



linear independence

Vectors u v w

آلكتب lc حقت وحدة من الإثنين
الباقيين لو تقدر فحنقول أنه

linear dependent
لوما تقدر فحنقول أنه

linear independent

Same test of the SPAN but After writing
the Vector equation we'll equal it to ZERO

$$k_1 v_1 + k_2 v_2 = \mathbf{0}$$

homogenous equation
Always Consistent

independent

$$k_1 = k_2 = \dots = k_n = 0$$

Trivial Solution

dependent

$$k \neq 0$$

Non Trivial

example: $V_1 = (1, -2, 3)$ $V_2 = (5, 6, -1)$
 $V_3 = (3, 2, 1)$

dependent or independent in \mathbb{R}^3 ?

1) $k_1 V_1 + k_2 V_2 + k_3 V_3 = \mathbf{0}$ ↗ matrix $\begin{bmatrix} c & c \\ c & c \end{bmatrix}$
order pairs $(0, 0)$

2) $k_1(1, -2, 3) + k_2(5, 6, -1) + k_3(3, 2, 1) = (0, 0, 0)$

3) $(k_1, -2k_1, 3k_1) + (5k_2, 6k_2, -k_2) + (3k_3, 2k_3, k_3) = (0, 0, 0)$

4) $(k_1 + 5k_2 + 3k_3, -2k_1 + 6k_2 + k_3, 3k_1 - k_2 + k_3) = (0, 0, 0)$

$$\begin{aligned} k_1 + 5k_2 + 3k_3 &= 0 \\ -2k_1 + 6k_2 + k_3 &= 0 \\ 3k_1 - k_2 + k_3 &= 0 \end{aligned}$$

مع الثوابت وكلو ← Augmented Matrices

↓
Gauss "REF"

$k_1 = 0 \quad k_2 = 0 \quad k_3 = 0$
 \downarrow
 dependent Non Trivial independent "Trivial"

det
Only for Square Matrices

det = 0 → "dependent" Non Trivial

det ≠ 0 → "independent" Trivial

zero Vector $\Rightarrow (0,0)$ or $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$ linearly dependent

Studying only one vector \Rightarrow linearly independent

If Vectors are scalarly multiple of each other \rightarrow linearly dependent

example: $u_1 = (-1, 2, 4)$ $u_2 = (5, -10, -20)$

$$u_1 = \frac{1}{5}(-1, 2, 4) = u_2 \cdot \frac{1}{5} \cdot (-1, 2, 4)$$

If the numbers of the Vectors $>$ degree of the subset

= linearly dependent

$\therefore \mathbb{R}^2 \rightarrow u_1, u_2, u_3$ linearly dependent

$\mathbb{P}^2 \rightarrow$
 linearly dependent