KPI Method for Dynamic Surface Reconstruction With B-splines based on Sparse Data

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Abstract High dimensional B-splines are catching tremendous attentions in fields of Isogeometry Analysis, dynamic surface reconstruction and so on. However, the actual measured data are usually sparse and nonuniform, which might not meet the requirement of traditional B-spline algorithms. In this paper, we present a novel dynamic surface reconstruction approach, which is a three-dimensional B-spline key points interpolation method (KPI), aimed at dealing with sparse distributed data. This method includes an algorithm of data set generation based on Kriging and a control point solving technology based on key points interpolation. The data set generation method can significantly reduce the number of parameters and generate a proper data set which may promisingly catch the trend of sparse data. In fact, the control points solving method ensures the three-dimensional B-spline function to interpolate the sparse data points precisely while approximating the other data points under least square premise. To demonstrate, the KPI method applied to a temperature data set. It is shown that the proposed three-dimensional B-spline preserves the dynamic characteristics and interpolate the temperature data with fewer control points than traditional B-spline interpolation algorithms. We argue that the KPI method provides an efficient way of interpolating sparse distributed data for dynamic surfaces reconstruction problems.

Keywords B-spline · Dynamic surface reconstruction · Sparse data · Kriging method · Key points interpolation · Parameter reduction

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1 Introduction

B-spline is widely used in the fields of reverse engineering and computer-aided design, due to its global smoothness and local regulation performance. Specifically, the two-dimensional B-spline function with parameters u and v, which has superior natures such as continuity and local control, is usually applied in surface fitting benefited by its simple form of a tensor product of two univariate B-spline curves [1-3].

Actually, a tensor product can be generalized to three or more dimensions by adding additional summations and B-spline basis functions. In this way, the parameters except u and v can be used to present many other continuous variables such as time, altitude, material, etc. [3-8]. For instance, Yang and Qian [7] introduced an integrated design and analysis approach for heterogeneous object realization with high-dimensional B-spline, which allows for direct interaction between the design and analysis model. Cameron and Richard [8] proposed a metamodel called HyPerModel, where the B-spline is used as the basis for design space metamodels. Koch [3] estimated a three-dimensional NURBS surface for time-depending problems based on lofting method, which can efficiently reduce the numerical complexity, and Koch and Schmidt [4] applied a similar method to locally International Reference Ionosphere improvement. Schmidt et al [6] presented a four-dimensional B-spline in space and time for modeling the electron density of the ionosphere in a certain region.

Among all the studies in three or higher-dimensional B-splines, the reconstruction of dynamic surfaces is a research hotspot [3-6]. Basically, the reconstruction of dynamic surfaces can be referred as a reverse engineering problem where the considered surface varies with time [3]. Dynamic surface fitting is widely used in aerospace, geoscience, climatology, image morphing and marine research. It is a process referring to the problem of fitting a time-varying surface through a series of data samples obtained in a period of time. The data points to be interpolated are measured at different time epochs in a certain region, and the resulted three-dimensional model gives the shapes between time epochs. Generally, the interpolation process of dynamic surface can be described as two procedures: first, for each time epoch, the corresponding data set is interpolated with a 2D B-spline surface, second, the surfaces of each epoch obtained in the first step is interpolated to capture the shape between the time epochs. The key point for B-spline interpolation technique, including three-dimensional B-spline-based dynamic surface interpolation method, is control points solving, which has been widely studied in [3,13,19].

During the process of reconstructing the dynamic surface, the majority of actual measured data are naturally sparse and nonuniform, such as the data obtained from persistent land sensors, geospatial data and LIDAR scanners [9-14, 17]. However, a well-distributed density data set and detailed information of adjacent relationship is crucial for a favorable B-spline interpolation method . If the data is too scatter and sparse, the reliability and precision of the control points solving result might be restricted greatly [9]. Consequently, some sparse data interpolation methods are proposed to deal with this problem. For example, Kriging is an easy and widely used way for surrogate modeling applications including sparse data interpolation, however, it is inefficient to obtain the surface while there are too many data points and may lost robustness in contrast with tensor-product splines [10]. Barthelmann et al. [11] implement sparse grids with the extrema of the Chebyshev polynomials instead of full grids to interpolate high-dimensional sparse data. Tang et al. [12] developed a multi-resolution learning framework for geospatial data interpolation and demonstrate the method in PM2.5 interpolation. Lee et al. [13] presented a multilevel B-spline-based scatter data interpolation method which can be applied to image warping and image morphing.

In this paper, by introducing key points interpolation, a novel three-dimensional B-spline surface reconstruction method for sparse distributed data is proposed. The main idea of this method includes a data set generation algorithm based on Kriging and a technique for B-spline control points solving based on key points interpolation. Firstly, the data set generation algorithm is composed with two parts, one part is a parameter simplification process which can significantly reduce the number of parameters, the other one is a data complement process based on Kriging which ensures the data to correspond to the trend of sparse data. Secondly, the B-spline control points solving method can interpolate any labeled points in the data set precisely while approximating the rest unlabeled points under least square premise. By combining the two parts mentioned above, the proposed method provides a dynamic surface reconstruction strategy which can integrate both the sparse distributed data interpolation for single surface and the time epochs interpolation.

The rest of the paper is organized as follows. In Section 2, an overview of KPI method is presented. In Section 3, we introduce a method for parameters reduction, a data set complement algorithm based on Kriging, and a B-spline control points solving method for key points interpolation. In Section 4, an experimental result is proposed to demonstrate the usefulness and quality of our method.. The concluding remarks are given in Section 5.

2 Overview of KPI method

Generally, the procedures for dynamic surface reconstruction with three-dimensional B-spline are closely related to B-spline surface reconstruction problems, which include parameterizing the sparse data set and solving control points of the dynamic surface[3,5,13]. Based on traditional B-spline interpolation algorithms, our KPI method is introduced as four steps.

