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CCCS 122 Computational Discrete Math
HW# 2 SPRING 2022

(7 pages)

PLO C4	CLO3.1	/100

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[1] Find the quotient and remainder of the following: $a = b \cdot q + r$ [6]

a) a=140, b=3

$a = b \cdot q + r \rightarrow q = \lfloor \frac{140}{3} \rfloor = 46 \rightarrow r = 140 \% 3 = 2$

$140 = 46 \cdot 3 + 2$

b) a=5000, b=13

$q = \lfloor \frac{5000}{13} \rfloor = 384 \rightarrow r = 5000 \% 13 = 8 \rightarrow 5000 = 384 \cdot 13 + 8$

c) a= −5000, b=13

$q = \lfloor \frac{-5000}{13} \rfloor = -385 \rightarrow r = a - (b \cdot q) = -5000 - (13 \cdot -385) = 5$

$-5000 = 13 \cdot -385 + 5$

[2] decide whether the following are true: [6]

a) $37 \equiv 4 \pmod 7$

$37 \% 7 \neq 4 \% 7 \rightarrow$ False

b) $66 \equiv 4 \pmod 7$

$66 \% 7 \neq 4 \% 7 \rightarrow$ False

c) $-73 \equiv 4 \pmod 7$

$-73 = -11 \cdot 7 + r \rightarrow r = -73 - (-11 \cdot 7) = 4 \rightarrow$ True

[3] Given a = 338, b=1078, answer the following: [8]

a) Perform prime factorization for a and b.

$338 \to 2$
$169 \to 13$
$13 \to 13$
1

- 338 = 2, 3, 5, 7, 11, 13, 17

$338 = 2 \cdot 13 \cdot 13 = 2 \cdot 13^2$

- 1078 = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 33

$1078 \to 2 \quad 539 \to 7 \quad 77 \to 7 \quad 11 \to 11 \quad 1$
$1078 = 2 \cdot 7 \cdot 7 \cdot 11 = 2 \cdot 7^2 \cdot 11$

b) gcd(a, b) = gcd(338, 1078)

$= 2^{min(1,1)} \cdot 7^{min(2,0)} \cdot 11^{min(1,0)} \cdot 13^{min(2,0)} = 2 \cdot 1 \cdot 1 \cdot 1 = 2$

c) lcm(a,b) =

$lcm(338, 1078) = 2^{max(1,1)} \cdot 7^{max(2,0)} \cdot 11^{max(1,0)} \cdot 13^{max(2,0)} = 2 \cdot 7^2 \cdot 11 \cdot 13^2 = 182,182$

[4] Given a = 968, b=539, answer the following: [8]

a) Perform prime factorization for a and b.

$968 \to 2$
$484 \to 2$
$242 \to 2$
$121 \to 11$
$11 \to 11$
1

- 968 = 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 33

$968 = 2^3 \cdot 11^2$

- 539 = 2, 3, 5, 7, 11, 13, 17, 19, 23

$539 \to 7$
$77 \to 7$
$11 \to 11$
1
$\to 1078 = 7^2 \cdot 11$

b) gcd(a, b) = (968, 539)

$= 2^{min(3,0)} \cdot 7^{min(2,0)} \cdot 11^{min(1,2)}$
$= 1 \cdot 1 \cdot 11 = 11$

c) lcm(a,b) =

$lcm(968, 539) = 2^{max(3,0)} \cdot 7^{max(2,0)} \cdot 11^{max(1,2)} = 8 \cdot 49 \cdot 121 = 47432$

[5] Compute gcd(a, b) of question [3] part (b) using the Euclidean Algorithm. [4]

a=bq+r 1078 =	gcd(b,r) =gcd(338,64)	gcd(b,r) gcd(64,18)	gcd(b,r) gcd(18,10)	gcd(b,r) gcd(10,8)	gcd(b,r) gcd(8,2)	gcd(b,r) gcd(1078,338)
338.3+64	338=64+18	64=18.3+10	18=10.1+8	10=8.1+2	8=2.4+0	= 2

[6] Compute gcd(a, b) of question [4] part (b) using the Euclidean Algorithm. [4]

a=bq+r	gcd(b,r) =gcd(539,429)	gcd(b,r) gcd(429,110)	gcd(b,r) gcd(110,99)	gcd(b,r) gcd(99,11)	gcd(b,r) gcd(968,539)
968= 539.1+429	539= 429.1+110	429 = 429=110.3+99	110= 99.1+11	99= 11.9+0	= 11

[7] **Encryption:** Consider the given mapping for alphabet and the single digit numbers to the equivalent number. Assume the encryption function is f(x) = x+17 mod 36. [8]

Symbol	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
Eq. number	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Symbol	T	U	V	W	X	Y	Z	0	1	2	3	4	5	6	7	8	9
Eq. number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35

a) Provide a decryption function f(y) that takes ciphertext as input and returns the original text.

