$$\varepsilon_0 = 8{,}86\cdot 10^{-12}\frac{As}{Vm}, \qquad \frac{1}{2\pi\varepsilon_0} = 1{,}8\cdot 10^{10}\frac{Vm}{As} \qquad \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^{9}\frac{Vm}{As}$$

Problem E3

There is an infinite long thin metal tube with the radius $R_1 = 0{,}1m$. The metal tube contains a homogeneous linear charge density $\sigma = 10^{-7} As/m$.

a./ Find and sketch the $E(r)$ function in coordinate system, and find the numerical value in the breakpoint $E(R_1)$.

b./Find the voltage of the tube relive to an external position $R_3 = 1m$.

c./ The whole system is in vacuum. A proton is released from the surface of the tube. Find the velocity of the proton at the radial position $R_3 = 1m$.

$m_{pr} = 1{,}67\cdot 10^{-27} kg$ and $q_{pr} = 1{,}6\cdot 10^{-19} As$

Solution:

a./ There is no insulating material, therefore $E \equiv E_{free}$ in this problem.

Gauss law in general: $\oint\limits_{Any\cdot surface} \mathbf{E}(\mathbf{r})\cdot d\mathbf{A} = \dfrac{Q}{\varepsilon_0}$ **Bold letters** mean vector

Gauss law in present case: $\oint\limits_{Tube} E(r)\cdot dA = \dfrac{l\sigma}{\varepsilon_0}$ Thin letters mean magnitude

$$2r\pi\cdot l\cdot E(r) = \frac{l\sigma}{\varepsilon_0}$$

$$E(r) = \frac{\sigma}{2\pi\varepsilon_0}\cdot\frac{1}{r}$$

Numerically:

$r < R_1$ $\qquad E(r) \equiv 0$

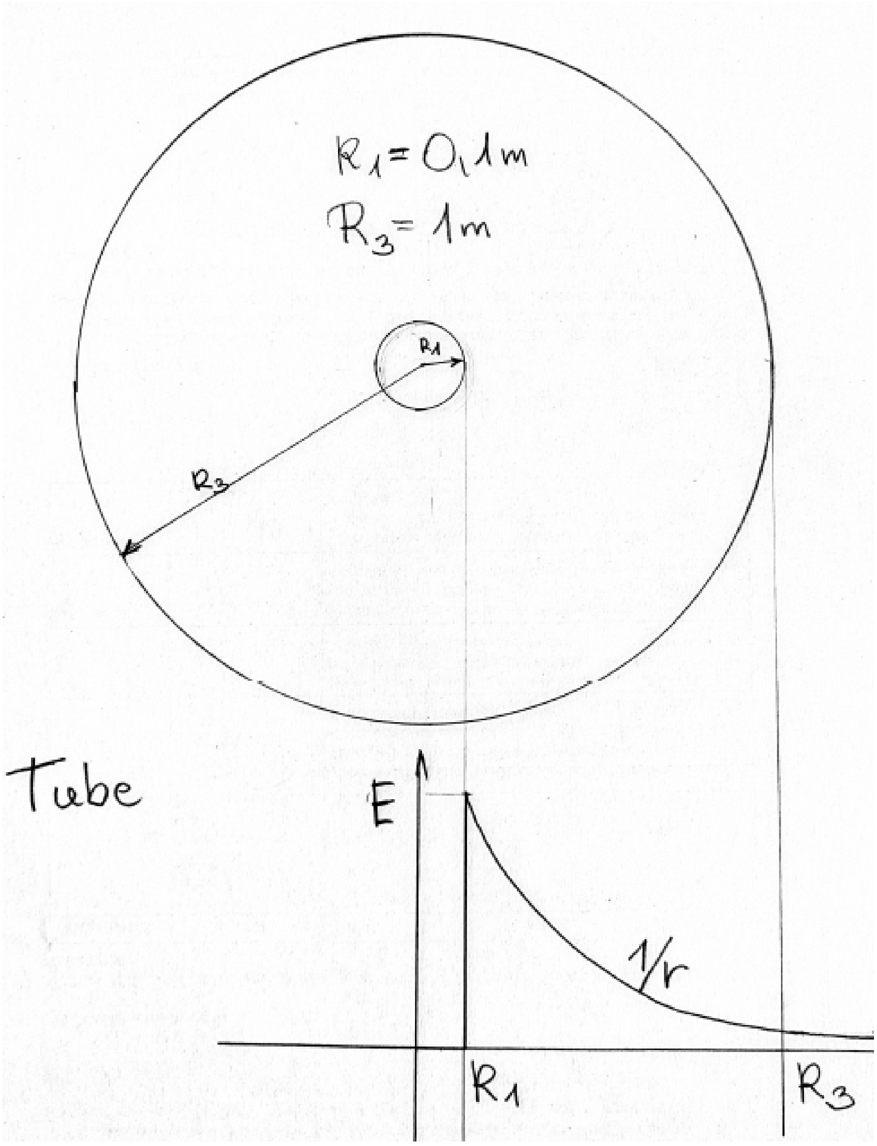
$R_1 \le r$ $\qquad E(r) = 1{,}8\cdot 10^{10}\frac{Vm}{As}\cdot 10^{-7}\frac{As}{m}\cdot\frac{1}{r} = 1800V\cdot\frac{1}{r}$ $\qquad \left[\frac{V}{m}\right]$

$$E(R_1) = 1800V\cdot\frac{1}{R_1} = \frac{1800V}{0{,}1m} = 18\frac{kV}{m}$$

b./

$$U(R_1,R_3) = -\int_{R_3}^{R_1} E(r)dr = -\int_{R_3}^{R_1} 1800V\cdot\frac{1}{r}dr = -1800V\int_{R_3}^{R_1}\cdot\frac{dr}{r} = 1800V\int_{R_1}^{R_3}\cdot\frac{dr}{r} = 1800V\ln\left(\frac{R_3}{R_1}\right)$$

$$U(R_1,R_2) = 1800V\ln\left(\frac{R_3}{R_1}\right) = 1800\ln(10)\cdot V \approx 4145V$$



c./

The conservation of the mechanical energy is used:

$$\frac{1}{2} m_{pr} v^2 = q_{pr} \cdot U(R_1, R_3)$$

$$v = \sqrt{\frac{2q_{pr} \cdot U(R_1, R_3)}{m_{pr}}} = \sqrt{\frac{2 \cdot 1{,}6 \cdot 10^{-19} \cdot 4145}{1{,}67 \cdot 10^{-27}}} = 8{,}91 \cdot 10^5 \frac{m}{s}$$