

Solution5BeautifulVersion

Lemma1:

Both edges e_{22} e_{11} should be cut by all the min cuts.

($\forall C$ u_1 and $u_2 \in S$ and v_1 and $v_2 \in T$)

Proof contradiction:

If they didn't then when we raised the capacities the maximum flow would not have increased because there would have existed a min cut which capacity value did not change. But the max flow value increased. This is a contradiction.

Lemma2:

Even after raising the capacity for e_{22} and e_{11} all the min cuts still remained min cuts.

Proof:

Before: $c(\text{min cuts}) = \text{val}(f_{\min})$

After raising the capacity: $\text{val}(f'_{\min}) = c(C'_{\min}) = c(\text{min cuts}) + \text{the new capacity}$.

Corollary:

The second largest capacity of a cut (in original graph) differs by ≥ 2 from the value of the min capacity.

This tells us that until we don't change the capacity of the min cuts more than ≥ 2 the flow will change the same amount.

The e_{12} and e_{21} also are cut by all the min cuts ($\forall C$ u_1 and $u_2 \in S$ and v_1 and $v_2 \in T$)
 \Rightarrow changing their value changes the min capacity of the cut.

$\Rightarrow \max(f) = 21 + 31 - 13 = 39$.