F(y) = x + 19

b) If the ciphertext is "6R99 T9IJJ", what was the original text?

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[8] Convert the following integers to the requested expansion. Show your steps. [8]

	Binary expansion
$(9301)_{10}$	9301=2.4650+1 4650=2.2325+0 2325=2.1162+1 1162=2.581+0 581=2.290+1 → 290=2.145+0 145=2.72+1 72=2.36+0 36=2.18+0 18=2.9+0 → 9=2.4+1 4=2.2+0 2=2.1+0 1=2.0+1 → 10010001010101
$(122AF)_{16}$	1 0010 0010 1010 1111

	Octal expansion
$(9301)_{10}$	9301=8.1162+5 1162=8.145+2 145=8.18+1 18=8.2+2 2=8.0+2 → 22125
$(122AF)_{16}$	10 010 001 010 101 111 221257

[9] Convert the following integers to decimal format: [6]

	Decimal expansion
$(1010\ 0010\ 1001)_2$	$1.2^{11}+0.2^{10}+1.2^{9}+0.2^{8}+0.2^{7}+0.2^{6}+1.2^{5}+0.2^{4}+1.2^{3}+0.2^{2}+0.2^{1}+1.2^{0}$ $2048+0+512+0+0+0+32+0+8+0+0+1=2601$
$(221257)_8$	$2.8^{5}+2.8^{4}+1.8^{3}+2.8^{2}+5.8^{1}+7.8^{0}$ $65536+8192+512+160+15=74415$
$(122AF)_{16}$	$1.16^{4}+2.16^{3}+2.16^{2}+10.16^{1}+15.16^{0}$ $65536+8192+512+160+15=74415$

[10] Indicate the rule of inference used in each of the following arguments: [6]

p $\wedge$ q

– Camels live in the desert, and they are mammals. Therefore, camels are mammals.

p, q ∴ $p \wedge q$

p q r

– It is either 40 C outside or high humidity. It is below 35 C. Therefore, it is a humid day.

$p \vee q$, $\neg p$ ∴ p

p q

– If it rains today, the university will close. The university is not closed today. Therefore, it didn't rain today.

$p \to q$, $\neg q$ ∴ $\neg p$

p q

– If it rains today, the university will close. It rained today. Therefore, the university will close.

$p \to q$, p ∴ q

p q

– If I solve all the questions in this assignment by myself, then I can get high grade in the exam. If I get high grade in the exam, I can pass the course with high grade. Therefore, If I solve all the questions in this assignment by myself, I will pass the course with high grade. $\to r$

$p \to q$, $p \to r$ ∴ $p \to r$

p

– We will take the exam on campus. Therefore, the exam will be taken on campus or online.

p ∴ $p \vee q$

p q

– If Ahmed does not have a job, then he is not a graduate. Ahmed is a graduate; therefore, he has a job.

$p \to q$, $\neg q$ ∴ $\neg p$

[11] Use rules of inference to show that the argument is valid: [10]

a. "If it does not rain or if it is not windy, then we can go fishing and we will have barbecue."
b. "If we go fishing, then we will have a big fishing competition."
c. "If we have a big fishing competition, someone wins the trophy e.g., one who catches the largest fish wins"
d. "Nobody wins the trophy"
e. Therefore, "it was windy"

Use: r: it rains w: it is windy f: we go fishing b: we have barbecue
c: we have big fish competition t: someone wins the trophy

First: Convert the statements (hypotheses) above to mathematical expressions.

a. $(\neg r \vee \neg w) \rightarrow (f \wedge b)$

b. $f \rightarrow c$

c. $c \rightarrow t$

d. $\neg t$

e. w

Second: show the steps of the deductive proof.

