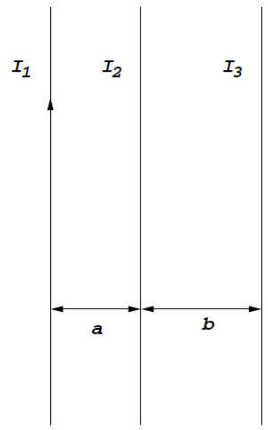
Problem M1

There are three long parallel wires as it is shown by the figure. There is no force on any of the wires. Give the currents in the second and the third wires if $I_1 = 10\ A$, $a = 0.1\ m$ and $b = 0.15\ m$.



Solution M1

The forces between the parallel wires are as follows:

$$F_{12} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a} l \qquad F_{13} = \frac{\mu_0}{2\pi}\frac{I_1 I_3}{a+b} l \qquad F_{23} = \frac{\mu_0}{2\pi}\frac{I_2 I_3}{b} l$$

Total force on wire 3 is $F_3 = F_{13} + F_{23}$

$$F_3 = \frac{\mu_0}{2\pi}\frac{I_1 I_3}{a+b} l + \frac{\mu_0}{2\pi}\frac{I_2 I_3}{b} l$$

Because of $F_3 = 0$ is zero. Bunch of factors cancel out.

$$\frac{\mu_0}{2\pi}\frac{I_1 I_3}{a+b} l + \frac{\mu_0}{2\pi}\frac{I_2 I_3}{b} l = 0$$

$$\frac{I_1}{a+b} + \frac{I_2}{b} = 0$$

$$I_2 = -I_1 \frac{b}{a+b} = -10\frac{15}{25} A = -6A$$

Total force on wire 1 is $F_1 = F_{12} + F_{13}$

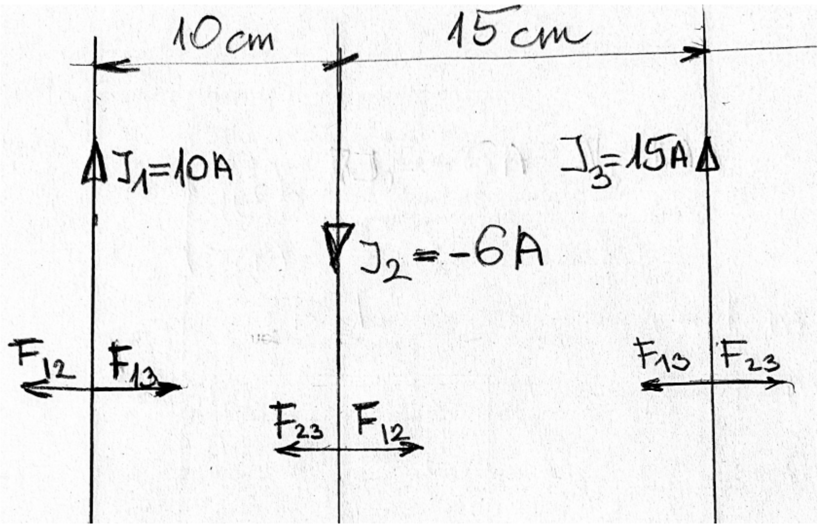
$$F_1 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a} l + \frac{\mu_0}{2\pi}\frac{I_1 I_3}{a+b} l$$

Because of $F_1 = 0$ is zero. Bunch of factors cancel out.

$$\frac{\mu_0}{2\pi}\frac{I_1 I_2}{a} l + \frac{\mu_0}{2\pi}\frac{I_1 I_3}{a+b} l = 0$$

$$\frac{I_2}{a} + \frac{I_3}{a+b} = 0$$

$$I_3 = -I_2 \frac{a+b}{a} = -(-6A)\frac{25}{10} = 15A$$



Checkpoint:
$F_2 = 0$ must be zero with the numerical results above: $I_2 - 6A$ and $I_3 = 15A$.
Since there are repulsive forces affecting the wire 2 from both sides, the resulting F_2 force is the difference of F_{12} and F_{23}. In addition this equilibrium is stable, because if either of the wires is approached closer, the increasing repulsion pushes back wire 2 to its original place.

Total force on wire 2 is $F_2 = F_{12} - F_{23}$

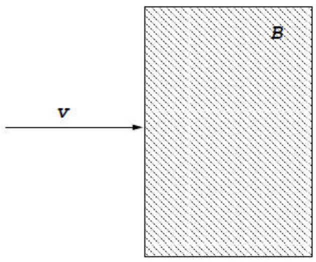
$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{a} l - \frac{\mu_0}{2\pi}\frac{I_2 I_3}{b} l$$

$$F_2 = \frac{\mu_0}{2\pi} l \left(\frac{I_1 I_2}{a} - \frac{I_2 I_3}{b} \right) = \frac{\mu_0}{2\pi} l \left(\frac{10 \cdot (-6)}{0,1} - \frac{(-6) \cdot 15}{0,15} \right) = 0$$

Q.E.D.

