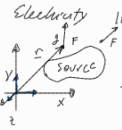
Electricity

Info | xuiuaGd



$\underline{E}(\underline{r}) = ?$

$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$

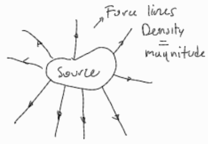
$[\underline{N}] \; \underline{F}(\underline{r}) = F_x(x,y,z)\,\underline{i} + F_y(x,y,z)\,\underline{j} + F_z(x,y,z)\,\underline{k}$

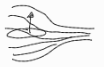
$\left[\frac{N}{C}\right] \; \frac{\underline{F}(r)}{q} = \underline{E}(\underline{r}) = E_x(x,y,z)\,\underline{i} + \dots + E_z\,\underline{j}$

$As = C \; \left[\frac{N}{A \cdot s} = \frac{V}{m}\right]?$

Force lines

Density = magnitude

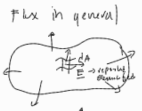




$\phi = A \cdot v \; \left[\frac{m^3}{s}\right]$

$Flux = \underline{A} \cdot \underline{E} = \Phi$

Flux in general



$\int_S \underline{E}(\underline{r}) \cdot d\underline{A} = \Phi$

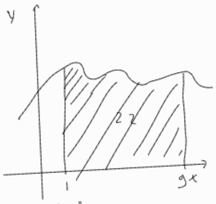
$\oint \underline{E}(\underline{r})\, d\underline{A} = \Phi$

Gauss's Law

$\varepsilon_0 = 8{,}86 \cdot 10^{-12} \frac{As}{Vm}$

$\oint \underline{E}(\underline{r}) \cdot d\underline{A} = \frac{Q}{\varepsilon_0} \; \left[\frac{As}{\frac{As}{Vm}} = Vm\right]$

$\left[\frac{V}{m} \cdot m^2 = Vm\right]$

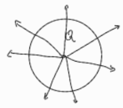


Spherical

Cylindrical (infinite)

Plane Parallel (infinite)

Point charge



$k = \frac{Q}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{Vm}{As}$

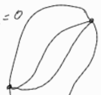
$4r^2\pi \cdot E_{(r)} = \frac{Q}{\varepsilon_0}$

$\Rightarrow E_{(r)} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}$

$F(r) = E(r) \cdot q = \frac{Qq}{4\pi\varepsilon_0 r^2}$

Columb's law

$\oint \underline{E}(\underline{r})\, d\underline{r} = 0$



$\underline{F} = q\underline{E}(\underline{r})$

$W = \int_A^B \underline{F}(\underline{r})\, d\underline{r}$

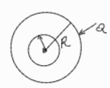
$\int_A^B \underline{E}\, d\underline{r} = \frac{W}{q}$

$\int_A^B (-\underline{E}(\underline{r}))\, d\underline{r} = U_B - U_A$

$\frac{F}{q} = E\left(\frac{N}{C}\right) \; \left[\frac{V}{m}\right] \quad Ed = V$

$E(r)$

Metal



$E_{in} \, 4r^2\pi = Q = 0$

$E_{in} = 0$

$E_{out} \cdot 4r^2\pi = Q$

$E_{out} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}$

$U(r) = -\int_\infty^R \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}\, dr = \frac{Q}{4\pi\varepsilon_0} \int_\infty^R \left(-\frac{1}{r^2}\right) dr$

$= \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r}\right]_\infty^R = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R}$

$C = \frac{Q}{U} = 4\pi\varepsilon_0 R = 0.11 \cdot 10^{-9} = \frac{1}{9} nF = 111 pF$

$F = \left[\frac{A \cdot s}{V m}\right]$

$Q = 10^{-8} As \quad R = 0.1 m$

$E(r = R) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R} = 9 \cdot 10^9 \cdot 10^{-8} \cdot 10^2 = 9 \frac{kV}{m}$

$U(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r} = 9 \cdot 10^9 \cdot 10^{-8} \cdot 10 = 0.9 kV$

