List of topics for the Final exam
ITC2, Spring 2022.

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1. Basic counting: **permutations, variations and combinations with and without repetition; examples**. Relationships between binomial coefficients, Pascal's triangle. Binomial theorem.

2. Basic concepts in graph theory: **graph, simple graph**, complement of a graph, **isomorphism, subgraph**, spanning subgraph, walk, **path, cycle, connected graph**, components.

3. **Trees: definition, four equivalent definitions**, properties of trees.

4. **Breadth-first search (BFS)**, properties of the BFS tree.

5. Concept of a **spanning tree**, its existence. Minimum total weight spanning tree, **Kruskal's algorithm**.

6. **Hamiltonian cycles and paths**. Necessary condition for the existence of a Hamiltonian cycle/path. Sufficient conditions: Dirac and Ore's theorem. **Eulerian trails and circuits**, necessary and sufficient conditions for their existence.

7. Colouring of graphs, **definitions of $\chi(G)$ and $\omega(G)$. Relation between $\chi(G)$ and $\omega(G)$**, possible values of their difference. Greedy colouring. Relation between $\chi(G)$ and $\Delta(G)$. Interval graphs, algorithm for optimal colouring of interval graphs. Concept of a **Bipartite graphs**, characterization of bipartite graphs with respect to odd cycles.

8. **Plane and planar graphs, Euler's formula**, number of edges in a planar graph, subdivisions, **Kuratowski's Theorem** (without proof), Fary's Theorem (without proof).

9. **Matchings. Independent edge set, edge cover, independent vertex set, vertex cover**, their relation with each other. **Definitions of $\nu(G), \rho(G), \alpha(G)$ and $\tau(G)$.** Gallai's theorems.

10. Matchings in a bipartite graph. **Augmenting path algorithm, Konig, Hall** and Frobenius theorems. Existence of perfect matchings in regular bipartite graphs.

11. Edge colouring of graphs, **$\chi_e(G)$ and its relation to $\Delta(G)$**. Vizing's theorem (without proof), Konig's theorem about the edge chromatic number of bipartite graphs.

12. Networks, **definition of flow, flow value, s,t-cut and its capacity. Algorithm for finding the maximum flow and minimum cut, Ford-Fulkerson theorem**, Edmonds-Karp theorem (without proof). Integrality lemma. Generalizations of the flow problem (multiple sources, vertex capacity, undirected graph).

13. **Definitions of k-connected, k-edge-connected, $\kappa(G)$ and $\lambda(G)$, and the relation between them**. Edge disjoint paths, disconnecting set of edges and the relation between them. Vertex disjoint paths, disconnecting set of vertices and the relation between them. Menger's theorems (local edge, vertex connectivity, for graph connectivity in both directed and undirected graphs).

14. Shortest path problem, **notion of a conservative weight function**. Bellman-Ford algorithm (without proof of correctness), **Dijkstra's algorithm** (without proof of correctness).

15. **Definition of an acyclic directed graph, topological ordering of vertices**. Algorithm for finding the shortest and longest paths in an acyclic directed graph.

16. **DFS algorithm**, DFS forest, classification of edges. Application of DFS for deciding acyclicity and determining topological order.