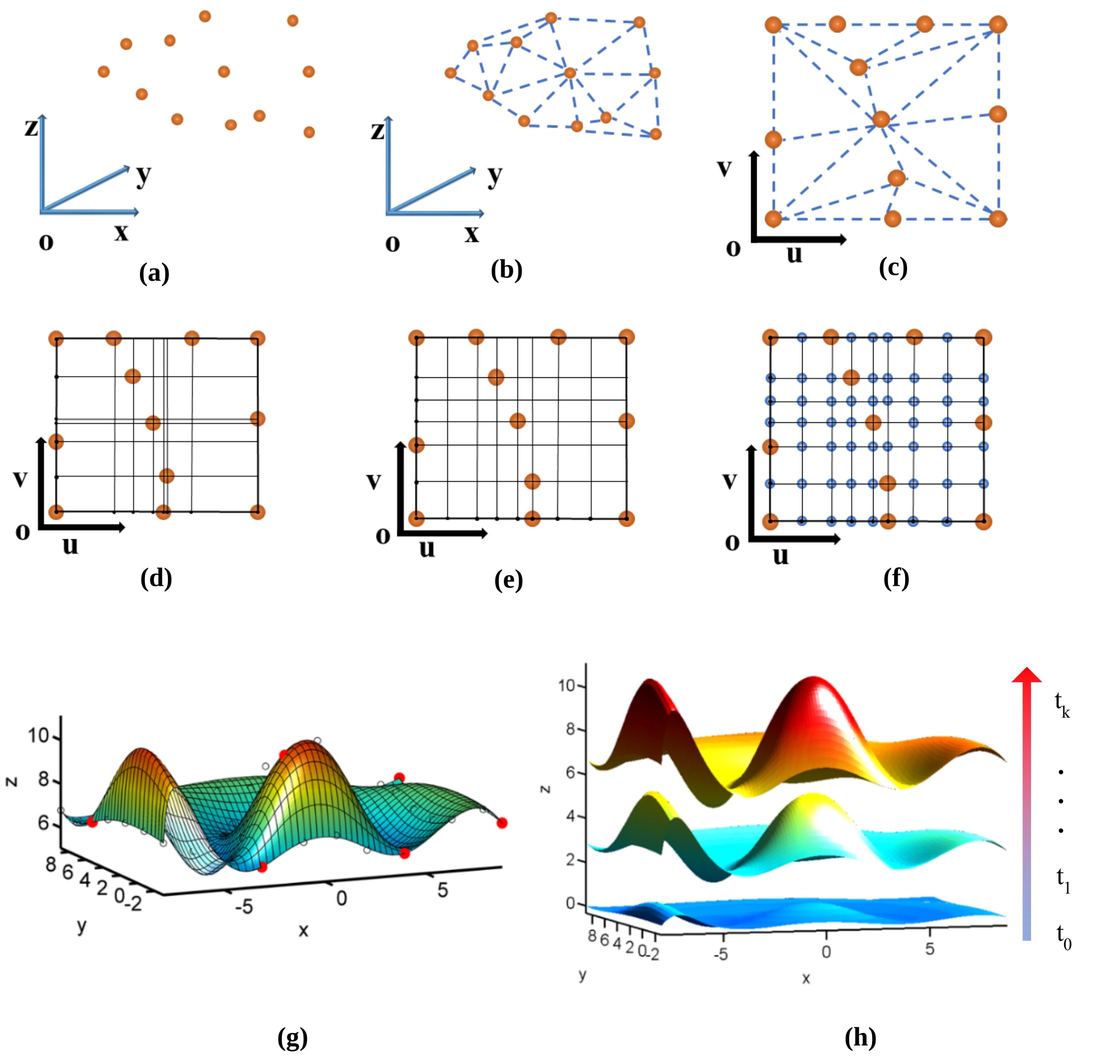


Fig. 1 The overview process in KPI method. (a) The sparse data of each time epoch. (b) Triangulation results of sparse data. (c)-(d) Parameterization of the triangulation mesh. (e) Mesh simplification method. (f) Data set completion with Kriging. (g) surface reconstruction based on key points interpolation. The red dots are original sparse data to be interpolated. (h) Dynamic surface reconstruction. The three surfaces are at different time t_0, t_1,... t_k,

Step 1. Triangulate the sparse data points (Fig. 1(a)) for each time epoch. For the forthcoming processes of parameterization and interpolation, the data of each surface should be triangulated to generate a mesh surface (Fig. 1(b)). Further, if the border of each surface is explicit (for example, meteorological data interpolation of a certain region), the border needs to be simplified until the number of points of the border and the number of inner points are of the same order of magnitude.

Step 2. Parameterize the data of each surface. When the adjunction information of the scatter points of each surface is obtained, it is necessary to parameterize the sparse data for further B-spline interpolation (Fig. 1(c)). The methods for point parameterization based on scatter points data are well studied. Here we select mean value coordination method [18], which is a simple and easy approach to generate the parameterization domain. We also present a method to decrease the number of parameters (Fig. 1(d)-Fig. 1(e)).

Step 3. Complete missing data points of the parameter domain. After step 2, a 2D mesh grid of parameters is obtained. However, there are not sufficient data points on the grid of parameter domain for each surface. More data points need to be generated on each grid cross-point of the parameter domain. Here we use the Kriging interpolation method to complete the data set (Fig. 1(f)).

Step 4. Solve the control points of three-dimensional B-spline based on key points interpolation. After completing the missing data of the parameter domain, the data set is interpolated with a new three-dimensional B-spline interpolation method based on key points interpolation (Fig. 1(g)-Fig. 1(h)). The method ensures to interpolate the original sparse data (red dots in Fig. 1(g)) of the data set precisely while approximate the data generated by kriging method (black circles in Fig. 1(g)) under least square premise. Compared with traditional B-spline interpolation algorithms, the proposed algorithm can interpolate the sparse data with fewer control points, reducing data storage capacity while ensuring the accuracy.

3 The details of the KPI method

In this section, our method is described in details. Let $\mathrm{Q} = \{\mathbf{q}_0, \boldsymbol{q}_1, \cdots, \boldsymbol{q}_{m_1}\}$ be a set of points in $\mathscr{R}^{\mathrm{d}}$, and $\tilde{Q} = \{\tilde{\boldsymbol{q}}_0, \tilde{\boldsymbol{q}}_1, \cdots, \tilde{\boldsymbol{q}}_{\mathrm{m}_2}\}$ be a subset of Q. Let f_1 be a function on the set Q, and f_2 be a function on $\tilde{Q}$. Suppose the valve of f_2 on $\tilde{Q}$ is given. $\mathrm{f}_1(\tilde{\boldsymbol{q}}_i) = f_2(\tilde{\boldsymbol{q}}_i), \forall \tilde{q}_i \in \tilde{S}$. The purpose of the interpolation is to find the function f_1 on Q with the given value on $\tilde{Q}$. In this paper, a three-dimensional B-spline function $\mathbf{M}$ which reconstructs dynamic surfaces is the function f_1 over Q, while the data points $\tilde{Q}$ is given as the form of sparsely sampled data. The details of KPI method are presented as follows:

3.1 Triangulation and border simplification of the original data set

To generate the parameter region of each surface for B-spline interpolation, we first use Delaunay triangulation method to generate the triangular mesh of each surface. Delaunay triangulation method is applied widely in many areas, including geographical information system, finite element analysis and surface reconstruction [14]. However, if the border of each surface is explicit (for example, the temperature or the topographic information of a certain region), the number of border points may be a few orders of magnitude higher than the sparse data (Fig. 2(a)), which may cause the triangulation result to be extremely nonuniform. In that case, the border should be simplified until the number of border points and the number of sparse data points of each surface are of the same order of magnitude. Let $\tilde{Q}$ be the sparse points, and B be the border points corresponding, $\mathrm{B} \not\subset \tilde{Q}$. To simplify the border points, we select the feature points $\tilde{B} \subseteq B$, which is shown as grey points in Fig. 2(b), to replace the border points. The feature points $\{\tilde{B}_0, \tilde{B}_1, \cdots, \tilde{B}_r\}$ are defined to be curvature extrema, cusp, inflection points, and the discontinuities of curvature [15]. Dmitry and Zsolt [16] presented a simple method to detect feature points of the border. After feature point detection, the border B can be replaced by the feature points (shown in Fig. 2(c)). Then triangulate the sparse data $\tilde{Q}$ and feature points $\tilde{B}$ of the border B. Fig. 2(d) shows the triangulation result of the data set $\tilde{Q}$ and $\tilde{B}$.

After border simplification and triangulation process, we obtain the triangular mesh M= (v, E, S) of each surface, where $v = \tilde{Q} \cup \tilde{B}$ represent the vertex sets and E present the edges of the triangulation mesh. The triangular mesh M shows the adjacent information of the sparse data.

