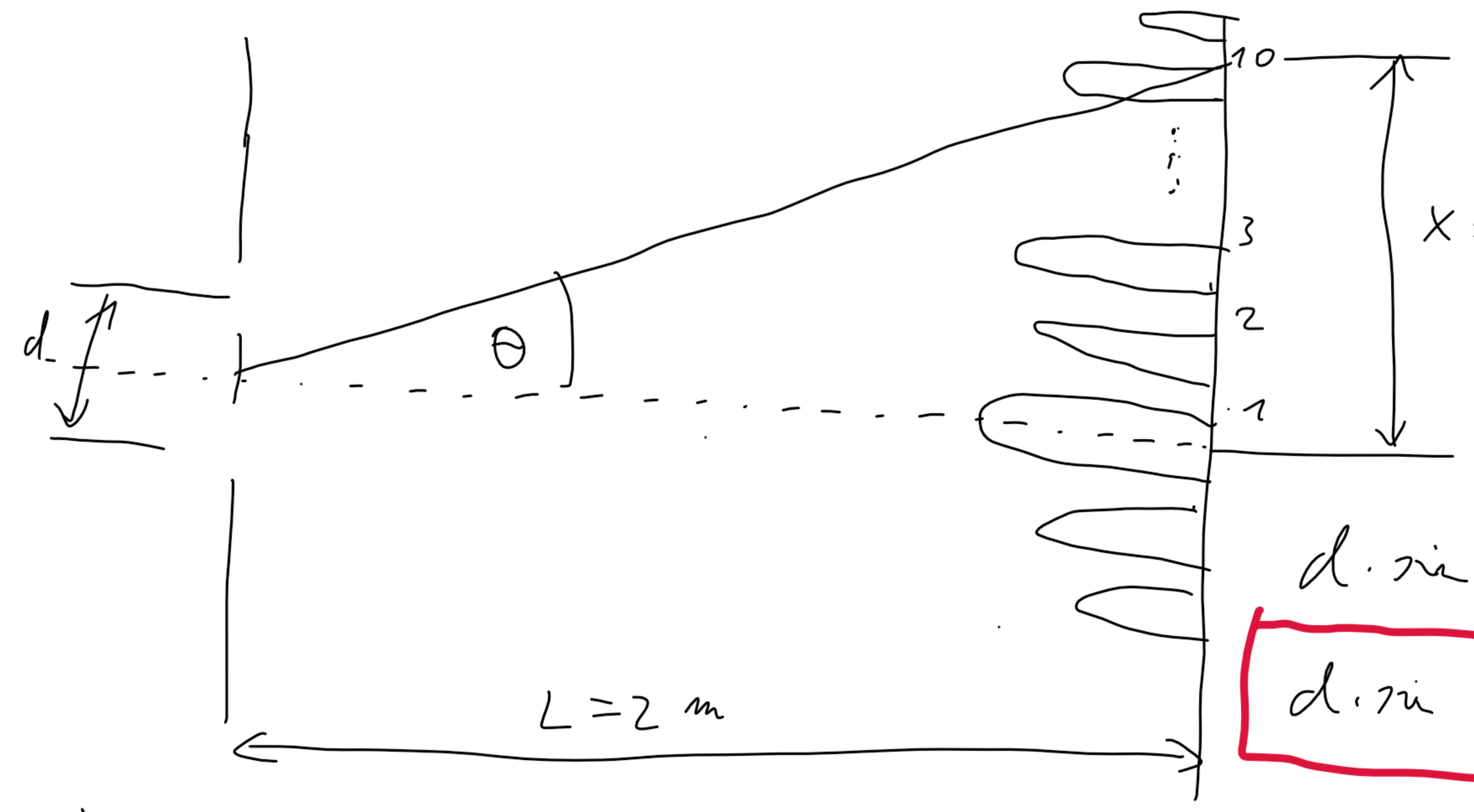
5. Young's double-slit experiment is performed with 589-nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.



$x = 7{,}26\ mm = 7{,}26 \cdot 10^{-3}\ m$

$m \in \mathbb{Z}$

$d \cdot \sin\theta = m\lambda$ BRIGHT

$\boxed{d \cdot \sin\theta = \left(m - \tfrac{1}{2}\right)\lambda}$ DARK.

