

1. Express the signals shown in Fig. 1 in terms of unit step functions.

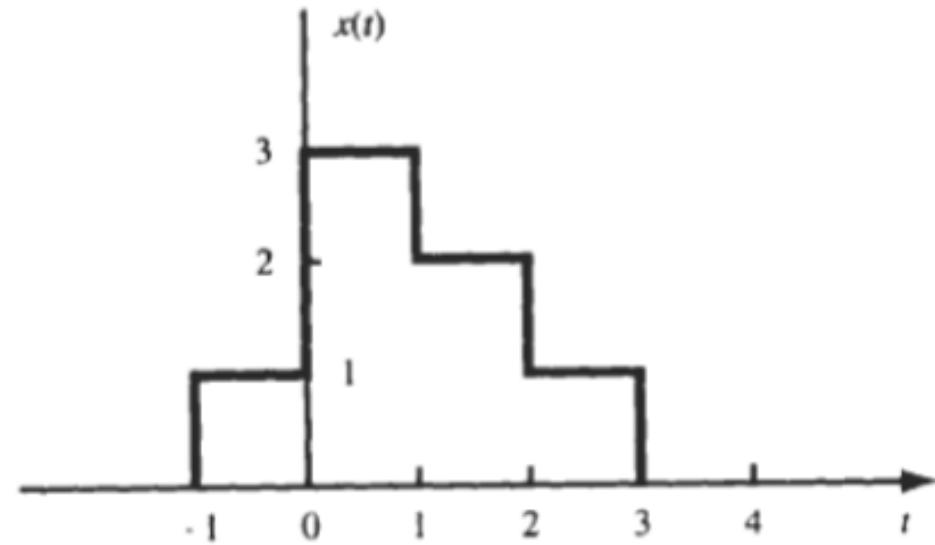
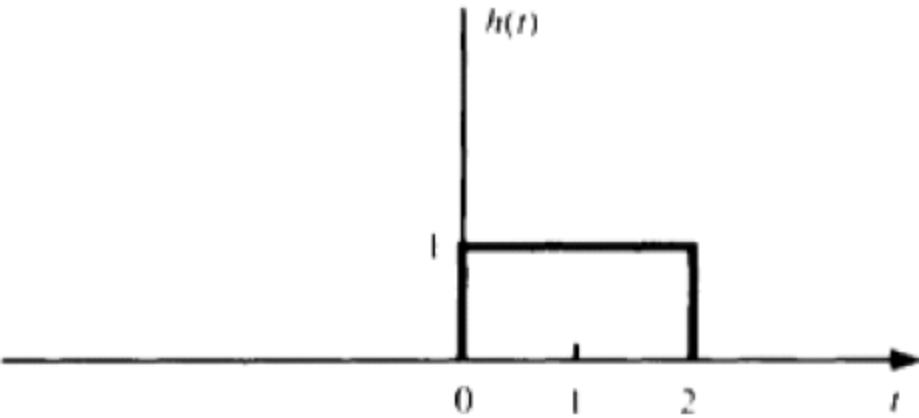
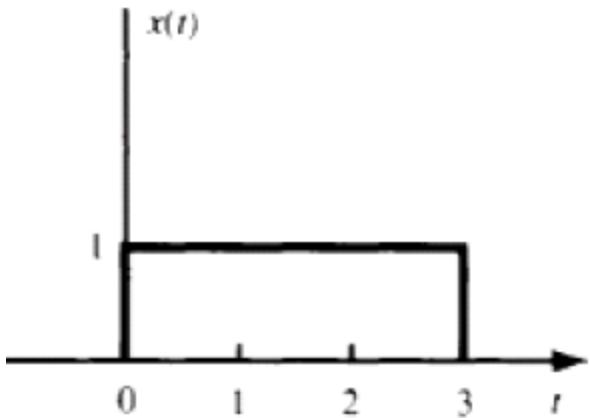


