in time dimension.

And I use "numba" package to implement parallel computing.

After numerical solving this PDE, plot the temperature distribution in 2-D system accounding to the time, as Figure 2 shows.

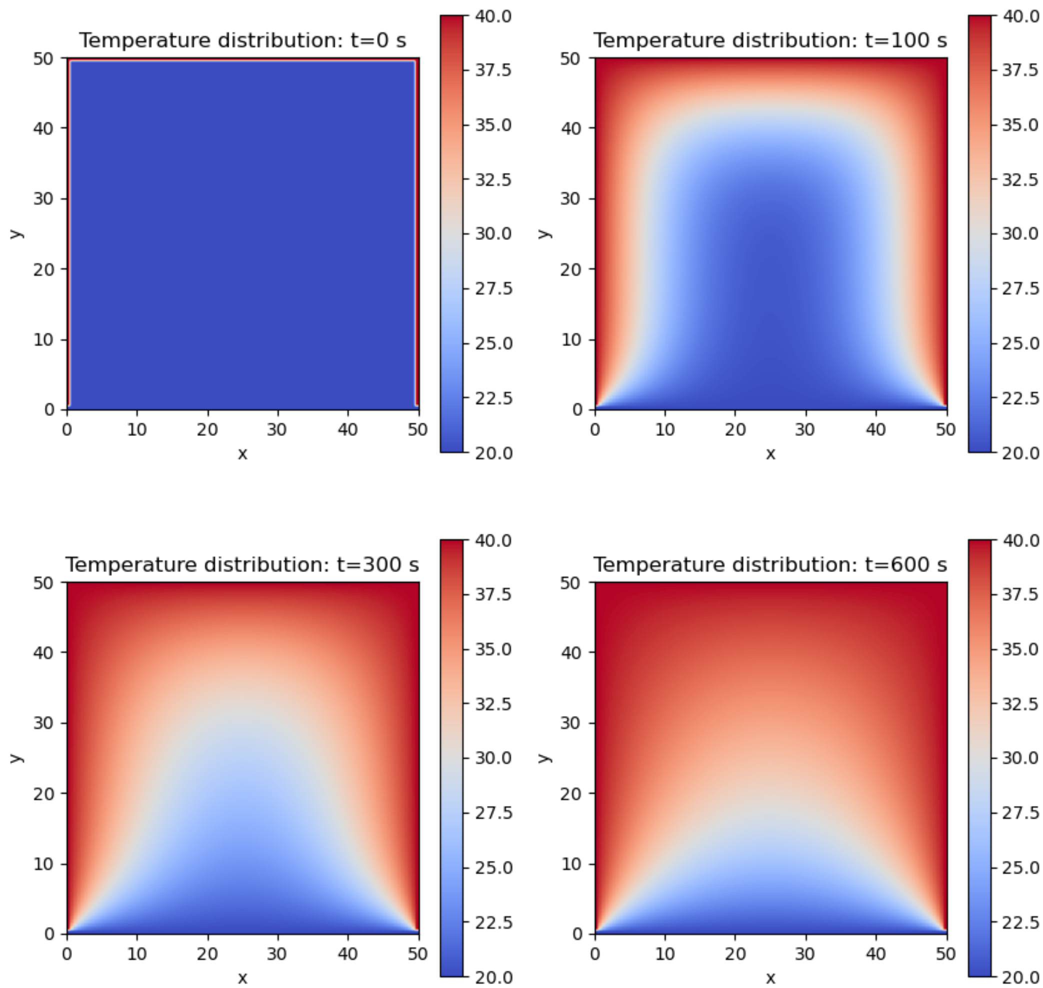

Figure 2: Temperature distribution with t=0s, t=100s, t=300s and t=600s

As we can see from Figure 2, in the begining of this 2-D system, only three slides with 40 degrees Celcius, all other places are with 20 degrees Celcius. As time increases, the temperature is continuously conducted inward from the three heat source sides, so that the "red" region of this 2-D system is getting bigger.

Finally, change the number of processes involved in the above 2D calculation and evaluate speedup and efficiency. Setting the number of process from 1 to 8, and calculating the time cost respectively. Then calculate the speedup and efficiency, and get the result as Figure 3 shows.

From Figure 3, we can see that the speedup and efficiency don't show the result as we expected. And I tried many times, the result may occur significant difference, and under most circumstances, 2-process has the best performance in speedup and efficiency. I don't know why these happen, but anyway, 2-8 processes can all speedup, i.e. cost less time than only one process.

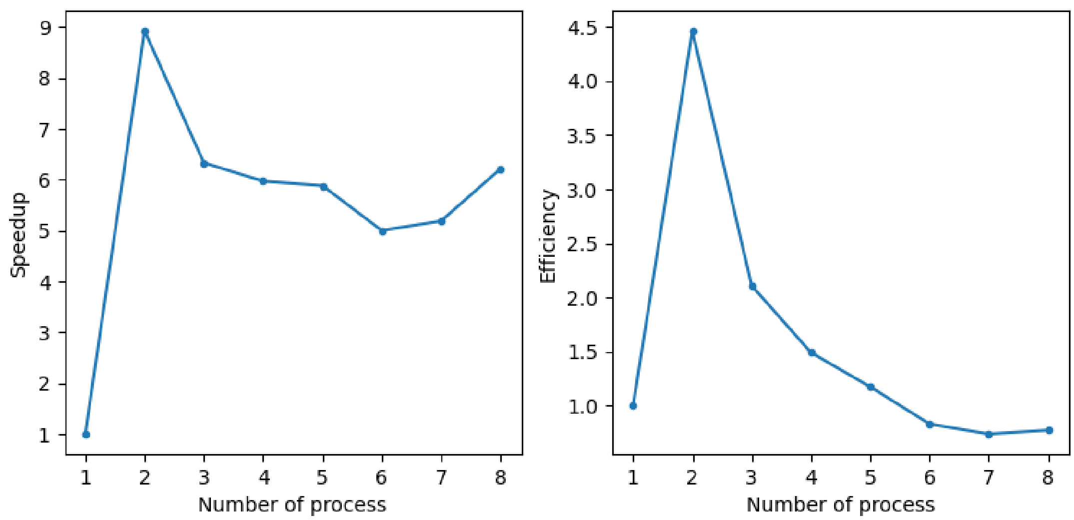

Figure 3: Speedup and Efficiency with the number of process