$\lambda = 589\ nm = 5{,}89 \cdot 10^{-7}\ m$

PRECISE:

$tg\,\theta = \frac{x}{L} = \frac{7.26 \cdot 10^{-3}}{2} \Rightarrow \theta = 0.208^\circ$

$d = \frac{\left(m - \frac{1}{2}\right)\lambda}{\sin\theta} = 0.00154\ m$

$d = 1.54\ mm$

APPROXIMATION:

$tg\,\theta \approx \sin\theta$

$d \cdot \frac{x}{L} = \left(m - \frac{1}{2}\right)\lambda$

$d = \frac{\left(m - \frac{1}{2}\right)\lambda L}{x} = 0{,}00154\ m$

$d = 1{,}54\ mm$

32. A diffraction grating has 4 200 rulings/cm. On a screen 2.00 m from the grating, it is found that for a particular order m, the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.59 mm. Determine the value of m.

$d \sin\theta \simeq m\lambda$

$m \in \mathbb{Z}$

$$d = \frac{1\,\text{cm}}{4200} = \frac{10^{-2}}{4200} = 2,38\cdot 10^{-6}\ \text{m} = 2,38\ \mu\text{m}$$

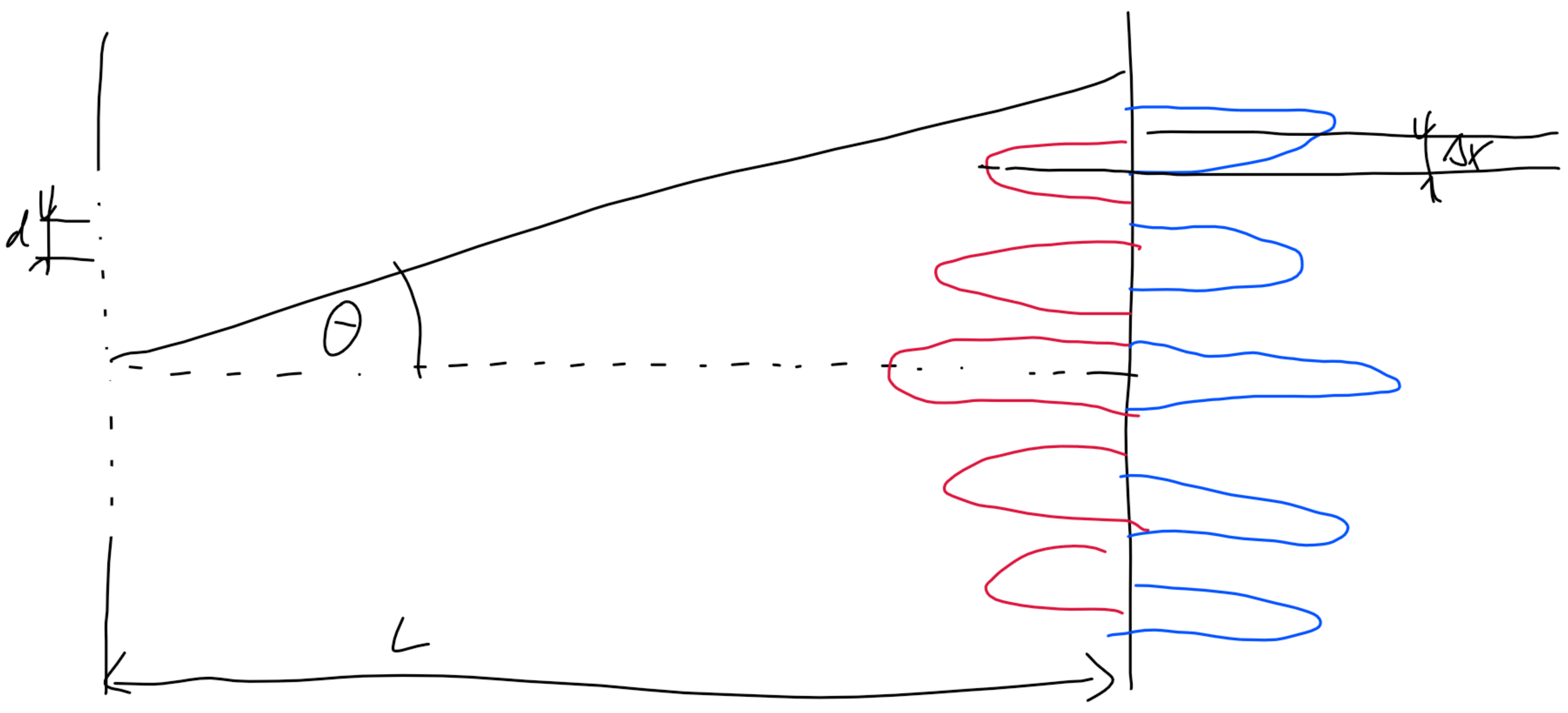
$$L = 2\ \text{m}$$

$$\lambda_1 = 585\ \text{nm} = 5,85\cdot 10^{-2}\ \text{m}$$

$$\lambda_2 = 589,6\ \text{nm} = 5,896\cdot 10^{-2}\ \text{m}$$

$$\Delta x = 1,59\ \text{mm} = 1,59\cdot 10^{-3}\ \text{m}$$

$$m = ?$$



$$d \sin\theta = m\lambda$$

$$m \in \mathbb{Z}$$

$$\theta \ll 1 \Rightarrow \sin\theta \approx \operatorname{tg}\theta = \frac{x}{L}$$

