1. Lecture

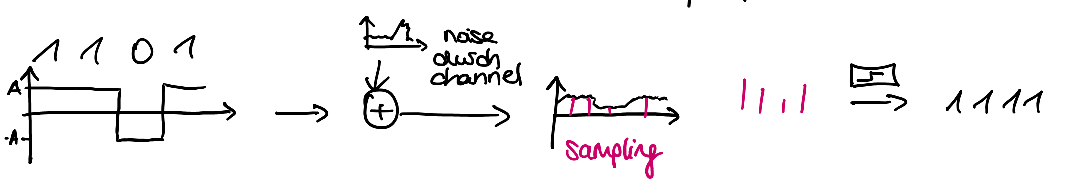
Mittwoch, 6. September 2023 15:04

LIMITS: 1) radio spectrum (teuer, begrenzt)
2) small devices → little batterie ↯ nicht geeignet für großen Verbrauch

1. ERROR CONTROL CODES

How to communicate reliably over an unreliable channel? – in QoS sense

=> $P(\text{error}) \le 10^{-\gamma}$ γ – QoS parameter (Quality of Service)



ERROR: random, independent (from past, no memory), depends on signal energy (proportional to amplitude)
→ wenn 0 niedriger gewesen wäre ↯ wäre Noise egal gewesen

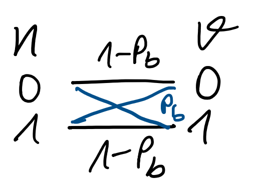
BINARY MODEL:

$u \in \{0,1\}$ → BSC → $v \in \{0,1\}$

binary symmetr. channel

P_b

Bit-Error-Wahrscheinlichkeit: $P_b = p(v=1 \mid u=0) = P(v=0 \mid u=1)$



≡

Additive Model:

$u \xrightarrow{} \oplus \rightarrow v = u \oplus \bar{e}$, $e \in \{0,1\}$, BNG

u	e	v		
0	0	0	OK	$1-P_b$
0	1	1	ERROR	P_b
1	0	1	OK	$1-P_b$
1	1	0	ERROR	P_b

?! $0 \le P_b \le 0{,}5$?!
$P_b > 0{,}5$: BSC → Inverter, $\downarrow P_b < 0{,}5$

Vectorial channel

$\bar{u} = (10101) \rightarrow \oplus \rightarrow v = \bar{u} \oplus \bar{e} = (11111)$, $\bar{e} = (01010)$

HAMMING DISTANZ: $d(\bar{u}, \bar{v}) = 2$ (Unterschiede)

$\rightarrow d(u, \bar{v}) = w(\bar{u} \oplus \bar{v}) = w(\bar{e})$ Gewicht (wo 1 steht)

10101
11111
01010

$P(\bar{v} = (11111) \mid \bar{u} = (10101))$
$= P_b^2 (1-P_b)^3$
$= P_b^2 (1-P_b)^{5-2}$ 5 positions – 2 errors

$$\boxed{P(\bar{v} \mid \bar{u}) = P_b^{d(\bar{u}, \bar{v})} \cdot (1-P_b)^{n - d(\bar{u}, \bar{v})} = P_b^{w(\bar{e})} (1-P_b)^{n - w(\bar{e})} = \left(\frac{P_b}{1-P_b}\right)^{w(\bar{e})} \cdot (1-P_b)^n}$$

binary

$0 \le P_b \le 0{,}5 \Rightarrow \frac{P_b}{1-P_b} < 1$ exp. decrease ⇒ kleine Errors schlimmer

$$P(w(\bar{e}) = i) = \binom{n}{i} \left(\frac{P_b}{1-P_b}\right)^i (1-P_b)^n$$

P: 1, ... no need to pay attention; exp. decay
small number of errors must correct

Correcting "t" # of errors

$$P(\text{errors}) = \sum_{i=t+1}^{n} \binom{n}{i} \left(\frac{P_b}{1-P_b}\right)^i (1-P_b)^n \le 10^{-\gamma} \quad (\text{QoS})$$

Given: P_b, γ, n, t

How to design a code of correcting "t" errors?

