Exercises for Topics 7-9

Micro- and macroeconomics

Extra Exercises: Costs and Supply

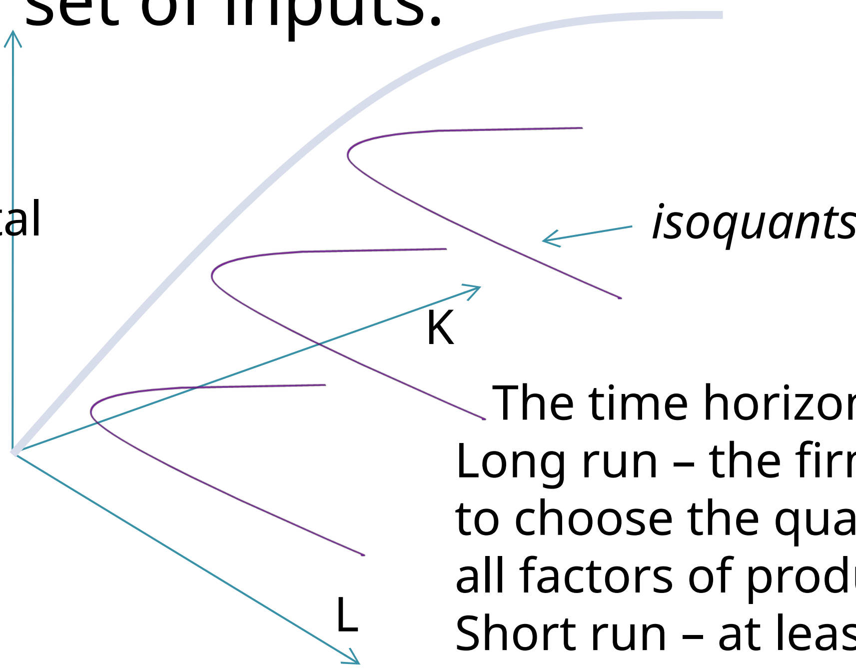
Micro- and Macroeconomics (BMEGT30A001, BMEGT30A410)

Reminder

The production function

- The production function ($Q = f(x_1, x_2, \ldots)$) shows how many goods or services a firm can produce utilizing its resources (factors of production) – the maximum output possible from a given set of inputs.

In our examples
K – Capital [← das Kapital (German)]
L – Labour
q – Quantity of output

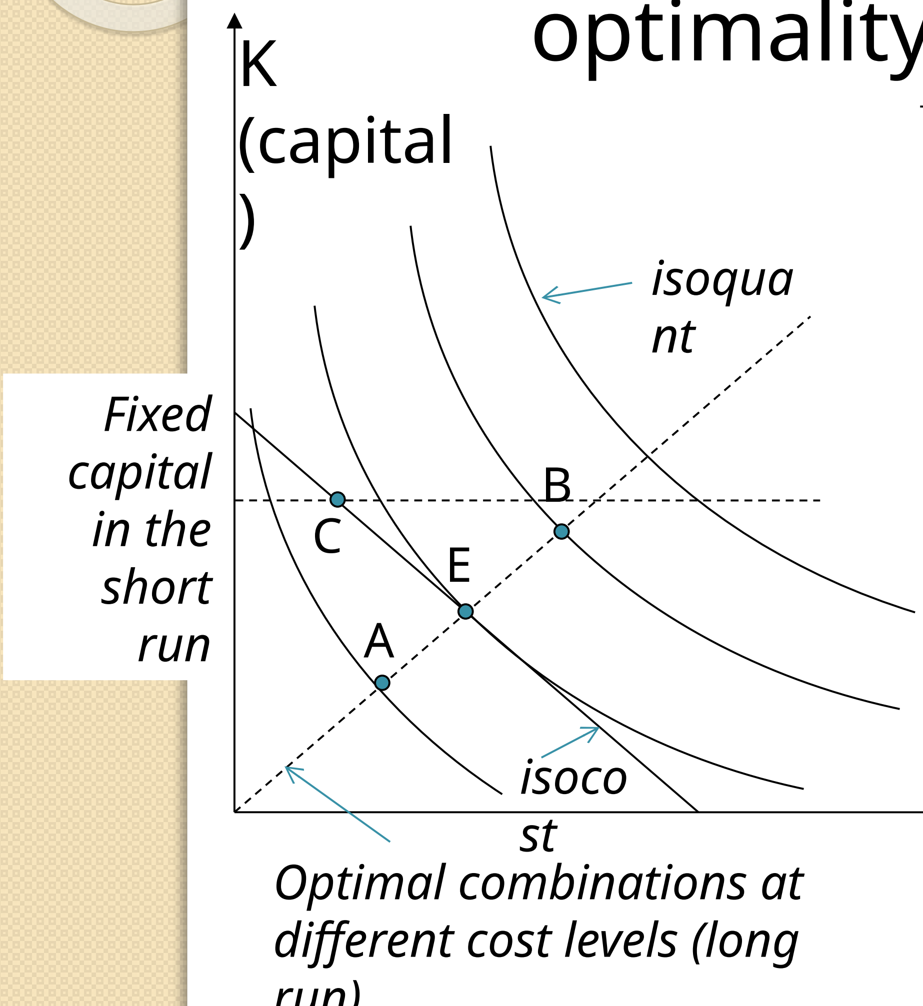


The time horizon:
Long run – the firm is free to choose the quantity of all factors of production
Short run – at least one of these factors is fixed

The optimal combination of inputs

A necessary condition for optimality:

$$\frac{P_L}{P_K} = \frac{\left(\frac{\partial Q}{\partial L}\right)}{\left(\frac{\partial Q}{\partial K}\right)} = \left|MRTS^{\dagger}\right|$$



(The isocost line is the line tangent to the isoquant at the optimum point)

Point C does not satisfy this condition so it cannot be the optimal (long-run) combination.