$E(R) = \frac{U(R)}{R}$

$\omega = 10^{-8} \frac{As}{m^2}$

$\oint \underline{E} \cdot d\underline{A} = \frac{Q_{ins}}{\varepsilon_0}$

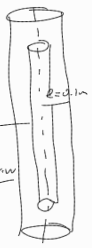
$2 r \pi l \, E_{in} = 0$

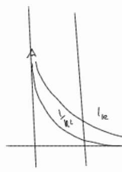
$E_{in} = 0$

Out

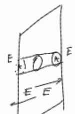
$\cancel{2} r \cancel{\pi} \cancel{l} \, E_{out} = \frac{2R\pi \cdot l \cdot w}{\varepsilon_0}$

$E_{out} = \frac{w R}{\varepsilon_0} \frac{1}{r}$





$E_{out}(r = R) = \frac{w}{\varepsilon_0} = \frac{10^{-8}}{8.86 \cdot 10^{-12}} = \frac{10^5}{8{,}86} = 1129 \frac{V}{m}$



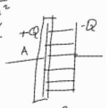
$w = 10^{-8} \frac{As}{m^2}$

$E \, 2\cancel{A} = \frac{w \cdot \cancel{A}}{\varepsilon_0}$

$E = \frac{w}{2\varepsilon_0}$

$A \cdot E = \frac{Q}{\varepsilon_0} \quad E = \frac{Q}{A\varepsilon_0}$

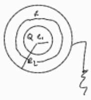
$E \cdot d = U$



$\frac{U}{d} = \frac{Q}{A\varepsilon_0}$

$E = \frac{U}{d}$

$C = \frac{Q}{U} = \frac{A\varepsilon_0}{d}$



$E \, 2r\pi l = \frac{Q}{\varepsilon_0}$

$E = \frac{Q}{2\pi\varepsilon_0 l} \frac{1}{R}$

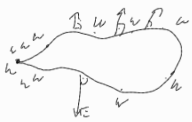
$U = -\int_{r_2}^{r_1} E_r\, dr = -\int_{R_1}^{R_1} \frac{Q}{2\pi\varepsilon_0 l} \frac{1}{r}\, dr$

$= \frac{Q}{2\pi\varepsilon_0 l} \int_{R_1}^{R_2} \frac{1}{r}\, dr = \frac{Q}{2\pi\varepsilon_0 l} \ln\left(\frac{R_2}{R_1}\right)$

$C = \frac{Q}{U} = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{R_2}{R_1}\right)} \Rightarrow \frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}$

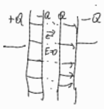
Metal

1. Equipotential

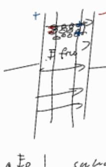


2. Hollow = bulky

3. Sharper surface $\Rightarrow$ Higher E



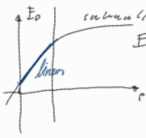
Dielectric Material



Polarization

$\underline{E} = E_{fee} - E_p$

$E_{fee} = \underline{E} + \underline{E}_p$



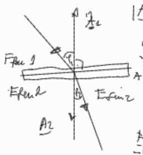
saturation / non linear Succeptibility

$E_p = \chi E$

$E_{fee} = \underline{E} + \chi \underline{E}$

$E_{fee} = (1+\chi)\underline{E} = \varepsilon_r \underline{E} \; \left[\frac{V}{m}\right]$

$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$



$|\underline{A}_1| = |\underline{A}_2| = A$

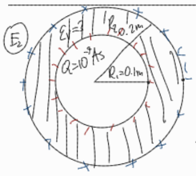
$\oint E_{fee} \cdot d\underline{A} = 0$

$E_{fee} A_1 + E_{fee} A_2 = 0$

$\underline{E}_{fee1} \underline{A}_1 = \underline{E}_{fee2} \underline{A}_2$

$|\underline{E}_{fee2}| \, \cancel{|\underline{A}_1|} \cos\varphi_1 = |\underline{E}_{fee1}| \, \cancel{|\underline{A}_1|} \cos\varphi_2$

