

# Image Frequency

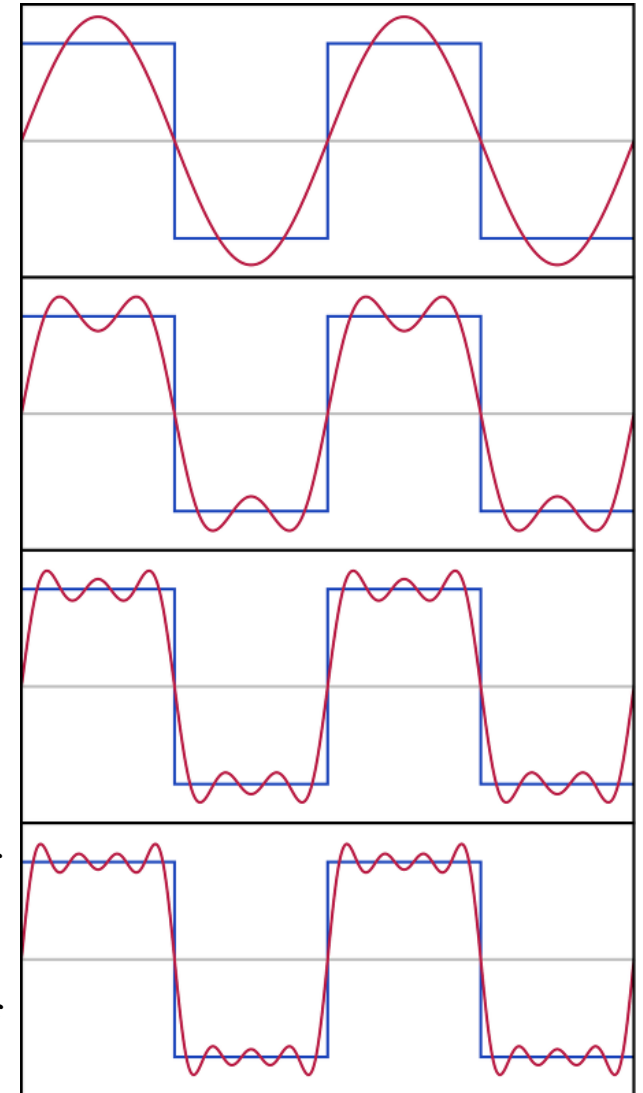
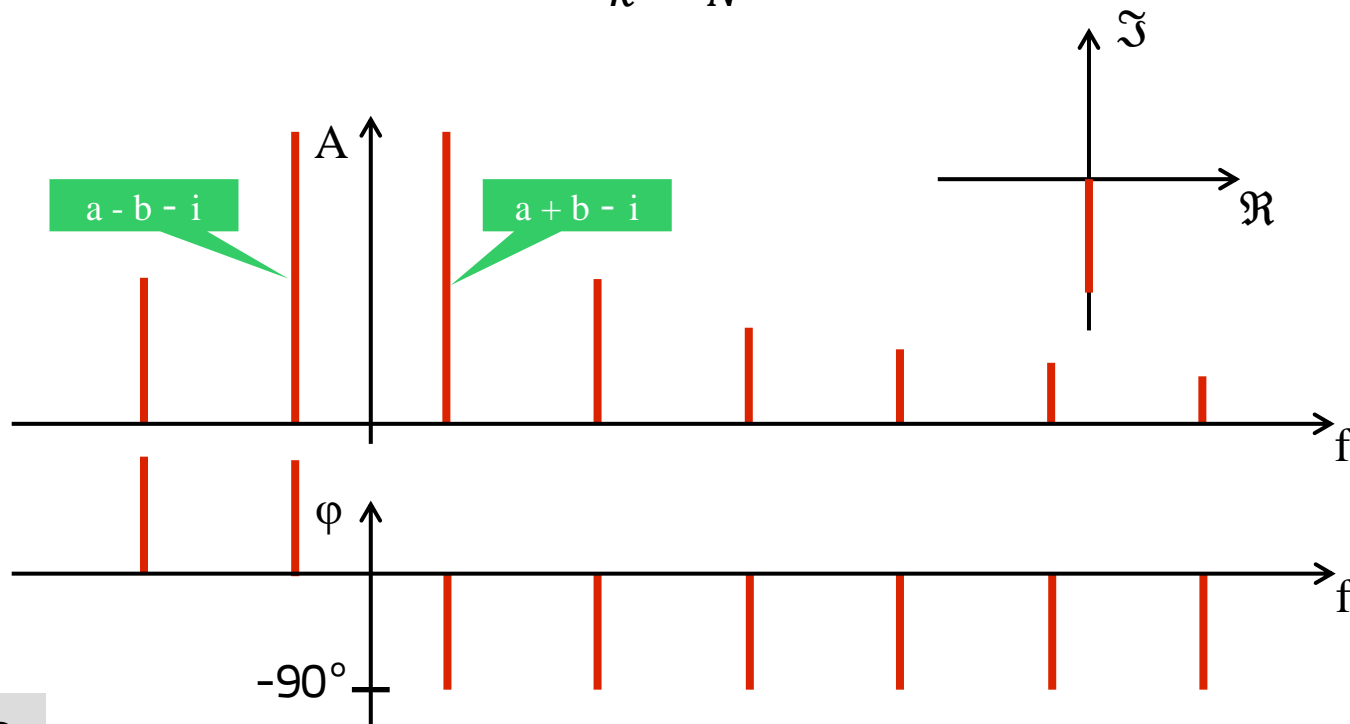
**Image Processing**  
**Dr. Márton Szemenyei**  
**Associate Professor**  
**2024**

# Frequency Domain



$$f(t) = a_0 + \sum_{k=1}^N a_k \cdot \sin(k\omega_0 \cdot t + \phi_k)$$

$$f(t) = \hat{f}_0 + \sum_{k=-N}^N \hat{f}_k \cdot e^{i \cdot k\omega_0 \cdot t}$$



# Fourier transform



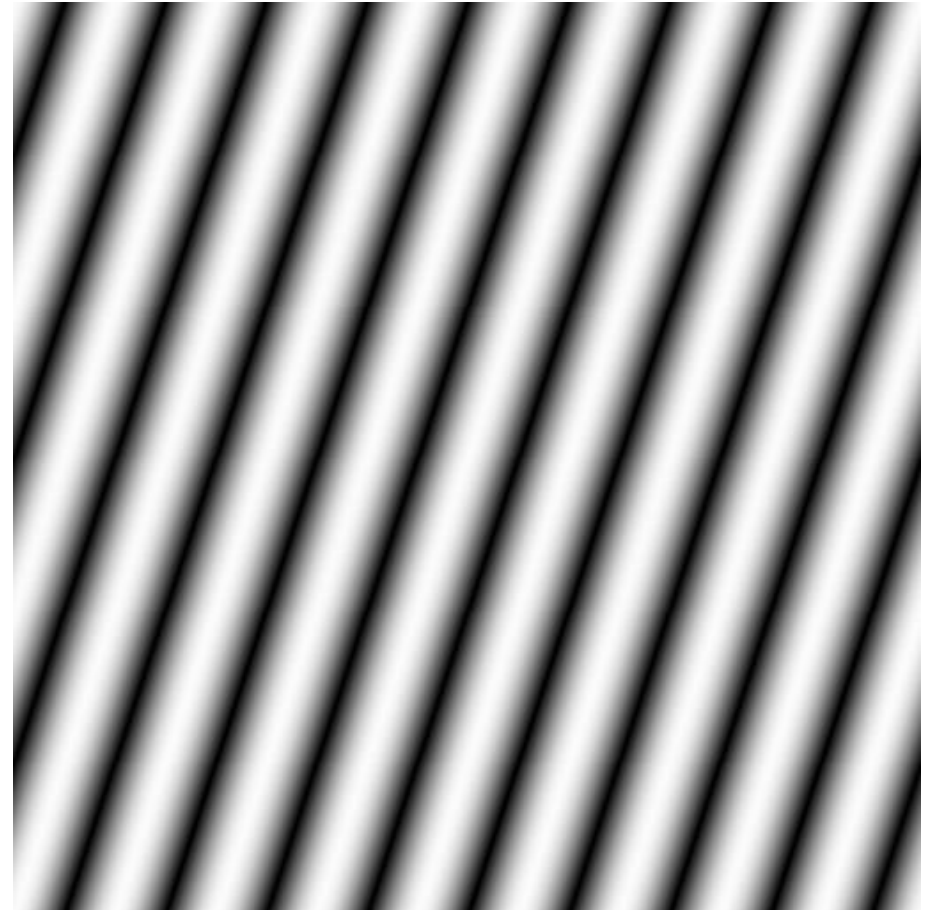
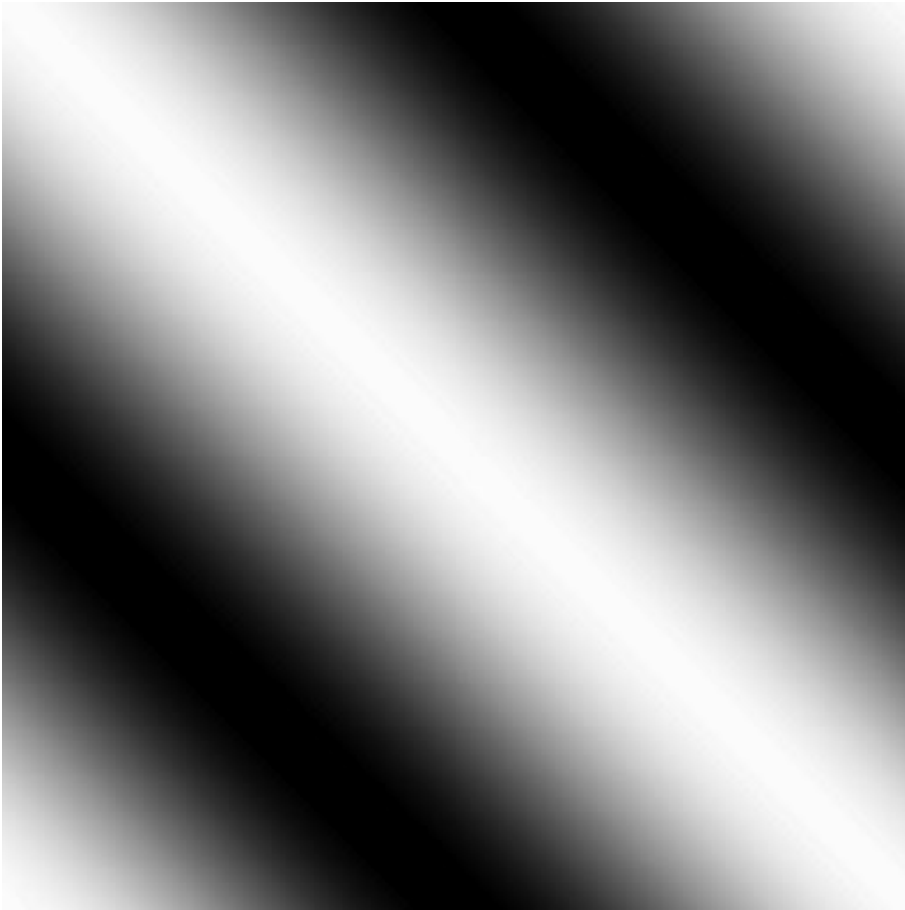
$$F(k, l) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \cdot e^{-i\left(kx\frac{2\pi}{M} + ly\frac{2\pi}{N}\right)}$$

$$f(x, y) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F(k, l) \cdot e^{i\left(kx\frac{2\pi}{M} + ly\frac{2\pi}{N}\right)}$$

Feature	Spectrum
Periodical	Discrete
Discrete	Periodical
Real	Symmetric
Symmetric	Real

# Image frequency

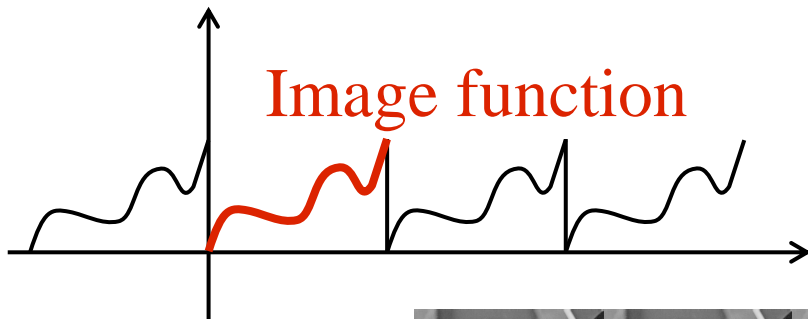
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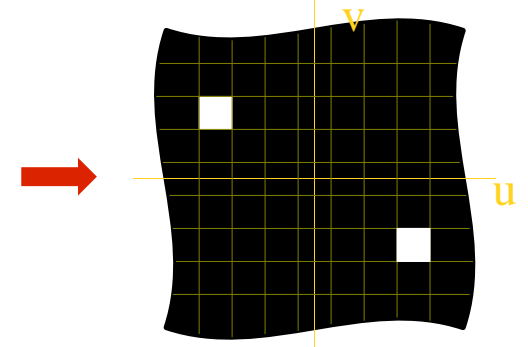
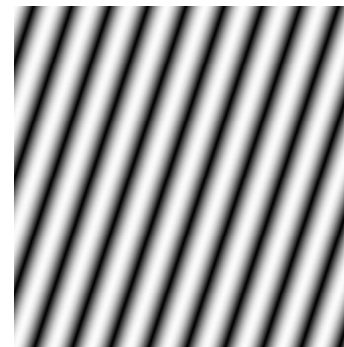
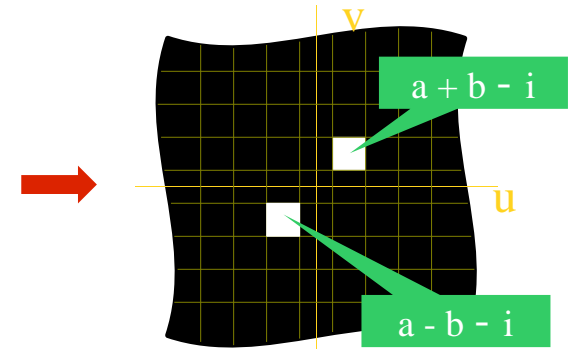
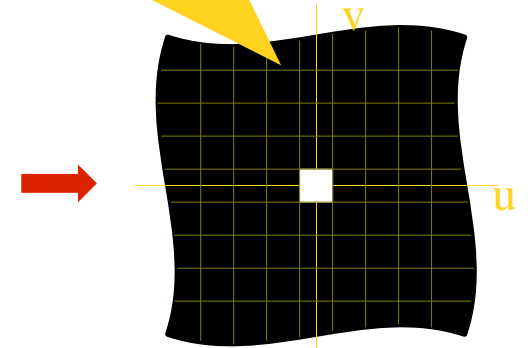
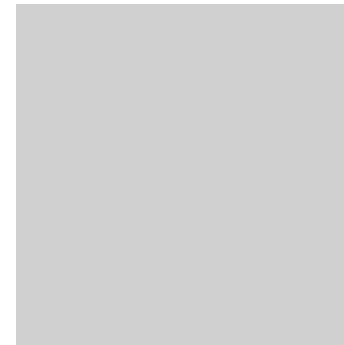
# Fourier Domain



$$f(x, y) = \sum_{m=-M}^M \sum_{n=-N}^N \hat{f}_{m,n} e^{imx} e^{iny}$$



Complex frequency image part

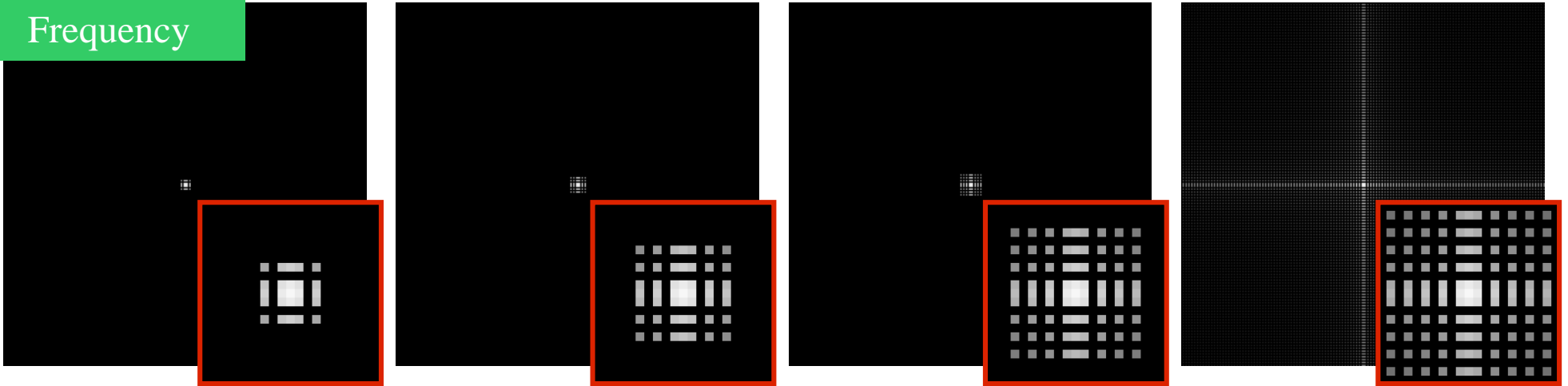


All elements are complex!