In the long run

†MRTS: the Marginal Rate of Technical Substitution

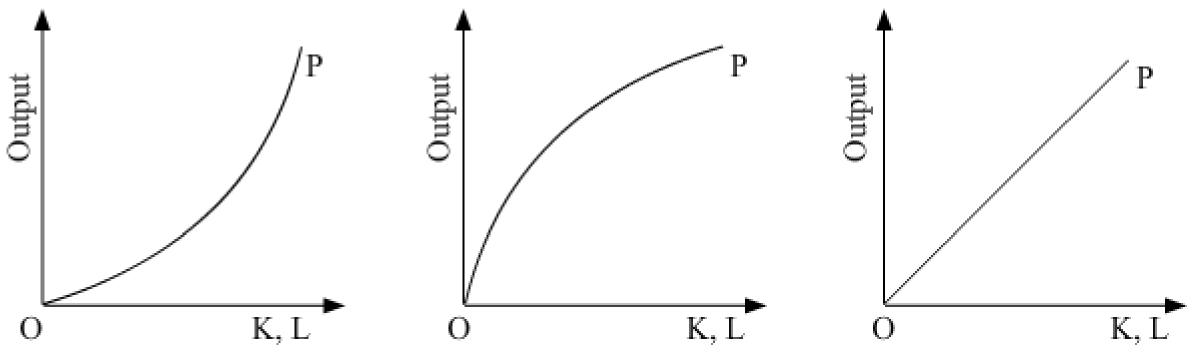
The optimum point is at „E"

[1] Or: Technical Rate of Substitution (TRS)

Returns to scale

Reminder

Increasing returns to scale mean that one (big) company can produce more efficiently than more than one (smaller) firms utilizing the same factors of production → emergence of natural monopolies



(a) Increasing Returns to Scale

(b) Decreasing Returns to Scale

(c) Constant Returns to Scale

Formally, „returns to scale”, refers to changes in output (quantity, q) resulting from a proportional change (λ) in all inputs:

a) If output increases by more than that proportional change, there are increasing returns to scale (economies of scale):

e.g. $Q = KL \quad Q(\lambda K, \lambda L) = (\lambda K)(\lambda L) = \lambda^2 Q(K, L)$

b) If output increases by less than that proportional change, there are decreasing returns to scale:

e.g. $Q = \sqrt[3]{KL} \quad Q(\lambda K, \lambda L) = \sqrt[3]{(\lambda K)(\lambda L)} = \lambda^{2/3} Q(K, L)$

c) If output increases by that same proportional change, there are constant returns to scale:

e.g. $Q = \sqrt{KL} \quad Q(\lambda K, \lambda L) = \sqrt{(\lambda K)(\lambda L)} = \lambda Q(K, L)$

A few useful formulae

$$Q = Q(K, L)$$

Production function

$$TC = FC + VC = p_K \cdot K + p_L \cdot L \qquad K = -\frac{p_L}{p_K} \cdot L + \frac{TC}{p_K}$$

Isocost eq.

$$AP_L = \frac{Q}{L}; \quad MP_L = \frac{\partial Q}{\partial L}$$

$$AP_K = \frac{Q}{K}; \quad MP_K = \frac{\partial Q}{\partial K}$$

Average and marginal products

$$MRTS = \frac{dK}{dL} = -\frac{MP_L}{MP_K} = \left[-\frac{p_L}{p_K}\right]$$

Optimum input combination

$$AC = \frac{TC}{Q}; \quad AVC = \frac{VC}{Q}; \quad AFC = \frac{FC}{Q}; \quad MC = \frac{dTC}{dQ} = \frac{dVC}{dQ}$$

Cost functions

$$TR = P \cdot Q; \quad AR = \frac{TR(Q)}{Q}; \quad MR = \frac{dTR(Q)}{dQ}$$

Revenue functions

Exercise 1

Our production function:

$q = F(K,L) = \sqrt{K_0 L}$, where $K_0 = 16$.

The input prices are $p_K = 10$ & $p_L = 40$.

a) Determine the short-run total cost function!

b) Determine the marginal product of labour and capital!

c) Determine the long-run total cost function!

Exercise 1a

Our production function:

$q = F(K, L) = \sqrt{K_0 L}$, where $K_0 = 16$.

The input prices are $p_K = 10$ & $p_L = 40$.

Which depends on the desired output volume

a) Determine the short-run total cost function!

