§1 Problems

Please justify your answers to these problems.

Exercise 1.1. Find all the roots of the following polynomials:

(a) $f(x) = 2x^4 - 6x^3 - 12x^2 + 16x$

(b) $g(t) = t^5 + t^4 - 6t^3 - 14t^2 - 11t - 3$

(c) $h(y) = 30y^3 + 11y^2 - 4y - 1$

(d) $p(x) = 25x^4 + 55x^3 - 192x^2 - 44x + 16$

(You may want to use a calculator for this last one.)

Exercise 1.2. Without trying to find its roots, explain why the polynomial $f(x) = 3x^4 + 5x^3 + 7x^2 + 4$ has no positive roots.

Exercise 1.3. Consider the polynomial $f(x) = x^4 - 12x^3 + 54x^2 - 108x + 81$. Notice that $f(3) = 0$, but no other factor of 81 is a root of f. Would it be correct to assume that f has no other integer (or rational) roots? Why or why not?

Exercise 1.4. Let $f(x) = 6x^3 + 25x^2 + 2x - 8$. Find the quotient and remainder when dividing f by $x - 1$. Explain how the result you get shows that there are no roots of f greater than 1. Then, find all the roots of f.