Problem M2

In the half of the space uniform magnetic field of $B = 0.5\ T$ perpendicular to the plane of the paper is present as it is indicated by the shaded area in the figure below. An ionized carbon atom is arriving to the boundary of the magnetic field with constant velocity of $v = 10^5\ m/s$ perpendicular to the magnetic field. The mass of a carbon atom is 12 times the mass of a proton. The mass of a proton is $m_p = 1.67 \times 10^{-27}\ kg$ and the charge of the ion is $Q_i = 1.6 \times 10^{-19}\ C$.



a.) What kind of motion will the electron have in the presence of uniform magnetic field? Make a plot in the figure!

b.) Where will the electron leave the area where the magnetic field is present?

c.) How large will the speed of the electron leaving the magnetic field?

d.) How much work has been done by the magnetic field on the electron?

e.) Find the kinetic energy of the carbon ion in (eV) units.

Solution M2

a.) In the lecture, this topic has been treated. We know that in homogeneous B magnetic field an ion goes around on a circle, provided the entering velocity is normal to the magnetic field. The Newton equation of this uniform circular motion is as follows:

$$m\frac{v^2}{r} = Q \cdot v \cdot B$$

The left hand side is the mass and the centripetal acceleration of the circular motion. The right hand side is the Lorentz force, which is just a simple product of magnitudes of the vector quantities, due to the perpendicular position of the entering velocity and the B magnetic field. The resulting force always points to the center of the circle.

After the velocity cancels out on the right, the formula of the radius results:

$$r = \frac{mv}{QB} = \frac{12 \cdot 1{,}67 \cdot 10^{-27} \cdot 10^5}{1{,}6 \cdot 10^{-19} \cdot 0{,}5} \approx \frac{12 \cdot 10^{-27} \cdot 10^5}{10^{-19} \cdot 0{,}5} = 2{,}4 \cdot 10^{-2}\, m$$

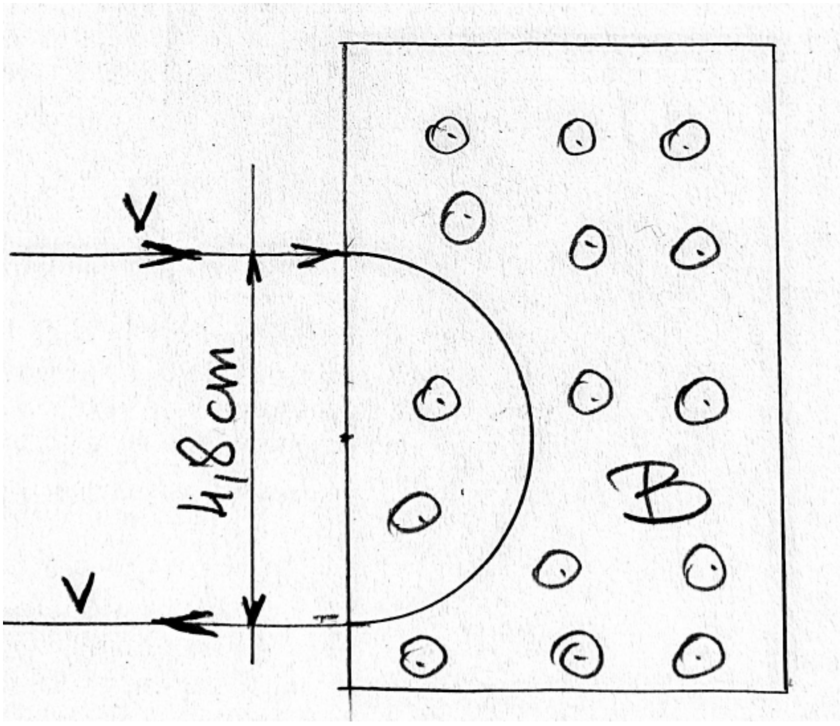
The cancellation of 1,67 and 1,6 can be done in this class always. The ion makes a half circle in the magnetic field, and exits from the magnetic field at 4,8 centimeter distance either up or down relative to the entrance.

The circulation angular frequency is called the cyclotron frequency (ω_c), as it was mentioned in the lecture. From the above formula one can conclude:

$$\frac{v}{r} = \omega_C = \frac{QB}{m}$$

This shows that the cyclotron frequency is independent of the entering velocity of the ion, provided the velocity is much less than the speed of light. This is of course true here. This independence made possible to construct the first particle accelerator, mentioned in the lecture in more in detail.

b.) The drawing shows that the magnetic field is directed out of the sheet of the paper, so by considering the direction of the Lorentz force $\mathbf{F}_L = Q(\mathbf{v} \times \mathbf{B})$ one can conclude that the half circle will be down relative to the entrance point. This means that the exit point will be 4,8 centimeter down from the entrance point.



