System Theory

1^{ST} HOME WORK

Name:	Imeri Klevis
Neptun code:	T4XGKO
Home work code:	kga4to
Deadline:	See the table of requirements on the website.

Attention! *The task sheets must be handed in together with the solution. Submitting only the final results is not sufficient, the full solution must be written in detail. For the calculation of the numerical results and for the plotting of figures one may use mathematical softwares (MATLAB, DERIVE, etc.) however all important steps of the solution must be detailed.*

	a	b	c	d	$\sum$	Corrector
1.1	/ 0.4	/ 0.4	/ 0.6	/ 1.6	/ 3	
1.2	/ 2.4	/ 0.6	/ 1.6	–	/ 4.6	
1.3	/ 1.6	/ 0.8	–	–	/ 2.4	
					/ 10*	

* The final result is the integer number obtained by the rounding of the sum of points given to the tasks.

1.1 The CT and the DT systems are given with the following impulse responses:

$$h(t) = 6\delta(t) + \varepsilon(t)\left\{4e^{-0.4t} + (3)\cdot e^{-0.2t}\right\}$$
$$h[k] = 3\varepsilon[k](0.7)^k \cos(0.2k + (0.6))$$

(a) Decide the $BIBO$ stability of the CT and the DT systems! Give explanation! (0.2+0.2 points)

(b) Modify one single parameter of the impulse responses to change their stability to the opposite! (0.2+0.2 points)

(c) The input signal of the DT system is given below. Find the response of the system for $k = 0$, $k = 1$ and $k = 2$! (0.6 points)

$$u[k] = \varepsilon[k]\left\{6 + (-3)\cdot(-0.3)^k\right\}$$

(d) Find the formula of the response of the systems if the input signals of the CT and DT systems are the CT and DT signals given below! (0.8+0.8 points)

$$u(t) = 8e^{1.3t}$$
$$u[k] = 8$$

1.2 The CT and the DT systems are given with the following state space description:
CT system:

$$\begin{aligned}
\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} &= \begin{bmatrix} -2.2 & 2.8 \\ -0.5 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1.4 \\ -1.6 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} -0.8 & 0.7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + (1.4)u(t)
\end{aligned}$$

DT system:

$$\begin{aligned}
\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} &= \begin{bmatrix} -0.4 & 1.7 \\ -0.1 & -0.6 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.7 \\ -0.8 \end{bmatrix} u[k] \\
y[k] &= \begin{bmatrix} -0.5 & -0.4 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + (-1.1)u[k]
\end{aligned}$$

(a) Find the formulae of the impulse responses of both systems! (1.2+1.2 points)

(b) With the step-by-step solution of the state equation, find the numerical values of the DT system impulse response for $k = 0$, $k = 1$ and $k = 2$ and compare them with the values given by the impulse response formula! (0.6 points)

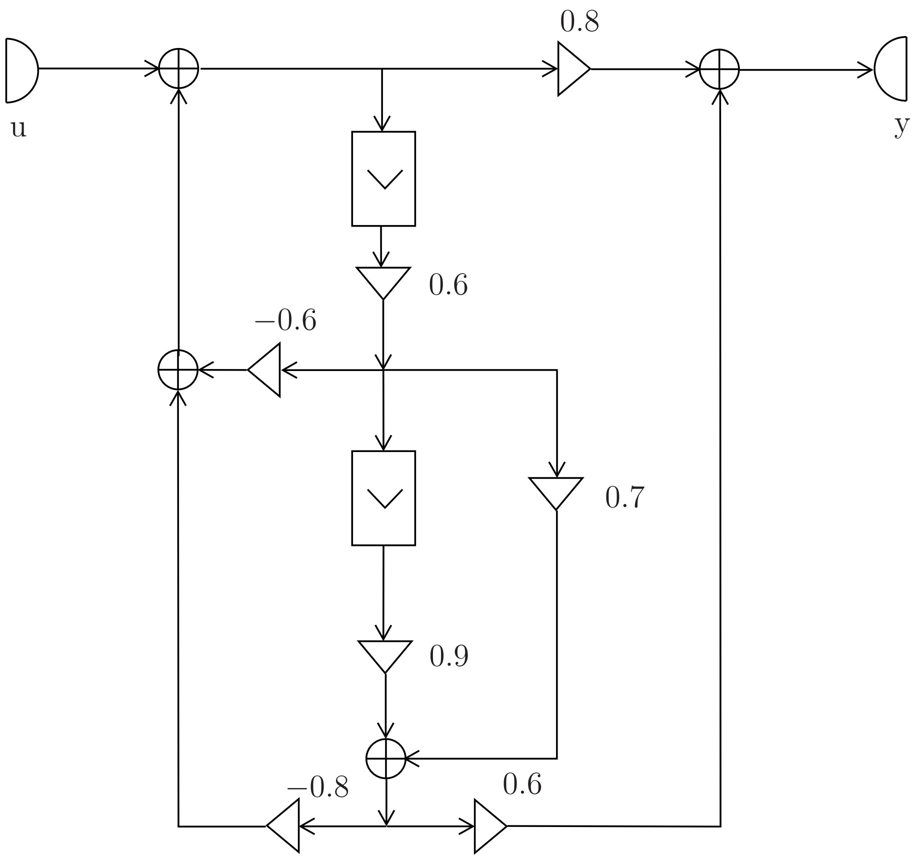
(c) Find the formulae and plot the time functions of the responses of both systems if the input signals of the CT and the DT systems are the ones given below! (0.8+0.8 points)

