1. Practice

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70% Anwesenheit

ILLÉS HORVÁTH
- Office hours WED 2 PM – 3 PM IB 115
- HORVATH.ILLESANTAL@GMAIL.COM
- www.math.bme.hu/~pollux/codingtech/

1
a) $u=(0010011)$ $e=(1000001)$ $P_b=0{,}1$

output vector:
```
  0010011
  1000001
  -------
  1010010
```

b) probability of error vector e

$$P(e) = 0{,}1^2 \cdot (1-0{,}1)^5 \approx 0{,}005905$$

2) $u=(0010011)$ $v=(1010010)$

a) Hamming distanz: 2

b) error vector $e = u + v \rightarrow$
```
  0 0 1 0 0 1 1
+ 1 0 1 0 0 1 0
  -------------
  1 0 0 0 0 0 1
```

3) $u=(00100111)$ $v=(10100101)$

a) $e = u+v = (10000010)$

b) Probability that output v when u $(P_b = 0{,}01)$

$$= P(e) = P(v|u) = P_b^2(1-P_b)^6 = 0{,}01^2 \cdot 0{,}99^6 \approx 0{,}00009415$$

$= P(e|u)$ independent

4) BSC $P_b = 0{,}2$
how many errors need to correct if QoS is less 0,001 & $n=30$ (block length)

probability that >t errors < 0,001

- i – incorrect bits number
- $30-i$ numbers of correct bits
- t is numbers of errors we can correct

$$\sum_{i=t+1}^{30} \binom{30}{i} 0{,}2^i\, 0{,}8^{30-i} \le 0{,}001$$ find smallest t for it

(i max, $30-i \times 0$)

can correct t errors ⇒ get error bei $t+1$

$t=12 \approx 0{,}00311$
$t=13 \approx 0{,}000902$
} nicht im test ⇒ schlecht zu rechnen

Voberssom: mehrmals übertragen ⇒ majority

5) numbers 5-dim. binary vectors mit Hamming dist. 3 from vector (01010)
→ differ in 3 position

$$\binom{5}{3} = \frac{5!}{3!\cdot 2!} = 10$$

(b) numbes of binary vectors inside sphere with radius 3 with center (01010)?

$$\sum_{i=0}^{3} \binom{5}{i} = 26$$

(c) largest distance → 5 2 only one point for each with distance 5

6 a) weight 0001000110001111 01000
$w=8$

7
$$\begin{pmatrix} 1&0&0&0&1&0&0&1 \\ 0&0&1&1&1&0&0&0 \\ 1&0&0&0&0&0&1&0 \\ 1&1&0&0&0&1&1&1 \end{pmatrix} \cdot \begin{pmatrix} 0\\1\\1\\0\\0\\0\\1\\0 \end{pmatrix} = \begin{pmatrix} 0\\1\\1\\0 \end{pmatrix}$$

Add columns 2, 3, 7 of matrix componentwise

8
$$(1\,0\,0\,1) \cdot \begin{pmatrix} 1&0&0&0&1&0&0&1 \\ 0&0&1&1&1&0&0&0 \\ 1&0&0&0&0&0&1&0 \\ 1&1&0&0&0&1&1&1 \end{pmatrix} = (01001110)$$

Add rows 1 & 4

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1 codewords: 10100, 01111, 11110, 00000

a) calculate n & k of code
$n=5$ $k=2$ (4 code words → 4 messages ≙ 2^2) $2^k =$ number of codewords $=4$
⇒ $C(5,2)$

b) 2 minimal Hamming dist. between codewords

c) how many errors can detect? correct?
$2-1=1$ $\left\lfloor \frac{1}{2} \right\rfloor = 0$
$(d_{min}-1)$ ⇒ can detect one error
$\left\lfloor \frac{d_{min}-1}{2} \right\rfloor$
Bsp.: (01110) can detect but not correct

2 Design a $C(5,2)$ code with max d_{min}

a) 32 vectors of length $n=5$ choose $2^k = 2^2 = 4$
→ check their minimal pairwise Hamming dist.