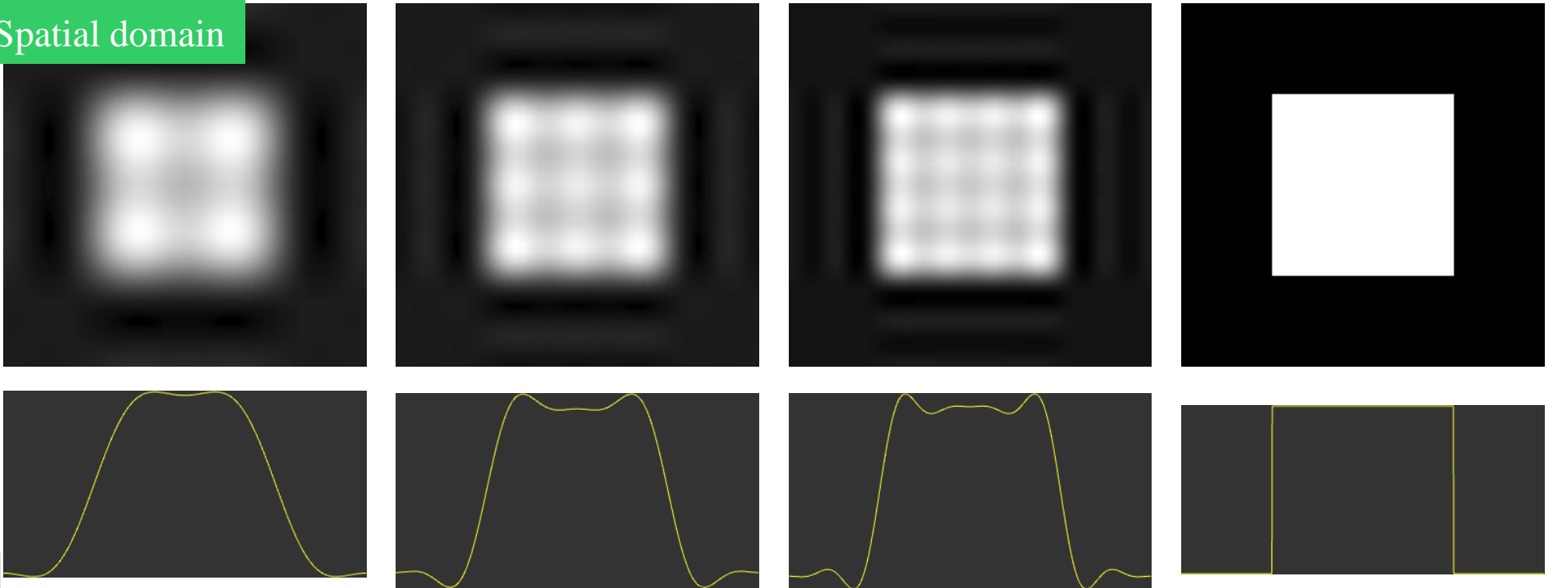
# "Square sign"



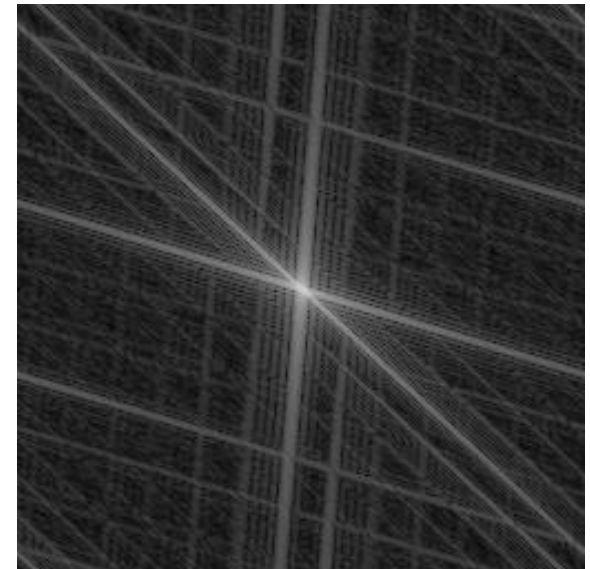
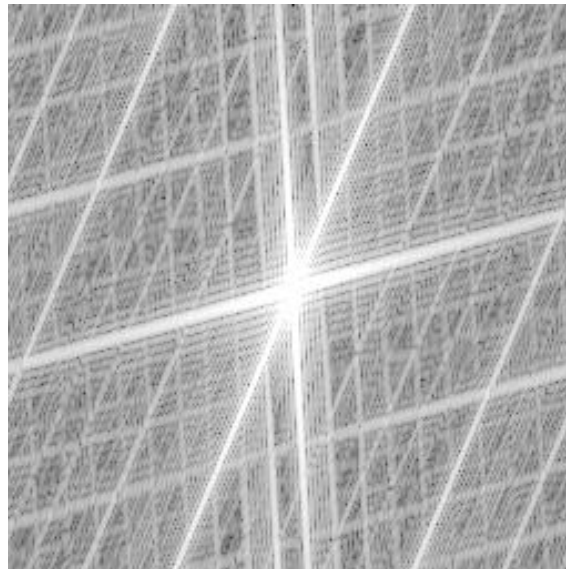
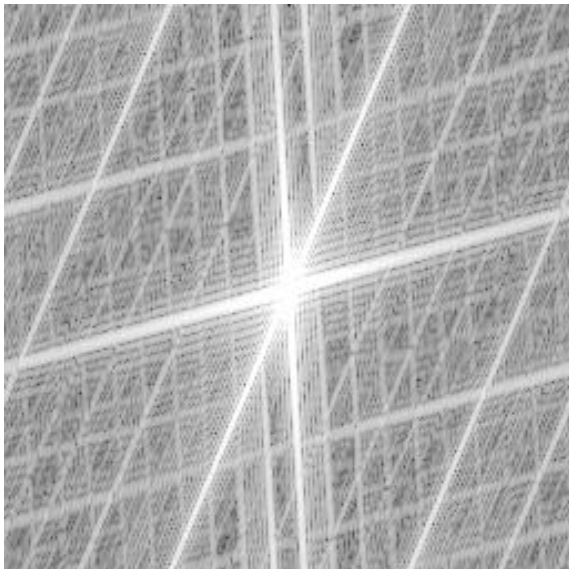
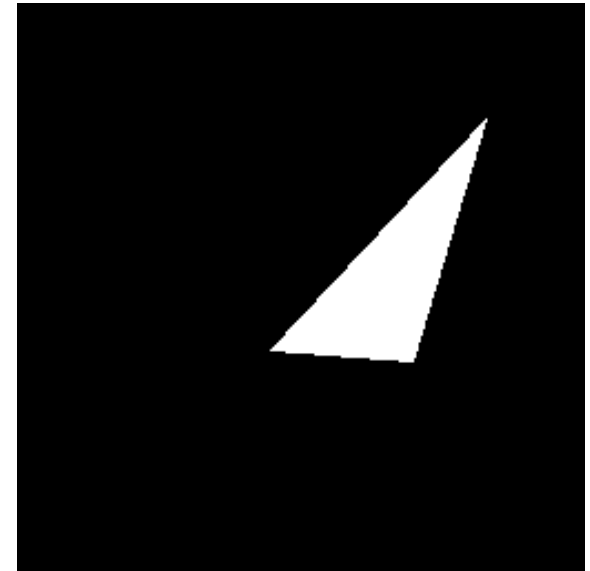
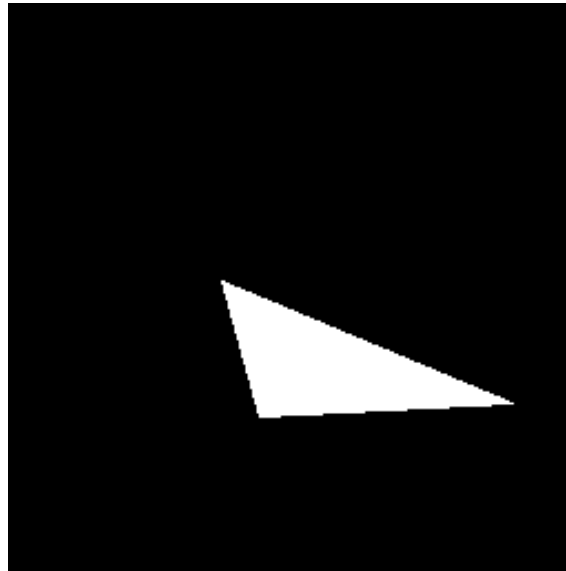
Frequency



Spatial domain

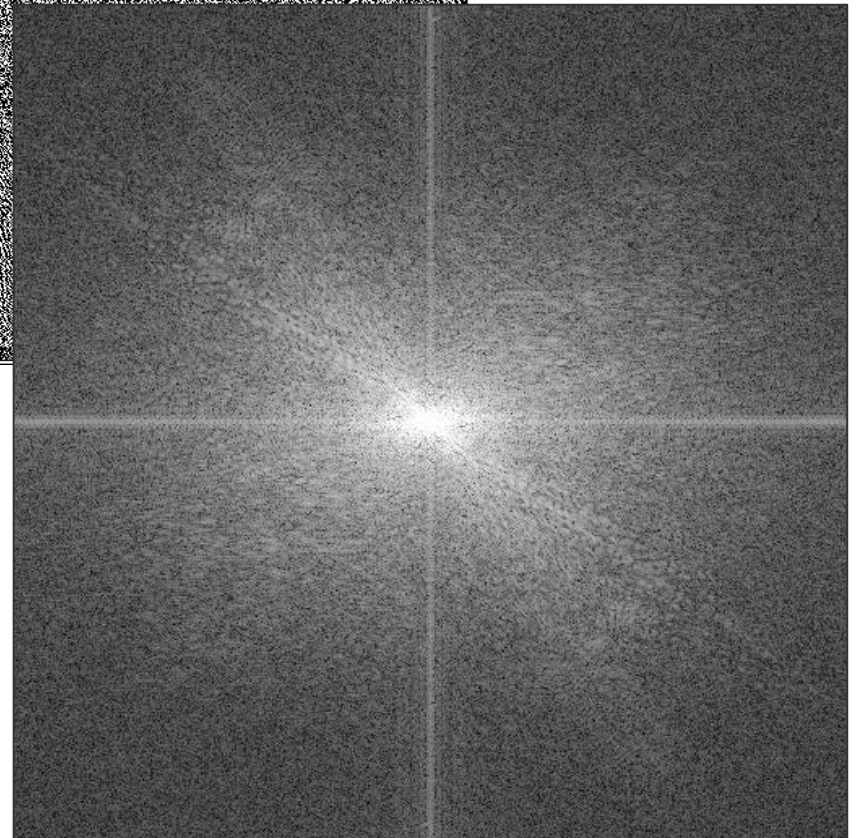
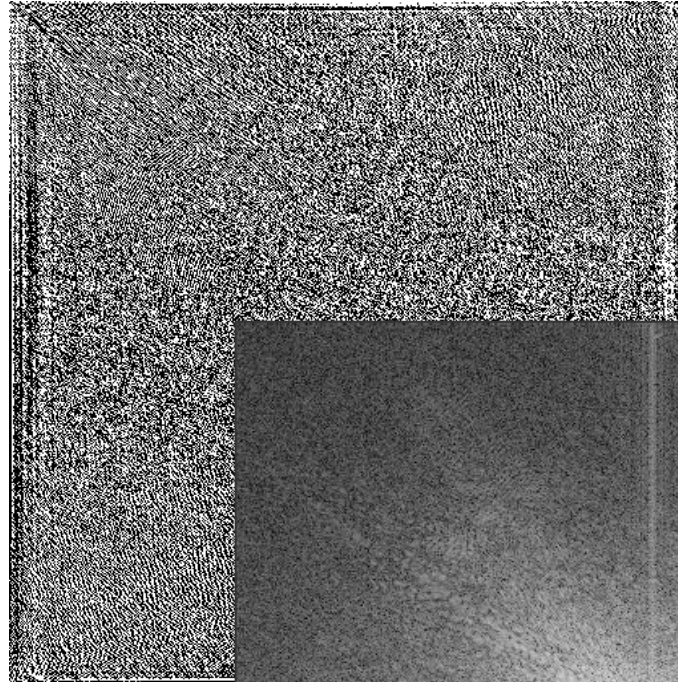


