Question 1:

Customers arrive at a cafe in a Poisson manner with an average rate of 2 per minute.

   (a) **Find the probability of 5 customers arriving within one minute.**

   (b) **Find the probability of no customer arriving within one minute.**

Question 2:

The joint PMF of two random variables X and Y is given by:

$$P_{XY}(x, y) = \begin{cases} \dfrac{1}{82} [4 \, x^2 + 3 \, y] & , x = 1, 2 , \ y = 3, 4 \\ \\ 0 & , otherwise \end{cases}$$

   (a) **What is the conditional PMF of Y given X?**

   (b) **What is the conditional PMF of X given Y?**

1/ $P_x[x] = \left[\dfrac{\lambda^x}{x!}\right] e^{-x}$

a. $P_x[x=5] = \dfrac{2^5}{5!} e^{-2} = \dfrac{4}{15} e^{-2} = 0.036$

b. $P_x[x=0] = \dfrac{2^0}{0!} e^{-2} = 1 e^{-2} = 0.135$

2/ a. $P_{y|x}(y \mid x) = \dfrac{P_{xy}(x,y)}{P_x(x)}$

$$P_x(x) = \sum_y P_{xy}(x,y) = \sum_{y=3}^{4} \frac{1}{82}\left[4x^2 + 3y\right]$$

$$= \frac{1}{82}\left[(4x^2+3(3)) + (4x^2+3(4))\right]$$

$$= \frac{1}{82}\left[8x^2 + 21\right]$$

$$P_x(x) = \begin{cases} \dfrac{1}{82}\left[8x^2+21\right] & , \; x=1,2 \\ \\ 0 & , \; \text{Otherwise} \end{cases}$$

$$P_{y|x}(y \mid x) = \frac{P_{xy}(x,y)}{P_x(x)} = \frac{\frac{1}{82}(4x^2+3y)}{\frac{1}{82}(8x^2+21)} = \frac{4x^2+3y}{8x^2+21}$$

$$P_{y|x}(y \mid x) = \frac{4x^2+3y}{8x^2+21}$$

b. $P_{X|Y}(x|y) = \dfrac{P_{XY}(x,y)}{P_Y(y)}$

$P_Y(y) = \sum_{x} P_{XY}(x,y) = \sum_{x=1}^{2} \dfrac{1}{82}\left[4x^2 + 3y\right]$

$= \dfrac{1}{82}\left[(4(1)^2 + 3y) + (4(2)^2 + 3y)\right]$

$= \dfrac{1}{82}\left[20 + 6y\right]$

$P_Y(y) \begin{cases} \dfrac{1}{82}\left[20 + 6y\right] & , \ y = 3,4 \\[4mm] 0 & , \ Otherwise \end{cases}$

$P_{X|Y}(x|y) = \dfrac{P_{XY}(x,y)}{P_Y(y)} = \dfrac{\frac{1}{82}(4x^2 + 3y)}{\frac{1}{82}(20 + 6y)} = \dfrac{4x^2 + 3y}{20 + 6y}$

$P_{X|Y}(x|y) = \dfrac{4x^2 + 3y}{20 + 6y}$

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