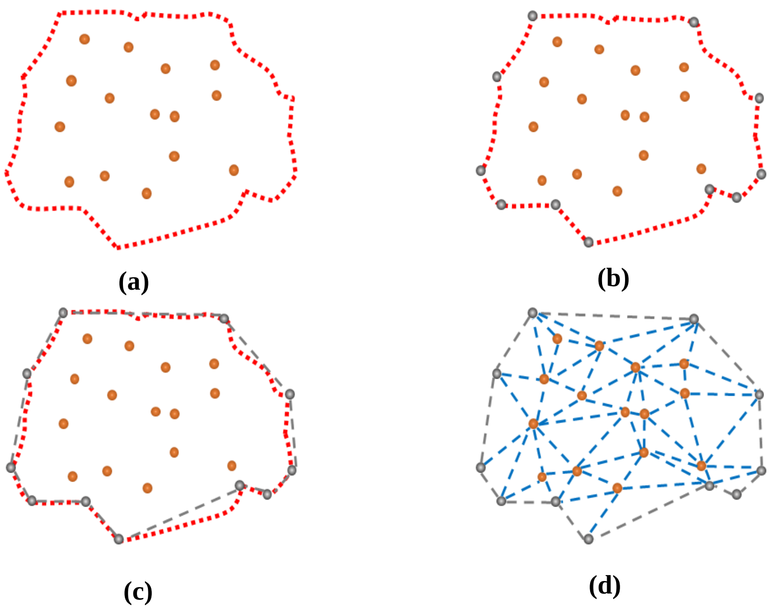


Fig. 2 Border simplification and triangulation of the sparse data points

3.2 Parameterization of the triangulation mesh

Since the adjacent relationship between the sparse data of each surface is obtained, the data can be parameterized. There are plenty of methods to parameterize triangulation meshes. Here we select mean value coordinates method, which is efficient in calculation and the resulting parameterization depends smoothly on the vertices of the triangulation [18]. The details of the method are presented as follows:

Let $\boldsymbol{v}_0, \boldsymbol{v}_1, \cdots, \boldsymbol{v}_k$ be the vertexes in the mesh with $\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_k$ arranged in an anticlockwise ordering around $\boldsymbol{v}_0$. Each vertex $\mathbf{v}_0$ can be expressed as a linear combination of its neighboring vertex $\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_k$ as follows

$$\sum_{i=1}^{k} \lambda_i \boldsymbol{v}_i = \boldsymbol{v}_0, \tag{1}$$

$$\sum_{i=1}^{k} \lambda_i = 1\,. \tag{2}$$

$\lambda_1, \cdots, \lambda_k \geq 0$. For the cases $k = 2, 3$, the weights λ_i are the barycentric coordinates. For the cases $k > 3$, the weights can be obtained by approximating the harmonic function using the mean value theorem

$$\lambda_i = \frac{\omega_i}{\Sigma_{j=1}^{k} \omega_j}, \tag{3}$$

$$\omega_i = \frac{\tan\left(\frac{\alpha_{i-1}}{2}\right) + \tan\left(\frac{\alpha_i}{2}\right)}{\| v_i - v_0 \|}, \tag{4}$$

Where α_i, $0 < \alpha_i < \pi$ is the adjacent angle at $\mathbf{v}_0$ in the triangle $[\boldsymbol{v}_0, \boldsymbol{v}_i, \boldsymbol{v}_{i+1}]$.

The parameter coordinates are (U_i, V_i) for each vertex $\mathbf{v}_i'$ in the parameter domain M' of triangular mesh M. Sort the parameters and the u and v parameters are $U_i, i = 0,1,\cdots,m$ and $V_j, j = 0,1,\cdots,m$. $U_{h_1} \leq U_{h_1+1}, V_{h_2} \leq V_{h_2+1}$, $h_1, h_2 \in \{0, 1, \cdots, m-1\}$. However, the number of points in the data sets Q in Section 3.3 will be astonishing if the parameters $\{U_i | i = 0,1,\cdots m\}$ and $\{V_j | j = 0,1,\cdots m\}$ are directly used as the parameters of the points to be interpolated. Obviously, the parameters are extremely nonuniform and dense. There are some methods to fix this problem. Les Piegl and Wayne Tiller [19] proposed a method remoting as many knots as possible to reduce the number of control points. H park et al. introduced a method, allowing the knots to be selected freely but leading to a more stable linear system, and the number of knot parameters in each B-spline curve is equal to the highest number of parameters [1]. Here we process the parameterization results of triangular meshes directly to reduce the number of parameters. The method is simple and costs less computation.

Here we select two constants C_1 and C_2. For each u parameter U_i $(i = 0,1,\cdots,m-1)$ of the parameter domain $M' = (v', E', S')$, $U_i \neq 0, U_{i+1} \neq 1$, if $|U_{i+1} - U_i| < C_1$, set $U_{i+1} = U_i$; For each v parameter V_j, $V_j \neq 0$, $V_{j+1} \neq 1$, if $|V_{j+1} - V_j| < C_2$, set $V_{j+1} = V_j$. The resulted parameters are $\widetilde{U_i}$ and $\widetilde{V_j}$, $i, j \in \{1,2,\cdots,m\}$. The number of knot parameters can be effectively reduced in the condition that for any two vertexes $\mathbf{v}_{k_1}', \mathbf{v}_{k_2}'$ of M',

$$|\widetilde{U}_{k_1} - \widetilde{U}_{k_2}| + |\widetilde{V}_{k_1} - \widetilde{V}_{k_2}| \neq 0. \tag{5}$$

Where $(\widetilde{U}_{k_1}, \widetilde{V}_{k_1})$ and $(\widetilde{U}_{k_2}, \widetilde{V}_{k_2})$ are the corresponding parameters of $\mathbf{v}_{k_1}'$ and $\mathbf{v}_{k_2}'$. It means that during the simplification process, there are not any two different vertexes that are merged into one. To make sure of this, it is necessary to choose proper C_1 and C_2.

In the parameter domain M', the edges are $E_i', i = 1,2,\cdots,r$. $\mathbf{v}_{i,1}'$ and $\mathbf{v}_{i,2}'$ are two vertexes of E_i'. The parameter coordinates of $\mathbf{v}_{i,1}'$ and $\mathbf{v}_{i,2}'$ are $(U_{i,1}, V_{i,1})$ and $(U_{i,2}, V_{i,2})$. We set C_1 and C_2 as two positive numbers smaller than the minimal length L of all the edges $\{E_i' | i = 1,2,\cdots,r\}$. It can be described as

$$0 < C_1, C_2 < L = \arg\min_{i=1\cdots r} |\mathbf{v}_{i,1}'\mathbf{v}_{i,2}'| = \arg\min_{i=1\cdots r} \sqrt{\left(U_{i,1} - U_{i,2}\right)^2 + \left(V_{i,1} - V_{i,2}\right)^2}. \quad (6)$$

Here we choose $C_1 = C_2 = \frac{L}{2}$. Iterate under the condition (5) then the knots can be reduced continuously. Generally, in a few iterations of the algorithm, the number of the knot parameters can be reduced to a suitable quantity. The generated parameters are denoted as $\overline{\boldsymbol{U}} = [\bar{u}_0, \bar{u}_1, \cdots, \bar{u}_{m_1}]$ and $\overline{\boldsymbol{V}} = [\bar{v}_0, \bar{v}_1, \cdots, \bar{v}_{m_2}]$.

3.3 Data set complement based on kriging interpolation method

It can be found that there are massive parameter nodes which do not correspond to any data points after we obtain the parameters from the last step. There are some methods to interpolate sparse distributed data with B-spline surface in the previous researches. For example, [13] imposed constraints on the control points to reduce freedom of the control points. However, the constrains may cause the surface to be unnatural since the constrains are often local and not directly corresponding to the distribution trends of the data. To make the data set more rational, here we introduce the kriging interpolation method to complete the data set for B-spline interpolation.

