34.36 Imagine a stratified system consisting of planar layers of transparent materials of different thicknesses. Show that the propagation direction of the emerging beam is determined by only the incident direction and the refractive indices of the initial and final layers (n_1 and n_f).

$$n_1 \sin\theta_{i1} = n_2 \sin\theta_{i2}$$

$$n_2 \sin\theta_{i2} = n_3 \sin\theta_{i3}$$

$$n_3 \sin\theta_{i3} = n_4 \sin\theta_{i4}$$

$$\vdots$$

$$n_\ell \sin\theta_{i\ell} = n_f \sin\theta_{if}$$

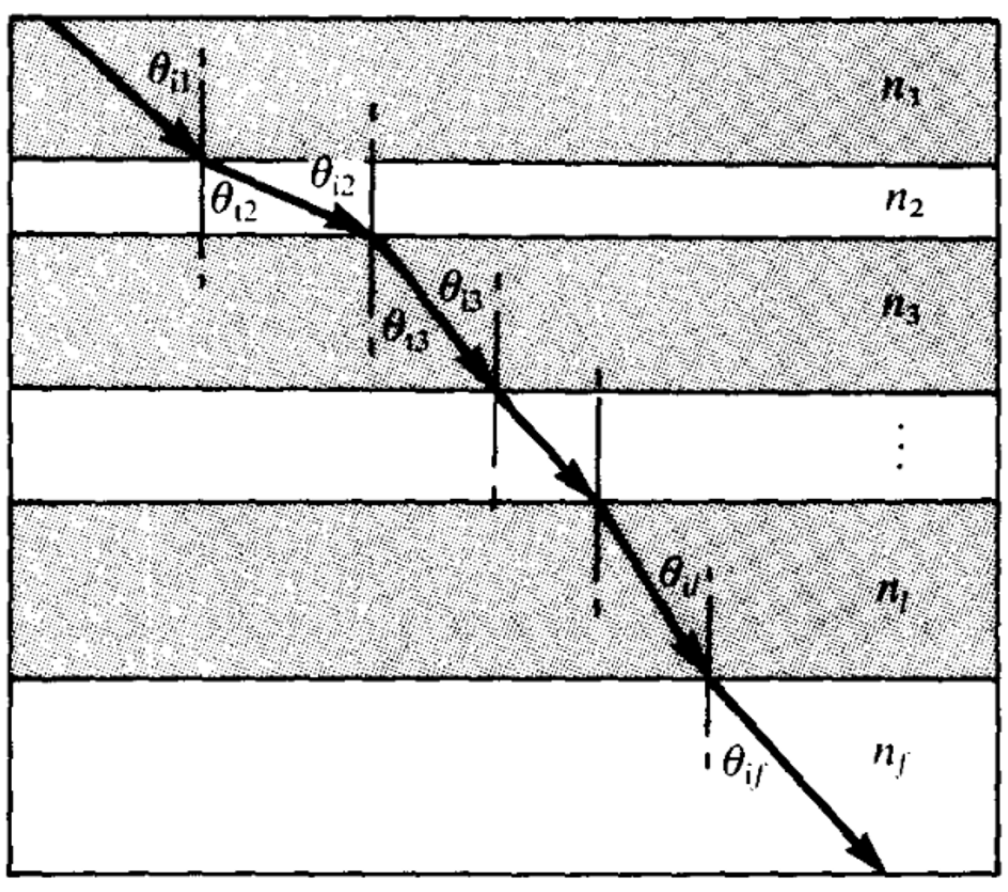
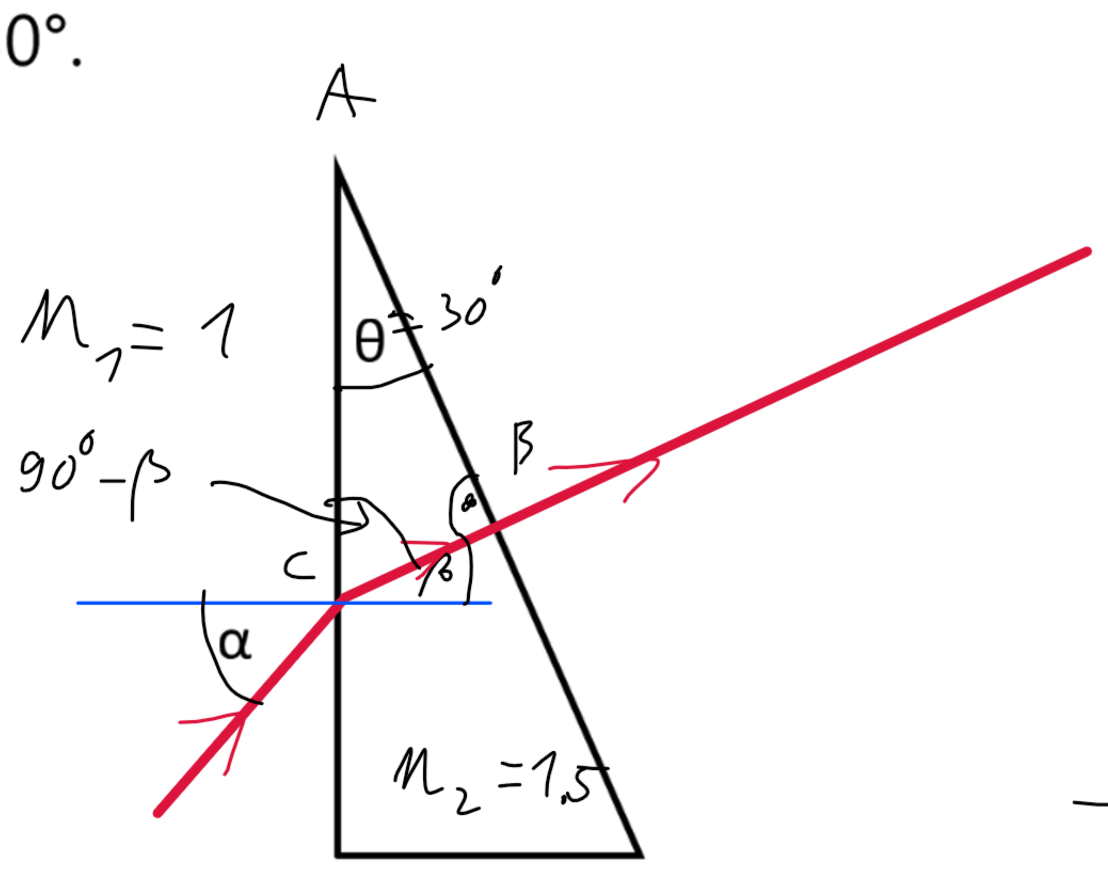


