

1. Express the signals shown in Fig. 1 in terms of unit step functions.

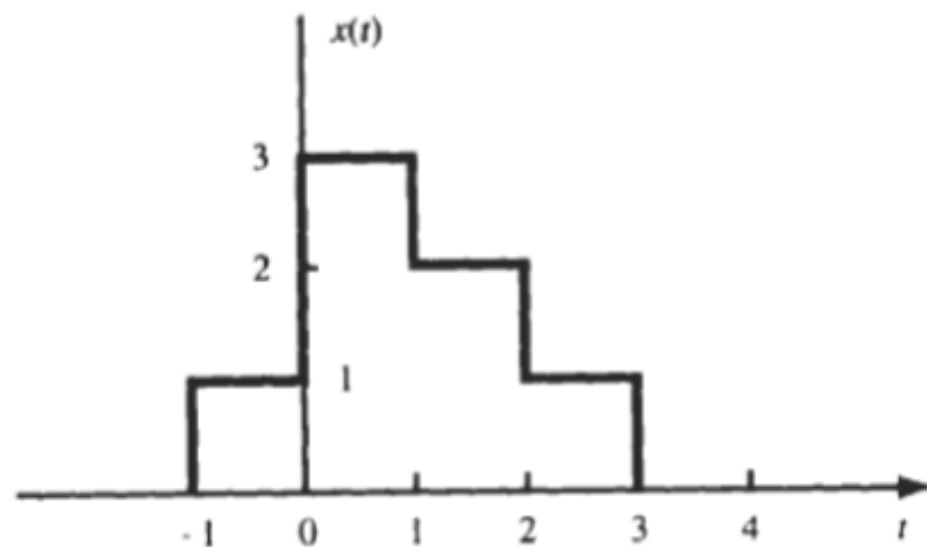
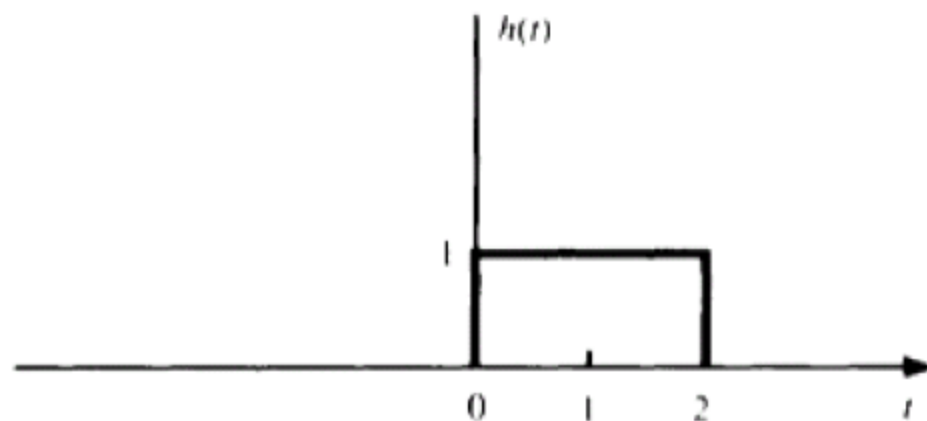
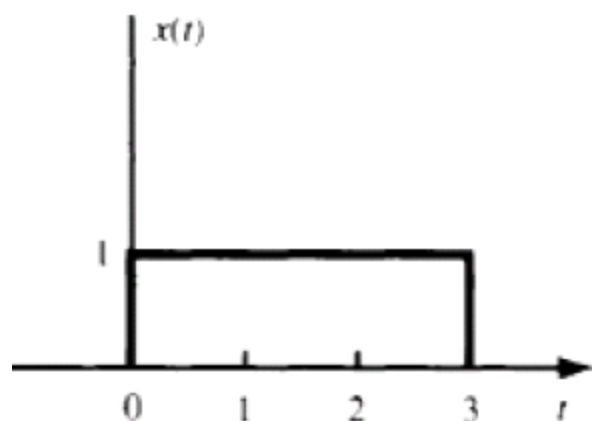


