



Deanonymization: Aggregated Data

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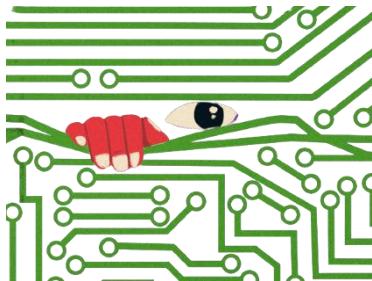
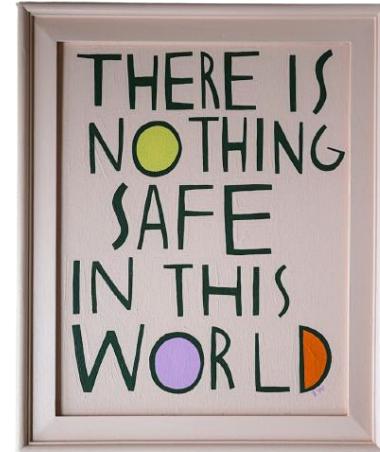
Agenda

- Dark Patterns
- Tracking
- GDPR
- Deidentification
- Machine Learning
- Anonymization
- Cryptography
- Entropy
- Database Reconstruction
- Query Auditing
 - On-line & Off-line
 - Theorems & Attack
- SAT
- Location Data
- Federated Learning

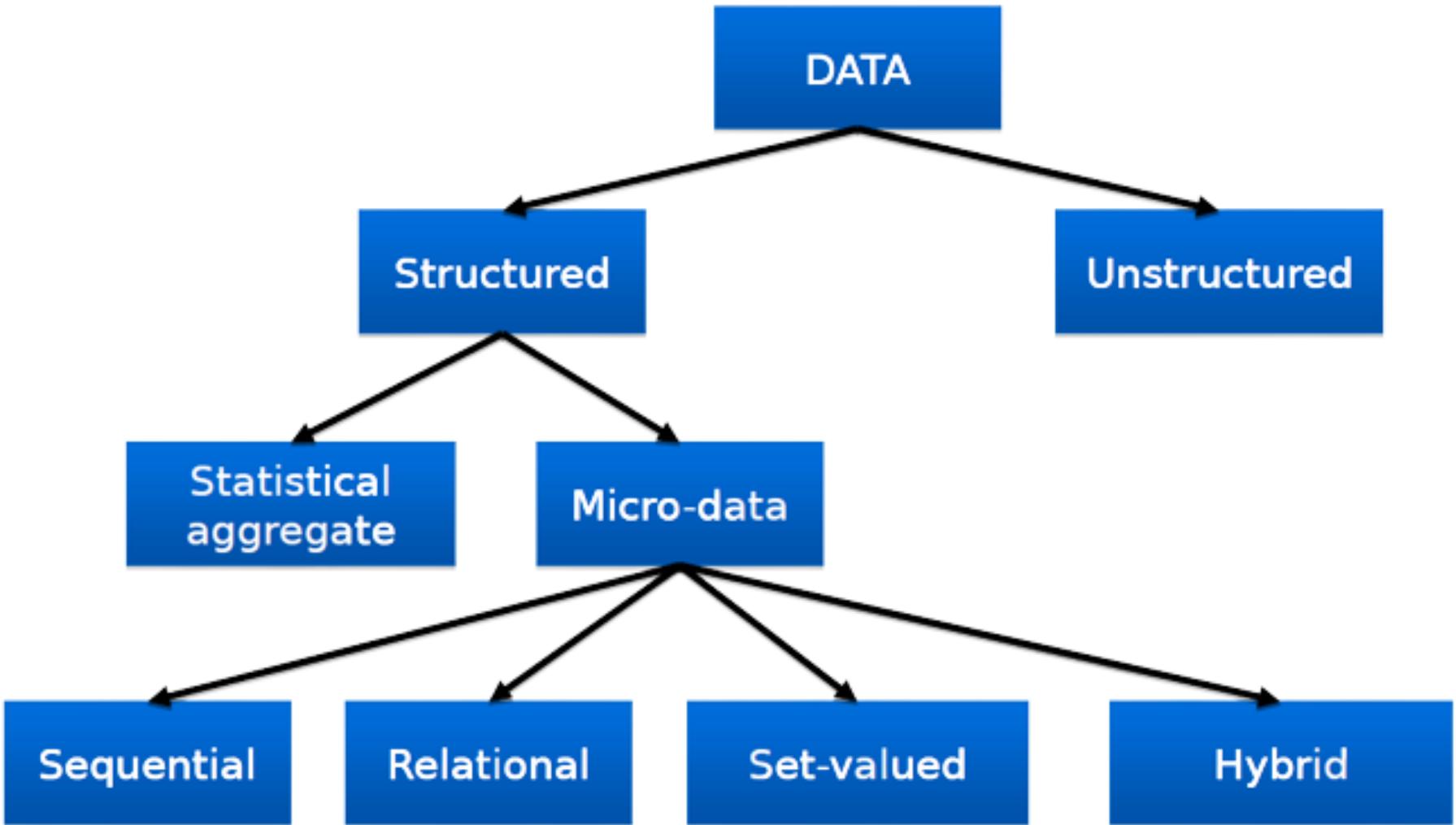


Recap

- All data type can be attacked.
 - Targeted / Untargeted / Mass Attack
- GDPR: if de-anonymization is reasonably possible, the data is personal (even it is encrypted).
 - By linking it to a physical person.
- Systematic (and automatic) de-anonymization is done by machine learning.
 - User is linked to anonymous data with a classifier.
 - One vs One / One vs Rest / All vs All



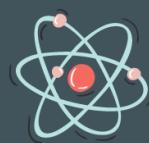
Data Types



Scientific Theory Definition

A scientific theory is an explanation of "why" or "how" that is based on experiments and facts.

big bang theory



atomic theory

theory of relativity

cell theory



theory of evolution

Predictions based on theories are falsifiable. That is, you use the scientific method to test them.

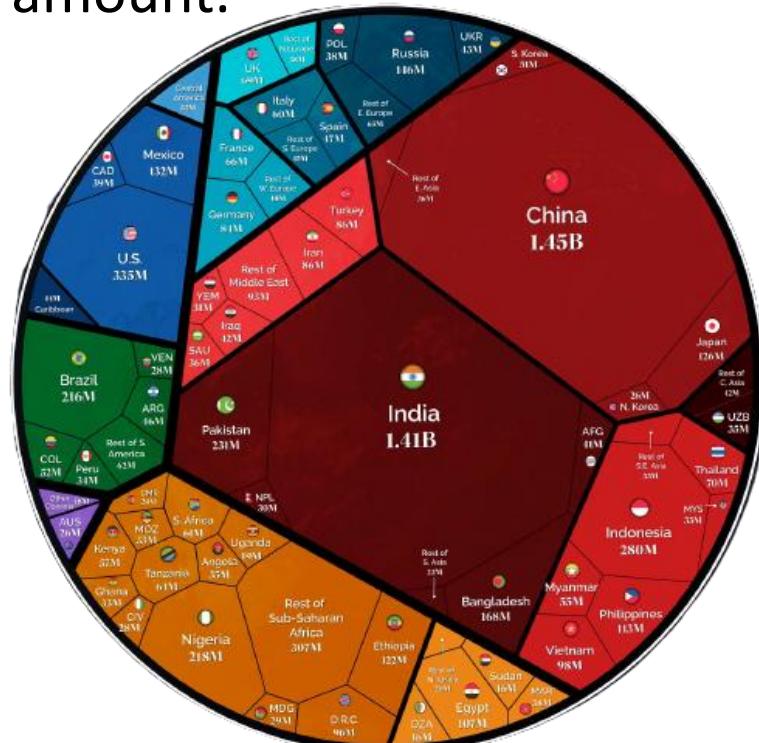
sciencenotes.org

Entropy



Entropy

- There are around 8 billion humans on the planet.
- The identity of a random, unknown person contains just under 33 bits of entropy ($2^{33} \approx 8$ billion).
- When we learn a new fact about a person, that fact reduces the entropy of their identity by a certain amount.
- $\Delta S = -\log_2 \Pr(X=x)$
- ΔS is the reduction in entropy, measured in bits.
- $\Pr(X=x)$ is simply the probability that the fact would be true of a random person.



