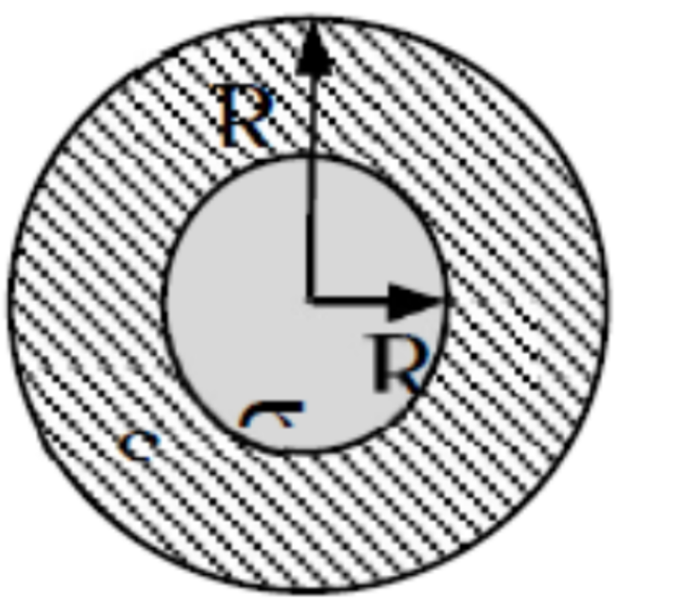
I. Are the statements below true or false?

Write a **T** or an **F** in the placeholder! (Good answer 1 point, bad answer -1 point, no answer 0 point)

1. [T] The electric field is conservative, so it can be described by a potential. **True**
2. [F] Static magnetic field is rotation free. **False**
3. [T] It is possible to induce electric field with static magnetic field. **True**
4. [F] The normal component of the E field is continuous on the interface of different dielectric materials **False**
5. [F] The capacitance of a metal sphere is proportional with its surface area

II. Solve the problems! (Prepare the solutions on separate sheets, then fold them into the problem sheet!)

1. The coaxial cable is a cylindrical capacitor essentially. Dielectric material is located between the electrodes. Find the capacitance (c=?) of a one meter long piece of such cable. Data: R_1=1mm, R_2=4mm, relative permittivity ε_r=2. Find the maximum safe voltage (U_{max}=?) one can apply to the capacitor, if the breakdown electric field of the dielectric material is $E_{critical}=10^8$ V/m. ($\varepsilon_0=8.86\ 10^{-12}$ As/Vm)



ELECTRIC FIELD BETWEEN R_1 AND R_2:

GAUSS LAW:

$$\oint\!\!\oint \underline{E}\cdot d\underline{A} = \frac{Q}{\varepsilon_0\varepsilon_r}$$

$$2\pi r\cdot\cancel{\ell}\cdot E(r) = \frac{\cancel{\ell}\cdot\sigma}{\varepsilon_0\varepsilon_r}$$

$$E(r) = \frac{1}{2\pi\varepsilon_0\varepsilon_r}\cdot\frac{\sigma}{r}$$

POTENTIAL DIFFERENCE BETWEEN R_1 AND R_2:

$$U = -\int_{R_2}^{R_1}\underline{E}(r)\cdot d\underline{r} = -\frac{\sigma}{2\pi\varepsilon_0\varepsilon_r}\int_{R_2}^{R_1}\frac{1}{r}\,dr = \frac{\sigma}{2\pi\varepsilon_0\varepsilon_r}\int_{R_1}^{R_2}\frac{1}{r}\,dr$$

$$U = \frac{\sigma}{2\pi\varepsilon_0\varepsilon_r}\cdot\ln\left(\frac{R_2}{R_1}\right)$$

CAPACITANCE:

$$Q = C\cdot U \Rightarrow C = \frac{Q}{U} = \frac{\cancel{\sigma}\cdot\ell}{\frac{\cancel{\sigma}}{2\pi\varepsilon_0\varepsilon_r}\cdot\ln\left(\frac{R_2}{R_1}\right)} = \frac{2\pi\varepsilon_0\varepsilon_r\ell}{\ln\left(\frac{R_2}{R_1}\right)}$$

$$C = \frac{2\pi\cdot 8.86\cdot 10^{-12}\cdot 2\cdot 1}{\ln\left(\frac{4\cdot\cancel{10^{-3}}}{1\cdot\cancel{10^{-3}}}\right)} = 8.031\cdot 10^{-11}\ F = \underline{\underline{80.31\ pF}}$$

