Optimization and regression Lab 1’ & 2’

All the equations must be verified by the tools: www.desmos.com/calculator

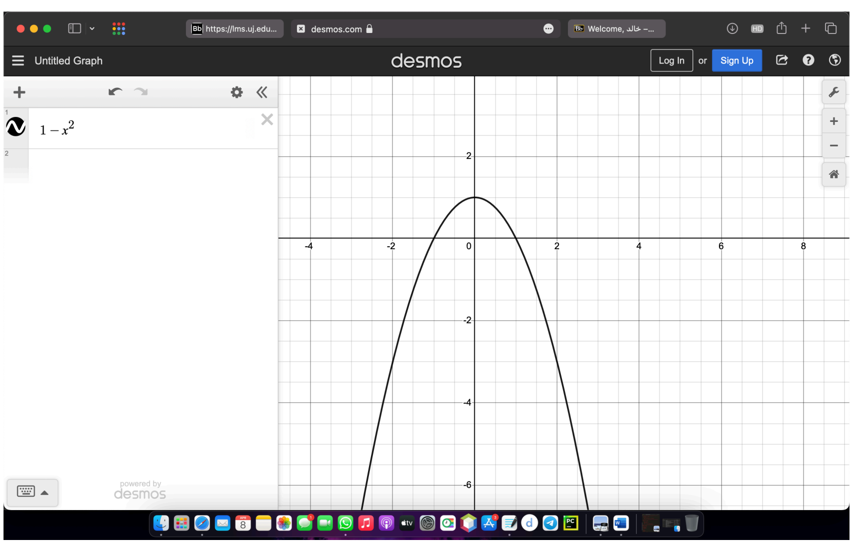
Exercise 1 :

Find all critical points :

1) what is the max of an area of a rectangle with perimeter equal to 12 ?

Area = XY >> MAX
2X + 2Y = 12 >>> X + Y = 6
Y = 6 – X
Area = F(x) = x(6-x)
F(x) = $6x - x^2$
F’(x) = $6 - 2x$>>> F’ = 0 >> X = 3
Area = X * Y >>> 3 x 3 = 9
Critical point = (3,3)

2) $1 - x^2$
F’(x) = -2x >> -2x = 0 >>> X = 0
The critical point: (0,0)

