12. lecture

Mittwoch, 8. November 2023 10:03

Exam: ! 10 Januar ! TEAMS
(17, 22, 24) Dez 12

Correlation based data compression

(voice, images) → next lecture altdueller Algo's für Bilder
↳ dataspeed

Source has a memory (present will determine the future)

x_k, $k = 0, 1, 2 \dots$ stochastic sequence

Auto-correlation function $R(\ell) = E(x_k x_{k-\ell})$
→ dependence of last sample & next

Intuition:
" I went into a bar and asked for a glass of win_ $\begin{cases} \to e & \text{(high propability ending)} \\ \to eger & \text{(low propability)} \end{cases}$ } low entropy (far away from real distribution)
past info

$\underbrace{H(x) \le L}_{\text{small}} \le H(x) + \varepsilon$ → L small → dataspeed small

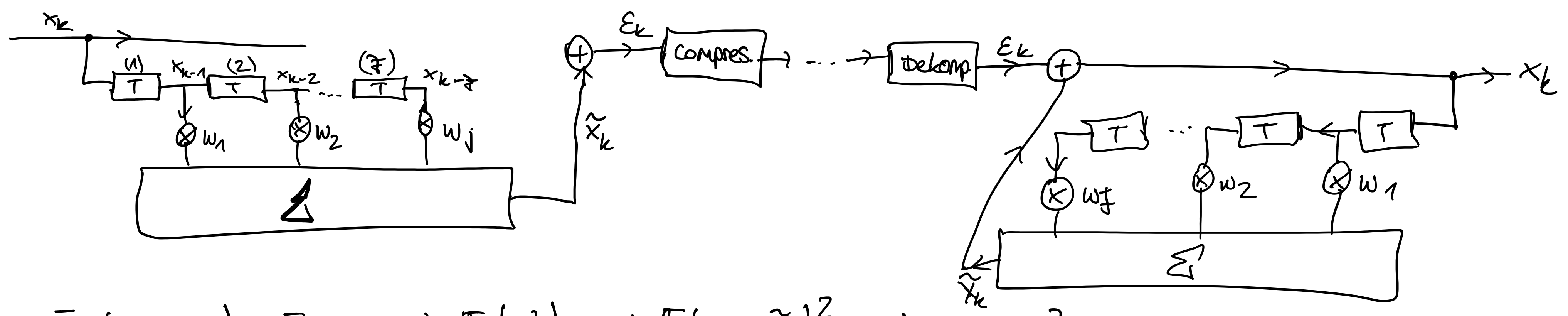
64 kbps (voice) ⟶ 32 kbps (aggressiv data compression)

$\tilde{x}_k = \sum_{j=1}^{\mathcal{F}} w_j \underbrace{x_{k-j}}_{\text{past samples}}$ (memory, how long past) $\qquad \varepsilon_k = x_k - \tilde{x}_k$

$\underset{\text{matrix}}{\bar{\bar{R}}}$: $R_{ij} := R(i-j) = \mathbb{E}(x_{k-i} x_{k-j})$

1) $\bar{\bar{R}} = \bar{\bar{R}}^T$ sym. Matrix
2) $\forall \bar{a}, \bar{b}: \bar{a}^T \bar{\bar{R}} \bar{b} = \bar{b}^T \bar{\bar{R}} \bar{a}$
3) $\bar{\bar{R}} \bar{s}_i = \lambda_i \bar{s}_i$, Eigenvektors $\bar{s}$
$\bar{s}_i^T \bar{s}_j = \begin{cases} 0 & \text{if } i \ne j \\ 1 & \text{if } i = j \end{cases}$ $\qquad \lambda_i \ge \emptyset$
4) $\bar{a}^T \bar{\bar{R}} \bar{a} \ge \emptyset$
5) $\sum_{i=1}^{\mathcal{F}} R_{ii} = \sum_{i=1}^{\mathcal{F}} \lambda_i$

APC
adaptive, predective coding



$\bar{w} = (w_1 \dots w_{\mathcal{F}})$ $\qquad \bar{w}_{opt} = \min_{\bar{w}} \mathbb{E}(\varepsilon_k^2) \sim \min_{\bar{w}} \mathbb{E}(x_k - \tilde{x}_k)^2 \sim \min_{\bar{w}} \mathbb{E}\left(x_k - \sum_{j=1}^{\mathcal{F}} w_j x_{k-j}\right)^2$

$$\mathbb{E}\left(x_k - \sum_{j=1}^{\mathcal{F}} w_j x_{k-j}\right)^2 = \mathbb{E}(x_k^2) - 2 \sum_{j=1}^{\mathcal{F}} w_j \underbrace{\mathbb{E}(x_k x_{k-j})}_{R(j)} + \sum_{j=1}^{\mathcal{F}} \sum_{i=1}^{\mathcal{F}} w_j w_i \underbrace{\mathbb{E}(x_{k-i} x_{k-j})}_{R(i-j)}$$

$$= \underset{\text{Null}}{R(0)} - 2 \sum_{j=1}^{\mathcal{F}} w_j R(j) + \sum_{j=1}^{\mathcal{F}} \sum_{i=1}^{\mathcal{F}} w_j w_i R(i-j)$$

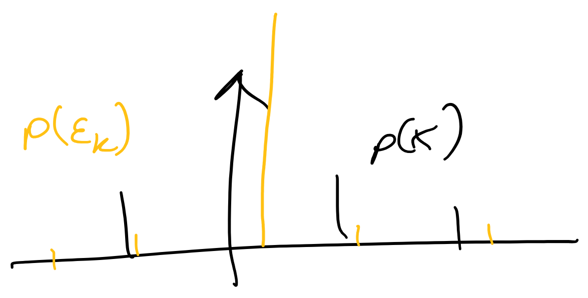
$\bar{r}$: $r_i = R(i)$
$\bar{\bar{R}}$: $R_{ij} = R(i-j)$

$$= R(0) - 2 \bar{r}^T \bar{w} + \bar{w}^T \bar{\bar{R}} \bar{w}$$

$\bar{w}_{opt}: \min_{\bar{w}} \bar{w}^T \bar{\bar{R}} \bar{w} - 2 \bar{r}^T \bar{w}$ → $\bar{w}_{opt}: \bar{\bar{R}} \bar{w} = \bar{r}$

if $R(\ell)$ → get $\bar{\bar{R}}$ & $\bar{r}$

$\mathbb{E}(x_k^2) \gg \mathbb{E}(\varepsilon_k^2)$
$H(x_k) \gg H(\varepsilon_k)$



uniform distribution
(deutlich anders als ~ ⇒ Entropie small)

R(ℓ) is unknown

$w_\ell(k+1) = w_\ell(k) - \Delta \left\{ x_k - \sum_{j=1}^{\mathcal{F}} w_j(k) x_{k-j} \right\} x_{k-\ell} \qquad \ell = 1, \dots, \mathcal{F}$ (optimiert sich selbst)

$\lim_{k \to \infty} \mathbb{E} \| \bar{w}(k) - \bar{w}_{opt} \|^2 = \emptyset$