$$\varepsilon_0 = 8{,}86 \cdot 10^{-12}\frac{As}{Vm}, \qquad \frac{1}{2\pi\varepsilon_0} = 1{,}8 \cdot 10^{10}\frac{Vm}{As} \qquad \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^{9}\frac{Vm}{As}$$

Problem E4

There is a thin metal sphere with the radius $R_1 = 0{,}1m$. This contains a charge of $Q = 10^{-7}As$.
a./ Find and sketch the $E(r)$ function in coordinate system, and find the numerical value in the breakpoint $E(R_1)$.
b./Find the voltage of the sphere relive to an external position $R_3 = 1m$.
c./ The whole system is in vacuum. There is a proton on the surface of the sphere. Find the velocity of the proton at the radial position $R_3 = 1m$.
$m_{proton} = 1{,}67 \cdot 10^{-27}kg$ and $q_{proton} = 1{,}6 \cdot 10^{-19}As$

Solution:

a./ There is no insulating material, therefore $E \equiv E_{free}$ in this problem.

Gauss law in general: $\oint\limits_{Any\cdot surface} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$ **Bold letters** mean vector

Gauss law in present case: $\oint\limits_{Sphere} E(r) \cdot dA = \frac{Q}{\varepsilon_0}$ Thin letters mean magnitude

$$4r^2\pi \cdot E(r) = \frac{Q}{\varepsilon_0}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2}$$

Numerically:

$r < R_1$ $\qquad E(r) \equiv 0$

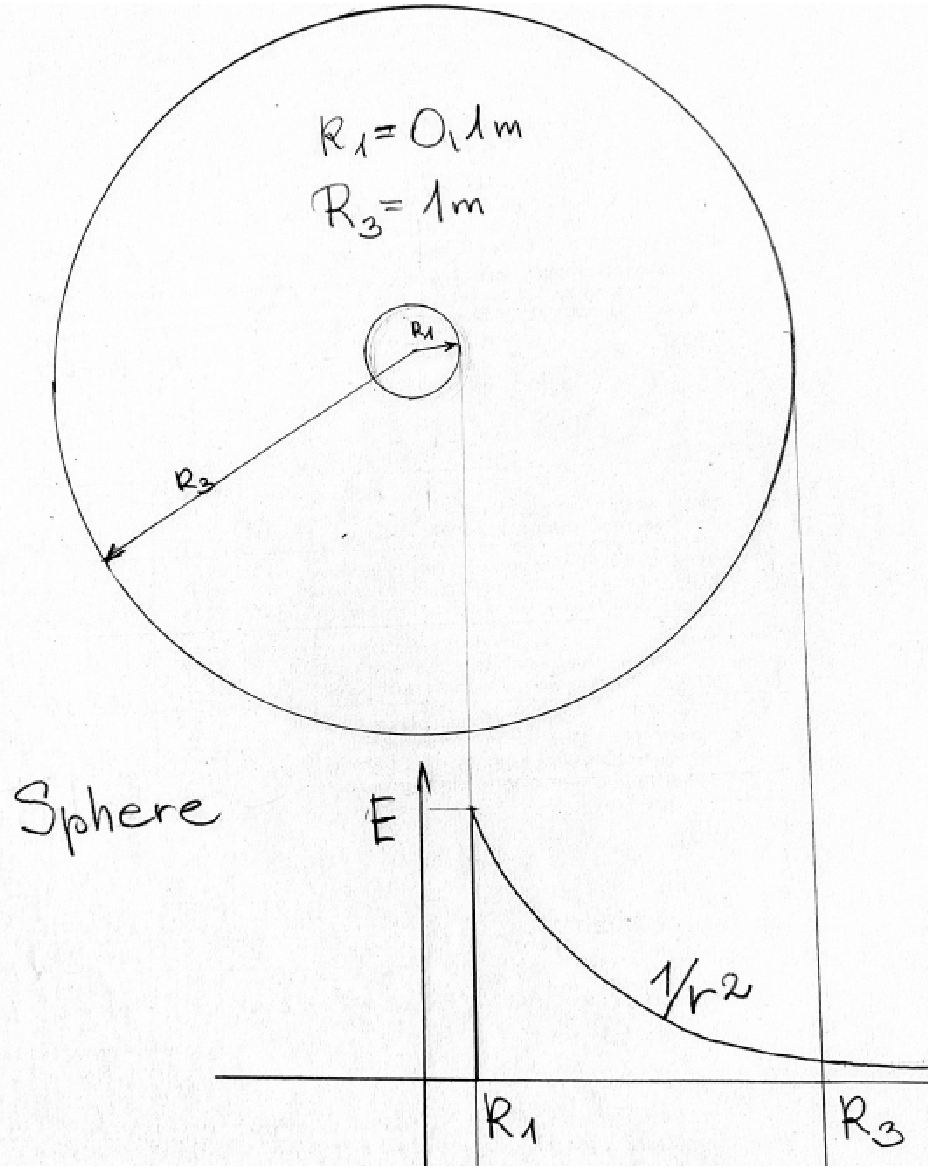
$R_1 \le r$ $\qquad E(r) = 9 \cdot 10^9 \frac{Vm}{As} \cdot 10^{-7}As \cdot \frac{1}{r^2} = 900Vm \cdot \frac{1}{r^2} \qquad \left[\frac{V}{m}\right]$

$$E(R_1) = 900Vm \cdot \frac{1}{R_1^2} = \frac{900Vm}{0{,}01m^2} = 90\frac{kV}{m}$$

b./

$$U(R_1, R_3) = -\int_{R_3}^{R_1} E(r)dr = \int_{R_3}^{R_1} 900Vm \cdot \left(-\frac{1}{r^2}\right)dr = 900Vm\left[\frac{1}{r}\right]_{r=R_3}^{r=R_1}$$

$$U(R_1, R_3) = 900Vm \cdot \left(\frac{1}{R_1} - \frac{1}{R_3}\right) = 900\left(\frac{1}{0{,}1} - \frac{1}{1}\right) = 900(10-1) = 8100V$$



c./

The conservation of the mechanical energy is used:

$$\frac{1}{2} m_{pr} v^2 = q_{pr} \cdot U(R_1, R_3)$$

$$v = \sqrt{\frac{2q_{pr} \cdot U(R_1, R_3)}{m_{pr}}} = \sqrt{\frac{2 \cdot 1{,}6 \cdot 10^{-19} \cdot 8100}{1{,}67 \cdot 10^{-27}}} = 1{,}25 \cdot 10^{6} \frac{m}{s}$$