# Position, orientation





# Fourier transform



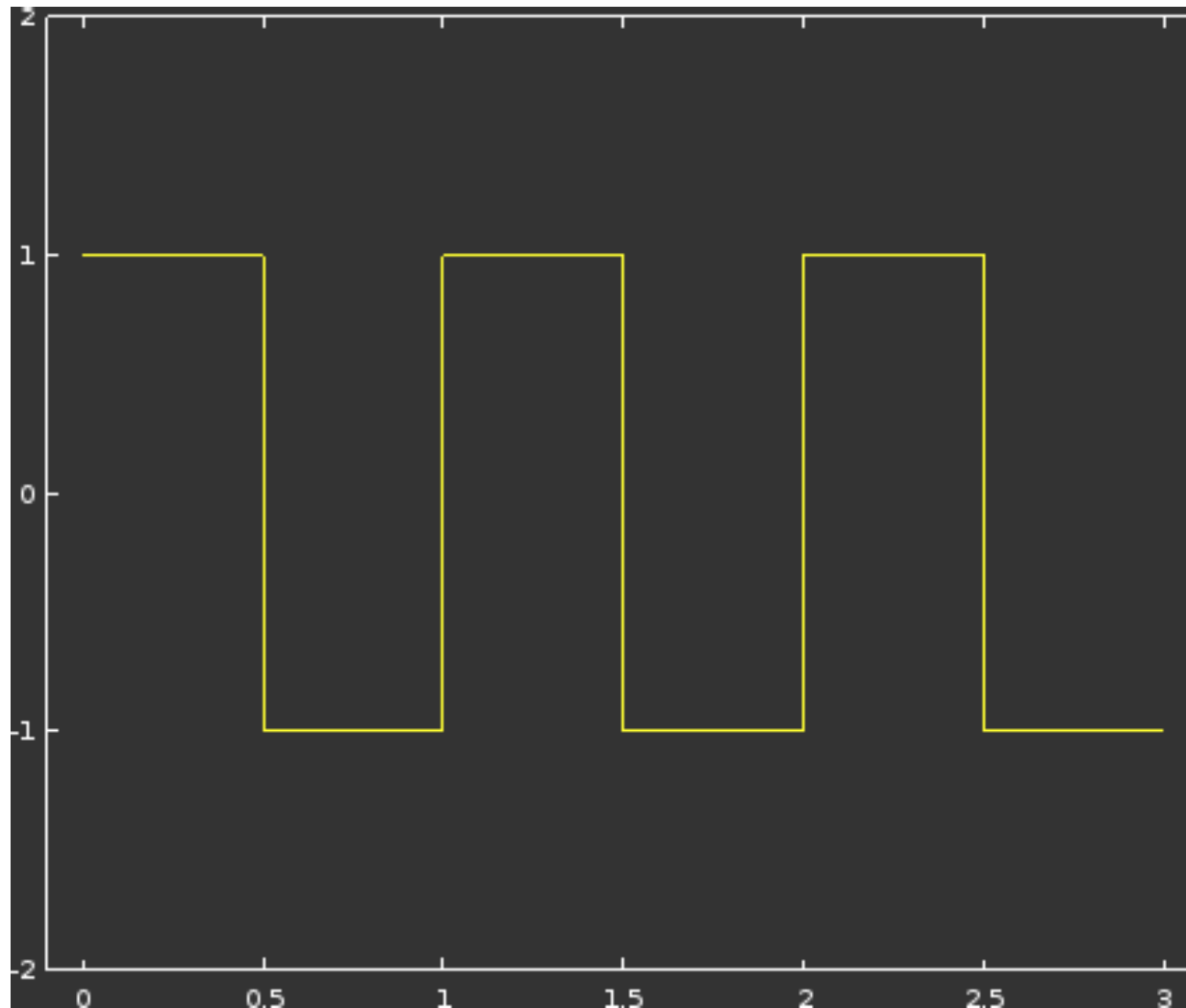
$$0.1 \cdot \log \left( 1 + \sqrt{\Re^2(I_{uv}) + \Im^2(I_{uv})} \right)$$



