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CCCS 122 Computational Discrete Math
HW# 1 SPRING 2022
(5 pages)

PLO C4	CLO3.1	/100

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[1] Use propositional equivalence laws to prove that $(p \to q) \wedge (\neg p \longrightarrow q) \equiv q$ [10]

Proof lines	***Name the law you are using***
$(\neg p \vee q) \wedge (\neg p \to q)$	Implication law
$(\neg p \vee q) \wedge (\neg\neg p \vee q)$	Implication law
$(\neg p \vee q) \wedge (p \vee q)$	double negation law
$q \wedge (\neg p \vee p)$	distributive law
$q \wedge t$	Negation law
q	Identity law

[2] Express this statement “If you take the course, you either pass or fail” in symbols using p is “you pass" and q is “you fail". [5]

$p \to q \oplus \neg q$

[3] Use *De Morgan's laws* to express this statement: "It is neither cold nor dry" in symbols, using *c* for "it is cold" and *d* for "it is dry,". [5]

$\neg(c \vee d)$

[4] Write the negation of the statement. (Don't write \It is not true that ....") [15]

a) It is Thursday and it is sunny.

It is not Thursday or it is not Cold

b) I will go to the play or read a book, but not both.

Hint: use the negation of $p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$

I will go to the Play and read a book, or I will not go to the Play and not read a book.

c) If it is windy, then we go to the desert. *Hint: use the negation of implication law*

It is windy and we will not go to the desert.

[5] Prove or disprove $A-(B \cap C) = (A-B) \cup (A-C)$ [5]

Hint: start using the definition of difference $A - B = A \cap \bar{B}$

Proof lines	***Name the law you are using***
$A-(B \cap C) = A \cap \overline{B \cap C}$	*by the definition of difference*
$A \cap (\bar{B} \cup \bar{C})$	De Morgan's law
$(A \cap \bar{B}) \cup (A \cap \bar{C})$	Distributive law
$(A-B) \cup (A-C)$	defintion of difference

[6] $P(x, y)$ means "$x + 2y = xy$", where x and y are integers. Determine the truth value of the statement. [5]

statement	Truth value
$\boldsymbol{P(0,0)}$	T
$\boldsymbol{P(1,-1)}$	T
$\boldsymbol{\exists y\, P(3,y)}$	T

statement	Truth value
$\exists x \exists y\, P(x,y)$	T
$\exists y \forall x\, P(x,y)$	F
$\neg\forall x \exists y\, \neg P(x,y)$	F

[7] Match the English statement with its equivalent logical expression (at least one match), given that *P(x, y)* means "*x is taking y*", where *x* represents students and *y* represents courses. [10]

[P1] $\exists x \forall y\, P(x,y)$

[P2] $\exists y \forall x\, P(x,y)$

[P3] $\forall x \exists y\, P(x,y)$

[P4] $\neg\exists x \exists y\, P(x,y)$

[P5] $\exists x \forall y\, \neg P(x,y)$

[P6] $\forall y \exists x\, P(x,y)$

[P7] $\exists y \forall x\, \neg P(x,y)$

[P8] $\neg\forall x \exists y\, P(x,y)$

[P9] $\neg\exists y \forall x\, P(x,y)$

[P10] $\forall x \exists y\, \neg P(x,y)$

Every course is being taken by at least one student.	(P6)
Some student is taking every course.	(P1)
No student is taking all courses.	(P10)
There is a course that all students are taking.	(P2)
Every student is taking at least one course.	(P3)
There is a course that no students are taking.	(P7)
Some students are taking no courses.	(P5)
No course is being taken by all students.	(P9)
Some courses are being taken by no students.	(P7)
No student is taking any course.	(P4)

[8] Given the first six elements of {an} as -3, 1, 13, 33, 61, 97, ... Find a formula that describes this sequence. (Write the final answer <u>only</u>) [10]

$a_n = (4*n^2+1) - 4$

[9] True or False [15]

question	**answer**
1. If f: N→N, and f(x) = 3 – x, then f(x) is a function	F
2. If f: N→Z, and f(x) = 3 – x, then f(x) is a function	T
3. $\lfloor -2.2 \rfloor = -2$	F
4. $\lceil 2.2 \rceil = 3$	T
5. $\lceil -2.2 \rceil = -2$	T
6. $\lceil \lfloor 1/2 \rfloor + \lceil -1/2 \rceil + 1/2 \rceil = 2$	F

7. If *f:A→A, A={a,b,c,d}, with these assignments f (a) = b, f (b) = a, f (c) = c, f (d) = d,*	*f is one-to-one*	T
	f is onto	T
8. If *f:A→A, A={a,b,c,d}, with these assignments f (a) = d, f (b) = b, f (c) = c, f (d) = d,*	*f is one-to-one*	F
	f is onto	F

9. $f(x) = -3x + 4$ is a bijection	F
10. $f(x) = -3x^2 + 7$ is a bijection	T
11. The negation of $\forall x\ ((x > -2) \vee (x < 2))$ is $\exists x\ \big((x \le -2) \wedge (x \ge 2)\big)$	T
12. The negation of $\exists x\ (1 < x \le 5)$ is $\forall x\ ((1 \ge x) \vee (x > 5))$	T
13. $\sum_{k=6}^{122} k = 7488$	T

