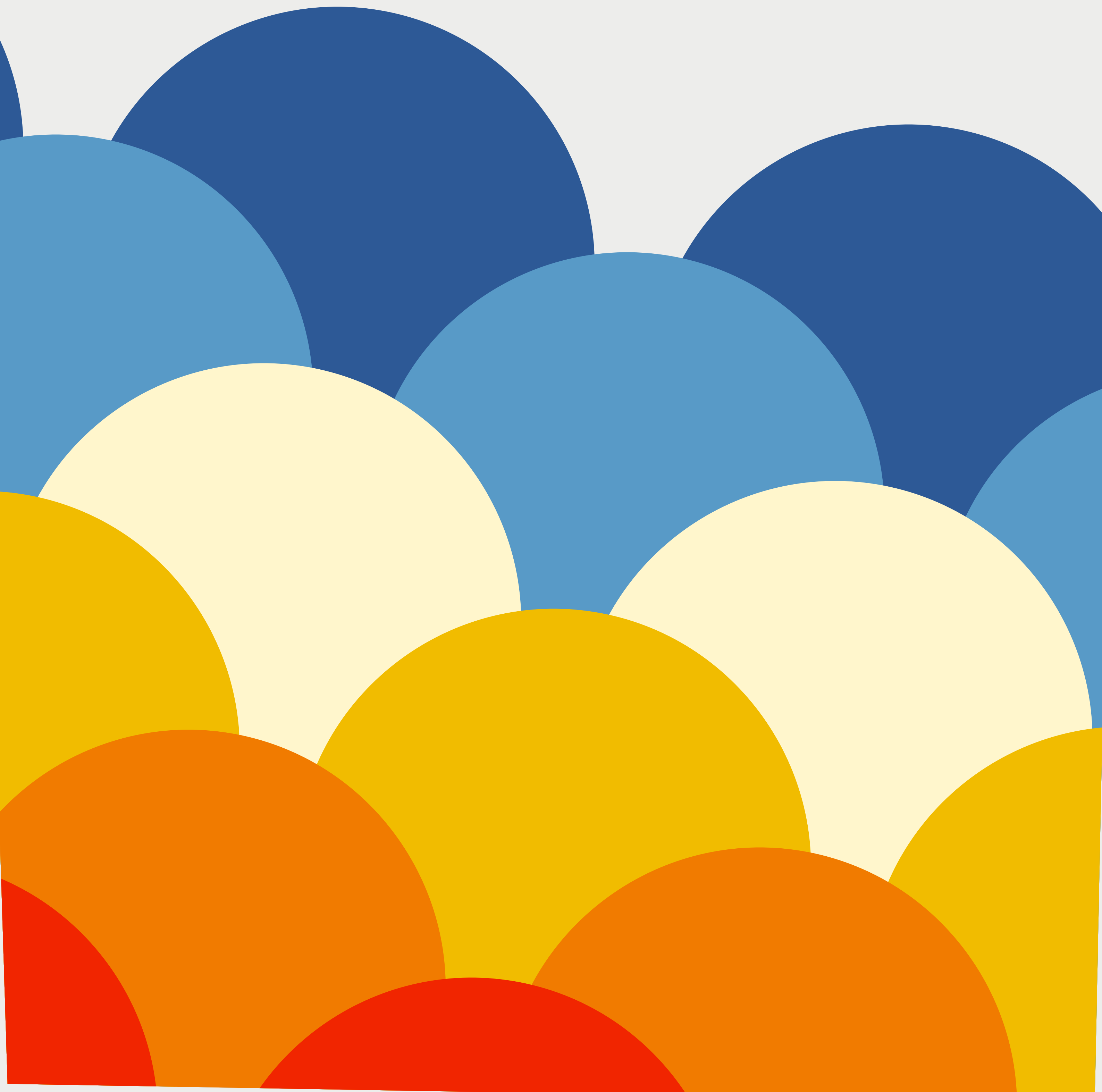

Cramer's Rule
↓
det of Cofficiant M
↓
det of each M
↓
$A_1 = \frac{det(A_1)}{det(A)}$

$A_2 = \frac{det(A_2)}{det(A)}$

linear Combination
↓
Found k
↓
Consistant
↓
You Can write it as a linear Combination

Span
↓
Same Process but equals b vector
↓
write the linear System

↓ $\square$M
- det=0 → inconsistant, Not Span → مالها حل
- det≠0 → Consistant, Span → لها حل