$$\begin{aligned}
u(t) &= 3\left\{\varepsilon(t) - \varepsilon(t-1.7)\right\} \\
u[k] &= \varepsilon[k]\left\{6 + (-3)\cdot(-0.3)^k\right\}
\end{aligned}$$

1.3 Consider the CT and the DT systems given by the common signal flow network!

(a) Give the state variable description of both systems in normal form! (0.8+0.8 points)

(b) Examine the stability of the CT and the DT networks as well! (0.4+0.4 points)


0.8
u
y
0.6
−0.6
0.7
0.9
−0.8
0.6

SYSTEM THEORY

2ND HOME WORK

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	a	b	c	d	e	$\sum$	Corrector
2	/ 2	/ 3.6	/ 2	/ 1.6	/ 0.8	/ 10	
						/ 10*	

* The final result is the integer number obtained by the rounding of the sum of points given to the tasks.

2.a Find the frequency response of the CT and the DT system given by the state space descriptions below! (1+1 points)
CT system:

$$\begin{aligned}\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} &= \begin{bmatrix} -2.2 & 2.8 \\ -0.5 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1.4 \\ -1.6 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} -0.8 & 0.7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + (1.4)u(t)\end{aligned}$$

DT system:

$$\begin{aligned}\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} &= \begin{bmatrix} -0.4 & 1.7 \\ -0.1 & -0.6 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.7 \\ -0.8 \end{bmatrix} u[k] \\ y[k] &= \begin{bmatrix} -0.5 & -0.4 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + (-1.1)u[k]\end{aligned}$$

2.b A periodic CT signal and a periodic DT signal are given below. Find the real form of the Fourier series of the DT signal and the real form of the Fourier polynomial of the CT signal which contains – in addition to the constant component – at least three nonzero harmonics! (1.8+1.8 points)

$$\begin{array}{lll} u(t) = -12t + 16\varepsilon(t)t, & -2 \le t \le 6, & u(t+8) = u(t) \\ u[k] = 5.5|k|, & -2 \le k \le 3, & u[k+6] = u[k] \end{array}$$

2.c The input signals of the CT and DT systems given by the state space descriptions are the periodic signals of the previous point. Find the response of the DT system and the Fourier polynomial approximation of the CT system response, and plot one period of the responses from $k = 0$ stroke and from $t = 0$ moment! (1+1 points)

2.d The input signals of the CT and DT systems given by the state space descriptions are the impulses which in one period (from $t = 0$ moment and from $k = 0$ stroke) are equal to the periodic signals given in the (2b) point, and the signal values are zero outside of this interval. Find the Fourier transform of the responses of the CT and DT systems! Plot the amplitude spectrum of the CT and DT sytems responses in the $0 < \omega \le 5$ and $0 \le \vartheta \le \pi$ intervals, respectively. (0.8+0.8 points)

2.e Based on the results of the previous point, find the bandwidth of the response of the CT system! Assume that the amplitude spectrum is negligible when its values are less than 5 % of its maximum! (It is possible that one needs to plot the amplitude spetrum in a wider interval than it was required in the previous problem.) (0.8 points)

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3^{RD} HOME WORK

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	a	b	$\sum$	Corrector
3.1	/ 1.6	/ 2.8	/ 4.4	
3.2	/ 2.4	–	/ 2.4	
3.3	/ 2.4	/ 0.8	/ 3.2	
			/ 10*	

* The final result is the integer number obtained by the rounding of the sum of points given to the tasks.