General form of the total cost function:

$$TC = p_K K + p_L L$$

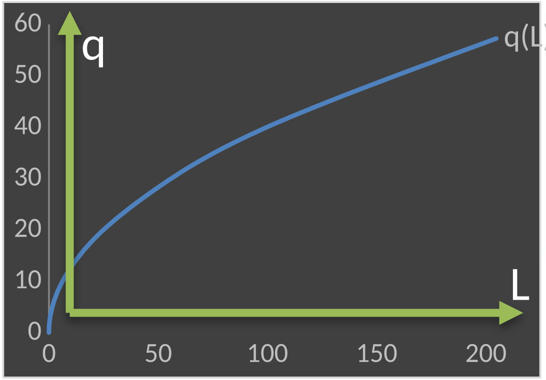
With our initial values:

SFC

$$TC = 10 \times 16 + 40L = 160 + 40L$$

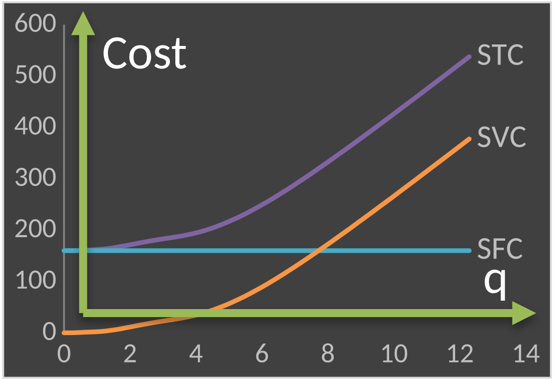
The short-run variable costs are driven by the number of workers.

Exercise 1a[cont.]



Our production function in the short run:

$$q_S = F(K_0, L) = \sqrt{K_0 L} = \sqrt{16L} = 4\sqrt{L}$$



We should express *L* as a function of *q*:

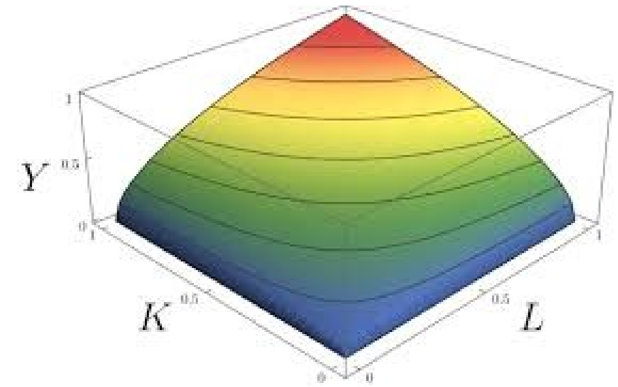
$$\sqrt{L} = q_S/4$$

$$L = q^2/16$$

Substitute this *L*(*q*) function into the total cost function:

$$TC = 160 + 40L = 160 + \frac{40q^2}{16} = 2.5q^2 + 160$$

Exercise 1b



Our production function:

$q = F(K, L) = \sqrt{KL}$

The input prices are $p_K = 10$ & $p_L = 40$.

b) Determine the marginal product of labour and capital!

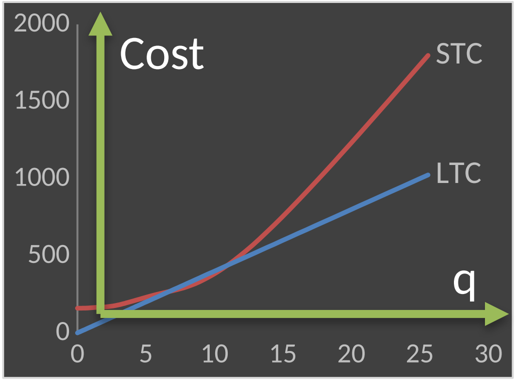
The marginal products are:

$$F_L = MP_L = \frac{\partial q}{\partial L} = \frac{\sqrt{K}}{2\sqrt{L}} \qquad F_K = MP_K = \frac{\partial q}{\partial K} = \frac{\sqrt{L}}{2\sqrt{K}}$$

$q = F(K,L) = \sqrt{KL} \quad p_K = 10, p_L = 40$

$MP_L = \frac{\sqrt{K}}{2\sqrt{L}} \quad MP_K = \frac{\sqrt{L}}{2\sqrt{K}}$

Exercise 1c



c) Determine the long-run total cost function!

$$|MRTS| = \frac{MP_L}{MP_K} = \frac{K}{L} = \frac{p_L}{p_K} = \frac{40}{10} = 4 \Rightarrow K = 4L$$

Substitute this $K(L)$ function into the production function:

$$q = F(K,L) = \sqrt{4L \times L} = 2L$$

The required number of workers (as a function of q): $L = q/2$

The required number of machines ($K(q)$): $K = 4L = 2q$

Finally, we can express the total cost as a function of q:

$$TC(q) = p_K K + p_L L = 10 \times 2q + 40 \times \frac{q}{2} = 20q + 20q = 40q$$

Exercise 2

Our production function (Sato's production function) is

$$q = \mathrm{F(K, L)} = \frac{K^2L^2}{K^3+L^3}$$

Does this function exhibit increasing, decreasing or constant returns to scale?

$$\mathrm{F(\lambda K, \lambda L)} = \frac{\lambda^2K^2\lambda^2L^2}{\lambda^3K^3+\lambda^3L^3} = \frac{\lambda^4}{\lambda^3}\frac{K^2L^2}{K^3+L^3} = \lambda q$$

Constant returns to scale.

Exercise 3

Our production function is:

$$q = F(K, L) = \sqrt{KL}\,.$$

The input prices are $p_K = 10$ & $p_L = 250$.

Determine the minimum (long-run) cost of producing $\hat{q} = 1000$ units of the output!