Example

- Western / Chinese Zodiac Sign: Ram & Goat
 - $\Delta S = -\log_2 \Pr(\text{Starsigns} = \text{Capricorn}) = -\log_2[1/(12 \times 12)] = 7.2$
- City: Szolnok
 - $\Delta S = -\log_2 \Pr(\text{ZIP} = 5000) = -\log_2[70 \text{ thousand} / 8 \text{ billion}] = 16.8$
- Job: Software Engineer
 - $\Delta S = -\log_2 \Pr(\text{Job} = \text{SE}) = -\log_2(30 \text{ million} / 8 \text{ billion}) = 8.0$
- Starsigns & City & Job
(assuming they are independent)
 - $7.2 + 16.8 + 8.0 = 33 \text{ bits}$
- Knowing all three pieces of information,
we can probably say exactly who the person is!
 - By combining several facts, you might learn nothing,
e.g., a starsign does not reduce the entropy if the
birthday is already known.





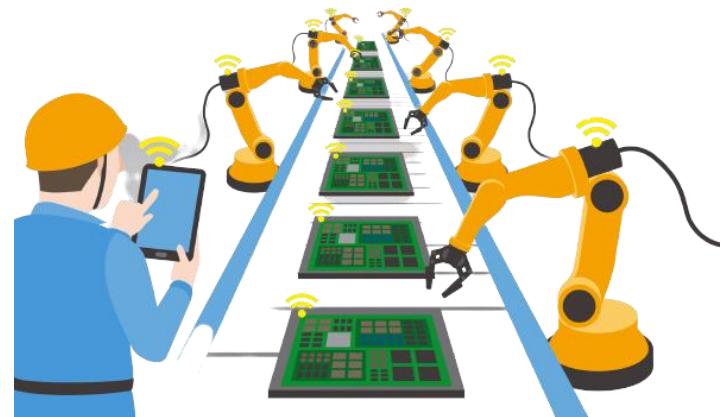
Database Reconstruction



Attack & Naïve Solution

- Ad-hoc anonymization techniques were automated from 1960.
 - Increased ability to analyze & the risk to isolate individuals.
- Attack: if the answer is 1 below, we singled out the victim.
 - ```
SELECT count(*) FROM table WHERE bday = '1990-01-01' AND gender = 'F' AND zip = 12345
```
  - This can lead to further attribute disclosure.

```
SELECT salary, count(*) FROM table WHERE bday = '1990-01-01' AND gender = 'F' AND zip = 12345 GROUP BY salary
```
- *K-threshold mechanism*: a certain number K of individuals must be present in aggregated data for the aggregate to be released.



# Following Attack & Naïve Solution

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- *Intersection Attack*
  - Suppose that there are  $N$  people in the CS department and  $N-1$  of them are men). Let  $K > 2$ .
  - `SELECT salary, count(*) FROM table WHERE dep = 'CS'`
  - `SELECT salary, count(*) FROM table WHERE dep = 'CS' AND gender = 'M'`
- Solution: distorting answers, e.g., by adding noise.



# More Attacks & Naïve Solution

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- *Averaging Attack*
  - If the analyst can make an unlimited number of queries to the database, the added noise can be removed through averaging.
- Defense: use sticky noise.
  - The same query would then produce the same noise.
- *Chaff Attack*
  - The analyst can still average out the noise by generating multiple different queries that all produce the same result, e.g., by adding conditions like  $age < 1000$ .
- And so on ...



```
Trust me, I used a die
rand = 4
```



# U.S. Census History

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- 1790: First census (without any safeguards).
- 1850: ‘Business data’ is not published directly.
- 1920: Manual suppression & compression is used to prevent indirect disclosure of ‘business data’.
  - Extended to ‘people data’ in 1940.
- 1930: Stops publishing small area data.
  - Cannot prevent indirect disclosure.
- 1970: Whole data table suppression about housing.
  - Extended to more tables in 1980.
- 1990: Data swapping, blank & impute protection is added.
  - To enable less whole table suppression.
- 2000: Rounding, top-coding, and more is added as further protection mechanisms.
- 2020: Differential Privacy is replaced all previous ad-hoc protection mechanism.



# Example

- Database is private.
- Statistics are public.
  - NA means not published because the value is < 3.
- Is this safe?
- Can we reconstruct the private dataset from the published statistics?

| Age | Sex | Race | Marital status |
|-----|-----|------|----------------|
| 8   | F   | B    | S              |
| 18  | M   | W    | S              |
| 21  | F   | W    | S              |
| 30  | M   | W    | M              |
| 31  | F   | B    | M              |
| 66  | F   | B    | M              |
| 84  | M   | B    | M              |

**PRIVATE**

| Age              | Count | Median | Mean |
|------------------|-------|--------|------|
| Total population | 7     | 30     | 38   |
| Female           | 4     | 30     | 32   |
| Male             | 3     | 30     | 26   |
| Black            | 4     | 35     | 48.5 |
| White            | 3     | 24     | 24   |
| Single adults    | 5     | NA     | NA   |
| Married adults   | 4     | 51     | 54   |
| Black female     | 3     | 36     | 36.7 |

**PUBLIC**



# Feasibility

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- Private data
  - Each record has 3 binary attributes (3 bits) and one multi-value attribute (age: 0 – 128 -> 7 bits).
  - For 7 records:  $(3 + 7) \cdot 7 = 70$  bits in total.
- Public statistics
  - Median: 7 bits
  - Mean: 7 bits
  - Count: 3 bits
  - Each query is released for 7 groups, which means  $(7+7+3) \cdot 7 = 119$  bits.
- We release 119 bits about a dataset which contains only 70 bits of information, so reconstruction seems possible!

**REDUNDANCY**



# Reconstruction Idea

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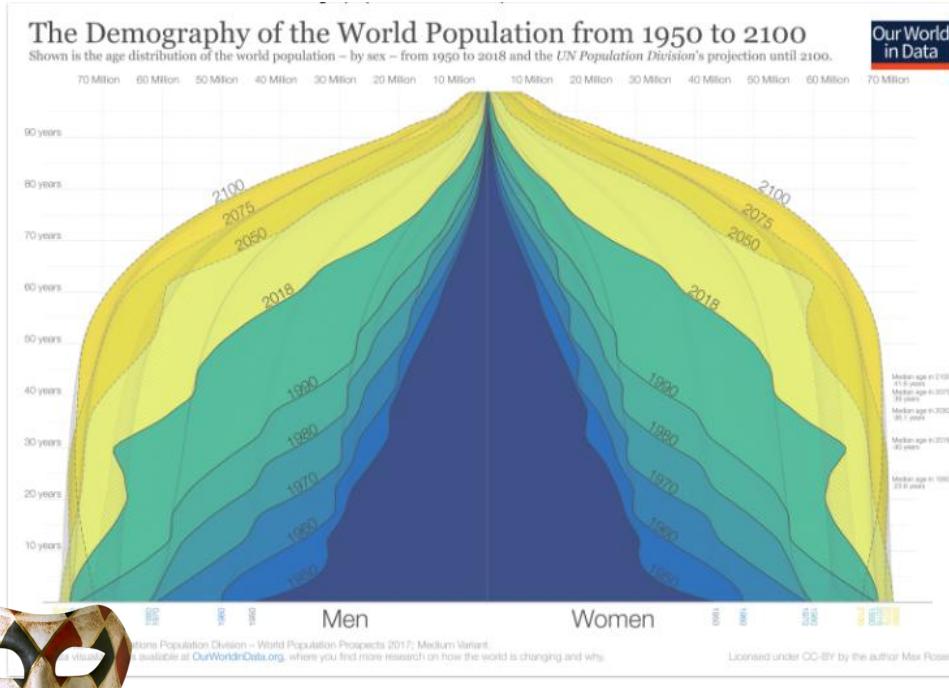
- We know that there are 3 whites in the private dataset.
  - If  $0 < \text{Age} < 128$ , then we have  $128^3 / 3 = 699051$  possible combinations.
- However, out of these, there are only 24 with mean 24 and median 24.
  - Applying the constraints imposed by the released statistics, the possible combinations can be substantially reduced!
- The more constraints (i.e., released statistics) we have the more likely we can find the private attributes!

| Age              | Count | Median | Mean |
|------------------|-------|--------|------|
| Total population | 7     | 30     | 38   |
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| Male             | 3     | 30     | 40   |
| Black            | 4     | 51     | 48.5 |
| White            | 3     | 24     | 24   |
| Single adults    | NA    | NA     | NA   |
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# Background Knowledge

- Instead of selecting an option uniformly from the possible options, the attacker can use other available (population level) statistics to further increase the guessing chance.

