### 11.1. Метод дифференциальной эволюции

Программно реализуйте метод дифференциальной эволюции и найдите с его помощью минимум заданной функции для всех указанных значений размерности $D$. Исследуйте влияние параметров на работу алгоритма. В отчет включить следующие данные:

1. Полную постановку задачи.
2. Исходный код программы.
3. График целевой функции для $D = 2$.
4. Наилучшее найденное оптимальное значение целевой функции и точка, в которой оно достигается.
5. Графики сходимости метода (по оси абсцисс — номер поколения, по оси ординат — текущее оптимальное значение погрешности, т. е. модуль разности точного и приближенного значения минимума). Ось ординат должна быть *логарифмической*! На одной плоскости изобразить вместе графики для трех «прогонов» алгоритма.
6. Среднее время работы алгоритма.
7. Для размерности $D = 2$ изобразить на плоскости линии уровня целевой функции и все точки популяции для четырех-шести различных поколений. Номера поколений выбрать таким образом, чтобы была видна сходимость метода.
8. Ответы на следующие вопросы:

   (a) Как зависит скорость сходимости от количества особей в популяции $N$?

   (b) Как зависит скорость сходимости от константы дифференцирования $F$?

   (c) Как зависит скорость сходимости от константы скрещивания $C$?

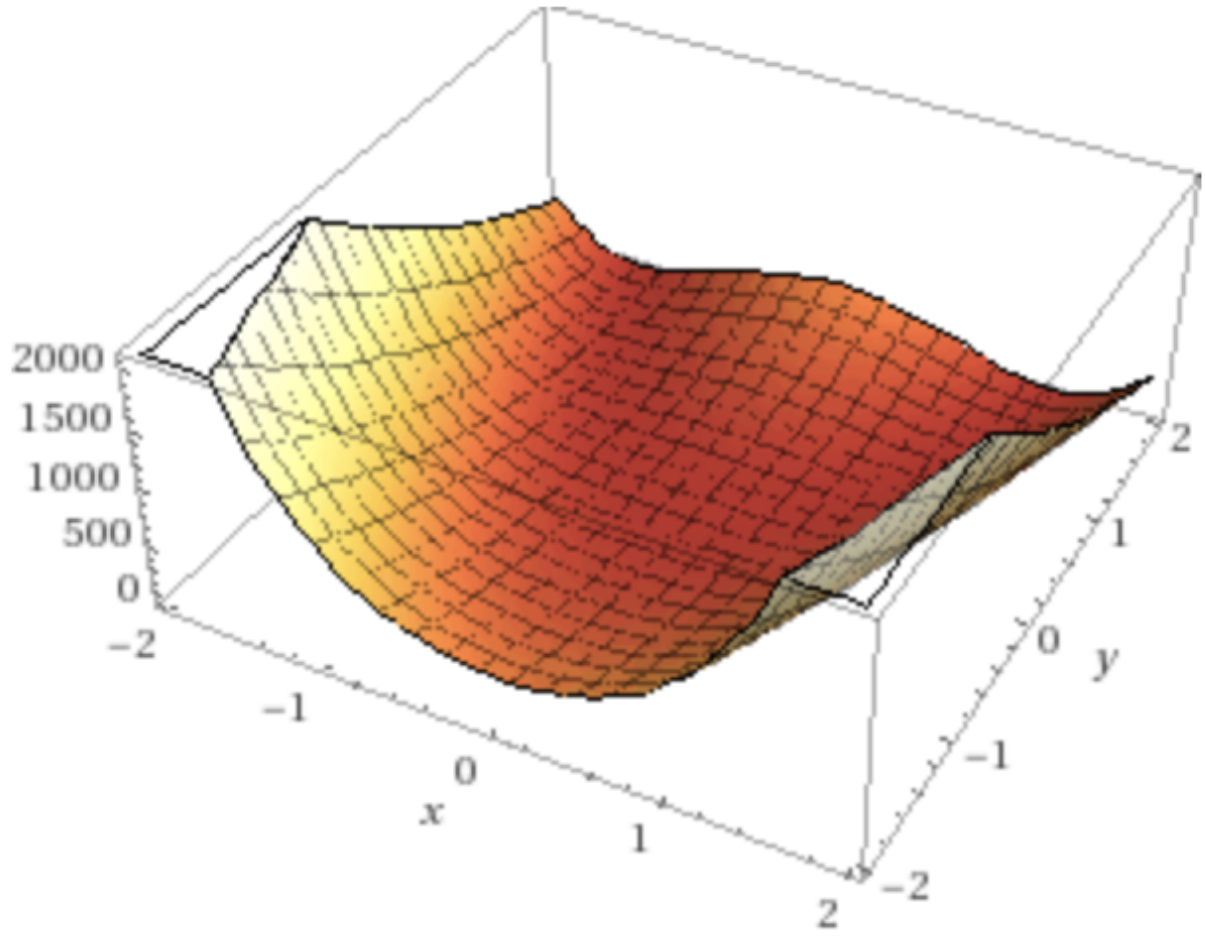
   (d) Какие значения параметров вам кажутся оптимальными?