Fig. 34-13

$$n_1 \sin\theta_{i1} = n_f \sin\theta_{if}$$

A ray of light enters a prism on one side and leaves on another side. When the ray leaves the prism, its direction does not change. Determine the α incidence angle, if the index of refraction of the glass is n = 1.5 and the angle of the prism is θ = 30°.



$$n_1 \sin\alpha = n_2 \cdot \sin\beta \qquad \mathrm{I}$$

$$ABC\Delta \Rightarrow \cancel{90^\circ} + 30^\circ + \cancel{90^\circ} - \beta = \cancel{180^\circ}$$

$$\beta = 30^\circ \qquad \mathrm{II}$$

$$1 \cdot \sin\alpha = 1.5 \cdot \sin 30^\circ$$

$$\sin\alpha = 1.5 \cdot \frac{1}{2}$$

$$\alpha = 48.59^\circ$$

59. The light beam in Figure P35.59 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

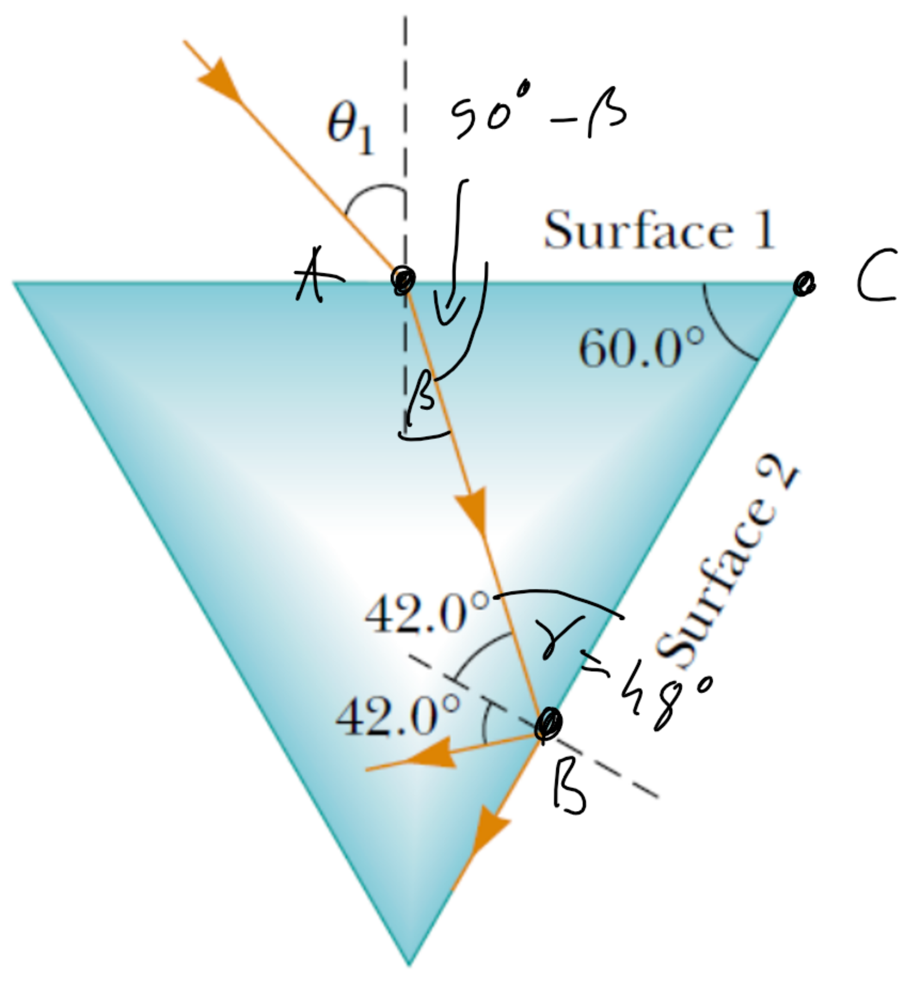


Figure P35.59

$n \sin \alpha = 1$

$\alpha = 42°$

$n = \frac{1}{\sin 42°} = 1.5$

$1 \cdot \sin \theta_1 = n \cdot \sin \beta$

$ABCA \Rightarrow 90° - \beta + 48° + 60° = 180°$

$\beta = 18°$

$\sin \theta_1 = 1.5 \cdot \sin 18°$

$\theta_1 = 27.61°$

40. An optical fiber has index of refraction n and diameter d. It is surrounded by air. Light is sent into the fiber along its axis, as shown in Figure P35.40. (a) Find the smallest outside radius R permitted for a bend in the fiber if no light is to escape. (b) **What If?** Does the result for part (a) predict reasonable behavior as d approaches zero? As n increases? As n approaches 1? (c) Evaluate R assuming the fiber diameter is 100 μm and its index of refraction is 1.40.

