Exercise-set 5.
Solutions

1. $\chi(G) \geq \lceil n/2 \rceil$ (at most 2 vertices can get the same color), and G can be colored with this many colors $\Longrightarrow \chi(G) = \lceil n/2 \rceil$.
2. $\omega(G) \geq 10$ (any 10 consecutive numbers form a clique) $\Longrightarrow \chi(G) \geq 10$, and G can be colored with 10 colors (periodically) $\Longrightarrow \chi(G) \leq 10$.
3. $\omega(G) \geq 5$ ($\{1, 8, 15, 22, 29\}$ is a clique) $\Longrightarrow \chi(G) \geq 5$, and G can be colored with 5 colors $\Longrightarrow \chi(G) \leq 5$.
4. $\omega(G) \geq 11$ ($\{10, 11, \ldots, 20\}$ is a clique) $\Longrightarrow \chi(G) \geq 11$, and G can be colored with 11 colors $\Longrightarrow \chi(G) \leq 11$.
5. $\omega(G) \geq 4$ (the powers of 2 form a clique) $\Longrightarrow \chi(G) \geq 4$, and G can be colored with 4 colors (using the same color between consecutive powers of 2) $\Longrightarrow \chi(G) \leq 4$.
6. $\omega(G) \geq 11$ (prime numbers and 1 form a clique) $\Longrightarrow \chi(G) \geq 11$, and G can be colored with 11 colors $\Longrightarrow \chi(G) \leq 11$.
7. $\omega(G) \geq 5 \Longrightarrow \chi(G) \geq 5$, and G can be colored with 5 colors $\Longrightarrow \chi(G) \leq 5$.
8. G is K_{10} with a perfect matching deleted. $\omega(G) \geq 5 \Longrightarrow \chi(G) \geq 5$, and G can be colored with 5 colors $\Longrightarrow \chi(G) \leq 5$.
9. $\omega(G) \geq 6$ ($\{1, 4, 7, 8, 9, 10\}$ is a clique) $\Longrightarrow \chi(G) \geq 6$, and G can be colored with 6 colors $\Longrightarrow \chi(G) \leq 6$.
10. YES. See exercise 22.
11. $\omega(G) \geq 50$ (even numbers form a clique) $\Longrightarrow \chi(G) \geq 50$, and G can be colored with 50 colors $\Longrightarrow \chi(G) \leq 50$.
12. $\omega(G) \geq 4 \Longrightarrow \chi(G) \geq 4$, and G can be colored with 4 colors $\Longrightarrow \chi(G) \leq 4$.
13. $\chi(G) = 4$. See exercise 25.
14. Use the greedy coloring in the original (increasing) order of the vertices.
15. Order the vertices: first the exceptional ones, then the rest, and use the greedy coloring.
16. Use the greedy coloring in the decreasing order of the degrees.
17. a) Vertices of the same color are independent.
    b) It is a good coloring, if each vertex in the complement of a maximum independent set of vertices gets a different color.
18. No (counterexample).
19. a) It is an interval graph ($\exists$ a representation of the vertices with intervals).
    b) It is not an interval graph, since $\chi(G) \neq \omega(G)$.
    c) It is not an interval graph, since there is no representation of the vertices with intervals.
    d) It is an interval graph.
    e) It is not an interval graph (the 5-cycle cannot be represented with intervals).
    f) It is an interval graph.
20. $\chi(G) = \omega(G) = 11$ (=max. # of intervals though a point)
21. It is an interval graph ($\exists$ a representation of the vertices with intervals).
22. $\omega = 10$ in the original graph. We can delete at most 2 vertices from a clique $\Longrightarrow \chi = \omega \geq 8$.