   *Подтвердите свои выводы экспериментальными данными!*

**Варианты целевых функций**

**11.1.1.** Функция Розенброка. $f(x^*) = 0, \quad x^* = (1, 1, \ldots, 1)^T$.

$$f(x) = \sum_{i=1}^{D-1} (100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2), \quad x_i \in [-2, 2] \quad D = 2, 10, 100.$$


2000
1500
1000
500
0
−2
−1
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1
2
x
2
1
0
−1
−2
y

In [1]:
```
import numpy as np
D = np.array([2, 10, 100], dtype=np.int)
limits = [-2.0, 2.0]
N_test = np.linspace(5, 25, 5, dtype=np.int)
C_test = np.linspace(0.1, 0.9, 5)
F_test = np.linspace(0.4, 1.3, 5)
```

In [30]:
```
def purpose_function(x):
    x_no_last = x[:-1]
    x_no_first = x[1:]
    return np.sum(100 * (x_no_last ** 2 - x_no_first) ** 2 + (1 - x_no_last) ** 2)


def purpose_function_population(population):
    N = population.shape[0]
    return np.min([purpose_function(population[i]) for i in range(N)])


def opt_plan(population):
    N = population.shape[0]
    opt_plan_index = np.argmax([purpose_function(population[i]) for i in range(N)])
    return population[opt_plan_index]


def get_population(D, N):
    result = np.random.uniform(low=limits[0], high=limits[1], size=(1,D))
    for i in range(N-1):
        item = np.random.uniform(low=limits[0], high=limits[1], size=(1,D))
        result = np.concatenate((result, item), axis=0)
    return result


def in_limits(x, limits):
    return np.all(np.logical_and(x <= limits[1], x >= limits[0]))
```

```python
def get_test_item(population, F, limits):
    size = population.shape[0]
    D = population.shape[1]
    indexes = np.random.randint(low=0, high=size, size=3)
    while(len(set(indexes)) != 3):
        indexes = np.random.randint(low=0, high=size, size=3)
    sample = population[indexes]
    beta = sample[2]
    delta = sample[1] - sample[0]
    omega = beta + F * delta
    if not in_limits(omega, limits):
        omega = get_population(D, 1).reshape((D, ))
    return omega


def cross(x, omega, C):
    mask = np.random.uniform(size=x.shape[0]) >= C
    return omega * mask + x * (1 - mask)


def evolution_step(population, F, C, limits):
    size = population.shape[0]
    for i in range(size):
        omega = get_test_item(population, F, limits)
        omega = cross(x=population[i], omega=omega, C=C)
        if purpose_function(omega) < purpose_function(population[i]):
            population[i] = omega


def full_process(D, N, F, C, limits, kNoBetter, log_populations=False):
    population = get_population(D, N)
    cur_purpose_function_value = purpose_function_population(population)
    log = [cur_purpose_function_value, ]
    log_populations = [population.copy(), ]
    cur_no_better = 0
    while cur_no_better < kNoBetter:
        evolution_step(population, F, C, limits)
        next_purpose_function_value = purpose_function_population(population)
        if cur_purpose_function_value <= next_purpose_function_value:
            cur_no_better += 1
        else:
            cur_purpose_function_value = next_purpose_function_value
            cur_no_better = 0
        if log_populations:
            log_populations.append(population.copy())
        log.append(cur_purpose_function_value)
        if len(log) > 10000:
            break
    return population, log, log_populations
```

```
def mega_test(D_test, N_test, F_test, C_test, limits, kNoBetter, log_populations=False, verbose=True):
    log = []
    for D in D_test:
        for N in N_test:
            for F in F_test:
                for C in C_test:
                    pop, logs, _ = full_process(D, N * D, F, C, limits, kNoBetter, log_populations)
                    log.append([D, N * D, F, C, opt_plan(pop), logs[-1], len(logs)])
                    if verbose:
                        print(log[-1])
    return log
```

