

**So. Last week we learned a little bit about asymptotes. I will give a mini review of the material right now.**

Firstly, what is an asymptote?

**A simplified definition**

An asymptote is an invisible line which a function f(x) approaches as x gets closer to a number or infinity.

Asymptotes can be generalized into two categories - Vertical Asymptotes and Horizontal Asymptotes.

**Vertical Asymptotes.**

Vertical Asymptotes can generally be found by finding the zeros of the denominator. It is really looking at how f(x) behaves as x approaches a number for which f(x) doesn’t exist.

For example,

For the function f(x) = 1/(x+1)

The zero of the denominator would be -1, so the vertical asymptote would take the equation of X=-1

It is called a vertical asymptote, because the equation of the asymptote is graphed as **vertical line**

Why is the zero of the denominator an asymptote?

For two reasons.

1. any rational number/function cannot exist with a denominator being zero. Any number divided by zero is undefined. So for that reason, the function cannot exist at points where the denominator equals 0. There is two possible results from this, a **asymptote** or a **hole (this occurs when at the point, the function is 0/0, when would this happen? Think about x/x or (x-2)(x+1)/(x-2)(x-1)).** We are examining the asymptote case.

2. The asymptote case happens when the numerator is anything but zero for that rational function. Take the function f(x) = 1/(x+1)

In this case, using what we know about limits, we know that as x approaches -1, in other words as the denominator approaches 0. Since the numerator is not zero, we see that f(x) approaches positive or negative infinity.

Next lesson we will go into more details about limits and behaviors of vertical asymptote.

**Horizontal/Slant/Quadratic Asymptotes.**

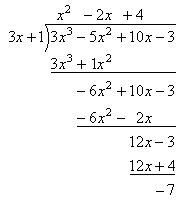
Horizontal/Slant/Quadratic Asymptotes can be found via finding the limit of the function as x approaches infinity or negative infinity. In other words, this is examining the end behaviors of the functions to see if it approaches a number(horizontal), a line(slant) or a quadratic equation at its ends.

It can be found via long division.

Look at this equation

F(x) =

Using long division,



we see that the equation can be rewritten as

x^2 - 2x + 4 + (-7)/(3x + 1)

**Now let’s imagine what happens to f(x) as x approaches positive infinity.**

First of all, we see the last component, -7/(3x+1) approaches 0 as x approaches positive infinity, so we can cross that out (you are adding 0 to a function). While the remaining terms, x2 and 2x approaches positive infinity as x approaches positive infinity.

In fact, the function f(x) approaches positive infinity **like** the function x2 – 2x +4.

If the function f(x) approaches positive infinity to a number, for example y=0, we call that a horizontal asymptote.

If the function f(x) approaches positive infinity to a linear equation, for example y=2x, we call that a slant/oblique asymptote.

If the function f(x) approaches positive infinity to a quadratic function, for example y=2x2, we call that a quadratic asymptote.

I am really generalizing here, in fact any equation can be an asymptote, there are asymptote that even looks like waves!

Now try out the following equations. Find their vertical and horizontal/slant asymptotes. Challenge: find the points where the function is 0/0, we call these point holes (more on this on next tutorial).

1.  2.  3. 

4.  5.  6. 

7.  8. 

9.  10. 

Next tutorial is going to be a review of material and introduce a little bit more about asymptotes and behavior of polynomials. No practice.

Furthermore, I am going to be away for at least three weeks starting from next week due to jaw surgery, however I am going to post practice online reviewing all the materials, hopefully every week : ) good luck.