Exercise-set 1.

1. Is there a simple or arbitrary graph with the following degree-sequences:
a) 1,2,2,3,3,3;
b) 1,1,2,2,3,4,4;
c) 1,3,3,4,5,6,6;
d) the degree of each vertex is different?

2. (MT'16) The graph G on 6 vertices can contain multiple edges, but no loops. We know that the degree of any two vertices of G are different. At least how many edges are there in G? (That is, for which integer k does it hold that there is a graph with this property with k edges, but not with less than k edges?)

3. (MT+'16) At least how many edges must a simple graph on 10 vertices have if it has 3 vertices of degree 9? (I.e., for which integer k does it hold that there is a graph with k edges having the above property, but for no graph with less than k edges has it?)

4. We directed the edges of a simple graph on 6 vertices at random in one direction. In the graph obtained the outdegrees of all the vertices were different. Draw the graph.

5. In the directed graph G on 100 vertices the outdegree of each vertex is at least as large as the indegree of the same vertex. The outdegree of a vertex is 18. What can the indegree of this vertex be?

6. How many vertices can a k-regular simple graph have if it has 15 edges? List all the possibilities, and draw (at least) one graph for each! (A graph is k-regular, if all of its degrees are k.)

7. Let G be a simple graph on n vertices ($n \geq 3$) with only one vertex of even degree. How many vertices of even degree are there in $\overline{G}$, the complement of G?

8. * (MT'15) In the simple graph G on 20 vertices 10 vertices have degree at most 7, and the other 10 vertices have degree at least 16. How many edges are there in G?

9. * (MT+'15) Does there exist a simple graph on 21 vertices for which it holds that both G and its complement $\overline{G}$ contain 9 vertices of degree 4 and 3 vertices of degree 10?

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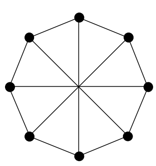
10. How many non-isomorphic simple graphs are there on 4 vertices?

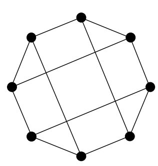
11. How many non-isomorphic simple graphs are there with 50 vertices and 1223 edges?

12. Draw all the pairwise non-isomorphic simple graphs with
a) $n = 5,\ e = 3$;
b) $n = 5,\ e = 7$;
c) $n = 5,\ e = 8$.
(Here n denotes the number of vertices, and e the number of edges.)

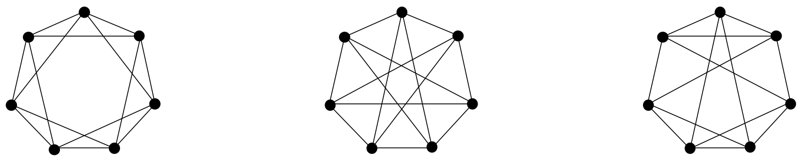
13. a) Is there a simple graph on 4 or 5 or 6 vertices which is isomorphic to its own complement?
b) Is there a 5-regular simple graph which is isomorphic to its own complement? (A graph is k-regular, if all of its degrees are k.)

14. Let graph G consist of the vertices and edges of a cube. Which of the graphs below are isomorphic to G?

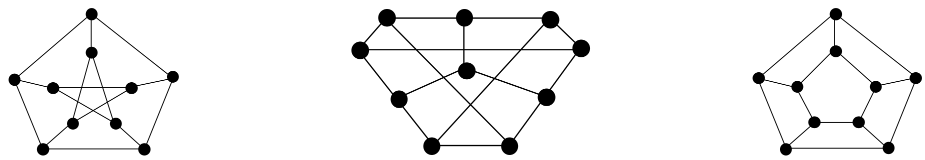




15. Which graphs are isomorphic from the three graphs below?



16. Let the vertices of the graph G be the 2-element subsets of the set $\{1, 2, 3, 4, 5\}$ and two vertices be adjacent if and only if the respective sets are disjoint. Which of the graphs below are isomorphic to G?



17. We place 2 white and 2 black knights on a 3×3 chessboard in such a way that the knights of the same color stand in opposite corners. Can we achive with the usual moves in chess that the knights stand in opposite corners, but the opposite ones are of different color? (During the moves at most one knight can stand on a square.)