Fig. 1

Answer  $x(t) = u(t + 1) + 2u(t) - u(t - 1) - u(t - 2) - u(t - 3)$

3. Evaluate  $y(t) = x(t) * h(t)$ , where  $x(t) = u(t) - u(t - 3)$  and  $h(t) = u(t) - u(t - 2)$



We first express  $x(t)$  and  $h(t)$  in functional form:

$$x(t) = u(t) - u(t - 3) \quad h(t) = u(t) - u(t - 2)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} [u(\tau) - u(\tau - 3)][u(t - \tau) - u(t - \tau - 2)] d\tau \\
&= \int_{-\infty}^{\infty} u(\tau)u(t - \tau) d\tau - \int_{-\infty}^{\infty} u(\tau)u(t - 2 - \tau) d\tau \\
&\quad - \int_{-\infty}^{\infty} u(\tau - 3)u(t - \tau) d\tau + \int_{-\infty}^{\infty} u(\tau - 3)u(t - 2 - \tau) d\tau
\end{aligned}$$

$$u(\tau)u(t - \tau) = \begin{cases} 1 & 0 < \tau < t, t > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$u(\tau)u(t - 2 - \tau) = \begin{cases} 1 & 0 < \tau < t - 2, t > 2 \\ 0 & \text{otherwise} \end{cases}$$

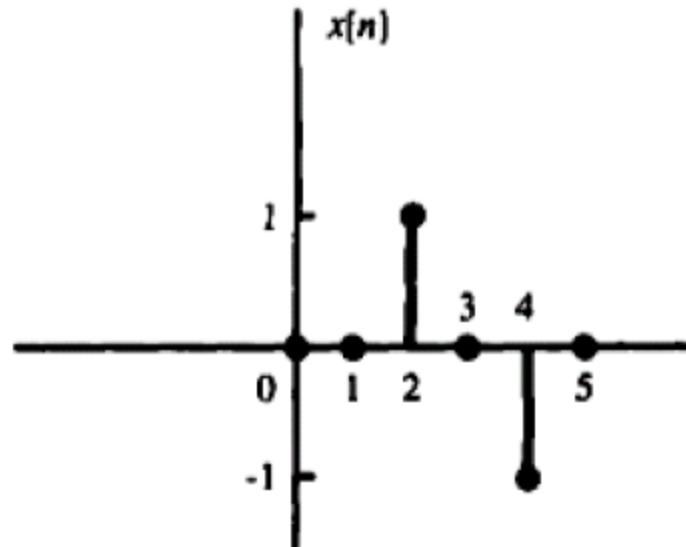
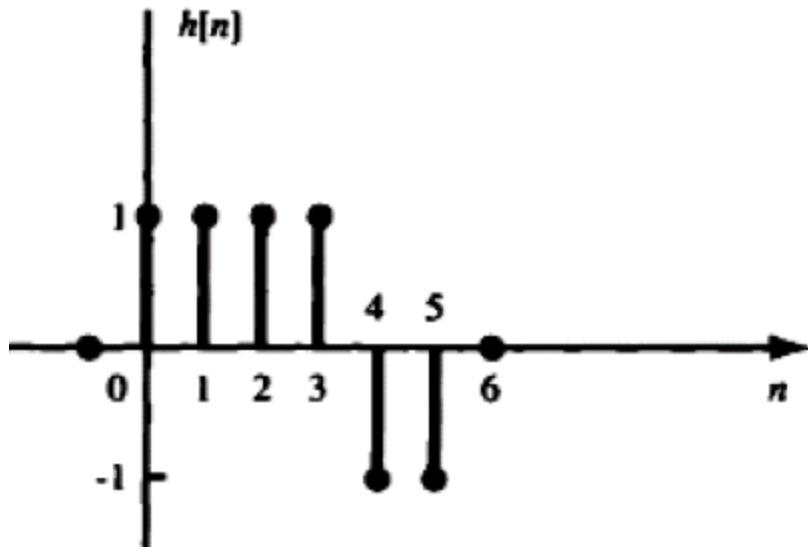
$$u(\tau - 3)u(t - \tau) = \begin{cases} 1 & 3 < \tau < t, t > 3 \\ 0 & \text{otherwise} \end{cases}$$