# Phase distortion



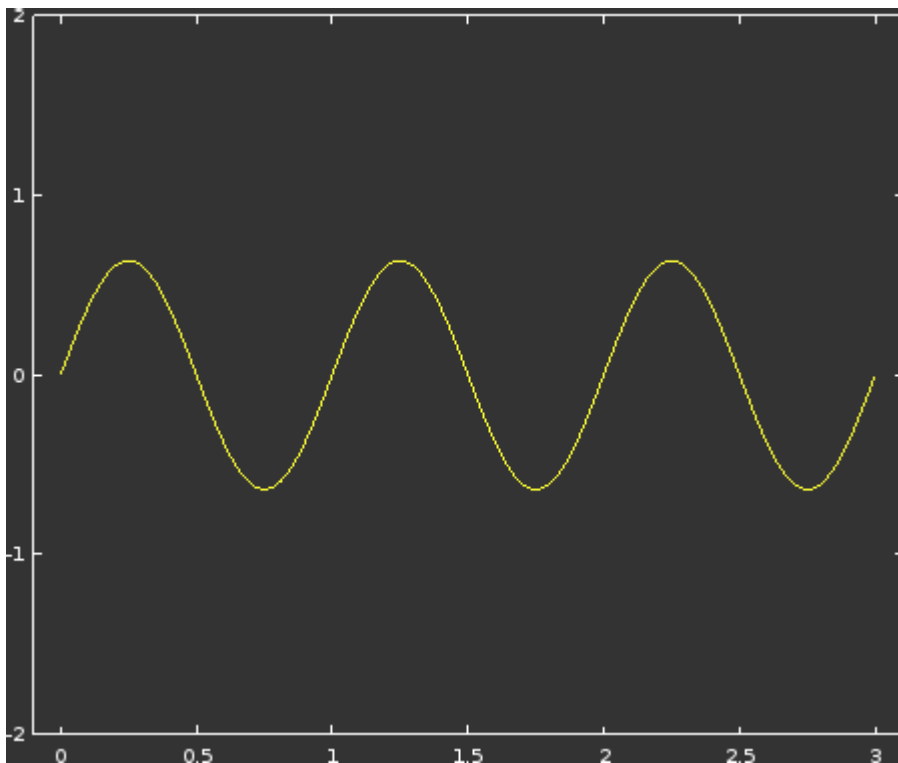
Original signal



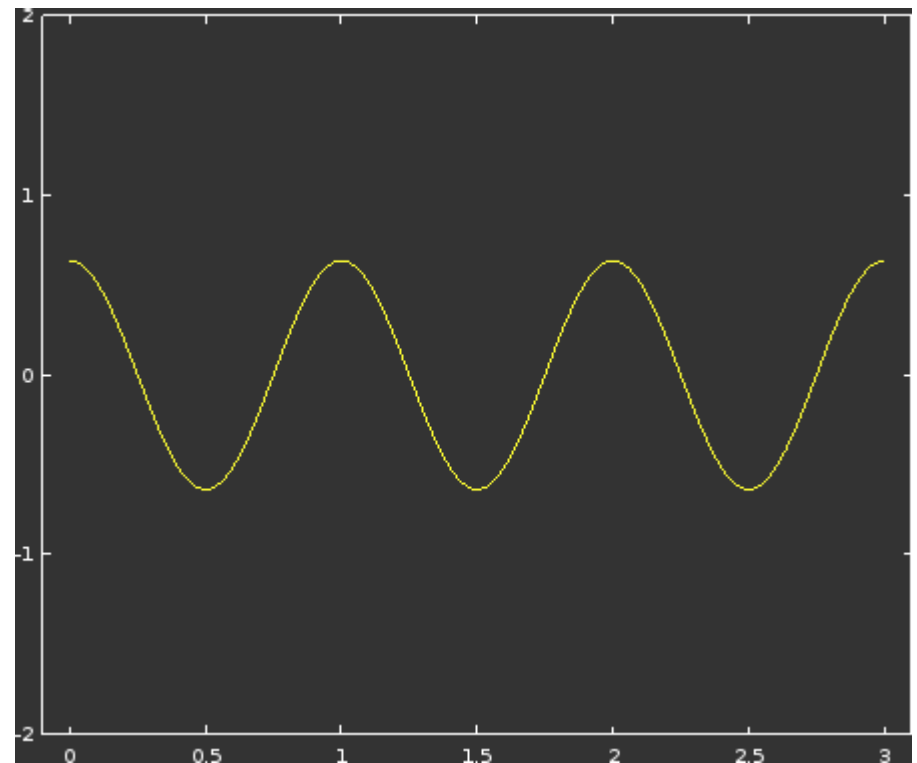
# Phase distortion



1 harmonic



-90°



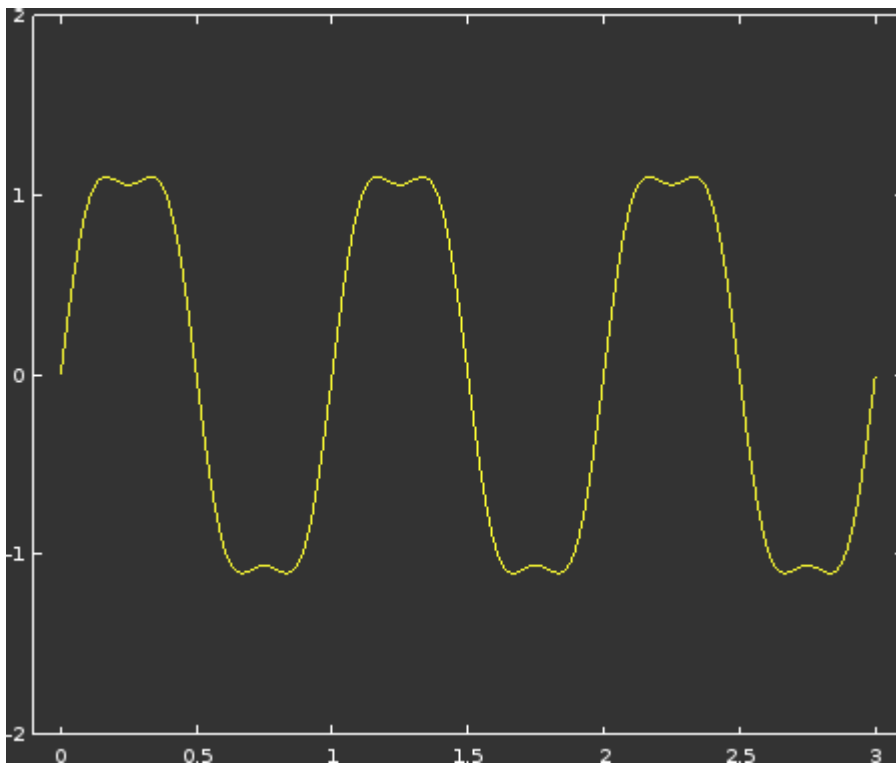
0°

phase

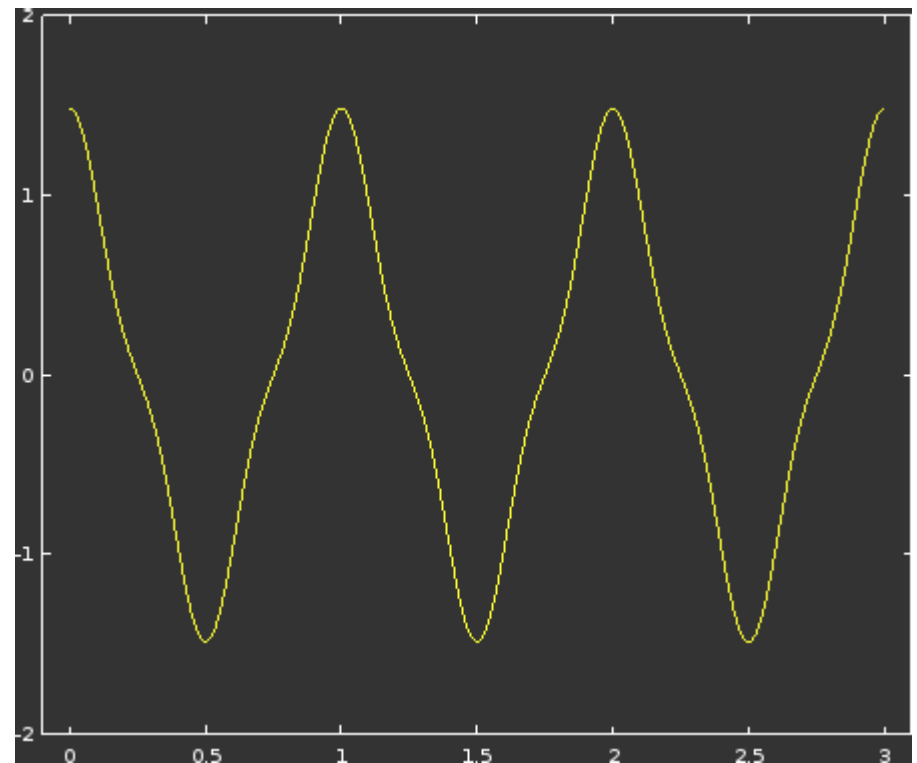
# Phase distortion



2 harmonics



$-90^\circ$



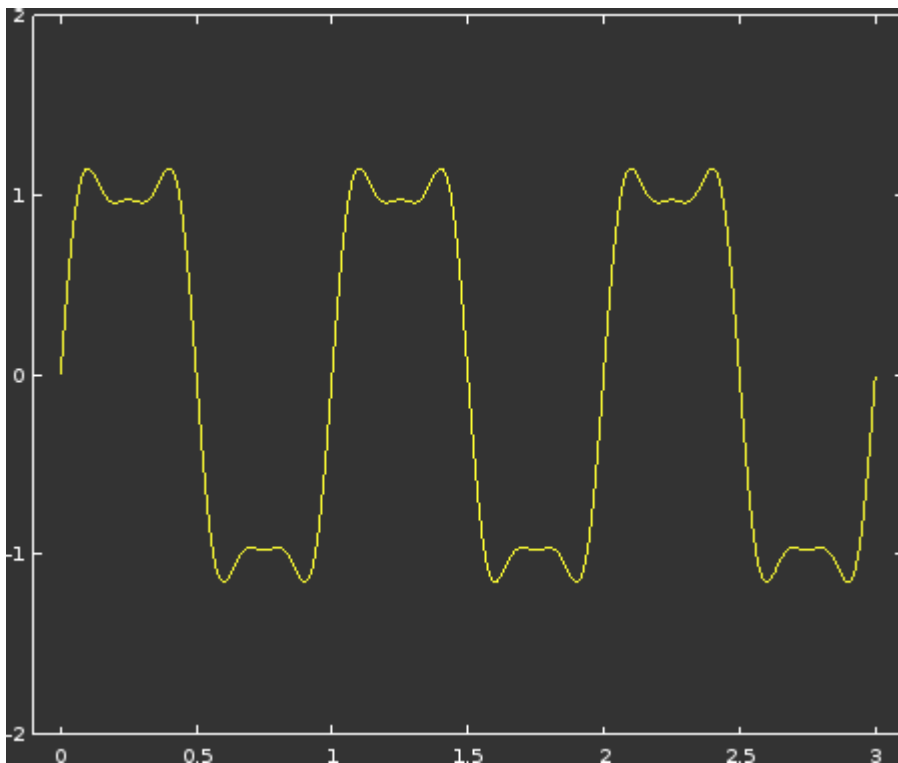
phase

$0^\circ$

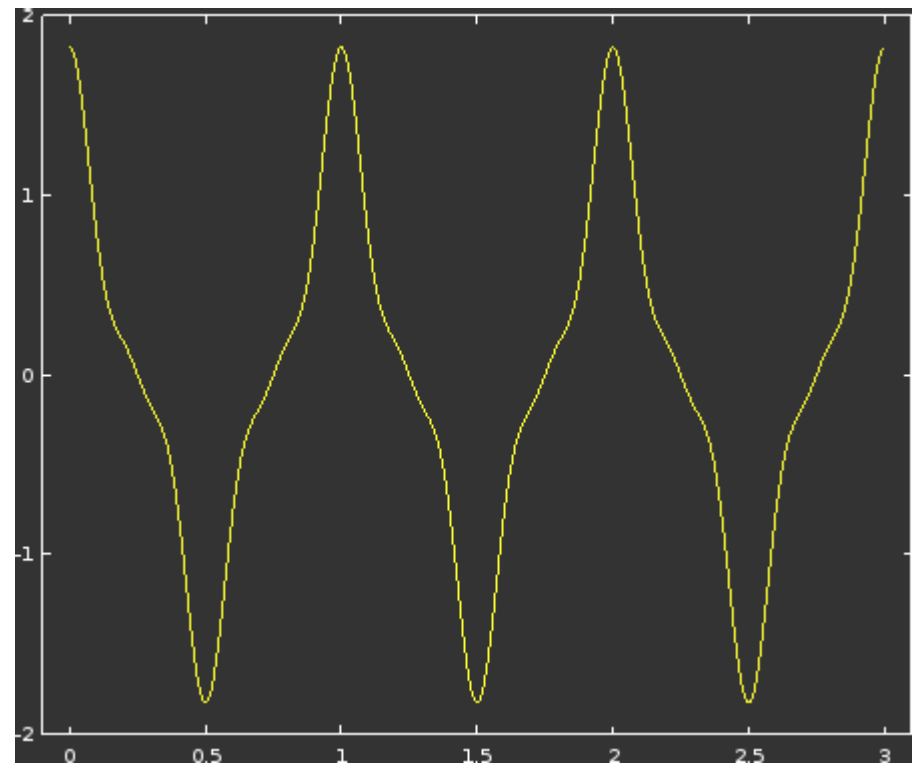
# Phase distortion



3 harmonics



$-90^\circ$



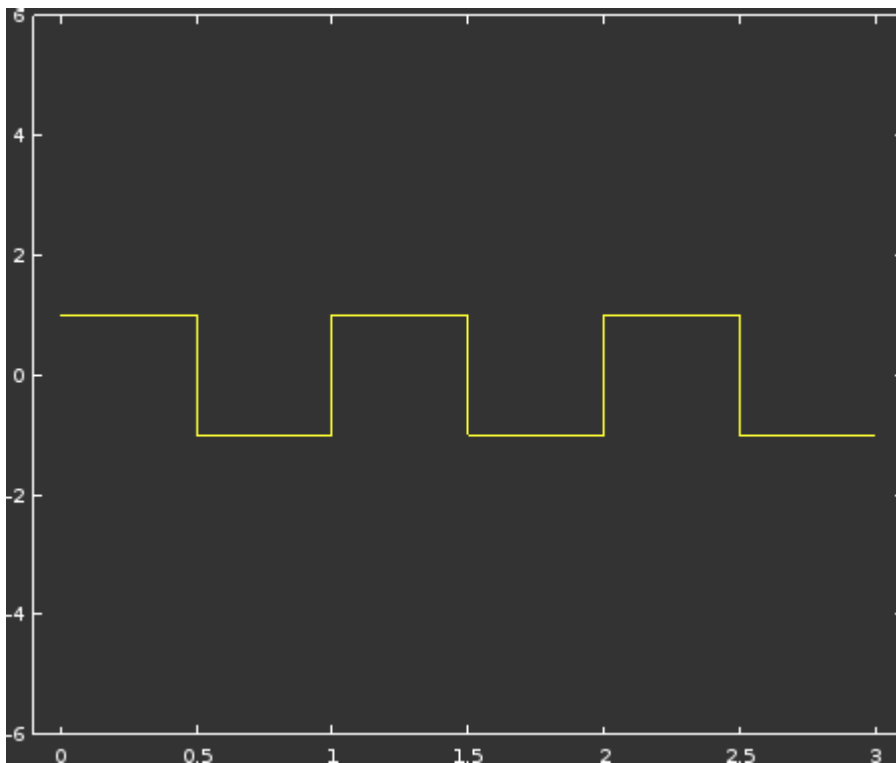
phase

$0^\circ$

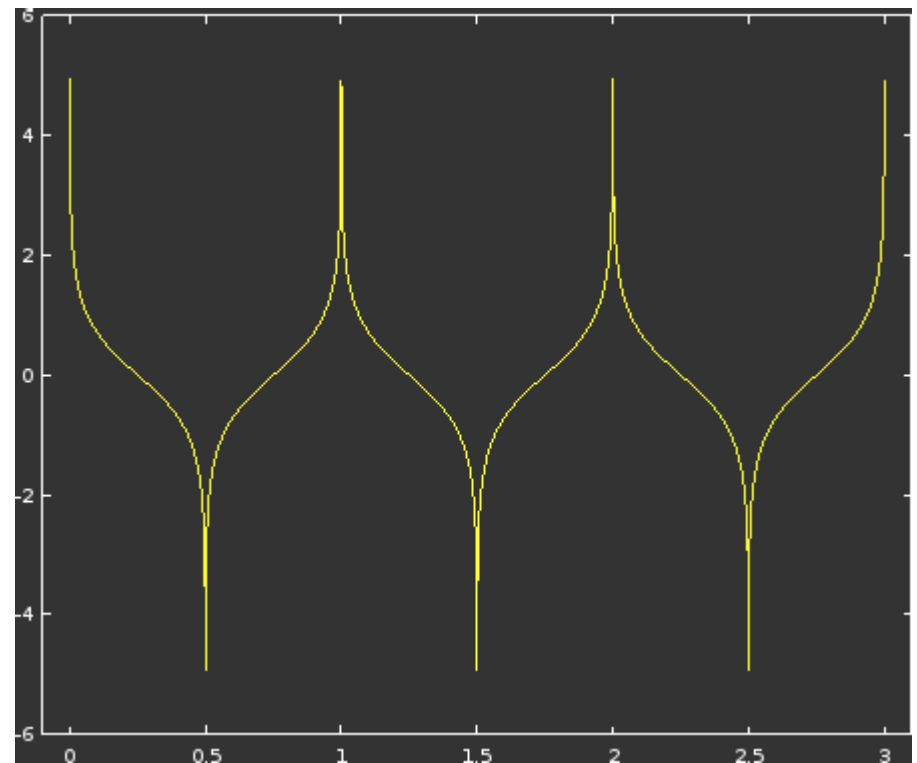
# Phase distortion



All harmonics



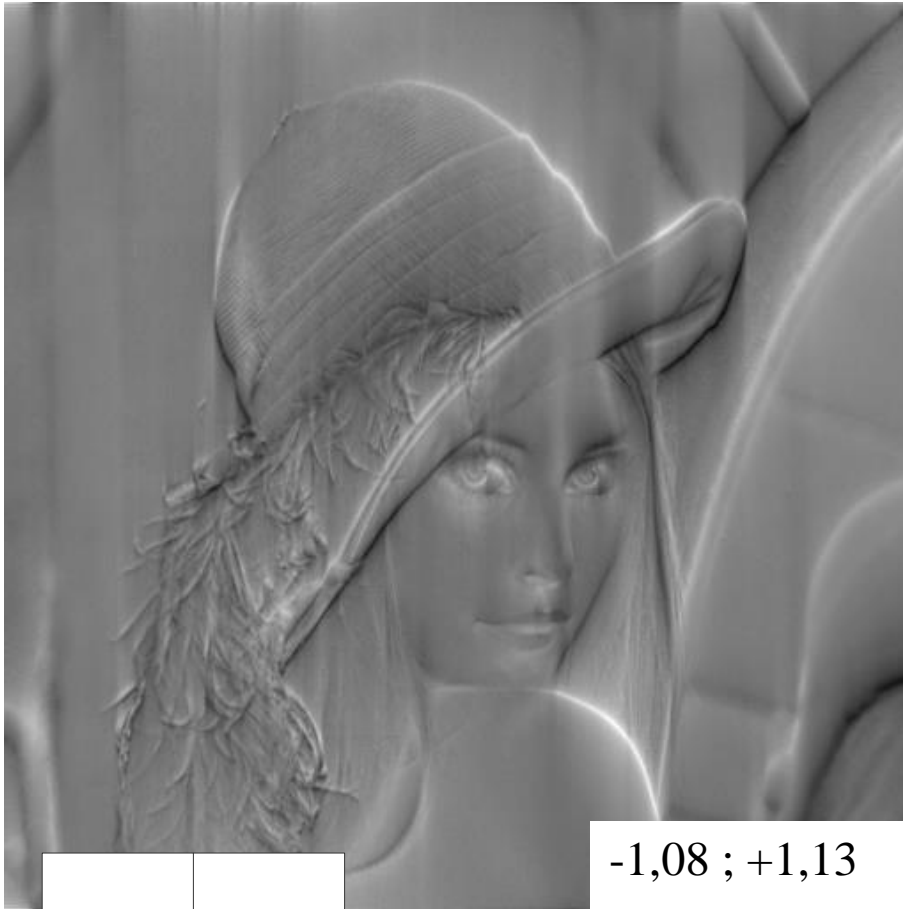
$-90^\circ$



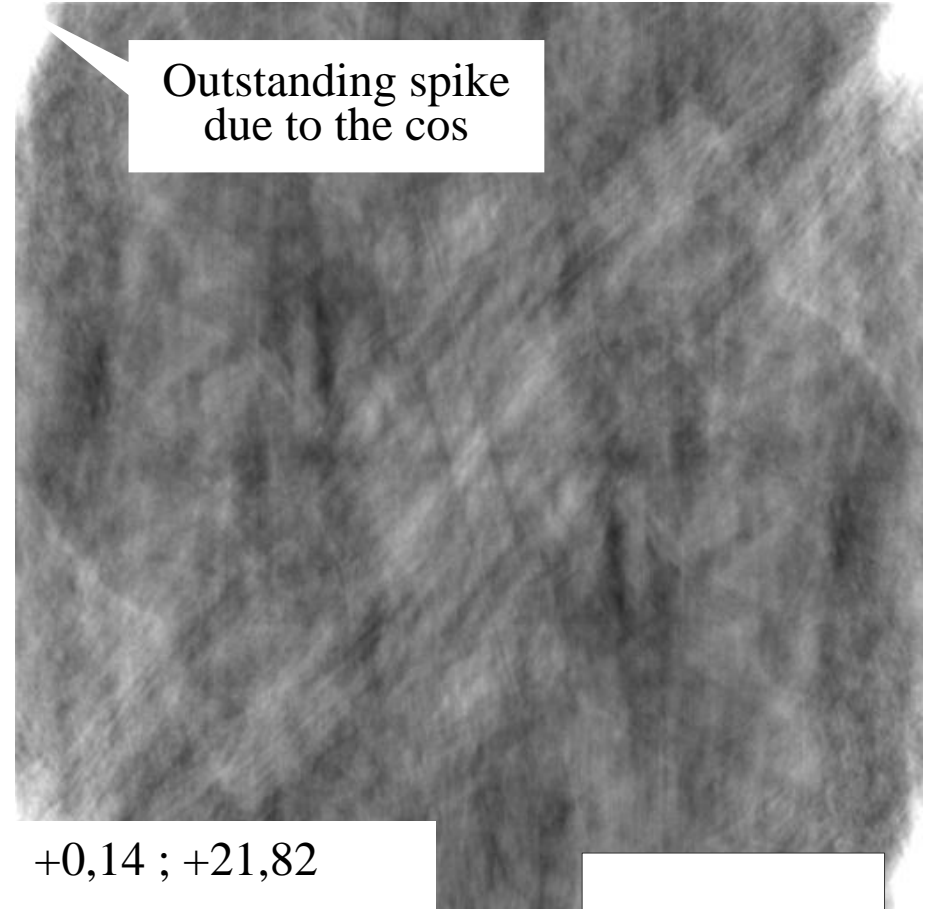
phase

$0^\circ$

# Phase distortion



-1,08 ; +1,13



Outstanding spike  
due to the cos

+0,14 ; +21,82

(Range of original image: 0 ; +1)

+90°

-90°

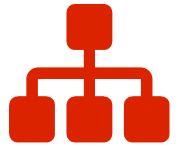
3

0°

3



# FFT



## 1D FFT

Data reordering

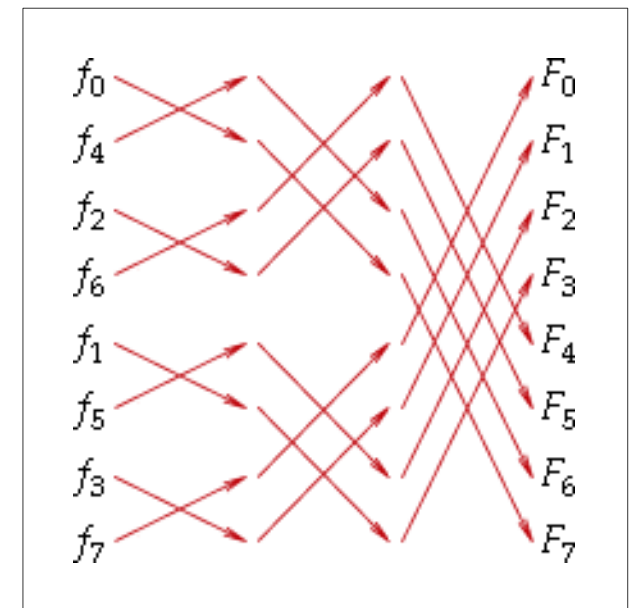
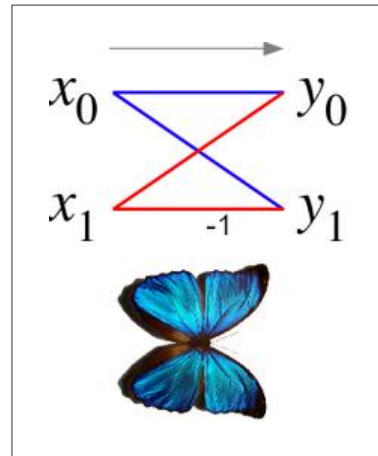
**data[ x ]  $\leftrightarrow$  data[ rev(x) ]** (e.g. data[0100<sub>(2)</sub>]  $\leftrightarrow$  data[0010<sub>(2)</sub>])