3.1 A CT and a DT systems are given with the following impulse responses:

$$\begin{aligned} h(t) &= 6\delta(t) + \varepsilon(t)\left\{4e^{-0.4t} + (3)\cdot e^{-0.2t}\right\} \\ h[k] &= 3\varepsilon[k](0.7)^k \cos(0.2k + (0.6)) \end{aligned}$$

(a) Find the transfer functions of the systems and write them in normal form! (0.8+0.8 points)

(b) The input signals of the systems are given below. Find the formulae of the responses of both systems! Plot the obtained time functions! (1.4+1.4 points)

$$\begin{aligned} u(t) &= 3\left\{\varepsilon(t) - \varepsilon(t-1.7)\right\} \\ u[k] &= \varepsilon[k]\left\{6 + (-3)\cdot(-0.3)^k\right\} \end{aligned}$$

3.2 The CT and DT systems are given with the following state space description:
CT system:

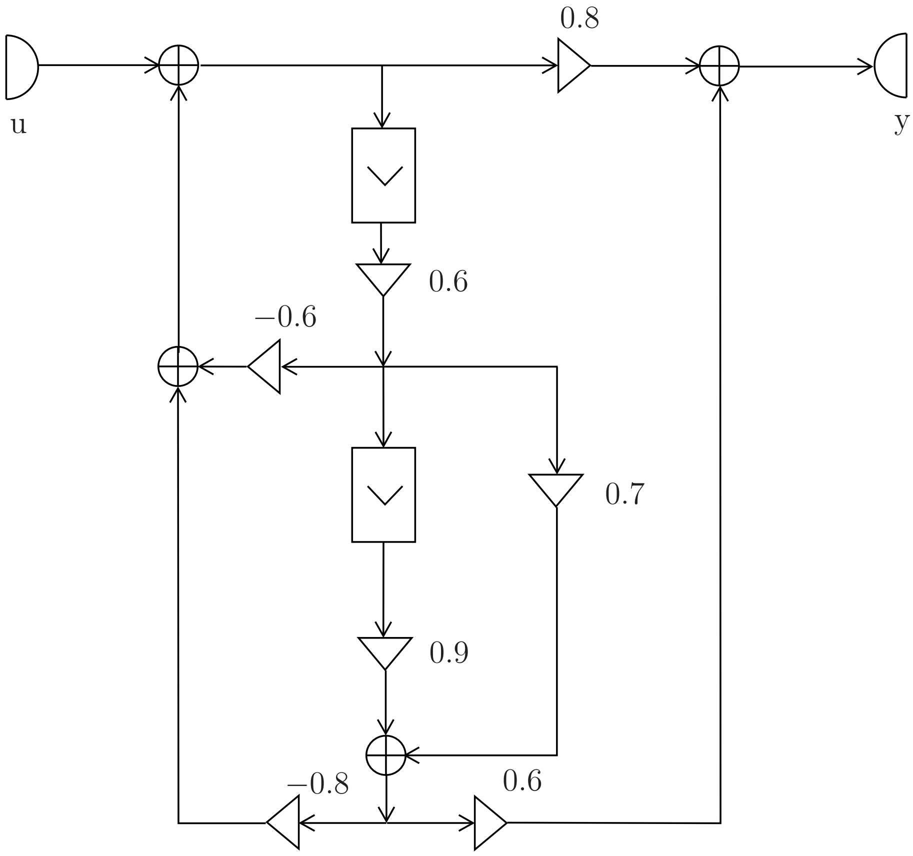
$$\begin{aligned} \begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} &= \begin{bmatrix} -2.2 & 2.8 \\ -0.5 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1.4 \\ -1.6 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} -0.8 & 0.7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + (1.4)u(t) \end{aligned}$$

DT system:

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} -0.4 & 1.7 \\ -0.1 & -0.6 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0.7 \\ -0.8 \end{bmatrix} u[k]$$
$$y[k] = \begin{bmatrix} -0.5 & -0.4 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + (-1.1)u[k]$$

Decide if the CT and DT systems are minimum phase systems or not! If any of the systems is not a minimum phase system, give their/its transfer function as the product of the transfer functions of a minimum phase and an all-pass systems! Give the poles and zeros of the transfer functions of the systems! (1.2+1.2 points)

3.3 The CT and the DT systems are given with the common signal flow network:



(a) Find the poles and zeros and give the pole-zero plot of the transfer functions of the systems! Based on the plots decide the stability of the CT and the DT networks! (1.2+1.2 points)

(b) Give a canonical (the number of the network components are minimal) DT network which, in a cascade connection with the given DT network realizes a FIR system! (0.8 points)