Problems of 2019. Duration of the exam: 150 minutes.

1. Before the König competition, we received four seemingly identical pills from the fairy godmother. Of the four tablets, two were type A and two were type B. If we take two pills of the same type, then we fall asleep immediately and do not wake up for the next for the next six hours. If you take just one type A and one type B pill, you will suddenly be transformed into a super-intelligent being, and are therefore sure to win this competition. How can we guarantee that at the results ceremony we will be the winners of the König competition, eating pogácsa with the organizers?

2. In the network $G(s, t, c)$, the capacity of some edges is x and the capacity of all the other edges does not depend on x. Assume that $M(22) = 1234$ and $M(33) = 1243$, where $M(x)$ denotes the maximum st-flow in the network. Determine the value of $M(42)$.

3. What is the maximum number of edges of a simple graph with 16 vertices, such that the deletion of any one edge results in a planar graph?

4. The edges of the complete graph K_n are colored with colors red, white and green, such that all colors are used, and the subgraph obtained with the edges of only one color is not connected for any color. Prove that there is a colorful triangle (with all three colors in it).

5. Prove that, for $n > 1$, irrespective of how we can assign pairwise different 0/1-vectors of length n to the vertices of a path of length 2^n, we will always find two edges on the path such that for each edge, the sum of the vectors (modulo two) of its endpoints is the same. Further prove that if the length of the 0/1 vectors was instead $(n + 1)$, then it is possible that the vectors obtained as the sum of the endpoints of edges are all pairwise different.

This time there were 20 participants, of whom 17 sumitted their answer sheets. The first prize of 30000 Ft was won by a girl this year. Problem 2 seemed to be the easiest for those who participated, while problem 5 was the hardest. But multiple correct solutions were handed in for every problem.