Extra Practice Problems.

1. Let A and B be independent events and C an event disjoint from both. Further let $P(A) = P(B) = P(C) = \frac{1}{3}$. What is $P(\overline{\overline{A} \cap B \cup C})$?

2. Upon inspection of the production line of a certain manufacturing unit, it was found that items had a faulty material with probability 0.15, a faulty size with probability 0.3 and had a surface fault with probability 0.2. It is further known that these faults are pairwise independent, but not totally independent. It is know that the probability that a product has all three faults is 0.02. What is the probability that a product has no faults?

3. There are two urns in the room, one with 2 green and 3 blue stones, and the other with 2 green and 2 blue stones. We pick one stone randomly from the first and place it in the second. Now we pick one stone at random from the second and place it in the first. What is the probability of drawing a blue stone if we draw from the (a) first urn (b) second urn?

4. In an examination hall, 75% of the students are from department A, 15% from B and 10% from C. The probability of getting a 5 on the exam is 0.4 for a student from department A, 0.7 from B and 0.6 from C. Given that a certain student got 5 on the exam, find the probabilities that he is from department A, B and C respectively.

5. Alice and Bob play the following game: They both roll a dice each, and if one of them rolled a number at least twice as much as the other, then the person with the smaller roll will pay, in dollars, an amount equal to three times the sum of the two dice rolls. (Eg. if the rolls were Alice 1, Bob 3, then Alice would pay Bob 12 dollars). If a roll is not at least twice the other, then no money is exchanged. What is the expected winnings of Alice?

6. There are three red and two white stones in an urn. We draw stones six times with replacement. Let X denote the number of times a red stone was drawn. Find the expected value of X and of $Z = (X + 2)(X - 2)$.

7. We pick a point uniformly at random from a unit square. What is the probability that the point is closer to the side of the square than to the diagonal?

8. Let $f(x) = \alpha \cdot x^4$, if $x \in (2, 3)$, and 0 otherwise. For what value of α will this be a density function? For this density function, find the distribution function. If X is the random variable with $f(x)$ as the density function, what are $P(X > 2)$ and $E(X)$?

9. Let Y be a random variable with the following density function, for some $\alpha \in \mathbb{R}$:
$$f_Y : y \mapsto \begin{cases} \frac{\alpha}{(1+y)^2} & \text{if } -5 < y < -2, \\ 0 & \text{otherwise.} \end{cases}$$
Determine $P(-4 < Y < -3)$. (2019, repeat midterm)

10. When trying to toss a crumpled waste paper into the bin, every try, independent of the other, has a probability of 0.2 of actually landing the paper in the bin. What is the expected number of tries? After the first time it lands in the bin, suppose we take it out and try to throw it in again. What is expected total number of tries (from the start) for landing the paper in the bin the second time?

11. Three devices are turned on at the same time. The time (in hours) for which each device functions without errors has exponential distribution with parameter $\lambda = \frac{1}{5}$, and is independent of the other devices. What is the probability that at least 1 of them will be working after 10 hours?

12. Let the time duration for the arrival of the first call on a telephone have the memoryless property. If we know that there will not be a call for 3 hours with probability 0.5, what is the expected time in which the first call will arrive?

13. At a product's help line, calls of customers arrive independent of each other and with equal probability. The probability of not receiving a call for 1 hour is 0.25.
(a) What is the expected number of calls in 3 hours?
(b) What is the probability that in a duration of 8 hours, there are at least 2 hours in which there was at most 1 call.

14. In a certain city, there are no traffic accidents on 25% of the days. Assume that a very large number of cars are driven in the city, there are approximately the same number of cars on different days, and each car independent of the others has the same probability of causing an accident. What is the probability that next week there will be exactly two days with more than 1 accident?

