Mean atomic weight of Earth, Moon, Venus, Mercury and Mars. Effect of mass of cores and density of planets

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MEAN ATOMIC WEIGHT OF EARTH, MOON, VENUS, MERCURY AND MARS. EFFECT OF MASS OF CORES AND DENSITY OF PLANETS. M Szurgot, Lodz University of Technology, Center of Mathematics and Physics, Al. Politechniki 11, 90 924 Lodz, Poland, (mszurgot@p.lodz.pl).

Introduction: Knowledge of mean atomic weight is useful to characterize terrestrial and extraterrestrial minerals and rocks, planets, moons, asteroids and comets. The aim of the paper was to determine and analyze mean atomic weight of Earth, Venus, Mars, Mercury and Earth's Moon. Literature data on the mean bulk elemental/oxide composition of the solar system internal planets and the Moon, as well as their constituents have been used to calculate mean atomic weight *Amean* values. Apart from the whole planets atomic weights, also mean atomic weights of their cores, mantles and crusts were determined.

Results and discussion: Tables 1 and 2 collect values of *Amean* calculated using data on chemical composition predicted by various researchers (e.g. [1-9]) in their models of planets and moon. To determine *Amean* values for moon, planets, their cores, mantles and crust following relationships were used:

$$Amean = 1/\sum(wi/Ai), \quad (1)$$

$$Amean=\sum(\mathrm{Ni/N})\cdot Ai), \quad (2)$$

$$wcore/Acore + wmantle/Amantle + wcrust/Acrust= =1/Amean, \quad (3)$$

$$wcore/Acore + wbsp/Absp = 1/Amean, \quad (4)$$

where *wi*(wt%) ia a mass fraction of *i* element/oxide, *Ni/N*(at%) is an atomic fraction of *i* element/oxide, *Ni is a* number of *i* atoms, N is a number of all atoms N = ∑Ni, *Ai* is the atomic weight of a given element, and/or a mean atomic weight of oxide. *wcore*(wt%), *wmantle*(wt%), *wcrust*(wt%) represent mass fraction of core, mantle and crust of a given planet or moon, *wbsp* is a mass fraction of a silicate part of a planet (bsp = BSP = Bulk Silicate Planet = Mantle + Crust), and *Acore*, *Amantle*, *Acrust*, *Absp* are mean atomic weights of cores, mantles, crusts and silicate parts of planets and moon. In addition, following relationships are valid:

$$\sum wi =1, \quad (5)$$

$$\sum(Ni/N) = 1, \quad (6)$$

$$wcore + wmantle + wcrust =1, \quad (7)$$

$$wcore + wbsp = 1. \quad (8)$$

Mean atomic weight is dimensionless, similary as relative atomic mass.

The comparison of *Amean* values determined in this paper with values established by other researchers reveals a satisfactory agreement (Tables 3, 4). The agreement is good not only for the whole planets and the moon (Table 3), but also for Earth's core and mantle (Table 4).

Table 1 Mean atomic weight *Amean* ± SD of the whole planets and the moon, their cores and bulk silicates (BSP = mantle+crust).

Planet	Global	Core	BSP
Mercury	35.8 ± 0.5	53.5 ± 3.5	21.1 ± 0.5
Venus	25.8 ± 0.6	51.2 ± 6.6	21.1 ± 0.9
Earth	26.5 ± 0.6	50.4 ± 2.1	21.1 ± 0.1
Moon	21.8 ± 0.4	50.3 ± 3.7	21.5 ± 0.4
Mars	25.2 ± 0.1	50.9 ± 1.0	22.1 ± 0.1
Mean		*51.3 ± 1.3*	*21.4 ± 0.4*

Table 2 Mean atomic weight of mantles and crusts of planets and the moon.

Planet	Mantle	Crust
Mercury	20.9 ± 0.2	21.0 ± 0.7
Venus	n.d.	21.9 ± 0.3
Earth	21.2 ± 0.3	21.3 ± 0.4
Moon	21.9 ± 0.4	21.7
Mars	n.d.	22.2 ± 0.4

n.d. = not determined

Table 3 Comparison of values of *Amean* of the whole planets and the moon determined in this paper (S15, and eq.(11)) with *Amean* values determined by other researchers: Anderson and Kovach [10] (AK67) using seismic data, and by Ringwood [11] (R66).

Planet	S15	eq.(11)	AK67	R66
Mercury	35.8 ± 0.5	35.5	36	
Venus	25.8 ± 0.6	26.3	26.4	
Earth	26.5 ± 0.6	26.4	27	25.8
Moon	21.8 ± 0.4	21.7	22	
Mars	25.2 ± 0.1	24.5	25.3	

Table 4 Comparison of *Amean* values of Earth's core and mantle with values determined by Anderson & Kovach (AK67), Anderson [1] (A89), and Birch [12] (B61).