[10] Fill the blank cells in the table [5]

Math form	List form
{x∈ **Z** | (-2 ≤ x ≤ 2) ∧ x is even}	= {-2, 0, 2}
{x∈ **Z** | $x^2 = 4 \vee x^2 = 9$}	{-2, -3, 2, 3}
{x∈ **Z** | $x^2 = 4 \wedge x^2 = 9$}	{ }
{x∈ $\mathbf{Z}^+$| $x^2 \le 9$}	{1, 2, 3}
{x∈Z | $x^2 \le 9$}	{−3,−2,−1, 0, 1, 2, 3}
{x∈Z | $x^2 \le 9 \wedge x \ne 0$}	{−3,−2,−1, 1, 2, 3}

[11] Let the *alphabet* be the universe, S = {a, k, m, z}, T = {k, x, z} fill the blanks with final answer. [5]

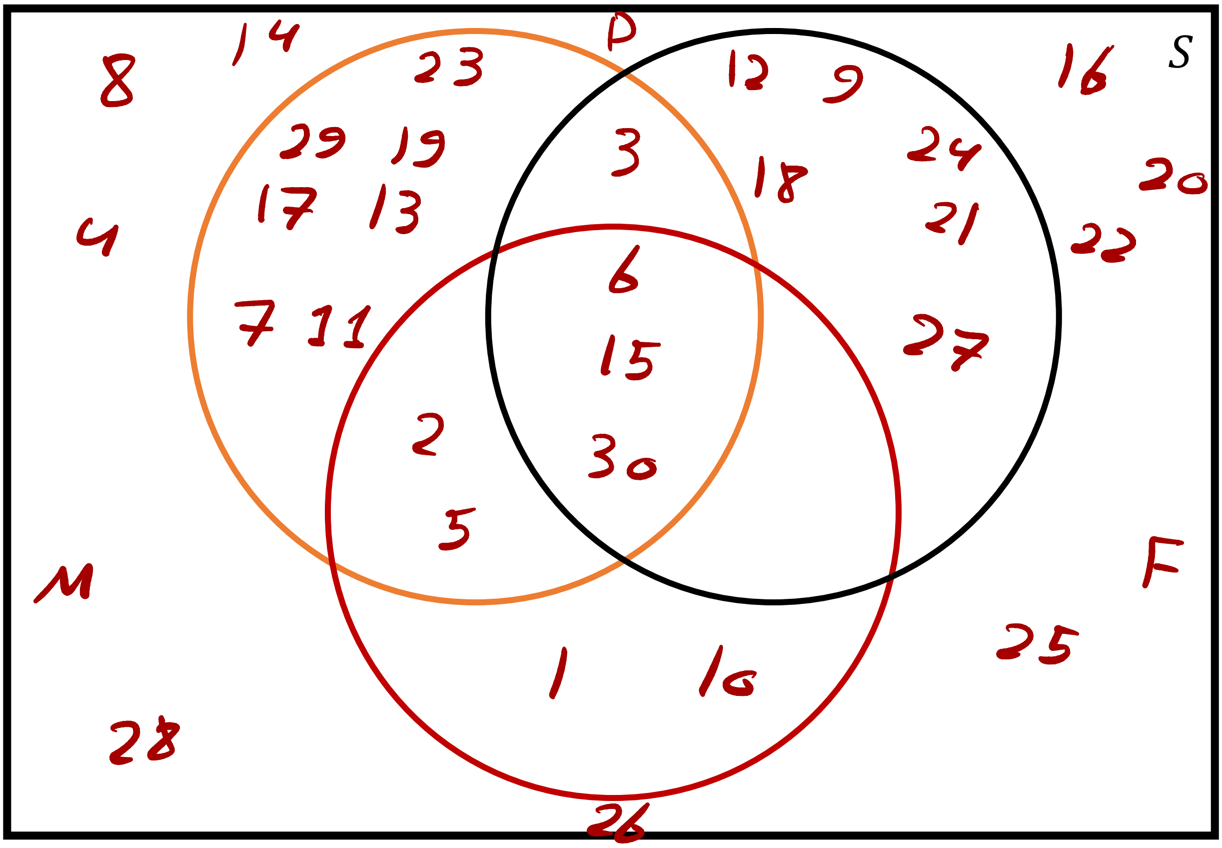
$\lvert \bar{S} \rvert$	= 4	$\lvert S \times T \rvert$	= 12
$\lvert S \cup T \rvert$	= 5	$\lvert \bar{S} \times T \rvert$	= 66
$\lvert S \cap \bar{T} \rvert$	= 2	$\lvert 2^S \times 2^T \rvert$	= 128

[12] Find $f \circ g$ and $g \circ f$, where $f(x) = x2 + 1$ and $g(x) = x + 2$, are functions from $R \to R$. [5]

$(f \circ g)(x) = f(g(x)) = f(x+2) = 2(x+2)+1 = 2x+4+1 = 2x+3$

$(g \circ f)(x) = g(f(x)) = g(2x+1) = 2(x+2)+1 = 2x+4+1 = 2x+5$

[13] **Do Question 18 in Lab 05** [5]



Best wishes, ■