c.)The direction of the Lorentz force is always normal to the velocity, which means that no tangential acceleration occurs. Therefore the magnitude of the velocity never changes by the effect of any magnetic field, not just by the homogeneous one right in this problem. So the speed of the exiting ion will be the same as that of the entering one, which is $v_{exit} = 10^5 m/s$.

d.) The direction of the Lorentz force is always normal to the velocity, which means that no work has ever been done by the magnetic field. Therefore the magnitude of the kinetic energy never changes by the effect of any magnetic field, not just by the homogeneous one right in this problem.

e.)So the velocity of the ion is constant $10^5 m/s$ all the way in this problem. The electron-volt (eV) unit is frequently used in atomic physics. The classically expressed Joule value should be divided with the elementary charge to create the eV result. This is very practical from two points of view. Firstly the Joule based numerical values are very small in the world of particles. This could be amended by using prefixes (femto, ato…). Secondly, the electron-volt unit has the further advantage, namely the value immediately expresses the beam-voltage of the particles. This means that a particle with elementary charge going through this voltage drop will gain this amount of energy.
According to the paragraph above the equation is as follows:

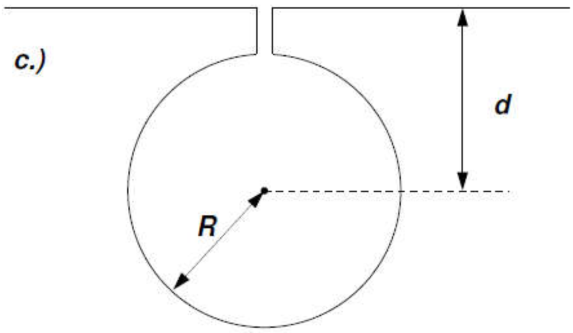
$$\frac{1}{2}mv^2 = Q \cdot U_{eV}$$

$$U_{eV} = \frac{mv^2}{2Q} = \frac{12 \cdot 1{,}67 \cdot 10^{-27} \cdot 10^{10}}{2 \cdot 1{,}6 \cdot 10^{-19}} \approx \frac{12 \cdot 10^{-17}}{2 \cdot 10^{-19}} = 600eV$$

The cancellation of 1,67 and 1,6 can be done in this class always.
So the "beam voltage" or the kinetic energy of the carbon ions is 600 electron-volts.

Problem M3

Determine the magnetic field at the center of the circle shown in the figure.



$$I = 12\ A$$
$$d = 0.2\ m$$
$$R = 0.1\ m$$

Solution M3

There are three important points we must know to solve this problem.

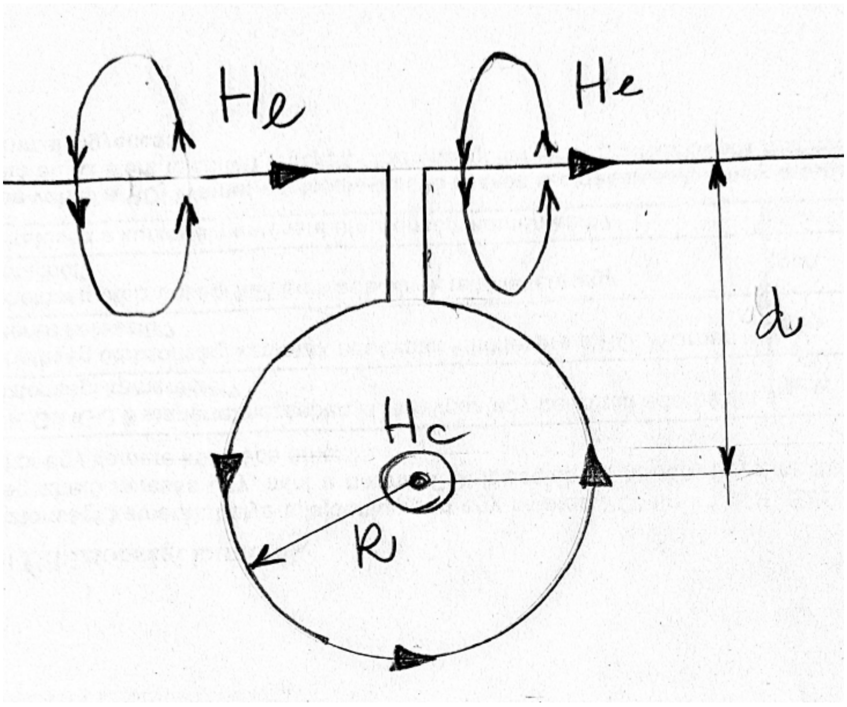
1. Magnetic field around a straight infinite wire. $H_{line} = \dfrac{I}{2r\pi}$. This is the direct consequence of the Ampere's law, which means that the curve integral of the magnetic field around the current is equal with the total current surrounded by the curve. The direction of the magnetic force lines creates a right hand screw rotation to the direction of the current.

