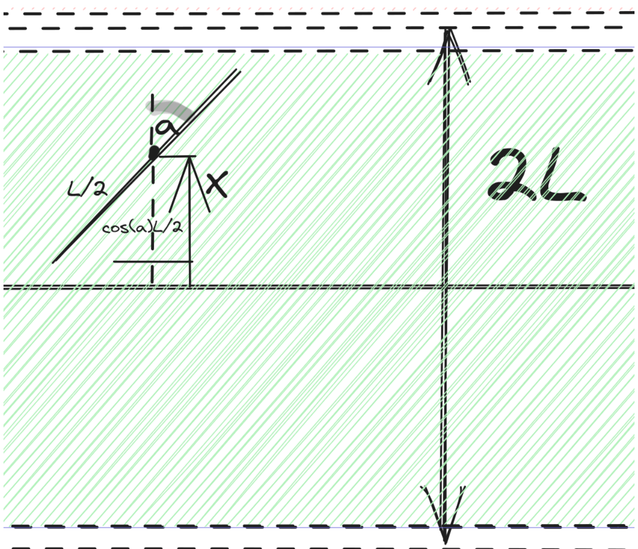
Buffon's Needle

PDF for the distance from the line and the angle of the needle are uniform distributions so we have:

$$f_x = \begin{cases} \frac{1}{L}, & [0, L] \\ 0, & otherwise \end{cases}$$

$$f_\alpha = \begin{cases} \frac{2}{\pi}, & [0, \pi/2] \\ 0, & otherwise \end{cases}$$

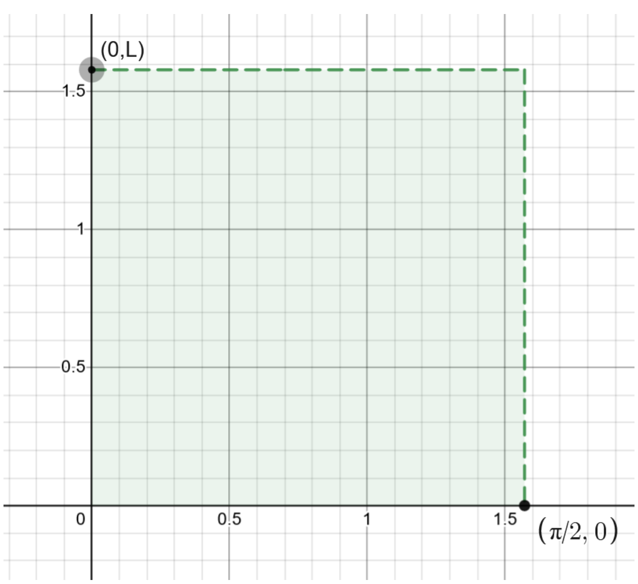
This is the setup for one line:



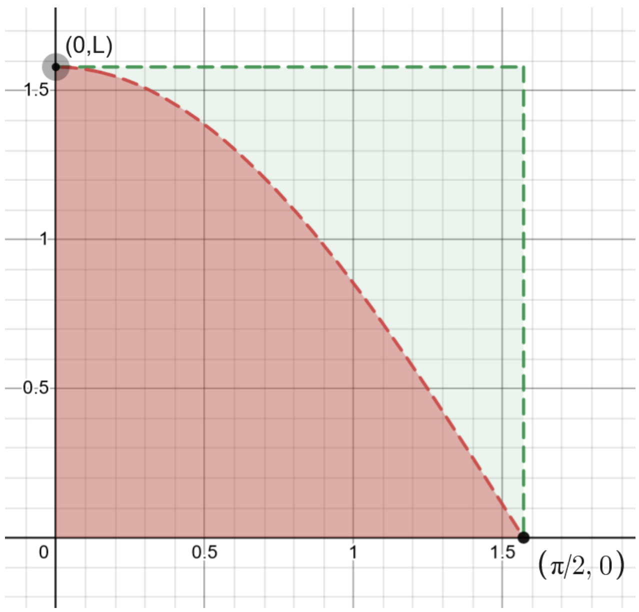
Mathematically we want:

$$P(X \leq cos(\alpha)\frac{L}{2})$$

We know that f_x and f_α are independent so our space is:



From the condition $X \leq cos(\alpha)\frac{L}{2}$ we have:

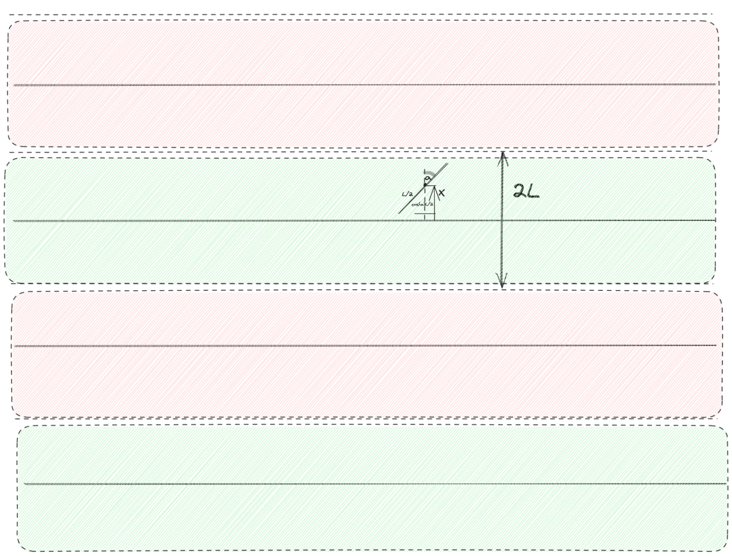


Now setting up the integral:

$$P\left(x \leq \cos\left(\alpha\right)\frac{L}{2}\right) = \int_0^{\frac{\pi}{2}}\int_0^{\cos(\alpha)\frac{L}{2}} f_x f_y dx d\alpha = \int_0^{\frac{\pi}{2}}\int_0^{\cos(\alpha)\frac{L}{2}} \frac{1}{L}\frac{2}{\pi} dx d\alpha = \frac{1}{\pi}$$

So now if we look at our whole space we have created a partition with the same probability. From the symmetry we have in fact:

$$P(that\ the\ needle\ touches\ the\ line) = n\frac{1}{\pi}$$



From this you can easily derive the most important constant π.

Notes: It is very beautiful how you can derive π from such random multiple events.

Here is the link to the Desmos graph for the paradox I created:
https://www.desmos.com/calculator/beylt2qnau