Butterfly operations

Log<sub>2</sub> N times

$$y_0 = x_0 + x_1 \omega^k$$

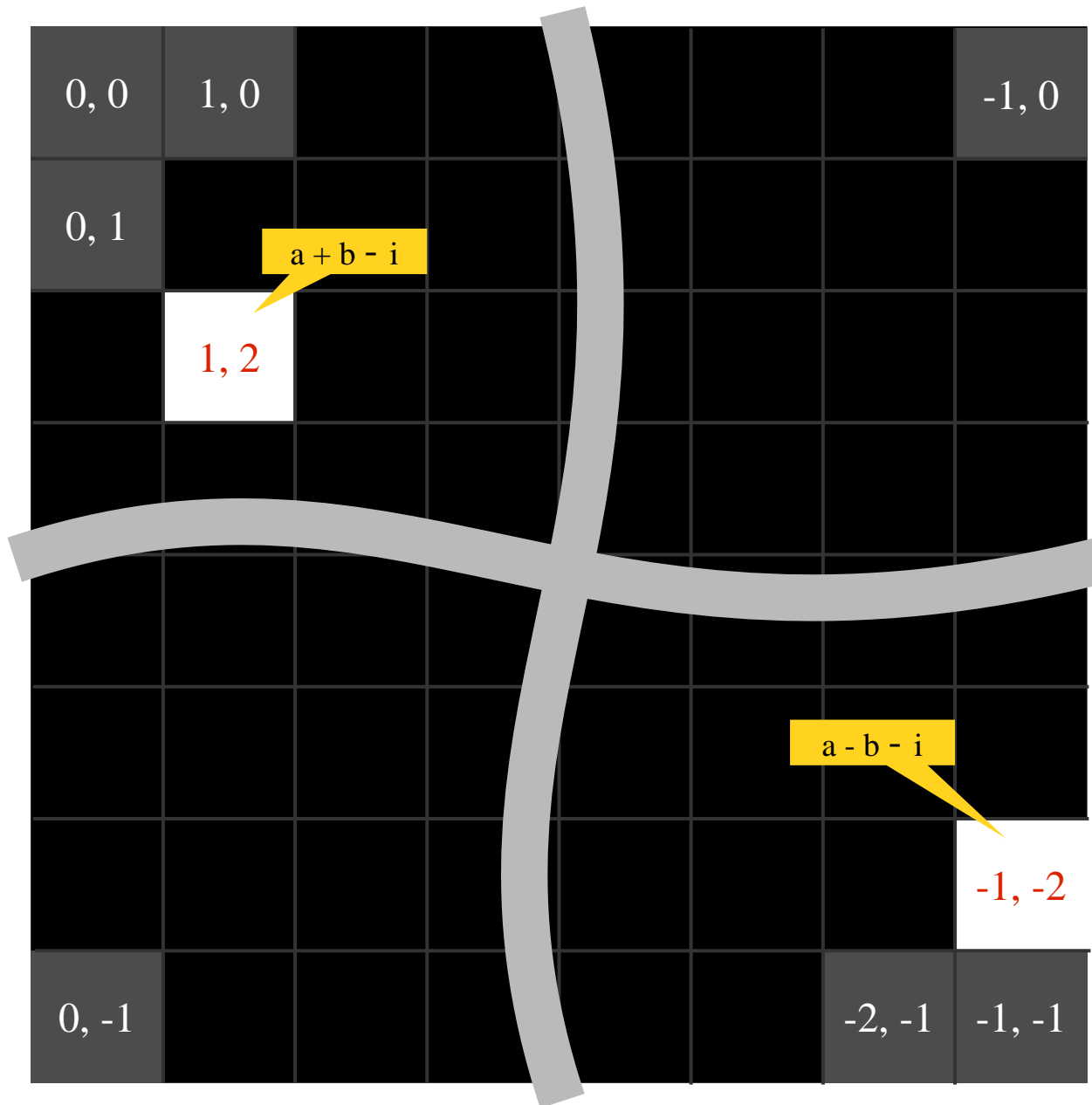
$$y_1 = x_0 - x_1 \omega^k$$



## 2D FFT

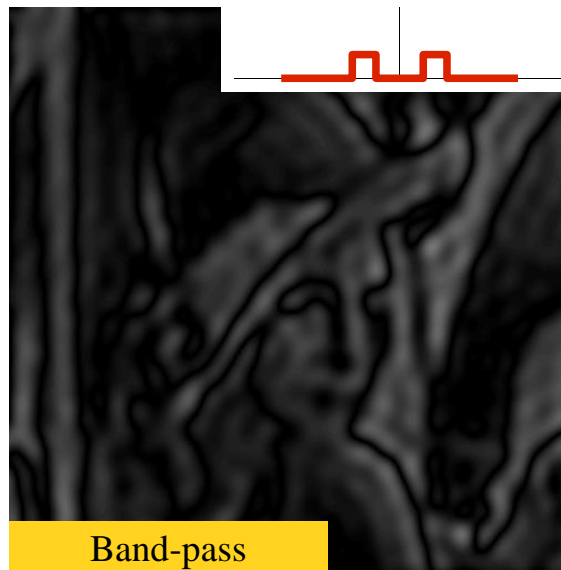
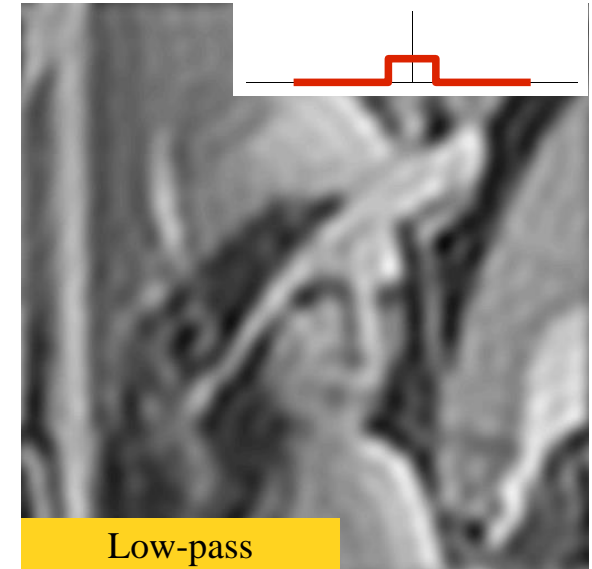
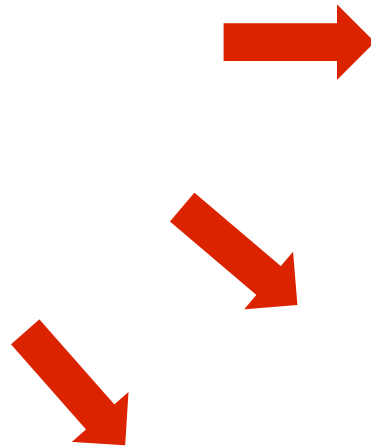
First horizontally, then vertically...

# FFT result

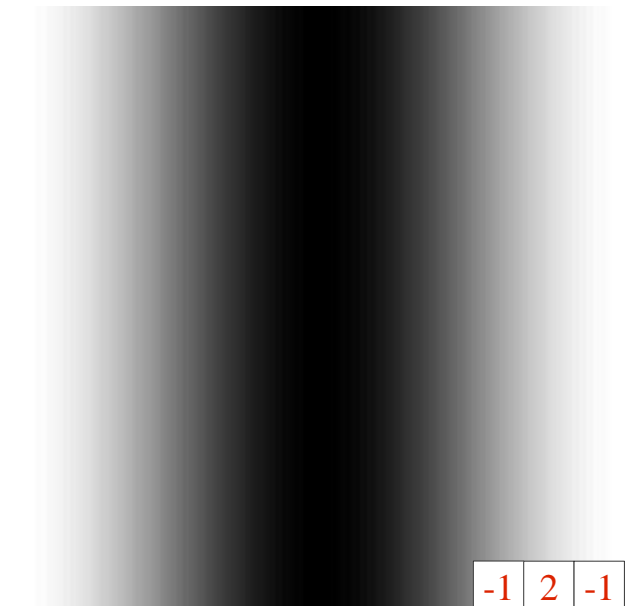
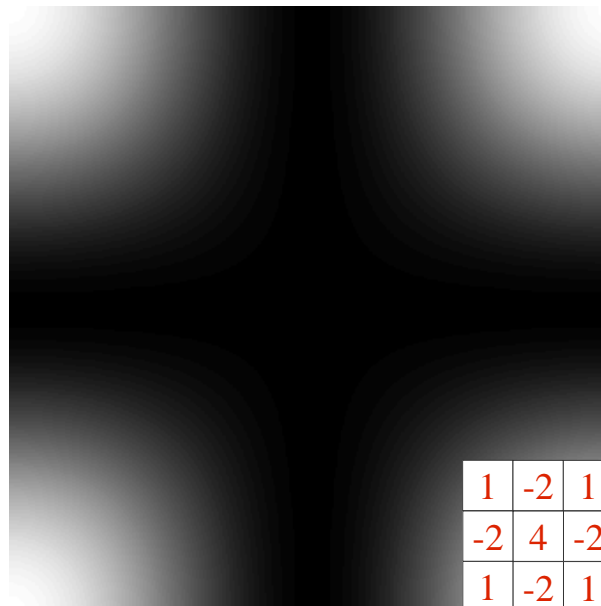
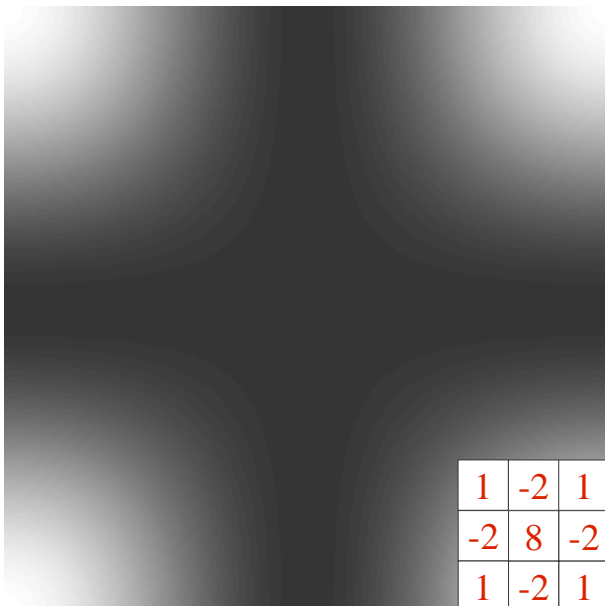
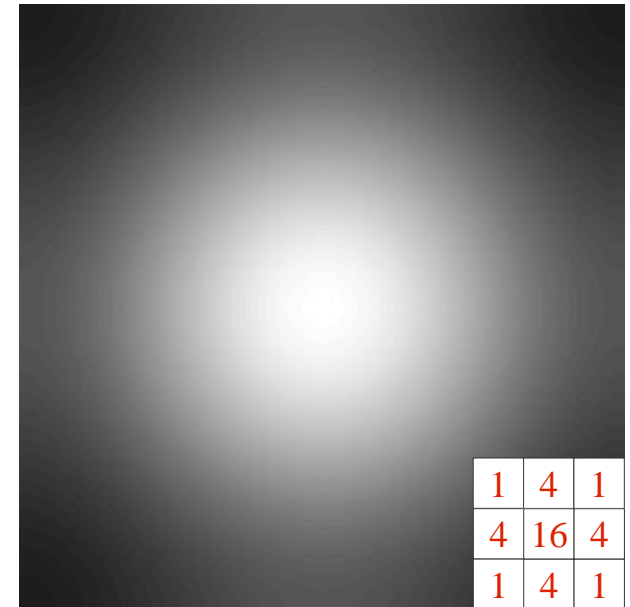
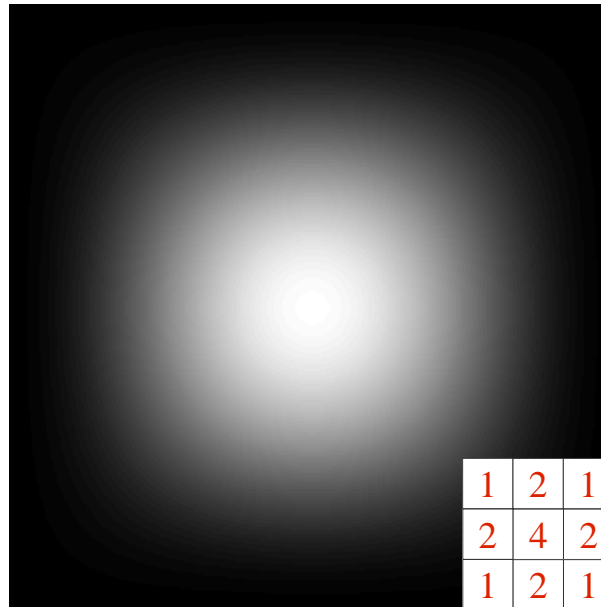
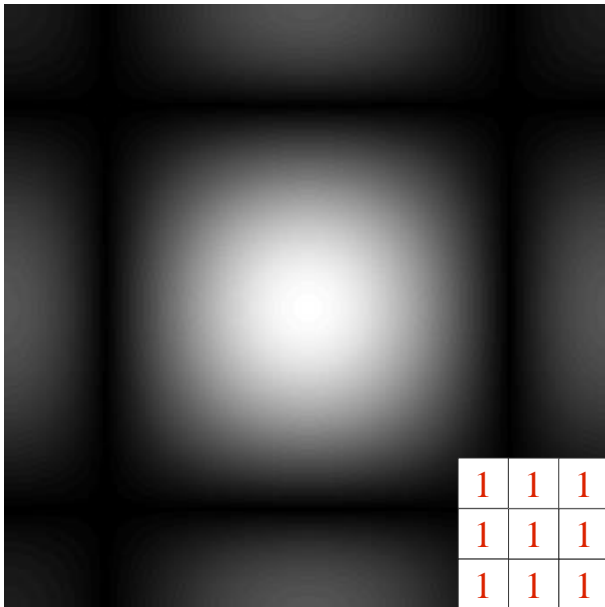


Due to periodicity  
( $N-1, N-1$ )

# Filtering



# Frequency diagram of filters

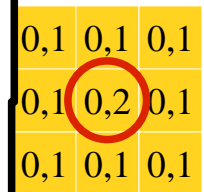
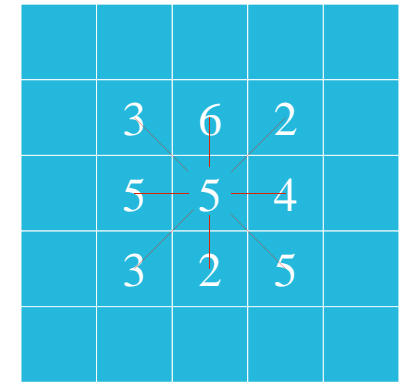
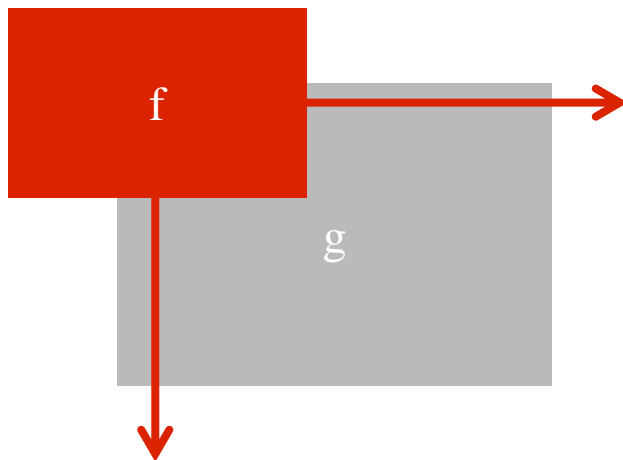


# Convolution

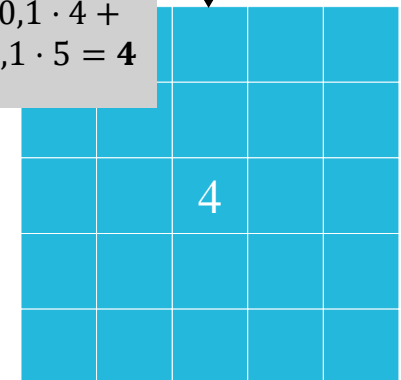


$$(f * g)(x, y) = \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} f(u, v) \cdot g(x - u, y - v)$$