MAX VOLTAGE:

$$E \leq E_{critical}$$

$$\frac{1}{2\pi\varepsilon_0\varepsilon_r}\cdot\frac{\sigma}{r} \leq E_{critical}$$

THIS IS MAX. WHEN r IS MIN.

$$\frac{1}{2\pi\varepsilon_0\varepsilon_r}\cdot\frac{\sigma}{R_1} \leq E_{critical}$$

$$\sigma \leq 2\pi\varepsilon_0\varepsilon_r\cdot R_1\cdot E_{critical}$$

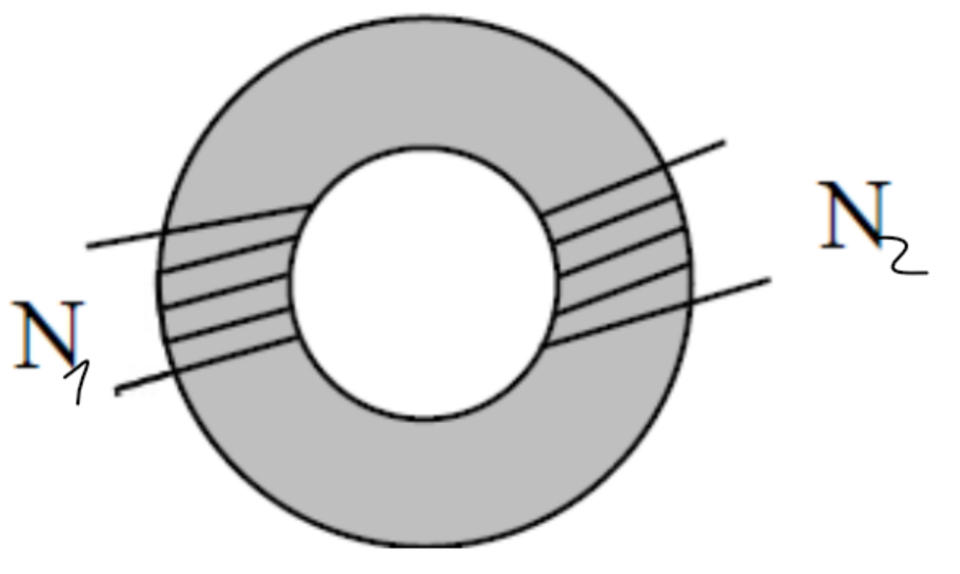
$$U = \frac{\sigma}{2\pi\varepsilon_0\varepsilon_r}\cdot\ln\left(\frac{R_2}{R_1}\right)$$

$$U \leq \frac{\cancel{2\pi\varepsilon_0\varepsilon_r}\cdot R_1\cdot E_{critical}}{\cancel{2\pi\varepsilon_0\varepsilon_r}}\ln\left(\frac{R_2}{R_1}\right) = R_1\cdot E_{critical}\ln\left(\frac{R_2}{R_1}\right)$$

$$U \leq 1\cdot 10^{-3}\cdot 10^{8}\ln\left(\frac{4\cdot\cancel{10^{-3}}}{1\cdot\cancel{10^{-3}}}\right) = 1.386\cdot 10^{5}\ V$$

$$U \leq \underline{\underline{138.6\ kV}}$$

2. A toroid transformer contains the primary and the secondary coils with the number of turns N_1 and N_2 respectively. Data: circumference $l=0.5$m, cross sectional area of the coil $A=10\text{cm}^2$, $N_1=10^2$, $N_2=10^3$, current in the primary coil is $I=10$A, relative permeability of the iron $\mu_r=10^3$. Find the mutual inductance (M=?) of the coils. The current is decreased at uniform pace all the way to zero in $\tau=10$ seconds. Find the induced voltage (U_2=?) in the secondary coil as the function of time. ($\mu_0=4\pi\ 10^{-7}$Vs/Am)



AMPERE'S LAW:

$$\oint \underline{B}\cdot \underline{dl} = \mu_0 \mu_r \sum I$$

$$Bl = \mu_0 \mu_r \cdot N_1 I$$

$$B = \mu_0 \mu_r \cdot \frac{N_1 I}{l}$$

TURN FLUX: $\Phi_T = B\cdot A = \mu_0 \mu_r \cdot \frac{N_1 A I}{l}$

THE VOLTAGE INDUCED IN 1 TURN OF THE SECONDARY COIL ACCORDING TO FARADAY'S LAW:

$$U_T = \frac{d\Phi_T}{dt} = \mu_0 \mu_r \cdot \frac{N_1 A}{l} \cdot \frac{dI}{dt}$$

