linear equation system

One Solution

Intersict in one Point

$$3x \quad \begin{aligned} x - y &= 1 \\ 3y &= 4 \end{aligned} = \begin{aligned} 3x - \cancel{3y} &= 3 \\ \cancel{3y} &= 4 \end{aligned}$$

$$\frac{3x}{3} = \frac{7}{3} \therefore x = \frac{7}{3}$$

$$\frac{7}{3} - y = 1$$

$$\frac{-y}{-1} = \frac{1 - 7/3}{-1}$$

$$y = 4/3$$

No Solution

Parellel

$$-3x \quad \begin{aligned} x + y &= 4 \\ 3x + 3y &= 6 \end{aligned} = \begin{aligned} \cancel{-3x} \cancel{-3y} &= -12 \\ \cancel{3x} \cancel{-3y} &= 6 \end{aligned}$$

$$? \; 0 = -6 \; ?$$

Contradictory

infinte Solutions

lines over each other

$$-4x \quad \begin{aligned} 4x - 2y &= 1 \\ 16x - 8y &= 4 \end{aligned} = \begin{aligned} \cancel{-16x} \cancel{-8y} &= \cancel{-4} \\ \cancel{16x} \cancel{-8y} &= \cancel{4} \end{aligned}$$

$$? \; 0 = 0 \; ?$$

Correct $\therefore$ $4x - 2y = 1 = \frac{4x}{4} = \frac{1 + 2y}{4}$

$$x = \frac{1}{4} + \frac{1}{2}y$$

$\therefore$ let $y = t$ $\rightarrow$ $1/4 + \frac{1}{2}t$, $y = t$

row echolon form

$$\text{row echolon form} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

reduced row echolon form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Notation	Solving area
* we have a leading 1 * we have to Make them zero	$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix} \Rightarrow x-2$ Multiply the leading one's row to Cancel the lower number and ADD it to the lower row
	$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & -2 & -4 & -18 \\ 3 & 6 & -5 & 0 \end{bmatrix} \Rightarrow x-3$
* we have to Make it a leading 1	$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & -2 & -4 & -18 \\ 0 & 3 & -11 & -27 \end{bmatrix} \rightarrow x - \frac{1}{2}$

and Continue until you Make it like this

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 0 & 1 & \frac{24}{.1} \end{bmatrix} \rightarrow \text{row echelon form}$$

Gassuion → row echlon form + back Substution

Gauss-Jurdon → reduced row echlon form

If the last Column of the Matrice is a zero Column then it is a homogenus System

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 0 \\ 9 & 11 & 0 \end{bmatrix}$$

Alwayes has a Solution

one Solution
(Trivial)

infinte number of Solutions
(non trivial)

* number of Variables > number of equations

Infinte number of Solutions "non trivial"

* number Of Variables < number Of equations

one Solution only "trivial"

* Multiplying a Matrix by its Identity = the same Matrix

Identity Always a square Matrix

$$2 \times 2 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad 3 \times 3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad 4 \times 4 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transpose $\rightarrow A^T \rightarrow A_{m \times n} \rightarrow A^T_{n \times m}$

example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

القطر الرئيسي تثبتو وتبدل الباقي

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $B^T \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

$A \cdot A^{-1} = I$ "Identity"

for 3X3

How to find the inverse of the Matrix?

1) Adjoint Method: $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$$

STEP 1) find the determenent

$$\begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix} \end{array}$$

$$|A| = 1(24-0) - 2(5) + 3(4)$$

$$|A| = 22$$

STEP 2) find the determenet for each element

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix} = \begin{bmatrix} 24 & -5 & -4 \\ 12 & 3 & -2 \\ -2 & 5 & 4 \end{bmatrix}$$

And So on...

STEP 3) Signs

$$\begin{bmatrix} 24 & -5 & -4 \\ 12 & 3 & -2 \\ -2 & 5 & 4 \end{bmatrix} = \text{Change the Signs}$$

STEP 4) get the Transpose

$$Z = \begin{bmatrix} 24 & +5 & -4 \\ -12 & 3 & +2 \\ -2 & -5 & 4 \end{bmatrix}$$

$$Adj(A) = Z^T$$

$$Z = \begin{bmatrix} 24 & +5 & -4 \\ -12 & 3 & +2 \\ -2 & -5 & 4 \end{bmatrix} \qquad Z^T = \begin{bmatrix} 24 & -12 & -2 \\ 5 & 3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 24 & -12 & -2 \\ 5 & 3 & -5 \\ -4 & 2 & 4 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 24/22 & \frac{-12}{24} & \frac{-2}{24} \\ 5/24 & 3/24 & -5/24 \\ -4/24 & 2/24 & 4/24 \end{bmatrix}$$

for 2×2

How to find the inverse of the Matrix?

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \longrightarrow A^{-1}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

Adj: بدل عناصر القطر الرئيسي
وغير اشارة الباقي

$$\text{Adj} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$|A| = -2$$

$$A^{-1} = \frac{-1}{2} \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \end{bmatrix}$$

$2X - 3Y = 8$

$4X + 5Y = 1$

divide them into $\underline{3}$ Matrecies

$$\begin{array}{cc} x & y \end{array}$$
$$A = \begin{vmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix} \quad B = \begin{vmatrix} x \\ y \end{vmatrix} \quad C = \begin{vmatrix} 8 \\ 1 \end{vmatrix}$$

قيمة X و Y = $A^{-1} \times C$

$$A^{-1} = \frac{1}{22}\begin{bmatrix} 5 & 3 \\ -4 & 2 \end{bmatrix} \qquad A^{-1} = \underset{2\times 2}{\begin{bmatrix} 5/22 & 3/22 \\ -4/22 & 2/22 \end{bmatrix}} \times \underset{2\times 1}{\begin{bmatrix} 8 \\ 1 \end{bmatrix}}$$

$$= \underset{2\times 1}{\begin{bmatrix} 43/22 \\ -1/11 \end{bmatrix}} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$\therefore X = 43/22$

$Y = -1/11$

minor → $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix}$

M_{22} ↓ row → Coulmn $= \begin{vmatrix} 0 & 1 \\ 4 & 1 \end{vmatrix} = 1 - 4 = -3$

Cofactor → $\begin{array}{ccccc} & + & - & + & \\ & 0 & 2 & 1 & \\ - & 3 & \overset{+}{-1} & 2 & - \\ & 4 & 0 & 1 & + \\ & + & - & & \end{array}$

$C_{12} = (-3 \times 1) - (4 \times -2) = 5$

$(-3) - (-8)$

So minor is getting the element's determenent without any sign or Anything else

Coffactor is getting the element's determenent with signs