Simplified Masters

$$T(n) = aT(\frac{n}{b}) + \Theta(n^d)$$
$$a \geq 1$$
$$b > 1$$
$$d \geq 0$$

- $a > b^d \rightarrow T(n) = \Theta(n^{\log_b a})$
- $a = b^d \rightarrow T(n) = \Theta(n^d \log_b n)$
- $a < b^d \rightarrow T(n) = \Theta(n^d)$

Masters

$$T(n) = aT(\frac{n}{b}) + f(n)$$
$$a \geq 1$$
$$b > 1$$

- $f(n) = \mathcal{O}(n^{log_b a - \epsilon}) \rightarrow T(n) = \Theta(n^{\log_b a}), \epsilon > 0$
- $f(n) = \Theta(n^{log_b a}) \rightarrow T(n) = \Theta(n^{log_b a} \log(n))$
- $f(n) = \Omega(n^{log_b a + \epsilon}) \rightarrow T(n) = \Theta(f(n)), \epsilon > 0$ and $af(\frac{n}{b}) \leq cf(n)$ for some $c < 1$ and big enough n
- case2 ext: $f(n) = \Theta(n^{log_b a} log^k(n)) \rightarrow T(n) = \Theta(n^{log_b a} log^{k+1}(n))$

Akra-Bazzi

$$T(n) = \sum_{i=1}^{k} a_i T(b_i n) + f(n), n > n_0$$

- $n_0 \geq \frac{1}{b_i}$, $n_0 \geq \frac{1}{1-b_i}$ for each i,
- $a_i > 0$ for each i,
- $0 < b_i$ for each i,
- $k \geq 1$,
- $f(n)$ is non-negative function,
- $c_1 f(n) \leq f(u) \leq c_2 f(n)$, for each u satisfying condition: $b_i n \leq u \leq n$

$$T(n) = \Theta(n^p(1 + \int_1^n \frac{f(u)}{u^{p+1}} du))$$

we get p from:

$$\sum_{i=1}^{k} a_i b_i^p = 1$$

Extended Akra-Bazzi

$$T(n) = \sum_{i=1}^{k} a_i T(b_i n + h_i(n)) + f(n), n > n_0$$

all of the conditions from Akra-Bazzi still hold plus:

$$|h_i(n)| = \mathcal{O}(\frac{n}{\log^2 n})$$

Annihilators

Steps:

- Write the recurrence in operator form.
- Extract the annihilator for the recurrence.
- Factor the annihilator (if necessary).
- Extract the generic solution form the annihilator.
- Solve for coefficients using base cases (if known).

Operator	Definition
addition	$(f+g)(n) := f(n) + g(n)$
subtraction	$(f-g)(n) := f(n) - g(n)$
multiplication	$(a \cdot f)(n) := a \cdot (f(n))$
shift	$Ef(n) := f(n+1)$
k-fold shift	$E^k f(n) := f(n+k)$
composition	$(X+Y)f := Xf + Yf$
	$(X-Y)f := Xf - Yf$
	$XYf := X(Yf) = Y(Xf)$
distribution	$X(f+g) = Xf + Xg$

Operator	Functions annihilated	
$E - 1$	α	
$E - a$	αa^n	
$(E-a)(E-b)$	$\alpha a^n + \beta b^n$	(if $a \neq$
$(E-a_0)(E-a_1)\cdots(E-a_k)$	$\sum_{i=0}^{k} \alpha_i a_i^n$	(if a_i distinc
$(E-1)^2$	$\alpha n + \beta$	
$(E-a)^2$	$(\alpha n + \beta)a^n$	
$(E-a)^2(E-b)$	$(\alpha n + \beta)a^n + \gamma b^n$	(if $a \neq$
$(E-a)^d$	$(\sum_{i=0}^{d-1} \alpha_i n^i)a^n$	

If X annihilates f, then X also annihilates Ef.
If X annihilates both f and g, then X also annihilates $f \pm g$.
If X annihilates f, then X also annihilates αf, for any constant

If X annihilates f and Y annihilates g, then XY annihilates $f \pm$