In [7]:
```
def get_best_params(log):
    opt_value = np.min([log[i][5] for i in range(len(log))])
    opt_indexes = [i for i in range(len(log)) if log[i][5] == opt_value]
    min_iter_count = np.min([log[i][6] * log[i][1] for i in opt_indexes])
    index = [i for i in opt_indexes if log[i][6] * log[i][1] == min_iter_count][0]
    return log[index][:4]
```

In [8]:
```
log2 = log[:125]
log10 = log[125:250]
log100 = log[250:]
```

In [9]:
```
best_params = [get_best_params(log2), get_best_params(log10), get_best_params(log100)]
```

In [10]:
```
best_params
```

Out[10]:
```
[[2, 10, 0.8500000000000001, 0.1], [10, 50, 0.625, 0.1], [100, 2000, 0.4, 0.9]]
```

In [11]:
```
for i in range(len(log2)):
    print(log2[i][:4] + log2[i][5:])
```

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```
import time
kTimes = 10
start = time.time()
for i in range(kTimes):
    bp_proc2, bp_log2, _ = full_process(best_params[0][0], best_params[0][1], best_params[0][2],
                                        best_params[0][3], limits, 50)
print('D=2 mean work time: {} seconds.'.format((time.time() - start) / kTimes))
```

```
D=2 mean work time: 0.19593801498413085 seconds.
```

```
start = time.time()
kTimes = 5
for i in range(kTimes):
    bp_proc10, bp_log10, _ = full_process(best_params[1][0], best_params[1][1], best_params[1][2],
                                 best_params[1][3], limits, 100)
print('D=10 mean work time: {} seconds.'.format((time.time() - start) / kTimes))
```

```
D=10 mean work time: 6.585433387756348 seconds.
```

```
kTimes = 3
start = time.time()
for i in range(kTimes):
    bp_proc100, bp_log100, _ = full_process(best_params[2][0], best_params[2][1], best_params[2][2],
                                            best_params[2][3], limits, 300)
print('D=100 mean work time: {} seconds.'.format((time.time() - start) / kTimes))
```

```
D=100 mean work time: 875.7185890674591 seconds.
```

In [34]:
```
bp_log2[-1], bp_log10[-1], bp_log100[-1]
```

Out[34]:
```
(0.0, 0.0, 93.82838291139522)
```

In [42]:
```
print(opt_plan(bp_proc2))
```

