

# Costs and Supply A

## Appendix:

The optimal combination of factors  
(supply) and consumer behaviour

**MICRO- AND  
MACROECONOMICS**

# The production function

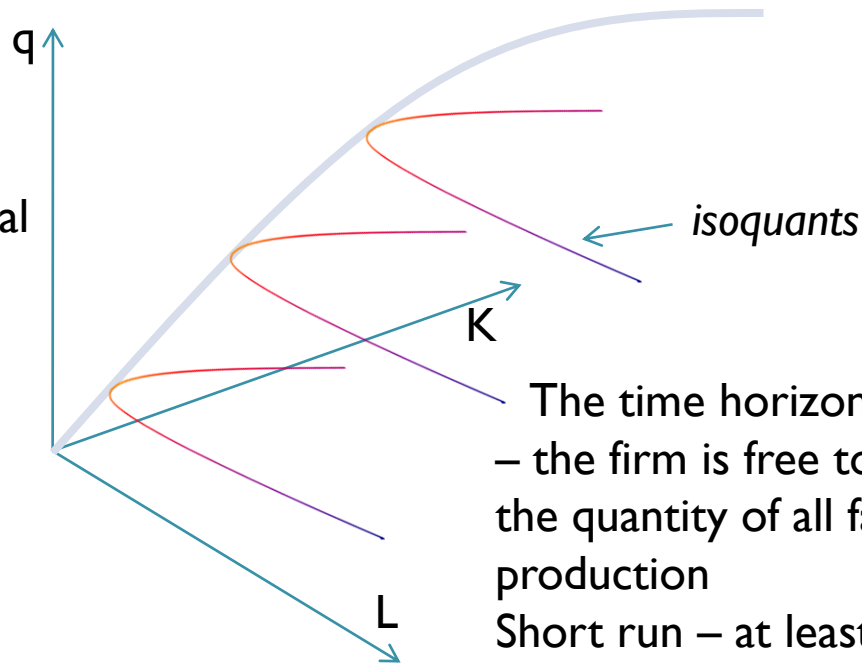
- The production function ( $Q = f(x_1, x_2, \dots)$ ) shows how many goods or services a firm can produce utilizing its resources (factors of production) – the maximum output possible from a given set of inputs.

In our examples

K – Capital [← das Kapital  
(German)]