2. Magnetic field in the center of a circle: $H_{circle} = \dfrac{I}{2r}$. In words: current over the diameter. This is the conclusion of the Biot-Savart law, by integrating the magnetic field in the center of the circle. In the lecture notes, this result comes out as a limit of an n sided regular polygon. The direction of the magnetic force lines creates a right hand screw rotation to the direction of the current in the circle.

3. Magnetic effect of a straight current, in the line of the straight current is zero. Those two wires which connect the horizontal straight wire to the circle have no magnetic effect to the center of the wire. Not just because it looks relatively short, that could be much longer, but because the line of these straight wires goes through the center of the circle, thus have no magnetic effect on it.

In the solution of this problem, the correct identification of the variables is crucial. In the exposition of the problem, we used the general variable *r*, which should be identified with the parameters defined by the concrete problem. The correspondence for the straight wire and the circular section are: $r := d$, and $r := R$ respectively.

In addition the directions of the above mentioned sections are opposite. Assume that the current flows from left to the right in the figure. Then the direction of the magnetic field generated by the straight wire section is pointing into the sheet of paper, due to the right hand screw rotation. The current in the circular section generates a counter clockwise rotation (CCW), so the magnetic fields points out of the sheet. Ultimately the magnitudes work against each other, and the stronger prevails.



$$H_{line} = \frac{I}{2d\pi} \qquad H_{circle} = \frac{I}{2R}$$

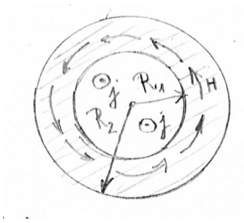
$$H_{tot} := H_{circle} - H_{line}$$

$$H_{tot} = \frac{I}{2R} - \frac{I}{2d\pi} = \frac{I}{2}\left(\frac{1}{R} - \frac{1}{d\pi}\right)$$

$$H_{tot} = \frac{I}{2}\left(\frac{1}{R} - \frac{1}{d\pi}\right) = \frac{12}{2}\left(\frac{1}{0{,}1} - \frac{1}{0{,}2\pi}\right) = 6\left(10 - \frac{5}{\pi}\right) = 50{,}4\frac{A}{m}$$

Ultimately the magnetic field of the circular section is stronger, so the direction of the resulting magnetic field points out from the sheet of the paper.

Problem M4



Infinite copper rod with the radius of $R_1 = 3cm$ is conducting a high current of $10^3 A$. The copper rod is coated with an insulating ferrite magnetic layer in $2cm$ thickness, so the total radius is equal with $R_2 = 5cm$. The magnetic material is characterized by $\mu_r = 10^3$ value.

a.) Find the current density in the copper, provided the distribution is uniform.
b.) Find and sketch the $H(r)$ magnetic field as the function of the radius. Determine the numerical values in the breakpoint $H(R_1)$ and $H(R_2)$ too.
c.) Find and sketch the $B(r)$ magnetic induction as the function of the radius. Determine the numerical values in the breakpoints: $B(R_1)$, $B(R_2)$.

Solution M4
a.) The current density j is as follows:

$$j = \frac{I}{R_1^2 \pi} = \frac{10^3}{\left(3 \cdot 10^{-2}\right)^2 \pi} = \frac{10^7}{9\pi} = 3{,}54 \cdot 10^5 \frac{A}{m^2}$$

b.) Ampere's law is the key to the magnetic field.

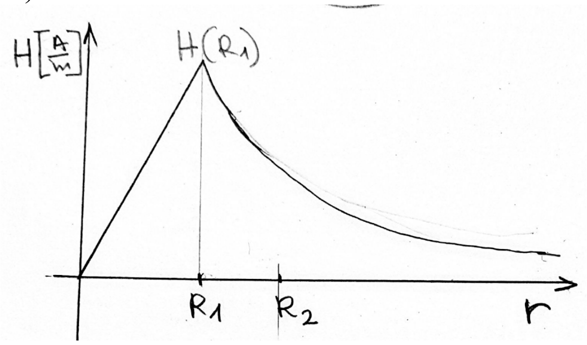
$$\oint_C \mathbf{H} \cdot \mathbf{dr} = \sum_i I_i$$

We know that the magnetic field is cylindrically symmetric around an infinite current. Therefore the integral on the left hand side is simply the product of the magnitude of the magnetic field and the circumference of the circle. The summa on the right hand side is the current; which is contained by the actual integration loop.

Inside the copper	Out of the copper
$2r\pi \cdot H_{in} = r^2 \pi \cdot j$	$2r\pi \cdot H_{out} = R_1^2 \pi \cdot j$
$H_{in} = \frac{j}{2} r$	$H_{out} = \frac{j}{2} \cdot \frac{R_1^2}{r}$

Visibly, the two domains make a continuous function for the total space.

b.)

