Exercise-set 10.

1. (MT+'10) In a network the capacity of the edge e is 3, the capacities of all the other edges are 2, and we know that the value of the maximum flow f is an odd integer. Is it true then that $f(e) = 3$?

2. In a network with rational capacities the value of the maximum flow is m. Is it true then that for each value $0 \le x \le m$ there is a flow of value x in this network?

3. (MT+'13) Let a directed graph G, the vertex $s \in V(G)$ and the capacity function $c : E(G) \to \mathbf{R}^+$ be given. For all $v \in V(G)$, $v \ne s$ let $m(v)$ denote the value of the maximum flow from s to v. Suppose that for some vertex $t \in V(G)$, $m(t) = 100$ holds, but for every vertex $v \in V(G)$, $v \ne s, t$, $m(v) > 100$. Show that in this case the total capacity of the edges arriving into t is 100.

4. Let a directed graph G and the capacity function $c : E(G) \to \mathbf{R}^+$ be given. Suppose that for the vertices s, t and $w \in V(G)$ there is a flow of value 100 from s to t and also from t to w. Prove that there exists a flow of value 100 from s to w as well.

5. In a network all the capacities are integers. Which of the statements below holds always?
   a) Each maximum flow in the network has an integer value.
   b) There is a maximum flow in the network which takes an integer value on each edge.
   c) Each maximum flow in the network takes an integer value on each edge.
   d) What about the same questions if we substitute „integer" for „even number" everywhere?

6. (MT+'21) * Let the network (G, s, t, c) be given, furthermore an edge e in G for which $c(e) > 0$ holds. Determine whether the statements below are true or not:
   a) If there is a minimum s,t-cut C for which e goes out of X, then $f(e) = c(e)$ holds for all maximum flows f.
   b) If $f(e) = c(e)$ holds for all maximum flows f, then there is a minimum s,t-cut C for which e goes out of X.

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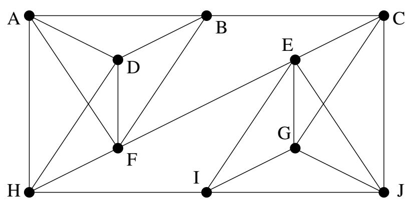
7. At most how many pairwise edge-disjoint and vertex-disjoint paths are there between the following points in the graph below:
   a) B and I, b) A and J, c) B and H.



8. (MT'12) The graph G on 15 vertices is constructed from three cycles, on 4, 5 and 6 vertices each, in such a way that each vertex of the 5-vertex cycle was connected (with one edge) to all the other vertices of the other two cycles. Let s be a vertex of the 4-vertex cycle, and t be a vertex of the 6-vertex cycle.
   a) At most how many pairwise vertex-disjoint paths are there in G between s and t?
   b) At most how many pairwise edge-disjoint paths are there in G between s and t?

9. (MT'12) The graph G contains a vertex from which 3 pairwise edge-disjoint paths go to any other vertex. Show that there are 3 pairwise edge-disjoint paths between any two vertices of G.