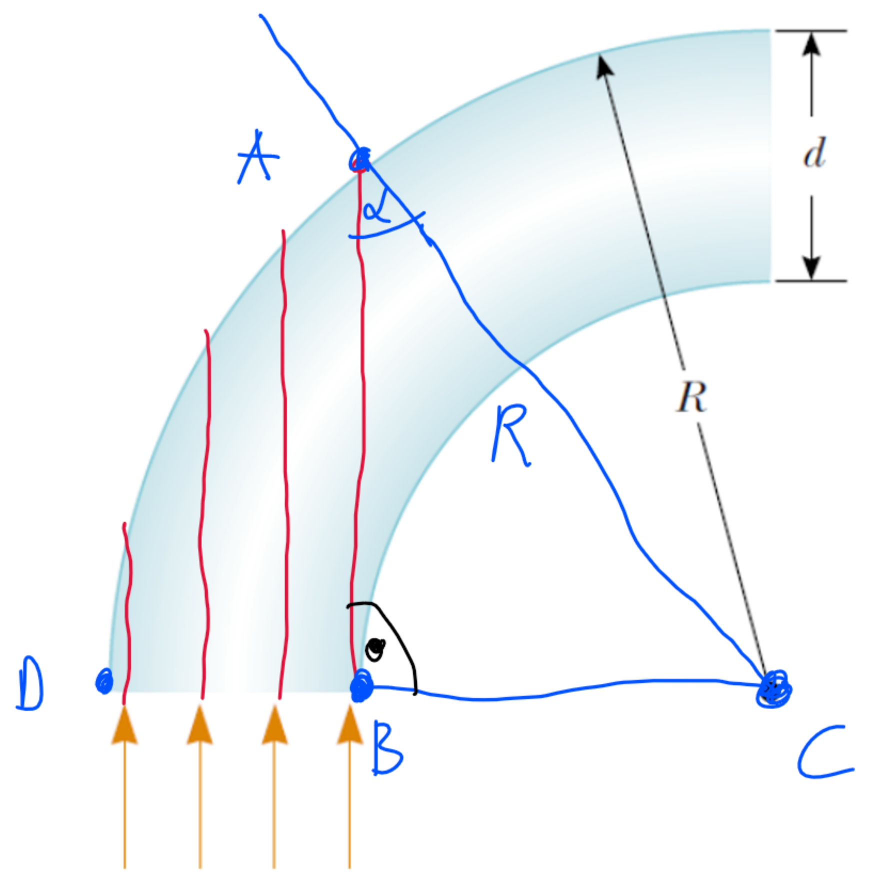


Figure P35.40

$$n \sin\alpha \geq 1$$

$$\left.\begin{array}{l}\overline{CD} = R \\ \overline{BD} = d\end{array}\right\} \overline{BC} = \overline{CD} - \overline{BD} = R - d$$

$$ABC\triangle \Rightarrow \sin\alpha = \frac{\overline{BC}}{\overline{AC}} = \frac{R-d}{R}$$

$$n\frac{R-d}{R} \geq 1$$

$$nR - nd \geq R$$

$$nR - R \geq nd$$

$$R \geq \frac{n}{n-1} d$$

b) $d \to 0 \Rightarrow R \to 0$ ✓

n INCREASES $\quad R \geq \dfrac{1}{\frac{n-1}{n}} d$

$$R \geq \frac{1}{1-\frac{1}{n}} d \quad ✓$$

$n \to 1 \Rightarrow R \to \infty$ ✓

c.) $n = 1.4$

$d = 100\,\mu m = 10^{-4}\,m$

$$R \geq \frac{n}{n-1} d = \frac{1.4}{0.4} \cdot 10^{-4} = 3.5 \cdot 10^{-4}\,m = 0.35\,mm = 350\,\mu m$$

$$R \geq 0.35\,mm$$

63. An object placed 10.0 cm from a concave spherical mirror produces a real image 8.00 cm from the mirror. If the object is moved to a new position 20.0 cm from the mirror, what is the position of the image? Is the latter image real or virtual?

$o = 10 \text{ cm}$

$i = 8 \text{ cm}$

$f > \emptyset$

$o' = 20 \text{ cm}$

$\Rightarrow i' = ?$

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} = \frac{o+i}{oi}$$

$$f = \frac{oi}{o+i} = \frac{10 \cdot 8}{10+8} = \frac{80}{18} = 4.\overline{44} \text{ cm}$$

$$\frac{1}{f} = \frac{1}{o'} + \frac{1}{i'} \Rightarrow \frac{1}{i'} = \frac{1}{f} - \frac{1}{o'} = \frac{o'-f}{fo'}$$

$$i' = \frac{fo'}{o'-f} = \frac{4.\overline{44} \cdot 20}{20 - 4.\overline{44}}$$

$$i' = 5.714 \text{ cm}$$

An object is placed 20 cm in front of a lens. The lens produces an image, which stands upside down. If we move the object 5 cm closer to the lens, the image moves 10 cm further away from the lens. Determine the focal distance of the lens and the object distance.

$$\frac{1}{f} = \frac{1}{\sigma} + \frac{1}{i} \quad \mathrm{I}$$

$\sigma = 20$ cm

$f > \emptyset \;,\; i > \emptyset$

$\sigma' = \sigma - 5 = 15$ cm

$i' = i + 10 \quad \mathrm{II}$

$$\frac{1}{f} = \frac{1}{\sigma'} + \frac{1}{i'} \quad \mathrm{III}$$

$$\frac{1}{\sigma} + \frac{1}{i} = \frac{1}{\sigma'} + \frac{1}{i'} \quad \mathrm{IV}$$

$$\frac{1}{\sigma} + \frac{1}{i} = \frac{1}{\sigma'} + \frac{1}{i+10} \quad \mathrm{V}$$

$$\frac{1}{20} + \frac{1}{i} = \frac{1}{15} + \frac{1}{i+10} \quad \Big/ \cdot 20 \cdot 15 \cdot i \cdot (i+10)$$

