حل مثال

3.1. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are

$$a_1 = a_{-1} = 2, a_3 = a_{-3}^* = 4j.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

3.1. Using the Fourier series synthesis eq. (3.38),

$$\begin{aligned} x(t) &= a_1 e^{j(2\pi/T)t} + a_{-1} e^{-j(2\pi/T)t} + a_3 e^{j3(2\pi/T)t} + a_{-3} e^{-j3(2\pi/T)t} \\ &= 2e^{j(2\pi/8)t} + 2e^{-j(2\pi/8)t} + 4je^{j3(2\pi/8)t} - 4je^{-j3(2\pi/8)t} \\ &= 4\cos(\frac{\pi}{4}t) - 8\sin(\frac{6\pi}{8}t) \\ &= 4\cos(\frac{\pi}{4}t) + 8\cos(\frac{3\pi}{4}t + \frac{\pi}{2}) \end{aligned}$$

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega t}}$$

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$e^{-j\varphi} = \cos\varphi - j\sin\varphi$$

$$\frac{e^{j\varphi} + e^{-j\varphi}}{2} = \cos\varphi$$

$$\frac{e^{j\varphi} - e^{-j\varphi}}{2j} = \sin\varphi$$

3.2. A discrete-time periodic signal $x[n]$ is real valued and has a fundamental period $N = 5$. The nonzero Fourier series coefficients for $x[n]$ are

$$a_0 = 1, \underline{a_2 = a^*_{-2} = e^{j\pi/4}}, a_4 = a^*_{-4} = 2e^{j\pi/3}.$$

3.2. Using the Fourier series synthesis eq. (3.95).

$$\begin{aligned}
x[n] &= a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2}e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4}e^{-j4(2\pi/N)n} \\
&= 1 + e^{j(\pi/4)}e^{j2(2\pi/5)n} + e^{-j(\pi/4)}e^{-2j(2\pi/5)n} \\
&\quad +2e^{j(\pi/3)}e^{j4(2\pi/N)n} + 2e^{-j(\pi/3)}a_{-4}e^{-j4(2\pi/N)n} \\
&= 1 + 2\cos(\frac{4\pi}{5}n + \frac{\pi}{4}) + 4\cos(\frac{8\pi}{5}n + \frac{\pi}{3}) \\
&= 1 + 2\sin(\frac{4\pi}{5}n + \frac{3\pi}{4}) + 4\sin(\frac{8\pi}{5}n + \frac{5\pi}{6})
\end{aligned}$$

3.3. For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

3.3. The given signal is

$$\begin{aligned} x(t) &= 2 + \boxed{\frac{1}{2}e^{j(2\pi/3)t} + \frac{1}{2}e^{-j(2\pi/3)t}} - \boxed{2je^{j(5\pi/3)t} + 2je^{-j(5\pi/3)t}} \\ &= 2 + \frac{1}{2}e^{\underline{j2(2\pi/6)t}} + \frac{1}{2}e^{-j2(2\pi/6)t} - 2je^{j5(2\pi/6)t} + 2je^{-j5(2\pi/6)t} \end{aligned}$$

From this, we may conclude that the fundamental frequency of $x(t)$ is $2\pi/6 = \pi/3$. The non-zero Fourier series coeffcients of $x(t)$ are:

$$a_0 = 2, \qquad a_2 = a_{-2} = \frac{1}{2}, \qquad a_5 = a_{-5}^* = -2j$$

3.38. Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \le n \le 2 \\ -1, & -2 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}.$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k],$$

determine the Fourier series coefficients of the output $y[n]$.