ERROR CONTROL CODING:

$0 \rightarrow \boxed{N\times} \rightarrow 00000 \rightarrow \oplus \rightarrow 01010 \rightarrow \boxed{\text{Majority decision}} \rightarrow 0$

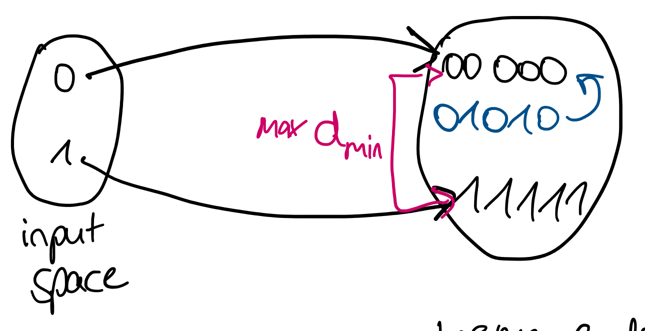
N is odd; repeat N times; 01010

$$P_b' = \sum_{i=\lfloor \frac{n}{2} \rfloor} \binom{n}{i} P_b^i (1-P_b)^{n-i} \le 10^{-\gamma}$$

abgerundet (geht hier nur wenn unter 2 Fehler)

Loss: $\frac{1}{n}$ (wegen mehrfach Übertragung)

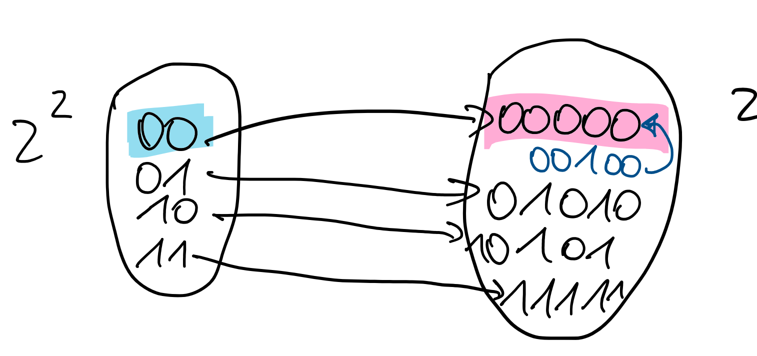
Geometric interpretation – space extension



2^5 output space

can detect error
→ what is closed code word (hamming distance)
→ correct

wenn code words far away von einander
⇒ besser detect & correct



detect error & correct

loss in data speed $\frac{2}{5}$

FORMAL DESCRIPTIONS:

Messages $\bar{u} \in \{0,1\}^k$ $\dim(\bar{u}) = k$ $\bar{u} = (\underbrace{10 \dots 1}_{k})$; $M = 2^k$

Code $C = \{\bar{c}_1, \bar{c}_2 \dots \bar{c}_n\}$ $\dim(\bar{c}) = n > k = \dim(\bar{u})$

$C(n,k)$, $\frac{\varepsilon}{n}$, $\underbrace{n-\varepsilon}_{\text{redundancy of code}}$

Coding: $\psi: \{0,1\}^k \rightarrow C$, $\psi(\bar{u}) = \bar{c}$

$\bar{c} \rightarrow \oplus \rightarrow \bar{v} = \bar{c} + \bar{e}$ ($\bar{\varepsilon}$) Received vector $\bar{v} \in \{0,1\}^n$

Detection: $\varphi: \{0,1\}^n \rightarrow C$ $\varphi(\bar{v}) = \bar{c}'$

Decoding: $\psi^{-1}: C \longrightarrow \{0,1\}^k$; $\psi^{-1}(\bar{v}) = \bar{c}'$ ¿$\bar{u}'$?

$\bar{u} \rightarrow$ Lookup Table $\xrightarrow{\bar{c}} \oplus$ ($\bar{e}$) $\xrightarrow{\bar{v}}$ SEARCH $\xrightarrow{\bar{c}'}$ LUT $\rightarrow \bar{u}'$

Lookup Table:

c	$\bar{c}$
00	00000
11	11111

max d_{min}

SEARCH: $\bar{c}'$: min $d(\bar{v}, \bar{c})$, $\bar{c} \in C$

LUT:

$\bar{c}$	$\bar{u}$
Code word	message

2 LUT + SEARCH ALGO

complexity analysis:

offline complexity $\binom{2^n}{2^k} \binom{2^k}{2}$

↯ NON REALTIME

online complexity $3 \cdot O(2^k)$