Question 1:

A random process Y(t) has the power spectral density:

$$S_{YY}(w) = \frac{16}{w^2 + 64}$$

(a) Find the autocorrelation function of Y(t).

(b) Find the average power in the process Y(t).

Question 2:

Let $P = \begin{bmatrix} 0.85 & 0.26 \\ 0.15 & 0.74 \end{bmatrix}$

Hint: $P\ \overline{X} = \overline{X}$

(a) Find stable probability matrix $(\overline{P})$?

(b) Find stable distribution matrix $(\overline{X})$?

Q1) **a.**

$$S_{rr}(\omega) = \frac{16}{\omega^2 + 64} \longrightarrow \frac{2a}{\omega^2 + a^2}$$

$$R_{xx}(\tau) = e^{-a|\tau|}$$

$$S_{rr}(\omega) = \frac{16}{\omega^2 + 64} = \frac{16}{\omega^2 + 8^2} = \frac{2 \times 8}{\omega^2 + 8^2}$$

$a = 8$ || $R_{xx}(\tau) = e^{-8|\tau|}$

b. $E[x^2(t)] = R_{xx}(0) = e^{-8|\tau|}$

$$= e^{-8|0|} = e^0 = 1 \quad \therefore \quad E[x^2(t)] = 1$$

Q2 a-b) $P\bar{x} = \bar{x}$ $\begin{bmatrix} 0.85 & 0.26 \\ 0.15 & 0.74 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix}$

$$= 0.85A + 0.26B = A \longrightarrow 1$$
$$0.15A + 0.74B = B \longrightarrow 2$$
$$A + B = 1 \longrightarrow 3$$

from ①

$$0.85A + 0.26B = A$$
$$0.26B = A - 0.85A$$
$$0.26B = 0.15A$$

$$B = \frac{0.15}{0.26}A$$

$$B = 0.57A$$

from ③ $A+B = 1$

$A + \dfrac{0.15}{0.26} A = 1$

$A + 0.57A = 1$

$1.57A = 1$

$A = \dfrac{1}{1.57} = 0.63$

So: $B = 0.57 \times 0.63$

$B = 0.35$

∴ a) $\overline{P} \begin{bmatrix} 0.63 & 0.63 \\ 0.35 & 0.35 \end{bmatrix}$

→ Final answer

∴ b) $\overline{\chi} \begin{bmatrix} 0.63 \\ 0.35 \end{bmatrix}$