Problem E5

There are 27 pieces of spherical mercury droplets originally far from each other. Each of them has the radius $R := 10^{-3} m$. They have all been charged to the potential of $U = 1kV$. The droplets are united to one piece of bigger droplet, with special care, not to touch them with any conducting material.
a./Find the potential of the united droplet.
b./ Find the amount of the mechanical work, needed to unite the droplets.

Solution:

a./ Capacitance of metal sphere: $C = 4\pi\varepsilon_0 R = \frac{1}{9 \cdot 10^9} 10^{-3} = \frac{1}{9} 10^{-12} F = \frac{1}{9} pF$

Charge on each sphere: $Q = CU = 4\pi\varepsilon_0 R \cdot U = \frac{1}{9} 10^{-12} \cdot 10^3 = \frac{1}{9} 10^{-9} As$

After uniting *

Charge $Q^* = 27Q$ Radius $R^* = 3R$ $(3^3 = 27)$

$Q^* = 27 \cdot 4\pi\varepsilon_0 R \cdot U = 27 \cdot \frac{1}{9} 10^{-9} As = 3 \cdot 10^{-9} As$ $C^* = 4\pi\varepsilon_0 3R$

$U^* = \frac{Q^*}{C^*} = \frac{27 \cdot 4\pi\varepsilon_0 R \cdot U}{4\pi\varepsilon_0 3R} = \frac{27}{3} U = 9kV$

b./

Electrostatic energy of a charged metal object is $E = \frac{1}{2} QU$.

Initially $E = 27 \cdot \frac{1}{2} QU = 27 \cdot \frac{1}{2} \cdot \left(\frac{1}{9} 10^{-9} \right) \cdot 10^3 = \frac{3}{2} 10^{-6} J$

Finally $E^* = \frac{1}{2} Q^* U^* = \frac{1}{2} 3 \cdot 10^{-9} \cdot 9 \cdot 10^3 = \frac{27}{2} 10^{-6} J$

Mechanical work $W = E^* - E = 12\mu J$

Work was needed to overcome the mutual repulsion of the charges.