3.38. The frequency response of the system may be evaluated as

$$H(e^{j\omega}) = -e^{2j\omega} - e^{j\omega} + 1 + e^{-j\omega} + e^{-2j\omega}.$$

For $x[n]$, $N = 4$ and $\omega_0 = \pi/2$. The FS coefficients of the input $x[n]$ are

$$a_k = \frac{1}{4}, \qquad \text{for all } n.$$

Therefore, the FS coefficients of the output are

$$b_k = a_k H(e^{jk\omega_0}) = \frac{1}{4}[1 - e^{jk\pi/2} + e^{-jk\pi/2}].$$

4.1. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

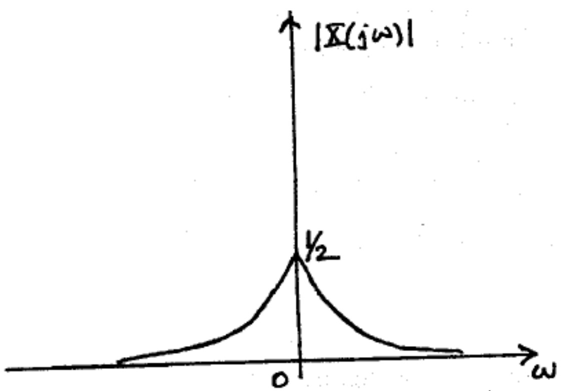
(a) $e^{-2(t-1)}u(t-1)$ **(b)** $e^{-2|t-1|}$

Sketch and label the magnitude of each Fourier transform.

4.1. **(a)** Let $x(t) = e^{-2(t-1)}u(t-1)$. Then the Fourier transform $X(j\omega)$ of $x(t)$ is:

$$
\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} e^{-2(t-1)} u(t-1) e^{-j\omega t} dt \\
&= \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t} dt \\
&= e^{-j\omega}/(2+j\omega)
\end{aligned}
$$

$|X(j\omega)|$ is as shown in Figure S4.1.

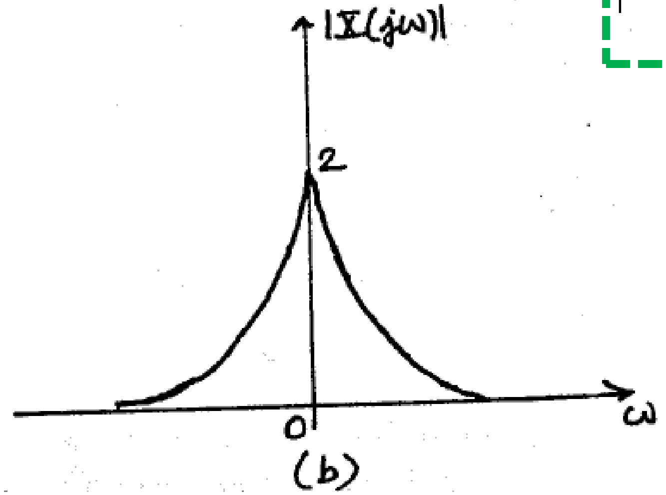


(a) **Figure S4.1**

(b) Let $x(t) = e^{-2|t-1|}$. Then the Fourier transform $X(j\omega)$ of $x(t)$ is:

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} e^{-2|t-1|} e^{-j\omega t} dt \\ &= \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t} dt + \int_{-\infty}^{1} e^{2(t-1)} e^{-j\omega t} dt \\ &= e^{-j\omega}/(2 + j\omega) + e^{-j\omega}/(2 - j\omega) \\ &= 4e^{-j\omega}/(4 + \omega^2) \end{aligned}$$

$$\boxed{|t-1| = \begin{cases} t-1 & t \geq 1 \\ -(t-1) = 1-t & t < 1 \end{cases}}$$



4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $\delta(t+1)+\delta(t-1)$ **(b)** $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

4.2. (a) Let $x_1(t)=\delta(t+1)+\delta(t-1)$. Then the Fourier transform $X_1(j\omega)$ of $x(t)$ is:

$$\delta(t-t_0)x(t)=\delta(t-t_0)x(t_0)$$

$$\int_{-\infty}^{\infty}\delta(t-t_0)=1$$

$$\begin{aligned} X_1(j\omega) &= \int_{-\infty}^{\infty}[\delta(t+1)+\delta(t-1)]e^{-j\omega t}dt \\ &= e^{j\omega}+e^{-j\omega}=2\cos\omega \end{aligned}$$

$|X_1(j\omega)|$ is as sketched in Figure S4.2.