Fig. 1

Answer $x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$

3. Evaluate $y(t) = x(t) * h(t)$, where $x(t) = u(t) - u(t - 3)$ and $h(t) = u(t) - u(t - 2)$



We first express $x(t)$ and $h(t)$ in functional form:

$$x(t) = u(t) - u(t - 3) \quad h(t) = u(t) - u(t - 2)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

$$= \int_{-\infty}^{\infty} [u(\tau) - u(\tau - 3)][u(t - \tau) - u(t - \tau - 2)] d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau)u(t - \tau) d\tau - \int_{-\infty}^{\infty} u(\tau)u(t - 2 - \tau) d\tau$$

$$- \int_{-\infty}^{\infty} u(\tau - 3)u(t - \tau) d\tau + \int_{-\infty}^{\infty} u(\tau - 3)u(t - 2 - \tau) d\tau$$

$$u(\tau)u(t - \tau) = \begin{cases} 1 & 0 < \tau < t, t > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$u(\tau)u(t - 2 - \tau) = \begin{cases} 1 & 0 < \tau < t - 2, t > 2 \\ 0 & \text{otherwise} \end{cases}$$

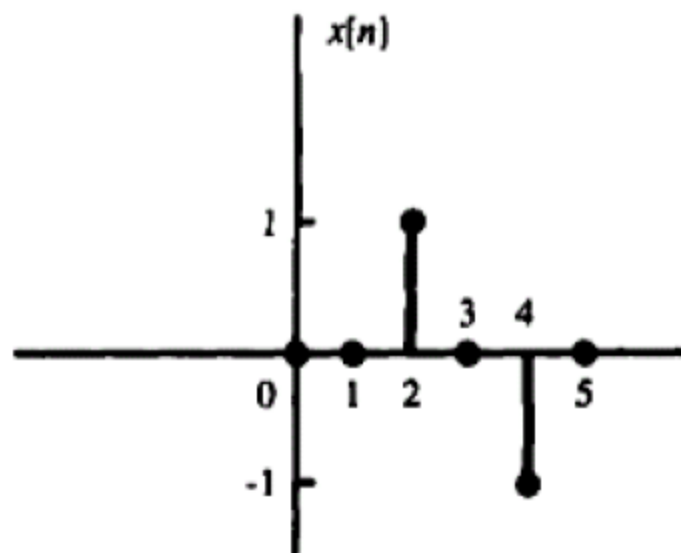
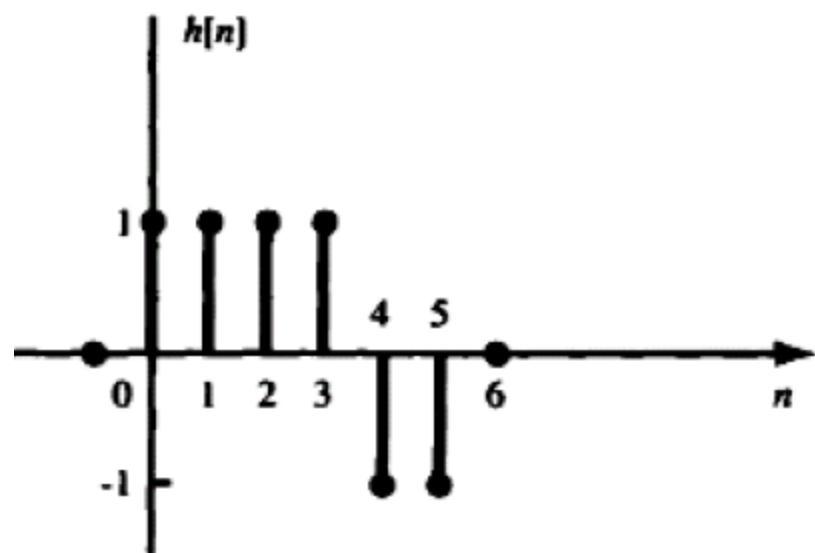
$$u(\tau - 3)u(t - \tau) = \begin{cases} 1 & 3 < \tau < t, t > 3 \\ 0 & \text{otherwise} \end{cases}$$

