Problems of 2018. Duration of the exam: 150 minutes.

1. Determine the maximum number of edges a graph G can have, if we know that it is simple, bipartite with 222 vertices, and $\nu(G) = 42$.
2. What are the integers $n \geq 1$ for which the edges of the complete graph K_n can be coloured by some colors, such that, edges in each color class form a spanning tree of K_n?
3. Assume that the edges of the complete graph K_n are colored with red, white and green colors such that the edges in each color class form a connected graph on the n vertices. Show that we can always find a triangle whose edges are coloured with the three different colours.
4. Let P be a set of n points in the plane such that no three of them lie on a straight line. Let H denote the set of intersection points of the lines defined by pairs of points in P, i.e., all points which lie on at least two such lines. Prove that there are at most n open segments defined by P that are disjoint from H.
5. Let G be a simple bipartite graph with n vertices in both of its partitions A, B. Further, for any $u \in A, v \in B$, if $uv \notin E$, then $d(u) + d(v) \geq n + 1$. Prove that G contains a Hamiltonian cycle.

In the notes they said that of the 46 participants, 43 participants chose to hand in a submission. There were many correct submissions for the first and second problems. The fourth and fifth problem each had two almost correct submissions. Nobody could solve problem 3. The first prize of 25000 Ft was awarded to two students, both who had three full correct solutions.