The kriging method is originated from the geo-statistic community. It is named after the pioneering work of D. G. Krige, which was formally developed by Matheron [20]. Kriging interprets the interpolation function as a random process, where the function value at each point in the domain is treated as a separate random variable that is correlated to all the others [10].

For each time epoch k, interpolate the data corresponding to the parameter $(\bar{u}_i, \bar{v}_j), i = 0,1,\cdots,m_1;\ j = 0,1,\cdots,m_2$ with kriging method. The generated data set $\{\mathbf{Q}_{i,j,k}\}, i = 0,\cdots,m_1, j = 0,\cdots,m_2, k = 0,\cdots,m_t$ preserves the observed value and conforms to the specification of B-spline fitting.

3.4 Three-dimensional B-spline control points solving based on Key Points Interpolation

A three-dimensional B-spline function can be expressed as follows [2-4]:

$$\mathbf{M}(u,v,t) = \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} N_{i,p}(u) N_{j,q}(v) N_{k,r}(t) \boldsymbol{P}_{i,j,k} \quad (7)$$

Where $\boldsymbol{P}_{i,j,k}$ are the control points and $N_{i,p}(u)$, $N_{j,q}(v)$ and $N_{k,r}(t)$ are B-spline basis function of degree p, q and r. The knot vectors are $\mathbf{U} = \{\mathrm{u}_0, \mathrm{u}_1, \cdots, \mathrm{u}_{\mathrm{n+p+1}}\}$, $\mathbf{V} = \{\mathrm{v}_0, \mathrm{v}_1, \cdots, \mathrm{v}_{\mathrm{m+q+1}}\}$ and $\mathbf{T} = \{\mathrm{t}_0, \mathrm{t}_1, \cdots, \mathrm{t}_{\mathrm{l+r+1}}\}$. In this way, the three-dimensional B-spline volume can be considered as an extension of bivariate B-spline surface.

For three-dimensional B-spline approximation, the traditional method can be expressed as:

$$\mathbf{M}(\mathrm{u}, \mathrm{v}, \mathrm{t}) = \sum_{\mathrm{i}=0}^{\mathrm{n}} \sum_{j=0}^{m} \sum_{k=0}^{l} N_{i,p}(u) N_{j,q}(v) N_{k,r}(t) \boldsymbol{P}_{i,j,k} = \sum_{\mathrm{i}=0}^{\mathrm{n}} \boldsymbol{c}_{i,j,k} N_{i,p}(u), \quad (8)$$

where

$$\boldsymbol{c}_{i,j,k} = \sum_{j=0}^{m} \boldsymbol{b}_{i,j,k} N_{j,q}(v)\,, \quad (9)$$

and

$$\boldsymbol{b}_{i,j,k} = \sum_{k=0}^{l} N_{k,r}(t) \boldsymbol{P}_{i,j,k}. \quad (10)$$

The process of control points solving of three-dimensional B-spline can be transformed into iterations of control points solving of B-spline curve[1]. a B-spline curve fitting can be expressed as follows:

$$\min \boldsymbol{f} = \sum_{i=0}^{m} [\boldsymbol{Q}_i - \boldsymbol{p}(\overline{u_\iota})]^2,$$

(11)

where $\mathbf{Q}_\mathrm{i}, i = 0, \cdots, m$ are the data points to be approximated, and $\bar{u}_i$ are the parameters corresponding. The unknown control points $\mathbf{b}_\mathrm{i}, \mathrm{i} = 0,1, \cdots, \mathrm{n}$ of the B-spline curve equation $\mathbf{p}(\mathrm{u}) = \sum_{\mathrm{i}=0}^{\mathrm{n}} N_{i,p}(u) \boldsymbol{b}_i$ can be calculated through least-square method [2].

When the number of control points $\mathrm{n} + 1$ are more than or equal to the number of data points $\mathrm{m} + 1$, control points can be computed by Eq. (8), Eq. (9) and Eq. (10) directly and the data points can be interpolated accurately.

It should be noticed here that as the number of data points to be interpolated increases, the control points should increase as well. However, in the data set we obtained from the above steps, there are only a few of data points from the original sparse data set that we need to interpolate precisely. To reduce the number of control points and improve computational efficiency, we develop a method which can interpolate the sparse points we designated but use less control points.

For curve interpolation, we directly set the original sparse data points as the key points and the rest of data points as the non-key points. Thus， we obtain the following equation:

$$\min \boldsymbol{f} = \sum_{i=0}^{m_1} \left[\boldsymbol{Q}_{i_1} - \boldsymbol{p}(\bar{u}_{i_1})\right]^2$$

$$\text{s.t.}\, \mathbf{Q}_{\mathrm{i}_2} = \boldsymbol{p}(\bar{u}_{i_2}), i = 0,1,\cdots,\bar{m}_1 \qquad (12)$$

where $\mathbf{Q}_{\mathrm{i}_1}, \mathrm{i} = 0,1,\cdots,\mathrm{m}_1$ are the non-key points and $\mathbf{Q}_{\mathrm{i}_2}, i = 0,1,\cdots,\bar{m}_1$ are the key points. $\bar{u}_{i_1}$ and $\bar{u}_{i_2}$ are the parameters of $\mathbf{Q}_{\mathrm{i}_1}$ and $\mathbf{Q}_{\mathrm{i}_2}$. Then the problem is transformed to an equality constrained quadratic program. By solving (12) to obtain the control points, a B-spline curve which approximates the non-key points and interpolates the key points is obtained.

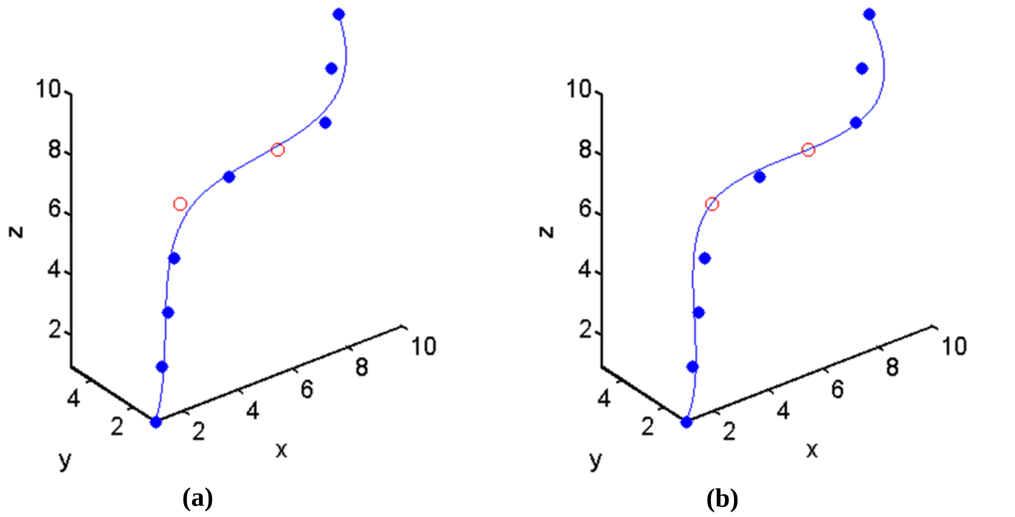


Fig. 3 Curve fitting with traditional method and KPI method

Fig. 3 shows two curves fitting with traditional method (Fig. 3(a)) and KPI method (Fig. 3(b)) respectively. Both curves fit 10 data points. The circles are key points and the solid dots are non-key points. Each of the two B-spline curves is presented by 5 control points. Fig. 3(b) shows that comparing with traditional B-spline approximation method, KPI method show higher precision on key points while the number of control points is the same as traditional method.