$$\delta(t-1)e^{-j\omega t}=\delta(t-1)e^{-j\omega}$$

$$\delta(t+1)e^{-j\omega t}=\delta(t+1)e^{j\omega}$$

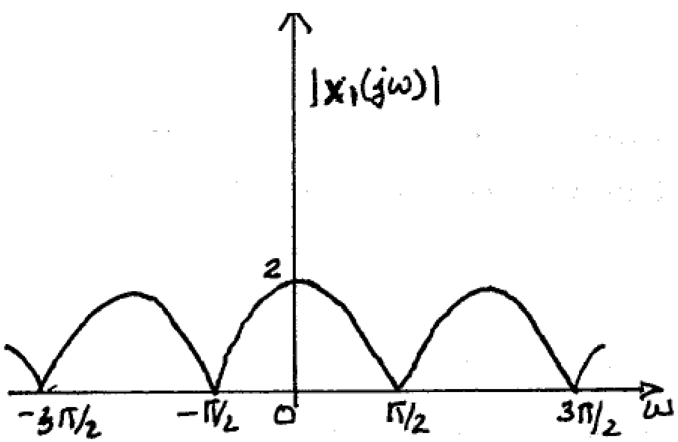


Figure S4.2

$$\delta(t-t_0)\, f(t) = f(t_0)\delta(t-t_0)$$

$$\delta(t+1)e^{-jwt} = e^{jw}\delta(t+1)$$

$$\int \left(e^{jw}\delta(t+1) + e^{-jw}\delta(t-1)\right)dt = \int e^{jw}\delta(t+1)dt + \int e^{-jw}\delta(t-1)dt$$

$$e^{jw}\int \delta(t+1)dt + e^{-jw}\int \delta(t-1)dt$$

$$= e^{jw} + e^{-jw}$$

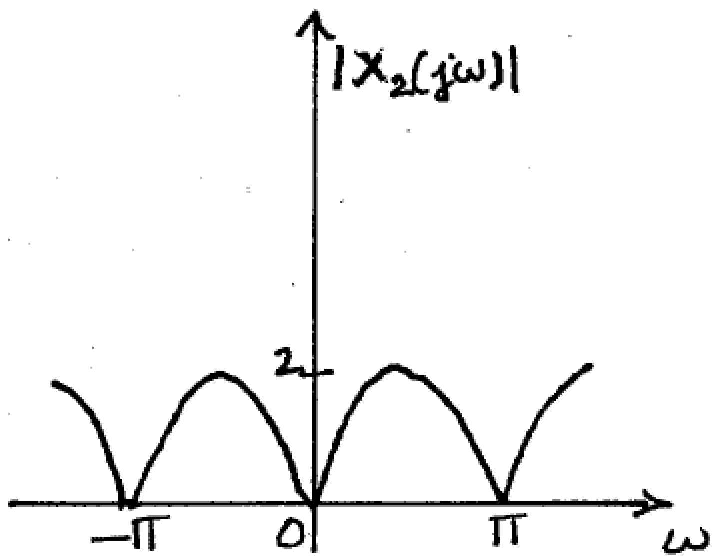
(b) The signal $x_2(t) = u(-2-t) + u(t-2)$ is as shown in the figure below. Clearly,

$$\frac{d}{dt}\{u(-2-t) + u(t-2)\} = \delta(t-2) - \delta(t+2)$$

Therefore,

$$\begin{aligned} X_2(j\omega) &= \int_{-\infty}^{\infty} [\delta(t-2) - \delta(t+2)] e^{-j\omega t} dt \\ &= e^{-2j\omega} - e^{2j\omega} = -2j\sin(2\omega) \end{aligned}$$

$|X_1(j\omega)|$ is as sketched in Figure S4.2.