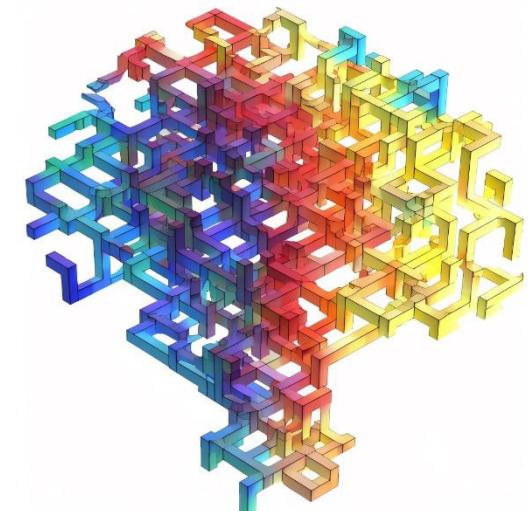


|   | A  | B  | C  | A  | B  | C  | A  | B  | C |
|---|----|----|----|----|----|----|----|----|---|
| 1 | 24 | 47 | 9  | 24 | 39 | 17 | 24 | 31 |   |
| 2 | 24 | 46 | 10 | 24 | 38 | 18 | 24 | 30 |   |
| 3 | 24 | 45 | 11 | 24 | 37 | 19 | 24 | 29 |   |
| 4 | 24 | 44 | 12 | 24 | 36 | 20 | 24 | 28 |   |
| 5 | 24 | 43 | 13 | 24 | 35 | 21 | 24 | 27 |   |
| 6 | 24 | 42 | 14 | 24 | 34 | 22 | 24 | 26 |   |
| 7 | 24 | 41 | 15 | 24 | 33 | 23 | 24 | 25 |   |
| 8 | 24 | 40 | 16 | 24 | 32 | 24 | 24 | 24 |   |

# Principle

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- Too many statistics published too accurately from a confidential database exposes the entire database with near certainty.
- Database can be reconstructed by treating the attributes of the persons as a collection of variables.
  - A set of constraints is then extracted from the published table.
- The database reconstruction finds a set of attributes that are consistent with the constraints.
  - There must be at least one solution because the statistics are known to be computed from a real database.
  - If the statistics are highly constraining, then there will be a single possible reconstruction.



# Problem Formalization

- Mean:  $A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 = 7 \cdot 38$ 
  - Assumed that  $0 < A_i < 128$
- Median:  $A_4 = 30$ 
  - Assumed that  $A_1 \leq A_2 \leq A_3 \leq A_4 \leq A_5 \leq A_6 \leq A_7$
- Count:  $S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 = 4$ 
  - Assumed that female  $\rightarrow 1$  & male  $\rightarrow 0$
- Count:  $R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 = 4$ 
  - Assumed that black  $\rightarrow 1$  & white  $\rightarrow 0$
- Count:  $M_k + M_{k+1} + \dots + M_7 = 3$ 
  - Assumed that single  $\rightarrow 0$  & married  $\rightarrow 1$
  - Assume that  $18 \leq A_k \leq A_{k+1} \leq \dots \leq A_7$
- ...

| Age              | Count | Median | Mean |
|------------------|-------|--------|------|
| Total population | 7     | 30     | 38   |
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| Black            | 4     | 51     | 48.5 |
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| Age | Sex | Race | Marital status |
|-----|-----|------|----------------|
| A1  | S1  | R1   | M1             |
| A2  | S2  | R2   | M2             |
| A3  | S3  | R3   | M3             |
| A4  | S4  | R4   | M4             |
| A5  | S5  | R5   | M5             |
| A5  | S6  | R6   | M6             |
| A7  | S7  | R7   | M7             |

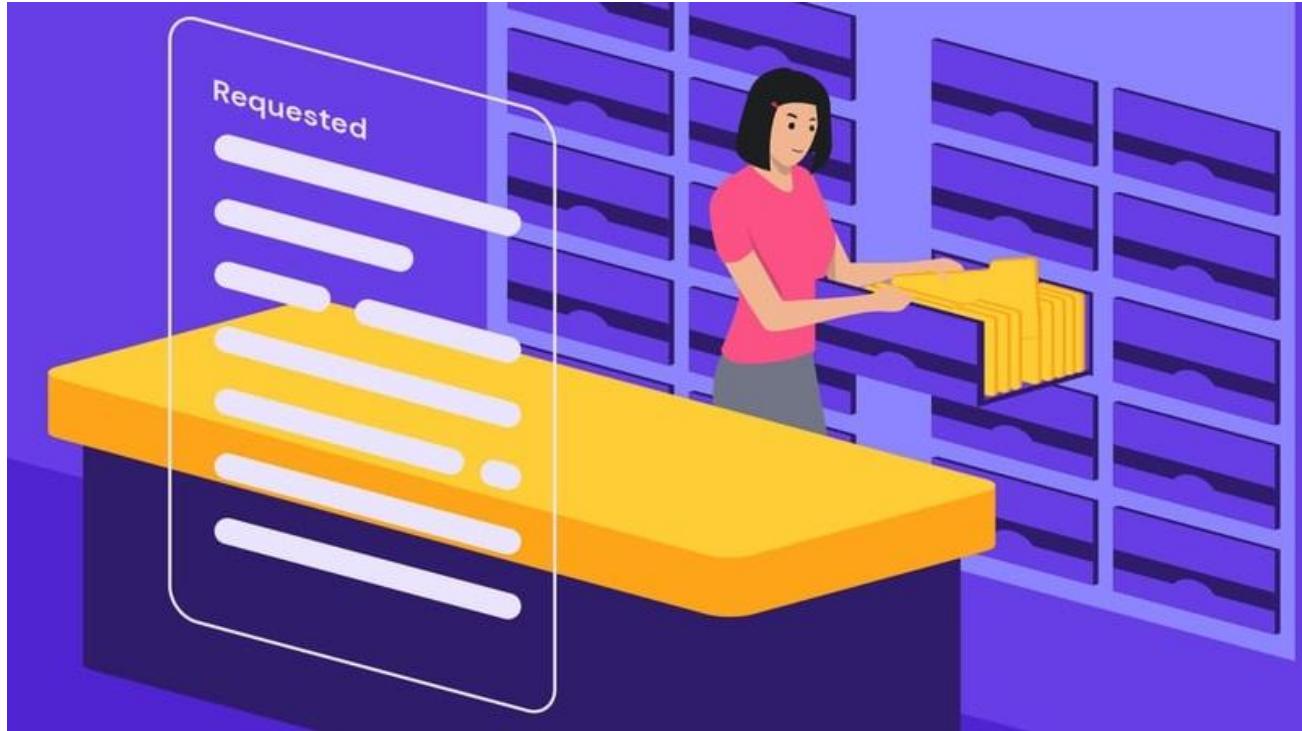


# U.S. Census Attack

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- Solving such a system of constraints is in general NP hard.
  - Using heuristics like SAT solvers and integer programming solvers.
- Same approach was effective against the full 2010 US Census.
- Internally they were able to reconstruct the microdata exactly for 46% of the population.
  - 71% if allowing errors in age of  $\pm 1$  year.
- Linking this with commercial databases allowed them to correctly re-identify over 50 million individuals by name.
- This large-scale reconstruction attacks lead to the adoption of differential privacy for disclosure avoidance in the 2020 U.S. census.
  - To be discussed in detail.





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# Query Auditing



# Problem

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- Objective: given a database with some disclosure policy.
  - Attribute X is private (e.g., diagnosis), but aggregated values of X over different subsets of the records may be available (e.g., SUM, COUNT, MEDIAN, MAX).
  - Detect or prevent violations of the disclosure policy (i.e., disclosure of private information).