Since the control points solving process for B-spline surfaces is an iteration of control points solving for B-spline curves, the appropriate choices of key points in each iteration are paramount to guarantee that the sparse data points to be interpolated precisely. For the data set $\mathbf{Q}_{\mathrm{i,j}}, i = 0, \cdots, m_1, j = 0, \cdots, m_2$, denote the points to be interpolated as point set $\mathcal{I}$ if they are from the original sparse point sets. The rest of the points are denoted as set $\mathcal{A}$ as they are the points to be approximated. The key points determination method is expressed as follows:

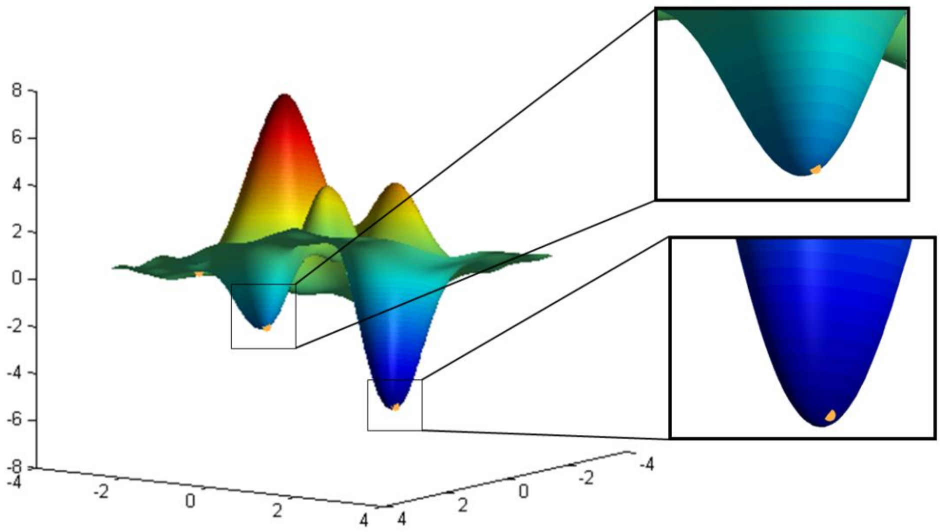
Step 1. fix $\mathrm{j} = \tilde{j}$, the key points in $\mathbf{Q}_{\mathrm{i},\tilde{j}}, i = 0,1, \cdots, m_1$ are the points in $\mathcal{I}$. fit the points with (12) And the control points $\mathbf{c}_{\mathrm{i,j}}$ $(i = 0,1, \cdots, n_1, j = 0,1, \cdots, m_2)$ of Eq. (8) can be obtained, while the knot vector is $\mathbf{u} = \{\mathrm{u}_0, \mathrm{u}_1, \cdots, \mathrm{u}_{\mathrm{n}_1+\mathrm{p}+1}\}$.

Step 2. The control points $\boldsymbol{c}_{i,j}, i = 0,1, \cdots, n_1, j = 0,1, \cdots, m_2, n_1 \leq m_1$ we generated in the last step are fitted and $\boldsymbol{b}_{i,j}, i = 0,1, \cdots, n_1, j = 0,1, \cdots, n_2$ are the control points to be generated in this step, while the knot vector is $\mathbf{v} = \{\mathrm{v}_0, \mathrm{v}_1, \cdots, \mathrm{v}_{\mathrm{n}_2+\mathrm{q}+1}\}$. For a point $\mathbf{c}_{\bar{\imath},\bar{\jmath}}$, if any data $\mathbf{Q}_{\mathrm{i},\bar{\jmath}}$ $(\mathrm{i} = i = 0,1, \cdots, m_1)$ whose u parameter is in the interval $[\mathrm{u}_{\bar{\imath}}, \mathrm{u}_{\bar{\imath}+\mathrm{p}+1})$ is a point in $\mathcal{I}$, then $\mathbf{c}_{\bar{\imath},\bar{\jmath}}$ is a key point. If the data points $\mathbf{Q}_{\mathrm{i},\bar{\jmath}}$ $(\mathrm{i} = i = 0,1, \cdots, m_1)$ whose u parameter is in $[\mathrm{u}_{\bar{\imath}}, \mathrm{u}_{\bar{\imath}+\mathrm{p}+1})$ are all from the set $\mathcal{A}$, then $\mathbf{c}_{\bar{\imath},\bar{\jmath}}$ is a non-key point.

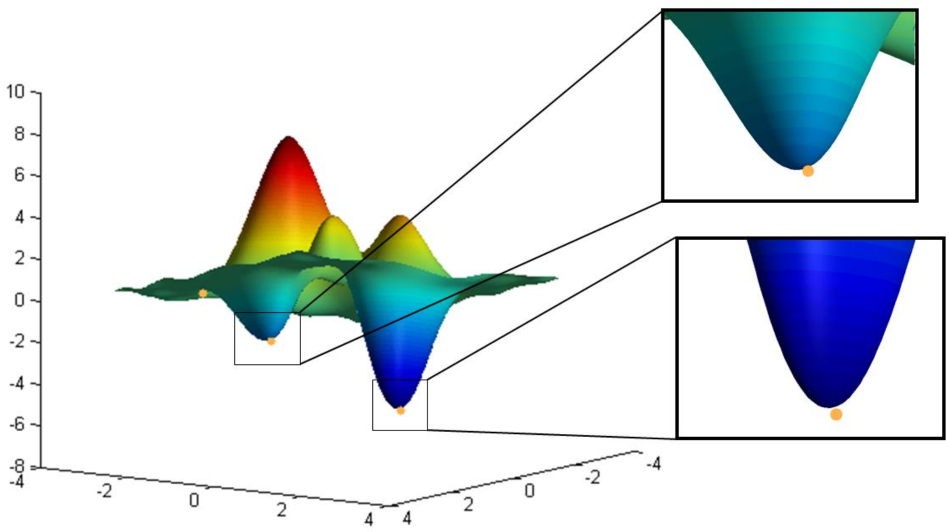
Assume that $\mathbf{Q}_{\mathrm{i_r,j_r}}$ is a point in $\mathcal{I}$, and the parameter of $\mathbf{Q}_{\mathrm{i_r,j_r}}$ is $(\mathrm{u_r}, \mathrm{v_r})$ where $\mathrm{u}_{\mathrm{k}_1} \leq \mathrm{u_r} < u_{k_1+1}, \mathrm{v}_{\mathrm{k}_2} \leq v_r < v_{\mathrm{k}_2+1}$. $\mathrm{u}_{\mathrm{k}_1}, v_{k_2}$ are knots in knot vector $\mathbf{u}, \mathbf{v}$. In step 1, according to the local support property of B-splines, the only control points that is related to the point $\mathbf{Q}_{\mathrm{i_r,j_r}}$ are $\mathrm{c}_{\mathrm{k}_1-\mathrm{p},\mathrm{j_r}}$, $\mathrm{c}_{\mathrm{k}_1-\mathrm{p}+1,\mathrm{j_r}}, \cdots, \mathrm{c}_{\mathrm{k}_1,\mathrm{j_r}}$[2]. To ensure that $\mathbf{Q}_{\mathrm{i_r,j_r}}$ is interpolated, step 2 needs to guarantee that the data points $\mathrm{c}_{\mathrm{k}_1-\mathrm{p},\mathrm{j_r}}$, $\mathrm{c}_{\mathrm{k}_1-\mathrm{p}+1,\mathrm{j_r}}, \cdots, \mathrm{c}_{\mathrm{k}_1,\mathrm{j_r}}$ we obtained from step 1 are interpolated. It needs to be notice that the number of key points in step 2 is more than the key points in step 1, thus generally more control points in step 2 are needed than step 1.