$|\underline{E}_{fee1}| =$



$E_{fee} = (1+\chi)E$

(a) Find $E_{fee}(r)$; $E(r)$ R_1; R_2

(b) Find $U(R_1, R_2)$

$\oint \underline{E}_{fee} \cdot d\underline{A} = \frac{Q}{\varepsilon_0}$

$E_{fee} \, 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$E_{fee} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} = k \frac{Q}{R^2}$

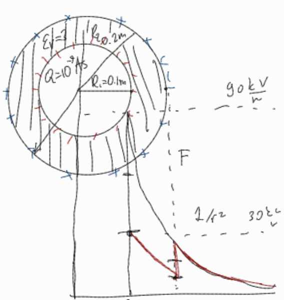
1.) $r < R_1 \quad Q = 0 \Rightarrow E_{fee} = 0$

2.) $R_1 < r \quad E_{fee} = 9 \cdot 10^9 \cdot \frac{10^{-7}}{r^2}$

$= 900 [Vm] \frac{1}{r^2}$

$E_{fee} \; R_1 = 90 \left[\frac{kW}{m}\right]$

3.) $E(r) = \frac{E_{fee}}{\varepsilon_r} = 300 \frac{1}{r^2} \; [Vm]$

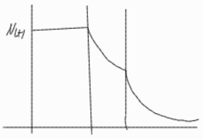


$U(R_1, R_2) = -\int_{R_2}^{R_1} E(r)\, dr = -\int_{R_1}^{R_2} 300 \frac{1}{r^2}\, dr$

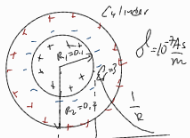
$= 300 \left[\frac{1}{r}\right]_{R_2}^{R_1} = -300 \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= \underset{(Vm)}{300} \left[\underset{\left(\frac{1}{m}\right)}{10 - 5}\right]^1 = 300 \cdot 5 V = 1500 V = 1.5 kV$

$U(R_1, \infty) = \frac{Q}{4\pi\varepsilon_0 R_1} = \frac{E(r)}{R_1} = \frac{9000 (Vm)}{2 m} = 4500 V$



Cylinder



$\oint E_{fee}(r) \cdot dA = \frac{l\delta}{\varepsilon_0}$

$E_{fee} \oint_{cylinder} dA = \frac{\lambda l}{\varepsilon_0}$

$E_{fee} \, 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0}$

$E_{fee} = \frac{l}{2\pi\varepsilon_0} \frac{1}{r}$

$E_{free}(r) = 2 \cdot 9 \cdot 10^9 \cdot 10^{-7} \frac{1}{r}$

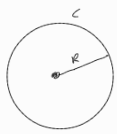
$= 1800 [V] \frac{1}{R_1} \left[\frac{1}{m}\right] = 18 \frac{kV}{m}$

$E(R_1) = \frac{E_{fee}}{\varepsilon_r} = 600 \frac{1}{R_1} \left[\frac{V}{m}\right] = 6 \frac{kV}{m}$

$U(R_1, R_2) = -\int_{R_2}^{R_1} 600 V \cdot \frac{1}{R}\, dr = 600 V \int_{R_1}^{R_2} \frac{1}{R}\, dr$

$= 600 V \ln\left(\frac{R_2}{R_1}\right) = 600 \cdot \ln 2 \cdot V = 416 V$

$\boxed{C = 4\pi\varepsilon_0 R}$



$C = \frac{Q}{U} \quad 4\pi\varepsilon_0 R \cdot U = Q$

$27 \cancel{\phi} r_s^3 = \cancel{\phi} r_b^3$

$27 r_s^3 = r_b^3$

$3 r_s = r_b$

$C = \frac{1}{9 \cdot 10^9} 10^{-3} = \frac{1}{9} pF$

$Q = \underbrace{4\pi\varepsilon_0 R}_{C} \cdot U = \frac{1}{9} 10^{-12} 10^3 = \frac{1}{9} 10^{-9} C$

