Problem set 13.
Approximation algorithms.

1. Consider the bin packing problem with the following weights: 6 elements each with weight $1/2 + \varepsilon$, 6 elements each with weight $1/4 + 2 \cdot \varepsilon$, 6 with weight $1/4 + \varepsilon$, and 12 with weight $1/4 - 2 \cdot \varepsilon$. (Here ε is a very small quantity, say, $\varepsilon = 1/1000$.)
   (a) What is the optimal solution in this case?
   (b) How many bins does FFD (First Fit Decreasing) use?

2. (a) What is the decision problem version of BINPACKING?
   (b) Show that this decision problem is NP-complete.

3. Let $t \geq 2$ be a whole number and assume that in the BINPACKING problem, weight of every element is smaller than $\frac{1}{t}$ (that is, $s_i < \frac{1}{t}$ for all $i \in 1, \ldots, n$). Design an approximation algorithm that runs in polynomial time and that uses atmost $\frac{t}{t-1} \cdot OPT + 1$ bins.