After the control points for each surface in each time epoch are obtained, the control points $\boldsymbol{P}_{i,j,k}$ for the dynamic surface can be generated by the third iteration of control points solving.

It is possible that there may be no solution for (12) since the equality constrain may not be satisfied if the control points are inadequate. Alternatively, the traditional knot insertion algorithms [2] can avoid this situation and generate proper solutions. After the control points $\boldsymbol{P}_{i,j,k}$ are obtained, the three-dimensional B-spline dynamic surface is generated. The model interpolates the sparse data and contains the dynamic characteristic information. To illustrate this, the proposed method and the traditional B-spline surface approximation method on a given sample is manifested in Fig. 4. The 25×25 data points are sampled from a peak function added random deviation. The data to be interpolated are marked as yellow points and the rest data points are the points to be approximated. The control points of two surfaces are 11×11. KPI method (Fig. 4(a)) interpolates the yellow points precisely while the traditional B-spline surface approximate method (Fig. 4(b)) can only approximate the yellow points as well as other data points in a least square sense with less accuracy.



(a)



(b)

Fig. 4 Surface fitting with KPI method and traditional method

4 Example study

To demonstrate the method, we apply the method to interpolate the temperature data of Shanxi Province, China, which is a simply connected region that contains 98 weather stations. The data were provided by China Meteorological Data Sharing Service System (http://data.cma.cn/en). The 98 weather stations provide local temperature every hour. The map of Shanxi Province is shown in Fig. 5. The red circles are the 98 weather stations within the border of Shanxi Province.

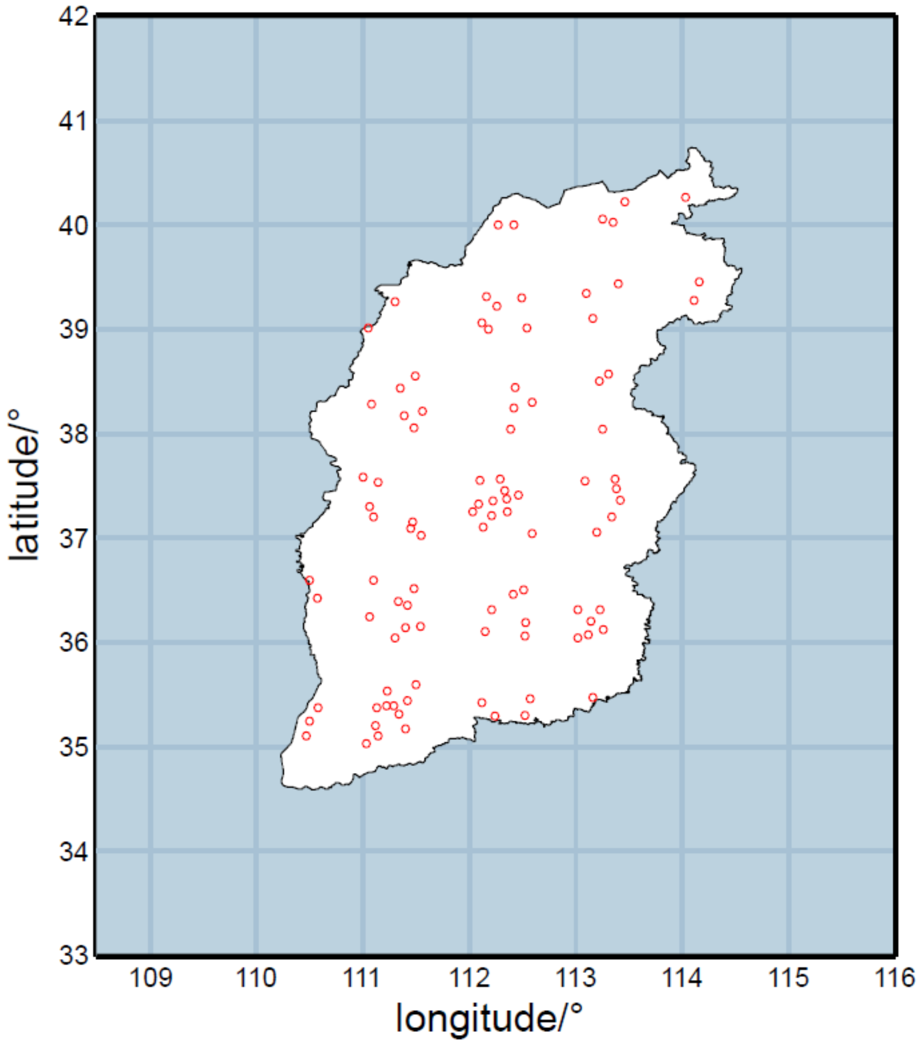


Fig. 5 The weather stations in Shanxi Province

We select a data set of a period from 1:00, October 15, 2017 to 24:00, October 17, 2017, each data point contains the longitude λ, latitude φ, temperature τ and time t of the corresponding station. The data points to be interpolated are denoted as

$\widetilde{\boldsymbol{Q}}_{it}$ $(i = 0,1,2,\cdots,97, t = 0,1,2,\cdots,71)$. For each moment t, the longitude and latitude of each station are invariant. The border of Shanxi Province is treated as the border of the surface . Fig. 6(a)-Fig. 6(c) shows the result of border simplification. The original border which is presented by 6102 points are simplified into 49 feature points (shown in Fig. 6(a)) and the simplified polygon (shown in Fig. 6(b)-Fig. 6(c)) can maintain the basic shape of the original border.