$$d\,\frac{x}{L} \approx m\lambda$$

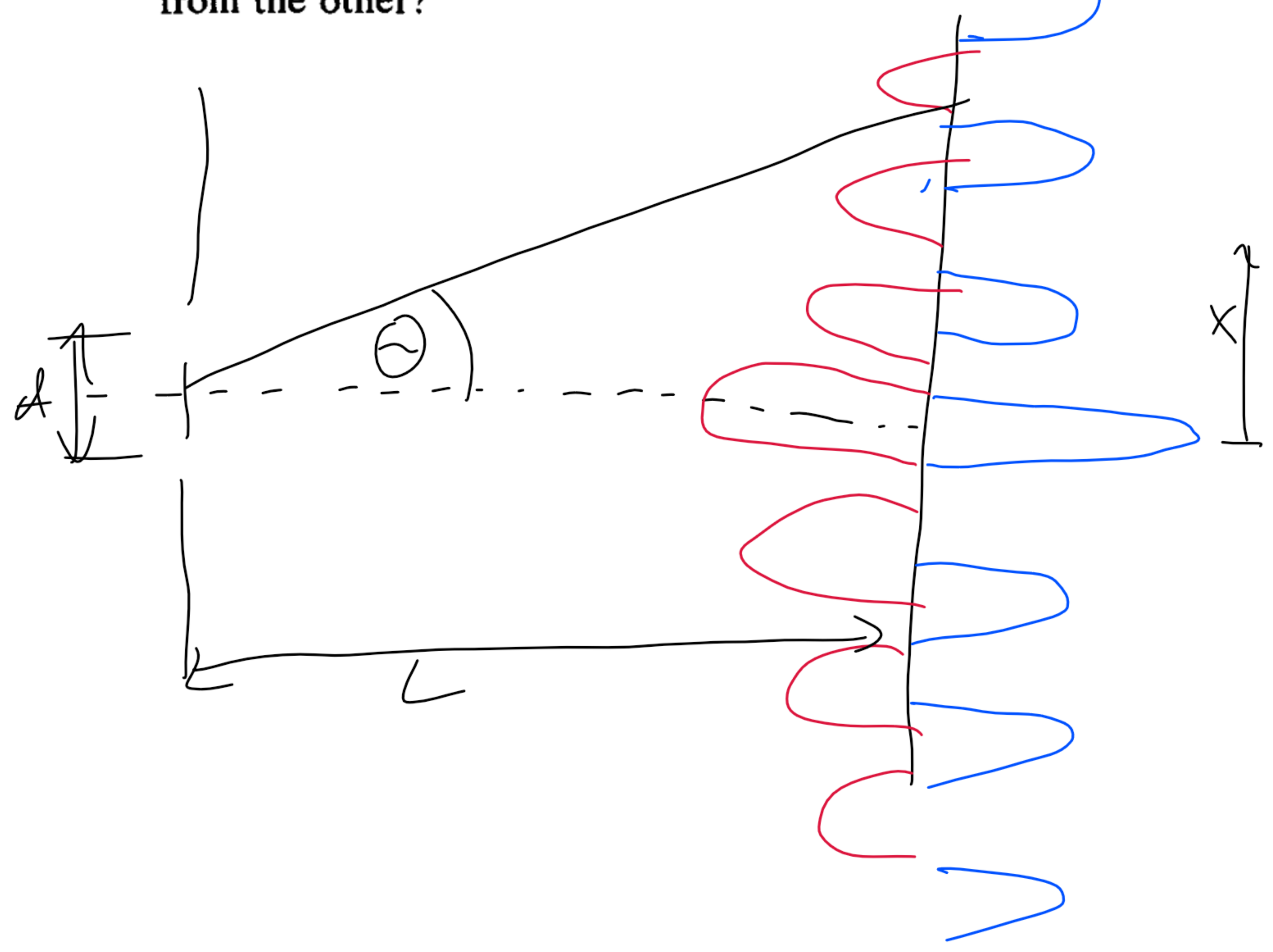
$$x \approx \frac{m\lambda L}{d}$$

$$\Delta x \approx \frac{m\lambda_2 L}{d} - \frac{m\lambda_1 L}{d} = \frac{mL}{d}(\lambda_2 - \lambda_1)$$

$$m = \frac{\Delta x\, d}{L(\lambda_2-\lambda_1)} = \frac{1,59\cdot\cancel{10^{-3}}\cdot 2,38\cdot\cancel{10^{-6}}}{2\cdot 0,6\cdot\cancel{10^{-9}}} = \frac{1,59\cdot 2,38}{2\cdot 0,6}$$

$$m \approx 3$$

36.12 In a Young's double-slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths, $\lambda = 750$ nm and $\lambda' = 900$ nm. At what minimum distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?



$d = 2\text{ mm} = 2 \cdot 10^{-3}\text{ m}$

$\lambda = 750\text{ nm} = 7.5 \cdot 10^{-7}\text{ m}$

$\lambda' = 900\text{ nm} = 9 \cdot 10^{-7}\text{ m}$

$L = 2\text{ m}$

$d \sin\theta = m\lambda$

$m \in \mathbb{Z}$

$d \sin\theta = m\lambda$

$d \sin\theta = \ell \lambda'$

$m, \ell \in \mathbb{Z}$

$m\lambda = \ell\lambda'$

$$\frac{m}{\ell} = \frac{\lambda'}{\lambda} = \frac{9 \cdot \cancel{10^{-7}}}{7.5 \cdot \cancel{10^{-7}}} = \frac{18}{15} = \frac{6}{5}$$

$m =$	6	12	18	24
$\ell =$	5	10	15	20

$d \sin\theta = m\lambda$

$d \,\mathrm{tg}\,\theta \simeq m\lambda$

$d \frac{x}{L} \simeq m\lambda$

$$x \simeq \frac{m\lambda L}{d} = \frac{6 \cdot 7.5 \cdot \cancel{10^{-7}} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{10^{-3}}} = 0.0045\text{ m}$$

10^{-4}

$x = 4.5$ mm

64. A plano-concave lens having index of refraction 1.50 is placed on a flat glass plate, as shown in Figure P37.64. Its curved surface, with radius of curvature 8.00 m, is on the bottom. The lens is illuminated from above with yellow sodium light of wavelength 589 nm, and a series of concentric bright and dark rings is observed by reflection. The interference pattern has a dark spot at the center, surrounded by 50 dark rings, of which the largest is at the outer edge of the lens. (a) What is the thickness of the air layer at the center of the interference pattern? (b) Calculate the radius of the outermost dark ring. (c) Find the focal length of the lens.