TOTAL INDUCED VOLTAGE IN N_2 TURNS OF THE SECONDARY COIL:

$$U_2 = N_2 \cdot U_T = \left(\mu_0 \cdot \mu_r \cdot \frac{N_1 \cdot N_2 A}{l}\right) \cdot \frac{dI}{dt}$$

MUTUAL INDUCTANCE:

$$U_2 = M \frac{dI}{dt}$$

$$\Rightarrow M = \mu_0 \cdot \mu_r \cdot \frac{N_1 \cdot N_2 A}{l} = 4\pi \cdot 10^{-7} \cdot 10^{3} \cdot \frac{10^{2} \cdot 10^{3} \cdot 10 \cdot 10^{-4}}{0.5}$$

$$M = \underline{\underline{0.2513\ H}}$$

INDUCED VOLTAGE

$$U_2 = M \cdot \frac{\Delta I}{\tau} = 0.2513 \cdot \frac{0-10}{10} = \underline{\underline{-0.2513\ V}}$$

I. Are the statements below true or false?
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6. F The speed of light in a medium is proportional to its index of refraction
7. F When a ray of light enters water from air, the refraction angle is greater than the incident angle.
8. F Electromagnetic waves cannot exist in perfect vacuum.
9. F The pointing vector is perpendicular to the direction of propagation.
10. T Materials with high magnetic permeability has a smaller skin depth.

II. Solve the problems! (Prepare the solutions on separate sheets, then fold them into the problem sheet!)

3. An object is placed in front of an optical system, consisting of two convex lens. The focal length of the first lens is $f_1 = 1$ m. The distance between the object and the first lens is 2 m, while the distance between the lenses is 3 m. Determine the focal length of the second lens, if the image is formed 3 m behind it!

$f_1 = 1\ m$

$o_1 = 2\ m$

$$\frac{1}{f_1} = \frac{1}{o_1} + \frac{1}{i_1} \Rightarrow \frac{1}{i_1} = \frac{1}{f_1} - \frac{1}{o_1} = \frac{o_1 - f_1}{f_1 o_1}$$