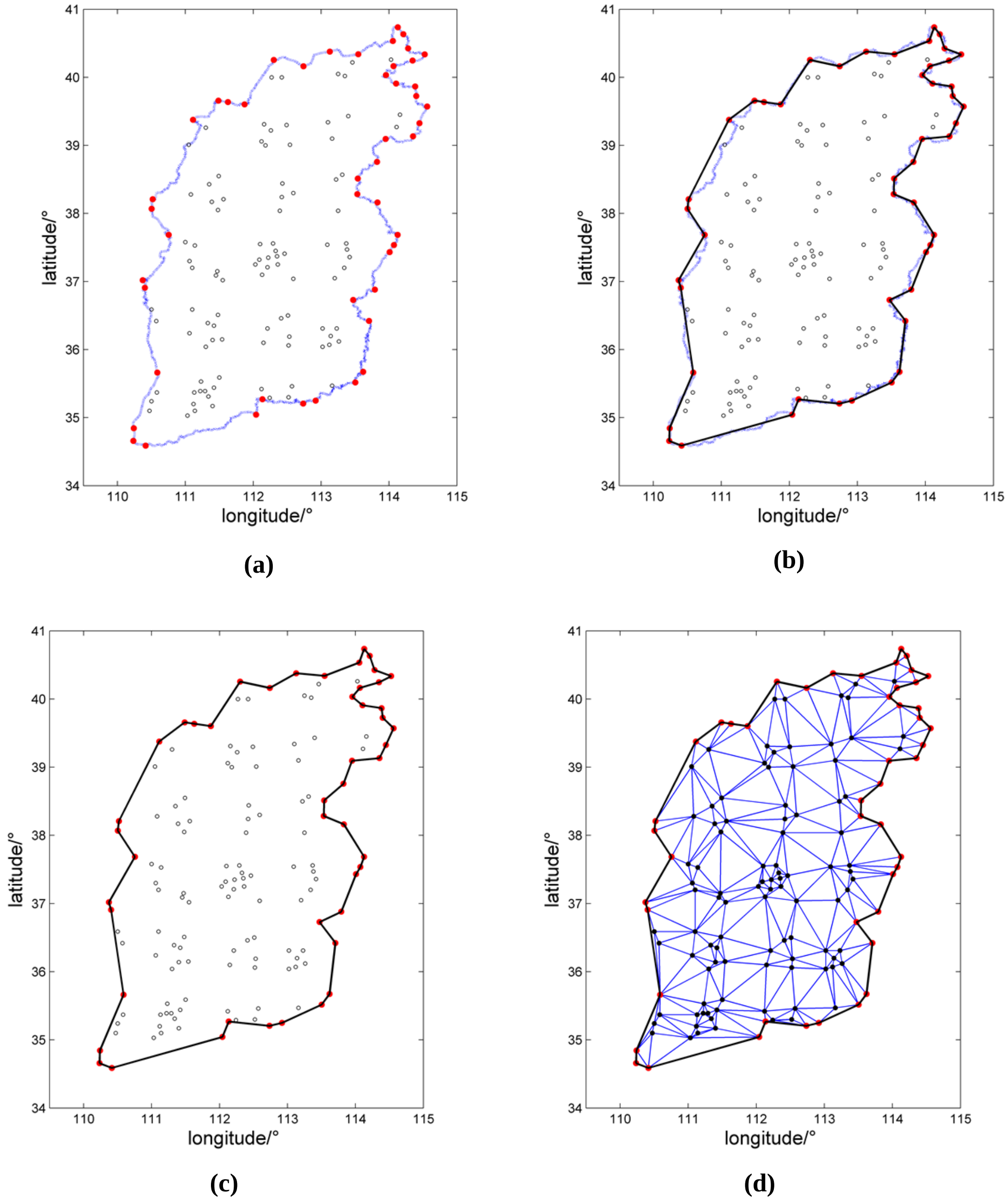
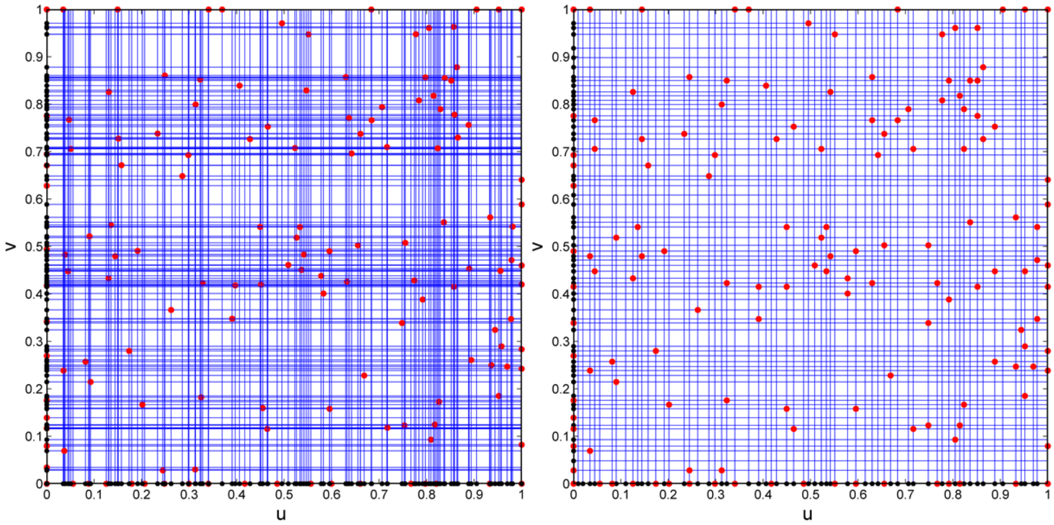


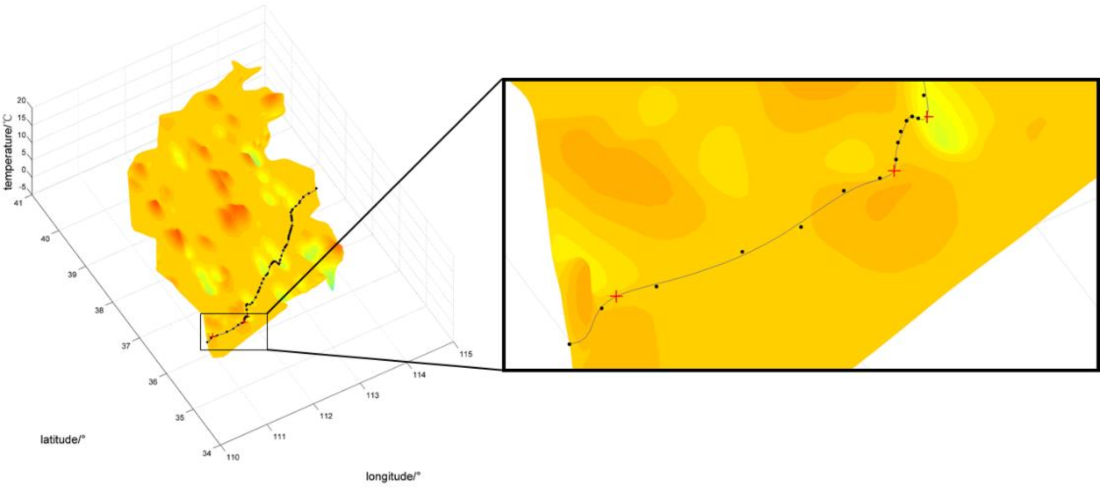
Fig. 6 Border simplification and triangulation of the data points



