Calculus 2

Orthogonality

1. Don't forget to substitute c
2. Don't forget the c of integration

Differecnial Equations

Linear First order differencial Equation

$$y' + p(t)y = g(t)$$

$$u(t)y' + u(t)p(t)y = u(t)g(t)$$

$$\ln u(t) = \int p(t)dt$$

$$u(t)y = \int g(t)u(t)dt + c$$

Homogenius First Order Linear Equations

$$\frac{dy}{dx} = F(\frac{y}{x})$$

$$let \;\; v = \frac{y}{x} \implies \frac{dy}{dx} = F(v) = v + x\frac{dv}{dx}$$

$$\frac{dv}{F(v) - v} = \frac{dx}{x}$$

Exact First Order Differeccial Equations

$$M(x,y)dx + N(x,y)dy = 0$$

$$M_y = N_x$$

$$\int M(x,y)dx + \int N(y)dy = c$$

Choose as $\mu(n)$

$$\frac{d\mu}{dx} = \frac{N_x - M_y}{M}\mu$$

Secound Order Homogenius Differecncial Equations

$$ay'' + by' + c = 0$$

$$y = c_1e^{r_1t} + c_2e^{r_2t}$$

$$y = c_1e^{r_1t} + c_2te^{r_1t}$$

$$y = e^{\alpha t}(c_1cos(\beta t) + c_2sin(\beta t))$$

$$\alpha + -\beta i$$

Euler Equations

$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0.$$

Secound Order Nonhomogenius Variation of Parameters

$$y'' + p(t)y' + q(t)y = g(t)$$

$$u_1 = -\int \frac{y_2\ g}{W(y_1, y_2)}dt \qquad u_2 = \int \frac{y_1\ g}{W(y_1, y_2)}dt$$

$$Y = y_1u_1 + y_2u_2$$

$$y = c_1y_1(t) + c_2y_2(t) + Y(t)$$

Series

The nth-Term Test

$$a_n \nrightarrow 0\ \ or\ \ d.n.e \implies \sum_{n=1}^{\infty} a_n\ \ diverges$$

Bounding

$$\sum_{n=1}^{\infty} a_n < \infty \quad a_n \geq 0 \iff S_n \leq bound$$

Integral Test

$a_n \geq 0 \quad a_n = f(n) \quad f(m+s) < f(m)$

$$\sum_{n=N}^{\infty} a_n \to c,\ \infty = \int_N^{\infty} f(x)dx \to c,\ \infty$$

Cauchy condesation test

$\{a_n\} \quad a_n \geq a_{n+1} \quad \lim_{n\to\infty}\{a_n\} = 0$

$$\sum a_n \to c \iff \sum 2^n a_{2^n} \to k$$

Direct Comparison Test

$\sum a_n \quad \sum b_n \quad 0 \leq a_n \leq b_n$

1. $\sum b_n \to c \implies \sum a_n \to k$
2. $\sum a_n \to \infty \implies \sum b_n \to \infty$

Limit Comparsison Test

$a_n > 0 \quad b_n > 0 \quad n \geq N$

1. $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = c \quad c > 0 \implies \sum a_n, \sum b_n \to c, \infty$
2. $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = 0 \quad \sum b_n \to c \implies \sum a_n \to k$

3. $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty \quad \sum b_n \to \infty \implies \sum a_n \to \infty$

2. a_n grows slower then a convergent
3. a_n grows faster then a divergent

Absolutelly/Conditionally convergent test

$$\sum |a_n| \to 0 \implies \sum a_n \to 0 \quad absolutely \quad convergent$$

$$\sum |a_n| \to \infty \ \ but \ \ \sum a_n \to 0 \quad Conditionally \quad convergent$$

The ratio test

$\sum a_n$

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = p$$

$p < 1$ converges absolutely
$p = 1$ inconclusive
$p > 1$ diverges

The root test

$\sum a_n$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = p$$

$p < 1$ converges absolutely
$p = 1$ inconclusive
$p > 1$ diverges

The Alternating Series Test

1. $a_n \geq 0$
2. $a_n \geq a_{n+1} \quad n \geq N$
3. $a_n \to 0$

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n \to c$$

The Alternating Series Extimation Theorem

$$\sum_{n=1}^{\infty} (-1)^{n+1} a_n \to L$$

$$L - S_n = (-1)^n a_{n+1} + (-1)^{n+1} a_{n+2} \ldots..$$

$$|L - S_n| < a_{n+1}$$

Rrearrangment Theorem for Absolute Convergent

> We can rearange absolute convergent series and theire sum is the same

Tailor Series

$$\sum_{n=0}^{\infty} \frac{f^n\,(a)}{n!}\,(x-a)^n$$

Fourier Series

Special Integrals

$$\int_{-\pi}^{\pi} \cos^2(x) = \pi \;\; \int_{-\pi}^{\pi} \sin^2(x) = \pi \;\; \int_{-\pi}^{\pi} \cos(x) = 0 \;\; \int_{-\pi}^{\pi} \sin(x) = 0$$

$$\int_{-\pi}^{\pi} \sin\,(px) \sin\,(qx) dx = \begin{cases} 0, & p \neq q \\ \pi, & q = p \end{cases}$$

$$\int_{-\pi}^{\pi} \sin\,(px) \cos\,(qx) dx = 0$$

$$\int_{-\pi}^{\pi} \cos\,(px) \cos\,(qx) dx = \begin{cases} 0, & p \neq q \\ \pi, & q = p \end{cases}$$

3 steps

$$f_n(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

$$\int_0^{2\pi} f(x) dx = \int_0^{2\pi} f_n(x) dx = a_0 x|_0^{2\pi} = 2\pi a_0$$

$$\int_0^{2\pi} f(x) \sin(kx) dx = \int_0^{2\pi} f_n(x) \sin(kx) dx = \pi b_k$$

$$\int_0^{2\pi} f(x) \cos(kx) dx = \int_0^{2\pi} f_n(x) \cos(kx) dx = \pi a_k$$

Final formula

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx$$

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin(kx) dx$$

$$a_k = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(kx) dx$$

$$f_n(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

Forier Series at the point of discontinuity c

$$f_n(c)_{n \to \infty} = \frac{f(c^+) - f(c^-)}{2}$$

Tricks:

$\sum k^2 = n(n+1)(2n+1)/6$
$n!/n^n \to 0$

$\sum 1(3)...(2n+1)/4^n 2^n n!$ use ratio test
In alternating series:

1. n'th terem test
2. $|a_n|$
3. Alternating series test
   At convergence of powerseries x=0 i kind of a special test when you check the radius of convergence
   Geometric series stats from 0
   $\int \frac{2}{x-1}\,dx = 2\ln|x-1| + C,$

   You can just put the series at the integral and find c

   $$\int \sec x dx = \ln|\sec x + \tan x| + c$$

   $$\int \tan x dx = \ln|\sec x| + c$$

   $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}(\frac{u}{a}) + c$$

   $$\int \frac{dx}{a^2 - x^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + c$$

   $$\int \frac{dx}{x\sqrt{a^2 - x^2}} = \frac{1}{a}\sec^{-1}(\frac{|x|}{a}) + c$$

Check:

1. geometric series
2. p series
3. limit
4. integral
5. telescoping
   Exercieses not done:
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