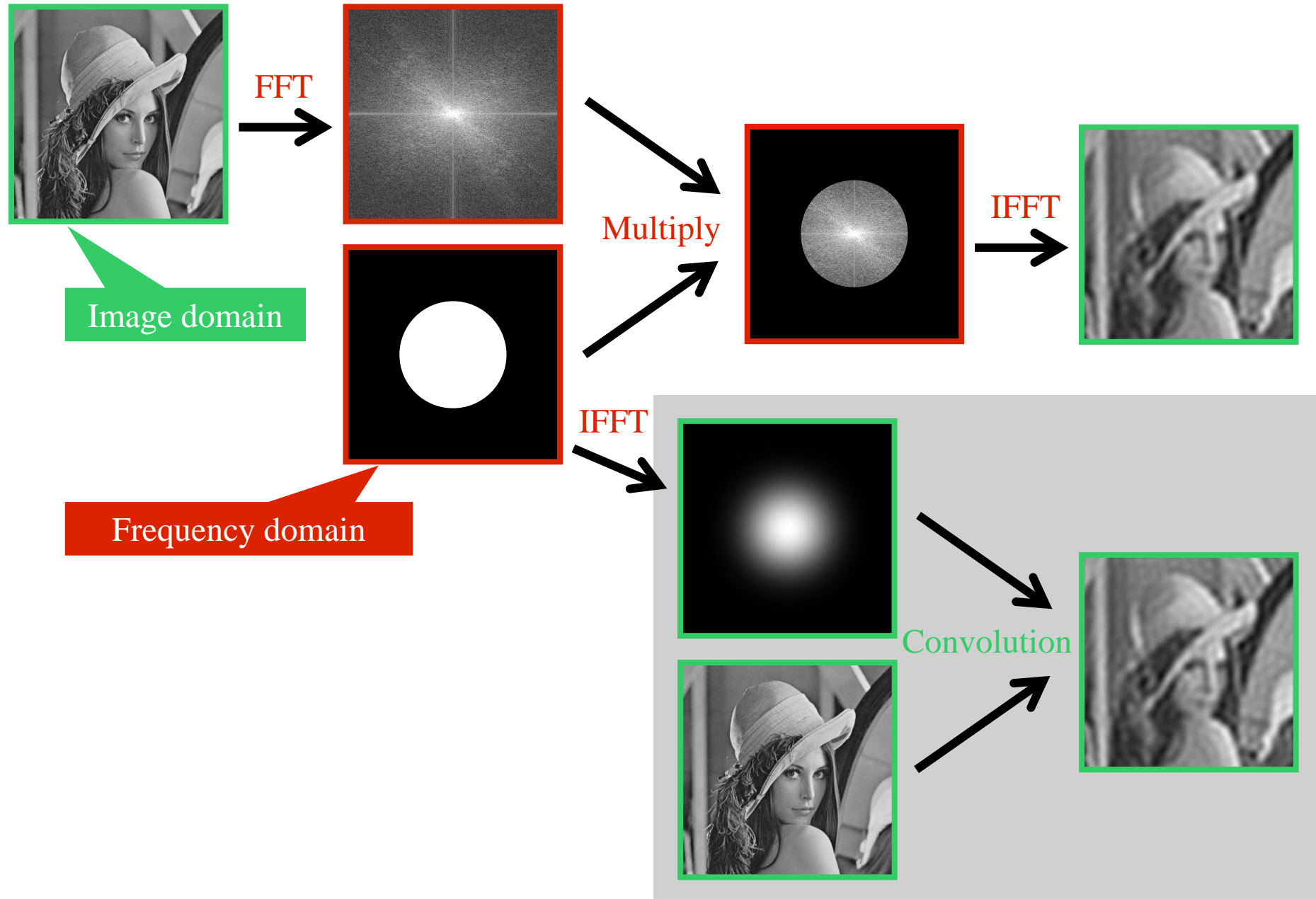
$$(k * I)(x, y) = \sum_{u=-1}^1 \sum_{v=-1}^1 k(u, v) \cdot I(x - u, y - v)$$



$$\begin{aligned} &0,1 \cdot 3 + 0,1 \cdot 6 + 0,1 \cdot 2 + \\ &+ 0,1 \cdot 5 + 0,2 \cdot 5 + 0,1 \cdot 4 + \\ &+ 0,1 \cdot 3 + 0,1 \cdot 2 + 0,1 \cdot 5 = 4 \end{aligned}$$

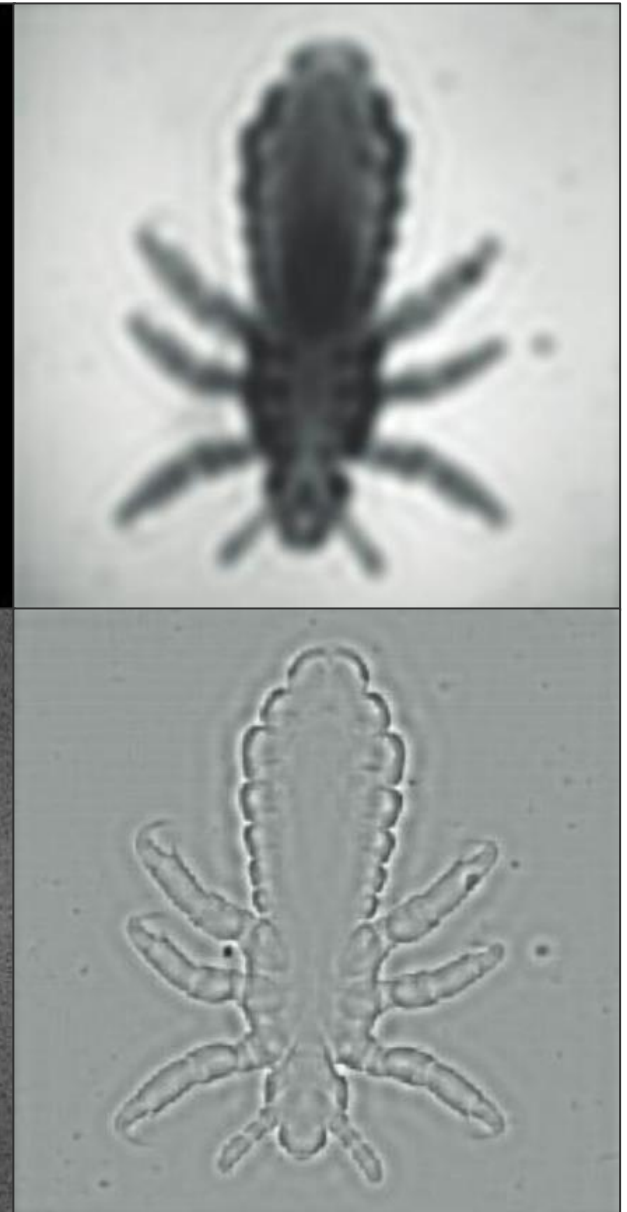
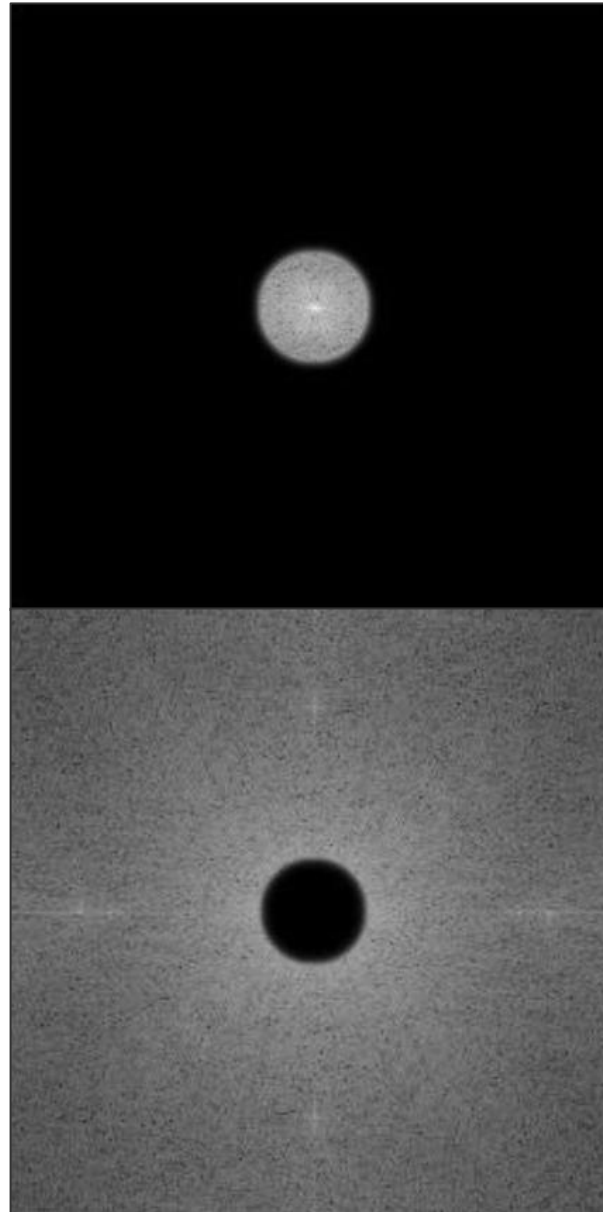
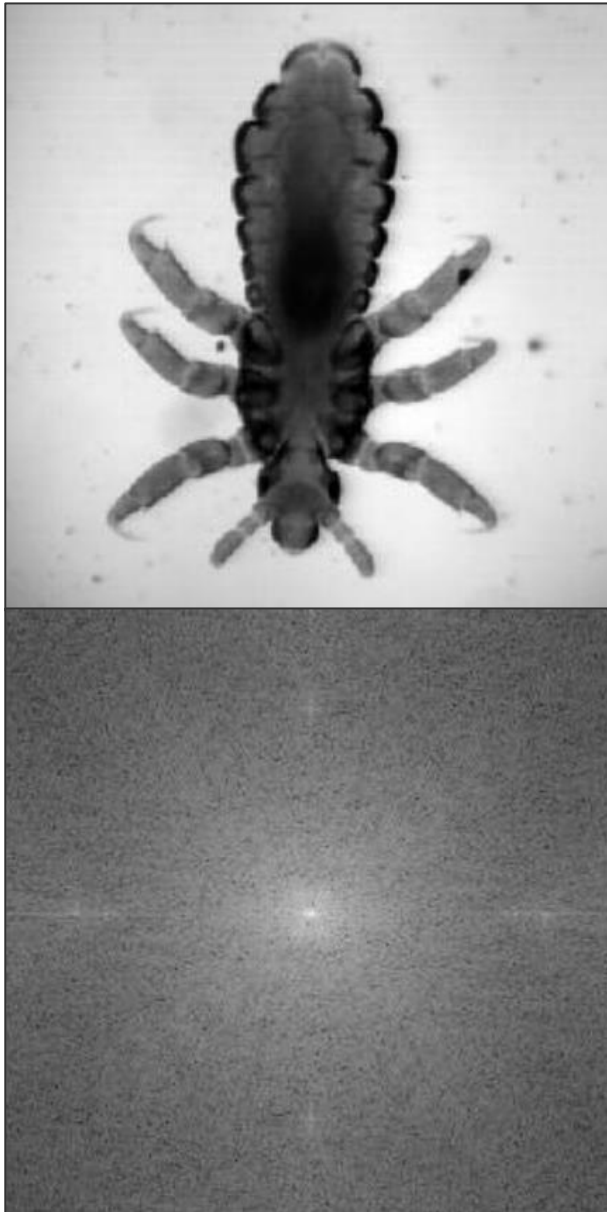


