Problems of 2020. Duration of the exam: **10 days!!**. Since this was held during the Corona times, it was a harder exam but it was a take-home and was due in 10 days.

1. After the relaxation of curfew regulations in the town of Pyranea, the citizens were obligated to wear a mask every day. As they do not want to unnecessarily restrict the already highly disciplined citizens, everyone is free to choose to wear either an orange mask or a black mask. The Pyrenians, while being orderly, hard-working and adaptable people, have a few other distinctive characteristics. For example, they are active on Facebook: all Pyrenians upload a selfie of themselves every morning and in the afternoon they look at the pictures uploaded by all their Facebook friends on that day. They then pick the mask colour used by a majority of their acquaintances to choose the colour of their mask for the next day. (Oddly enough, each Pyrenian has an odd number of Pyrenian Facebook friends.)
   Prove that sooner or later there will be a day when all the Pyrenians will be wearing a mask of the same colour as the one they wore two days before.
   (It's common knowledge that in Pyranea, Facebook friends cannot be deleted or altered.)

2. At the suggestion of the president of Pyranea, each of the n citizens of his country are assigned an identification number (a positive integer). The identification number of the president just happens to be his favourite number, while the next $n-1$ consecutive integers are assigned to the remaining citizens. Prove that there exists a non-empty subset of Pyrenians such that the sum of their identification numbers is divisible by $\binom{n+1}{2}$.

3. Show that a 50-regular simple graph on 101 vertices must have a Hamiltonian cycle.