ProblemSet10

Questions

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1. 7.b should be $2x^2 + 7/2$
2. 12 I think i solved it very different because I don't see what kind of theorem should be used there regarding the recitation.
3. 13 could not solve.
4. 14 I think I am right but not sure.
5. 17 hot to i turn the integral to the form of the Φ?

12

- 1 get out P(1)=1/3
- 2 --> P(2 and 3)=2/3
- 3 --> send you back
  231 => 2.5*2 + 1 = 6

  > On average you need wait 2.5 times to get back

X:= the amount of time needed to get out
Y:= the number of times i picked till i get
$Y \sim Geo(1/3)$
$E(Y) = 3$
$E(X) = E(2.5(Y-1)+1) = 6$

13.

$Var\left(aX+b\right) = a^2 Var\left(X\right)$
$E\left(aX+b\right) = aE\left(X\right)+b$
$Var\left(3X+2Y\right) = 9Var\left(X\right)+4Var\left(Y\right) = 13$
$V \sim N(0,13)$
$W \sim N(0,5)$

$$Cov(3X+2Y, 2X-Y) = Cov(3X, 2X) + 0 + 0 + Cov(2Y, Y) \neq 0$$

$$E(V|W) = \int_{Range(V)} v f_{V|W}(v|w) dv$$

$$f_{V|W}(v|w) = \frac{f_{V,W}(v,w)}{f_W(w)}$$

E(3X+2Y|2X-Y)
3X+2Y=(2X-Y)+X+3Y

14.

$X\ Pois(\lambda)$:= the number of costumers calling a day = the number of costumers in the que.

1_f

Y:= the number of woman in the que.

$Y|X \sim Bin(X,p)$

$E[Y|X] = Xp$

$Var(Y|X) + E^2(Y|X) = E(Y^2|X) = Xp(1-p) + (Xp)^2$

15.

$$f_y = \begin{cases} 1 & (0,1) \\ 0 & otherwise \end{cases} \$\$\$\$ F_{X|Y}(x) = \begin{cases} 1 & y < x \\ \frac{x-y^2}{y-y^2} & x < y < \sqrt{x} \\ 0 & y > \sqrt{x} \end{cases}$$

$$F_x(0.5) = \int_{-\infty}^{\infty} F_{X|Y}(0.5|Y=y) f_Y\, dy$$

$$\int_0^{0.5} 1 * 1dy + \int_{0.5}^{\sqrt{0.5}} \frac{(0.5-y^2)}{y-y^2} * 1dy = 0.6130$$

16.

X:= the probability that the coin lands in heads.

$f_X(x) = 6x - 6x^2 \quad (0,1)$

Y:= the number of heads

$Y|X \sim Bin(4,X)$

$$P(Y=2) = \int_{-\infty}^{\infty} P(Y=2|X=x) f_X dx = \int_0^1 \binom{4}{2} (x)^2 (1-x)^2 \left(6x - 6x^2\right) dx = 0.2571$$

17.

T:= the time in which a security bug is detected.

$$F_T(t) = \begin{cases} 1 - e^{-t^2/2} & t > 0, \\ 0 & otherwise. \end{cases}$$

$$f_T(t) = \begin{cases} te^{-t^2/2} & t > 0, \\ 0 & otherwise. \end{cases}$$

E:= exploitation

$P(E|T) = 1 - e^{-t}$

$$P(E) = \int_{-\infty}^{\infty} P(E|T=t) f_T(t) dt$$

$$= \int_0^{100} \left(1 - e^{-t}\right) t e^{-\frac{t^2}{2}} dt = 0.6557$$

18.

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T:= the time the tourist spends
T~U(1/2,3/2)
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$f_T = 1 \ \ [1/2, 3/2]$

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V:= catch virus
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$P(V|T=t) = \left(t - \frac{1}{2}\right)^2$

$$P(V) = \int_{\frac{1}{2}}^{\frac{3}{2}} \left(t - \frac{1}{2}\right)^2 \cdot 1 dt = \frac{1}{3}$$