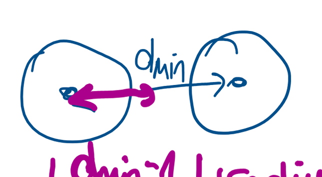
$d_{min} = 3$
- (00000)
- (00111)
- (11100)
- (11011)

$d_{min} \ge 4$ not possible
0 [0000]
0 [1111] match in least in 2 positions

b) LUT

c'	u'
00000	00
11100	01
00111	10
11011	11

c) $d_{min}-1 = 2$ detection
$=1$ correction
$\left\lfloor \frac{d_{min}-1}{2} \right\rfloor$



$\left\lfloor \frac{d_{min}-1}{2} \right\rfloor$ radius
sogl. das nicht überlappen

3) ~~ut~~ $u=(11)$ $e=(001000)$ $n=6$ $k=2$

$\bar{c} = (111111)$ (LUT)

$\bar{v} = \bar{c} + \bar{e} = (110111)$

$c' = (111111)$ min dist.

$u' = (11)$ (LUT)

4) $u=(01)$ $e=(001011)$ $d_{min}=3$ ≙ only correct 1 error

$\bar{c} = (010101)$ ~~$\bar{v}$~~ $\bar{v} = (011110)$ $\bar{c}' = (111111)$ $u' = (11)$

5) (a) $\{111, 100, 010, 001\} \Rightarrow (3, 2, 2)$ (n k d_{min}) $\frac{k}{n} = \frac{2}{3}$

(b) $\{00000, 01111, 10100, 11011\} \Rightarrow (5,2,2)$ $\frac{2}{5}$ ≙ code worst
same d_{min} but schlechtes code rate

(c) $\{00000\} \Rightarrow (5, 0, d)$ ⇒ code rate $=0$
$2^k = 1$ d undef.
⇒ no useful info can be transmitted

6 "Freshman", "Sophomore", "Junior", "Senior" ⇒ 2^2 $k=2$
single error correct., 5-bit binary encoding
↓ $d_{min}=3$ ↓ $n=5$
$C(5,2,3)$

7 $\underbrace{(D_1, D_2, D_3)}_{\text{data bits}}$ $\underbrace{(P_1, P_2, P_3)}_{\text{parity bits}}$
$P_1 = D_1 + D_2$ $P_2 = D_2 + D_3$ $P_3 = D_3 + D_1$

(a) n, k? – $n=6$ $k=3$

(b)
```
000000
001011
010110
100101
101110
110011
111000
```

c) $d_{min} = 3$ → smallest weight of codes words except all "0"
linear code

8) syndrome bits E_1, E_2, E_3
$E_1 = D_1 + D_2 + P_1$ $E_2 = D_2 + D_3 + P_2$ $E_3 = D_3 + D_1 + P_3$
$= D_1 + D_2 + D_1 + D_2$

$E_1 E_2 E_3$ → fewest errors nehmen wir
000 → no error
010 → P_2 (one P – 1×"1") one error von P_2
101 → D_1 (one D – 2×"1")
111 → multiple errors

9) Maximum Distance Separable

$c_1 = (00000)$ $c_2 = (11111)$

Singleton: $d_{min} \le n-k+1$
$5 \le 5-1+1$
$5 = 5$ ✓ MDS code → nice

10) not MDS code $n=6$ $k=3$ $d_{min}=3$

11) $n=15$ corrects $t=1$ error what k?
for perfect code

Hamming: t errors correct

$$\sum_{i=0}^{t} \binom{n}{i} \le 2^{n-k}$$

$$\sum_{0}^{1} \binom{n}{i} \le 2^{n-k}$$

$$\binom{15}{0} + \binom{15}{1} \le 2^{15-k}$$

$$15 + 1 \le 2^{15-k}$$

$$16 = 2^{15-k} \;\; ≙ \; k = 11$$