L – Labour

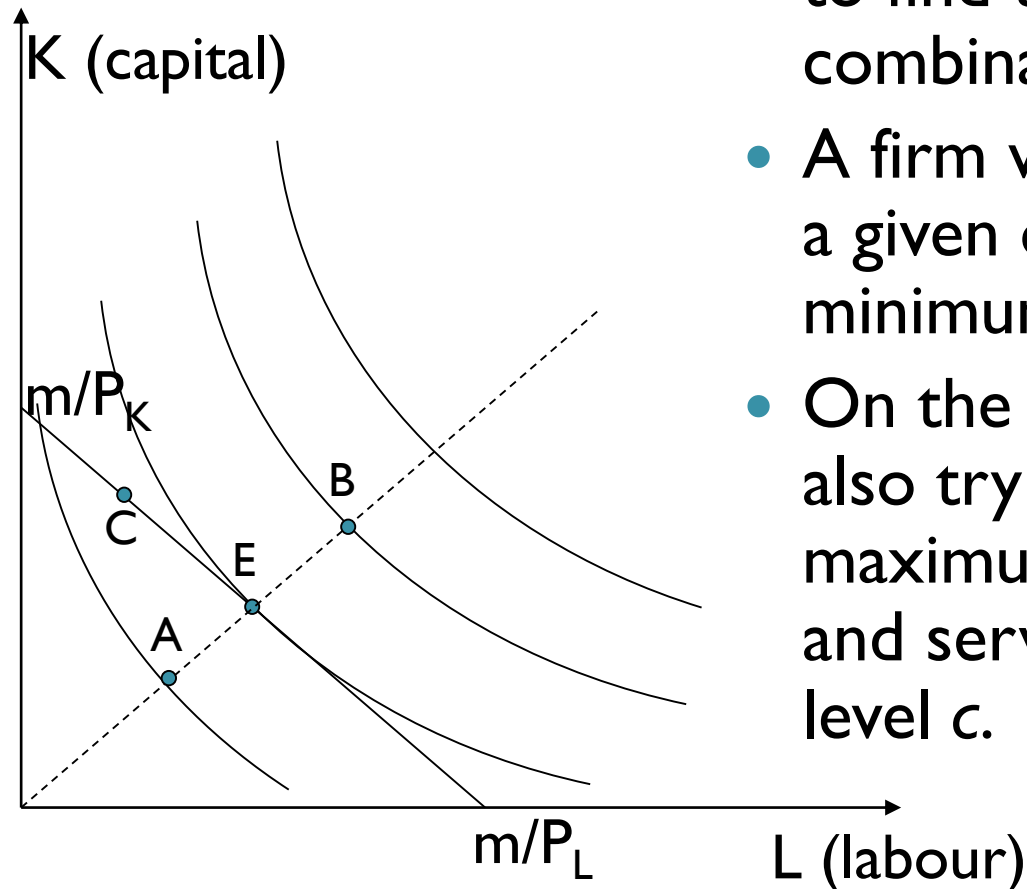
q – Quantity of products



The time horizon: Long run  
– the firm is free to choose  
the quantity of all factors of  
production

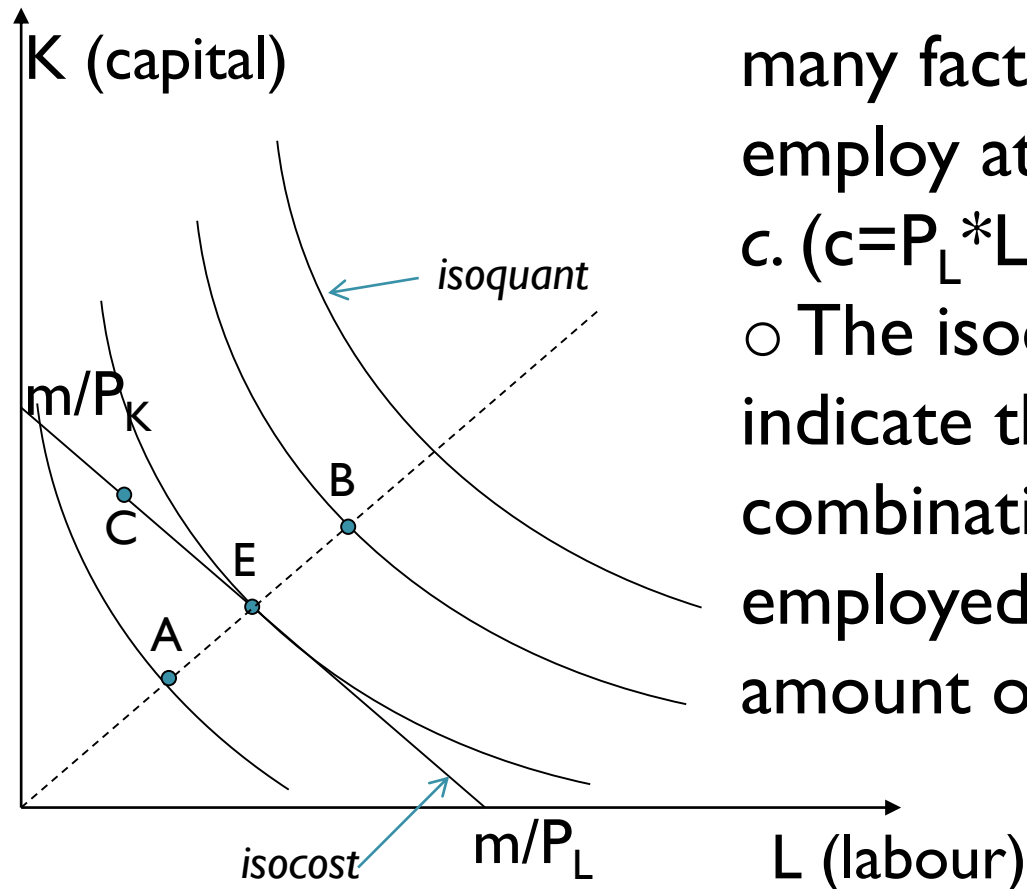
Short run – at least one of  
these factors is fixed

# The optimal combination of factors of production



- In the long run, the firm tries to find the optimal combination of factors:
- A firm would try to produce a given quantity „ $q$ ” at the minimum cost ( $LTC(q)$ ).
- On the other hand, it would also try to produce the maximum amount of goods and services at a given cost level  $c$ .

# The optimal combination of factors of production II.



- Isocost lines show how many factors the firm can employ at a given cost level  $c$ . ( $c = P_L * L + P_K * K$ )
- The isoquant curves indicate the factor combinations that can be employed to produce a given amount of goods.

