$$\varepsilon_0 = 8{,}86 \cdot 10^{-12} \frac{As}{Vm}, \qquad \frac{1}{2\pi\varepsilon_0} = 1{,}8 \cdot 10^{10} \frac{Vm}{As} \qquad \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^{9} \frac{Vm}{As}$$

Problem E2

There is thin metal sphere with the radius $R_1 = 0{,}1m$. This is surrounded by an insulator coating up to the radius $R_2 = 0{,}2m$ with relative permittivity $\varepsilon_r = 3$. The metal sphere contains a charge of $Q = 10^{-7} As$.

a./ Find and sketch the following functions: $E_{free}(r)$ and $E(r)$ in the same coordinate system, and find the numerical values in the three following breakpoints. $E_{free}(R_1)$, $E(R_1)$, $E(R_2)$.

b./Find the voltage of the metal sphere relive to the external surface of the insulator coating.

Solution:

a./

Gauss law in general: $\oint_{Any\cdot surface} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$ **Bold letters** mean vector

Gauss law in present case: $\oint_{Sphere} E_{free}(r) \cdot dA = \frac{Q}{\varepsilon_0}$ Thin letters mean magnitude

$$4r^2\pi \cdot E_{free}(r) = \frac{Q}{\varepsilon_0} \qquad E_{free}(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2}$$

Numerically:

$r < R_1$ $\qquad E(r) \equiv 0$

$R_1 \le r$ $\qquad E_{free}(r) = 9 \cdot 10^9 \frac{Vm}{As} \cdot 10^{-7} As \cdot \frac{1}{r^2} = 900Vm \cdot \frac{1}{r^2} \qquad \left[\frac{V}{m}\right]$

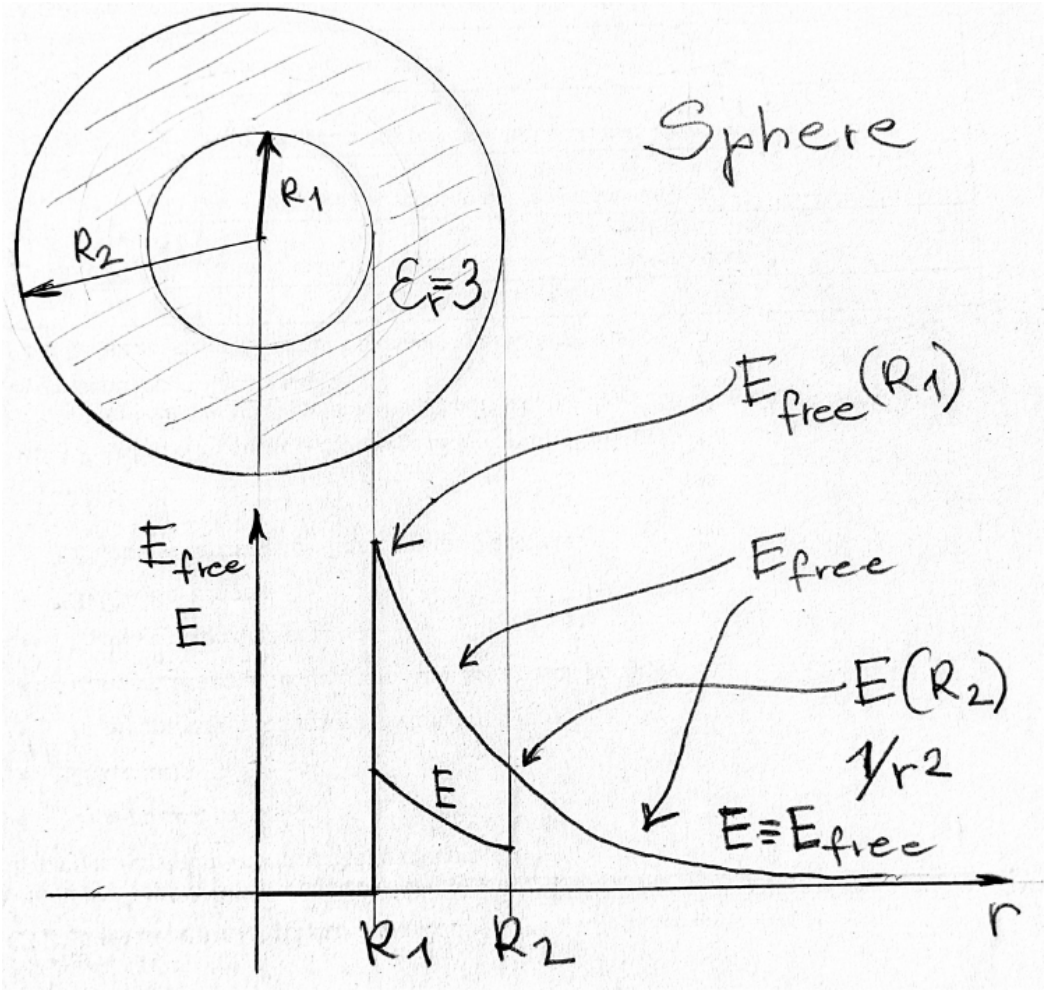
$R_1 \le r < R_2$ $\qquad E(r) = \frac{E_{free}(r)}{\varepsilon_r} = 300Vm \cdot \frac{1}{r^2} \qquad \left[\frac{V}{m}\right]$

$R_2 \le r$ $\qquad E(r) = 900Vm \cdot \frac{1}{r^2} \qquad \left[\frac{V}{m}\right]$

$$E_{free}(R_1) = 900Vm \cdot \frac{1}{R_1^2} = \frac{900Vm}{0{,}01m^2} = 90\frac{kV}{m}$$

$$E(R_1) = 300Vm \cdot \frac{1}{0{,}01m^2} = 30\frac{kV}{m}$$

$$E(R_2) = E_{free}(R_2) = 900Vm \cdot \frac{1}{0{,}04m^2} = 22{,}5\frac{kV}{m}$$



b./

$$U(R_1,R_2) = -\int_{R_2}^{R_1} E(r)dr = \int_{R_2}^{R_1} 300Vm \cdot \left(-\frac{1}{r^2}\right)dr = 300Vm \int_{R_2}^{R_1} \left(-\frac{1}{r^2}\right)dr = 300Vm \cdot \left[\frac{1}{r}\right]_{r=R_2}^{r=R_1}$$

$$U(R_1,R_2) = 300Vm \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 300\left(\frac{1}{0{,}1} - \frac{1}{0{,}2}\right) = 300(10-5) = 1500V$$