↓ Augmented Matrix
- REF
- RREF

→ inconsistant, Not Span → مالها حل

→ Consistant, Span → لها حل

linear Independence
↓
Same Process but equals zero
↓
Always Consistant
↓
get k's Value
- k's = 0 → Independent → Trivial
- k's ≠ 0 → dependent → Non Trivial

$\vec{0}$ = dependent

one vector only = Independent

multiple of each other = dependent

num of $\vec{v}$ > degree of Subset = dependent (doesn't span)

det only for Square Matrices
- det=0 → "dependent" Non Trival
- det≠0 → "independent" Trival

basis
- Span
- linearly Independence

the name of the subspace

Coordinate Vector

$(V)_S = (c_1, c_2, c_3)$

Same Process but equal to the givin Vector
↓
RREF → $(V)_S$ = (عدد، عدد، عدد)

THEOREM 4.8
A Property of Linearly Dependent Sets

A set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$, $k \geq 2$, is linearly dependent if and only if at least one of the vectors $\mathbf{v}_j$ can be written as a linear combination of the other vectors in S.

PROOF To prove the theorem in one direction, assume S is a linearly dependent set. Then there exist scalars $c_1, c_2, c_3, \ldots, c_k$ (not all zero) such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + \cdots + c_k\mathbf{v}_k = \mathbf{0}.$$

Because one of the coefficients must be nonzero, no generality is lost by assuming $c_1 \neq 0$. Then solving for $\mathbf{v}_1$ as a linear combination of the other vectors produces

$$c_1\mathbf{v}_1 = -c_2\mathbf{v}_2 - c_3\mathbf{v}_3 - \cdots - c_k\mathbf{v}_k$$

$$\mathbf{v}_1 = -\frac{c_2}{c_1}\mathbf{v}_2 - \frac{c_3}{c_1}\mathbf{v}_3 - \cdots - \frac{c_k}{c_1}\mathbf{v}_k.$$

Conversely, suppose the vector $\mathbf{v}_1$ in S is a linear combination of the other vectors. That is,

$$\mathbf{v}_1 = c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + \cdots + c_k\mathbf{v}_k.$$

Then the equation $-\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + \cdots + c_k\mathbf{v}_k = \mathbf{0}$ has at least one coefficient, -1, that is nonzero, and you can conclude that S is linearly dependent.

THEOREM 4.6
The Intersection of Two Subspaces Is a Subspace

If V and W are both subspaces of a vector space U, then the intersection of V and W (denoted by $V \cap W$) is also a subspace of U.

PROOF Because V and W are both subspaces of U, you know that both contain the zero vector, which means that $V \cap W$ is nonempty. To show that $V \cap W$ is closed under addition, let $\mathbf{v}_1$ and $\mathbf{v}_2$ be any two vectors in $V \cap W$. Then, because V and W are both subspaces of U, you know that both are closed under addition. Because $\mathbf{v}_1$ and $\mathbf{v}_2$ are both in V, their sum $\mathbf{v}_1 + \mathbf{v}_2$ must be in V. Similarly, $\mathbf{v}_1 + \mathbf{v}_2$ is in W because $\mathbf{v}_1$ and $\mathbf{v}_2$ are both in W. But this implies that $\mathbf{v}_1 + \mathbf{v}_2$ is in $V \cap W$, and it follows that $V \cap W$ is closed under addition. It is left to you to show (by a similar argument) that $V \cap W$ is closed under scalar multiplication. (See Exercise 48.)

REMARK: Theorem 4.6 states that the *intersection* of two subspaces is a subspace. In Exercise 44 you are asked to show that the *union* of two subspaces is not (in general) a subspace.

Subspace of R^n

R^n is a convenient source for examples of vector spaces, and the remainder of this section is devoted to looking at subspaces of R^n.

EXAMPLE 6 Determining Subspaces of R^2

matrix.

THEOREM 3.8
Determinant of an Inverse Matrix

If A is invertible, then

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

$$|A^{-1}| = \frac{1}{|A|}$$

PROOF Because A is invertible, $AA^{-1} = I$, and you can apply Theorem 3.5 to conclude that $|A||A^{-1}| = |I| = 1$. Because A is invertible, you also know that $|A| \neq 0$, and you can divide each side by $|A|$ to obtain

$$|A^{-1}| = \frac{1}{|A|}.$$

$$\frac{|A||A^{-1}|}{|A|} = \frac{I \to 1}{|A|} = \frac{1}{|A|}$$