**Problem set 10.**

Kruskal's and Prim's algorithm.

1. The edges of an undirected graph are as follows: $ab(2), ac(3), bd(2), cd(1), de(2), df(4), ef(1), eg(2), fg(2), fh(1), gh(3)$.
(a) What are the edges selected by Prim's algorithm, and in which order, when run from vertex $a$? Show the table for $Q$ with every step of the run. Also show the final set of parents obtained.
(b) What are the edges selected by the Kruskal's algorithm and in which order?

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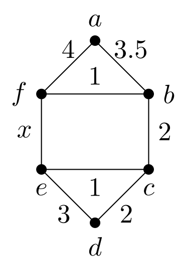
2. Give an algorithm to find the maximum weight spanning tree.

3. We run the Prim's algorithm from vertex $a$ on the undirected graph $G$ given below (where the edge weight $x$ is unknown) and we find that edges $ab, bd, de, bc$ are inserted, in that order, into the minimum spanning tree. Prove that the value of $x$ can only be 2.
Edges of $G$: $ab(1), ac(x), bc(2), bd(x), cd(3), ce(4), de(1)$.

4. Kruskal's algorithm is run on the graph given below, where the edge weight $x$ is unknown. The first three edges that the procedure selects are: $bf, ec, bc$. Give all possible values of $x$ (briefly justifying your answer) and describe which edges are selected, why, and their order of selection.




5. A big film production wants to shoot a film in a city. A map of the city is given as an undirected graph, where the vertices are the nodes and the edges are the direct roads between them. The filmmakers would like to close as many roads as possible during the filming, but we charge money (greater than zero) to close each section, we know the specific amount for each road. (The filmmakers have money to close all roads.) Give an algorithm that determines which roads should be closed if we want the city to function during the closure (we need to be able to get from everywhere to everywhere) and we want to maximize our revenue. The running time of your algorithm should be $O(n^2)$, where $n$ is the number of nodes.

6. We are given the adjacency list of a simple, connected graph $G = (V, E)$. The edges of the graph are weighted, the weight function $c : E \to \{-1, 1\}$. Give an algorithm that determines in $O(|V| + |E|)$ steps, the minimum total weight of a subgraph of $G$ that is connected and contains all points of $G$.

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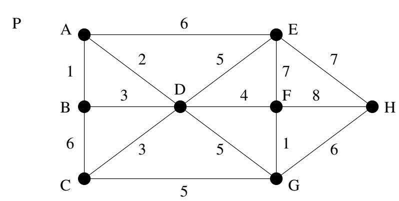
7. We are given the adjacency matrix of the connected, weighted, undirected, graph of a city's road network: the vertices are the nodes, the edges are the direct roads between the nodes, and the weight of the edges indicates how many workers can clear the given section of road in 1 hour. After a night snow storm, we would like to know the minimum number of workers we need to hire in total, so that all the nodes of the city can be reached after 8am from the main square (which is a vertex in the graph) on roads clear of snow. Assume that the snowfall stopped and the workers can start the work at 7 am. Which algorithm can be applied, if we want to get an answer in $O(n^2)$ running time, where $n$ is the number of nodes? Describe why it can applied and how.

8. (a) What are the edges selected by Prim's algorithm, and in which order, when run from vertex $A$? Show the table for $Q$ with every step of the run. Also show the final set of parents obtained.
(b) What are the edges selected by Kruskal's algorithm and in which order?



9. Give an example of an undirected, connected graph $G$ with 5 vertices and a run of the Prim's algorithm on the graph where the weight of an edge chosen later by the algorithm is smaller than that of an edge chosen earlier.

10. Give an example of a connected, undirected, edge-weighted graph with 5 vertices and a starting vertex where the edge weights are all different and where the 4 edges with the lowest weights are not the ones that are included in the minimum spanning tree at the end of the Prim's algorithm for finding the minimum spanning tree.

11. We are given the adjacency matrix of an undirected graph $G(V, E)$, where each edge has a positive weight. Each vertex of the graph is either a warehouse or a store, with the edge weights representing the corresponding distances. Find a subgraph $H$ of $G$ that contains all vertices, and in which each store has at least one warehouse, from which we can deliver supplies (i.e. there are paths in the subgraph $H$ between them). Give an algorithm with running time $O(n^2)$ for finding the minimum weight subgraph $H$.