$Q^* = 27 Q \quad R^* = 3R$

$Q^* = 27 \frac{1}{9} \cdot 10^{-9} C = 3 \cdot 10^{-9} C$

$E_{mem} = \frac{Q}{2} U$

$E = 27 \cdot \frac{Q}{2} U = 27 \frac{1}{2} \left(\frac{1}{9} \cdot 10^{-9}\right) \cdot 10^3 = \frac{3}{2} \cdot 10^{-6} J$

$E^* = \frac{1}{2} Q^* \cdot U^* = \frac{1}{2} \, 3 \cdot 10^{-9} \cdot 9 \cdot 10^3 = \frac{27}{2} 10^{-6} J$

$\frac{27-3}{2} 10^{-6} J = 12 \mu J$

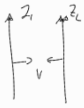
$C(R) \quad U$

$Q\,C(R) \Rightarrow Q^*$

$R \Rightarrow R^* \quad U = \frac{Q^*(R)}{C(R^*)}$

$E = \frac{1}{2} QU$

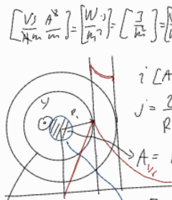
Exercises

1.) 

$F_{12} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r} \ell$ $\quad 2 \cdot 10^{-7} \frac{Vs}{Am}$

$\frac{F_{12}}{\ell} = 2 \cdot 10^{-7} \frac{I_1 I_2}{r}$

$\left[\frac{Vs}{Am}\frac{A^2}{m}\right] = \left[\frac{W}{m}\right] = \left[\frac{J}{m}\right] = \left[\frac{N}{m}\right]$



$i\,[A]$

$j = \frac{i}{R_1^2 \pi} = 1$

$A = \pi R_1^2$

$\oint H\, dr = \sum i \Rightarrow 2r\pi H_{in} = \sum i$

$2r\pi H = A j$

$H_{in} = r \frac{1}{2} j \Leftarrow H = \frac{\pi r^2}{2r\pi} j$

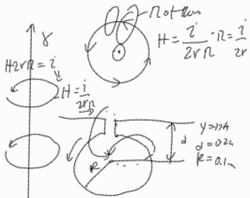
$R_1 \to \infty$

$2 r \pi H_{out} = \pi R_1^2 j$

$H_{out} = \frac{1}{2} \frac{R^2}{r} j = c \frac{1}{r}$

$R_1 \to R_2$

$B = \mu_0 \mu_r H$



$H = \frac{i}{2r\pi} \cdot \pi = \frac{i}{2r}$

$H 2r\pi = i$

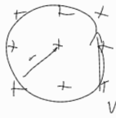
$H = \frac{i}{2r\pi}$

$y = 12A$, $d = 0.2 \ldots$, $R = 0.1m$

$H_\ell = \frac{i}{2r}$ $\quad H_c = \frac{i}{2r\pi}$

$H_e = H_\ell - H_C = \frac{i}{2r}\left(1 - \frac{1}{\pi}\right) = \frac{12}{0.2}(1 - 0.31)$

$= \frac{12}{3.4} = 3.5 \frac{A}{m}$

2.) 

$F_L = Q(v \times B)$

$Q = 1.6 \cdot 10^{-19} A_s$

$m = 1.67 \cdot 10^{-27} kg$

$V = \omega r$

$m \frac{v^2}{r} = Q v \cdot B$

$\frac{v}{r} = \omega = \frac{QB}{m} = \frac{Q}{m} \Omega$

$\omega = -\frac{Q}{m} B$

$m = 1.67 \cdot 10^{-27} kg$

$v = \frac{m v}{Q \pi}$

$\frac{1}{2} m v^2 = Q V$

$= 10^{-16} J$

$\frac{10^{-16} J}{Q} = \ldots$

$\frac{V \cdot e}{e} = \frac{m}{1.6 \cdot 10^{-19}}$

$u_{AB} = \int_A^B E(r)\, dr = \int_{R_1}^{R_2} 600 \frac{1}{r} dr = 600V \left[\ln(r)\right]_{R_1}^{R_2}$

$= 600[V] \cdot \ln\left(\frac{R_2}{R_1}\right) = 416V$.