4.3. Determine the Fourier transform of each of the following periodic signals:
(a) $\sin(2\pi t + \frac{\pi}{4})$ **(b)** $1 + \cos(6\pi t + \frac{\pi}{8})$

4.3. (a) The signal $x_1(t) = \sin(2\pi t + \pi/4)$ is periodic with a fundamental period of $T = 1$. This translates to a fundamental frequency of $\omega_0 = 2\pi$. The nonzero Fourier series coefficients of this signal may be found by writing it in the form

$$\begin{aligned} x_1(t) &= \frac{1}{2j}\left(e^{j(2\pi t+\pi/4)} - e^{-j(2\pi t+\pi/4)}\right) \\ &= \frac{1}{2j}e^{j\pi/4}e^{j2\pi t} - \frac{1}{2j}e^{-j\pi/4}e^{-j2\pi t} \end{aligned}$$

Therefore, the nonzero Fourier series coefficients of $x_1(t)$ are

$$a_1 = \frac{1}{2j}e^{j\pi/4} \qquad a_{-1} = -\frac{1}{2j}e^{-j\pi/4}$$

From Section 4.2, we know that for periodic signals, the Fourier transform consists of a train of impulses occurring at $k\omega_0$. Furthermore, the area under each impulse is 2π times the Fourier series coefficient a_k. Therefore, for $x_1(t)$, the corresponding Fourier transform $X_1(j\omega)$ is given by

$$\begin{aligned} X_1(j\omega) &= 2\pi a_1\delta(\omega - \omega_0) + 2\pi a_{-1}\delta(\omega + \omega_0) \\ &= (\pi/j)e^{j\pi/4}\delta(\omega - 2\pi) - (\pi/j)e^{-j\pi/4}\delta(\omega + 2\pi) \end{aligned}$$

(b) The signal $x_2(t) = 1 + \cos(6\pi t + \pi/8)$ is periodic with a fundamental period of $T = 1/3$. This translates to a fundamental frequency of $\omega_0 = 6\pi$. The nonzero Fourier series coefficients of this signal may be found by writing it in the form

$$\boxed{6\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{3}}$$

$$\begin{aligned} x_2(t) &= 1 + \frac{1}{2}\left(e^{j(6\pi t+\pi/8)} - e^{-j(6\pi t+\pi/8)}\right) \\ &= 1 + \frac{1}{2}e^{j\pi/8}e^{j6\pi t} + \frac{1}{2}e^{-j\pi/8}e^{-j6\pi t} \end{aligned}$$

Therefore, the nonzero Fourier series coefficients of $x_2(t)$ are

$$a_0 = 1, \qquad a_1 = \frac{1}{2}e^{j\pi/8}, \qquad a_{-1} = \frac{1}{2}e^{-j\pi/8}$$

From Section 4.2, we know that for periodic signals, the Fourier transform consists of a train of impulses occurring at $k\omega_0$. Furthermore, the area under each impulse is 2π times the Fourier series coefficient a_k. Therefore, for $x_2(t)$, the corresponding Fourier transform $X_2(j\omega)$ is given by

$$\begin{aligned} X_2(j\omega) &= \underline{2\pi a_0\delta(\omega)} + 2\pi a_1\delta(\omega - \omega_0) + 2\pi a_{-1}\delta(\omega + \omega_0) \\ &= 2\pi\delta(\omega) + \pi e^{j\pi/8}\delta(\omega - 6\pi) + \pi e^{-j\pi/8}\delta(\omega + 6\pi) \end{aligned}$$

4.4. Use the Fourier transform synthesis equation (4.8) to determine the inverse Fourier transforms of:

(a) $X_1(j\omega) = 2\pi\,\delta(\omega) + \pi\,\delta(\omega - 4\pi) + \pi\,\delta(\omega + 4\pi)$

(b) $X_2(j\omega) = \begin{cases} 2, & 0 \le \omega \le 2 \\ -2, & -2 \le \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$

$$\boxed{x(t) = \left(1/2\pi\right)\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega}$$

4.4. **(a)** The inverse Fourier transform is

$$\begin{aligned} x_1(t) &= (1/2\pi)\int_{-\infty}^{\infty}[2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)]e^{j\omega t}d\omega \\ &= (1/2\pi)[2\pi e^{j0t} + \pi e^{j4\pi t} + \pi e^{-j4\pi t}] \\ &= 1 + (1/2)e^{j4\pi t} + (1/2)e^{-j4\pi t} = 1 + \cos(4\pi t) \end{aligned}$$

