Exercise-set 2.
Solutions

1. a) 5.
   c) Can be 1, 2, 3, 4 or ∞.
2. a) 2.
   c) Can be 1, 2 or ∞.
3. a) For non-simple graphs no.
   b) For simple graphs yes: they cannot have (at least) 2 components.
4. The graph cannot have 3 components.
5. There are at most 2 components.
6. In $\overline{G}$ the degrees are the same. It cannot have two components.
7. If G is not connected, then the edges between the components of it make $\overline{G}$ connected.
8. In every connected component of a graph there is an even number of odd-degree vertices.
9. There are 1, 2 3 an 6 of them, respectively.
10. a) $n_1 \cdot 1 + n_2 \cdot 2 + 5 \cdot 3 = 2(n_1 + n_2 + 5 - 1) \implies n_1 = 7$.
11. One of the degrees is 1. $d \cdot 9 + 92 \cdot 1 = 200 \implies d = 12$.
12. $n/2 \cdot (1 + k) = 2n - 2 \implies 4 = n(3 - k)$. Contradiction, if $n \geq 5$.
13. All the other vertices have degree 1.
14. The tree has an even number of vertices.
15. $10(n-1) = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 22$.
16. Necessary: $n - 1 = \binom{n}{2} - (n-1) \implies n = 1$ or $n = 4$. Both are possible.
17. 20 ($e = n - k$).
18. A forest would have 17 edges.
19. Otherwise the sum of the degrees would be too large: $1 \cdot \Delta + (\Delta - 1) \cdot 1 + (n - \Delta) \cdot 2 > 2(n-1)$.
20. a) no;
    b) yes.
21. A graph is a spanning tree and 3 more edges, each of which forms a cycle with the tree.
22. The graph contains a cycle, of length at least 3.
23. Two edges cannot form a cycle in a simple graph.
24. A degree one vertex in a spanning tree is like that.