$m = 1.67 \cdot 10^{-27} kg$

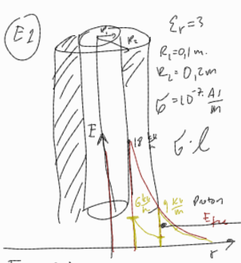
$q = 1.6 \cdot 10^{-19} A_s$

$u(R_1, R_3) = \int_{R_1}^{R_3} [1800V] \frac{1}{r} dr = 1800V \ln\left(\frac{R_3}{R_1}\right) = 2897V$

$QV = \frac{1}{2} m v^2 \Rightarrow V = \sqrt{\frac{2QV}{m}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 2897}{1.67 \cdot 10^{-27}}}$

$= \sqrt{5745 \cdot 10^{8}} = 76 \cdot 10^{4} \frac{m}{s}$

(E1)



$\varepsilon_r = 3$

$R_1 = 0.1m$

$R_2 = 0.2m$

$\sigma = 10^{-7} \frac{As}{m}$

$\sigma \cdot \ell$

$R_3 = 1m$

Swinging ring exp.

$E_{free}(r)$, $E(r)$ $\quad E_{free}(R_1) = ?$ $\quad E(R_1) = ?\ E(R_2)$

$\oint E\, dA = \frac{Q_0}{\varepsilon_0} = \frac{\sigma \cdot \ell}{\varepsilon_0}$

$E_r 2 r \pi \ell = \frac{\sigma \ell}{\varepsilon_0}$ $\quad k = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9$

$E(r) = \frac{1}{2\pi\varepsilon_0 r} \sigma = 2 \cdot 9 \cdot 10^{9} \cdot 10^{-7} \left(\frac{1}{r}\right)[V] = 18 \cdot 10^{2} [V] \left(\frac{1}{r}\right)$

$E = \frac{E_f}{\varepsilon_r} = 6 \cdot 10^{2} [V] \left(\frac{1}{r}\right)$

$\oint E_n\, dA = \frac{Q_0}{\varepsilon_r}$

$\oint E_n\, dA = 0$

$E_{fr} = 0 \quad r < R_1$

$E_{fr}(R_1) = 18 \cdot 10^{2} \frac{1}{10^{-1}} \left[\frac{V}{m}\right] = 18 \cdot 10^{3} \left[\frac{V}{m}\right] = 18 \left[\frac{kV}{m}\right]$

$E(R_1) = 6 \left[\frac{kV}{m}\right]$

$E(R_2) = 6 \cdot 10^{2} \frac{1}{2 \cdot 10^{-1}} \left[\frac{V}{m}\right] = 3 \cdot 10^{2} \left(\frac{V}{m}\right)$

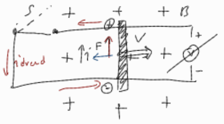
Magnetic induction

Motion related

ϕ_m $\quad U_i = \frac{d\phi_m}{dt}$

Faraday induction law

Lenz's law



$F_L = Q(v \times B) = \frac{F_L}{Q} = E_L = v \times B$

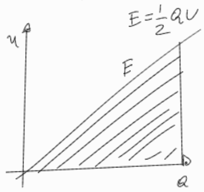
$u = E_L \cdot l = B v l$ $\quad i = \frac{Bvl}{R}$

$F_1 = i l \times B \leftarrow$

$P_{el} = \frac{(Bvl)^2}{R}$

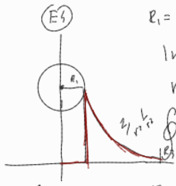
$P_{mech} = F v = \frac{(Blv)^2}{R}$

$p = \frac{Blv}{R} l B = i^2 R$



$E = \frac{1}{2} Q U$

(E3)