Exercise 3 (solution)

Our production function is: $q = F(K, L) = \sqrt{KL}$. The input prices are $p_K = 10$ & $p_L = 250$. Determine the minimum (long-run) cost of producing $\hat{q} = 1000$ units of the output!

$$F_L = MP_L = \frac{\partial q}{\partial L} = \frac{\sqrt{K}}{2\sqrt{L}}$$

$$F_K = MP_K = \frac{\partial q}{\partial K} = \frac{\sqrt{L}}{2\sqrt{K}}$$

$$|MRTS| = \frac{MP_L}{MP_K} = \frac{K}{L} = \frac{p_L}{p_K} = \frac{250}{10} = 25$$

Exercise 3 (solution, cont.)

$$|MRTS| = \frac{MP_L}{MP_K} = \frac{K}{L} = \frac{p_L}{p_K} = \frac{250}{10} = 25 \ \Rightarrow K = 25L$$

Substitute this ratio into the production function:

$$q = F(K, L) = \sqrt{25L \times L} = 5L$$

The required number of workers (as a function of q): $L = q/5$

The required number of machines ($K(q)$): $K = 25L = 5q$

Use these values to determine the total cost function $TC(q)$:

$$TC(q) = p_K K + p_L L = 10 \times 5q + 250 \times \frac{q}{5} = 100q$$

The cost of producing 1000 units of the output:

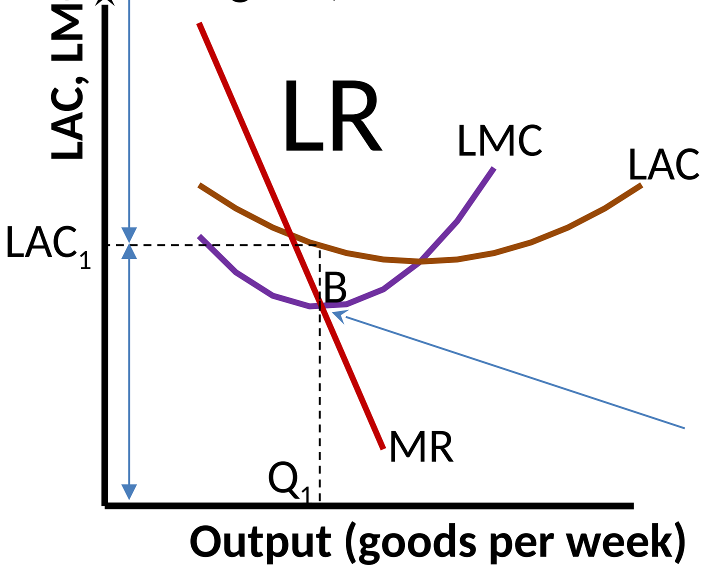
$$\mathrm{TC}(\hat{q} = 1000) = 100{,}000$$

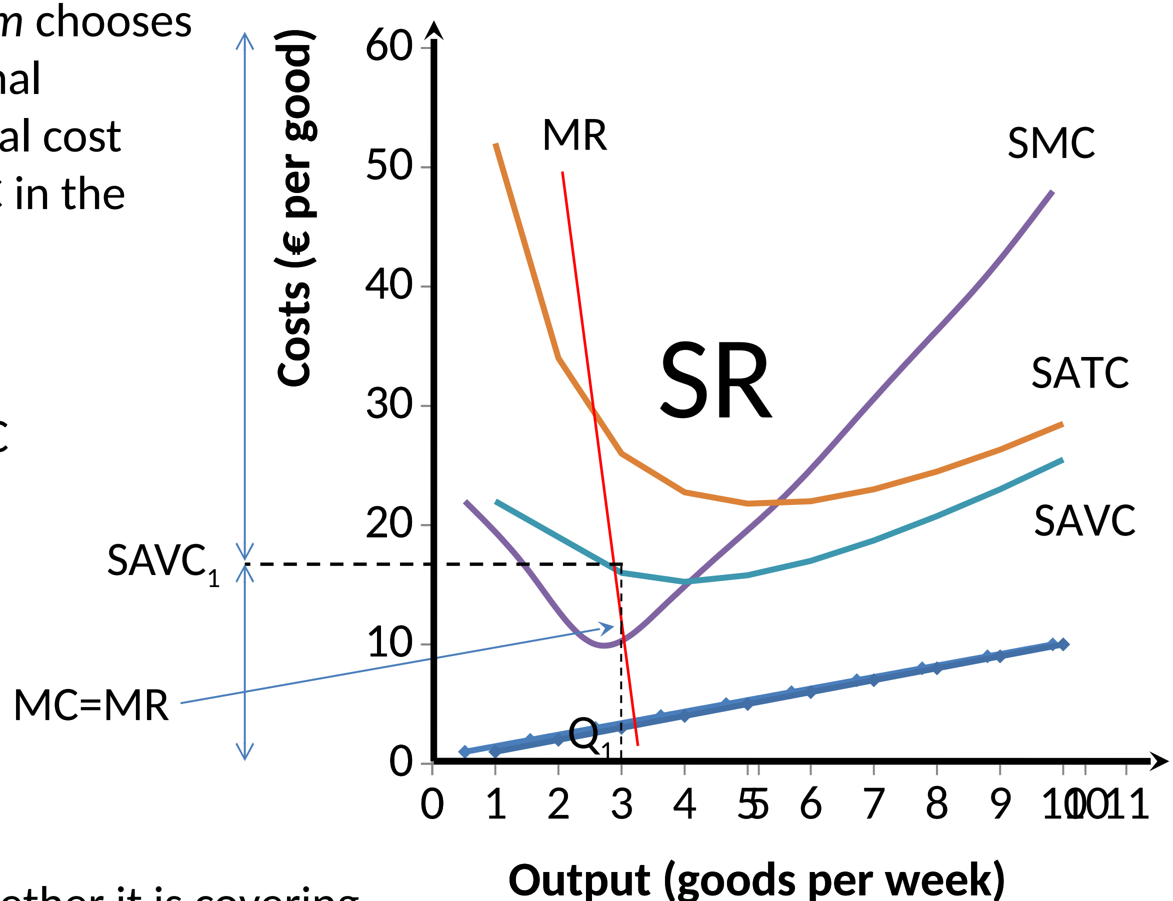
Extra Exercises:
Perfect Competition, Monopoly