β

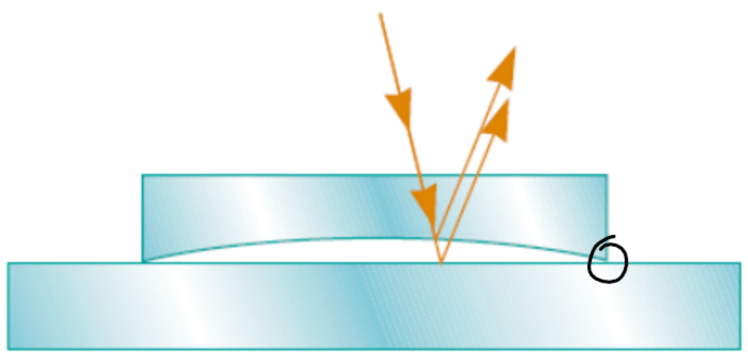
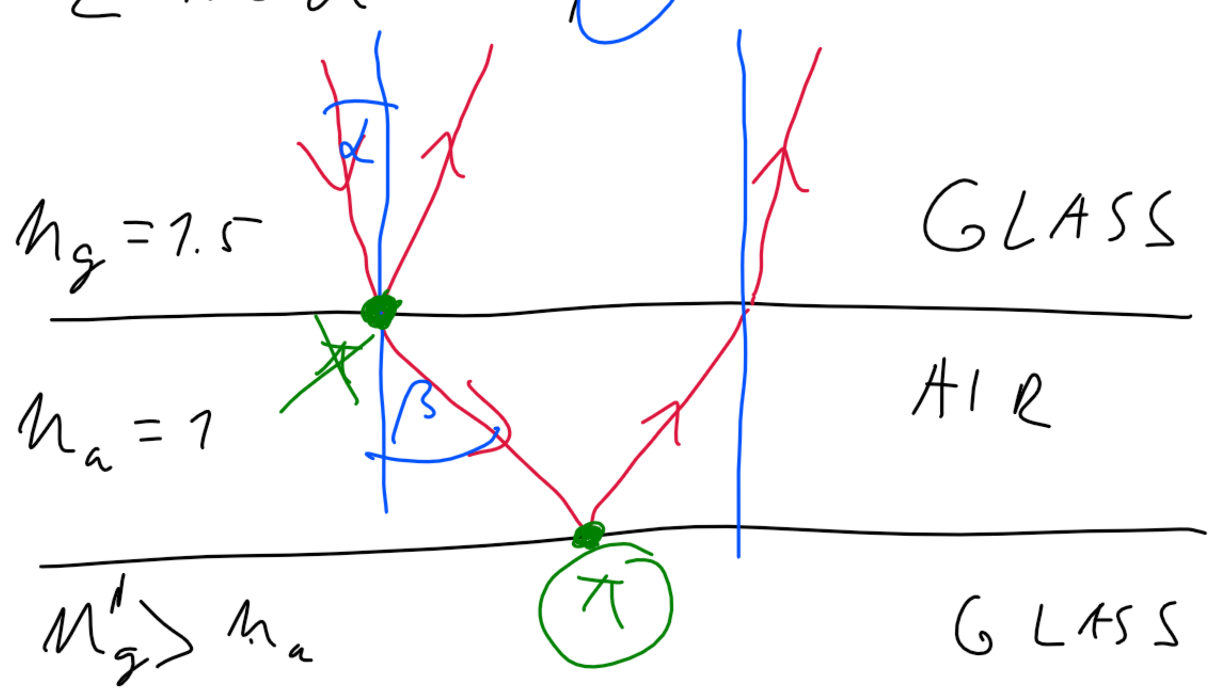


