Exercise-set 5.

1. Determine the chromatic number of the complement of the cycle on n vertices.
2. Let the vertices of the graph G be the numbers 1,2,...,2015, and two vertices be adjacent if and only if the difference of the corresponding numbers is at most 9. Determine $\chi(G)$, the chromatic number of G.
3. (MT+'16) Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 30\}$. Let the vertices $x, y \in V(G)$ be adjacent in G if the difference of the numbers x and y is at least 7. Determine $\chi(G)$, the chromatic number of G.
4. (MT++'03) Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 100\}$. Let the vertices $x, y \in V(G)$ be adjacent in G if $x \neq y$ and $100 \leq x \cdot y \leq 400$. Determine the value of $\chi(G)$.
5. (MT++'09) Let the vertices of the graph G be the numbers 1,2,...,15, and two vertices be adjacent if and only if one of the corresponding numbers divides the other. Determine $\chi(G)$, the chromatic number of G.
6. Let the vertices of the graph G be the numbers 1,2,...,30, and two vertices be adjacent if and only if the corresponding numbers are relatively prime. Determine $\chi(G)$, the chromatic number of G.
7. (MT'16) Let the vertex set of the graph G on 9 vertices be the vertices of the unit cube together with the center of it, i.e. $V(G) = \{(x, y, z) : x, y, z \text{ are } 0 \text{ or } 1\} \cup \{(1/2, 1/2, 1/2)\}$. Let two vertices of G be adjacent if they differ either in the first or the second coordinate, or both. (E.g. $(0, 0, 1)$ is adjacent to $(0, 1, 1)$ and $(1, 1, 0)$ but not to $(0, 0, 0)$.) Determine $\chi(G)$, the chromatic number of G.
8. (MT++'15) In a simple graph G on 10 vertices the degree of each vertex is 8. Determine the chromatic number of G.
9. (MT'19) Let the vertex set of the complete graph K_{10} be $V(K_{10}) = \{1, 2, ..., 10\}$. We obtain the graph G by deleting the edges $\{1, 2\}, \{1, 3\}, \{2, 3\}$ and $\{4, 5\}, \{4, 6\}, \{5, 6\}$ from K_{10}. Determine $\chi(G)$, the chromatic number of G.
10. (MT+'19) We add two edges to a bipartite graph on 10 vertices. Is it possible (with an appropriate choice of the graph and the added edges) that the chromatic number of the graph obtained is 4?
11. (MT++'19) Let the vertices of the graph G be the numbers $1, 2, \ldots, 100$, and two (different) vertices be adjacent if and only if at least one of 2, 3 or 5 is a common divisor of the respective numbers. Determine $\chi(G)$, the chromatic number of the graph G.
12. (MT+'20) The simple graph G on 9 vertices consists of a cycle on 3 vertices and a cycle on 7 vertices with exactly one vertex in common. Determine the chromatic number of the complement of G.
13. (MT++'20) We delete the edges of a Hamilton cycle from K_8, the complete graph on 8 vertices. Determine the chromatic number of the graph obtained.

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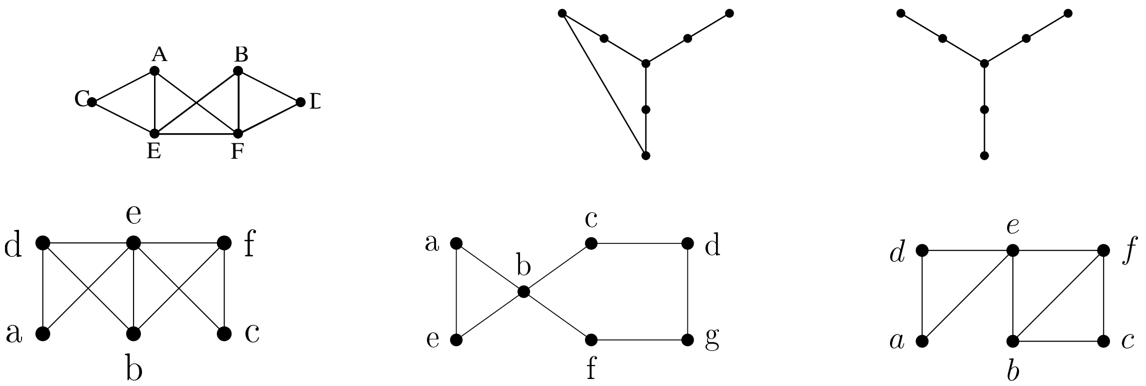
14. Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 2021\}$. Suppose that every vertex of G is adjacent to at most 10 smaller numbers. Prove that $\chi(G) \leq 11$.
15. In the simple graph G apart from 100 exceptional vertices the degree of each vertex is at most 99. Prove that $\chi(G) \leq 100$.

16. (MT++'15) A simple graph G on 10 vertices contains one vertex of degree 5, one of degree 4, one of degree 3, and the rest of the vertices have degree 2. Show that G can be colored with 3 colors.

17. Show that in a loop-free graph G on n vertices the following hold, where $\alpha(G)$ denotes the size of a maximum independent set in G:
a) $\chi(G) \cdot \alpha(G) \geq n$,
b) $\chi(G) + \alpha(G) \leq n + 1$.

18. Is it true that every simple graph G has a coloring with $\chi(G)$ colors in which one color class contains $\alpha(G)$ vertices?

19. (MT+'07, MT'15, MT+'17, MT++'17, MT+'18) Decide whether the following graphs are interval graphs or not. (A graph is an interval graph if the vertices of it are closed intervals on the real line, and and two vertices are adjacent if and only if the corresponding intervals intersect.)



20. (MT+'11) Consider those intervals on the number line whose endpoints are both integers between 1 and 100, their length is at least 1 and at most 4, and at least one of their endpoints is an even number. Determine the chromatic number of the interval graph determined by them.

21. (MT'16) Delete 4 edges from the complete graph on 8 vertices, in such a way that all of them are incident to a given vertex. Determine whether the graph obtained is an interval graph or not.

22. (MT'14) The chromatic number of an interval graph belonging to a given system of intervals is 10. Prove that if we delete a few intervals from the system in such a way that no three of them have a common point, then the interval graph belonging to the remaining intervals has chromatic number at least 8.