

Sample Midterm 2

1. (a) The array of a heap contains the following elements (in this order) 3, 8, 4, 12, 10, 6, 21, 33. Draw the heap as a binary tree.
(b) Insert 5 into this heap and then perform a DELETETMIN. You must show all the steps performed in these two operations.
2. Given an array of $n \geq 2$ distinct numbers, we want to determine if there are two numbers such that their difference is exactly 42. Give an algorithm that solves this problem with at most $O(n \log n)$ comparisons.
3. The preorder of a binary search tree is 3, 10, 4, 8, 7, 9. Draw this binary search tree on 6 vertices and justify why this is the way to draw the tree.
4. We insert $1, 2, \dots, n$, in this order, into an empty 23-tree. Show that in the resulting tree, the number of 3-nodes is at most $O(\log n)$.
5. Show that the following language is in coNP:
DEG4SPANNING-TREE = $\{G \mid G \text{ is a simple, connected, undirected graph whose every spanning tree contains a degree 4 vertex}\}$
6. Determine if the following language is in P or is NP-complete:
kINDPDT-SET-CLIQUE = $\{(G, k) \mid G \text{ is a simple, undirected graph that contains a clique on } k \text{ vertices and also a vertex independent set on } k \text{ vertices}\}$
7. Given n numbers, we define the middle number as the $\lceil n/2 \rceil$ -th number when these numbers are arranged in increasing order. We are initially given integers a_1, a_2, \dots, a_n , where we know that a_1 is their middle number, but all other numbers are unordered. Design a data structure that can be constructed with these numbers with at most $O(n)$ comparisons. Also, it must support the following two operations, both with running time $O(\log k)$ when the data structure has k elements:
INSERT: insert a new element into the data structure,
DELETE-MIDDLE: deletes the current middle number in the data structure.