- $\text{SUM}(\text{Blood S.})$



- $\text{SUM}(\text{Blood S.})$

WHERE Sex = Female



- $\text{SUM}(\text{Blood S.});$

$\text{SUM}(\text{Blood S.})$



WHERE Sex = Male

| Name     | Sex    | ZIP  | Blood sugar | Diagnosis  |
|----------|--------|------|-------------|------------|
| John S.  | Male   | 1123 | 4.3         | Meningitis |
| John D.  | Male   | 1123 | 5.2         | Crohn      |
| Jerry K. | Male   | 1114 | 6.1         | Alzheimer  |
| Jack. D. | Male   | 8423 | 3.2         | Crohn      |
| Eve A.   | Female | 1234 | 7.1         | Facture    |



# Detection vs Prevention

- Detection aka off-line query auditing
  - The process of examining queries that were answered in the past to determine whether answers to these queries could have been used to obtain confidential information forbidden by the disclosure policy.
- Prevention aka on-line query auditing
  - Examine current query in real-time and deny queries that could potentially cause a breach of privacy.
- Both can be realized by suppressing information.

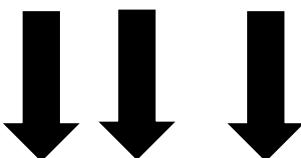


# On-line vs Off-line

Database



1. Answer
2. Answer
- ...
- n. Answer



1. Answer
2. No Reply
- ...
- n. Answer

1. Query
2. Query
- ...
- n. Query

- Query 1
- Query 2
- ...
- Query n



- Answer 1
- Answer 2
- ...
- Answer n

- No Reply
- OR
- Answer 1
- ...
- Answer n



Auditor

1. Query
2. Query
- ...
- n. Query

- Query 1
- Query 2
- ...
- Query n



Querier



# Off-line Auditor for SUM over Reals

- $\text{SUM}(\text{Blood S.}) \text{ WHERE Sex = Male}$ 
  - $x_1 + x_2 + x_3 + x_4 = 18.8$
- $\text{SUM}(\text{Blood S.}) \text{ WHERE ZIP = 1123}$ 
  - $x_1 + x_2 = 9.5$
- $\text{SUM}(\text{Blood S.}) \text{ WHERE ZIP > 1200}$ 
  - $x_4 + x_5 = 10.3$
- Queries can be represented by a binary matrix.
- An auditor essentially needs to solve a system of linear equations.
  - If the matrix is invertible, all  $x_i$ 's are disclosed (complexity  $O(n^3)$ ).
  - Otherwise, the matrix can be diagonalized (complexity  $O(n^3)$ ), where some  $x_i$ 's are disclosed.

| Name     | Sex    | ZIP  | Blood sugar |
|----------|--------|------|-------------|
| John S.  | Male   | 1123 | 4.3         |
| John D.  | Male   | 1123 | 5.2         |
| Jerry K. | Male   | 1114 | 6.1         |
| Jack. D. | Male   | 8423 | 3.2         |
| Eve A.   | Female | 1234 | 7.1         |

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 18.8 \\ 9.5 \\ 10.3 \end{bmatrix}$$



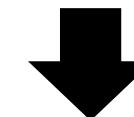
# Off-line Auditor for SUM over Boolean

- $\text{SUM}(\text{HIV}) \text{ WHERE Sex} = \text{Male}$ 
  - $x_1 + x_2 + x_3 + x_4 = 2$
- $\text{SUM}(\text{HIV}) \text{ WHERE ZIP} = 1123$ 
  - $x_1 + x_2 = 1$
- $\text{SUM}(\text{HIV}) \text{ WHERE ZIP} > 1200$ 
  - $x_4 + x_5 = 1$
- Instead of solving a system of linear equations over integers, it is possible to relax the constraints for reals.
  - The final solutions need to be rounded:  
 $x_i = 1$  if  $x_i > \frac{1}{2}$ , otherwise  $x_i=0$ .

| Name     | Sex    | ZIP  | HIV   |
|----------|--------|------|-------|
| John S.  | Male   | 1123 | True  |
| John D.  | Male   | 1123 | False |
| Jerry K. | Male   | 1114 | True  |
| Jack. D. | Male   | 8423 | False |
| Eve A.   | Female | 1234 | True  |

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

~~where  $x_i \in \{0, 1\}$  for all  $i$~~

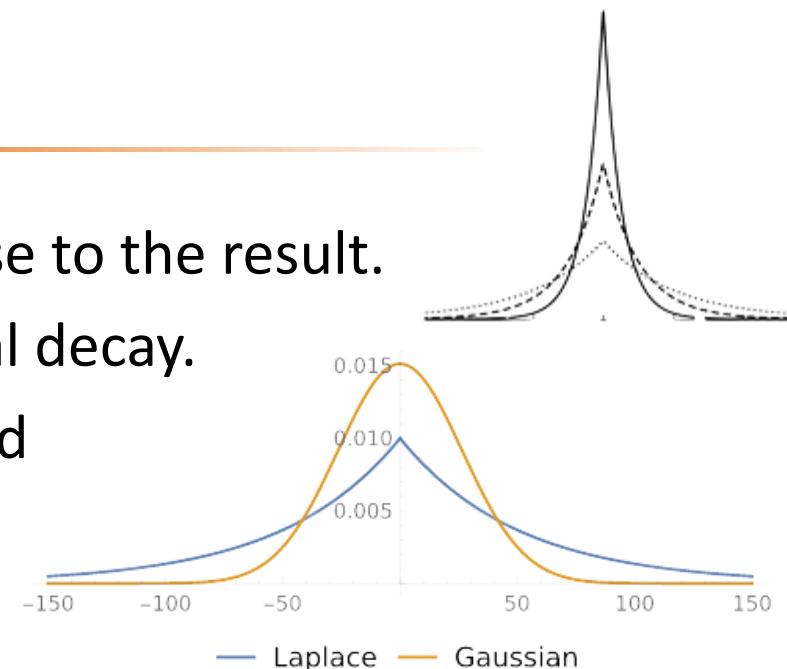


where  $0 \leq x_i \leq 1$  for all  $i$



# Perturbation

- Instead of denying answers, add noise to the result.
- Noise has zero mean and exponential decay.
- The variance of the noise is calibrated to the desired level of privacy.
  - Larger variance  $\rightarrow$  stronger privacy.
  - Smaller variance  $\rightarrow$  weaker privacy.



- Suppose the absolute error caused by the noise is at most  $E$ .
- This can still be solved efficiently with any LP-solver.

$$\begin{bmatrix} 2-E \\ 1-E \\ 1-E \end{bmatrix} \leq \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \leq \begin{bmatrix} 2+E \\ 1+E \\ 1+E \end{bmatrix}$$

where  $0 \leq x_i \leq 1$  for all  $i$



# Model

---

- Dataset D is a collection of n records, i.e.,  $D = \{d_1, d_2, \dots, d_n\}$ .
- Each record consist of some identifying information  $z$  and some secret  $b$ , i.e.,  $d_i = \{z_i, b_i\}$ .
  - Identifying information: ZIP, gender, Bday, etc.
  - Secret: disease, sexual orientation, etc.

|       |       |
|-------|-------|
| $z_1$ | $b_1$ |
| :     | :     |
| $z_n$ | $b_n$ |

- The attacker's goal is to learn some piece of secret information about as many individuals as possible.

- Simplification
  - $b_i$  is binary, i.e.,  $b_i \in \{0,1\}$ .
  - $z_i$  is unique for all  $1 \leq i \leq n$ .
  - Attacker knows  $z_i$  for all  $1 \leq i \leq n$ .