$R_1 = 0.1m$ $\quad Q = 10^{-7} Am$ $\quad E(r)$

$\ln s = 0 \quad r < R_1$

$r \geq R_1$

$\oint E\, dA = \frac{\sum Q_0}{\varepsilon_0}$

$E = \frac{Q_0}{4\pi\varepsilon_0} \frac{1}{r^2}$

$u(R_1, R_3) = \int_{R_1}^{R_3} [900 Vm]\left(-\frac{1}{r^2}\right) dr$

$= 900Vm \left[\frac{1}{r}\right]_{R_3}^{R_1}$

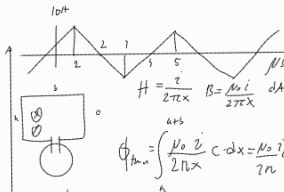
$= 900Vm \left(\frac{1}{0.1} - \frac{1}{1}\right) = 8100V$

$E(r) = 9 \cdot 10^{9} \cdot 10^{-7} \frac{1}{r^2} [Vm] = 9 \cdot 10^{3} \frac{1}{r^2} [Vm]$

$E(R_1) = 9 \cdot 10^{3} \cdot 10^{2} \left[\frac{V}{m}\right] = 90 \cdot \left[\frac{kV}{m}\right]$

$qV = \frac{1}{2} m v^2 \Rightarrow V = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 8100V}{1.67 \cdot 10^{-27}}} = 1.25 \cdot 10^{6} \frac{m}{s}$

$N = 100$



$H = \frac{i}{2\pi x}$ $\quad B = \frac{\mu_0 i}{2\pi x}$ $\quad dA = c \cdot dx$

$\phi_{Am} = \int_a^{a+b} \frac{\mu_0 i}{2\pi x} c \cdot dx = \frac{\mu_0}{2\pi} i c \ln\frac{a+b}{a}$

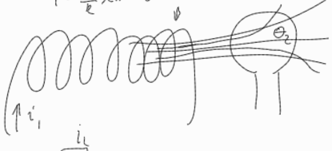
$\frac{V_i}{A} = \frac{\phi}{A}$

$u_{turn} = \frac{d\phi_{turn}}{dt} = \frac{\mu_0 c}{2} \ln\left(\frac{a+b}{a}\right)\frac{di}{dt}$

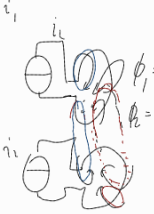
$u_{coil} = \frac{\mu_0 N c}{2\pi} \ln\left(\frac{a+b}{a}\right)\frac{di}{dt} = 9.6V$

$\oint H\, dr = \sum i$

$\frac{10A}{1\mu s} = 10^{7} \frac{A}{s}$ $\quad M = 9.6 \cdot 10^{-7}$



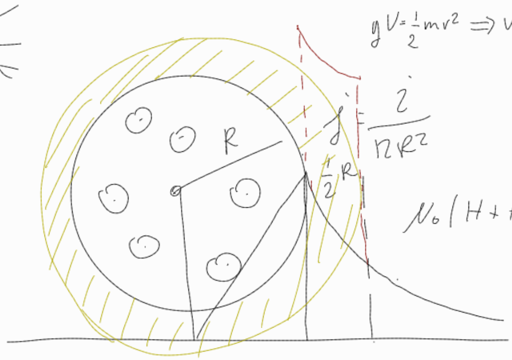
i_1



$\phi_1 = L_1 i_1 + M i_2$

$\phi_2 = L_2 i_2 + M i_1$

i_2



$j = \frac{i}{\pi R^2}$

$\mu_0 (H + H_m) = B$

$J = i u t$

$dJ = i L \frac{di}{dt} dt$

$dJ = i L di$

$J = \frac{1}{2} L i^2$

$\int_0^J dJ = \int_0^i i L\, di$

A.L $\quad r < R$

$\oint H\, dr = \sum i$

$H\, 2r\cancel{\pi} = j\, r^2 \cancel{\pi}$

$H = \frac{j}{2} r$

$r \geq R$

$\oint H\, dr = \sum i$

$H\, 2r\cancel{\pi} = j\, \cancel{\pi} R^2$

$H = \frac{j}{2} R^2 \frac{1}{r}$