(b) The inverse Fourier transform is

$$\begin{aligned} x_2(t) &= (1/2\pi)\int_{-\infty}^{\infty} X_2(j\omega)e^{j\omega t}d\omega \\ &= (1/2\pi)\int_0^2 2e^{j\omega t}d\omega + (1/2\pi)\int_{-2}^{0}(-2)e^{j\omega t}d\omega \\ &= (e^{j2t} - 1)/(\pi j t) - (1 - e^{-j2t})/(\pi j t) \\ &= -(4j\sin^2 t)/(\pi t) \end{aligned}$$

$$\delta(\omega-\omega_0)F(\omega)=\delta(\omega-\omega_0)F(\omega_0)$$

$$\int\delta(\omega-\omega_0)F(\omega)d\omega=\int\delta(\omega-\omega_0)F(\omega_0)d\omega=F(\omega_0)\int\delta(\omega-\omega_0)d\omega=F(\omega_0)$$

$$\int 2\pi\delta(\omega)e^{j\omega t}d\omega=2\pi e^{j(0)t}\int\delta(\omega)d\omega=2\pi$$

$$\delta(\omega)e^{j\omega t}=\delta(\omega)e^{j(0)t}=\delta(\omega)$$

4.11. Given the relationships

$$y(t) = x(t) * h(t)$$

and

$$g(t) = x(3t) * h(3t),$$

and given that $x(t)$ has Fourier transform $X(j\omega)$ and $h(t)$ has Fourier transform $H(j\omega)$, use Fourier transform properties to show that $g(t)$ has the form

$$g(t) = Ay(Bt).$$

Determine the values of A and B.

4.11. We know that

$$x(3t) \stackrel{FT}{\longleftrightarrow} \frac{1}{3}X(j\frac{\omega}{3}), \qquad h(3t) \stackrel{FT}{\longleftrightarrow} \frac{1}{3}H(j\frac{\omega}{3})$$

Therefore,

$$G(j\omega) = \mathcal{F}T\{x(3t) * h(3t)\} = \frac{1}{9}X(j\frac{\omega}{3})H(j\frac{\omega}{3})$$

Now note that

$$Y(j\omega) = \mathcal{F}T\{x(t) * h(t)\} = X(j\omega)H(j\omega)$$

From this, we may write

$$Y(j\frac{\omega}{3}) = X\left(j\frac{\omega}{3}\right) H\left(j\frac{\omega}{3}\right)$$

Using this in eq. (**), we have

$$\boxed{G(j\omega) = \frac{1}{9}Y(j\frac{\omega}{3})}$$

and

$$g(t) = \frac{1}{3}y(3t).$$

Therefore, $A = \frac{1}{3}$ and $B = 3$.

4.21. Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t}\cos\omega_0 t]u(t),\ \alpha > 0$

(b) $e^{-3|t|}\sin 2t$

(c) $x(t) = \begin{cases} 1 + \cos\pi t, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k \delta(t - kT),\ |\alpha| < 1$

(e) $[te^{-2t}\sin 4t]u(t)$

(f) $\left[\frac{\sin\pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$

(g) $x(t)$ as shown in Figure P4.21(a)

(h) $x(t)$ as shown in Figure P4.21(b)

