Circuit of quantum teleportation

Kitti Olah

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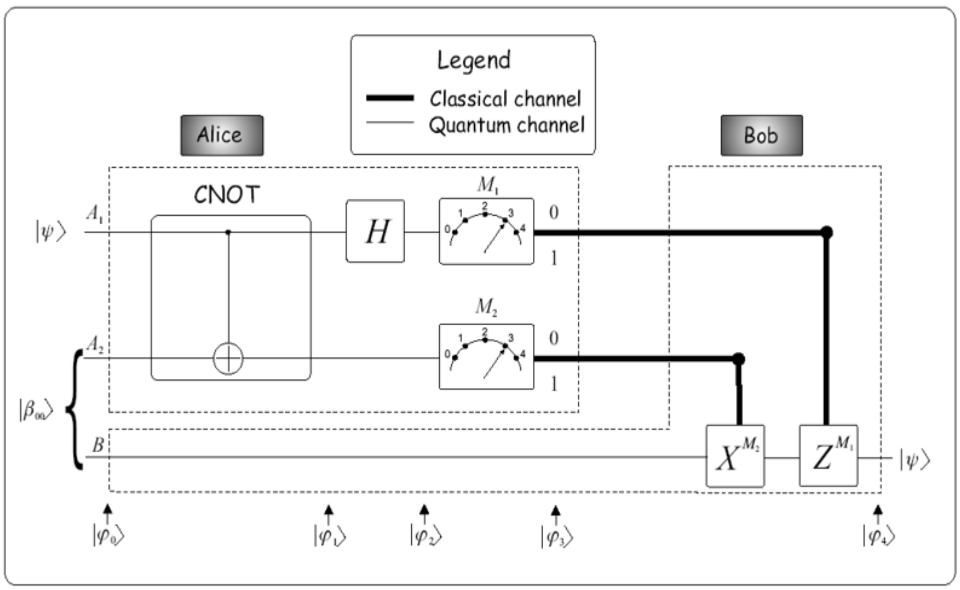

Alice has an unknown quantum state $|\psi\rangle = a|0\rangle + b|1\rangle$, which she wants to send to Bob. The initialization is the first step of teleportation, where they they will use a previously shared Bell-pair $|\beta_{00}\rangle$. In this case the Bell pair is:

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

The initial state of the whole system including $|\psi\rangle$ is:

$$|\varphi_0\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left[a|0_{A_1}\rangle\left(|0_{A_2}0_B\rangle + |1_{A_2}1_B\rangle\right) + b|1_{A_1}\rangle\left(|0_{A_2}0_B\rangle + |1_{A_2}1_B\rangle\right)\right]$$

After Alice uses a CNOT gate to entangle A_1 and A_2, we will get the following state:

$$|\varphi_1\rangle = \frac{1}{\sqrt{2}}\left[a|0_{A_1}\rangle\left(|0_{A_2}0_B\rangle + |1_{A_2}1_B\rangle\right) + b|1_{A_1}\rangle\left(|1_{A_2}0_B\rangle + |0_{A_2}1_B\rangle\right)\right].$$

Alice uses a Hadamard gate to superpose this state:

$$|\varphi_2\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left[a(|0_{A_1}\rangle + |1_{A_1}\rangle)(|0_{A_2}0_B\rangle + |1_{A_2}1_B\rangle) + b(|0_{A_1}\rangle - |1_{A_1}\rangle)(|1_{A_2}0_B\rangle + |0_{A_2}1_B\rangle)\right].$$

We can simplify this state by grouping together the qubits belonging to Alice and Bob. This will result in a representation that helps us to easily determine the outcomes of the M_1 and M_2 measurements

$$\begin{aligned}|\varphi_2\rangle = \frac{1}{2}[\,&|0_{A_1}0_{A_2}\rangle\,(a\,|0_B\rangle + b\,|1_B\rangle) + |0_{A_1}1_{A_2}\rangle\,(a\,|1_B\rangle + b\,|0_B\rangle) + \\ &|1_{A_1}0_{A_2}\rangle\,(a\,|0_B\rangle - b\,|1_B\rangle) + |1_{A_1}1_{A_2}\rangle\,(a\,|1_B\rangle) - b\,|0_B\rangle].\end{aligned}$$