# Convolution

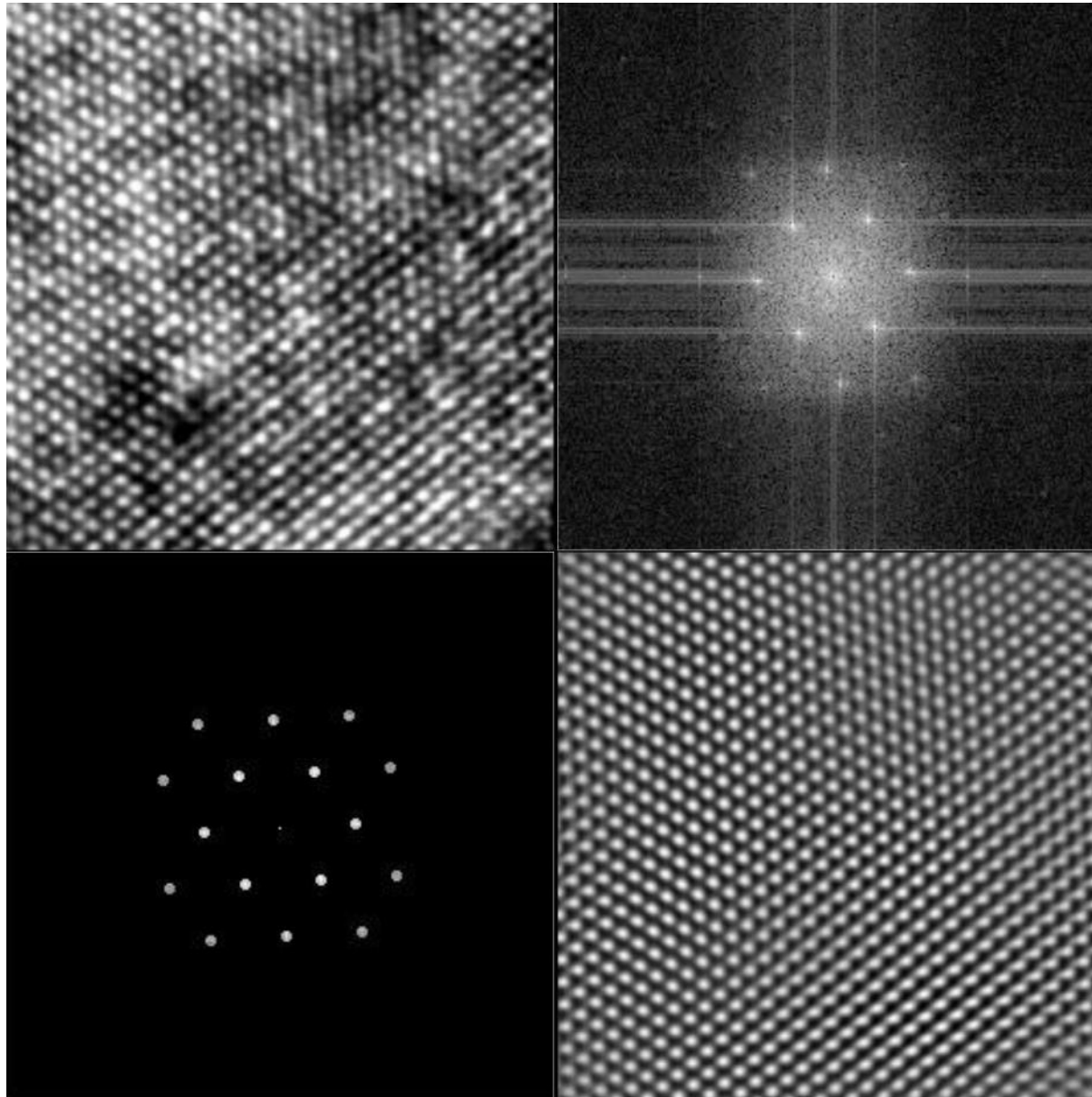




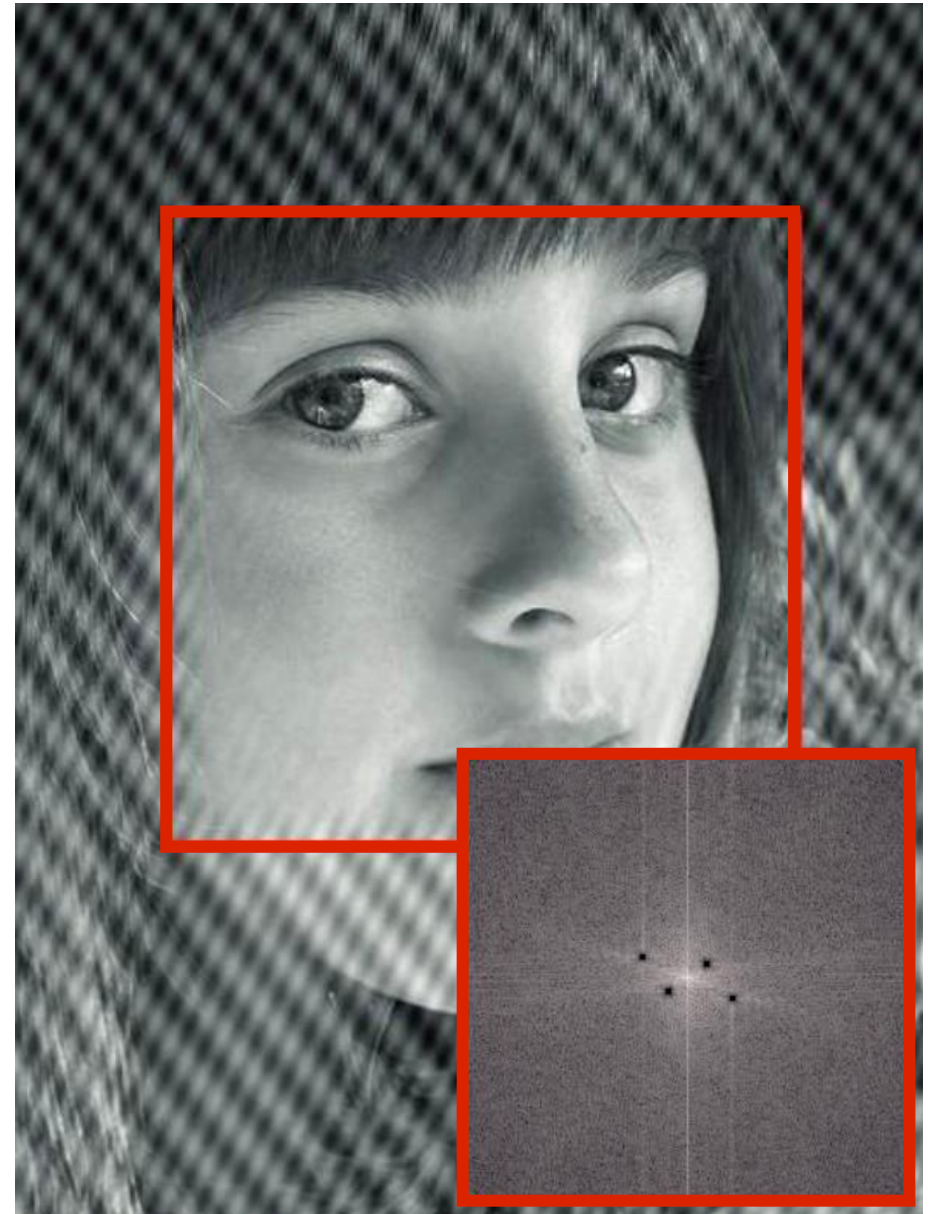
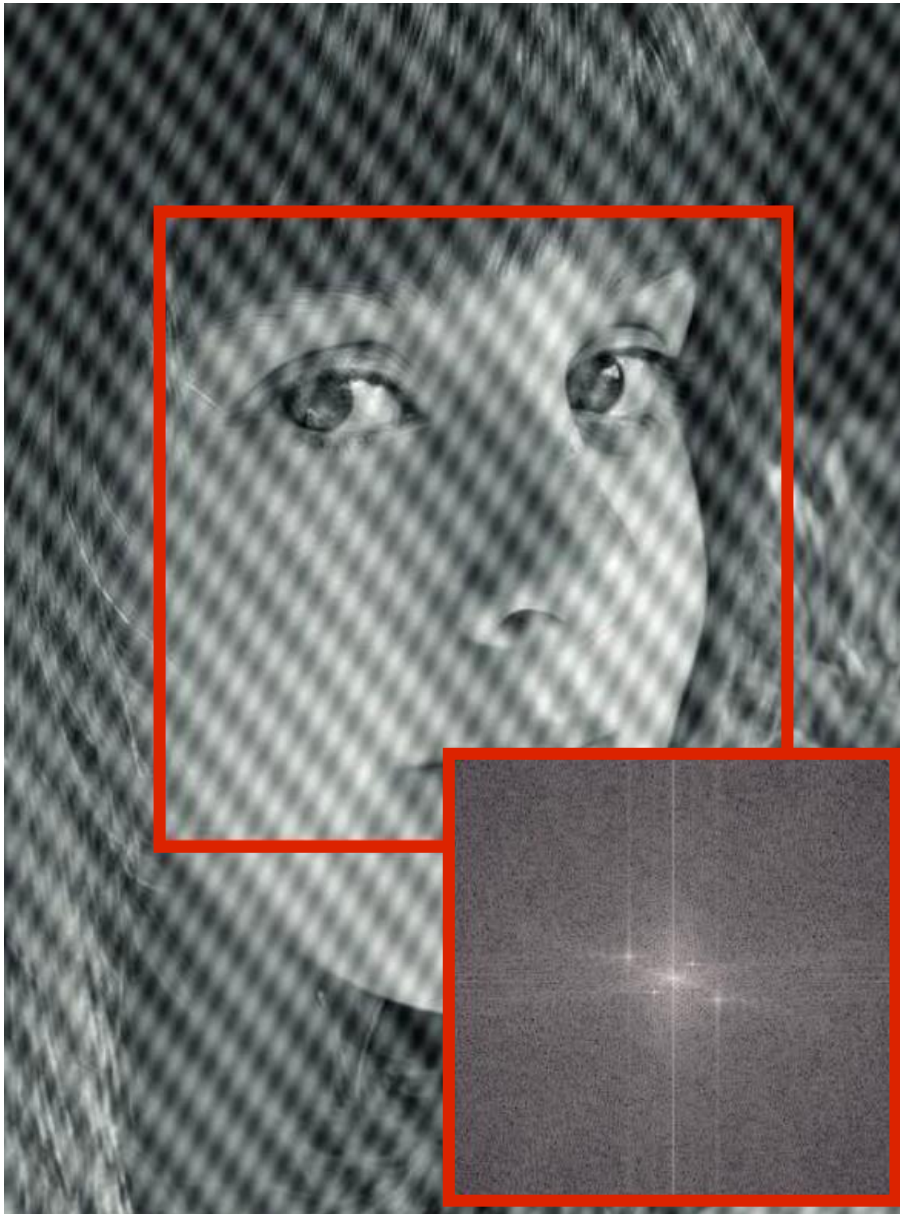
# Filtering



# Filtering

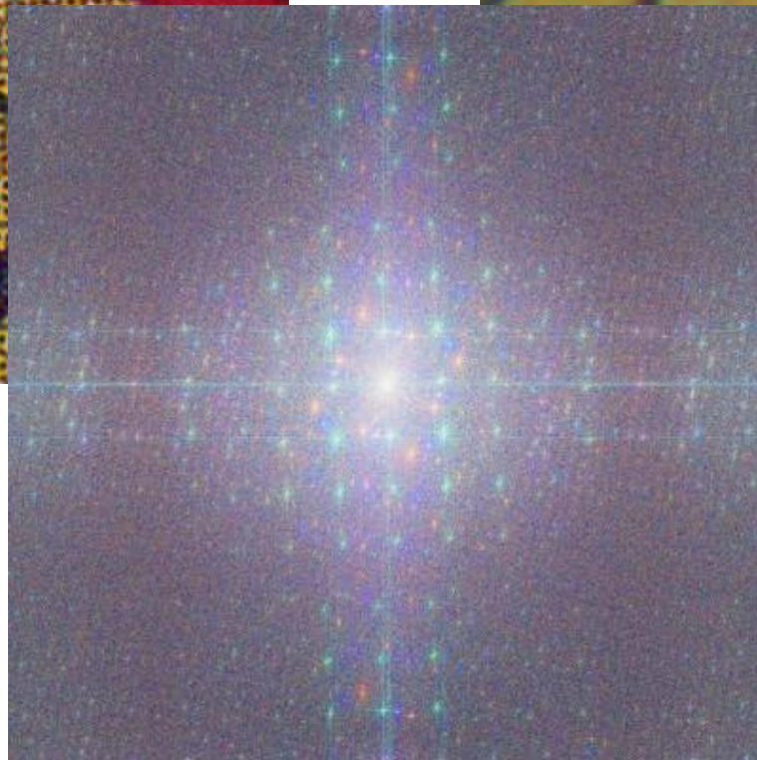


# Periodic noise

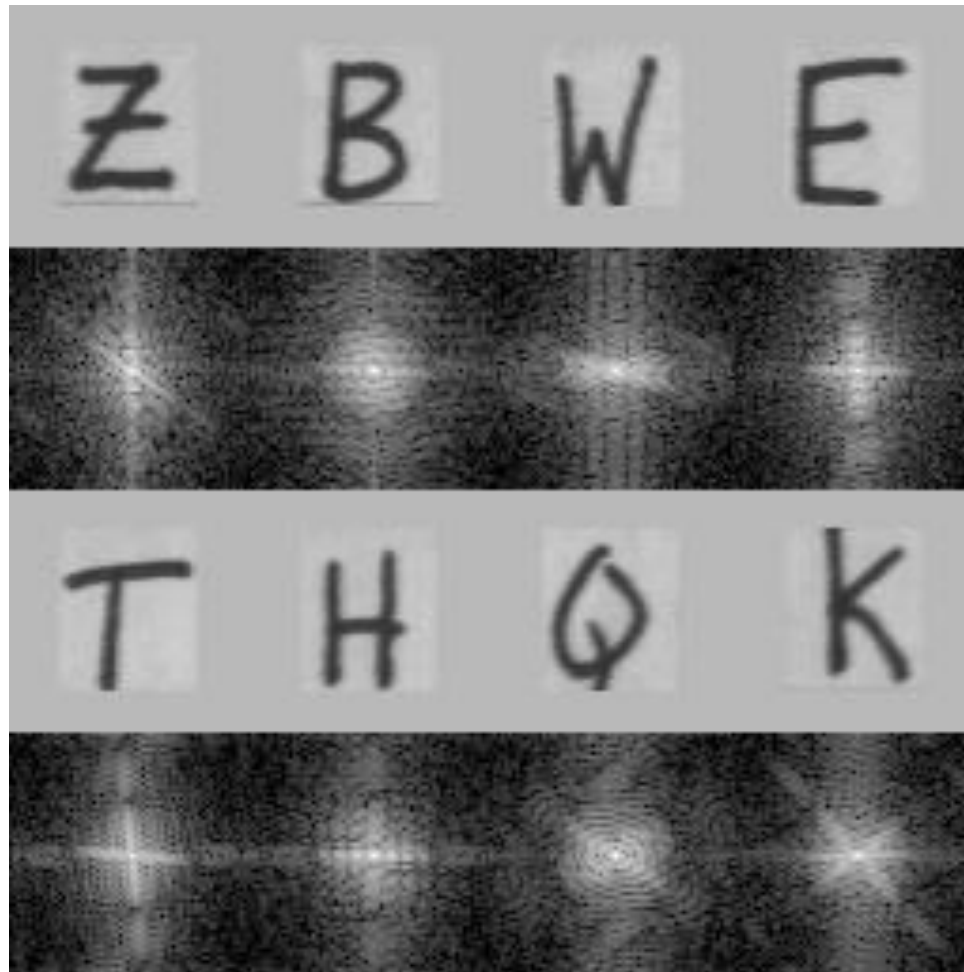




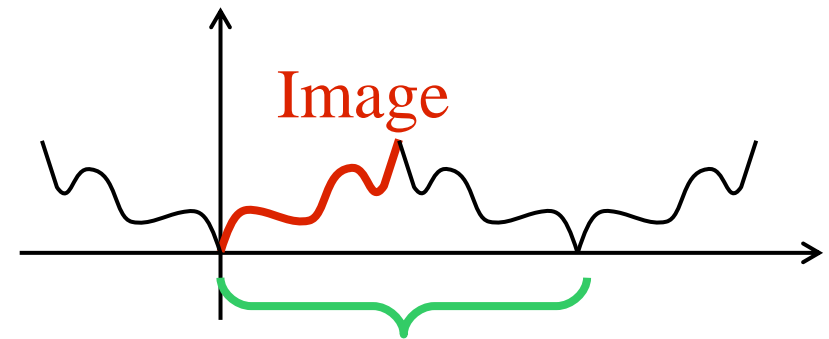
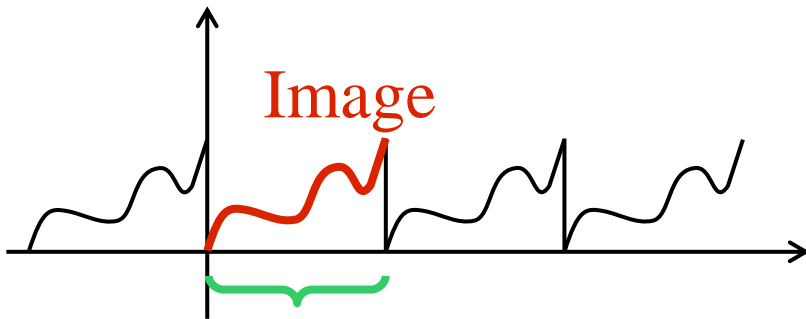
# Halftone



# Shape recognition



# DCT





# DCT



Real values

Energy compression

Simpler

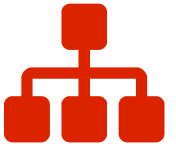
1D

$$C[k] = \sum_{x=0}^{N-1} i[x] \cos \left[ \frac{\pi}{N} \left( x + \frac{1}{2} \right) k \right]$$

2D

$$C[k, l] = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} i[x, y] \cos \left[ \frac{\pi}{N} \left( x + \frac{1}{2} \right) k \right] \cos \left[ \frac{\pi}{M} \left( y + \frac{1}{2} \right) l \right]$$