```
[1. 1.]
```

In [43]:
```
print(opt_plan(bp_proc10))
```

```
[1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

In [44]:
```
print(opt_plan(bp_proc100))
```

```
[6.52412292e-01 4.32555678e-01 1.95869173e-01 4.68087712e-02
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 1.02084364e-02 1.02084230e-02 1.02084199e-02 1.02084229e-02
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 1.30399898e+00 1.27346240e+00 1.34715724e+00 1.11673598e+00
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```

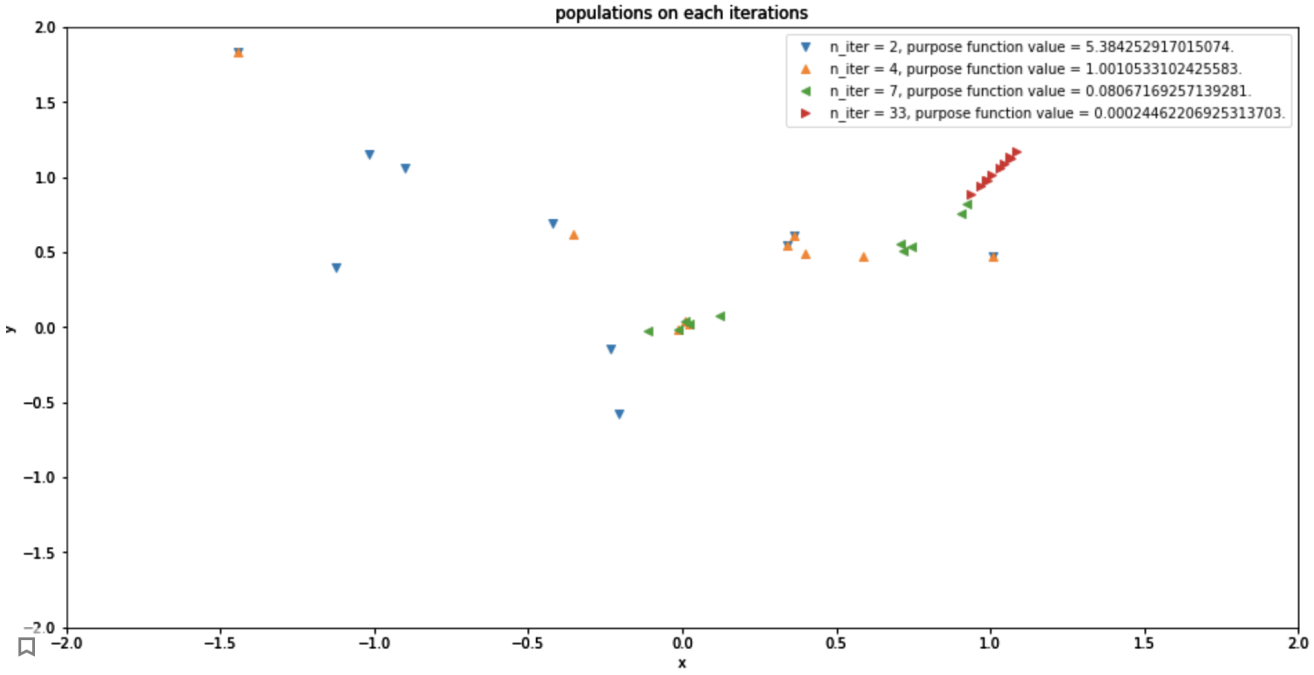
populations on each iterations
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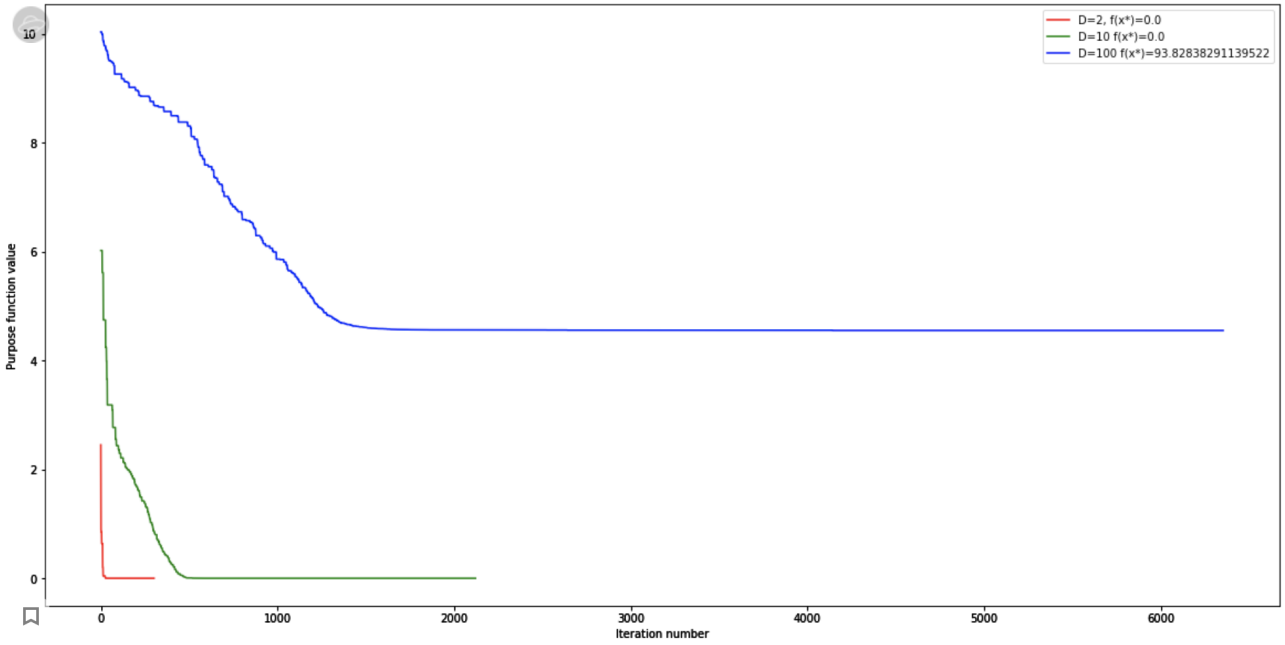
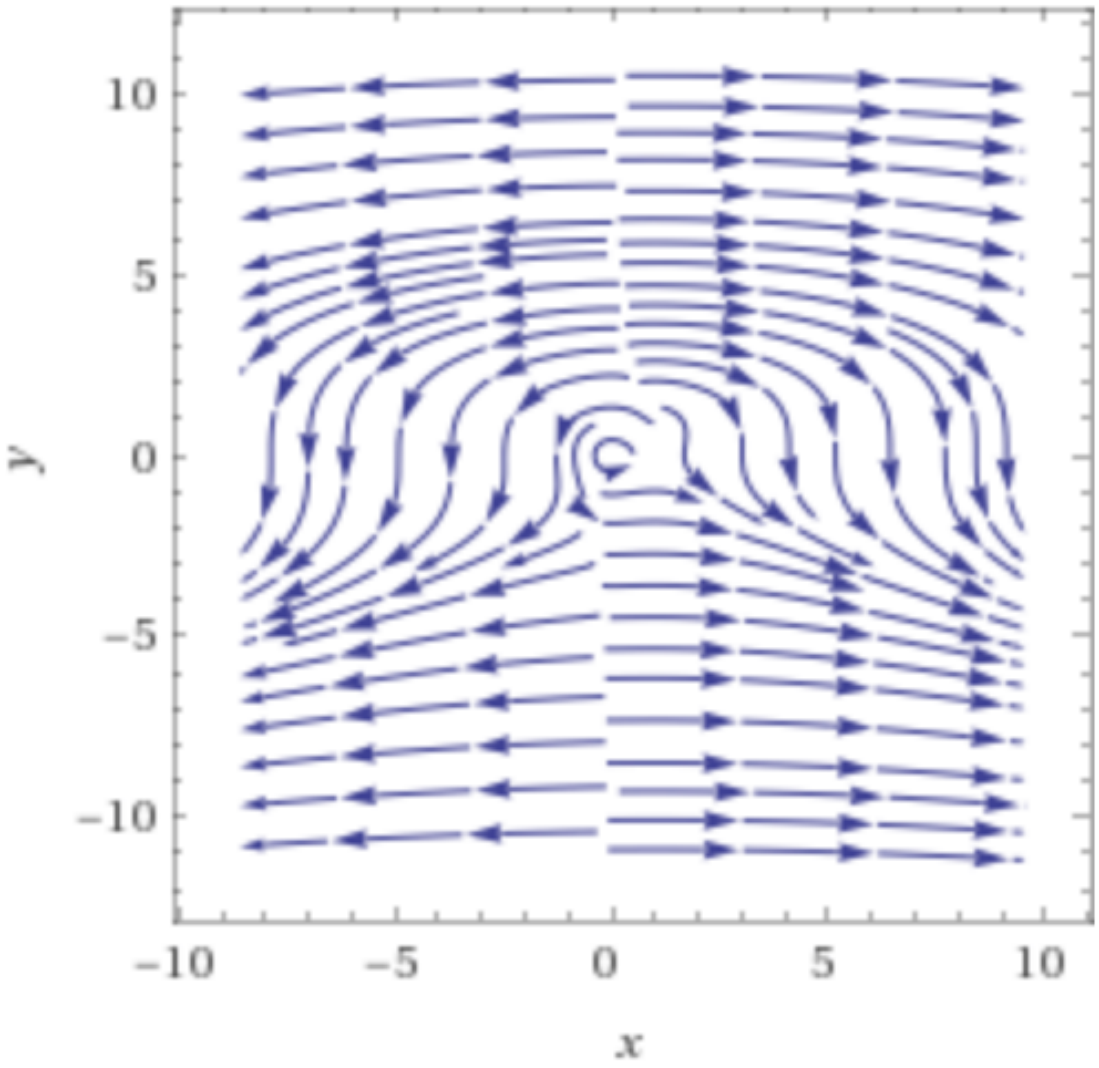
n_iter = 4, purpose function value = 1.0010533102425583.
n_iter = 7, purpose function value = 0.08067169257139281.
n_iter = 33, purpose function value = 0.00024462206925313703.