$$u(\tau - 3)u(t - 2 - \tau) = \begin{cases} 1 & 3 < \tau < t - 2, t > 5 \\ 0 & \text{otherwise} \end{cases}$$

we can express $y(t)$ as

$$\begin{aligned} y(t) &= \left(\int_0^t d\tau \right) u(t) - \left(\int_0^{t-2} d\tau \right) u(t-2) \\ &\quad - \left(\int_3^t d\tau \right) u(t-3) + \left(\int_3^{t-2} d\tau \right) u(t-5) \\ &= tu(t) - (t-2)u(t-2) - (t-3)u(t-3) + (t-5)u(t-5) \end{aligned}$$

5. The impulse response $h[n]$ of a discrete-time LTI system. (a). Determine and sketch the output $y[n]$ of this system to the input $x[n]$.

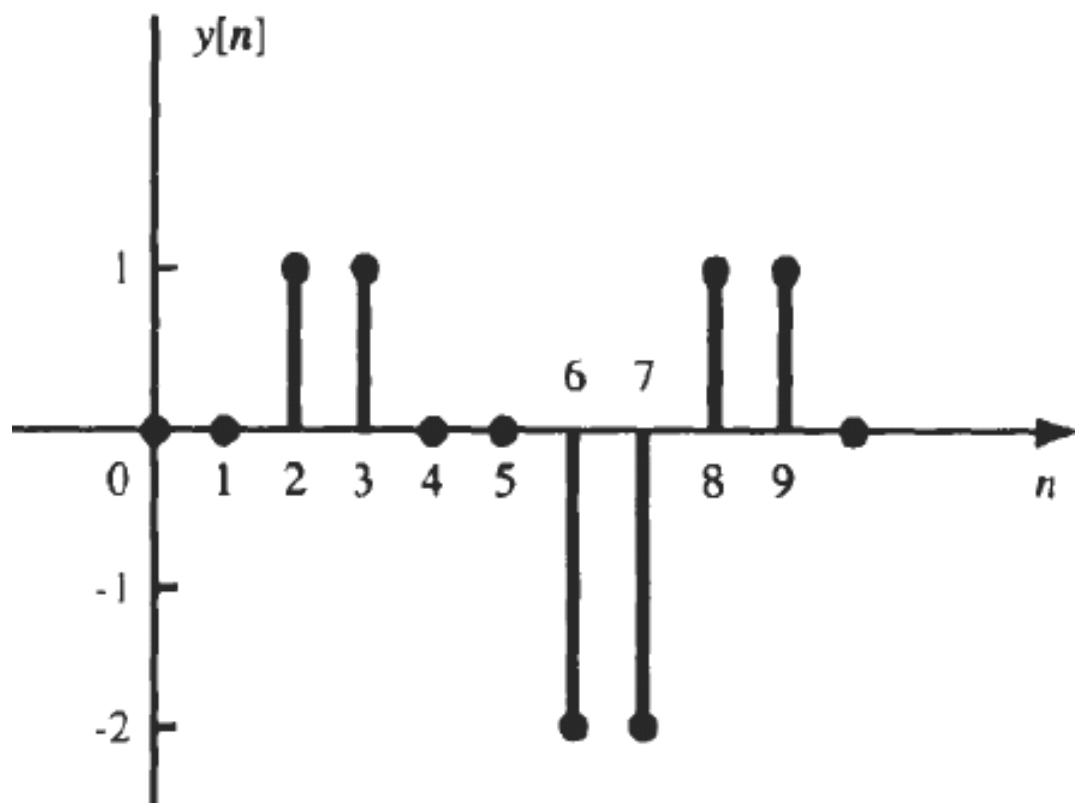


$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-4] - \delta[n-5],$$

$$x[n] = \delta[n-2] - \delta[n-4]$$

$$\begin{aligned} x[n] * h[n] &= x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-4] - \delta[n-5]\} \\ &= x[n] + x[n-1] + x[n-2] + x[n-3] - x[n-4] - x[n-5] \end{aligned}$$