# Dwork and Roth

---

- Fundamental Law of Information Recovery
  - Giving overly accurate answers to too many questions will inevitably destroy privacy.
- The querier can specify queries which are a subset of  $[n]$ .
- Subset query:  $S \in \{0, 1\}^n$  vector with 1 for the indices included in the subset, and 0 otherwise.
  - For instance, “NAME = John S. OR Jack D.”
- The true answer to query  $S$  is  $A(S)$ .
- The auditor’s output is  $r(S)$ .
- $r(S) = A(S)$  could easily lead to privacy violations.
  - Query  $S = \{s_k\}$  where  $s_k=1$  if  $k=i$  and 0 otherwise would reveal the secret bit of individual  $i$ .

| Name     | Sex    | ZIP  | Blood sugar | Diagnosis  |
|----------|--------|------|-------------|------------|
| John S.  | Male   | 1123 | 4.3         | Meningitis |
| John D.  | Male   | 1123 | 5.2         | Crohn      |
| Jerry K. | Male   | 1114 | 6.1         | Alzheimer  |
| Jack. D. | Male   | 8423 | 3.2         | Crohn      |
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# Dinur and Nissim

- An algorithm is blatantly non-private if an adversary can construct a database  $c \in \{0, 1\}^n$  such that it matches the true database  $D$  in all but  $o(n)$  entries.
  - $f(x)=o(g(x))$ : For every constant  $k > 0$ , the inequality  $0 \leq f(x) \leq k \cdot g(x)$  holds asymptotically.  
The error could be arbitrarily small.
- Auditor output a noised version of  $A(S)$ , such that  $|r(S) - A(S)| < E$ .
- If the analyst is allowed to ask  $2^n$  subset queries, and the curator adds noise with some bound  $E$ , then the adversary can reconstruct the database in all but  $4 \cdot E$  positions.
  - Adding 5% error to each answer means 80% reconstruction rate.

what are other words for blatantly?



flagrantly, brazenly, openly, publicly, shamelessly, noisily, vociferously, obtrusively, conspicuously, garishly

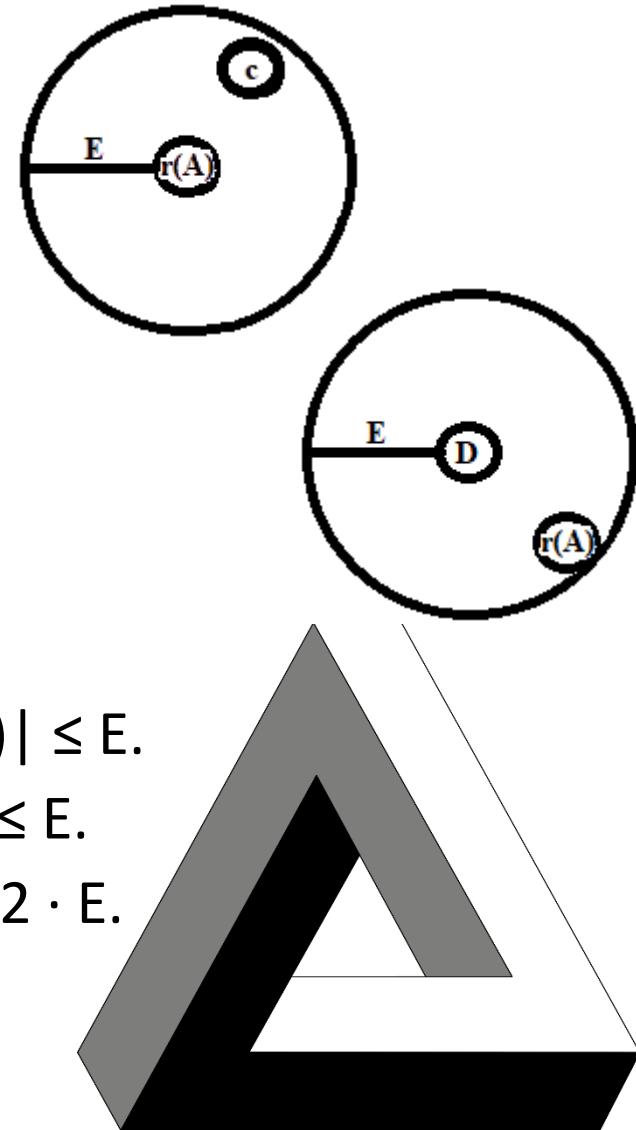


NOISE



# Proof

- The querier asks all  $2^n$  subsets queries.
- For each candidate database  $c \in \{0, 1\}^n$ , if there exists a query set  $S$  such that  $|\sum_S c_i - r(S)| > E$ , then rule out  $c$ , otherwise, output  $c$ .
  - The true database  $D$  satisfies this condition, thus, there will be some output databases.
- Let  $I_0$  be the set of indices where the secret bit is 0, i.e.,  $I_0 = \{i \mid b_i = 0\}$ .
- Due to the adversary's strategy,  $|\sum_{I_0} c_i - r(I_0)| \leq E$ .
- Due to the auditor's strategy,  $|\sum_{I_0} b_i - r(I_0)| \leq E$ .
- Due to the triangle inequality,  $|\sum_{I_0} b_i - c_i| \leq 2 \cdot E$ .
- The same holds for  $I_1$  where  $I_1 = \{i \mid b_i = 1\}$ , consequently  $|\sum_{\{I_0\} \cup \{I_1\}} b_i - c_i| \leq 4 \cdot E$ .



# Stronger Attack

- The attack requires exponentially many queries.
- If the querier is allowed to ask  $O(n)$  random subset queries, and the auditor adds noise with some bound  $E = O(\alpha \cdot \sqrt{n})$ , then a computationally efficient adversary can reconstruct the database in all but  $O(\alpha^2)$  positions.
  - $f(x)=O(g(x))$ : There exists a constant  $k > 0$  such that the inequality  $0 \leq f(x) \leq k \cdot g(x)$  holds asymptotically. The error cannot be arbitrarily high.
- Also holds when the auditor makes a constant fraction of the responses with arbitrary noise magnitudes.
- If the queries are chosen randomly (i.e., each record is covered by a query with probability  $\frac{1}{2}$ ), then  $O(n \cdot \log^2 n)$  queries are needed for almost full disclosure.



# Diffix Attack

- In 2017, Aircloak released Diffix where an unlimited numbers of queries are allowed while preserving user privacy.
  - Provided the first bounty program for anonymized data reidentification. An effective reconstruction attack would be awarded with \$5,000.
- It limit the ability of an adversary to use the full power of SQL to access the database.
- It has a collection of data-dependent ad-hoc methods to prevent leaking information about individuals or very small subsets of users.
- It adds a data-dependent noise term with a constant variance.
- It adds a query-dependent noise term whose variance depends on the complexity of the query.
  - Creating subset queries via accumulated conditions such as “NAME = John S. OR Jack D.” would introduce too much noise.

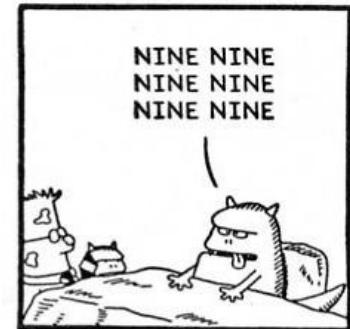
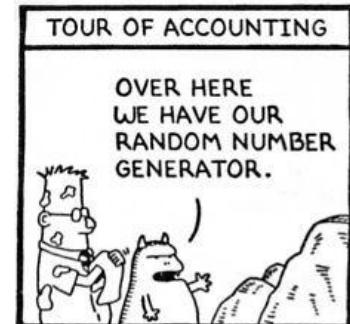


