* $\det(kA) = k^n \det(A)$

k = Constant
n = number of rows

* $\det(A+b) \neq \det(A) + \det(B)$

* square matrix is invertable if and only if $\det \neq 0$

* $\det(AB) = \det(A) \cdot \det(B)$

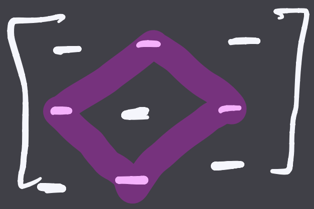
* $\det(A^{-1}) = \dfrac{1}{\det(A)}$

* Adjoint Matrix: ① 1) get the Coffactor for each element
2) put it in a Matrix
3) get the transpose of it
4) A^T = Adjoint

* Adjoint ② Matrix:

1) find the determinent for each element

2) put it in a Matrix



3) Change the Signs Of the diamond Only

4) get the transpose

5) A^T = Adjoint

* finding the inverse using the Adjoint

$$A^{-1} = \frac{1}{\det(A)} \text{Adjoint}(A)$$

example of finding the inverse using the Adjoint:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$$

1) find the determiment

$1(10-12) = -2$ $\quad$ $0(12-0) = 0$ $\quad$ $1(24-0) = 24$

$|A| = 22$

2) find the determinent for each element and put it in a Matrix "Don't Multiply the number"

$$\begin{bmatrix} 24 & -5 & -4 \\ 0 & 3 & -2 \\ -2 & 5 & 4 \end{bmatrix}$$

3) Change the Signs Of the diamond

$$\begin{bmatrix} 24 & -5 & -4 \\ 0 & 3 & -2 \\ -2 & 5 & 4 \end{bmatrix} = \begin{bmatrix} 24 & +5 & -4 \\ 0 & 3 & +2 \\ -2 & -5 & 4 \end{bmatrix}$$

4) get the transpose

$$\begin{bmatrix} 24 & +5 & -4 \\ 0 & 3 & +2 \\ -2 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 24 & 0 & -2 \\ 5 & 3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$$

A^T = Adjoint

5) Use the therom

$$A^{-1} = \frac{1}{\det(A)} \text{Adjoint}(A) = \frac{1}{22} \begin{bmatrix} 24 & 0 & -2 \\ 5 & 3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 24/22 & 0 & -1/11 \\ 5/22 & 3/22 & -5/22 \\ -4/22 & 1/11 & 4/22 \end{bmatrix}$$

Cramer's Rule

1) write the Coffactor Matrix and get the determinent

2) write as many Matrices as the number Of the Variables "but replace the Variable's column with the Constant table numbers"

3) get the determinant for each Matrix

4) use Cramer's Rule = $\dfrac{\det(A_1)}{\det(A)}$

And

$$\frac{\det(A_2)}{\det(A)}$$

And

$$\frac{\det(A_3)}{\det(A)}$$

example for Cramer's Rule:

$$\begin{aligned} x_1 + \quad\quad + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned}$$

$$1)\ A = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \end{array} \qquad \det(A) = 20 + 0 + 24 = 44$$

$$\overset{A_1}{\begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix}} \overset{A_2}{\begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix}} \overset{A_3}{\begin{bmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix}}$$

$$A_1 = \det(A_1) = -40$$

$$A_2 = \det(A_2) = 72$$

$$A_3 = \det(A_3) = 152$$

$$A_1 = \frac{\det(A_1)}{\det(A)} = \frac{-40}{44}$$

$$A_2 = \frac{\det(A_2)}{\det(A)} = \frac{72}{44}$$

$$A_3 = \frac{\det(A_3)}{\det(A)} = \frac{152}{44}$$

EXAMPLE 5 Finding the Area of a Triangle

EXAMPLE 6 Finding an Equation of the Line Passing Through Two Points

Find an equation of the line passing through the points $(2, 4)$ and $(-1, 3)$.

EXAMPLE 8 Finding an Equation of the Plane Passing Through Three Points