Introduction to the Theory of Computing 2.

Exercise-set 8.
Solutions

1. $\Delta(G)=4 \implies \chi_e(G)\geq 4$, and the edges of G can be colored with 4 colors $\implies \chi_e(G)\leq 4$.
2. a) Vertices of the same color are independent.
   b) Edges of the same color are independent.
3. $\chi_e(K_5)\geq e/\nu = 10/2 = 5$ and $\chi_e(K_5)\leq \Delta(K_5)+1=5$, so $\chi_e(K_5)=5$.
   $\chi_e(K_6)\geq \chi_e(K_5)=5$, and the edges of K_6 can be colored with 5 colors $\implies \chi_e(K_6)\leq 5$.
   (In general, $\chi_e(K_{2n+1})=2n+1$ and $\chi_e(K_{2n})=2n-1$.)
4. $\chi_e(K_{20})=19$ (ex. 2.), and a round corresponds to edges of the same color.
5. $\chi_e(G)\geq \chi_e(K_5)=5$, and and the edges of G can be colored with 5 colors $\implies \chi_e(G)\leq 5$.
6. a) $\chi_e(G)\geq e/\nu = 15/2 > 7$ and the edges of G can be colored with 8 colors $\implies \chi_e(G)=8$.
   b) $\chi_e(G)\geq e/\nu = 15/2 > 7$ and the edges of G can be colored with 8 colors $\implies \chi_e(G)=8$.
7. $|E(G)|=1999\cdot 10/2=9995,\ \nu(G)\leq 1999/2=999 \implies \chi_e(G)\geq 9995/999>10$ and $\chi_e(G)\leq \Delta(G)+1=11 \implies \chi_e(G)=11$.
8. $\chi_e(G)\geq e/\nu=(2k\cdot 3+2)/2k>3$ (since $|V(G)|$ is odd) and $\chi_e(G)\leq\Delta(G)+1=4 \implies \chi_e(G)=4$.
9. For a k-regular graph on 9 vertices $\chi_e(G)=k+1$, and $\overline{G}$ is $8-k$-regular $\implies \chi_e(\overline{G})\geq 9-k$.
10. Any color class of edges forms a perfect matching (covers all the vertices).
11. $\nu(G)\geq e/\chi_e\geq 16/5>3$ (since $\chi_e(G)\leq\Delta(G)+1=4$), and $\nu(G)\leq 9/2$.
12. a) The edges of G can be colored with 3 colors; 2 colors for the Hamilton cycle (since it has an even length, because $|E|=3|V|/2$ is an integer), and one for the remaining edges.
    b) The edges of it cannot be colored with 3 colors.
13. $\chi_e(G),\chi_e(G-v)\in\{8,9\}$.
    If $\chi_e(G-v)=9 \implies \chi_e(G)=9$.
    If $\chi_e(G-v)=8$, then an 8-coloring of the edges of $G-v$ can be extended to an 8-coloring of the edges of G.
14. If we delete the edge $\{a,b\}$ then by exercise 8, $\chi_e(G)=4$.
15. Yes, if we delete a matching (see exercise 3.).
16. The edges of the original graph can be colored with $\chi_e(G')+2$ colors.
17. Yes, $K_5\setminus\{\text{one edge}\}$ is like that.
18. $\chi_e(G)\geq 4$ and and the edges of G can be colored with 4 colors $\implies \chi_e(G)\leq 4$.
19. $G=$(rows, colums; selected squares) is a 3-regular bipartite graph $\implies \chi_e(G)=3$.
20. G is bipartite $\implies \chi_e(G)=\Delta(G)=6$; or give a concrete edge-coloring.
21. G is bipartite $\implies \chi_e(G)=\Delta(G)=3$.
22. The other degree is 3, and trees are bipartite $\implies \chi_e(G)=\Delta(G)=3$.
23. $G=$ two vertex-disjoint cycles (which are bipartite) and a 100-regular bipartite graph $\implies \chi_e(G)=2+100=102$.
24. $\chi_e(G)=\Delta(G)=9$
25. There are 36 minimum weight spanning trees of weight 19.
26. There are 99! minimum weight spanning trees of weight $2+3+\cdots+100=5049$.
27. The weight of a minimum weight spanning tree is 150.
28. One of them is a path from 1 to 10 of weight 18.
29. By Kruskal's algorithm: when we get to e, we cannot create a cycle.
30. By Kruskal's algorithm: the other edges of C can be selected before e.
31. There must be a cut in G consisting of edges of weight 100 only.
32. Order the edges of G such that the edges of the given spanning tree come first among the edges of the same weight.