# The optimal combination of factors of production III.

A necessary condition for optimality:

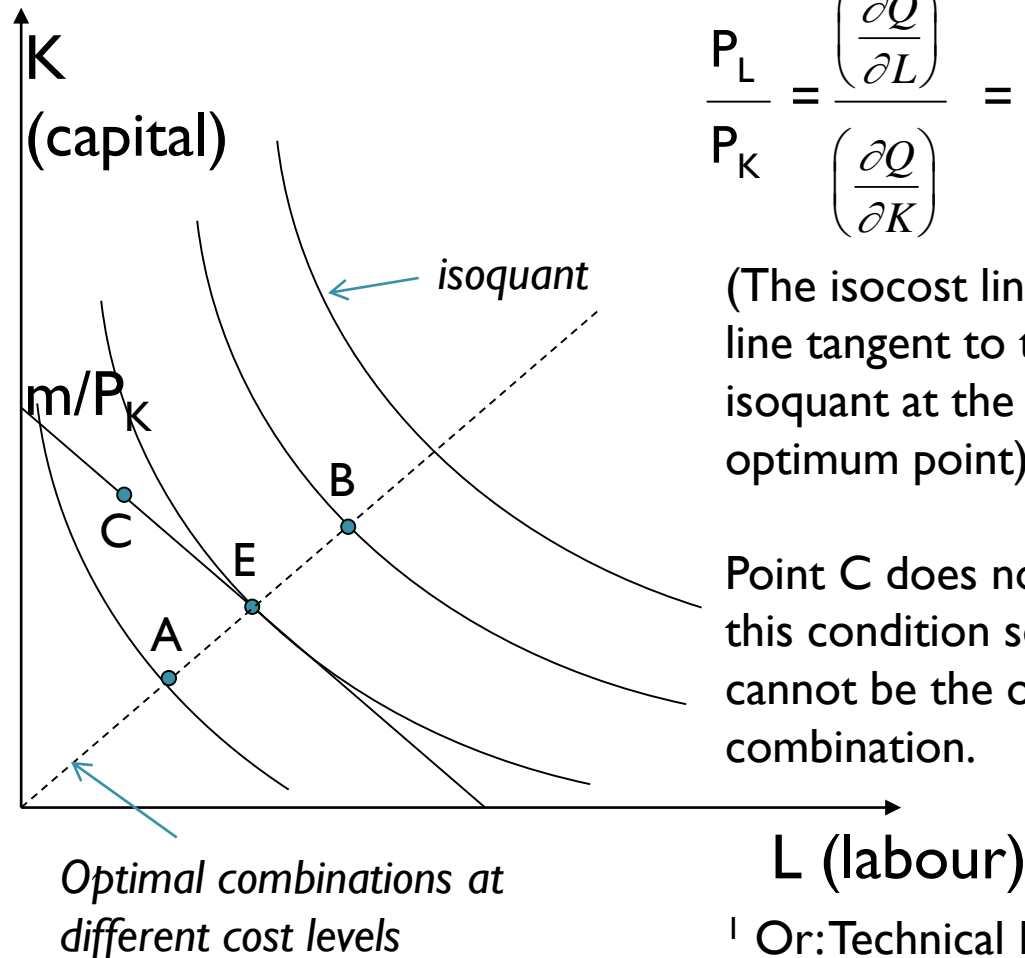
$$\frac{P_L}{P_K} = \frac{\left(\frac{\partial Q}{\partial L}\right)}{\left(\frac{\partial Q}{\partial K}\right)} = \text{MRTS}^*$$

(The isocost line is the line tangent to the isoquant at the optimum point)

Point C does not satisfy this condition so it cannot be the optimal combination.

\*MRTS: the Marginal Rate of Technical Substitution

The optimum point is at „E”



