* Vector Space Properties : 1) non empty ,,, At least 1 element

0, 1, 100 ← Real numbers

$\begin{bmatrix} a_1 & a_3 \\ a_2 & a_4 \end{bmatrix}$ ← Matrices

$x^2 - x + 5$ ← Polynomial

(1, 0, 5) ← (1, 0) ← ordered Pairs

# Vector Space

Addition

Scaler Multiplication

Let **u**, **v**, and **w** be vectors in the plane, and let $c$ and $d$ be scalars.

| | | |
|---|---|---|
| 1) | 1. **u** + **v** is a vector in the plane. | Closure under addition |
| 3 | 2. **u** + **v** = **v** + **u** | Commutative property of addition |
| 4 | 3. (**u** + **v**) + **w** = **u** + (**v** + **w**) | Associative property of addition |
| 5 | 4. **u** + **0** = **u** | Additive identity property |
| | 5. **u** + (−**u**) = **0** | Additive inverse property |
| 2) | 6. $c$**u** is a vector in the plane. | Closure under scalar multiplication |
| | 7. $c$(**u** + **v**) = $c$**u** + $c$**v** | Distributive property |
| | 8. ($c$ + $d$)**u** = $c$**u** + $d$**u** | Distributive property |
| | 9. $c$($d$**u**) = ($cd$)**u** | Associative property of multiplication |
| | 10. 1(**u**) = **u** | Multiplicative identity property |

1) we're Studying R vector Space "Clouser under addition"

-2 + 7 = 5 → in the R vector Space ✓

"Clouser under Scaler Multiplication"

6 ) we're Studying the Ordered Pairs Vector Space

cU ⇒ -4(3, 5) ⇒ (-12, -20) → in the Ordered Pairs Vector Space ✓

2) Commutative law قانون الإبدال

U+V = V+U ⇒ 5 + 6 = 11 ⇒ 6+5 = 11 ✓

3) Associative law

$U+(V+W)=(U+V)+W \Rightarrow 5+(2+1)\overset{8}{=}(5+2)+1$ ✓

4) Additive identity Property "الصفر ما بأثر على الفكتور"

$(3,4)+(0,0)=(3,4)$ ✓

5) Additive inverse "النظير الجمعي" ⟹ الي يخليهم = صفر

$U+(-U)=0 \Rightarrow 5+(-5)=0$ ✓

7) Distributive law "قانون التوزيع"

$k(U+V)=kU+kV \Rightarrow 2(3+5)\overset{16}{=}2(3)+2(5)$ ✓

→ real number

→ Objects

8) Distributive law "قانون التوزيع"

$(k+m)U=kU+mU$ ✓

Real numbers    Object

9) Associative Property of multiplication

$k(mU)=(km)U$ → "اضربهم بأي طريقة" ✓

real numbers    Object

10) Multiplicative identity Property "لما تضرب الا وبجكت في 1 يعطيك نفسه" ✓

$1U=U$ → real number

Object

example: * Set Of 2x2 matrix with used operations

the 10 laws

Step 1) Closure under addition "1" ✓

$$U+V = \underbrace{\begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}}_{R} + \underbrace{\begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}}_{R} = \overbrace{\underbrace{\begin{bmatrix} u_1+v_1 & u_2+v_2 \\ u_3+v_3 & u_4+v_4 \end{bmatrix}}_{R}}^{2\times 2}$$

Step 2) Closure under Scaler multiplication "6" ✓

$$\underbrace{K}_{R}U = K\begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}^{R} = \underbrace{\begin{bmatrix} ku_1 & ku_2 \\ ku_3 & ku_4 \end{bmatrix}^{R}}_{2\times 2}$$

Step 3) Commutative law ✓ "من خصائص المصفوفات"

$$U+V = V+U$$

$$\begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix} + \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix} + \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}$$

Step 4) Associative law ✓ "من خصائص المصفوفات"

Step 5) zero Vector ✓ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Step 6) Additive inverse ✓

Step 7, 8, 9) "من خصائص المصفوفات" ✓

Step 10) Multiplicative identity Property ✓

$$1u = u$$

$$1\begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix} = \overset{2\times 2}{\begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix}}$$

Set Of 2x2 matrices is a Vector Space

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$R^2 \to$ Ordered Pairs $(x, y)$

$R^3 \to$ Ordered Pairs $(x, y, z)$

$R^n \to$ Ordered Pairs $(x, y, \ldots n)$

How to Check If it is a Vector Space or not?

Check the 10 laws $\to$ Fails in 1 law $\Rightarrow$ Not a Vector Space

$\to$ Passes All the laws $\Rightarrow$ Vector Space

$\to$ Start by $1 \to 6 \to 10 \to 2, 3, 4, \ldots$

$$
\begin{aligned}
R &= \text{set of all real numbers} \\
R^2 &= \text{set of all ordered pairs} \\
R^3 &= \text{set of all ordered triples} \\
R^n &= \text{set of all } n\text{-tuples} \\
C(-\infty, \infty) &= \text{set of all continuous functions defined on the real number line} \\
C[a, b] &= \text{set of all continuous functions defined on a closed interval } [a, b] \\
P &= \text{set of all polynomials} \\
P_n &= \text{set of all polynomials of degree} \leq n \\
M_{m,n} &= \text{set of all } m \times n \text{ matrices} \\
M_{n,n} &= \text{set of all } n \times n \text{ square matrices}
\end{aligned}
$$