15. There are many mice living underground in a certain field, assume the weight of every single one of them is 10 decagrams. The probability that we find exactly one mouse running on the field is $2e^{-2}$. A cat goes to the field and catches every single mouse it sees running there. What is the standard deviation, in decagrams, of the total weight of the mice caught by the cat?

16. For a randomly selected rabbit in a field, its weight X in kilograms has the following distribution function:
$$F_X(x) = \begin{cases} 0, & \text{if } x < 2, \\ \dfrac{(x-2)^2}{64}, & \text{if } 2 \leq x \leq 10, \\ 1, & \text{if } x > 10. \end{cases}$$
Determine the density function of X and also find its expected value and standard deviation.

17. There are 2 gaming apps and 3 social media apps on a smartphone. The phone's operating system first randomly selects one of the five apps to update, then if it was a social media app, it randomly selects one more of the remaining four to update, and if it was a gaming app, it selects one of the three social media apps to update. Denote by X the number of gaming apps updated and by Y the number of social media apps updated. Determine the joint distribution and the marginal distributions of X and Y. Also find $Var(Y)$ and $E(XY)$. Are X and Y independent?

18. Lajoska really likes trams, so on a nice day during the summer holidays he goes to Horizon street and observes three number 3 trams running one after the other. Assume that the trams are, independently of each other, CAF with 70% probability and Hanoverian with 30% probability. Let X denote the number of CAF trams observed by Lajoska and Y denote the number of Hanoverian trams observed by him.
   (a) What is the distribution of Y and with what parameters?
   (b) Determine $Var(X)$ and $Var(Y)$.
   (c) Determine the joint distribution of X and Y.
   Note: If it is easier, we can solve (a) and (b) after (c).
   (d) Determine $E(XY)$.
   (e) Are X and Y independent?

19. You are given an urn with 1 white and 1 black stone. We keep drawing, with replacement, stones until we get a black stone. At any turn, if the drawn stone is white, then we put the stone back with one additional white stone. Let X denote the number of draws until (and including) the black stone. What is the distribution, expected value and standard deviation of X?

20. Let $X \sim U(0,1)$, and $Y = \frac{1}{X+3}$. What is the distribution and density function of Y? What is $P(Y \geq \frac{7}{24})$?

21. The density function of random variable X is given by, $f_X(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$. Generate the random variable $Y \in U(0,1)$ using X.

Extra Practice Problems - Final Answers.

1. 0.7778
2. 0.465
3. (a) 0.584 (b) 0.52
4. 0.6452, 0.2258, 0.1290
5. 0
6. $E(X) = 3.6$, $E(Z) = 10.4$
7. $\tan(\frac{\pi}{8}) = 0.4142$
8. $\alpha = \frac{5}{211}$, $P(X > 2) = 1$, $E(X) = 2.626$, $F_X(x) = \begin{cases} 0 & x \leq 2 \\ \frac{1}{211}(x^5 - 32) & 2 < x \leq 3 \\ 1 & 3 < x \end{cases}$
9. $\alpha = \frac{4}{3}$, $\frac{1}{8}$
10. 5, 10.
11. 0.3535
12. 4.328.
13. (a) 4.159, (b) 0.9910.
14. 0.2583.
15. 14.14
16. -, 7.333, 1.886
17. $Var(Y) = \frac{21}{100}, E(XY) = \frac{7}{10}$, no
18. (a) $Bin(3, \frac{3}{10})$, (b) $Var(X) = Var(Y) = 0.7937$, (c) -, (d) $\frac{63}{50}$, (e) no.
19. $P(X = n) = \frac{1}{n(n+1)}$. $E(X) \to \infty$, not defined.
20. $F(y) = \begin{cases} 0 & \text{if } y < \frac{1}{4}, \\ 4 - \frac{1}{y} & \text{if } \frac{1}{4} \leq y \leq \frac{1}{3}, \\ 1 & \text{otherwise} \end{cases}$

    $\frac{3}{7}$.
21. Let $Y = \frac{X^2}{4}$.