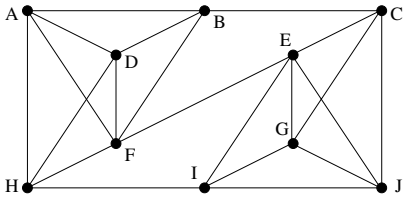
Exercise-set 11.

1. (MT+'11) For which values of k is it true that the following graph is
   a) k-edge-connected;
   b) k-vertex-connected?



2. Determine the vertex- and edge connectivity numbers ($\kappa(G)$ and $\lambda(G)$) of the following graphs:
   a) the graph consisting of the vertices and edges of a cube,
   b) the complete bipartite graph $K_{m,n}$, where $m \geq n$.

3. The vertices of an 18-vertex graph G can be divided into 3 classes of six vertices each, in such a way that 2 vertices are adjacent if and only if they are in different classes. Determine the largest integer k for which G is k-vertex-connected ($\kappa(G)$), and the largest integer l for which G is l-edge-connected ($\lambda(G)$).

4. Show that a k-(vertex-)connected graph G on n vertices has at least $kn/2$ edges.

5. Prove that an $n/2$-(vertex-)connected graph on n vertices contains a Hamilton cycle.

6. Construct a simple graph which is 2-vertex-connected, 3-edge-connected and has minimum degree 4.

7. (MT'14) We connect two disjoint complete graphs on 5 vertices with 3 edges, in such a way that the resulting graph G is simple. Is it true in all cases that G is
   a) 3-(vertex)-connected;
   b) 3-edge-connected?

8. (MT'17) A simple graph on 10 vertices has 40 edges. Determine the largest integer k for which G is surely k-vertex-connected.

9. Show that if a graph is 3-(vertex-)connected, then it contains a cycle of even length.

10. (MT'07) Let G be a 3-(vertex-)connected graph with 100 vertices and let $x, y \in V(G)$ be two different vertices. Show that there is a path from x to y whose length (i.e. the number of edges in it) is not greater than 33.

11. a) Let G be a k-connected graph, and G' be a graph obtained by adding a new vertex of degree at least k to G. Show that if G' is a simple graph, then it is k-(vertex-)connected as well.
    b) Let G be a k-connected graph, and $A = \{a_1, \ldots, a_k\}$ and $B = \{b_1, \ldots, b_k\}$ be two disjoint point sets in it. Prove that there are k (completely) vertex-disjoint paths in G connecting A and B.