$$S_n^2 = rac{1}{n-1} \sum_{i=1}^n (X_i - ar{X}_n)^2$$

$$E(S_c^2) = E\left(rac{1}{n-1} \sum_{i=1}^n (X_i - ar{X}_n)^2
ight)$$

$$=rac{1}{n-1}E\left(\sum_{i=1}^n(X_i-ar{X}_n)^2
ight)$$

$$L = rac{1}{n-1} E \left(\sum_{i=1}^n (X_i^2 - 2 X_i ar{X}_n + ar{X}_n^2)
ight)$$

$$=rac{1}{n-1}\sum_{i=1}^{n}\left(E(X_{i}^{2})-2E(X_{i}ar{X}_{n})+E(ar{X_{n}}^{2})
ight)$$

$$=^{1.} rac{1}{n-1} \sum_{i=1}^n \left(E(X_i^2) - E(ar{X_n}^2)
ight)$$

$$=^{2\cdot}rac{1}{n-1}\sum_{i=1}^n(\sigma^2+\mu^2-rac{\sigma^2}{n}-\mu^2)$$

$$=rac{1}{n-1}\sum_{i=1}^n(rac{n-1}{n}\sigma^2)$$

$$=\sigma^2$$

$$E({ar{X_n}}^2) = rac{1}{n} E((X_1 + \dots + X_n) ar{X_n})$$

$$=rac{1}{n}nE(X_iar{X}_n)$$

$$=E(X_iar{X}_n)$$

$$1. \implies E(ar{X_n}^2) = E(X_i ar{X}_n) \quad 0 < i \leq n$$

$$2. \ \ E(ar{X_n}^2) = Var(ar{X_n}) + E(ar{X_n}) = ^{E(ar{X_n}) = E(X_i)} = rac{\sigma^2}{n} + \mu^2$$

$$Var(ar{X_n})=rac{1}{n^2}Var(X_1+\cdots+X_n)=^{i.i.d}=rac{n}{n^2}Var(X_1)=rac{\sigma^2}{n}$$

$$S_n^2 = rac{1}{n-1} \sum_{i=1}^n (X_i - ar{X}_n)^2$$

$$E(S_c^2) = E\left(rac{1}{n-1} \sum_{i=1}^n (X_i - ar{X}_n)^2
ight)$$

$$=rac{1}{n-1}E\left(\sum_{i=1}^n(X_i-ar{X}_n)^2
ight)$$

$$=rac{1}{n-1}E\left(\sum_{i=1}^{n}(X_{i}^{2}-2X_{i}ar{X}_{n}+ar{X}_{n}^{2})
ight)$$

$$=rac{1}{n-1}\sum_{i=1}^{n}\left(E(X_{i}^{2})-2E(X_{i}ar{X}_{n})+E(ar{X_{n}}^{2})
ight)$$

$$=rac{1}{n-1}\sum_{i=1}^{n}(\sigma^{2}+\mu^{2}-rac{\sigma^{2}}{n}-rac{n-1}{n}\mu^{2})$$

$$=rac{1}{n-1}\sum_{i=1}^n\left(rac{n-1}{n}\sigma^2+rac{1}{n}\mu^2
ight)$$

$$E(X_iar{X}_n) = rac{1}{n}E\left(X_i^2 + \sum_{j
eq i}^n X_iX_j
ight)$$

$$=rac{1}{n}(E(X_i^2)+\sum_{i
eq i}^n E(X_i)E(X_j))$$

$$=rac{\sigma^2}{n}+rac{n-1}{n}\mu^2$$
 i.i.d

$$E(ar{X_n}^2) = rac{1}{n} E((X_1 + \cdots + X_n)ar{X_n})$$

$$=rac{1}{n}nE(X_iar{X}_n)$$

$$=E(X_iar{X}_n)$$