Figure P37.64

$2 n d \cos \beta$



$2 n d \cos\beta = m \lambda$ DARK

$2 n d \cos\beta = \left(m - \frac{1}{2}\right) \lambda$ BRIGHT

$2d = m\lambda$ DARK

$m \in \mathbb{Z}$

$2d = \left(m - \frac{1}{2}\right)\lambda$ BRIGHT

OUTERMOST RING: $d = \emptyset$

2nd RING: $2d = 1 \cdot \lambda$

$d = \frac{\lambda}{2}$

3rd RING: $2d = 2\lambda$

$d = \lambda$

$\vdots$

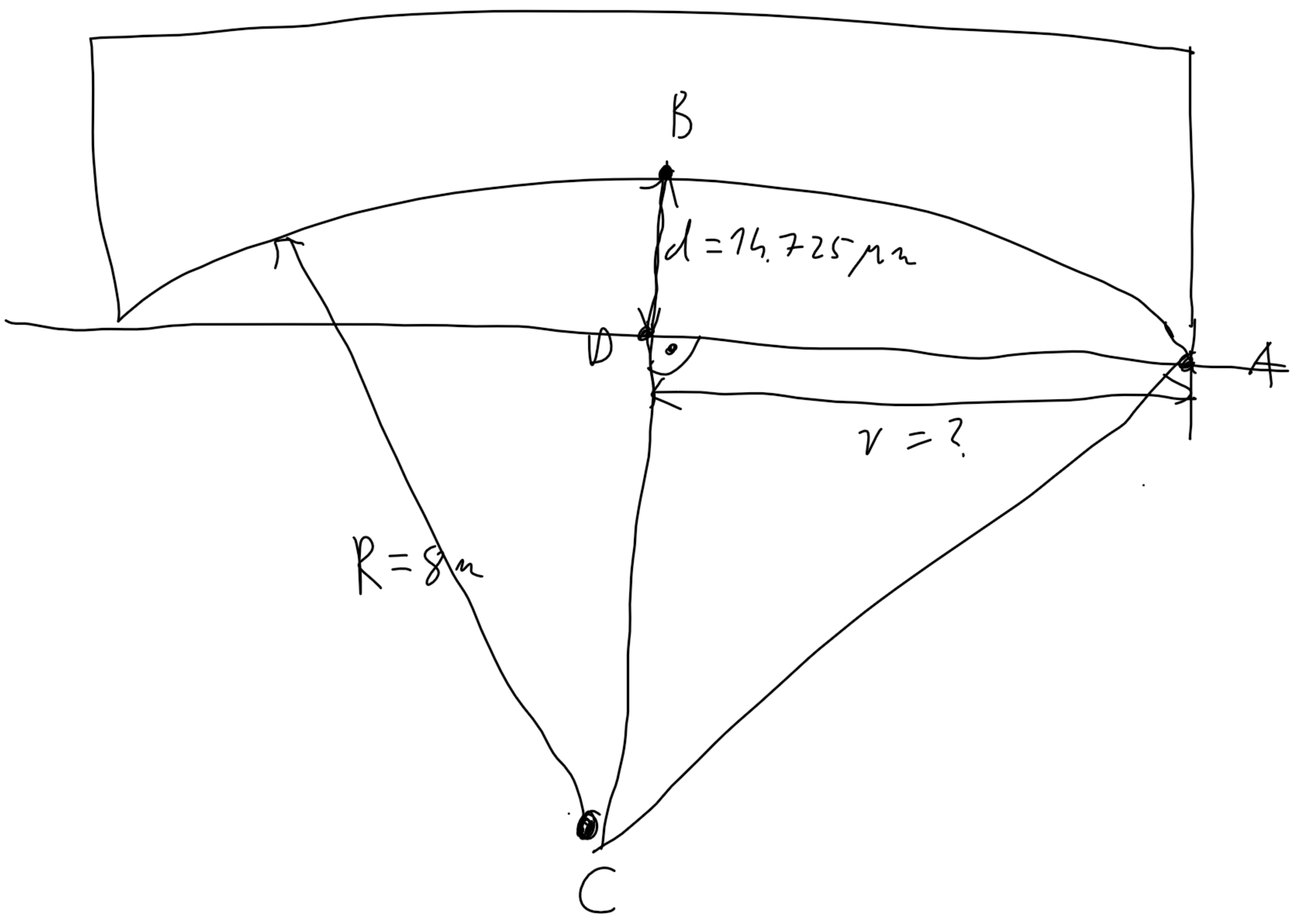
50th RING: $2d = 49\lambda$

$d = \frac{49}{2}\lambda$

CENTRAL DARK SPOT $\Rightarrow$ $2d = 50\lambda$

$d = 25\lambda = 25 \cdot 589 \cdot 10^{-9} = 14.725 \cdot 10^{-6}\ m$

$d = 14.725\ \mu m$



ADC $\Delta \Rightarrow$

$\overline{AD}^2 + \overline{DC}^2 = \overline{CA}^2$

$r^2 + (R - d)^2 = R^2$

$r^2 + \cancel{R^2} - 2Rd + d^2 = \cancel{R^2}$

$r^2 = 2Rd - d^2$

$r = \sqrt{2Rd - d^2}$

$r \approx \sqrt{2Rd}$

$r = 0.0153\ m$

$r = 1.53\ cm$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5 - 1) \cdot \left(\cancel{\frac{1}{\infty}} + \frac{1}{-8}\right) = -0.5 \cdot \frac{1}{8} = -\frac{1}{16}$$

$f = -16\ m$