$$\begin{aligned}
 y[n] &= \delta[n-2] - \delta[n-4] + \delta[n-3] - \delta[n-5] + \delta[n-4] - \delta[n-6] + \delta[n-5] - \delta[n-7] \\
 &\quad - \delta[n-6] + \delta[n-8] - \delta[n-7] + \delta[n-9] \\
 &= \delta[n-2] + \delta[n-3] - 2\delta[n-6] - 2\delta[n-7] + \delta[n-8] + \delta[n-9] \\
 y[n] &= \{0, 0, 1, 1, 0, 0, -2, -2, 1, 1\}
 \end{aligned}$$



1.13. Consider the continuous-time signal

$$x(t) = \delta(t + 2) - \delta(t - 2).$$

Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^t x(\tau) d\tau.$$

$$y(t) = \int_{-\infty}^t x(\tau) dt = \int_{-\infty}^t (\delta(\tau + 2) - \delta(\tau - 2)) dt = \begin{cases} 0, & t < -2 \\ 1, & -2 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

Therefore,

$$E_\infty = \int_{-2}^2 dt = 4$$

1.14. Consider a periodic signal

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ -2, & 1 < t < 2 \end{cases}$$

with period $T = 2$. The derivative of this signal is related to the “impulse train”

$$g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

with period $T = 2$. It can be shown that

$$\frac{dx(t)}{dt} = A_1 g(t - t_1) + A_2 g(t - t_2).$$

Determine the values of A_1 , t_1 , A_2 , and t_2 .

1.14. The signal $x(t)$ and its derivative $g(t)$ are shown in Figure S1.14.

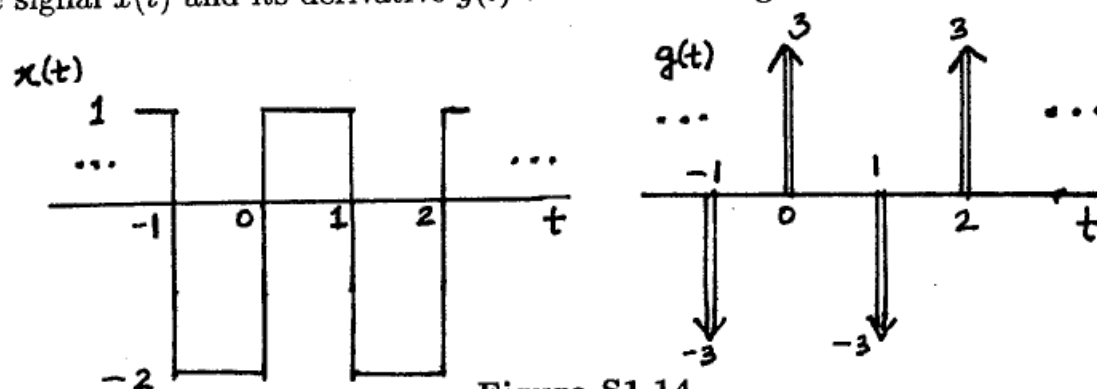


Figure S1.14

Therefore,

$$g(t) = 3 \sum_{k=-\infty}^{\infty} \delta(t - 2k) - 3 \sum_{k=-\infty}^{\infty} \delta(t - 2k - 1)$$

This implies that $A_1 = 3$, $t_1 = 0$, $A_2 = -3$, and $t_2 = 1$.

2.1. Let

$$x[n] = \delta[n] + 2\delta[n - 1] - \delta[n - 3] \quad \text{and} \quad h[n] = 2\delta[n + 1] + 2\delta[n - 1].$$

Compute and plot each of the following convolutions:

(a) $y_1[n] = x[n] * h[n]$ **(b)** $y_2[n] = x[n + 2] * h[n]$

(c) $y_3[n] = x[n] * h[n + 2]$

(a) We know that

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \quad (\text{S2.1-1})$$

The signals $x[n]$ and $h[n]$ are as shown in Figure S2.1.

