Matrix trace

A: Squre Matrix $\therefore$ trace = مجموع عناصر القطر الرئيسي

e.g: $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$ $\therefore tr(A) = 2+2+1 = 5$

trace Properties

1. $Tr(KA) = K\,Tr(A)$

   Constant

2. $Tr(A \pm B) = Tr(A) \pm Tr(B)$

3. $Tr(I_n) = n$

   Identity degree

4. $Tr(AB) = Tr(BA)$

5. $Tr(A^T) = A$

e.g: $tr(A)=3$, $tr(B)=-1$, A,B 3×3

get the following :

1) $tr(2A) = 2Tr(A) = 2\times3 = 6$

2) $tr(3A-2B) = 3Tr(A) - 2Tr(B) = (3\times3) - (2\times-1) = 11$

3) $tr(A^T) = Tr(A) = 3$

4) $tr(\frac{1}{2}(A+A^T)) = Tr(\frac{1}{2}A) + Tr(\frac{1}{2}A^T) = \frac{1}{2}tr(A) + \frac{1}{2}tr(A)$

$= \frac{1}{2}(3) + \frac{1}{2}(3) = 3$

☆ Note: if $A,B \rightarrow$ Skew matrix $\rightarrow tr(A \cdot B) = tr(B \cdot A) = 0$

↳ القطر الرئيسي عبارة عن اصفار

linear transformation

Let V and W be vector spaces. The function $T: V \to W$ is called a **linear transformation** of V into W if the following two properties are true for all $\mathbf{u}$ and $\mathbf{v}$ in V and for any scalar c.

1. $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ ✓
2. $T(c\mathbf{u}) = cT(\mathbf{u})$ ✓

then we can say that T is a linear transformation

☆ if V = W then T is called linear operator

domain, vector spaces, codomain

Q 1:

vector space → matrix

vector space → R

Q. Let $T: M_n \to \mathbb{R}$ such that $T(A) = tr(A)$, show that T is linear Transformation

VS لازم اول

Step 1) let $A, B \in M_n$

trace

1. $T(A+B) = Tr(A+B) = Tr(A) + Tr(B)$
$= T(A) + T(B)$ ✓

2. $T(KA) = Tr(kA) = k\,Tr(A) = kT(A)$ ✓

or B

✓ T is linear transformation ✓

Q2.

Q2 If $T: M_n \to \mathbb{R}$ such that $T(A) = \det(A)$. Is T linear transformation

1. let $A, B \in M_n$

1. $T(A+B) = \det(A+B) \neq \det(A) + \det(B)$

2. $T(KA) = \det(KA) \neq K\det(A)$

T is not linear transformation

يكفي اختلال شرطا واحد لعدم اعتبار T ← l.T

Notes :

domain = codomain

1. let V be any vector space, let $T: V \to V$ such that $T(u) = u$ for all $u \in V$, then T is linear Transformation, and we call T the Identity operator on V.
2. If $T: V \to W$ such that $T(u) = 0_w$, Then T is called Zero transformation

---

Q3.

Q3 Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(x,y) = (2x+y,\ y-x)$, show that T is linear Transformation

domain for the function
&
Codomain for the image

* let $U, V \in \mathbb{R}^2 \Rightarrow U = (x_1, y_1),\ V = (x_2, y_2)$

$U+V = (x_1+x_2,\ y_1+y_2)$

1. $T(U+V) = (2(x_1+x_2) + y_1+y_2,\ y_1+y_2 - x_1+x_2)$

ترتيب فقط $= (2x_1+y_1+2x_2+y_2,\ y_1-x_1+y_2-x_2)$

$T(U) + T(V)$

الشرط الأول تحقق ✓

2. $kU = (kx_1 + ky_1)$ عامل مشترك

$T(kU) = (2kx_1 + ky_1, ky_1 - kx_1) = k(2x_1 + y_1, y_1 - x_1)$

T is linear transformation $= kT(U)$ ✓

Qu.

For any vector $\mathbf{v} = (v_1, v_2)$ in R^2, let $T: R^2 \to R^2$ be defined by
$T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2)$.

(a) Find the image of $\mathbf{v} = (-1, 2)$.

$$\overset{v_1}{} \quad \overset{v_2}{}$$
$$T(-1, 2) = (-1-2, -1+2(2)) = (-3, 3)$$

(b) Find the preimage of $\mathbf{w} = (\overset{v_1}{-1}, \overset{v_2}{11})$.

RREF

$v_1 - v_2 = \text{Value}(v_1)$
$v_1 + 2v_2 = \text{Value}(v_2)$

$$\left.\begin{aligned} v_1 - v_2 &= -1 \\ v_1 + 2v_2 &= 11 \end{aligned}\right\} \quad \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 11 \end{bmatrix}$$

$$R_1 \times -1 \to R_2 \begin{bmatrix} 1 & -1 & -1 \\ 0 & 3 & 12 \end{bmatrix} R_2 \times 1/3 \to R_2 \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 4 \end{bmatrix} R_2 + R_1 \to R_1 \begin{array}{c} \begin{array}{ccc} v_1 & v_2 & B \end{array} \\ \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix} \end{array}$$

$v_1 = 3$
$v_2 = 4$

$\therefore$ Preimage $= (3, 4)$