Micro- and Macroeconomics
(BMEGT30A001, BMEGT30A410)

Marginal condition	Average condition	
	Short run	Long run
Produce output Where MR = MC	If P < SAVC shut down temporarily	If P < LAC exit industry

To maximize profits *any firm* chooses the output at which marginal revenue MR equals marginal cost (SMC in the short run, LMC in the long run).

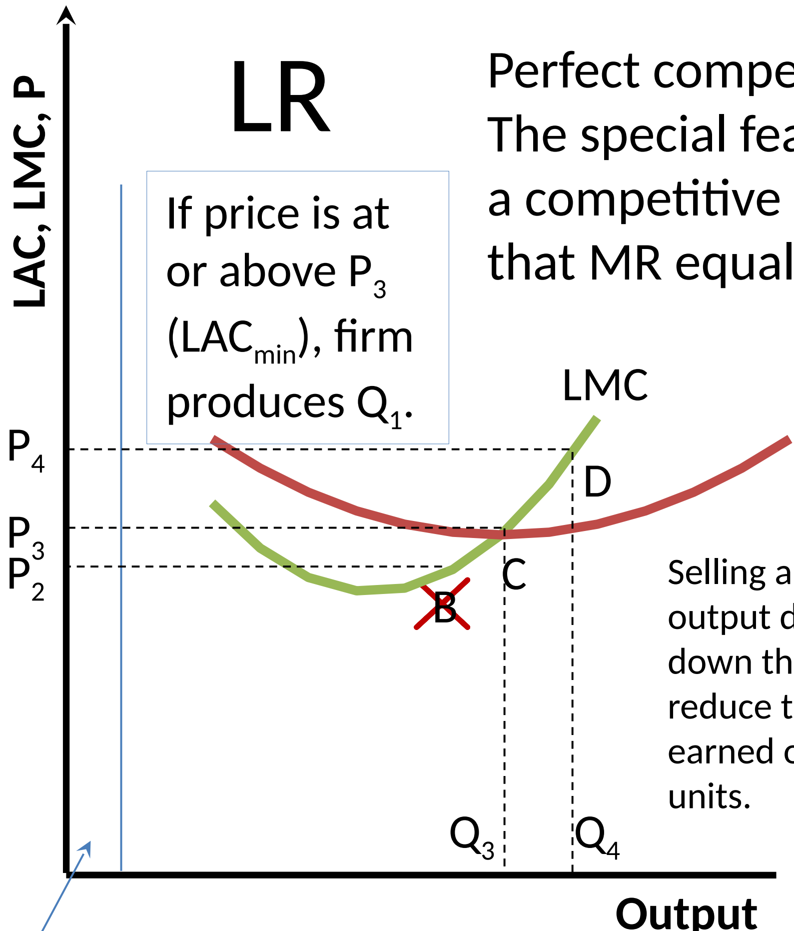




It then checks whether it is covering average costs (SAVC in the short run and LAC in the long run).

Reminder

Marginal condition	Average condition	
	Short run	Long run
Produce output Where P = MC	If P < SAVC shut down temporarily	If P < LAC exit industry



Perfect competition: The special feature of a competitive firm is that MR equals price.