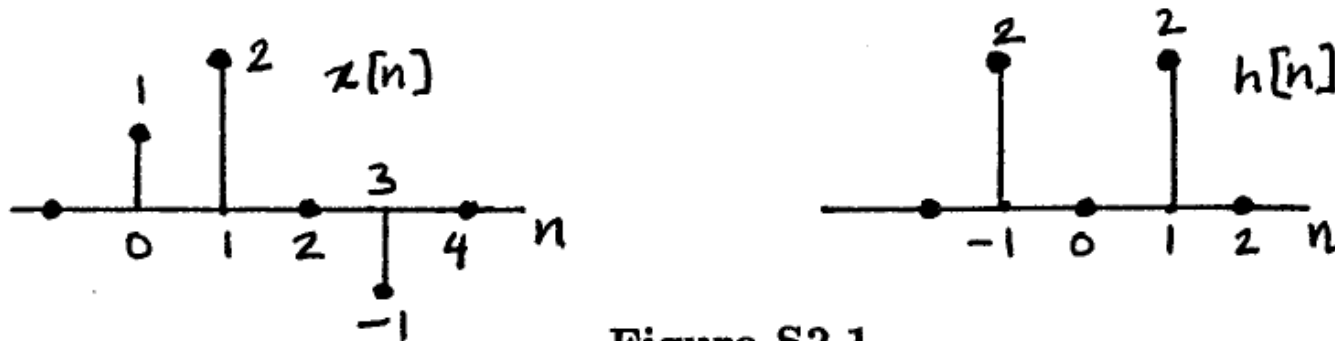


Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$\begin{aligned} y_1[n] &= h[-1]x[n+1] + h[1]x[n-1] \\ &= 2x[n+1] + 2x[n-1] \end{aligned}$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq. (S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

(c) We may rewrite eq. (S2.1-1) as

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq. (S2.1), we see that

$$y_3[n] = y_1[n+2]$$

2.22. For each of the following pairs of waveforms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results.

(a) $\left. \begin{array}{l} x(t) = e^{-\alpha t} u(t) \\ h(t) = e^{-\beta t} u(t) \end{array} \right\}$ (Do this both when $\alpha \neq \beta$ and when $\alpha = \beta$.)

(a) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau \\ &= \int_0^t e^{-\alpha \tau} e^{-\beta(t-\tau)} d\tau, \quad t \geq 0 \end{aligned}$$

Then

$$y(t) = \begin{cases} \frac{e^{-\beta t} \{e^{-(\alpha-\beta)t} - 1\}}{\beta - \alpha} u(t) & \alpha \neq \beta \\ te^{-\beta t} u(t) & \alpha = \beta \end{cases}.$$

$$\begin{aligned} \text{(b)} \quad x(t) &= u(t) - 2u(t-2) + u(t-5) \\ h(t) &= e^{2t}u(1-t) \end{aligned}$$

(b) The desired convolution is

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_0^2 h(t-\tau)d\tau - \int_2^5 h(t-\tau)d\tau. \end{aligned}$$

This may be written as

$$y(t) = \begin{cases} \int_0^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, & t \leq 1 \\ \int_{t-1}^2 e^{2(t-\tau)}d\tau - \int_2^5 e^{2(t-\tau)}d\tau, & 1 \leq t \leq 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}d\tau, & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$

Therefore,

$$y(t) = \begin{cases} (1/2)[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}], & t \leq 1 \\ (1/2)[e^2 + e^{2(t-5)} - 2e^{2(t-2)}], & 1 \leq t \leq 3 \\ (1/2)[e^{2(t-5)} - e^2], & 3 \leq t \leq 6 \\ 0, & 6 < t \end{cases}$$