$$\cancel{15i(i+10)} + 300(i+10) = \cancel{20}\,5\, i(i+10) + 300i$$

$$\cancel{300i} + 3000 = 5i^2 + 50i + \cancel{300i}$$

$$i^2 + 10i - 600 = \emptyset$$

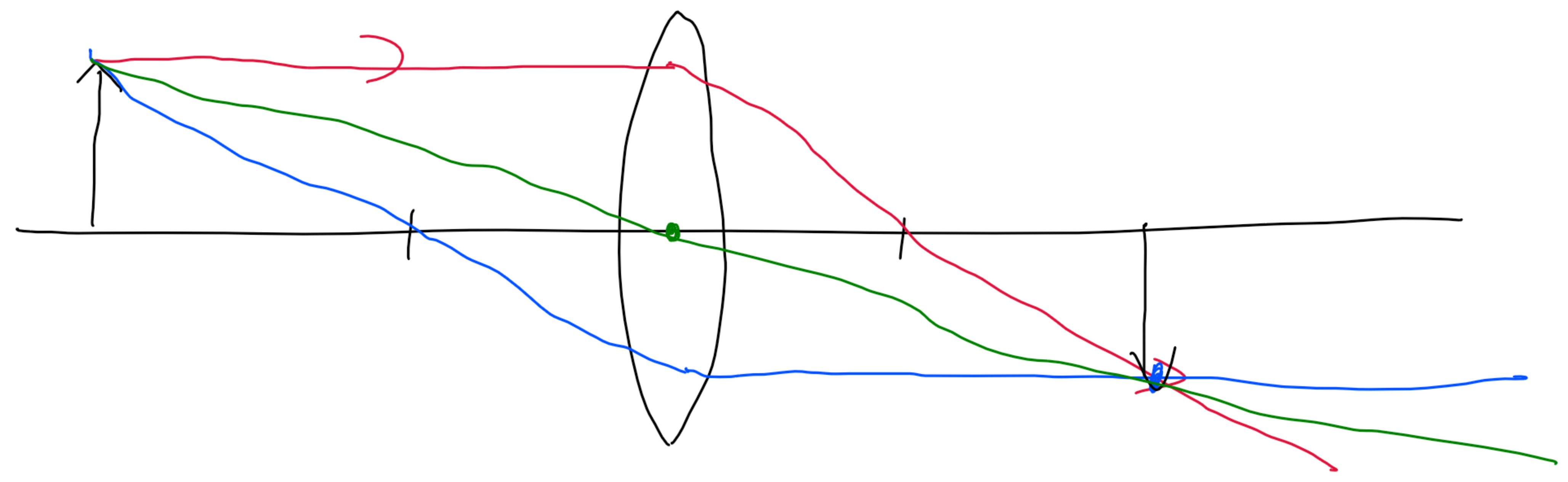
$$i_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{100 + 4 \cdot 1 \cdot 600}}{2} = \frac{-10 \pm \sqrt{2500}}{2}$$

$$i_{12} = \frac{-10 \pm 50}{2} \quad \begin{cases} 20 \text{ cm} \checkmark \\ \cancel{-30 \text{ cm}} \end{cases}$$

