Application of Quantum Annealing in Optimizing Trading Trajectories

ZHANG Juntao - 20908272

November 23, 2022

Abstract

Quantum annealing is a sub-empirical algorithm with quantum fluctuation characteristics, which can find the global optimal solution when the objective function has multiple sets of candidate solutions. Quantum annealing has applications in many different fields, we will describe its application in optimizing trading trajectories.

1 Description

Managers of large portfolios typically need to optimize their portfolios over a multiple-period horizon. So "The Optimal Trading Trajectory Problem" comes, the model can be written as Figure 1 shows. In this problem: (i) A sequence of single-period optimal positions is rarely multi-period optimal; (ii) Rebalancing the portfolio to align with every single period's optimal weight is typically prohibitively expensive. As the number of assets in the portfolio increases, the computational difficulty increases exponentially, the problem is intractable even for the world's most powerful supercomputers.

$$w = \operatorname{argmax}_w \left\{ \sum_{t=1}^{T} \left(\underbrace{\mu_t^T w_t}_{\text{returns}} - \frac{\gamma}{2} \underbrace{w_t^T \Sigma_t w_t}_{\text{risk}} - \underbrace{\Delta w_t^T \Lambda_t \Delta w_t}_{\substack{\text{direct costs,}\\ \text{temp.impact}}} - \underbrace{\Delta w_t^T \Lambda_t' \Delta w_t}_{\text{perm.impact}} \right) \right\}$$

s.t.:

mean-variance portfolio optimization

$$\forall t: \sum_{n=1}^{N} w_{n,t} \leq K \, ; \forall t, \forall n: w_{n,t} \leq K'$$

transaction cost and market impacts

Figure 1: The multi-period integer optimization problem

The application of quantum annealing in solving this problem shows advantages. Firstly, convert the multi-period integer optimization problem to standard QUBO(Quadratic Unconstrained Binary Optimization) form.

$$\min_x x^T Q x$$
$$x \in \{0,1\}^N, Q \in \mathbb{R}^{N \times N}$$

Figure 2: Standard QUBO form

Then through quantum annealing, manipulate the environment around the system to let the device find the ”ground state”, which corresponds to the optimal portfolio selection.

In the completed experiments, the experimental procedure is as follows:

(i) Generate a random problem: number of assets; time horizon; and total investable assets.

(ii) Solve using quantum annealer.

(iii) Find exact minimum solution using an exhaustive search.

(iv) Evaluate performance by how far the quantum solution is from the exact solution

And the experimental results:

The quantum annealer solution is typically within a small and acceptable margin of error of the exact globally minimal solution. What’s more, quantum annealer provides higher speed than the classic computer.