1. Cross product

What are the coordinates of a vector $\mathbf{v} = [1, 0, 0] \times [1, 1, 0]$?

2. Theorem of Pythagoras

Using coordinate geometry and vector algebra, prove the theorem of Pythagoras:

$$|\mathbf{v}|^2 = c^2 = a^2 + b^2,$$

where $\mathbf{v} = [a, b]$.

3. Implicit equation of a line

In the 2D Euclidean plane, the parametric equation of a line is

$$\mathbf{r} = [r_x, r_y] = [p_x, p_y] + [v_x, v_y] \cdot t.$$

What is the implicit equation of this line?

4. Dual numbers

```
float value = 1;
Dnum t(value, 1);
Dnum F = t * t * 2 + t * 3 + 4;
```

After having F evaluated, what are the values of F.f and F.d?

5. Curvature

A torus is defined by radii R and r. What is the Gaussian curvature in those surface points, where the distance from the axis is maximal?

6. Lagrange curve

The control points are defined as r_1 = [-2, 4], r_2 = [0, 0], and r_3 = [2, 4]. The corresponding knots are t_1 = -2, t_2 = 0, and t_3 = 2, respectively. Using Lagrange interpolation, what are the coordinates of a point corresponding to parameter t = 1?

7. Quadratic surfaces

The implicit equation of a quadratic surface is

$$f(x, y, z) = a{\cdot}x^2 + b{\cdot}x{\cdot}y + c = 0.$$

What are the elements of matrix **Q** in the equivalent quadratic form

$$f(x, y, z] = [x, y, z, 1]\ \mathbf{Q}\ [x, y, z, 1]^\mathsf{T}$$

if **Q** is required to be symmetric?

8. Surface normal

The implicit equation of an ellipsoid is $f(x, y, z) = x^2 / 4 + y^2 + z^2 - 1 = 0$.

What are the coordinates of the surface normal at point $\left[\sqrt{2}, \quad \frac{1}{2}, \quad \frac{1}{2}\right]$?

9. Affine transformations

What are the elements of a transformation matrix that represents a counter-clockwise rotation by 90 degrees around the point [1, 1]?

10. Ambient space

In the ambient space embedding the projective geometry, a point is defined by P = [X, Y, w] and a line is defined by L = [n_x, n_y, c]. What are the coordinates of a vector in the ambient space that represents a line that is perpendicular to line L and goes through point P?

11. Parametric surfaces

A flag is defined by the following parametric surface:

$$x(u, v) = 2u,$$
$$y(u, v) = \sin(2\pi u) / (2\pi),$$
$$z(u, v) = v,$$

where $u, v \in [0, 1]$.

What are the coordinates of the normal corresponding to the surface point defined by $[u, v] = [0.5, 0.5]$?

12. Barycentric coordinates

In Cartesian coordinates, the vertices of a triangle are defined as $\mathbf{r}_1 = [x_1, y_1]$, $\mathbf{r}_2 = [x_2, y_2]$, and $\mathbf{r}_3 = [x_3, y_3]$. How can we express the barycentric coordinates of an internal point $\mathbf{r} = [x, y]$ from the Cartesian coordinates?