	S15	AK67	A89	B61
Core	50.4±2.1	47	50 ± 4	47
Mantle	21.2±0.3	22.4±0.5	22.4 22 ± 1	22.5±0.5

Table 1 reveals that the matter forming cores, and bulk silicates exhibit close values of their mean atomic weights. For cores of four inner planets and Moon we have *Amean* = 51.3 ± 1.3, and for their bulk silicates *Amean* = 21.4 ± 0.4. Only Mercury's core *Amean* is somewhat higher (*Amean* = 53.5 ± 3.5) than that of Earth, Venus and Mars, for which *Amean* = 51.5 ±1.4.

Knowledge of mean atomic weight of terrestrial planets and moon enables one searching for various factors determining *Amean* and useful dependiences. Important relationships have been established.

First relationship is between mean atomic weight of planets and uncompressed density $d(g/cm^3)$ of planets (Fig. 1). It can be expressed by the equation (R^2 = 0.99, RMSE = 0.54)

$$Amean = (7.51\pm0.13)\cdot d + (-2.74\pm0.55). \quad (9)$$

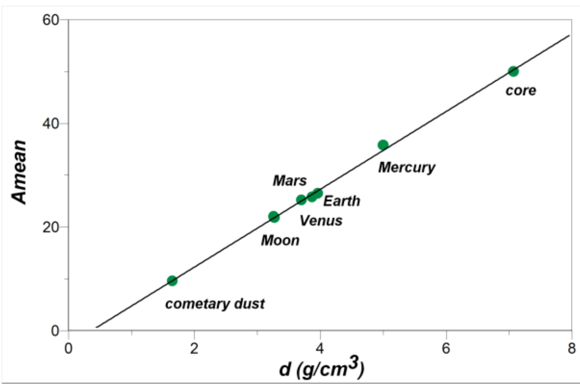

Fig. 1 Relationship between mean atomic weight and uncompressed density of a planet. *Amean* of dust of P/Halley comet, and *Amean* of planetary core material are also inserted.

Second relationship is between mean atomic weight of planets and mass fraction of their cores (Fig. 2, eqs. (10), (11)). The core mass fraction *wcore* = *Mcore/Mplanet,* where *Mcore* is the mass of core, and *Mplanet is* the mass of a planet. The relationship can be expressed by two fits.

First is the empirical equation (with RMSE = 0.74):

$$Amean = (23.8\pm2.8)\cdot wcore^{2}+(5.6\pm2.9)\cdot wcore +(21.9\pm0.5). \quad (10)$$

According to eq. (10) *for wcore* = 1 (planet without silicates) we have *Amean* = 51.3 ± *6.2* (range 45.1-57.5) and for *wcore* = 0 (planet without a metallic core) we have *Amean* = 21.9 ± *0.5 (range 21.4-22.4).* For the Earth *wcore = 0.325* so *Amean = 26.2.*

The second equation presenting effect of mass fraction of core *wcore* on *Amean* follows from eqs. (4) and (8). Substituting *wbsp*=1-*wcore*, and mean values: *Abse* = 21.4, and *Acore* = 51.3 into eq. (4) gives the equation:

$$Amean = 1/ [-0.02724\cdot wcore + 0.04673], \quad (11)$$

where coefficient 0.02724 = (1/*Absp*) - (1/*Acore*), and coefficient 0.04673 = 1/*Absp*. *Amean* values determined by eq. (11) are listed in third column of Table 3. The following values of *wcore* were used in these calculations: 0.68 for Mercury, 0.32 for Venus, 0.325 for Earth, 0.025 for Moon, and 0.217 for Mars.

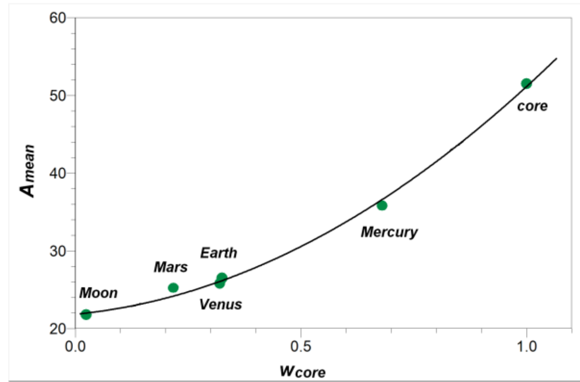

Fig. 2 Relationship between mean atomic weight of a planet and mass fraction of the core.

The reverse effects: the effect of *Amean* on *wcore*, and effect of *Amean* on *d* have been also established. They are given by the equations:

$$wcore = (-36.716 \pm 1.15)/Amean + (1.716\pm0.041), \quad (12)$$

$$d = (0.133 \pm 0.002)\cdot Amean + (0.37 \pm 0.07), \quad (13)$$

which results from eqs. (11) and (9). Values of RMSE: 0.03 for eq.(12), and 0.07 for eq. (13)). So we are able to predict *wcore* and *d*, if *Amean* is known.

Conclusions: Mean atomic weight of inner planets and the Moon can be determined using chemical composition, planetary uncompressed density, and/or mass of core. Mass fraction of cores and uncompressed planetary density can be predicted by *Amean* data.

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