# FCT



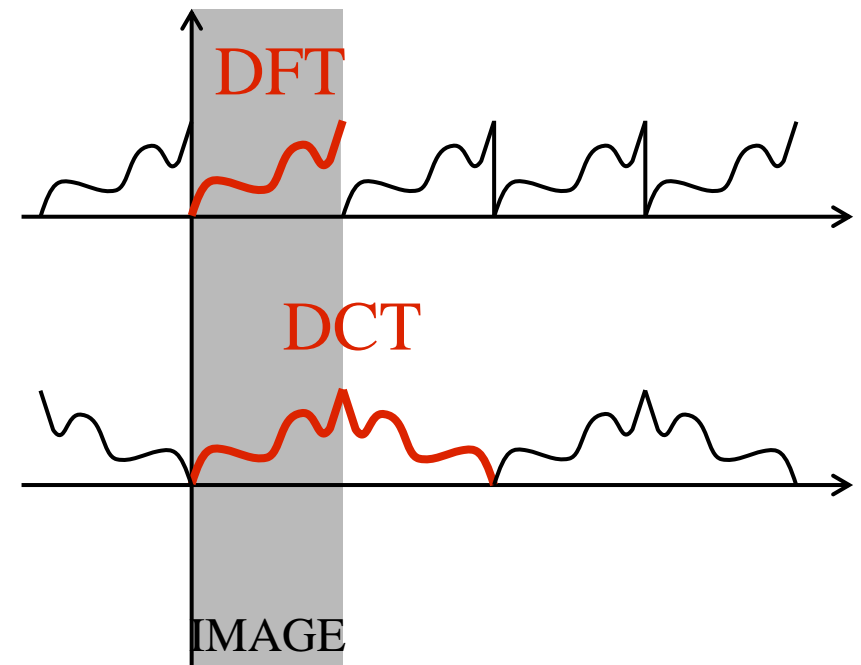
## 1. Symmetric function

$$y[n] = \begin{cases} x[n] & , ha\ 0 \leq n < N \\ x[2N - 1 - n] & , ha\ N \leq n < 2N \end{cases}$$

## 2. FFT (on 2N data)

## 3. The real part

$$C[n] = \Re \left( e^{\frac{-i\pi k}{2N}} * Y[n] \right)$$



# JPEG

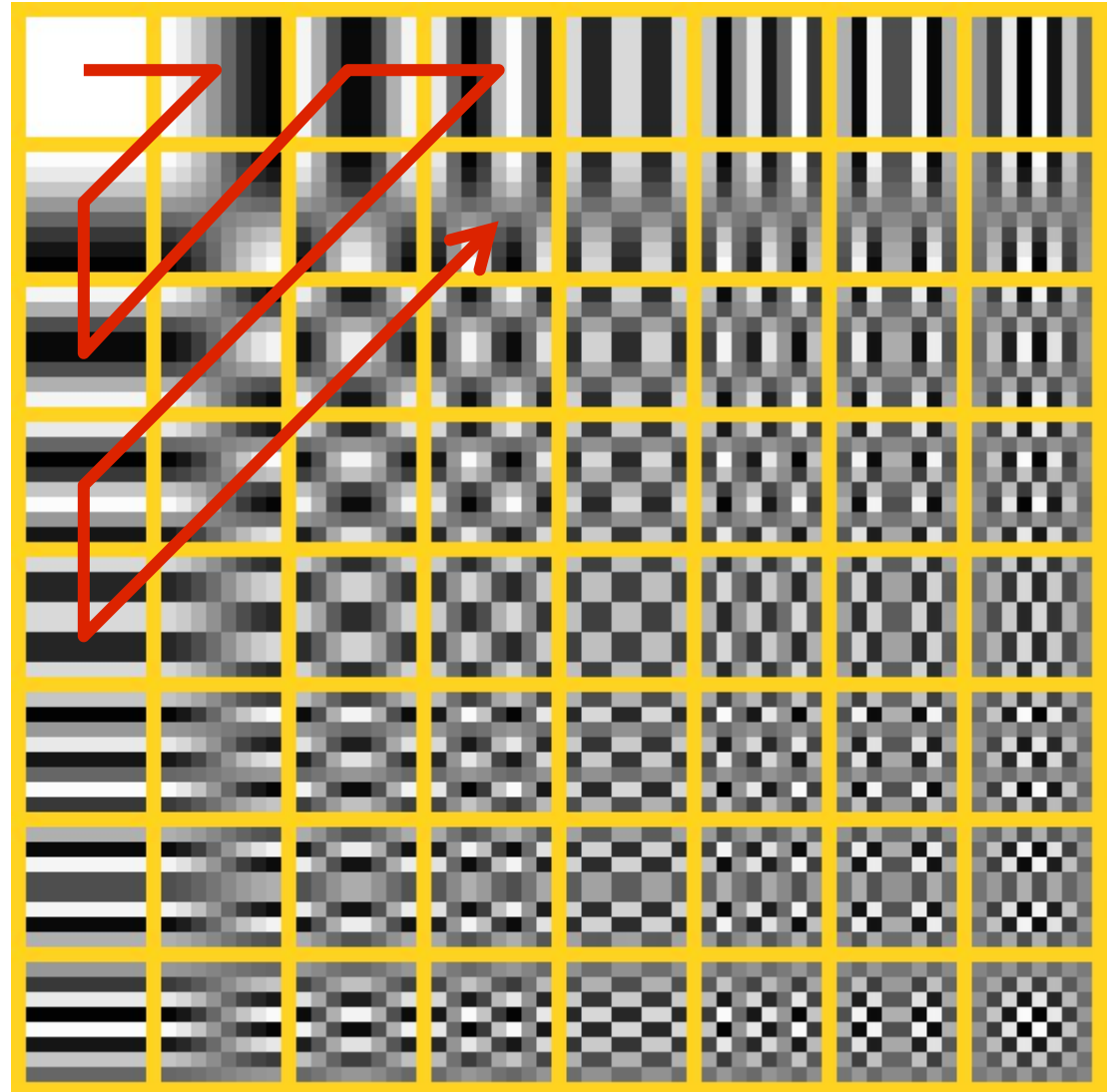


DCT-II

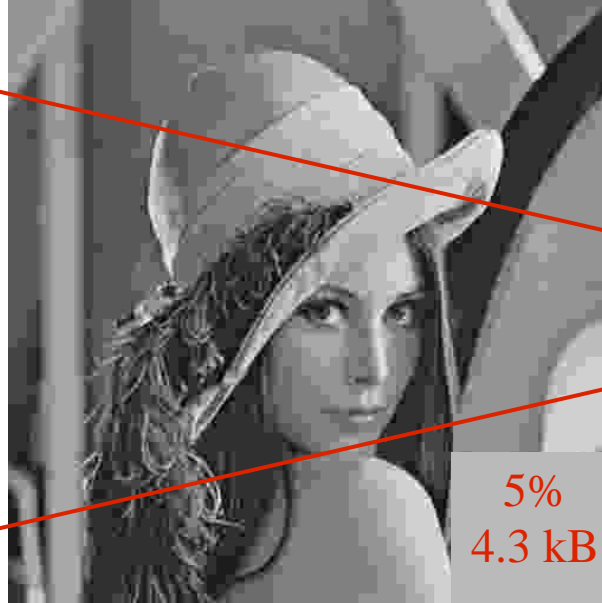
8x8 windows

8x8 C-table

Only the first few  
values are stored



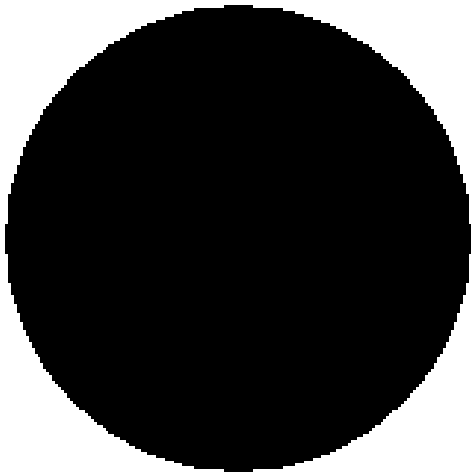
# JPEG compression



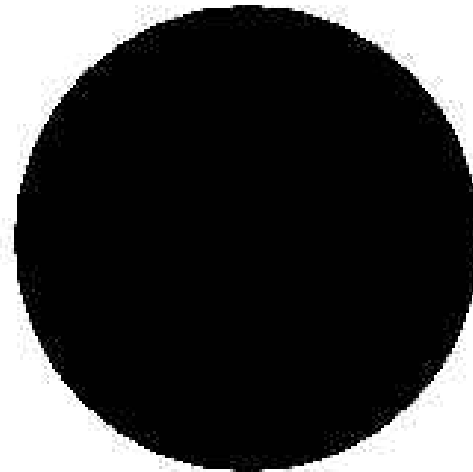
# Impact of compression



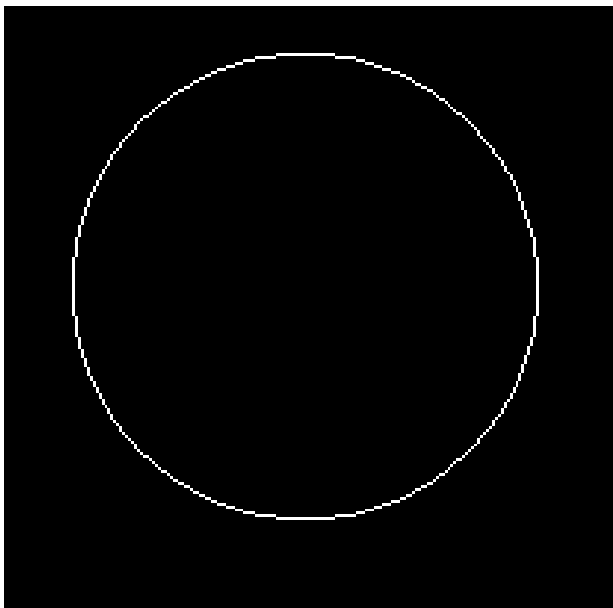
# Lossy JPEG



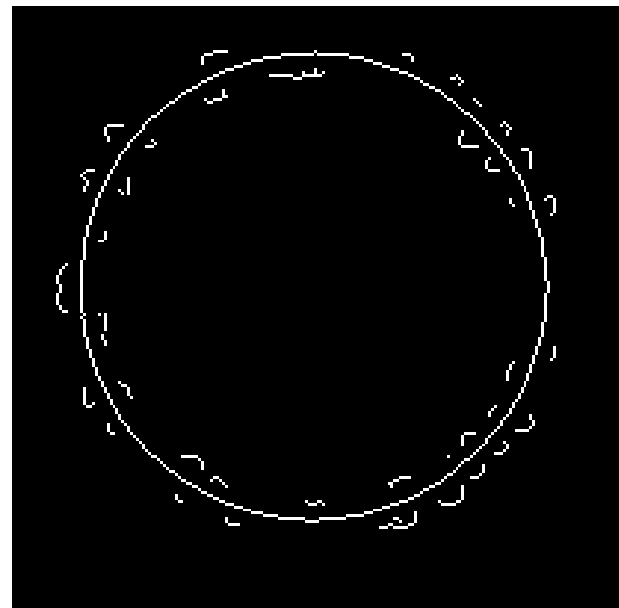
Original



Lossy JPEG

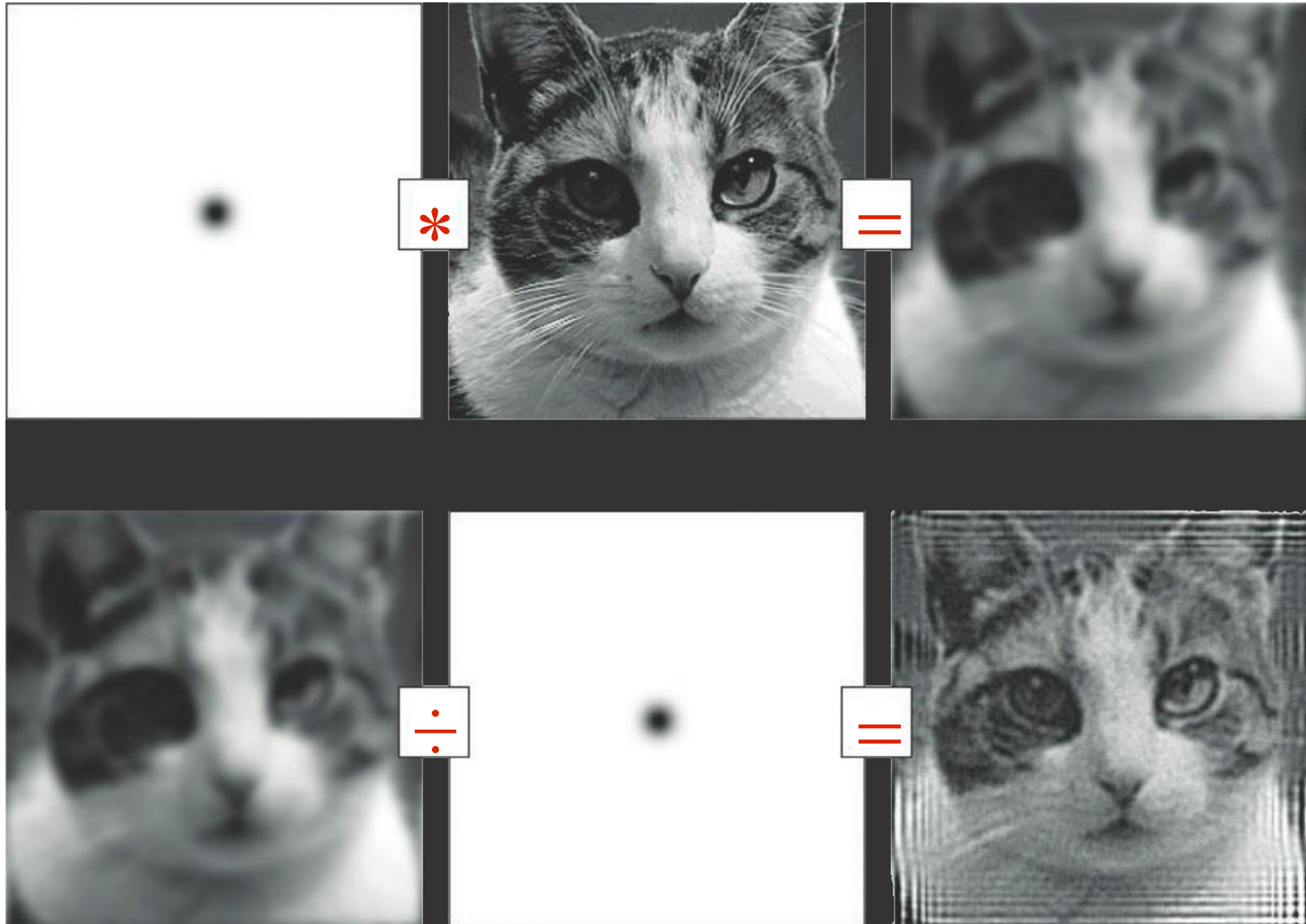


Canny  
edge  
detection





# Deconvolution



# Simple deconvolution

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$$g(x, y) = f(x, y) * h(x, y)$$

$$f(x, y) = g(x, y) \div h(x, y)$$

$$F(u, v) = \left[ \frac{1}{H(u, v)} \right] \cdot G(u, v)$$

# Wiener deconvolution



$$g(x, y) = f(x, y) * h(x, y) + \varepsilon$$

$$f(x, y) = g(x, y) \div h(x, y)$$

$$F(u, v) \approx \left[ \frac{1}{H(u, v)} \right] \cdot \left[ \frac{|H(u, v)|^2}{|H(u, v)|^2 + \frac{1}{SNR(u, v)}} \right] \cdot G(u, v)$$