Analysis of matrices
Preliminary exam topics, 2024

1. Basic properties, operations of matrices, determinant, rank.

2. Rank-one matrices. Minimal rank one decomposition: algorithms, applications (properties of the rank).

3. Inverse of a matrix, adjoint matrix, Sherman-Morrison formula.

4. Inverse of special matrices (diagonal, triangular, permutation matrix). Elementary transformations and their matrix forms. Sylvester's law of nullity.

5. Projections, theorem of Egerváry. Biorthogonal system and how to make it a complete system.

6. Types of generalized inverses (generalized, reflexive, Moore-Penrose) and their properties.

7. Special matrices: symmetric, tridiagonal, uniform tridiagonal, Toeplitz, upper Toeplitz, nilpotent. Chebyshev polynomials.

8. Determinant, adjoint and the inverse of a symmetric uniform tridiagonal matrix.

9. Block matrix. Symmetric 2 by 2 partition, determinant, inverse expressed by the blocks.

10. Persymmetrical partition and its application to the inverse computation of symmetric tridiagonal and one-pair matrices.

11. Solutions of linear system of equations (homogeneous, inhomogeneous case).

12. Linear maps, similar matrices. Eigenvalues, eigenvectors, eigenspaces. Characteristic polynomial. Diagonalizable matrix.

13. Spectral decomposition. Eigenvalues of special matrices (Hermitian, skew Hermitian, unitary, projection).

14. Theorem of Schur. A can be diagonalized by unitary matrix iff A is normal.

15. Singular values, singular value decomposition.

16. Cayley-Hamilton theorem. Minimal polynomial, reduced adjoint.

17. Connection between the minimal polynomial and the property that the matrix can be diagonalized.

18. Functions of matrices. Lagrange polynomials. Properties of the matrix $L_j(A)$.

19. Spectral decomposition of $f(A)$. Special cases: projections and cyclic matrices.

20. Multiple roots of the minimal polynomial. Hermite interpolation.

21. Jordan form of matrices.

22. System of linear differential equations (homogeneous, inhomogeneous). Case of constant coefficients.