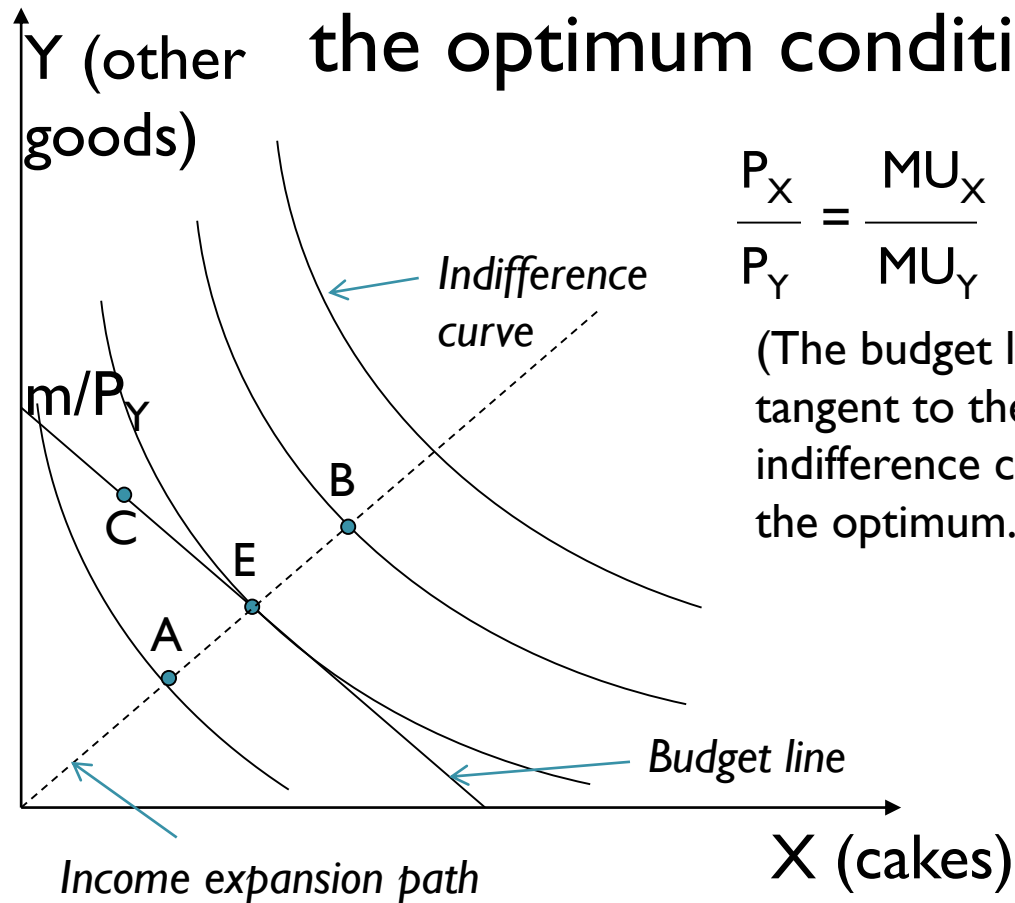
† Or: Technical Rate of Substitution (TRS)



# Consumer behaviour

Reminder

The optimum condition is similar to the optimum condition of the firm:



$$\frac{P_X}{P_Y} = \frac{MU_X}{MU_Y} = MRS^*$$

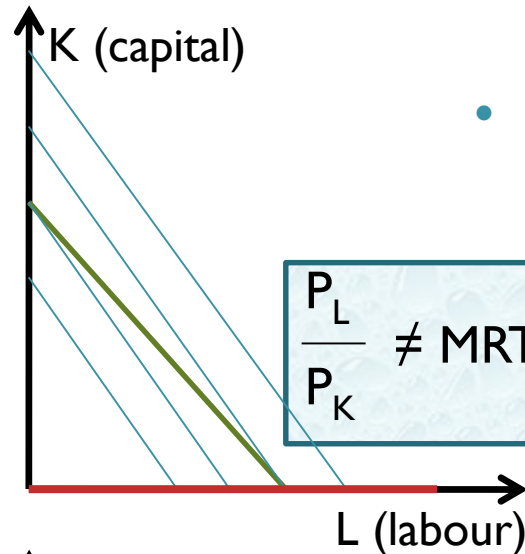
(The budget line is a tangent to the indifference curve at the optimum.)

**MRS:**  
Marginal Rate  
of Substitution

The optimum  
point is at "E"

