The Konig Denes competition is held in May every year. The duration of the exam is 150 minutes. There is a cash prize awarded to the top three contestants. It is a written exam with 3-5 problems on it.

1. (KD'2018, Problem 5) Let G be a simple bipartite graph with n vertices in both of its partitions A, B. Further, for any $u \in A, v \in B$, if $uv \notin E$, then $d(u)+d(v) \geq n+1$. Prove that G contains a Hamiltonian cycle. (Note: In the notes they said that in 2018, of the 46 participants, 43 participants chose to hand in a submission. There were many correct submissions for the first and second problems. The fourth and fifth problem each had two almost correct submissions. Nobody could solve problem 3. The first prize of 25000 Ft was awarded to two students, both who had three full correct solutions.)

2. (KD'2020, Problem 3) Prove that every simple 50-regular graph on 101 vertices has a Hamiltonian cycle. (Out of the 28 submissions, 5 solved this correctly).

3. (KD'2023, Problem 3) Show that a simple connected graph G on 10 vertices with the degree sequence $1, 1, 4, 4, 4, 4, 4, 4, 4, 4$, has a spanning tree with at most 4 leaves. (28 students handed in a sumbission on this contest, this was not the easiest or the hardest problem in this test, and multiple solutions were present for this problem.)

   Now some problems from another topic.

4. (KD'2019, Problem 2) In the network $G(s, t, c)$, the capacity of some edges is x and the capacity of all the other edges does not depend on x. Assume that $M(22) = 1234$ and $M(33) = 1243$, where $M(x)$ denotes the maximum st-flow in the network. Determine the value of $M(42)$.

5. (KD'2022, Problem 2) Assume that in the network (G, s, t, c), the maximum st-flow has capacity 24. If the capacity of both the edges u_1v_1 and u_2v_2 increases by 32, then the maximum flow increases to 88. Is it possible to determine, with this information, the maximum flow in the original network, if the capacity of the edge u_1v_2 is decreased by 13, while the capacity of the edge u_2v_1 is increased by 31? If yes, then how much is this maximum flow? (Assume that the capacity of the edge u_1v_2 in the original network is at least 13).

   Back to Hamiltonian paths! (I don't know how I missed this before).

6. (KD'2022, Problem 4) Is it true that simple graph on 10 vertices with degree sequence $3, 3, 4, 4, 4, 5, 5, 5, 5, 6$ surely has a Hamiltonian path?