(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

(j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

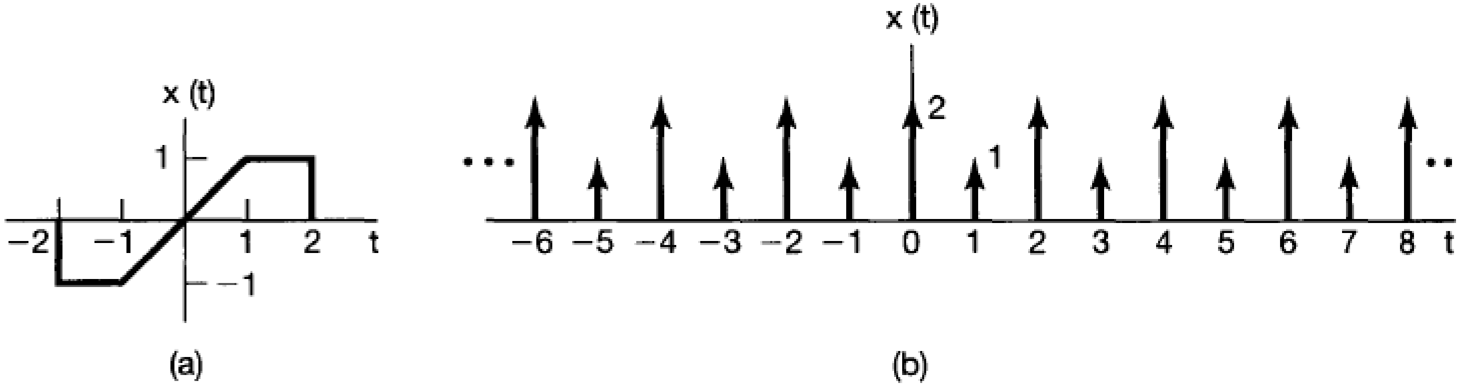


Figure P4.21

4.21. (a) The given signal is

$$e^{-\alpha t}\cos(\omega_0 t)u(t) = \frac{1}{2}e^{-\alpha t}e^{j\omega_0 t}u(t) + \frac{1}{2}e^{-\alpha t}e^{-j\omega_0 t}u(t).$$

Therefore,

$$X(j\omega) = \frac{1}{2(\alpha - j\omega_0 + j\omega)} - \frac{1}{2(\alpha - j\omega_0 + j\omega)}.$$

(b) The given signal is

$$x(t) = \boxed{e^{-3t}\sin(2t)u(t)} + \boxed{e^{3t}\sin(2t)u(-t).}$$

x1(t) x2(t)

We have

$$x_1(t) = e^{-3t}\sin(2t)u(t) \stackrel{FT}{\longleftrightarrow} X_1(j\omega) = \frac{1/2j}{3 - j2 + j\omega} - \frac{1/2j}{3 + j2 + j\omega}.$$

Also,

$$\boxed{x_2(t) = e^{3t}\sin(2t)u(-t) = -x_1(-t)} \stackrel{FT}{\longleftrightarrow} X_2(j\omega) = -X_1(-j\omega) = \frac{1/2j}{3 - j2 - j\omega} - \frac{1/2j}{3 + j2 - j\omega}.$$

Therefore,

$$X(j\omega) = X_1(j\omega + X_2(j\omega) = \frac{3j}{9 + (\omega + 2)^2} - \frac{3j}{9 + (\omega - 2)^2}.$$

$$\frac{1}{2}e^{-\alpha t}e^{j\omega_0 t} = \frac{1}{2}e^{-(\alpha - j\omega_0)t}$$

$$x_1(t) = e^{3t}\sin(2t)u(t)$$

$$t \to -t$$

$$x_1(-t) = e^{-3t}\sin(-2t)u(-t)$$

$$\boxed{\sin(-\alpha) = -\sin(\alpha)}$$

$$\sin(-2t) = -\sin(2t)$$

$$\Rightarrow x_1(-t) = -e^{-3t}\sin(2t)u(-t)$$

$$x_2(t) = e^{-3t}\sin(2t)u(-t)$$

$$\Rightarrow x_2(t) = -x_1(-t)$$

(c) Using the Fourier transform analysis equation (4.9) we have

$$\frac{1}{2}e^{-\alpha t}e^{j\omega_0 t} = \frac{1}{2}e^{-(\alpha - j\omega_0)t} \quad X(j\omega) = \frac{2\sin\omega}{\omega} + \frac{\sin\omega}{\pi - \omega} - \frac{\sin\omega}{\pi + \omega}.$$