$$i_1 = \frac{f_1 o_1}{o_1 - f_1} = \frac{1 \cdot 2}{2-1} = \frac{2}{1} = 2\ m$$

$d = 3\ m$

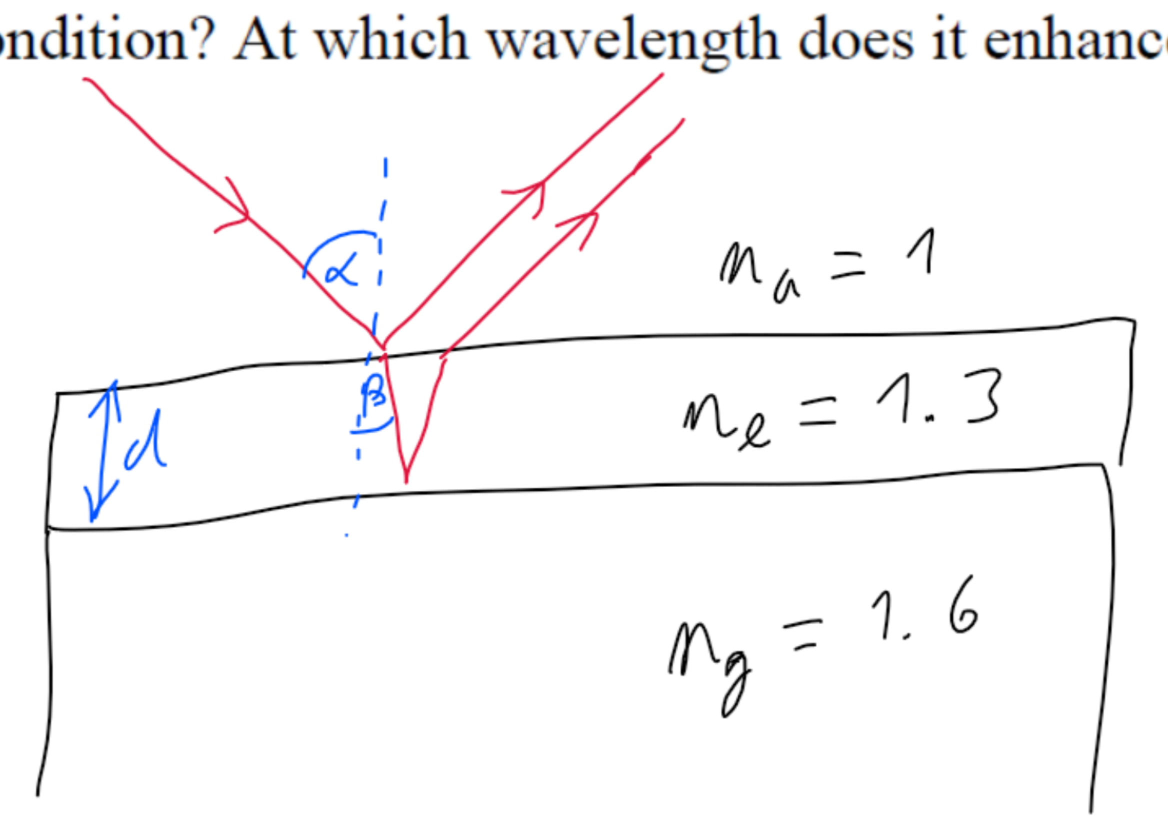
$o_2 = d - i_1 = 3 - 2 = 1\ m$

$i_2 = 3\ m$

$$\frac{1}{f_2} = \frac{1}{o_2} + \frac{1}{i_2} = \frac{i_2 + o_2}{o_2 i_2}$$

$$f_2 = \frac{o_2 i_2}{i_2 + o_2} = \frac{1 \cdot 3}{3+1} = \frac{3}{4}\ m = 0.75\ m$$

3. A thin layer is deposited to a glass substrate. Their refractive indices are $n_l = 1.3$ and $n_g = 1.6$, respectively. Determine the thickness of the layer if at normal incidence, it enhances reflection at 433 nm, and decreases it when the wavelength of light is 520 nm. Are there other possible layer thicknesses that satisfy the above condition? At which wavelength does it enhance reflection if the incidence angle is 60°?



$$2n_\ell \cdot d \cdot \cos\beta = \begin{cases} m\lambda & \text{CONSTR.} \\ \left(m - \frac{1}{2}\right)\lambda & \text{DESTR.} \end{cases} \qquad m \in \mathbb{Z}^+$$

$\lambda_1 = 433\ nm \Rightarrow$ CONSTR.

$\lambda_2 = 520\ nm \Rightarrow$ DESTR.

$$\Rightarrow m \cdot \lambda_1 = \left(k - \frac{1}{2}\right) \cdot \lambda_2 \qquad m, k \in \mathbb{Z}^+$$

$$\frac{m}{k - \frac{1}{2}} = \frac{\lambda_2}{\lambda_1} = \frac{520}{433} \approx \frac{6}{5}$$

$$\frac{2m}{2k-1} = \frac{6}{5} \Rightarrow$$

m	3	9	15	...	$3+6\ell$
k	3	8	13	...	$3+5\ell$

$\ell \in \mathbb{Z}^+$

NORMAL INCIDENCE: $\alpha = 0° \Rightarrow \beta = 0° \Rightarrow \cos\beta = 1$

$$\Rightarrow 2n_\ell \cdot d = m\lambda_1$$

$$d = \frac{m\lambda_1}{2n_\ell}$$

$$d_0 = \frac{3\lambda_1}{2n_\ell} = \frac{3 \cdot 433}{2 \cdot 1.3} = \frac{1300}{2.6} = 500\ nm$$

$$d_1 = \frac{9 \cdot \lambda_1}{2n_\ell} = \quad = 1500\ nm$$

$$d_2 = \frac{15 \cdot \lambda_1}{2n_\ell} = \quad = 2500\ nm$$

$$\vdots$$

$$d_\ell = \frac{(3+6\ell)\lambda_1}{2n_\ell} = \quad = 500 + \ell \cdot 1000\ nm$$

IF $\alpha = 60°$

$$n_a \cdot \sin 60° = n_\ell \cdot \sin\beta \Rightarrow \beta = \arcsin\left(\frac{n_a}{n_\ell} \cdot \sin 60°\right)$$

$$\beta = 41.77°$$

$$\Rightarrow \cos\beta = 0.7458$$

$$\left.\begin{aligned} 2n_\ell \cdot 1 &= m\lambda_1 \\ 2n_\ell \cdot \cos\beta &= m\lambda_1' \end{aligned}\right\} \qquad \frac{\lambda_1'}{\lambda_1} = \cos\beta \Rightarrow \lambda_1' = \lambda_1 \cdot \cos\beta$$