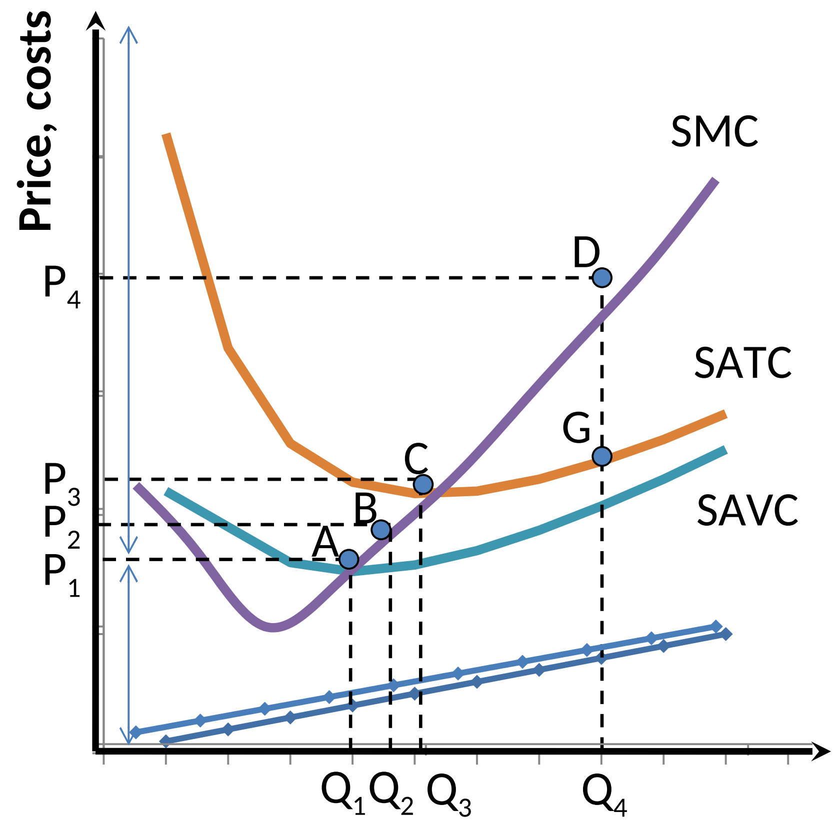
Selling an extra unit of output does not bid down the price and reduce the revenue earned on previous units.

If price is less than P_3 (LAC_{min}), firm goes out of business

The price at which the extra unit is sold is the change in total revenue.

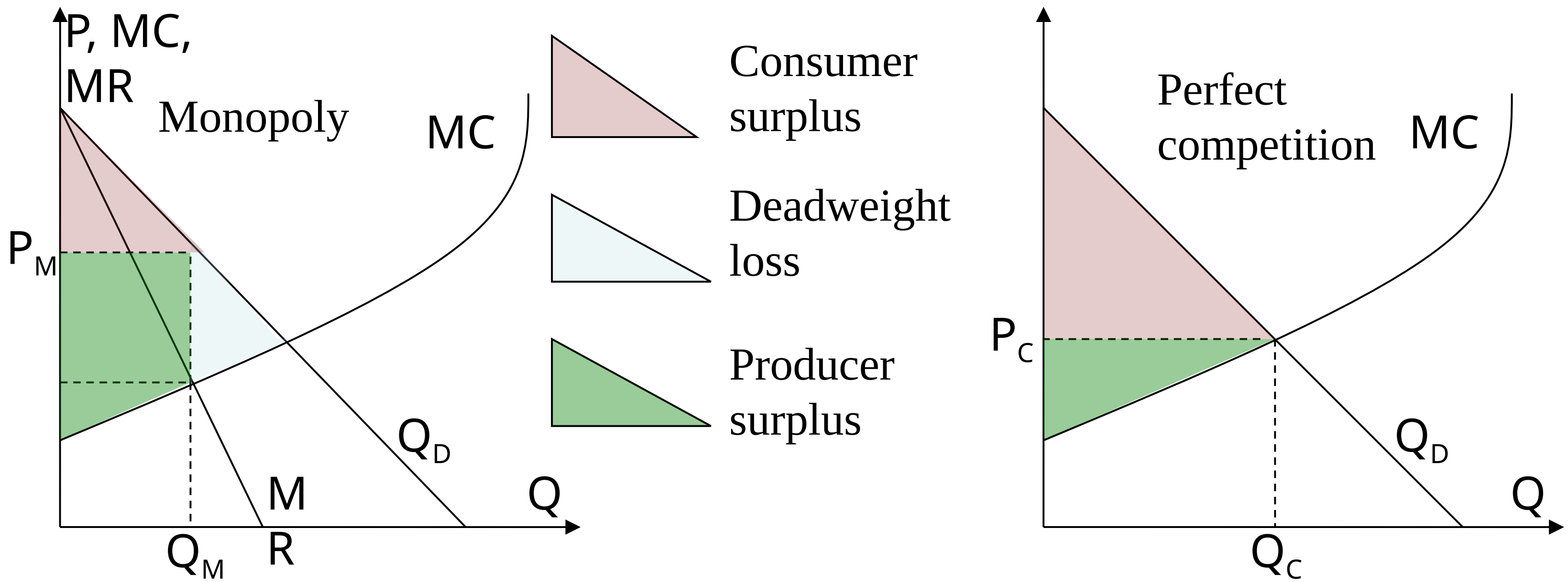
Reminder

SR



Output (goods per week)

Comparison of monopoly and perfect competition, deadweight loss



- The deadweight loss is the economic benefit forgone by the society as the result of the lower quantity of output supplied by the firm with market power.
- In theory, both the consumers and the firm could increase their surplus if the firm could sell (Q_C-Q_M) units of the output at price (P_C) *after* selling Q_M goods at the monopoly price (P_M). If the monopolist could charge individualized prices, the deadweight loss could be reduced (or in an extreme case, eliminated).

  → Discriminating monopolist, price discrimination

Exercise 1: Perfect Competition

In a perfectly competitive industry, the demand function is given by the equation: $Q(P) = 20850 - P$.

The total cost function of a typical firm in the industry is given by the equation: $\mathrm{TC}(q) = 10q^2 + 50q + 25000$.

All firms are *equal* (identical products and cost functions).

The current market price is 1350.

How many firms are in the industry in the *short run* and in the *long run*?

Exercise 1: Short run

$Q(P) = 20850 - P$

$\mathrm{TC}(q) = 10q^2 + 50q + 25000$

Current price: 1350.

