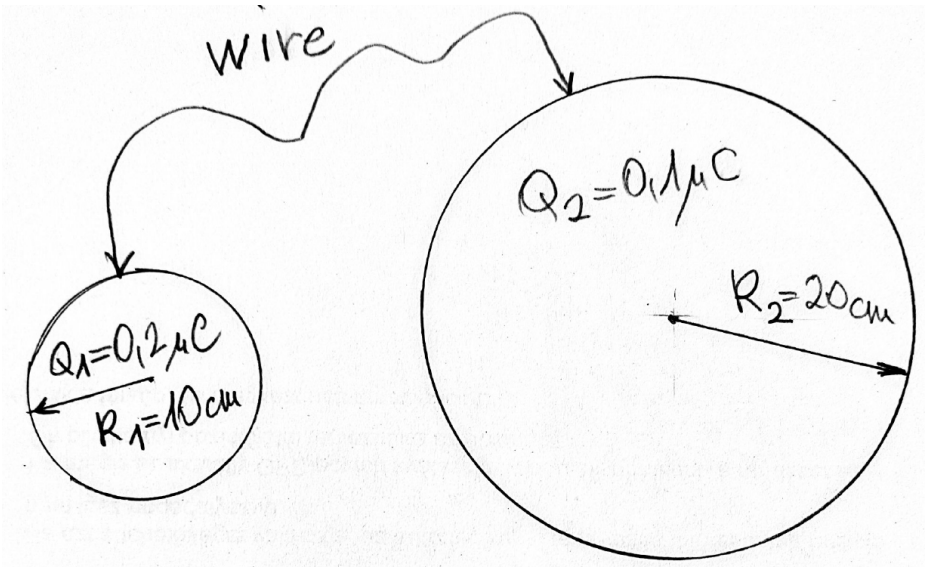
Problem E6

There are two metal spheres with the following radii: $R_1 = 10cm$ and $R_2 = 20cm$. The charges on the spheres are $Q_1 = 2 \cdot 10^{-7}\,As$ and $Q_2 = 10^{-7}\,As$ respectively. They have been touched simultaneously by means of a connecting wire.
a./ Find the potentials of the spheres prior to connecting.
b./ Find the total energy of the spheres prior to connecting.
c./ Find the potentials of the spheres after connecting.
d./ Find the charges on the spheres after connecting.
e./ Find the total energy of the spheres after to connecting.
f./ Did the total energy conserve or change? If yes or no, why did it happen?



Solution:

a./ Capacitance of metal spheres:

$$C_1 = 4\pi\varepsilon_0 R_1 = \frac{1}{9 \cdot 10^9} 10^{-1} = \frac{1}{9} 10^{-10}\,F$$

$$C_2 = 4\pi\varepsilon_0 R_2 = \frac{1}{9 \cdot 10^9} 2 \cdot 10^{-1} = \frac{2}{9} 10^{-10}\,F$$

Potentials of the spheres:

$$U_1 = \frac{Q_1}{C_1} = \frac{2 \cdot 10^{-7}}{\frac{1}{9} 10^{-10}} = 18 \cdot 10^3 V = 18\,kV$$

$$U_2 = \frac{Q_2}{C_2} = \frac{10^{-7}}{\frac{2}{9} 10^{-10}} = \frac{9}{2} \cdot 10^3 V = 4{,}5kV$$

b./ Electrostatic energy of a charged metal object is $E = \frac{1}{2} QU$.

$$E_1 = \frac{1}{2} Q_1 U_1 = \frac{1}{2} 2 \cdot 10^{-7} \cdot 18 \cdot 10^3 = 1{,}8 \cdot 10^{-3} J$$

$$E_2 = \frac{1}{2} Q_2 U_2 = \frac{1}{2} 10^{-7} \cdot 4{,}5 \cdot 10^3 = 2{,}25 \cdot 10^{-4} J$$

$$E_{tot} = E_1 + E_2 = 2{,}025 mJ$$

c./ The total charge is unchanged at the connection. The capacitances are added together.
(* means after connection)

$$Q^* = Q_1 + Q_2 = 3 \cdot 10^{-7} As$$

$$C^* = C_1 + C_2 = \left(\frac{1}{9} + \frac{2}{9} \right) 10^{-10} F = \frac{1}{3} 10^{-10} F$$

$$U^* = \frac{Q^*}{C^*} = \frac{3 \cdot 10^{-7}}{\frac{1}{3} 10^{-10}} = 9 \cdot 10^3 V = 9 kV$$

Another approach:

$$U^* = \frac{Q^*}{C^*} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{4\pi\varepsilon_0 R_1 U_1 + 4\pi\varepsilon_0 R_1 U_2}{4\pi\varepsilon_0 R_1 + 4\pi\varepsilon_0 R_1} = \frac{R_1 U_1 + R_2 U_2}{R_1 + R_3} = \frac{0{,}1 \cdot 18 + 0{,}2 \cdot 4{,}5}{0{,}3} kV = 9 kV$$

The potential is $U^* = 9 kV$ of both spheres.

d./ Charges on the spheres after connecting:
(* means after connection)

$$Q_1^* = C_1 U^* = \frac{1}{9} 10^{-10} \cdot 9 \cdot 10^3 = 10^{-7} As = 0{,}1 \mu C$$

$$Q_2^* = C_2 U^* = \frac{2}{9} 10^{-10} \cdot 9 \cdot 10^3 = 2 \cdot 10^{-7} As = 0{,}2 \mu C$$

e./ Total energy after connecting:
(* means after connection)

$$E_1^* = \frac{1}{2} Q_1^* U^* = \frac{1}{2} \cdot 10^{-7} \cdot 9 \cdot 10^3 = 4{,}5 \cdot 10^{-4} J$$

$$E_2^* = \frac{1}{2} Q_2^* U^* = \frac{1}{2} 2 \cdot 10^{-7} \cdot 9 \cdot 10^3 = 9 \cdot 10^{-4} J$$

$$E_{tot}^* = E_1^* + E_2^* = 13{,}5 \cdot 10^{-4} J = 1{,}35 mJ$$

Another approach:

$$E_{tot}^* = \frac{1}{2} (C_1 + C_2) U^{*2} = \frac{1}{2} \left(\frac{1}{9} + \frac{2}{9} \right) 10^{-10} \cdot 9^2 10^6 = \frac{9}{2} (1 + 2) 10^{-10} \cdot 9^2 10^6 = 13{,}5 \cdot 10^{-4} J$$

f./ Energy conservation?
Prior to connecting: $E_{tot} = E_1 + E_2 = 2{,}025 mJ$
After connecting: $E_{tot}^* = E_1^* + E_2^* = 1{,}35 mJ$
The total energy diminished. The missing energy has been emitted in electromagnetic wave.