$$\varepsilon_0 = 8{,}86 \cdot 10^{-12} \frac{As}{Vm}, \qquad \frac{1}{2\pi\varepsilon_0} = 1{,}8 \cdot 10^{10} \frac{Vm}{As} \qquad \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^{9} \frac{Vm}{As}$$

Problem E1

There is an infinite long thin metal tube with the radius $R_1 = 0{,}1m$. This is surrounded by an insulator coating up to the radius $R_2 = 0{,}2m$ with relative permittivity $\varepsilon_r = 3$. The metal tube contains a homogeneous linear charge density $\sigma = 10^{-7} As/m$.
a./ Find and sketch the following functions: $E_{free}(r)$ and $E(r)$ in the same coordinate system, and find the numerical values in the three following breakpoints. $E_{free}(R_1)$, $E(R_1)$, $E(R_2)$.
b./Find the voltage of the inner tube relive to the external surface of the insulator coating.

Solution:

a./

Gauss law in general: $\oint\limits_{Any\cdot surface} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$ **Bold letters** mean vector

Gauss law in present case: $\oint\limits_{Tube} E_{free}(r) \cdot dA = \frac{l\sigma}{\varepsilon_0}$ Thin letters mean magnitude

$$2r\pi \cdot l \cdot E_{free}(r) = \frac{l\sigma}{\varepsilon_0} \qquad E_{free}(r) = \frac{\sigma}{2\pi\varepsilon_0} \cdot \frac{1}{r}$$

Numerically:

$r < R_1$ $\qquad E(r) \equiv 0$

$R_1 \le r$ $\qquad E_{free}(r) = 1{,}8 \cdot 10^{10} \frac{Vm}{As} \cdot 10^{-7} \frac{As}{m} \cdot \frac{1}{r} = 1800V \cdot \frac{1}{r} \qquad \left[\frac{V}{m}\right]$

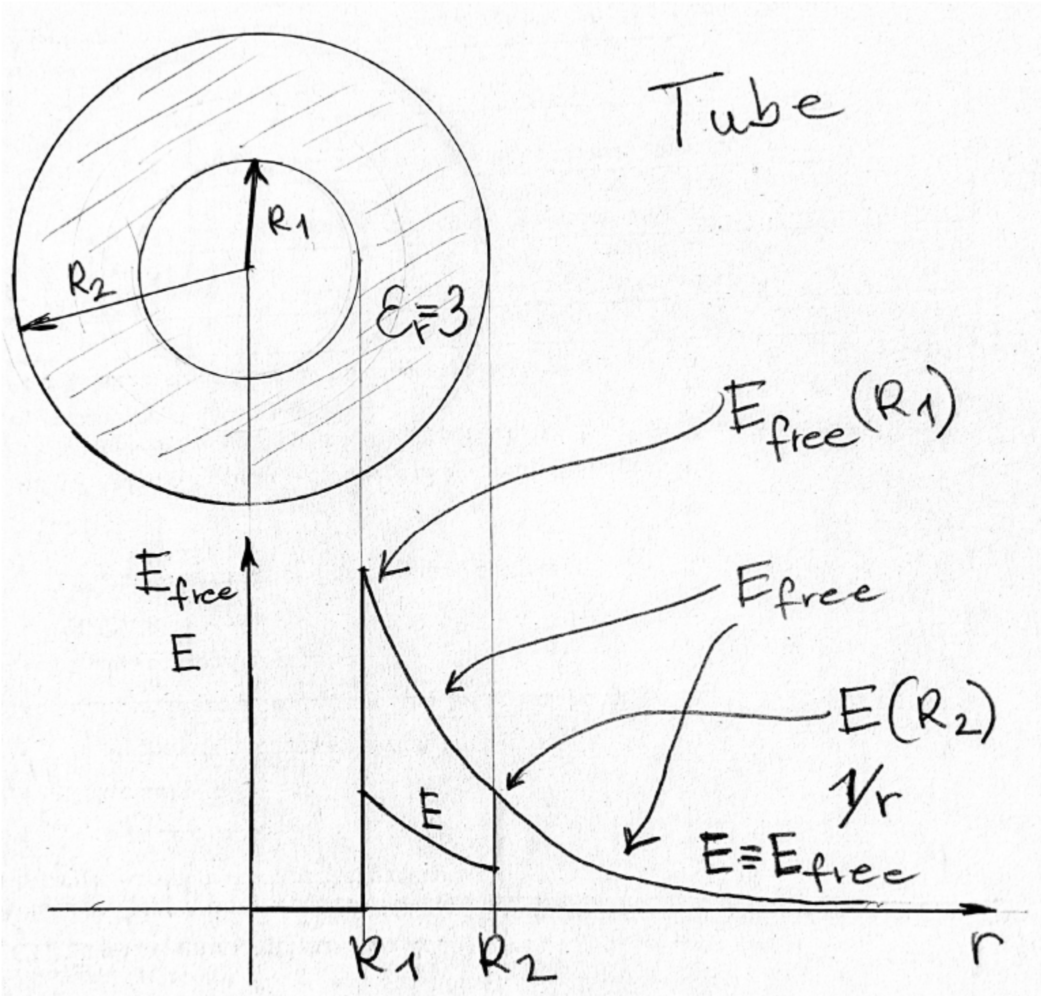
$R_1 \le r < R_2$ $\qquad E(r) = \frac{E_{free}(r)}{\varepsilon_r} = 600V \cdot \frac{1}{r} \qquad \left[\frac{V}{m}\right]$

$R_2 \le r$ $\qquad E(r) = 1800V \cdot \frac{1}{r} \qquad \left[\frac{V}{m}\right]$

$$E_{free}(R_1) = 1800V \cdot \frac{1}{R_1} = \frac{1800V}{0{,}1m} = 18\frac{kV}{m}$$

$$E(R_1) = 600V \cdot \frac{1}{0{,}1} = 6\frac{kV}{m}$$

$$E(R_2) = E_{free}(R_2) = 1800V \cdot \frac{1}{0{,}2} = 9\frac{kV}{m}$$



b./

$$U(R_1,R_2) = -\int_{R_2}^{R_1} E(r)dr = -\int_{R_2}^{R_1} 600V \cdot \frac{1}{r} dr = -600V \int_{R_2}^{R_1} \cdot \frac{dr}{r} = 600V \int_{R_1}^{R_2} \cdot \frac{dr}{r} = 600V \ln\left(\frac{R_2}{R_1}\right)$$

$$U(R_1,R_2) = 600V \ln\left(\frac{R_2}{R_1}\right) = 600 \ln 2 \cdot V \approx 416V$$