(d) Using the Fourier transform analysis equation (4.9) we have

$$X(j\omega) = \frac{1}{1 - \alpha e^{-j\omega T}}.$$

(e) We have

$$x(t) = (1/2j)te^{-2t}e^{j4t}u(t) - (1/2j)te^{-2t}e^{-j4t}u(t).$$

Therefore,

$$X(j\omega) = \frac{1/2j}{(2 - j4 + j\omega)^2} - \frac{1/2j}{(2 + j4 - j\omega)^2}.$$

$$x_1(t) = (1/2j)e^{-2t}e^{j4t}u(t) - (1/2j)e^{-2t}e^{-j4t}u(t)$$

$$x(t) = tx_1(t)$$

$$f(t) = \sum \alpha^k \delta(t - KT) = \delta(t) + \alpha\delta(t - T) + \alpha^2\delta(t - 2T) + \ldots$$

$$\int \alpha \boxed{\delta(t - T)e^{-j\omega t}}\, dt = \alpha e^{-j\omega T} \int \delta(t - T)\, dt = \alpha e^{-j\omega T}$$

$$\delta(t - T)e^{-j\omega t} = \delta(t - T)e^{-j\omega T}$$

$$F(j\omega) = 1 + \alpha e^{-j\omega T} + \alpha^2 e^{-j2\omega T} + \alpha^3 e^{-j3\omega T} + \ldots$$

(f) We have

$$x_1(t) = \frac{\sin \pi t}{\pi t} \overset{FT}{\longleftrightarrow} X_1(j\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}.$$

Also

$$x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)} \overset{FT}{\longleftrightarrow} X_2(j\omega) = \begin{cases} e^{-2\omega}, & |\omega| < 2\pi \\ 0, & \text{otherwise} \end{cases}.$$

$$x(t) = x_1(t)x_2(t) \overset{FT}{\longleftrightarrow} X(j\omega) = \frac{1}{2\pi}\{X_1(j\omega) * X_2(j\omega)\}.$$

Therefore,

$$X(j\omega) = \begin{cases} e^{-j\omega}, & |\omega| < \pi \\ (1/2\pi)(3\pi+\omega)e^{-j\omega}, & -3\pi < \omega < -\pi \\ (1/2\pi)(3\pi-\omega)e^{-j\omega}, & \pi < \omega < 3\pi \\ 0, & \text{otherwise} \end{cases}.$$

(g) Using the Fourier transform analysis eq. (4.9) we obtain

$$X(j\omega) = \frac{2j}{\omega}\left[\cos 2\omega - \frac{\sin \omega}{\omega}\right].$$

$$\boxed{X(j\omega) = \int_{-2}^{-1} (-1)e^{-j\omega t}\,dt + \int_{-1}^{1} te^{-j\omega t}\,dt + \int_{1}^{2} (1)e^{-j\omega t}\,dt}$$

(h) If

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k),$$

then

$$x(t) = 2x_1(t) + x_1(t-1).$$

Therefore,

$$X(j\omega) = X_1(j\omega)[2 + e^{-\omega}] = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)[2 + (-1)^k].$$

9.7. How many signals have a Laplace transform that may be expressed as

$$\frac{(s-1)}{(s+2)(s+3)(s^2+s+1)}$$

in its region of convergence?

9.7. We may find different signals with the given Laplace transform by choosing different regions of convergence. The poles of the given Laplace transform are

$$s_0 = -2, \quad s_1 = -3, \quad s_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}j, \quad s_3 = -\frac{1}{2} - \frac{\sqrt{3}}{2}j.$$

Based on the locations of these poles, we my choose from the following regions of convergence:

(i) $\mathcal{R}e\{s\} > -\frac{1}{2}$

(ii) $-2 < \mathcal{R}e\{s\} < -\frac{1}{2}$

(iii) $-3 < \mathcal{R}e\{s\} < -2$

(iv) $\mathcal{R}e\{s\} < -3$

9.9. Given that

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \qquad \Re e\{s\} > \Re e\{-a\},$$

determine the inverse Laplace transform of

$$X(s) = \frac{2(s+2)}{s^2+7s+12}, \qquad \Re e\{s\} > -3.$$

9.9. Using partial fraction expansion

$$X(s) = \frac{4}{s+4} - \frac{2}{s+3}.$$

Taking the inverse Laplace transform,

$$x(t) = 4e^{-4t}u(t) - 2e^{-3t}u(t).$$

9.13. Let

$$g(t) = x(t) + \alpha x(-t),$$

where

$$x(t) = \beta e^{-t}u(t)$$

and the Laplace transform of $g(t)$ is

$$\boxed{G(s) = \frac{s}{s^2 - 1},} \qquad -1 < \Re e\{s\} < 1.$$

Determine the values of the constants α and β.