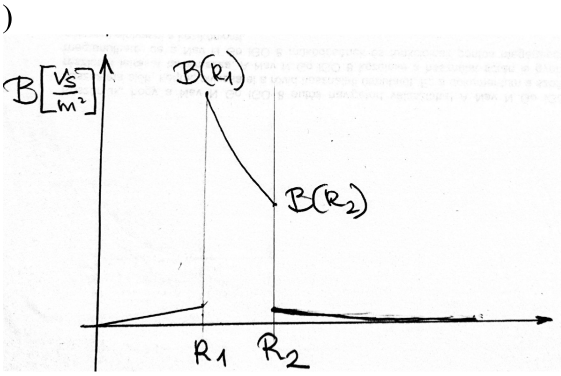


The magnetic field $H(r)$

$$H(R_1) = \frac{j}{2} R_1 = \frac{3{,}54 \cdot 10^5}{2} 3 \cdot 10^{-2} = 5310 \frac{A}{m}$$

$$H(R_2) = \frac{j}{2} \frac{R_1^2}{R_2} = \frac{3{,}54 \cdot 10^5}{2} \frac{9 \cdot 10^{-4}}{5 \cdot 10^{-2}} = 3186 \frac{A}{m}$$

c.)



The magnetic induction $B(r)$

$$B(R_1) = \mu_0 \mu_r H(R_1) = 4\pi \cdot 10^{-7} \cdot 10^3 \cdot 5310 \frac{Vs}{m^2} = 6{,}67T$$

$$B(R_2) = \mu_0 \mu_r H(R_2) = 4\pi \cdot 10^{-7} \cdot 10^3 \cdot 3186 \frac{Vs}{m^2} = 4{,}00T$$

Problem M5

Infinite straight wire carries a current with the time function $I(t)$ shown in the Fig.1. In the plane of the wire, there is a rectangular coil-frame with the number of turns $N = 100$. The induced voltage in the frame is displayed by the oscilloscope connected to its terminals. The relevant geometrical data are shown in Fig.2, along with other details.

a.) Find the mutual induction coefficient between the wire and the frame.

b.) Find and sketch the time function of the induced voltage displayed by the oscilloscope.

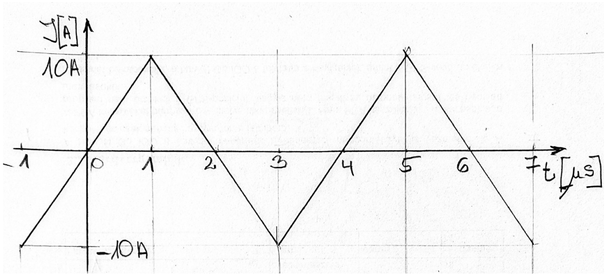


Fig.1.

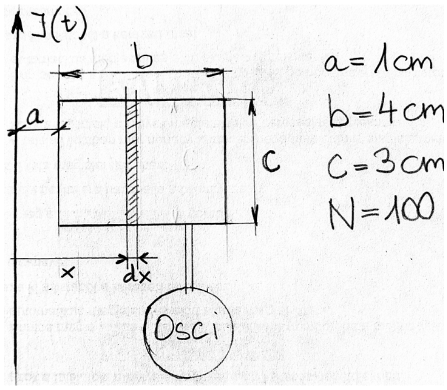


Fig.2.

Solution M5

The turn-flux in the coil should be found first.

The magnetic field and so the magnetic induction diminishes as one goes away from the current.

$$H = \frac{I}{2\pi \cdot x} \qquad\qquad B = \frac{\mu_0 I}{2\pi \cdot x}$$

The magnetic field is circular symmetric around the current, thus it is normal to the surface of the frame. The elementary surface therefore is parallel with the magnetic field. So the flux contribution is merely the product the *B* and *dA*.

$$dA = c \cdot dx \qquad\qquad d\Phi = B \cdot dA = \frac{\mu_0 I}{2\pi \cdot x} \cdot c \cdot dx$$

The magnitude of *B* changes by position, so integration is needed to find the total-turn-flux.

$$\Phi_{turn} = \int_a^{a+b} d\Phi = \frac{\mu_0 Ic}{2\pi} \cdot \int_a^{a+b} \frac{dx}{x} = \frac{\mu_0 Ic}{2\pi} \cdot \ln\left(\frac{a+b}{a}\right)$$

The turn-voltage is the time derivative of the turn-flux. At this point we are not making considerations if to use minus sign. At the end of the solution we clarify this detail.

$$U_{turn} = \frac{d\Phi_{turn}}{dt} = \frac{\mu_0 c}{2\pi} \cdot \ln\left(\frac{a+b}{a}\right)\frac{dI}{dt}$$

The coil-voltage is the *N* times larger value than the turn-voltage.

$$U_{coil} = NU_{turn} = \frac{\mu_0 Nc}{2\pi} \cdot \ln\left(\frac{a+b}{a}\right)\frac{dI}{dt}$$

The general formula of the induced voltage in a coupled system is written below. This contains *M*, which is called the mutual induction coefficient.

$$U_{coil} = M\frac{dI}{dt}$$

By comparison one can find:

$$M = \frac{\mu_0 Nc}{2\pi} \cdot \ln\left(\frac{a+b}{a}\right)$$

After numerical evaluation:

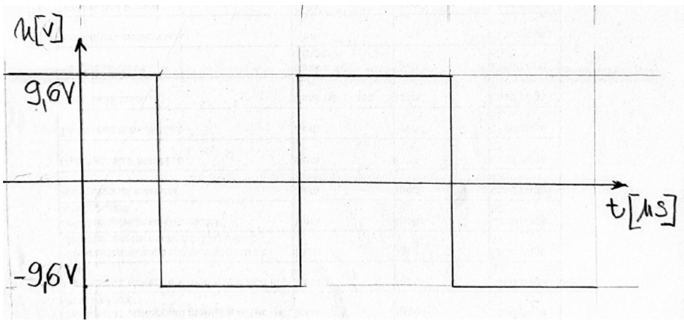
$$M = \frac{4\pi \cdot 10^{-7} \cdot 10^{2} \cdot 3 \cdot 10^{-2}}{2\pi} \cdot \ln(5) \approx 6 \cdot 1{,}6 \cdot 10^{-7} = 9{,}6 \cdot 10^{-7}\frac{Vs}{A} = 0{,}96\mu Henry$$

Now have a look at the Fig. 1. The time derivative of the current function obviously consists of constant sections with alternating polarity. So the induced voltage is rectangular function. The slopes of the current in the two parts are:

$$\frac{dI}{dt} = \pm 10^{7}\frac{A}{s}$$

By this way, the magnitude of the induced voltage also results.

$$U_{coil} = 9{,}6 \cdot 10^{-7}\frac{Vs}{A} \cdot (\pm)10^{7}\frac{A}{s} = \pm 9{,}6V$$

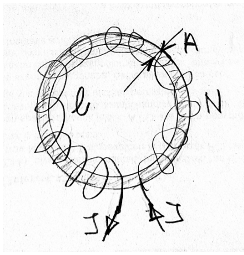


Now find out the condition that the above graph will be the right polarity of the voltage. The current is considered positive in upside direction. So in the positive current-slope domains the *B* magnetic induction will increase in down-direction in the coil-frame. Accordingly the flux will also increase in the coil, provided the surface-normal of the coil-frame is also directed down-direction. This is the case when the turn-direction of the coil is clockwise (CW). Now we have to consider Faraday induction law:

$$U_{turn} = -\frac{d\Phi_{turn}}{dt}$$

The minus in the above equation means that if the flux increases down-direction then the clockwise turned coil will show just the negative of the Fig.3. In order to see the displayed voltage-graph, the turning direction of the coil should be switched to CCW. This time the starting terminal of the coil goes to the ground of the oscilloscope, and the final terminal goes the sensitive-point of the oscilloscope. The more positive sensitive-point lifts up the bright spot on the screen.

Problem M6



There is a toroid coil with mean circumference of $l = 60cm$, cross sectional area $A = 10cm^2$ and the number of turns $N_1 = 10^3$. The cavity of the coil is filled with an iron core with relative permeability of $\mu_r = 10^3$. The current in the coil is $I = 10A$, which goes to zero in linear slope in $\Delta t = 10s$ time. $\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

a. Find the self induction coefficient of the first coil $(L_1 = ?)$.
b. Find the magnitude of the induced voltage in the first coil as a function of time $U_1(t)$.
A second coil is wound around the first coil tightly with the number of turns $N_2 = 600$.
c. Find the mutual induction coefficient of the coils $(M = ?)$
d. Find the magnitude of the induced voltage in the second coil as a function of time $U_2(t)$.

Solution M6
The magnetic field inside the toroid coil can be calculated by means of Ampere's law. The integration path is the central circular line (C) of the cavity in the coil.

$$\oint_C \mathbf{H} \cdot \mathbf{dr} = \sum_i I_i$$

The central circular line stretches a circular surface. If the current crosses this surface several times, each crossing always counts one more in the right hand side of the equation. So ultimately the equation looks like this:

$$H \cdot l = N_1 I$$

So the magnetic field is as follows:

$$H = \frac{N_1 I}{l} \qquad \left[\frac{A}{m}\right]$$

The magnitude of the magnetic induction vector is B.

$$B = \mu_0 \mu_r \frac{N_1 I}{l} \qquad \left[\frac{Vs}{m^2} = Tesla\right]$$

The turn flux is as follows:

$$\Phi_{turn} = BA = \frac{\mu_0 \mu_r N_1 A I}{l}$$

The time derivative of the turn flux is the turn voltage, according to the Faraday induction law.

$$U_{turn} = \frac{d\Phi_{turn}}{dt} = \frac{\mu_0 \mu_r N_1 A}{l} \frac{dI}{dt}$$

The turn voltage is the fundamental starting point to answer all the questions in the problem. The coil voltage in the first coil is simply N_1 times larger than the turn voltage. So:

$$U_1 = N_1 U_{turn} = \frac{\mu_0 \mu_r N_1^2 A}{l} \frac{dI}{dt}$$

Let us compare this with the expression of the induced voltage in the first coil.

