### BEKK-GARCH Analysis of JSE Stock Volatility

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June 2025

## **Executive Summary**

This report presents a multivariate volatility analysis of three JSE-listed stocks — Capitec Bank (CPI.JO), Anglo American (AGL.JO), and Bid Corporation Ltd (BID.JO) — using the BEKK-GARCH(1,1) model. The goal is to quantify dynamic volatilities, covariances, and spillover effects across sectors from January 2020 to June 2025.

### **Key Findings:**

- Volatility clustering is present in all three stocks, with Capitec showing the most extreme spikes.
- Statistically significant spillovers exist from Bid Corp to Capitec and from Capitec to Anglo American.
- Portfolio volatility is time-varying, with major spikes around macroeconomic events.
- BEKK-GARCH effectively captures interdependence and dynamic risk, though it has limitations in scalability and correlation modeling.

#### Introduction

Volatility in financial markets is a central concern for investors, risk managers, and policymakers. Asset prices often exhibit clustering of volatility and spillover effects, where shocks in one sector or asset can influence others. These dynamics are particularly pronounced in emerging markets like South Africa, where macroeconomic events, global commodity cycles, and domestic financial health interact in complex ways.

This report investigates the time-varying volatility and interdependence among three Johannesburg Stock Exchange (JSE) stocks — Capitec Bank (CPI.JO), Anglo American (AGL.JO), and Bid Corporation Ltd (BID.JO). These firms represent distinct sectors: financial services, mining, and diversified industrials, respectively. Their systemic importance and liquidity make them ideal candidates for multivariate volatility modeling.

Using the BEKK-GARCH(1,1) framework, we aim to capture conditional variances, covariances, and spillovers across these assets over a 5.5-year period (January 2020 to June 2025). This timeframe includes the COVID-19 pandemic, post-pandemic recovery, and recent macroeconomic shifts — offering a rich context for analyzing risk transmission and portfolio behavior

### **Problem Statement and Model Objective**

The financial markets exhibit volatility clustering and spillover effects, significantly impacting portfolio risk management. This study focuses on three Johannesburg Stock Exchange (JSE)-listed stocks: AGL.JO (Anglo American), BID.JO (Bidvest Group), and CPI.JO (Capitec Holdings), representing mining, diversified industrial, and financial sectors, respectively. The objective is to model multivariate volatility using the BEKK-GARCH (1,1) approach to quantify dynamic volatilities, covariances, and correlations, enabling optimized portfolio risk assessment over the period January 2020 to June 2025. This 5.5-year span, including the COVID-19 period, provides insights into market dynamics and diversification benefits.

#### Portfolio Selection And Sectoral Diversification

Stock	Sector	JSE Ticker
Capitec Bank	Financials (Banking)	CPI.JO
Anglo American	Basic Materials(Mining)	AGL.JO
Bid Corporation Ltd	Consumer(Food Distribution)	BID.JO

environment

#### 1. Sectoral diversification

- Each company comes from a distinct sector, which helps minimize unsystematic risk:
- oBid Corp Ltd provides exposure to the consumer demand cycle (referring to the fluctuations in economic activity, specifically consumer spending, over time), especially in food services and logistics
- o Capitec represents the banking and retail lending

o Anglo American is a leading player in global mining and resource extraction

## 2. Liquidity and Market Relevance

- All three are highly liquid, JSE Top 40 constituents
- They are suitable for time-series modeling because of consistent and deep historical data

### 3. Systemic Importance

- Capitec reflects household credit health and domestic economic strength.
- Anglo American captures global commodity cycles and geopolitical risk
- Bid Corp links to consumption, trade logistics, and global food services, which will help spotting spillovers from demand/supply shocks

### 4. Potential for Volatility Spillovers

- Bid Corp could absorb or transmit shocks due to changes in consumer spending and supply chain disruptions.
- Capitec may transmit shocks during monetary policy changes since it is part of the financial sector.
- Anglo American deals with commodities so it is sensitive to global demand and China-related shocks.

## Methodology

### **Modeling Steps**

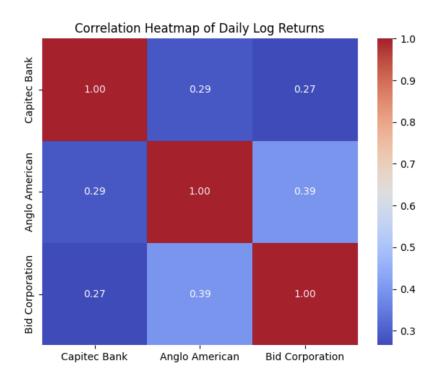
Data was sourced from Yahoo Finance, comprising daily adjusted close prices for AGL.JO, CPI.JO, and BID.JO from June 1, 2020, to June 20, 2025. Log returns were computed as:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$



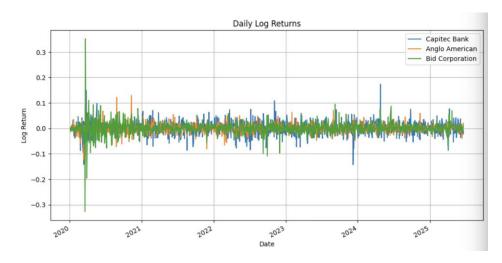
- Capitec Bank shows steady price growth most of the time, but sharp drops during financial stress.
- Anglo American, being commodity-linked, displays more cyclical behavior tied to global demand, meaning that
  during periods of economic expansion, the company experiences increased demand for its mined commodities,
  leading to higher revenues and potentially higher stock prices, however, demand and prices decrease during
  economic downturn such as the COVID 19 disrupting the supply and demand around 2020. Anglo prices might
  swing up and down often because commodity prices change rapidly.
  - o Anglo American Full Year Results 2024
- Bid Corp is relatively stable but sensitive to global supply chain shocks. Bid Corp had displayed strong
  performance especially around 2024 as the food inflation during 2024 saw a significant slowdown.
  - South Africa inflation drops to lowest in over four years
  - o Bidcorp gets South Africa and UK boost
  - o Bidcorp reports increase in four-month trading profit

• These differences support our hypothesis that each stock behaves differently in terms of how volatile (unstable) its price is over time, which makes them suitable for multivariate modeling.



- While we expect some correlation between each due to overall market movements. Each stock has its own distinct driver.
- The correlations between the log returns of the three stocks range from 0.27 to 0.39, indicating weak to moderate linear dependence. This suggests that the assets do not move in perfect unison, which is favorable for

diversification. However, the fact that correlation is not zero implies there's still some co-movement and that's where volatility spillovers might emerge, especially during periods of market stress.

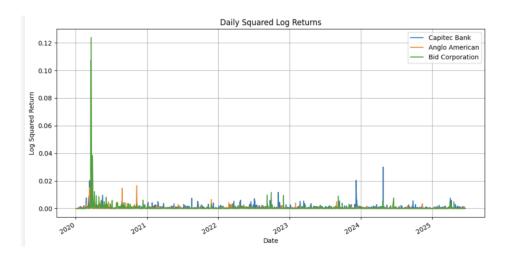


## Interpretation:

- The daily log returns tend to fluctuate around zero, as expected in financial return series.
- We observe:
  - Spikes in returns, both positive and negative, especially during major events like the COVID crash, energy crisis.

A disaster of politics: The energy supply crisis in South Africa

These patterns are precisely what GARCH models are designed to capture: time-varying volatility.



- The squared log returns are used to measure volatility. Higher values on the y-axis represent larger deviations (more volatility) in daily returns.
- Squaring emphasizes extreme values and removes direction (so both large losses and large gains are shown as spikes).
- Capitec Bank (Blue)
  - Moderate but frequent spikes from 2020 to 2025.
  - o Notably high spikes around 2023 and 2024, suggesting short periods of intense volatility.
- Anglo American (Orange)
  - Generally lower volatility.
  - o Occasional spikes, particularly early on, but overall, more subdued compared to the other two companies.
- Bid Corporation (Green)
  - Massive spike in early 2020, likely tied to the COVID-19 market crash.
  - o After that, volatility settles but remains relatively frequent in smaller magnitudes.
- Early 2020 spike in Bid Corporation dominates the plot, which is consistent with global market turmoil during the COVID outbreak.

- Capitec and Bid Corp show occasional volatility bursts, hinting at either market events or company-specific shocks.
- Anglo American appears to have more stable returns, possibly due to diversified global operations in commodities.
- There is no sustained high volatility. We see clusters of spikes, which is consistent with volatility clustering (a stylized fact in financial time series).

```
ADF Test for Capitec Bank:
  Test Statistic = -19.6360
  p-value = 0.0000
  → Stationary (Reject H0)

ADF Test for Anglo American:
  Test Statistic = -16.0247
  p-value = 0.0000
  → Stationary (Reject H0)

ADF Test for Bid Corporation:
  Test Statistic = -15.2787
  p-value = 0.0000
  → Stationary (Reject H0)
```

- Before modeling volatility, it is essential to confirm that the return series is stationary. Stationarity implies that the statistical properties of the series, such as mean and variance, remain constant over time. This ensures the suitability of models like GARCH and BEKK-GARCH, which rely on the assumption of stationarity.
- The Augmented Dickey-Fuller (ADF) test was conducted on the log return series of all selected stocks.
  - The null hypothesis of the ADF test is that the series is not stationary. The results yielded p-values below the 0.05 threshold, indicating strong evidence against the null hypothesis of a unit root. Thus, the return series are considered stationary, making them appropriate for subsequent volatility modeling.

```
Ljung-Box Test for Squared Returns - Capitec Bank
lb_stat lb_pvalue
10 210.634769 9.720317e-40

Ljung-Box Test for Squared Returns - Anglo American
lb_stat lb_pvalue
10 159.678019 3.776041e-29

Ljung-Box Test for Squared Returns - Bid Corporation
lb_stat lb_pvalue
10 441.146494 1.614981e-88
```

- H0: there is no autocorrelation up to the chosen lag.
- H1: there is presence of autocorrelation. significant level = 0.05,
- The Ljung-Box test applied to the squared returns of all three stocks produced p-values effectively equal to zero, implies we should reject null hypothesis, indicating strong and significant autocorrelation in volatility. This confirms the presence of volatility clustering and justifies the use of GARCH-family models, such as BEKK-GARCH, to capture the time-varying conditional variance and cross-asset spillover effects.

```
ARCH Test for Capitec Bank:

LM Stat = 118.0177, p-value = 0.0000

→ ARCH effects detected

ARCH Test for Anglo American:

LM Stat = 114.1358, p-value = 0.0000

→ ARCH effects detected

ARCH Test for Bid Corporation:

LM Stat = 544.6369, p-value = 0.0000

→ ARCH effects detected
```

- All three stocks have very low p-values (p = 0.0000), which means the null hypothesis of no ARCH effects is rejected at any conventional significance level.
- The high LM statistics (especially for Bid Corporation at 544.64) confirm strong presence of ARCH effects, meaning there is time-varying volatility in the return series.
- Significant autocorrelation in squared returns (Ljung-Box test) and confirmation from the ARCH-LM test provided evidence of ARCH effects, justifying the use of BEKK-GARCH to capture volatility clustering and spillovers.

#### **BEKK-GARCH**

The BEKK (1,1) model is a tool used to understand how the volatility (risk) of multiple financial assets changes over time, and how the volatility of one asset can influence others.

BEKK (1,1) Formula

The BEKK (1,1) form for the conditional covariance matrix Ht is:

$$H_t = C^T C + A^T \epsilon_{t-1} \epsilon_{t-1}^T A + B^T H_{t-1} B$$

#### where:

- C: constant lower triangular matrix (gives the long-term average covariance and ensures positive definiteness of *Ht*)
- A: ARCH effects (models how short-term past shocks impact current volatility and covariance)
- B : GARCH effects (captures persistence of past volatility and covariances)

By estimating these matrices, we can track how volatility behaves and propagates over time, offering insights into how shocks in one stock affects the volatility of other stocks.

### Why use BEKK?

- The BEKK (1,1) captures dynamic covariances
  - Unlike the univariate GARCH (1,1), the BEKK (1,1) allows covariances between assets to evolve over time, not just variances.
- Models' volatility spillovers
  - A shock in one asset can influence the volatility of other assets in the portfolio, helping quantify interdependence.

- · Positive definiteness is guaranteed
  - The BEKK (1,1) formula ensures that the estimated covariance matrix remains mathematically valid for use in portfolio risk.
  - o This is essential for unconstraint optimization in multi-dimensions.

Portfolio variance was calculated as  $\sigma_{p,t}^2 = w^T H_t w$ 

Where  $w = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$ 

are equal weights, and volatility was derived as  $\sigma_{p,t} = \sqrt{\sigma_{p,t}^2}$ .

## **Assumptions**

- · Returns are stationary.
- Presence of ARCH Effects
- Positive Definite Covariance Matrix (H<sub>t</sub>)
- Normality or Conditional Normality of Errors
- Returns are often assumed to follow a normal distribution, conditional on past information.
  - Why? For the model to perform well under Maximum Likelihood Estimation (MLE).
  - $\circ$  In real life  $\to$  returns often have fat tails  $\to$  so sometimes other distributions (like t-distribution) are used.
- Correct Model Specification
  - o The chosen BEKK (1,1) structure assumes that one lag of past shocks (A) and past volatility (B) is enough.
  - Why? Misspecification can lead to bad results.
- BEKK-GARCH (1,1) adequately captures volatility dynamics.
- No Perfect Multicollinearity
  - The return series of the selected assets shouldn't be perfectly correlated.
  - o Why? Perfect correlation breaks the estimation process.

#### **Model Estimation**

#### Step 1: Model Definition

For an N-dimensional return vector  $r_t$ , we model:

$$r_t = \mu + \varepsilon_t$$

where:

- μ is the constant mean vector,
- ε<sub>t</sub> | F<sub>t-1</sub> ~ N(0, H<sub>t</sub>),

#### Step 3: Likelihood Function

For T observations, the joint likelihood is:

$$L(\theta) = \prod_{t=1}^{T} f(r_t \mid \mathcal{F}_{t-1}; \theta)$$

where  $\theta = \{\mu, C, A, B\}$  represents all model parameters.

#### Step 4: Log-Likelihood Function

Taking logarithms, the log-likelihood becomes:

$$\begin{split} \ln L(\theta) &= \sum_{t=1}^{T} \ln f(r_t \mid \mathcal{F}_{t-1}; \theta) \\ &= -\frac{TN}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \ln |H_t| - \frac{1}{2} \sum_{t=1}^{T} (r_t - \mu)' H_t^{-1}(r_t - \mu) \end{split}$$

## Step 2: Conditional Density of Returns

Since  $\varepsilon_t$  is multivariate normal, the conditional density of  $r_t$  is:

$$f(r_t \mid \mathcal{F}_{t-1}) = \frac{1}{(2\pi)^{N/2} |H_t|^{1/2}} \exp \left(-\frac{1}{2}(r_t - \mu)' H_t^{-1}(r_t - \mu)\right)$$

where  $|H_t|$  denotes the determinant of  $H_t$ .

#### Step 5: Residuals and Recursive Computation

Define residuals:

$$\varepsilon_t = r_t - \mu$$

The recursion for  $H_t$  is:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B$$

computed recursively from t = 1 to T.

### Step 6: Estimation Objective

Estimate  $\theta$  by maximizing the log-likelihood:

$$\hat{\theta} = \arg \max_{\theta} \ln L(\theta)$$

### Step 7:

Loaded the MTS package in R  $\rightarrow$  which provides the BEKK11() function for estimating BEKK (1,1) models.

The model uses Maximum Likelihood Estimation (MLE) to find the values of C, A, and B matrices that maximize the likelihood of observing the given return data.

Internally, gradient based optimization algorithms (like BFGS or similar) are used to solve this.

### **Model Diagnostics**

- Log-likelihood: Successfully maximized after rescaling log returns by 100.
- Convergence: Achieved with stable Hessian after addressing numerical instability.
- ADF, Ljung-Box, ARCH-LM: All tests confirm stationarity and presence of ARCH effects, validating BEKK-GARCH suitability.

### **Estimates Diagonostic.**

The BEKK (1,1) model output provides estimates along with their standard errors, t-values, and p-values. Parameters with p-values below 0.05 were considered statistically significant, confirming their contribution to the model. The log-likelihood value was also examined to confirm a good model fit.

R Output

Matrix A (ARCH - Short-term shock effects):

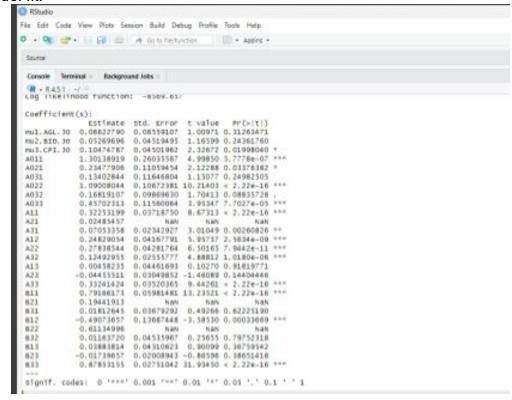
CPI BID AGL

CPI 0.3225 0.2483 0.0046

BID 0.0249 0.2784 -0.0446

AGL 0.0705 0.1249 0.3324

Matrix A: Shock effect

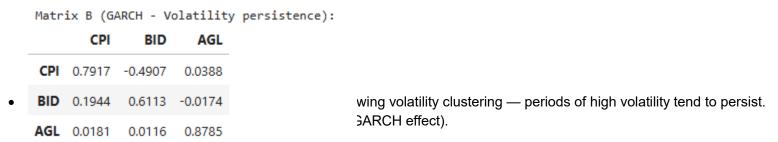


Matrix A captures the impact of past shocks on current volatility (ARCH effect).

- A<sub>12</sub>, A<sub>31</sub> and A<sub>32</sub> are statistically significant.
- Other off-diagonal entries are small and statistically insignificant. There is weak evidence that there is a spillover between stocks.
- Capitec volatility is influenced by Bid Corp's shocks.
- Anglo American volatility influenced by Capitec shocks
- Anglo American volatility influenced by Bid Corp shocks

## Matrix B: Volatility Persistence

Matrix B represents the effect of past volatility on current volatility (GARCH effect)



o Bid Corp is moderately persistent, but uncertainty exists.

- B<sub>12</sub> and B<sub>13</sub> are statistically significant
  - o Capitec's volatility inversely reacts to Bid Corps's past volatility
  - o B<sub>13</sub> is small so Capitec's volatility is not significantly influenced by Anglo American volatility.
- All the other entries were not statistically significant and small so there is weak evidence of volatility spillovers
  - o B<sub>21</sub> = 0.1944 is not statistically significant but Bid Corp weakly responds to Capitec's past volatility

0

### **Interpretation Summary**

- Each asset's volatility is mostly influenced by its own past volatility and shocks.
- There are significant shock spillovers from Bid Corp to Capitec and from Capitec to Anglo American.
- Volatility clustering is clearly present in all three assets.
- Spillover from one asset's volatility to another is minimal, however, there is some volatility transmission from Capitec to Bid Corp.

### **Encountered problems:**

The log returns were not scaled, and as a result the log returns ended up being very small and these small values caused numerical instabilities in optimization since BEKK (1,1) uses gradient based methods

- The Hessian was ill-conditioned
- Daily log returns were now multiplied by 100 prior to model estimation. This rescaling was done to improve numerical stability during the optimization process, as raw log returns are often very small and can lead to illconditioned likelihood functions.
- Rescaling did not affect the model dynamics or interpretation but ensured better convergence behavior and more interpretable volatility estimates (now expressed in percentage.

### **Empirical Analysis**

## Portfolio Conditional Variance Forecasting

Given the estimated conditional covariance matrix  $H_t$  from the BEKK(1,1) model, the conditional variance of a portfolio with weight vector w is:

$$\sigma_{p,t}^2 = w^\top H_t w$$

where:

- σ<sup>2</sup><sub>p,t</sub> is the conditional variance of the portfolio at time t,
- w is the vector of portfolio weights,
- H<sub>t</sub> is the estimated conditional covariance matrix at time t.

The conditional volatility is then:

$$\sigma_{p,t} = \sqrt{w^{\top}H_tw}$$

This allows for the assessment of how portfolio risk evolves over time. The volatility forecasts provide critical inputs for portfolio risk management, including Value-at-Risk (VaR), stress testing, and optimal asset allocation.

#### Derivation

For a portfolio return:

$$R_{n,t} = w^{\top} r_t$$

the conditional variance is:

$$\operatorname{Var}(R_{n,t} \mid \mathcal{F}_{t-1}) = \operatorname{Var}(w^{\top}r_t) = w^{\top} \operatorname{Var}(r_t)w = w^{\top}H_tw$$

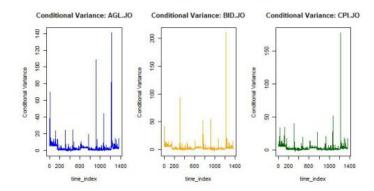
Thus, the portfolio conditional variance is:

$$\sigma_{p,t}^2 = w^\top H_t w$$

and the corresponding conditional volatility is:

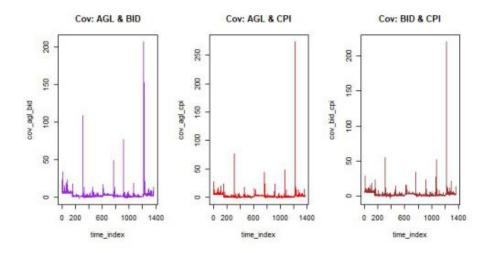
$$\sigma_{p,t} = \sqrt{w^{\top}H_tw}$$

#### **Conditional Variance**



- The AGL.JO plot clearly shows volatility clustering, where long calm periods interrupted by sharp spikes (possible external shocks). Moderate overall volatility level.
  - o The plot has many spikes in volatility, meaning it's frequently volatile, possibly reacting to news or sector risk
- The BID.JO plot also displays clustering and spikes, with some very large jumps, suggesting exposure to large, abrupt shocks.
- The CPI.JO plot has the highest volatility spike in the dataset, suggesting high sensitivity to certain market events. Otherwise relatively low and stable.
- Each asset's volatility is time-varying and reacts strongly to market events, supporting the use of the BEKK-GARCH model.
- While AGL and BID have similar patterns of modest volatility interrupted by spikes, CPI experiences more extreme
  episodes reflecting possibly higher market sensitivity or company-specific risks.

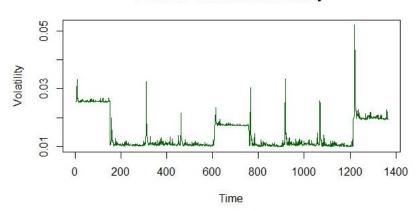
#### **Condition Covariance**



- All three plots show fluctuations over time, which confirms that asset relationships are not constant.
- This justifies the use of a multivariate GARCH model like BEKK (1,1), which is designed to capture these dynamics.
- Sharp spikes could correspond to economic news, crises, or high volatility events.
  - o These spikes suggest temporary periods of stronger co-movement or contagion.
- The covariances hover around relatively modest values indicating low to moderate average cross-asset volatility co-movement.
  - o The generally low average covariance levels align with the weak correlation

## **Portfolio Conditional Volatility**

#### Portfolio Conditional Volatility



- Portfolio risk is not constant; it evolves over time.
  - There are many peaks and troughs
  - o Volatility is sometimes very low (near 0.01), and other times it shoots above 0.10.
  - o These spikes and declines tell you that volatility is changing
  - o This reflects the influence of both individual asset volatilities and their covariances.
- Spikes are followed by periods of gradual decay (GARCH effect).
- Major volatility spikes occur around time points 350, 800, and 1200.
- These may correspond to macro events, political shifts, or sector shocks.
- The largest spike around t ≈ 1200 indicates a market-wide shock, affecting the whole portfolio.
- Forecasted portfolio volatility help us anticipate periods of elevated risk.

- By forecasting how the portfolio's risk changes over time, we can see when the portfolio is becoming more risky before it happens.
- Knowing this early helps investors or managers adjust how much they invest in each asset to reduce potential losses.

## **Financial Interpretation**

- In normal market conditions, diversification reduces portfolio risk, but during market stress, volatility in one stock
  can spread to other stocks in the portfolio making the whole portfolio risky even when stocks from different sectors
  were chosen.
- Portfolios need to be adjusted when markets become risky so keeping the same mix of stocks all the time will not be beneficial.
- Tools such as futures and options for hedging can help reduce big losses.
- Knowing how risky the portfolio might become helps us decide how much risk we can take.
- These forecasts can help set limits to avoid taking on too much risk.

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#### Limitations

Computationally Intensive-BEKK uses a lot of computing power, especially as you add more stocks. Estimation takes longer and may fail if the dataset is too large.

Difficult to Interpret -The parameter estimates (matrices A, B, C) are hard to interpret directly, especially for non-specialists. You don't get a clear picture of how correlations between stocks change over time.

Fixed Spillover structure BEKK assumes that the way shocks affect each other stays the same over time -But relationships between stocks can change during market stress BEKK doesn't fully capture that.

Only Captures Volatility, not changing Correlation- BEKK shows how volatilities interact, but not how correlations rise and fall (for that we can use DCC-GARCH).

Challenging with Large Portfolios -Works okay for 2 or 3 assets → but adding more makes the model too complex

#### Conclusion

This project gave us a deeper understanding of how risk behaves across different sectors of the JSE, especially during periods of Market Stress like the COVID-19 pandemic. By applying the BEKK-GARCH model, we were able to track how volatility in one stock can spill over into others, even when they belong to different industries. That's a powerful reminder that diversification isn't always a safety net, especially when markets are under stress.

While the model has its limitations, such as being computationally heavy and tricky to interpret, it still offers valuable insights for portfolio managers. Knowing when risk is building up allows for smarter decisions, whether that's rebalancing assets, setting tighter risk limits.

In short, this analysis shows that risk isn't static. It evolves, and so should our strategies. By forecasting volatility and understanding how assets interact, we can build portfolios that are not just diversified, but resilient

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