y
x


D=2, f(x*)=0.0
D=10 f(x*)=0.0
D=100 f(x*)=93.82838291139522
Purpose function value
Iteration number

| stream plot | $(-y + x\,y^2, x - x^2)$ |
|---|---|

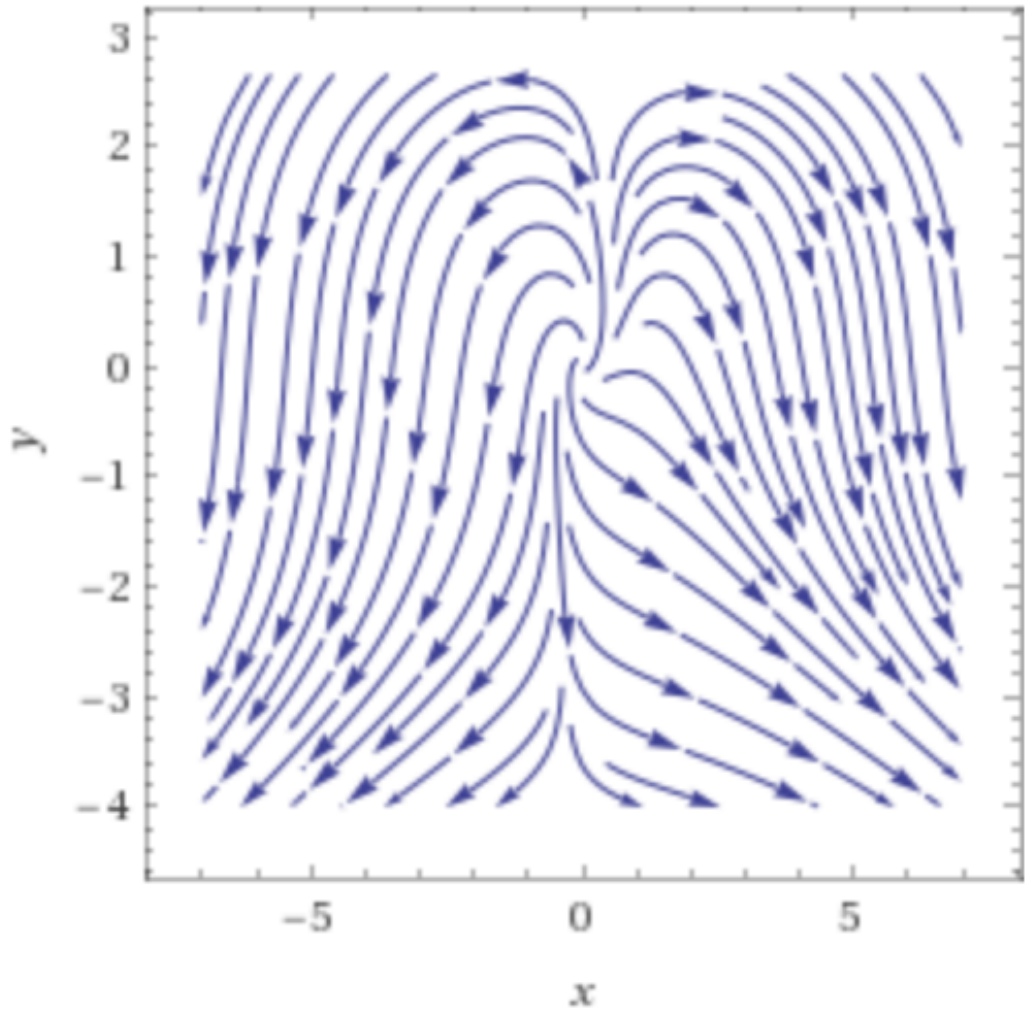
Plot:



Input interpretation:

| | |
|---|---|
| stream plot | $(x - y + x\,y^2,\ x - x^2 + y)$ |

Plot:



## Input interpretation:

| | |
|---|---|
| stream plot | $(-x - y + x y^2, x - x^2 - y)$ |

## Plot: