



SubSpace

Closure under addition

zero vector

Closure under scalar multiplication

Example: $(a_1, a_2, 0)$ of \mathbb{R}^3

$$1) \text{ Closure under addition} \Rightarrow (a_1, a_2, 0) + (a_3, a_4, 0) = (a_1 + a_3, a_2 + a_4, 0) \in \mathbb{R}^3$$

اجمع عناصرتين طائفتين اذا يتحقق \Rightarrow
دادد المجموعتين لا تساوي ام لا
وطبعاً يتحقق الصورة المطلوبة في المدار

$$2) \text{ Closure under scalar multiplication} \Rightarrow K(a_1, a_2, 0) = (ka_1, ka_2, 0) \in \mathbb{R}^3$$

اضرب k في الجملة طائف اذا \Rightarrow
يتحقق لا فد المجموعات لا

$$\text{وطبعاً يتحقق الصورة المطلوبة في المدار}$$

$$= \frac{(ka_1)}{\downarrow R} \quad \frac{(ka_2)}{\downarrow R}$$

$\therefore (a_1, a_2, 0)$ subspace of \mathbb{R}^3

Example 2) Diagonal Matrices Of $M_{n \times n}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

\downarrow
square
matrix

1) CUA \Rightarrow diagonal Matrices مكتوب ✓

2) CUSM \Rightarrow diagonal Matrices مكتوب ✓

diagonal Matrices is a Subspace of $M_{n \times n}$

Example 3) P_2 of P_n $\rightarrow 1 + a_1x + a_2x^2 + \dots + a_nx^n$
 \circlearrowleft
الدرجة الأولى

Counter example: CUA $\Rightarrow (3+x+x^2) + (6+2x-x^2)$
 $= 9+3x+0 \Rightarrow$ جمع من جموعة الدرجات الأولى

\therefore Not a Subspace of P_n

Note: You can use Counter examples if he gave you "P₂ or P₃ or P_n" from P_n

linear Combination

دجج هي فكتور للخطوة على الفكتور المطلوب حال:

هل اذا دمجت U و V حصلت على W ؟

example: $W = (2, 2, 2)$ other vectors $V = (0, -2, 2)$
 $U = (1, 3, -1)$

$$W = k_1 U + k_2 V$$

$$(2, 2, 2) = k_1 (0, -2, 2) + k_2 (1, 3, -1)$$

$$(2, 2, 2) = k_1 (0, -2, 2) + k_2 (1, 3, -1)$$

صفر ضرب أحادي ضرب أحادي ضرب

$$(2, 2, 2) = (0, -2k_1, 2k_1) + (k_2, 3k_2, -k_2)$$

$$(2, 2, 2) = (k_2, -2k_1 + 3k_2, 2k_1 - k_2)$$

$$(2, 2, 2) = (k_2, -2k_1 + 3k_2, 2k_1 - k_2)$$

$$\begin{aligned} k_2 &= 2 \\ -2k_1 + 3k_2 &= 2 \\ 2k_1 - k_2 &= 2 \end{aligned}$$

جib al qimma k₁, k₂ Back Substitution
Gauss Jordan or ...

linear Combination \Rightarrow

$$(2, 2, 2) = 2U + 2V$$

Note: if you can find the value of k
then it is consistent and you can
write it as a linear combination
if not then it is not consistent
and you can't write it as a linear combination

Span

$k_1(1,0) + k_2(0,1)$ Span of \mathbb{R}^2 ?

Yes: $2(2,0) + 5(0,1)$
 $= (4,5)$ And so on...

Example: $V_1 = (1, 1, 2)$ $V_2 = (1, 0, 1)$ $V_3 = (2, 1, 3)$
Span \mathbb{R}^3 ?

$$k_1 V_1 + k_2 V_2 + k_3 V_3 = b \rightarrow \text{من كليسي}$$

$$\overbrace{k_1(1, 0, 2)}^{} + \overbrace{k_2(1, 0, 1)}^{} + \overbrace{k_3(2, 1, 3)}^{} = (b_1, b_2, b_3)$$

$$(k_1, k_1, 2k_1) + (k_2, 0, k_2) + (2k_3, k_3, 3k_3) = (b_1, b_2, b_3)$$

$$= (k_1+k_2+2k_3, k_1+k_3, 2k_1+k_2+3k_3) = (b_1, b_2, b_3)$$

$$\begin{aligned} k_1 + k_2 + 2k_3 &= b_1 \\ k_1 + k_3 &= b_2 \\ 2k_1 + k_2 + 3k_3 &= b_3 \end{aligned}$$

doesnt Span \mathbb{R}^3

\det
Only for square
matrices

$\det = 0$
Inconsistent
not Span
 \mathbb{W}

Augmented
Matrices

$\det \neq 0$
Consistent
It spans \mathbb{W}

Gauss ✓
Gauss Jordan
Inconsistent
not Span
 \mathbb{W}

Consistent
It spans \mathbb{W}