Step	Reason
1. $c \rightarrow t$	Hypothesis
2. $\neg t$	Hypothesis
3. $\neg c$	Modus tollens Steps 1&2
4. $f \rightarrow c$	Hypothesis
5. $\neg f$	Modus tollens Steps 3&4
6. $(\neg r \vee \neg w) \rightarrow (f \wedge b)$	Hypothesis
7. $(\neg r \vee \neg w) \rightarrow f$	Simplifications
8. $\neg(\neg r \vee \neg w)$	Modus tollens Steps 5&7
9. $(\neg\neg r \wedge \neg\neg w)$	Logical equivalence
10. $(w \wedge r)$	Logical equivalence
11. w	Simplification

[12] Use rules of inference to show that the argument is valid [6]

"Khalid, a student in this class, is left-handed."
"Everyone who is left-handed has beautiful handwriting."
Therefore, "someone in the class has beautiful handwriting."

Use: $c(x)$ "x is in this class" $f(x)$ "x is left-handed" $h(x)$ "x has beautiful handwriting"

$C(\text{Khaled}) \wedge F(\text{Khaled})$
$\forall (f(x) \rightarrow h(x))$

Step 1	$\forall (f(x) \rightarrow h(x))$	Hypothesis
Step 2	$f(x) \longrightarrow h(x)$	Univ. instantiation (Using Step 1)
Step 3	$C(\text{khaled}) \wedge F(\text{khaled})$	Hypothesis
Step 4	$F(\text{khaled}) \wedge C(\text{khaled})$	
Step 5	$h(x)$	Modus Ponens (Using Steps 2&4)

Existential generalization $\therefore \exists x\ h(x)$

[13] Consider the following theorem: "*if x and y are odd integers, then x + y is even*." [8]

a) Give a direct proof of this theorem.

1. ASSUME P IS TRUE:
X=2K+1 WHERE K IS INTEGER
Y=2K+1 WHERE K IS INTEGER

2. WE TRY TO PROVE THAT Q IS ALSO TRUE
X+Y=2K+1+2K+1
=4K+2
=2(2K+1)
=2*(ANY INTEGER)
=EVEN

3. P-->Q IS TRUE

b) Give a proof by contradiction for this theorem.

1. ASSUME THE NEGATION OF CONCLUSION¬Q IS TRUE
X+Y IS ODD
X+Y=2K+1

2. X=2K+1 WHERE K IS INTEGER
X=X+Y
Y=0
Y=2K+1 WHERE K IS INTEGER
Y=X+Y
X=0
THEN X AND Y ARE EVEN THEN IT IS FALSE
THEN P IS FALSE

3. (P^¬Q)
FALSE AND TRUE = FALSE

4. P—>Q IS TRUE

[14] Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer. [8]

LET P(N) BE THE PROPOSITION $N^2 + 1 \geqslant 2^n$

1. AT N = 1 ,P(1) IS TRUE BECAUSE $1^2 + 1 \geqslant 2^1$
2. SUPPOSE P(K) IS TRUE FOR THE POSITIVE INTEGER N=K $K^2 + 1 \geqslant 2^K$
3. WE WANT TO PROVE THAT THE RELATION IS TRUE FOR N = K+1

$(K+1)^2 + 1 \geqslant 2^{k+1}$

$= 2^{k+1} \leqslant (K+1)^2 + 1$

$= 2^{k+1} \leqslant K^2 + 2 + 2K$

$2^{k+1} = 2.2^k$

$2^{k+1} \leqslant 2*(K^2 + 1)$

$2^{k+1} \leqslant 2K^2 + 2$

$2^{k+1} \leqslant K^2 + 2 + K^2$

SINCE K^2 NOT $\leqslant$ 2K , FOR ALL VALUE OF K

2^{k+1} NOT $\leqslant K^2 + 2 + 2K$

2^{k+1} NOT $\leqslant (K+1)^2 + 1$

THE RELATION IS NOT TRUE FOR ALL VALUE OF K

[15] Repeat previous question using strong induction with $1 \leq n \leq 4$. [4]

LET P(N) BE THE PROPOSITION $N^2 + 1 \geqslant 2^n$

1. IF N = 1 , P(1) IS TRUE BECAUSE $1^2 + 1 \geqslant 2^1$
2. $P(1) = 1^2 + 1 \geqslant 2^1$ TRUE

   $P(2) = 2^2 + 1 \geqslant 2^2$ TRUE

   $P(3) = 3^2 + 1 \geqslant 2^2$ TRUE

   $P(4) = 4^2 + 1 \geqslant 2^4$ TRUE

   $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$ ALL ARE TRUE
3. P(K+1) ,P(5) == $5^2 + 1 \geqslant 5^4$ IS FALSE

Best wishes,
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