Introduction to the Theory of Computing 2.

Exercise-set 6.
Solutions

1. a) yes,
   b) no,
   c) no,
   d) yes.

2. a) $S, G, E, A, H, B, F, C, D$.
   b) No.

3. a) no,
   b) yes,
   c) yes.

4. a) The edge not in the BFS spanning tree started from s whose endpoints are closest to s determines such a cycle, if the first common ancestor of its endpoints is s.
   b) Start a BFS in $G - e$ from one of the endpoints of e.

5. 99 (must be a tree).

6. Not true.

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7. a) $\nu(G) = 4,\ \tau(G) = 4,\ \alpha(G) = 6,\ \rho(G) = 6$.
   b) $\nu(G) = 5,\ \tau(G) = 5,\ \alpha(G) = 7,\ \rho(G) = 7$.
   a) $\nu(G) = 4,\ \tau(G) = 4,\ \alpha(G) = 6,\ \rho(G) = 6$.

8. $\chi(G) = 3,\ \nu(G) = 9,\ \tau(G) = 12,\ \alpha(G) = 6,\ \rho(G) = 9$.

9. $G = K_{668} \cup K_{668,669} \implies \chi(G) = 668,\ \nu(G) = 334+668 = 1002,\ \tau(G) = 667+668 = 1335,\ \alpha(G) = 1 + 669 = 670,\ \rho(G) = 334 + 669 = 1003$.

10. $\nu(G) = 20 = \tau(G)$.

11. $\alpha(G) = 86,\ \tau(G) = 14,\ \nu(G) = 14,\ \rho(G) = 86$.

12. $\nu(G) = 25,\ \alpha(G) = 75$.

13. a) $\{b, c, g, h\}$ ($\nu(G) = 4$).
    b) $\{b, d, f, h\}$ ($\nu(G) = 4$).

14. a) $\nu(G) = 4 = \tau(G)$.
    b) $\nu(G) = 4 = \tau(G)$.

15. a) By contradiction: otherwise the matching would not be maximum.
    b) Follows from a).
    c) Follows from b) and Gallai's theorem.

16. a) True.
    b) False.
    c) No.

17. G contains a Hamilton cycle $\implies$ $\nu(G) \geq \lfloor 2k + 1/2 \rfloor = k$, and $\nu(G) \leq (2k + 1)/2 = k$.

18. Yes (the graph contains a Hamilton cycle).

19. If we add the edge $\{u, v\}$ to G then it contains a Hamilton cycle.

20. If we add a new vertex to G connected to all the old ones then the new graph contains a Hamilton cycle + ex. 17.

21. $\det M \neq 0 \implies \exists$ a nonzero elementary product, corresponding to a perfect matching.

22. $|E(G)| \leq \frac{20 \cdot 19}{2} + 20 \cdot 80 = 1790$, and this is possible (example).

23. $\tau(G) = 50$.

24. We can choose the independent vertices greedily, one by one.

25. $\nu(G) \leq 7 \implies \rho(G) \geq 13$.