# “Special technologies”

- isocost
- isoquant
- growth path

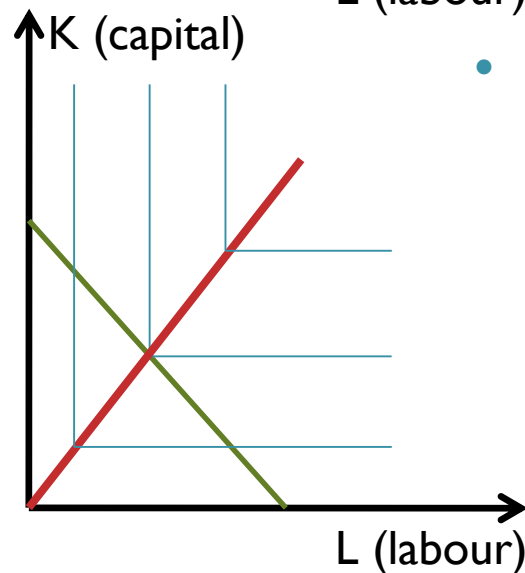


- The factors of production are perfect substitutes  $\rightarrow$  the firm would only use one input type (here: labour)

$$Q = \alpha L + \beta K$$

if  $\alpha/\beta > p_L/p_K$ , the firm would only use labour;

if  $\alpha/\beta < p_L/p_K$ , the firm would only use capital.



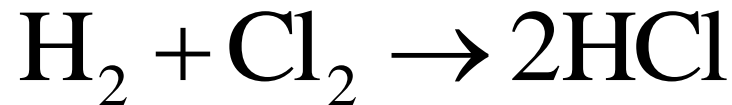
- Leontief production function: The factors of production are perfect complements  $\rightarrow$  the factors must be used in fixed proportions

$$Q = \min\{\alpha L; \beta K\}$$

if the opt. condition  $L = \beta/\alpha K$  is not satisfied, the company is wasting its resources.

# Leontief production function

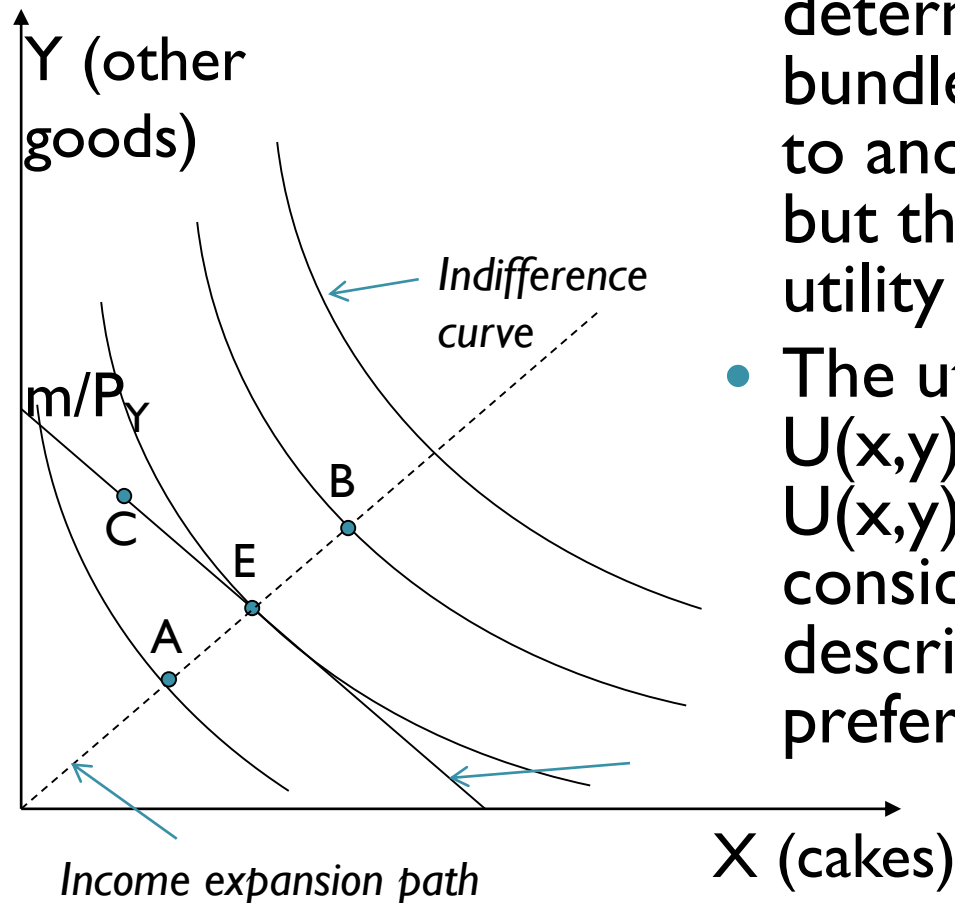
- This type of production function is characteristic of the chemical industry and chemical reactions
- Example: The direct synthesis of hydrogen chloride using hydrogen and pure chlorine gas