aircloak

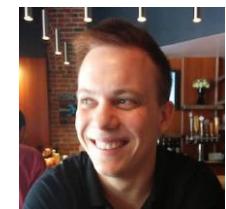


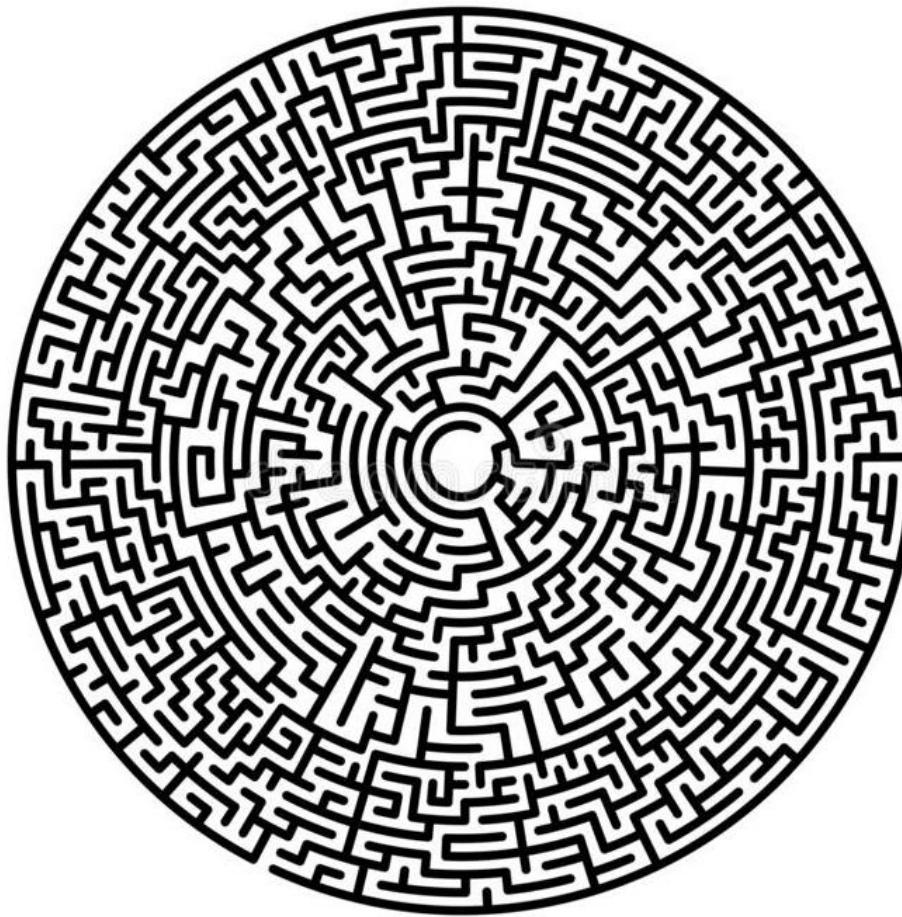
# Aloni & Nissim

- Instead of choosing a subset and then producing conditions to specify it, they created a query involving a low number of conditions which specifies a “random” set.
- Each user in the dataset has a unique client ID.
- They use functions specified by four variables.
  - mult, exp, d, pred
- Does the d-th digit of  $(\text{mult} \cdot \text{client-id})^{\text{exp}}$  satisfy pred?
  - Mult=17, exp=0.5, d=3, pred=Is the digit even?  
Client ID=1  $\rightarrow (17 \cdot 1)^{0.5} = 4.123\dots$ , so it is included  
Client ID=2  $\rightarrow (17 \cdot 2)^{0.5} = 5.830\dots$ , so it is excluded
  - ...



- As a response, Aircloak further restrict the queries allowed by Diffix.
  - Columns like clientId, where most of the values correspond to a single user, are tagged as ‘isolating’, and mathematical functions can no longer be used on such columns.
- The hope was that this modification would prevent the extraction of entropy from an identifying column via hashing.
- Without uniquely identifying column from the database, another way is needed to single out rows.
  - Recap: almost 90 percent of Americans could be identified with only a date of birth, ZIP code, and gender.
- Use the same idea by choosing multiple non-isolating columns which, when taken together, can isolate rows in the database.





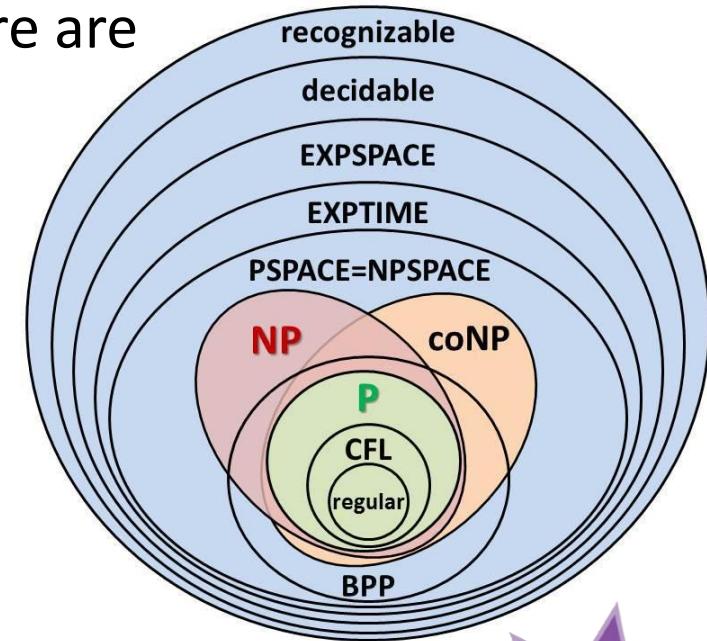
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SAT



# Auditing Results

- If the attribute values are real, then there are efficient polynomial auditors for {SUM}, {MEDIAN}, {AVG}, and {MAX, MIN}.
- If the attribute values are boolean, then the problem for {SUM} is in coNP.
  - If the queries are 1-dimensional, then there is an efficient polynomial auditor.  
1D: the total number of HIV-positive people in various age groups.  
2D: the total number of HIV-positive depending on age and blood sugar.
- There is no polynomial time full-disclosure auditing algorithm for {SUM, MAX} queries unless P=NP.



**DeID 5  
Results**

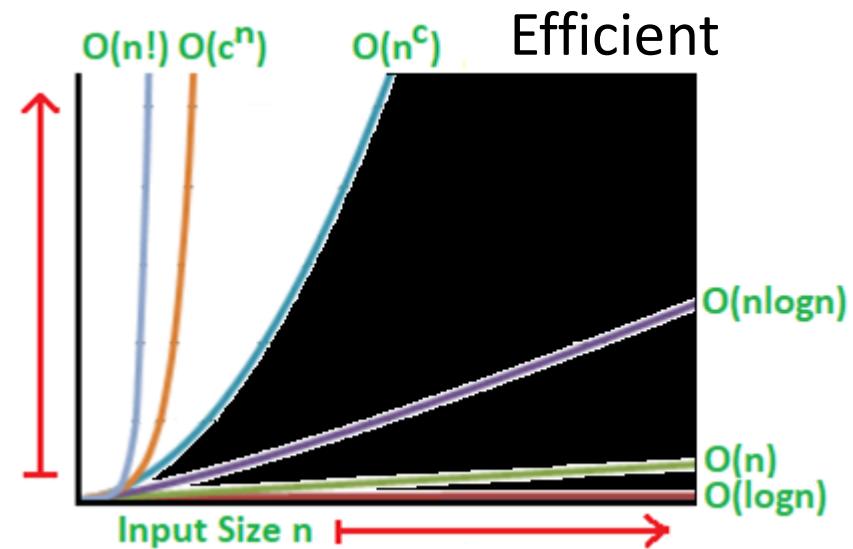


# Complexity

- P: Solvable in polynomial time.
- NP: “YES” answers checkable in polynomial time by a non-deterministic Turing machine.
  - coNP: same with “NO”.
- NP-complete: Every problem in NP could be reduced to it.

- Examples

- P: decide if a number is prime.
  - NP: decide if X edge jumps are enough to visit in a given graph all desired nodes.
  - NP-complete: decide if a logical formula could be evaluated to True.



# Boolean Satisfiability Problem

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- For a given Boolean formula, replacing each variable with either true or false can make the formula evaluate to true?
  - $A \wedge B$  is satisfiable.
  - $A \wedge \neg A$  is unsatisfiable.
- There is no known algorithm that efficiently solves each SAT problem, and it is generally believed that no such algorithm exists (P vs NP).
- However, heuristic SAT-algorithms can handle tens of thousands of variables and formulas consisting of millions of symbols.
- SAT is the first problem that was proven to be NP-complete (Cook-Levin theorem).
  - Many problems can be cast to SAT solving and hence use these highly optimized solvers.



**SAT**





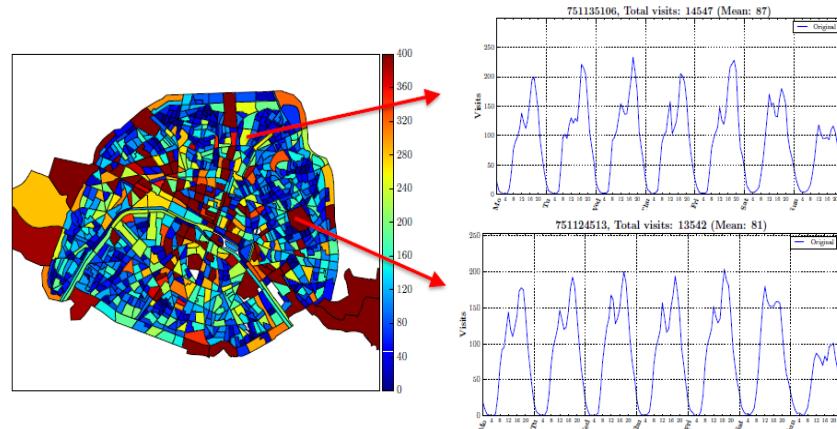
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## Location Data



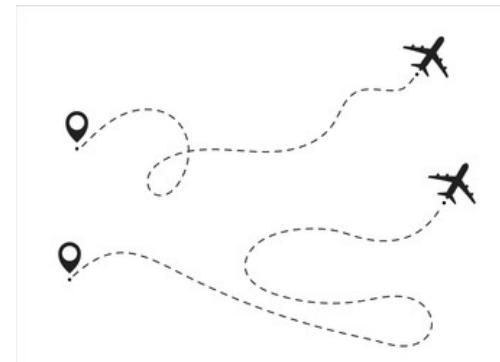
# Membership Test

- Telecom companies wish to publish spatio-temporal densities computed from call detail records (i.e., number of cell phones at each tower depending on the time).
  - To optimize traffic, identify the spots of new services, etc.
- $x = \{1, \dots, 168\}$  denotes the hour of a week.
- $T[x]$  is the number of people who visited the location in slot  $x$ .
- Can this aggregate leak information about a single person?