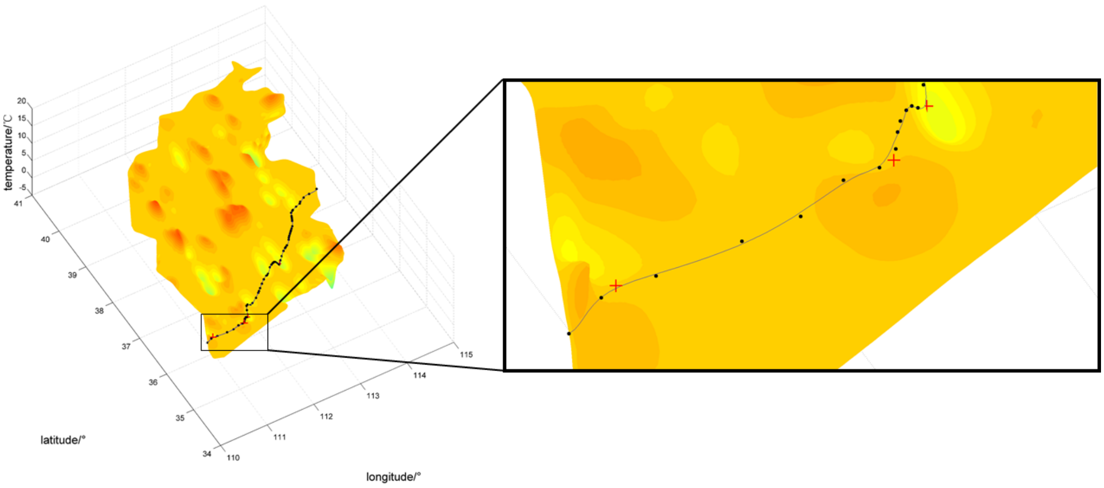
(a) **(b)**

Fig. 7 Original parameters and parameter reduction result

To generate the parameters $\overline{\boldsymbol{U}}$ and $\overline{\boldsymbol{V}}$ in Section 3.2, 98 station points and 49 feature points of the original border are triangulated with Delaunay triangulation method in a two-dimensional planar, where the x and y coordinates are the longitude and the latitude respectively. The triangulation result is shown in Fig. 6(d). The next step is to generate the parameters of each point based on the triangulation result in Fig. 6(d) with mean value coordinates method. The number of parameters **U** and parameters **V** are 124 and 120 respectively. From Fig. 7(a) we can see that the parameters **U** and **V** are extremely dense and nonuniform. Process the parameters with the method we proposed in Section 3.2 and the ultimate numbers of parameters $\overline{\boldsymbol{U}}$ and $\overline{\boldsymbol{V}}$ are 66 and 68 respectively (shown in Fig.7(b)). After completing the data set with Kriging method, the data set $\mathbf{Q}_{\mathrm{i,j,k}}\ (i = 0,\cdots,65, j = 0,\cdots,67, k = 0,\cdots,71)$ is generated. The data set $\mathbf{Q}_{\mathrm{i,j,k}}\ (i = 0,\cdots,65, j = 0,\cdots,67, k = 0,\cdots,71)$ consists of $\widetilde{\boldsymbol{Q}}_{it}\ (i = 0,1,2,\cdots,97, t = 0,1,2,\cdots,71)$ and the generated data $\widetilde{\widetilde{\boldsymbol{Q}}} = \{\mathbf{Q}_{\mathrm{i,j,l}}\} \cap \{\widetilde{\boldsymbol{Q}}_{it}\}$. With the method we proposed in Section 3.4, the data $\mathbf{Q}_{\mathrm{i,j,k}}\ (i = 0,\cdots,65, j = 0,\cdots,67, k = 0,\cdots,71)$ is interpolated of degree 3, while the points to be interpolated are 98×72 original sparse data points $\widetilde{\boldsymbol{Q}}_{it}\ (i = 0,1,2,\cdots,97, t = 0,1,2,\cdots,71)$ provided by the weather stations. The control points are $\mathbf{P}_{\mathrm{i,j,k}}(i = 0,\cdots,40, j = 0,\cdots,45, k = 0,\cdots,73)$.



(a)



(b)

Fig. 8 Isolines of KPI method and traditional approximation method

Fig. 8(a) and Fig 8(b) depict iso-u lines of KPI method and traditional B-spline approximation method, while the crosses are the original data provided by the weather stations and the dots denote the data generated by Kriging. The numbers of control points and u, v, t knot vectors of the two methods are the same. Using KPI method, the original data is interpolated precisely while the traditional B-spline fitting method cannot recognize the original data and the generated data. The resulted B-spline model can be recorded as three knot vectors u, v and t and a series of control points $\mathbf{P}_{\mathrm{i,j,k}}(i = 0,\cdots,40, j = 0,\cdots,45, k = 0,\cdots,73)$.

The performance data of KPI method and traditional methods on the example are shown in Table 1. The table presents the maximum errors and mean square deviations of KPI method, traditional B-spline approximation method and traditional B-spine interpolation method on the sparse data $\widetilde{\boldsymbol{Q}}_{it}$. The result indicates that if the control points are set as the same number, KPI method performs better because all the sparse points are interpolated precisely while the maximum deviation and standard deviation of traditional approximation method are larger. The traditional interpolation method performs as well as KPI method in precision. However, the traditional interpolation method requires more control points, which will cost more computation and data storage capacity.

Table 1 Table 1 Performance results of KPI method and traditional B-spline fitting methods

Method	Number of control points	Maximum deviation	Standard deviation
KPI method	41×46×74	9.9492×10^{-14}	2.7706×10^{-14}
Traditional method(approximation)	41×46×74	0.7661	0.0819
Traditional method(interpolation)	126×122×74	7.1497×10^{-14}	2.0480×10^{-14}

Fig. 9 shows the temperature interpolation results of Shanxi Province. Fig. 9 (a), Fig. 9(b) and Fig. 9(c) are the temperatures at 4:35, October 15, 2017, 14:00, October 15, 2017 and 10:15, October 16, 2017.

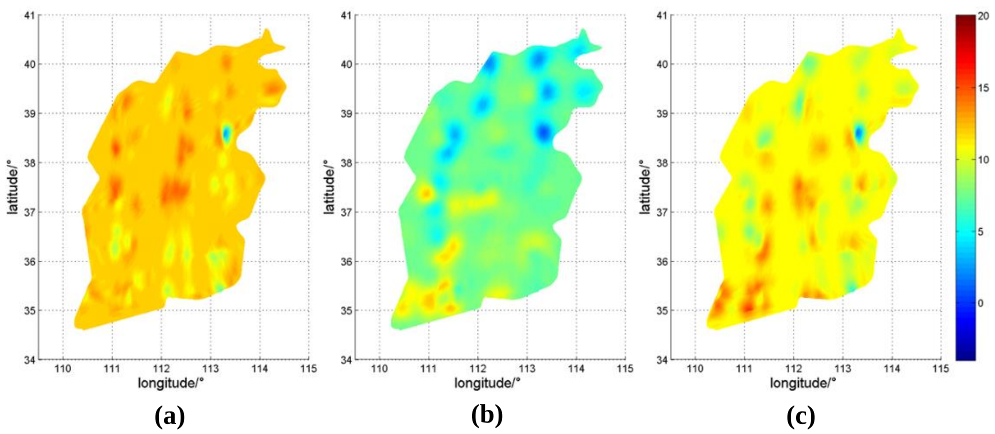


Fig.9 Temperature interpolation results of Shanxi Province with KPI method

5 Conclusions

A novel interpolation method (KPI) is introduced in this paper to reconstruct dynamic surface with three-dimensional B-spline based on sparse data. In this method, the data points of each time epoch are first triangulated with Delaunay triangulation method.

Then the data of each surface is parameterized with mean value coordination method, and the parameters can be reduced with a simple method. After the data set is completed, the key points interpolation-based control points solving method is implemented. At last, the dynamic surface based on sparse data is reconstructed.

The advantages of this dynamic surface reconstruction method based on key points interpolation are summarized as follows.

(1) The parameter reduction method can reduce the number of parameters generated by mean value coordinates significantly. Our method is simple and easy, it only needs linear operation. The generated parameters lead to smaller data set generated by Kriging, costing less computation.

(2) The data set generated by Kriging ensures the data to be fitted by B-splines to correspond to the trend of sparse data. Compared with traditional sparse data interpolation algorithms with B-splines which impose constrains on the control points, the proposed method generates a rational data set, thus, the generated B-splines becomes more rational.

(3) The control points solving method based on key points interpolation generates a three-dimensional B-spline which interpolates the sparse data while approximate the data generated by Kriging. Compared with traditional B-spline interpolation algorithms, the proposed algorithm can interpolate the sparse data with fewer control points, reducing data storage capacity while ensuring the accuracy.

Future works might focus on the following aspects. First, interpolation of sparse data considering diverse types of data distributions should be investigated. Second, the application of KPI method should be extended to higher dimensional problems such as photo morphing and animation. In addition, solid modeling and analysis and optimization methods based on KPI may also be considered.

Acknowledgement This work is supported by National Key Research and Development Program of China (Grants No.2018YFB1107402, No.2017YFB0701702.) and NSFC (Grant No.11290141, No.11571028).

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