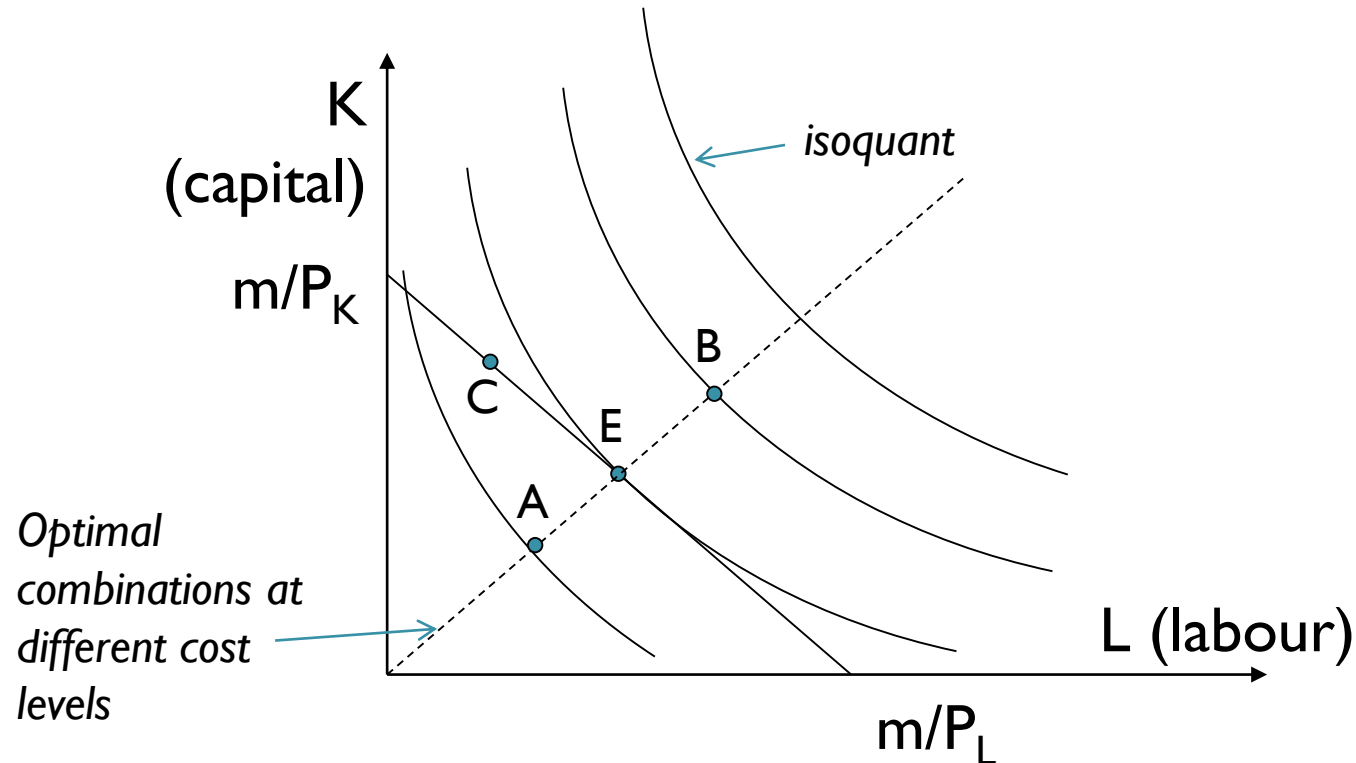
- Prod. func.:  $Q_{\text{HCl}} = 2 \min\{\text{Cl}_2; \text{H}_2\}$
- The inputs of this production function are hydrogen and chlorine gas, the output is hydrogen chloride (but: several other inputs might be needed, e.g. a factory, facilities (capital), chemists/technicians (labour)).



# Differences between utility functions and production functions



- According to the theory of ordinal utility, we can only determine whether one bundle of goods is preferred to another (by a consumer), but the nominal value of utility has no significance.
- The utility functions  $U(x,y)=xy$ ;  $U(x,y)=(xy)^2$  and  $U(x,y)=\ln(x)+\ln(y)$  are considered equivalent, describing the same preference map.



- On the other hand the isoquants of production represent the quantity of goods produced by the firm. Production functions  $Q(L,K)=(KL)^{0.5}$  and  $Q(L,K)=KL$  are not considered equivalent, the technology represented by the latter is more efficient and therefore preferred by the firm.

➔ Returns to scale

Thank  
you