$$u(\tau - 3)u(t - 2 - \tau) = \begin{cases} 1 & 3 < \tau < t - 2, t > 5 \\ 0 & \text{otherwise} \end{cases}$$

we can express  $y(t)$  as

$$\begin{aligned}y(t) &= \left( \int_0^t d\tau \right) u(t) - \left( \int_0^{t-2} d\tau \right) u(t-2) \\&\quad - \left( \int_3^t d\tau \right) u(t-3) + \left( \int_3^{t-2} d\tau \right) u(t-5) \\&= tu(t) - (t-2)u(t-2) - (t-3)u(t-3) + (t-5)u(t-5)\end{aligned}$$

5. The impulse response  $h[n]$  of a discrete-time LTI system. (a). Determine and sketch the output  $y[n]$  of this system to the input  $x[n]$ .

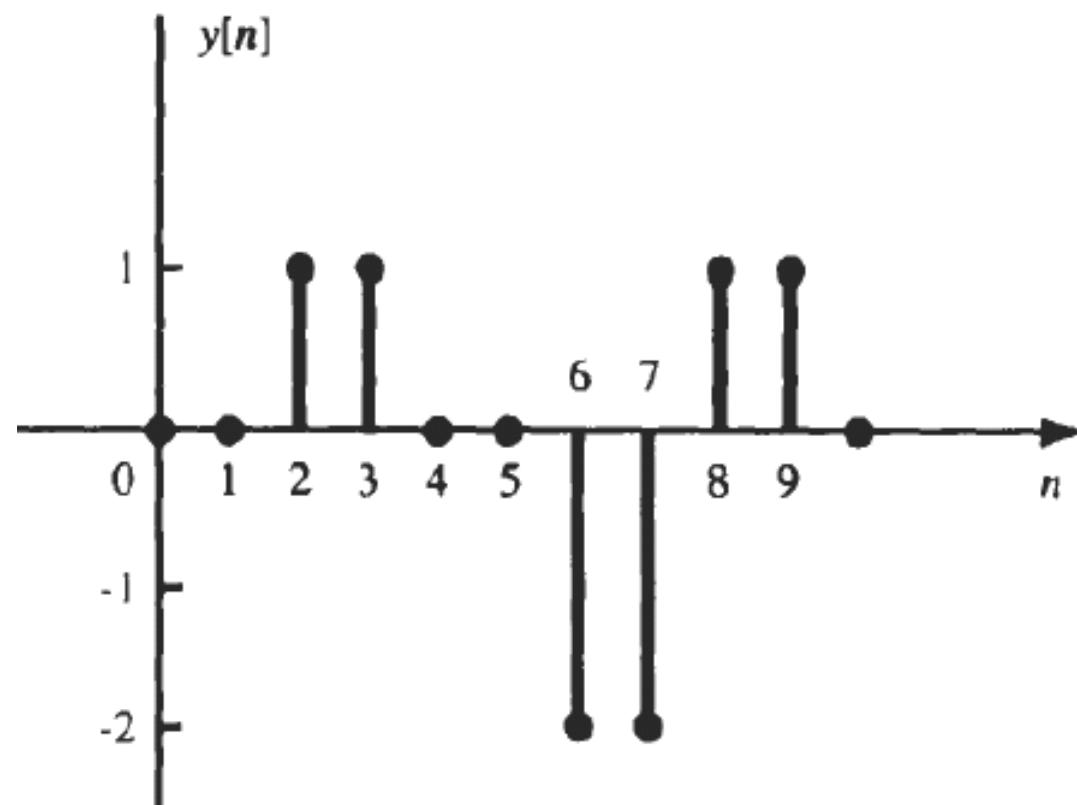


$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-4] - \delta[n-5],$$

$$x[n] = \delta[n-2] - \delta[n-4]$$

$$\begin{aligned} x[n] * h[n] &= x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-4] - \delta[n-5]\} \\ &= x[n] + x[n-1] + x[n-2] + x[n-3] - x[n-4] - x[n-5] \end{aligned}$$