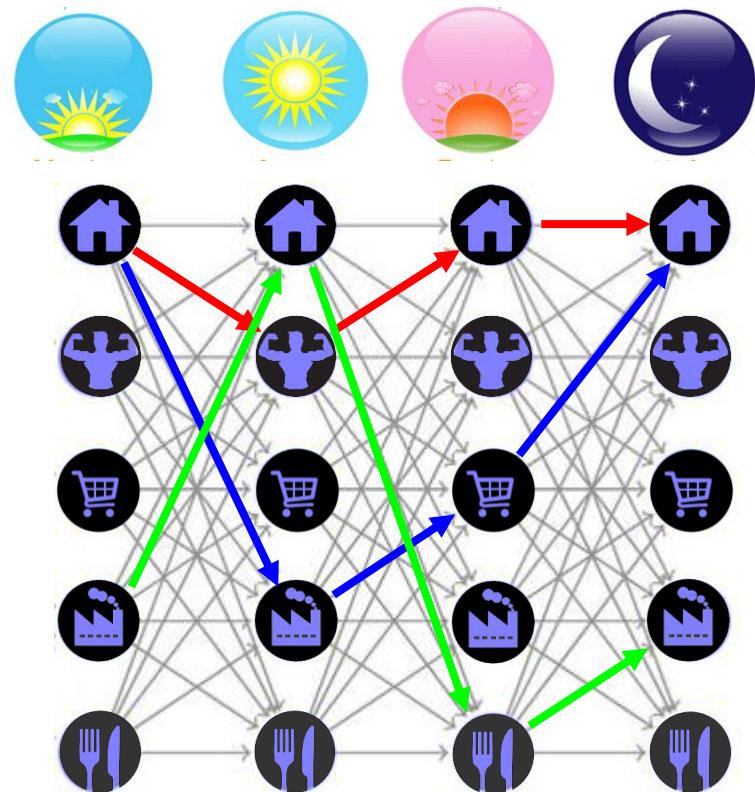
# Location Trajectories

- Knowing only the number of visits per tower in each hour, it is possible to reconstruct all the towers of a person with more than 70% of accuracy.
  - Predictability: the next tower of a person is geographically close to the previous tower.
  - Regularity: every person visits the same (or very similar) towers each day because of daily routines.
  - Uniqueness: 4-5 towers visited by an individual are unique to that individual (there are no other individuals who visited these towers).



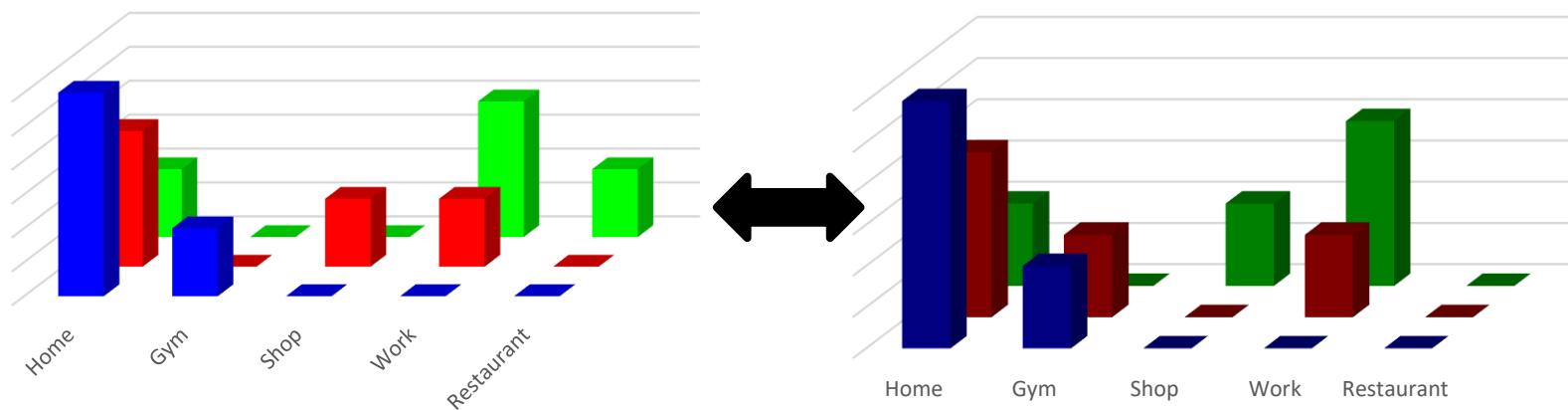
# Trace Reconstruction

- Reconstruct traces within each day independently.
  - Exploit predictability.
- Assign weights between locations following geo-distance.
- Find an optimal assignment of visits between consecutive slots such that
  - Objective: the total weights of the assignment is minimized.
  - Constrain: at each time slot the given number of individuals are present in each location.
- Solving the reconstruction problem recovers the globally most likely moving patterns.



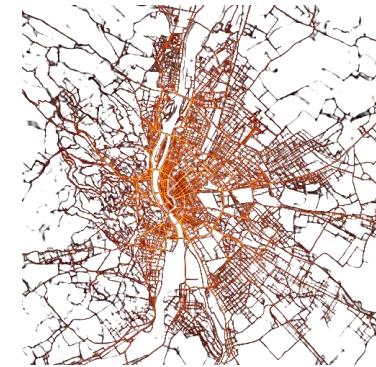
# Linking Traces

- Link traces together across several days.
  - Exploit regularity.
- Restart the reconstruction at the beginning of each day.
- For every reconstructed trace within a day, compute the frequency of towers.
  - Get a distribution of towers per trace.
- Those traces belong to the same person, whose tower distribution is similar (according to some similarity measure).



# Matching Individuals

- Assign individuals to reconstructed traces.
  - Exploit uniqueness of location data.
- Do not know which trace belongs to whom.
- 4 location visits of a person is unique with a chance of 95% within a population of 1.5 million users.
  - If I know 4 locations and the time when (s)he were there, then (s)he is the only such person with these positions.
- Crawl Facebook for locations & time coordinates.
- Find the trace which has these 4 points.



**DeID 6**  
**Example**





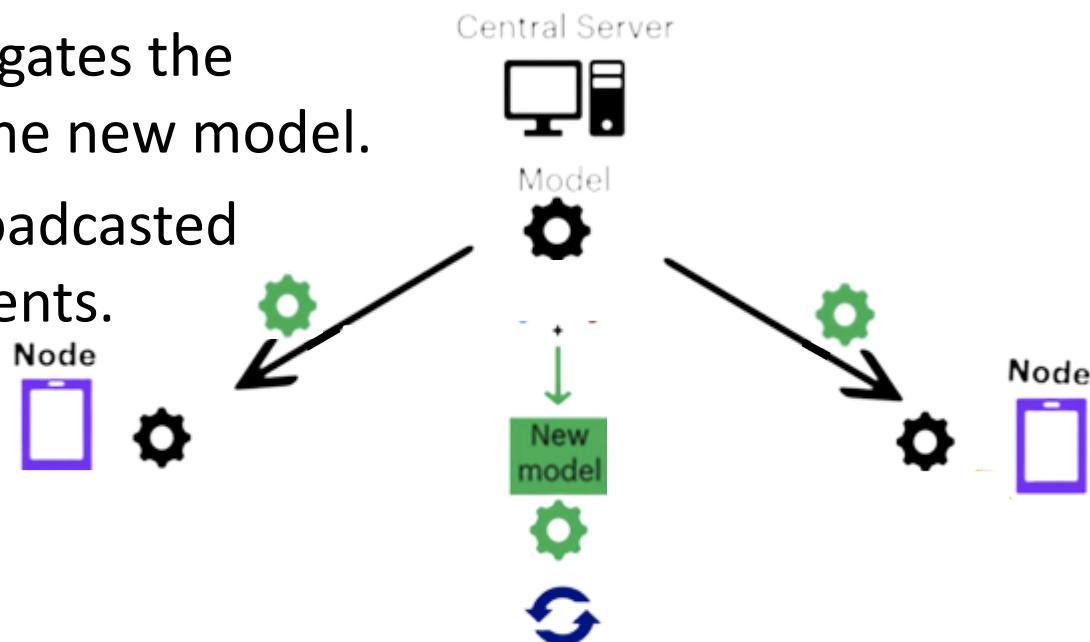
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# Federated Learning