$$\boxed{U_1 = L_1 \frac{dI}{dt}}$$

The self induction coefficient shows up here as $L\left[\frac{Vs}{A} = Henry\right]$

Accordingly:

$$\boxed{L_1 = \frac{\mu_0 \mu_r N_1^2 A}{l}}$$

Similarly: The coil voltage in the second coil is simply N_2 times larger than the turn voltage.

$$U_2 = N_2 U_{turn} = \frac{\mu_0 \mu_r N_1 N_2 A}{l} \frac{dI}{dt}$$

Let us compare this with the expression of the induced voltage in the first coil.

$$\boxed{U_2 = M \frac{dI}{dt}}$$

The mutual induction coefficient shows up here as $M\left[\frac{Vs}{A} = Henry\right]$

Accordingly:

$$\boxed{M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}}$$

Let us take the partial questions if this problem one by one.

a. Find the self induction coefficient of the first coil $(L_1 = ?)$.

$$L_1 = \frac{\mu_0 \mu_r N_1^2 A}{l} = \frac{4\pi \cdot 10^{-7} \cdot 10^3 \cdot 10^6 \cdot 10^{-3}}{0{,}6} = \frac{2\pi}{3} = 2{,}1 Henry$$

This is quite large self inductance.

b. Find the magnitude of the induced voltage in the first coil as a function of time $U_1(t)$.

$$U_1 = L_1 \frac{dI}{dt} = 2{,}1 \frac{Vs}{A} \frac{10A}{10s} = 2{,}1 V_{DC}$$

The induced voltage is constant voltage shortly DC.

A second coil is wound around the first coil tightly with the number of turns $N_2 = 600$.

c. Find the mutual induction coefficient of the coils $(M = ?)$

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l} = \frac{4\pi \cdot 10^{-7} \cdot 10^3 \cdot 10^3 \cdot 6 \cdot 10^2 \cdot 10^{-3}}{0{,}6} = \frac{2\pi}{5} = 1{,}256 Henry$$

d. Find the magnitude of the induced voltage in the second coil as a function of time $U_2(t)$.

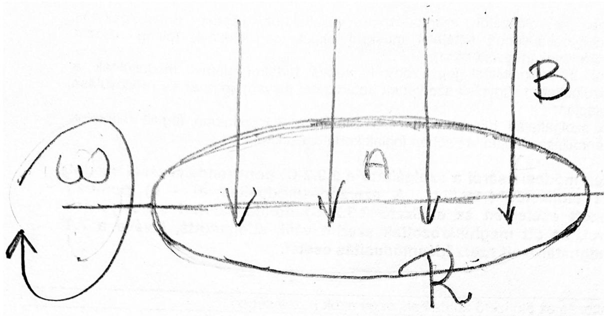
$$U_2 = M \frac{dI}{dt} = 1{,}256 \frac{Vs}{A} \frac{10A}{10s} = 1{,}256 V_{DC}$$

The induced voltage is constant voltage shortly DC.

Problem M7

A circular loop with the cross sectional area $A = 300cm^2$ is rotating with constant angular velocity $\omega = 10 rad/s$ in the presence of uniform magnetic field of $B = 1T$. The axis of the rotation is the diameter position of the loop, and the diameter is normal to the magnetic field. The resistance of the loop is $R = 0{,}01\Omega$. How large is the average torque to be applied to keep the loop rotating? Prove that the mechanical power to rotate the ring is equal with the generated heat of the induced current.

Solution M7



The magnetic flux through the ring is: $\Phi(t) = B \cdot A \cdot \cos(\omega t)$ $[Vs]$

The induced voltage in the ring is: $U(t) = -\frac{d\Phi}{dt} = B \cdot A \cdot \omega \cdot \sin(\omega t)$ $[V]$

The current in the ring is: $I(t) = \frac{U(t)}{R} = \frac{BA\omega}{R}\sin(\omega t)$ $[A]$

The magnetic moment of the ring is: $m = IA = \frac{BA^2\omega}{R}\sin(\omega t)$ $[Am^2]$

Torque (M) on a magnetic dipole by the effect of magnetic induction B field:

$$M(t) = |\mathbf{m} \times \mathbf{B}| = \frac{BA^2\omega}{R}\sin(\omega t) \cdot B \cdot \sin(\omega t) \qquad [Nm]$$

$$M(t) = |\mathbf{m} \times \mathbf{B}| = \frac{B^2A^2\omega}{R}\sin^2(\omega t)$$

The torque is expressed as a function of time. The sine square function is well known to have a mean value of one half. So the questioned average torque is the half of the amplitude, which is: $M_{ave} = \frac{B^2A^2\omega}{2R}$. Verify the dimension. $\left[\frac{V^2s^2}{m^4}m^4\frac{1}{s}\frac{A}{V} = VAs = Nm\right]$

$$M_{ave} = \frac{B^2A^2\omega}{2R} = \frac{1^2\left(3\cdot10^{-2}\right)^2 10}{2\cdot10^{-2}} = \frac{9\cdot10^{-3}}{2\cdot10^{-2}} = 0{,}45Nm$$

To go further, we can express the mechanical power, which overcomes this opposition.

$$P_{mech}(t) = M(t)\cdot\omega = \frac{B^2A^2\omega^2}{R}\sin^2(\omega t)$$

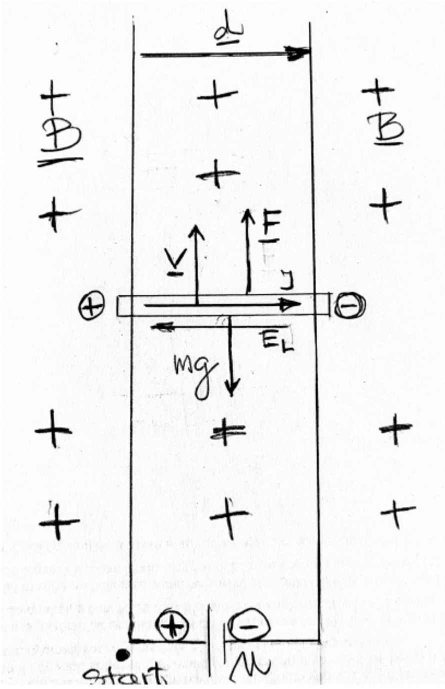
Here we look back to the above formula as follows: $U(t) = B \cdot A \cdot \omega \cdot \sin(\omega t)$

Let us substitute this to the formula of mechanical power:

$$P_{mech}(t) = \frac{U^2(t)}{R}$$

On the right hand side one can see the electric heat dissipated power, which is the same as the mechanical power to keep the ring rotated.

Problem M8



There is a long vertical rail in the presence of a homogeneous magnetic field of $B = 1T$ which is perpendicular to the plain of the rail. A small metal rod with a mass of $m = 10^{-2} kg$ can slide on the rail without any friction. The resistance of the rod is $R = 1\Omega$ and the resistance of the rail is negligible. The distance between the rails is $d = 0{,}1m$. The lower end of the rail is connected to a $U = 2V$ battery as it is shown in the figure. After the initial transient, the rod will travel with a constant velocity up or down direction.

a.) Find the stationary current value in the rod.

b.) Find the magnitude and the direction of the velocity of the rod.

c.) Find the resistance value, which results the velocity to be zero, if all other data are the same.

Solution M8

The rod can slide freely on the vertical rails, keeping contact with both. Initially the battery will drive current through the resistor from left to the right direction. This implies that **d** vector is directed from left to the right. The current will interact with the magnetic field by Lorentz force. $\mathbf{F} = I\mathbf{d} \times \mathbf{B}$. The cross product shows that the direction of the force is pointing upward. The Lorentz force and the gravity force together will accelerate the rod in the initial moments of the experiment. This problem is asking about the stationary state when the transient phenomena are over. Then there is no more acceleration, and the rod will travel with a constant velocity. This stationary velocity is reached when the total force exerted to the rod becomes zero.

$$I \cdot d \cdot B - mg = 0$$

This equation determines the current in the rod. With the actual numerical values the current can be calculated.

$$I = \frac{mg}{d \cdot B} = \frac{10^{-2} \cdot 10}{0{,}1 \cdot 1} = 1A$$

So the current in the rod is *1A*, and this value will not change in the course of this problem. The next detail to find out is the direction and the velocity of the travelling rod. Let us take the upward reference direction as positive. Due to the motion of the rod, the induced Lorentz electric field is expressed yet another Lorentz formula as follows: $\mathbf{E}_L = \mathbf{v} \times \mathbf{B}$. This points to the left, thus pushing the positive particles to the left side, making the left side the positive side of the induced voltage. In Kirchhoff loop equations however, the polarity of the voltage is considered positive value, as we go from the positive terminal to the negative one. So the induced voltage showing up in the loop equation is the dot product as follows:

$$U_{ind} = -\mathbf{d} \cdot \mathbf{E}_L = -\mathbf{d}(\mathbf{v} \times \mathbf{B}) = Bdv$$

Now it is possible write the Kirchhoff loop equation. We start from the lower left corner and go around clockwise direction.

$$RI + Bdv - U = 0$$

The velocity can be readily expressed, since the current value (*1A*) is known.

$$Bdv = U - RI$$

$$v = \frac{U - RI}{Bd} = \frac{2 - 1 \cdot 1}{1 \cdot 0{,}1} = \frac{1}{0{,}1} = 10\frac{m}{s}$$

This result shows that the rod will travel upward direction with 10 m/s velocity. From the formula above it is clear that direction and speed are determined by the resistance of the rod. Keeping all other values fixed, at 2 ohm value the velocity goes to zero, while lower or higher value makes to rod to elevate or to descend respectively.