$$\begin{aligned}
 y[n] &= \delta[n-2] - \delta[n-4] + \delta[n-3] - \delta[n-5] + \delta[n-4] - \delta[n-6] + \delta[n-5] - \delta[n-7] \\
 &\quad - \delta[n-6] + \delta[n-8] - \delta[n-7] + \delta[n-9] \\
 &= \delta[n-2] + \delta[n-3] - 2\delta[n-6] - 2\delta[n-7] + \delta[n-8] + \delta[n-9] \\
 y[n] &= \{0, 0, 1, 1, 0, 0, -2, -2, 1, 1\}
 \end{aligned}$$



**1.13.** Consider the continuous-time signal

$$x(t) = \delta(t + 2) - \delta(t - 2).$$

Calculate the value of  $E_\infty$  for the signal

$$y(t) = \int_{-\infty}^t x(\tau) d\tau.$$

$$y(t) = \int_{-\infty}^t x(\tau) dt = \int_{-\infty}^t (\delta(\tau + 2) - \delta(\tau - 2)) dt = \begin{cases} 0, & t < -2 \\ 1, & -2 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

Therefore,

$$E_\infty = \int_{-2}^2 dt = 4$$

**1.14.** Consider a periodic signal

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ -2, & 1 < t < 2 \end{cases}$$

with period  $T = 2$ . The derivative of this signal is related to the “impulse train”

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

with period  $T = 2$ . It can be shown that

$$\frac{dx(t)}{dt} = A_1 g(t - t_1) + A_2 g(t - t_2).$$

Determine the values of  $A_1$ ,  $t_1$ ,  $A_2$ , and  $t_2$ .

**1.14.** The signal  $x(t)$  and its derivative  $g(t)$  are shown in Figure S1.14.

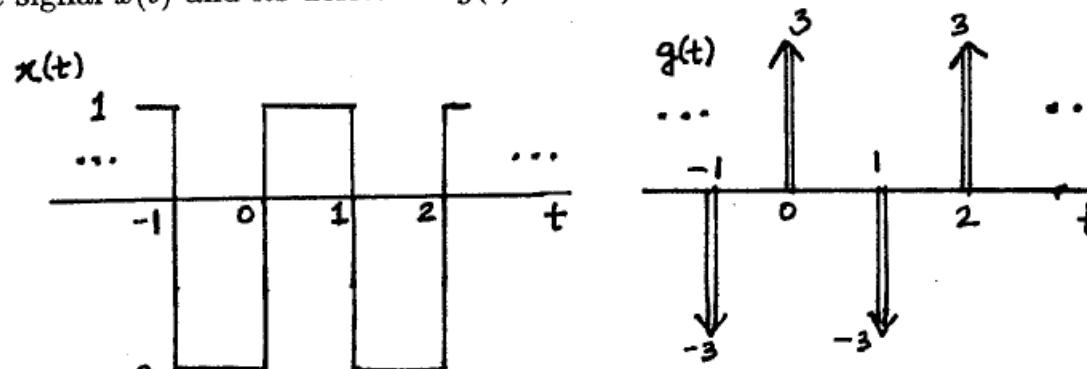


Figure S1.14

Therefore,

$$g(t) = 3 \sum_{k=-\infty}^{\infty} \delta(t - 2k) - 3 \sum_{k=-\infty}^{\infty} \delta(t - 2k - 1)$$

This implies that  $A_1 = 3$ ,  $t_1 = 0$ ,  $A_2 = -3$ , and  $t_2 = 1$ .