- $MC = 20q + 50 = P = 1350$
- $q = 65$
- $Q = 20850 - 1350 = 19500$
- $n = Q/q = 19500/65 = 300$

Market demand:
$Q^*(P=1350)$

Firms' optimum:
q^*: $MC = P$
(When $P \geq SAVC_{min}$)

Number of firms:
$n = Q/q$

Exercise 1: Long run

$$Q(P) = 20850 - P$$

$$\mathrm{TC}(q) = 10q^2 + 50q + 25000$$

- $AC = 10q + 50 + 25000/q = MC = 20q + 50$
- $q = 50$
- $MC = 20 \cdot 50 + 50 = 1050$
- $Q = 20850 - 1050 = 19800$
- $n = 19800/50 = 396$

Long-run equilibrium:
q^*: $P = MC = AC$

Exercise 2: Monopoly and PC

- The demand function facing a monopolist: Q = 500 − 0.5P.
- The total cost function of the monopolist: TC = $1.5Q^2 + 160Q + 20000$.

a. Determine the optimal (profit-maximizing) quantity of output and price for the monopolist!
b. If the same industry was perfectly competitive (identical demand and cost conditions), what would be the market price and industry output?
c. Determine the deadweight loss associated with monopoly pricing in this industry!

Exercise 2: a. Monopoly

- $MC = 3Q + 160$
- $MR = 1000 – 4Q$
- $MR = MC$
- $3Q + 160 = 1000 – 4Q$
- $Q = 120$
- $P = 760$

The demand function facing a monopolist (= market demand): $Q = 500 – 0.5P$.

Inverse demand function: $P = 1000 – 2Q$.

Total revenue function: $TR = P \cdot Q = 1000Q – 2Q^2$.

The total cost function of the monopolist: $TC = 1.5Q^2 + 160Q + 20000$.

Exercise 2: b. Competition

Perfect competition: P = MC (at the industry level):

- P = 3Q + 160

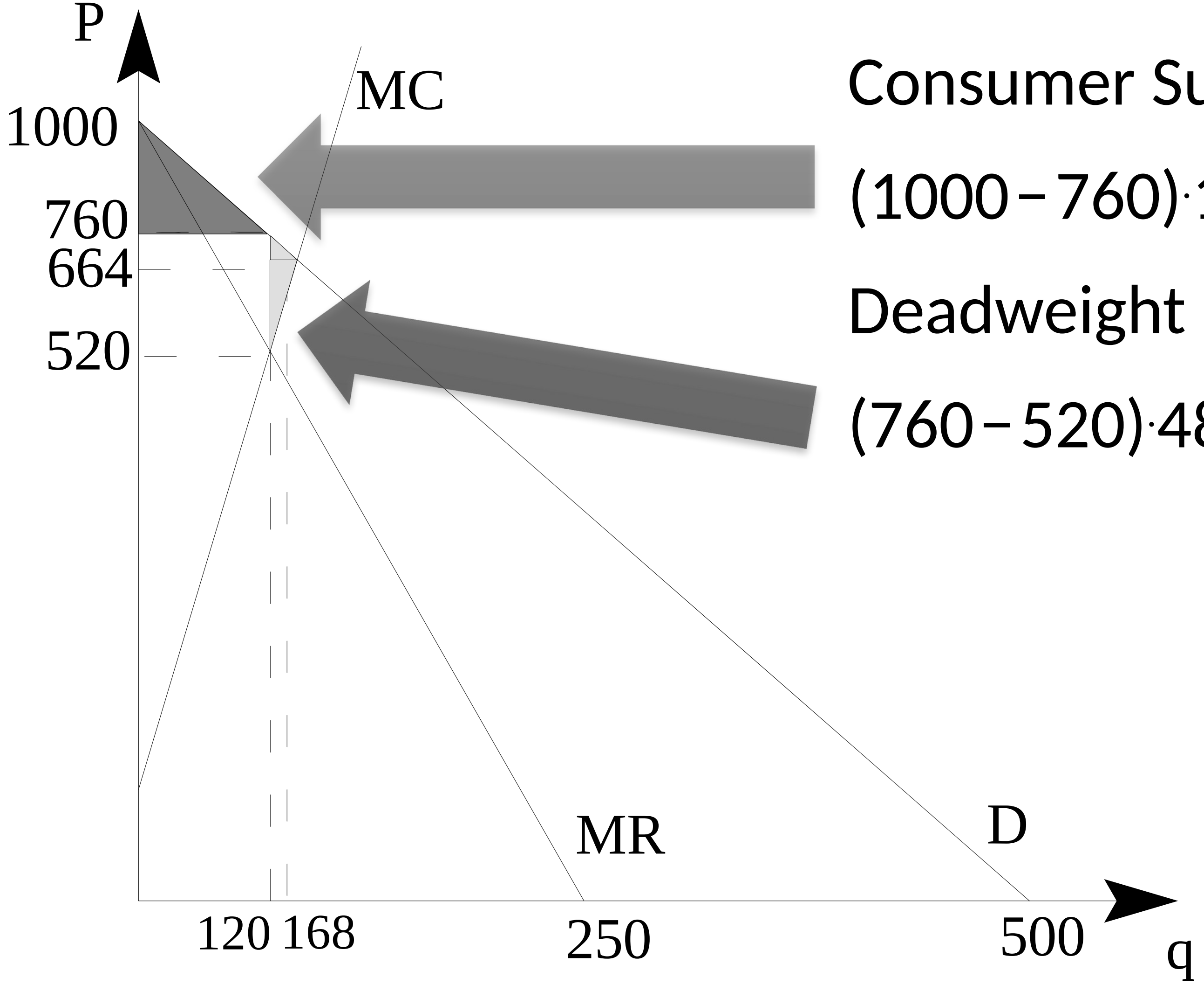
Market equilibrium (supply equals demand):

- 3Q + 160 = 1000 − 2Q
- Q = 168
- P = 664

The market demand function: Q = 500 − 0.5P ~ P = 1000 − 2Q

The total cost function characterizing the industry: TC = $1.5Q^2 + 160Q + 20000$.

c. Consumer Surplus and Deadweight Loss



Consumer Surplus:

$(1000 - 760) \cdot 120/2 = 14400$

Deadweight Loss:

$(760 - 520) \cdot 48/2 = 5760$

Bonus Content:
Exotic Production Functions

Micro- and Macroeconomics
(BMEGT30A001, BMEGT30A410)

Exercise 1

Our production function (CES production function; CES: Constant elasticity of substitution) is

$$F(K,L) = B[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}]^{-\frac{1}{\rho}}.$$

Does this function exhibit increasing, decreasing or constant returns to scale?

Exercise 1 (solution)

$$F(\lambda K, \lambda L) = B[\alpha(\lambda K)^{-\rho} + (1-\alpha)(\lambda L)^{-\rho}]^{-\frac{1}{\rho}} =$$

$$= [\lambda^{-\rho}]^{-\frac{1}{\rho}} B[\alpha K^{-\rho} + (1-\alpha) L^{-\rho}]^{-\frac{1}{\rho}} = \lambda q$$

Constant returns to scale.

Exercise 2

Our production function is $q = Ae^{0.5K^2+L^2}$, where K is the number of machines, while L is the number of workers, *e* is Euler's number and A is an unknown constant.

Determine the optimal capital intensity *K*(*L*) if p_L and p_K are the input prices!

Exercise 2 (solution)

Our production function is $q = Ae^{0.5K^2+L^2}$, where K is the number of machines, while L is the number of workers, *e* is Euler's number and A is an unknown constant.

Determine the optimal capital intensity *K*(*L*) if p_L and p_K are the input prices!

$$F_L = MP_L = \frac{\partial q}{\partial L} = Ae^{0.5K^2+L^2}2L$$

$$F_K = MP_K = \frac{\partial q}{\partial K} = Ae^{0.5K^2+L^2}K$$

$$|MRTS| = \frac{MP_L}{MP_K} = \frac{2L \times Ae^{0.5K^2+L^2}}{K \times Ae^{0.5K^2+L^2}} = \frac{2L}{K} = \frac{p_L}{p_K}$$

$$L = \frac{Kp_L}{2p_K}$$

$$K = \frac{2Lp_K}{p_L}$$

Exercise 3

Our production function is: $q = F(K,L) = \ln(K^{\alpha}L^{1-\alpha})$.
Determine the total cost function if input prices are p_L & p_K!

$$q = \alpha \ln K + (1-\alpha)\ln L$$

$$F_L = MP_L = \frac{\partial q}{\partial L} = \frac{1-\alpha}{L}$$

$$F_K = MP_K = \frac{\partial q}{\partial K} = \frac{\alpha}{K}$$

$$|MRTS| = \frac{MP_L}{MP_K} = \frac{1-\alpha}{\alpha}\frac{K}{L} = \frac{p_L}{p_K}$$

$$L = \frac{1-\alpha}{\alpha}\frac{p_K}{p_L}K$$

$$q = \ln\left(K^{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}K\right)^{1-\alpha}\right) =$$

$$= \ln\left(K\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}\right)^{1-\alpha}\right)$$

Exercise 3 (cont.)

$$K = L\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}\right)^{-1}$$

$$q = \ln\left(L\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}\right)^{-\alpha}\right)$$

$$L = e^q\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}\right)^{\alpha}$$

$$K = e^q\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}\right)^{\alpha-1}$$

$$L = \frac{1-\alpha}{\alpha}\frac{p_K}{p_L}K$$

$$q = \ln\left(K^{\alpha}\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}K\right)^{1-\alpha}\right) =$$

$$= \ln\left(K\left(\frac{1-\alpha}{\alpha}\frac{p_K}{p_L}\right)^{1-\alpha}\right)$$

Exercise 3 (solution)

$$K = e^q \left(\frac{1-\alpha}{\alpha} \frac{p_K}{p_L} \right)^{\alpha - 1} \quad L = e^q \left(\frac{1-\alpha}{\alpha} \frac{p_K}{p_L} \right)^{\alpha}$$

Our production function is: $q = F(K, L) = \ln(K^\alpha L^{1-\alpha})$.

Determine the total cost function if input prices are p_L & p_K!

$$TC(q) = p_K K + p_L L =$$

$$= p_K e^q \left(\frac{1-\alpha}{\alpha} \frac{p_K}{p_L} \right)^{\alpha - 1} + p_L e^q \left(\frac{1-\alpha}{\alpha} \frac{p_K}{p_L} \right)^{\alpha}$$