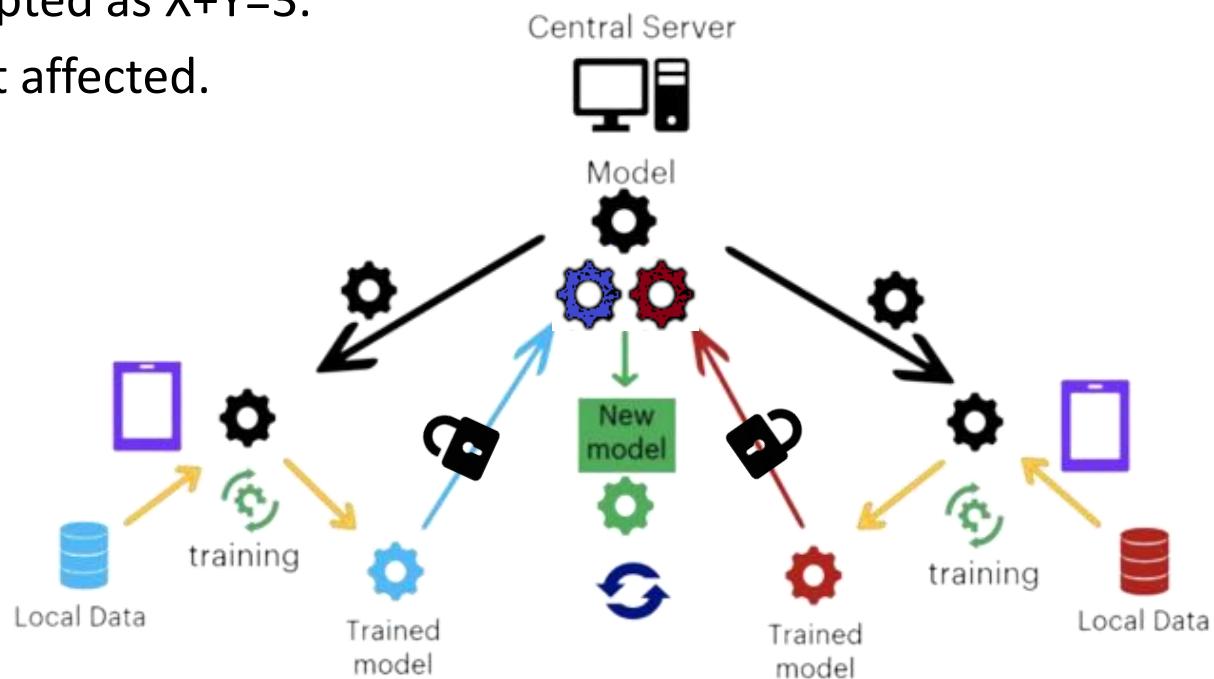
# Federated Learning

- 0) The aggregator server initializes the model,  
i.e., determine the hyperparameters.
- 1) The aggregator broadcast the model to some clients.
- 2) Those clients train that model on their local dataset  
and send the update to the aggregator.
- 3) The aggregator aggregates the  
model updates into the new model.
- 4) The final model is broadcasted  
to all participating clients.
- 5) Repeat from 1)  
until convergence  
is reached.



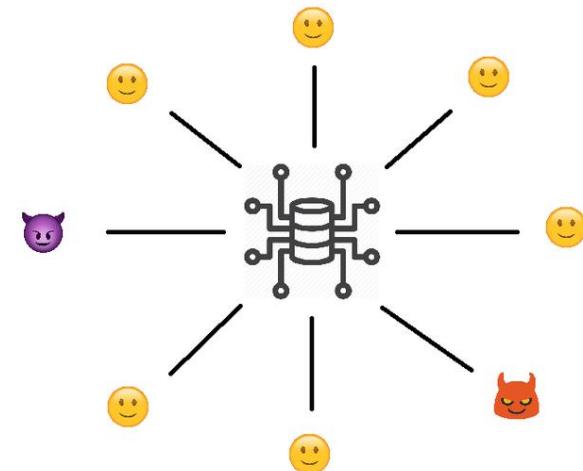
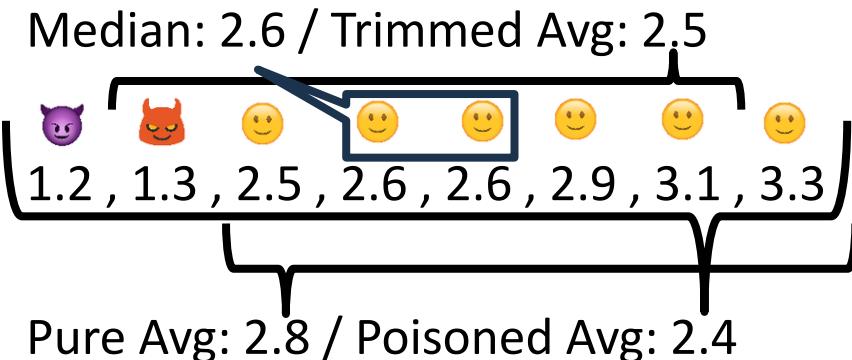
# Secure Aggregation

- In FL the server learns the individual changes; hence, it can get information about underlying sensitive data.
- Secure Aggregation hides individual gradients with masks which cancel out after aggregation.
  - Relies on cryptography.  
E.g.,  $1+2=3$  encrypted as  $X+Y=3$ .
  - Final model is not affected.
- Attribution is not possible without background knowledge.



# Byzantine Resilience

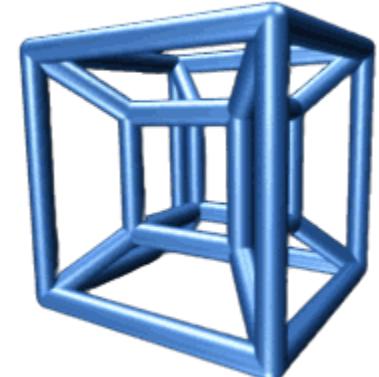
- The ability of the system to withstand malicious behaviors exhibited by participating devices (Byzantine faults).
- It ensuring the integrity of the collective model updates.
- Usually achieved by employing robust algorithms that can tolerate and mitigate the impact of such adversarial actions.
  - Trimmed Mean
  - Median



# Take Away

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- Entropy decreases fast due to the curse of dimensionality.
  - More information decreases the anonymity set.
- Too many too accurate statistics could expose the entire database.
  - Reconstruction attacks are possible not just theoretically.
  - Possible defense is suppression or noise injection.
- Query auditing can be on-line (prevention) or off-line (detection).
  - For non-linear or arbitrary queries, one can use SAT solvers.
- A practical attack may recover exact location data using regularity, predictability, and uniqueness.
- Federated Learning is also vulnerable, Secure Aggregation and Byzantine Resilience are possible privacy and security defense mechanisms.



# Control Questions

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- What can you measure with entropy in relation to database reconstruction? What is the corresponding formula?
- What is the Fundamental Law of Information Recovery and when is an algorithm Blatantly Non-private?
- What are Federated Learning, Secure Aggregation, and Byzantine Resilience?



# References

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- [Tweetorial: Reconstruction-abetted re-identification attacks and other traditional vulnerabilities](#)
- [The algorithmic foundations of differential privacy](#)
- [Revealing information while preserving privacy](#)
- [Linear program reconstruction in practice](#)
- [A survey of Query auditing techniques for data privacy](#)
- [Knock Knock, Who's There? Membership Inference on Aggregate Location Data](#)
- [Where You Are Is Who You Are: User Identification by Matching Statistics](#)
- [Algorithms for Private Data Analysis: Reconstruction Attacks](#)
- [Diffix Cedar bounty prize](#)