**2.1.** Let

$$x[n] = \delta[n] + 2\delta[n - 1] - \delta[n - 3] \quad \text{and} \quad h[n] = 2\delta[n + 1] + 2\delta[n - 1].$$

Compute and plot each of the following convolutions:

- (a)  $y_1[n] = x[n] * h[n]$
- (b)  $y_2[n] = x[n + 2] * h[n]$
- (c)  $y_3[n] = x[n] * h[n + 2]$

(a) We know that

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \quad (\text{S2.1-1})$$

The signals  $x[n]$  and  $h[n]$  are as shown in Figure S2.1.

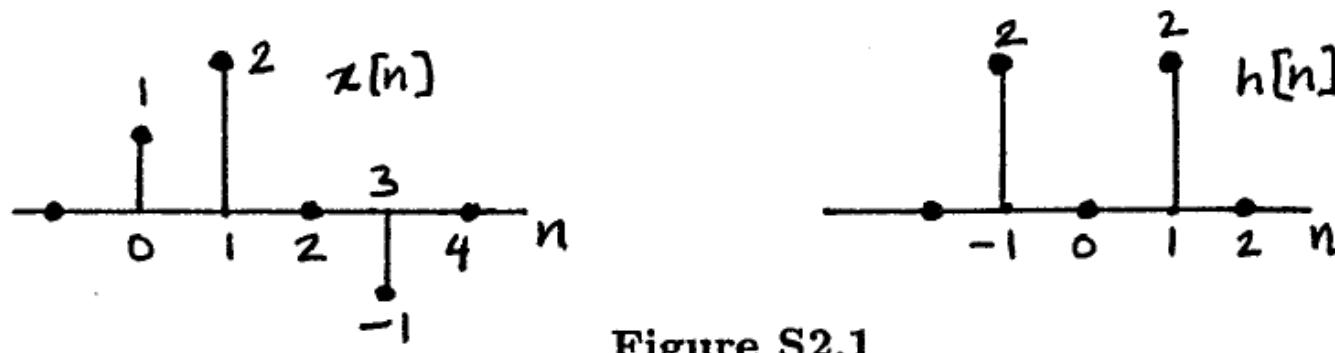


Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$\begin{aligned} y_1[n] &= h[-1]x[n+1] + h[1]x[n-1] \\ &= 2x[n+1] + 2x[n-1] \end{aligned}$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq. (S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

(c) We may rewrite eq. (S2.1-1) as

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq. (S2.1), we see that

$$y_3[n] = y_1[n+2]$$

- 2.22.** For each of the following pairs of waveforms, use the convolution integral to find the response  $y(t)$  of the LTI system with impulse response  $h(t)$  to the input  $x(t)$ . Sketch your results.

(a)  $\left. \begin{array}{l} x(t) = e^{-\alpha t} u(t) \\ h(t) = e^{-\beta t} u(t) \end{array} \right\}$  (Do this both when  $\alpha \neq \beta$  and when  $\alpha = \beta$ .)

(a) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau \\ &= \int_0^t e^{-\alpha t} e^{-\beta(t-\tau)} d\tau, \quad t \geq 0 \end{aligned}$$

Then

$$y(t) = \begin{cases} \frac{e^{-\beta t} \{e^{-(\alpha-\beta)t} - 1\}}{\beta - \alpha} u(t) & \alpha \neq \beta \\ te^{-\beta t} u(t) & \alpha = \beta \end{cases} .$$

$$\mathbf{(b)} \quad x(t) = u(t) - 2u(t-2) + u(t-5)$$

$$h(t) = e^{2t}u(1-t)$$

(b) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_0^2 h(t-\tau)d\tau - \int_2^5 h(t-\tau)d\tau. \end{aligned}$$

This may be written as

$$y(t) = \begin{cases} \int_0^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, & t \leq 1 \\ \int_{t-1}^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, & 1 \leq t \leq 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}d\tau, & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

Therefore,

$$y(t) = \begin{cases} (1/2)[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t \leq 1 \\ (1/2)[e^2 + e^{2(t-5)} - 2e^{2(t-2)}], & 1 \leq t \leq 3 \\ (1/2)[e^{2(t-5)} - e^2], & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$