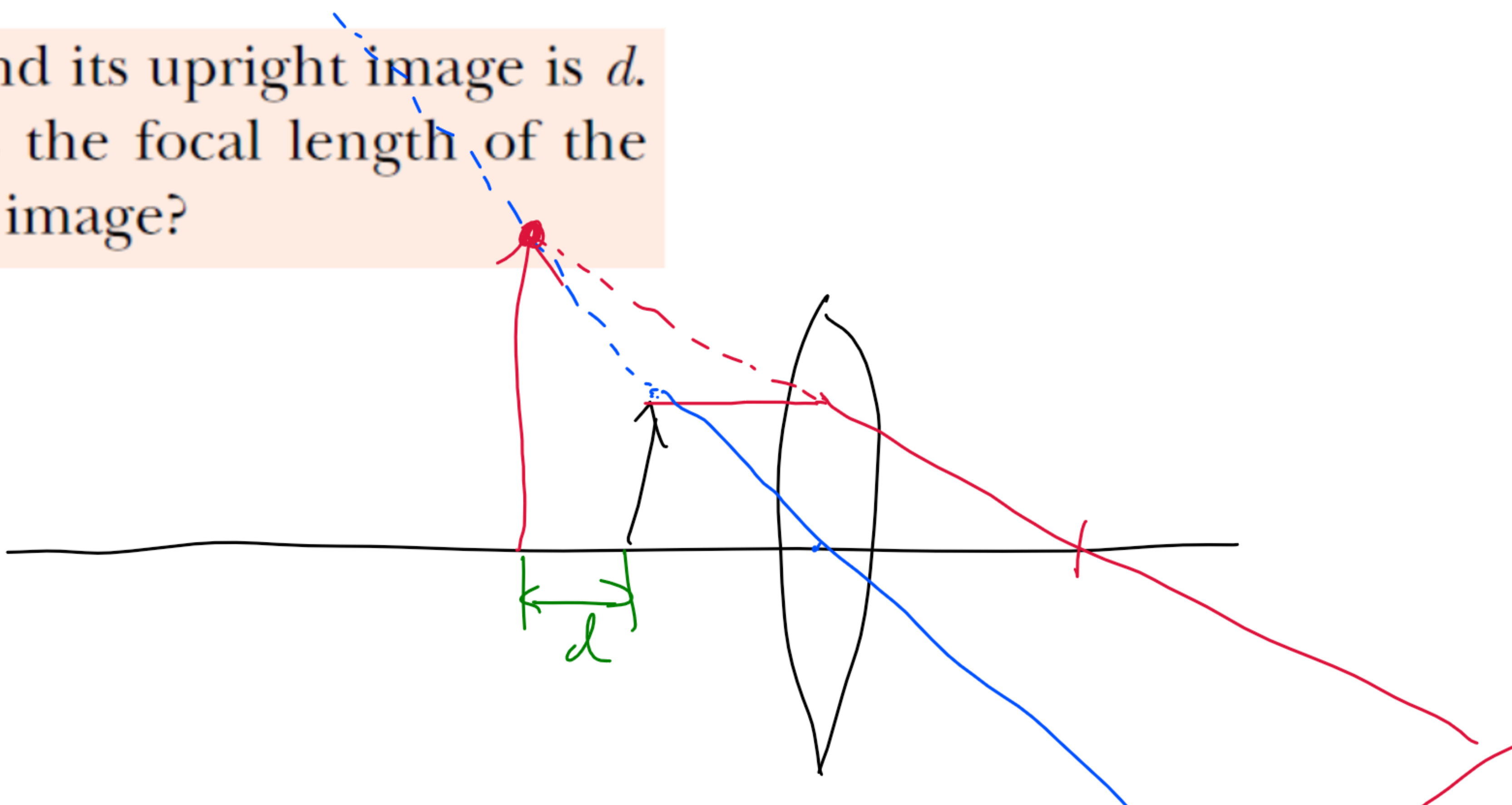
$$\Rightarrow i' = 20 + 10 = 30 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{\sigma} + \frac{1}{i} = \frac{\sigma + i}{\sigma i}$$

$$f = \frac{\sigma i}{\sigma + i} = \frac{20 \cdot 20}{20 + 20} = \frac{400}{40} = 10 \text{ cm}$$



58. The distance between an object and its upright image is d. If the magnification is M, what is the focal length of the lens that is being used to form the image?



$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$M = -\frac{i}{o}$$

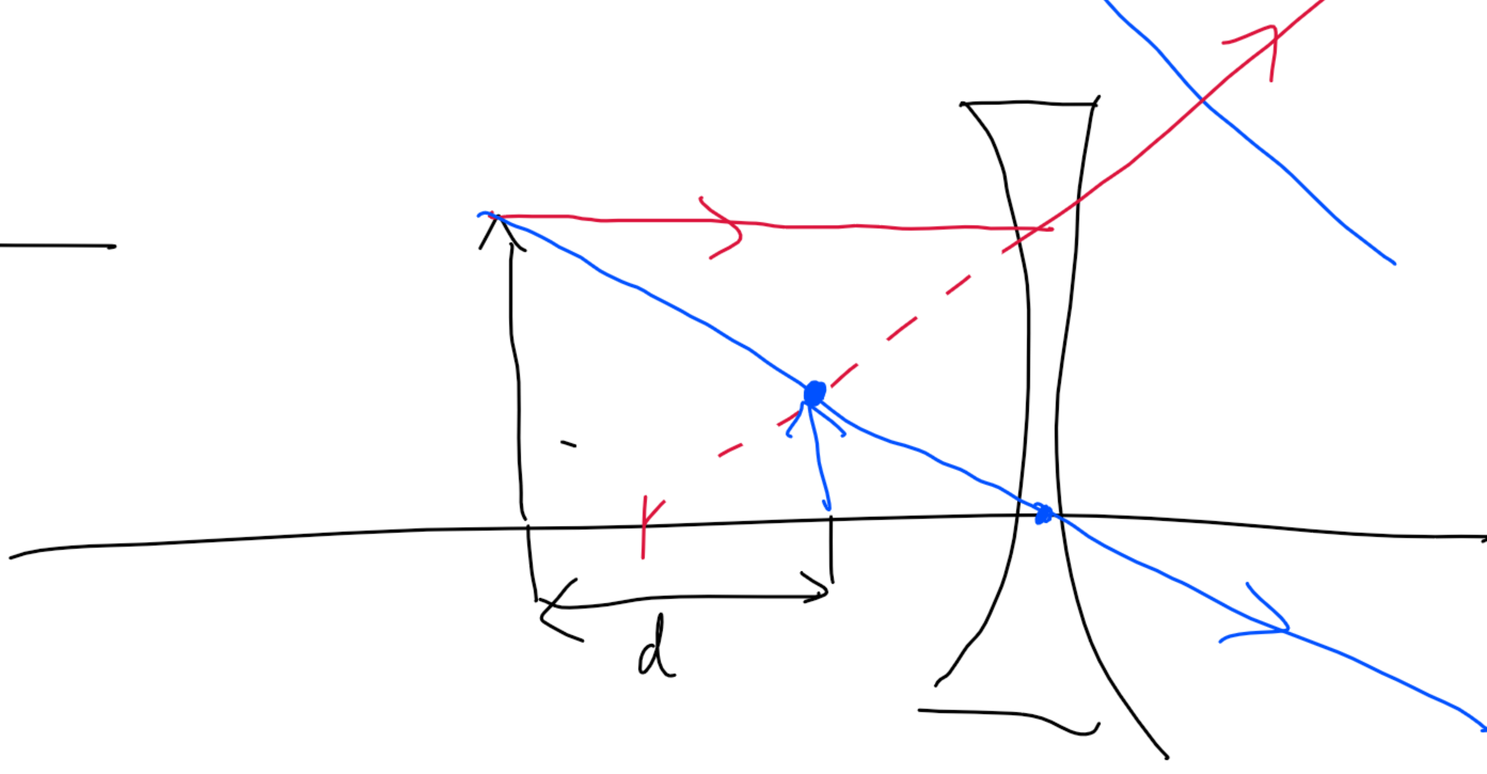
$$i = -(o \pm d)$$

$$\frac{1}{f} = \frac{1}{o} - \frac{1}{o \pm d}$$

$$M = \frac{o \pm d}{o}$$

$$\frac{1}{f} = \frac{\cancel{o} \pm d - \cancel{o}}{o(o \pm d)}$$

$$oM = o \pm d$$



$$\frac{1}{f} = \pm \frac{d}{o^2 \pm o d}$$

$$(M-1)\, o = \pm d \Rightarrow o = \pm \frac{d}{M-1}$$

$$f = \pm \frac{o^2 \pm o d}{d} = \pm \frac{\frac{d^{\cancel{2}}}{(M-1)^2} + \frac{d^{\cancel{2}}}{M-1}}{\cancel{d}} = \pm d \left[\frac{1}{(M-1)^2} + \frac{1}{M-1} \right]$$

$$f = \pm d \frac{\cancel{1} + M \cancel{-1}}{(M-1)^2} = \pm d \frac{M}{M^2 - 2M + 1}$$

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62. The object in Figure P36.62 is midway between the lens and the mirror. The mirror's radius of curvature is 20.0 cm, and the lens has a focal length of – 16.7 cm. Considering only the light that leaves the object and travels first toward the mirror, locate the final image formed by this system. Is this image real or virtual? Is it upright or inverted? What is the overall magnification?

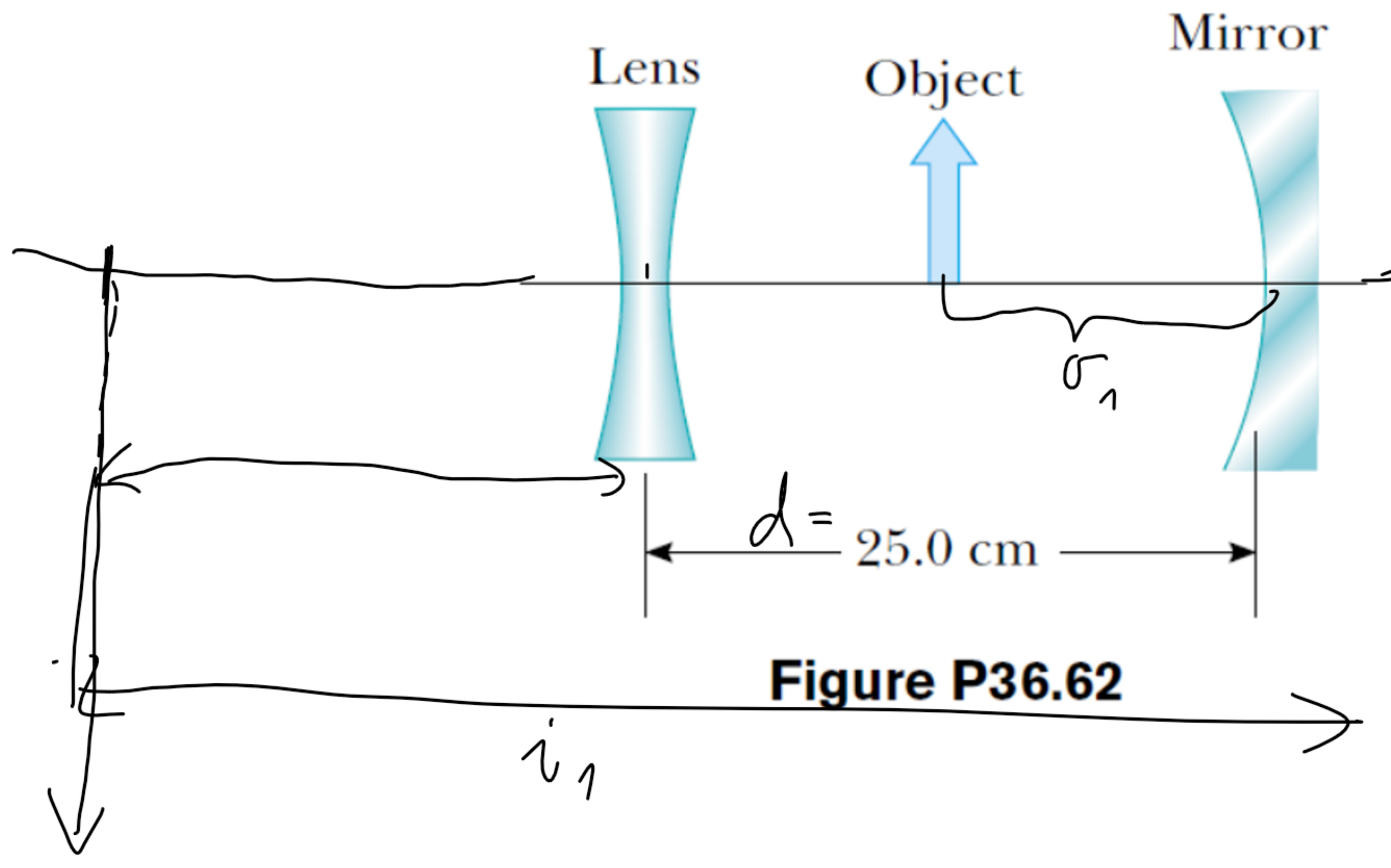

Figure P36.62

$R = 20$ cm

$f_m = \frac{R}{2} = 10$ cm

$f_e = -16.7$ cm

$d = 25$ cm

$o_1 = \frac{d}{2} = 12.5$ cm

$$\frac{1}{f_m} = \frac{1}{o_1} + \frac{1}{i_1} \Rightarrow \frac{1}{i_1} = \frac{1}{f_m} - \frac{1}{o_1} = \frac{o_1 - f_m}{f_m o_1}$$

$$i_1 = \frac{f_m o_1}{o_1 - f_m} = \frac{10 \cdot 12.5}{12.5 - 10}$$

$$i_1 = \frac{125}{2.5} = 50 \text{ cm}$$

$o_2 = -25$ cm

$$\frac{1}{f_e} = \frac{1}{o_2} + \frac{1}{i_2} \Rightarrow \frac{1}{i_2} = \frac{1}{f_e} - \frac{1}{o_2} = \frac{o_2 - f_e}{f_e o_2}$$

$$i_2 = \frac{f_e o_2}{o_2 - f_e} = \frac{-16.7 \cdot (-25)}{-25 + 16.7} = \frac{417.5}{-8.3}$$

$$i_2 = -50.3 \text{ cm}$$

$$M_1 = -\frac{i_1}{o_1} = -\frac{50}{12.5} = -4$$

$$M_2 = -\frac{i_2}{o_2} = -\frac{-50.3}{-25} = -2$$

$$M = M_1 \cdot M_2 = -4(-2) = +8$$