9.13. We have

$$\boxed{X(s) = \frac{\beta}{s+1},} \qquad \Re e\{s\} > -1.$$

Also,

$$\boxed{G(s) = X(s) + \alpha X(-s),} \qquad -1 < \Re e\{s\} < 1.$$

Therefore,

$$\boxed{G(s) = \beta\left[\frac{1 - s + \alpha s + \alpha}{1 - s^2}\right]}$$

Comparing with the given equation for $G(s)$,

$$\alpha = -1, \qquad \beta = \frac{1}{2}.$$

9.15. Consider two right-sided signals $x(t)$ and $y(t)$ related through the differential equations

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

and

$$\frac{dy(t)}{dt} = 2x(t).$$

Determine $Y(s)$ and $X(s)$, along with their regions of convergence.

9.15. Taking the Laplace transforms of both sides of the two differential equations, we have

$$\boxed{sX(s) = -2Y(s) + 1} \quad \text{and} \quad \boxed{sY(s) = 2X(s).}$$

Solving for $X(s)$ and $Y(s)$, we obtain

$$X(s) = \frac{s}{s^2+4} \quad \text{and} \quad Y(s) = 2s^2 + 4.$$

The region of convergence for both $X(s)$ and $Y(s)$ is $\mathcal{R}e\{s\} > 0$ because both are right-sided signals.

10.3. Let

$$x[n] = (-1)^n u[n] + \alpha^n u[-n - n_0].$$

Determine the constraints on the complex number α and the integer n_0, given that the ROC of $X(z)$ is

$$\boxed{1 < |z| < 2.}$$

10.3. By using eq. (9.3), we can easily show that

$$\alpha^n u[-n - n_0] \overset{z}{\longleftrightarrow} \frac{-z^{-n_0}}{1 - \alpha z^{-1}}, \qquad |z| < |\alpha|.$$

We then obtain

$$X(z) = \frac{1}{1 + z^{-1}} + \frac{-z^{-n_0-1}}{1 - \alpha z^{-1}}, \qquad \boxed{1 < |z| < |\alpha|.}$$

Therefore, $|\alpha|$ has to be 2. n_0 can take on any value.

10.4. Consider the signal

$$x[n] = \begin{cases} (\frac{1}{3})^n \cos(\frac{\pi}{4}n), & n \le 0 \\ 0, & n > 0 \end{cases}.$$

Determine the poles and ROC for $X(z)$.

10.4. Using eq. (9.3), we have

$$\begin{aligned}
X(z) &= \sum_{\boxed{n=-\infty}}^{\boxed{0}} (\frac{1}{3})^n \cos(\frac{\pi}{4}n) z^{-n} \\
&= (1/2) \sum_{n=-\infty}^{0} (\frac{1}{3})^n e^{j\pi n/4} z^{-n} + (1/2) \sum_{n=-\infty}^{0} (\frac{1}{3})^n e^{-j\pi n/4} z^{-n} \\
&= (1/2) \sum_{n=0}^{\infty} (\frac{1}{3})^{-n} e^{-j\pi n/4} z^{n} + (1/2) \sum_{n=0}^{\infty} (\frac{1}{3})^{-n} e^{j\pi n/4} z^{n} \\
&= (1/2) \frac{1}{1 - 3e^{-j\pi/4} z} + (1/2) \frac{1}{1 - 3e^{j\pi/4} z}, \qquad |z| < \frac{1}{3}
\end{aligned}$$

The poles are at $z = \frac{1}{3}e^{j\pi/4}$ and $z = \frac{1}{3}e^{-j\pi/4}$.