Midterm

Mittwoch, 25. Oktober 2023 09:53

45 Min.

2nd Nov 6pm QBF12
Laptop → electronical (Moodle guide) + calculator

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① Binary Linear Code

$\bar{\bar{G}}_{2\times5} = \begin{pmatrix} 1&0&1&1&1 \\ 0&1&1&1&0 \end{pmatrix}$ → systematic code: $I, B = G$

a) type of code: $n=5$ $k=2$ $C(5,2)$

b) error correcting / detecting

d_{min} - min $d(\bar{c}, \bar{c}')$ ~ w_{min}

$$\left.\begin{aligned} (0,0)\cdot\bar{\bar{G}} &= (00000) \\ (0,1)\cdot\bar{\bar{G}} &= (01110) \\ (1,0)\cdot\bar{\bar{G}} &= (10111) \\ (1,1)\cdot\bar{\bar{G}} &= (11001) \end{aligned}\right\} \begin{aligned} w_{min} &= 3 \\ &= d_{min} \end{aligned}$$

↯ $d_{min} - 1 = 2$ detecting
$\left\lfloor \frac{d_{min}-1}{2} \right\rfloor = 1$ correcting

c) $\bar{e} = (01100)$ $\bar{e}_{detected}$?

$\bar{\bar{H}}_{3\times5} = \begin{pmatrix} 1&1&1&0&0 \\ 1&1&0&1&0 \\ 1&0&0&0&1 \end{pmatrix}$

$\bar{\bar{H}}\cdot\bar{e}^T = \bar{s}^T = \begin{pmatrix} 1&1&1&0&0 \\ 1&1&0&1&0 \\ 1&0&0&0&1 \end{pmatrix} \cdot \begin{pmatrix} 0\\1\\1\\0\\0 \end{pmatrix} = \begin{pmatrix} 0\\1\\0 \end{pmatrix}$

$E_{(010)} = ((01100), (01100 + 01110), (01100 + 10111), (01100 + 11001))$
$= \{(01100), \underline{(00010)}, (11011), (10101)\}$

↓ „team" leader → $\bar{e}_{detected} = (00010)$ //
↯ misdetected

d) if BSC $P_b = 0.1$ what is $P(\bar{e})$ $\bar{e} = (01100)$

$P(\bar{e}) = P_b^2 \cdot (1-P_b)^3$
$= 0.9 * 0.1 * 0.1 * 0.9 * 0.9$
$= 0.1^2 * 0.9^3$ //

Short text questions (4 or 5)
→ true or false

- Reed-Sol. Codes $\overset{\text{able to correct}}{\vee}$ $t = n-k$ #errors. : False → $t = \left\lfloor \frac{n-k}{2} \right\rfloor$
- numbers of errors in E_S same as number of codewords → true
- system. code ^{use} Linear Feedback Shift Register to do coding → false
- can binary linear code $C(15,11)$ can be binary Hamming code? → $\sum_{i=0}^{t} \binom{n}{i} = 2^{n-k}$

$t = 1$ (Hamming code) ⇒ true

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2) Reed-Solomon-Code

a) correcting every double error - $C(n,k)$? over $GF(q)$

$q - 1 = n$ $t = \left\lfloor \frac{n-k}{2} \right\rfloor$

→ $2t = 4 = n-k$

q	n	k
2	1	—
3	2	—
5	4	0
7	6	2 ←

$\underline{\underline{C(6,2)}}$ → GF(7) → $\overset{\text{largest}}{\text{primitive element}}$ = 5 (given to us)

b) Generator / Parity-Matrix

$\bar{\bar{G}}_{2\times6} = \begin{pmatrix} 1&1&1&1&1&1 \\ 1&5&4&6&2&3 \end{pmatrix}$ $\bar{\bar{H}}_{4\times6} = \begin{pmatrix} 1&5&4&6&2&3 \\ 1&4&2&1&4&2 \\ 1&6&1&6&1&6 \\ 1&2&4&1&2&4 \end{pmatrix}$

c) $\bar{c}$? if $\bar{u} = (2,2)$

$\bar{c} = (2,2) \begin{pmatrix} 1&1&1&1&1&1 \\ 1&5&4&6&2&3 \end{pmatrix}$
$= (4\ 5\ 3\ 0\ 6\ 1)$ //

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3) Reed-Solomon $GF(2^M)$

a) $GF(2^3)$, $p(y) = y^3 + y + 1$ - given

$y \in GF(2^3)$ primitive element ($\overset{y}{\overleftarrow{2}}$ immer primitive element)

$3+4 = y+1+y^2 = y^2+y+1 = 7$
$4+5 = \cancel{y^2}+\cancel{y^2}+1 = 1$

	binary	polyn.
0	000	$0\cdot y^2 + 0\cdot y^1 + 0\cdot y^0 = 0$
1	001	$1\cdot y^0 = 1$
2	010	$1\cdot y^1 = y$
3	011	$y + 1$
4	100	y^2
5	101	$y^2 + 1$
6	110	$y^2 + y$
7	111	$y^2 + y + 1$
8		

Power Table:

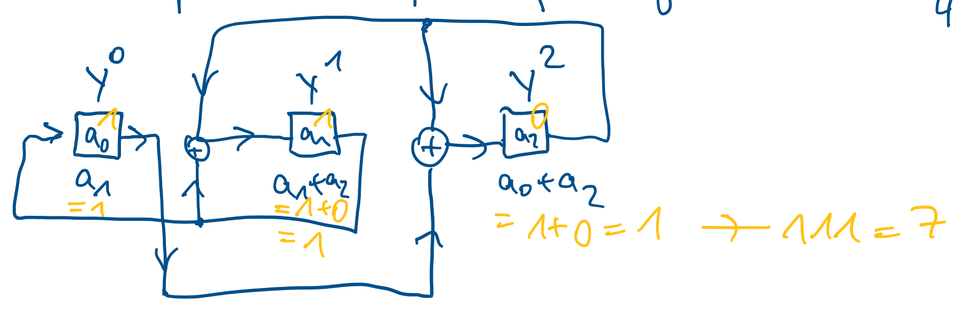
y^0	1
y^1	y
y^2	y^2
y^3	$y+1$
y^4	y^2+y
y^5	y^2+y+1
y^6	y^2+1
y^7	1

} will be given

$y^3 : (y^3+y+1)$
$\to y^3 = 1\cdot(y^3+y+1) + y + 1$

a) $3 * 4 = (y+1)\,y^2$
$= y^3 \cdot y^2$
$= y^5 = y^2+y+1 = 7$

b) How implement by Shift Registers?



$4 * \alpha = 4\,(a_0 + a_1 y + a_2 y^2)$
$= y^2 (a_0 + a_1 y + a_2 y^2)$
$= a_0 y^2 + a_1 y^3 + a_2 y^4$
$= a_0 y^2 + a_1 (y+1) + a_2 (y^2+y)$
$= a_1 + (a_1+a_2)\,y^1 + (a_0+a_2)\,y^2$

⇒ connection ~~pattern~~ pattern durch 4 festgelegt

$\alpha = 3$

bis
Reed-Solomon q-prime