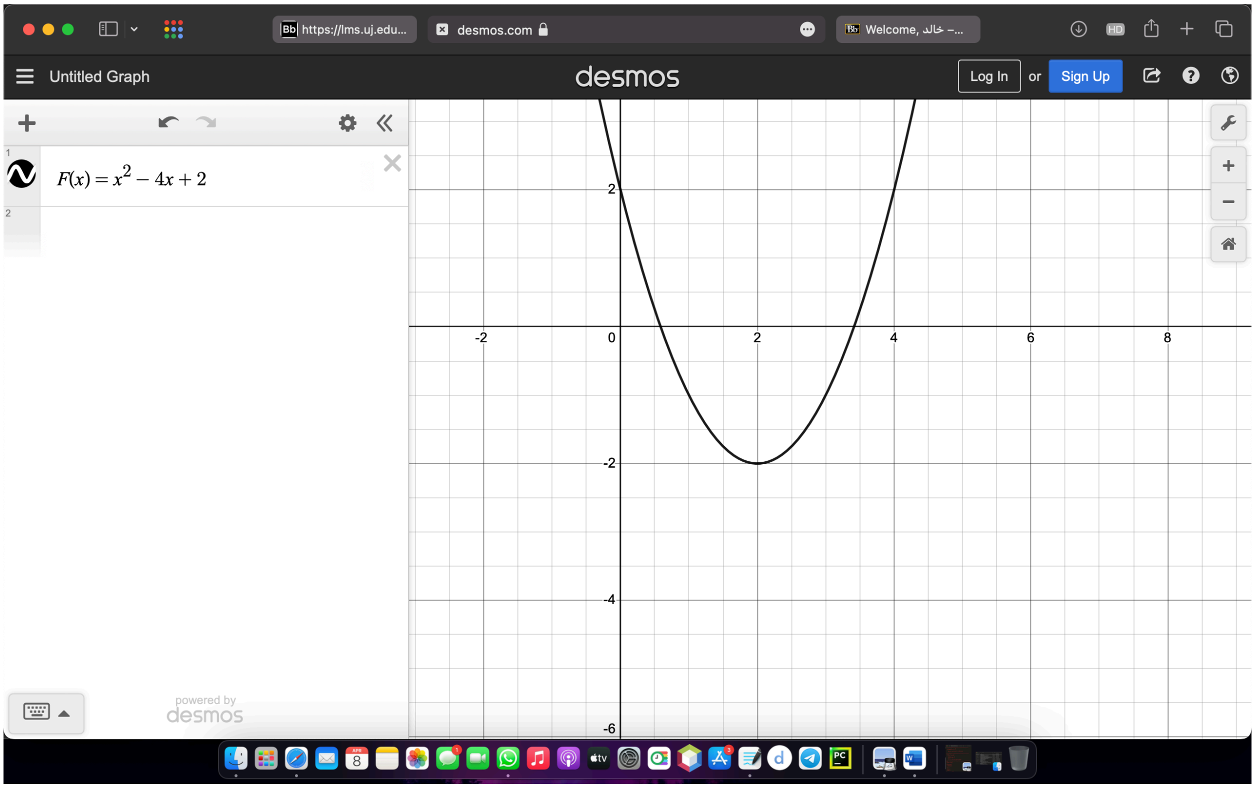


3)

$F(x) = x^2 - 4x + 2$

F’(x) = 2x – 4 >> 2x-4 =0 >>> 2x=4 >>> x = 2

The critical point: X = (2,0)



4) $f(x, y) = 1 + x^2 + y^2$

The partial derivative with respect to x is:

$\partial f/\partial x = 2x$

The partial derivative with respect to y is:

$\partial f/\partial y = 2y$

To find where both partial derivatives are equal to zero, we can solve the system of equations:

$2x = 0$

$2y = 0$

The only critical point is: (0,0)

5)

$$f(x, y) = 1 - (x - 2)^2 + (y + 3)^2$$

The partial derivative with respect to x is:

$\partial f/\partial x$ = -2(x - 2)

The partial derivative with respect to y is:

$\partial f/\partial y$ = 2(y + 3)

To find where both partial derivatives are equal to zero, we can solve the system of equations:

-2(x - 2) = 0

2(y + 3) = 0

The only critical point is : (2,-3) and (2,3)

6)

$$f(x, y) = (3x - 2)^2 + (y - 4)^2$$

The partial derivative with respect to x is:

$\partial f/\partial x$ = 2(3x − 2)(3)

The partial derivative with respect to y is:

$\partial f/\partial y$ = 2(y − 4)

To find where both partial derivatives are equal to zero, we can solve the system of equations:

2(3x − 2)(3) = 0

2(y − 4) = 0

Solving these equations gives us x = 2/3 and y = 4.

The only critical point is: ($\frac{2}{3}$,4)

Exercise 2 :

Find the critical points of the function and test for extrema or saddle points by using algebraic techniques.

1)

$$f(x, y) = 1 + x^2 + y^2$$

The partial derivative with respect to x is:

$\partial f/\partial x = 2x$

The partial derivative with respect to y is:

$\partial f/\partial y = 2y$

Setting both partial derivatives to zero, we get:

$2x = 0$ and $2y = 0$

which gives $x = 0$ and $y = 0$.

the only critical point of $f(x,y) = 1 + x^2 + y^2$ is (0, 0).

maximum, minimum, or saddle point test:

The second partial derivative with respect to x is:

$\partial^2 f/\partial x^2 = 2$

The second partial derivative with respect to y is:

$\partial^2 f/\partial y^2 = 2$

The mixed partial derivative is:

$\partial^2 f/\partial x \partial y = 0$

The second partial derivative test tells us to evaluate the function at the critical point and examine the signs of the second partial derivatives.

At the critical point (0,0), we have:

$f(0,0) = 1$

$\partial^{2}f/\partial x$^2(0,0) = 2 > 0

$\partial^{2}f/\partial y$^2(0,0) = 2 > 0

$\partial^{2}f/\partial x\partial y(0,0) = 0$

Since the second partial derivatives are both positive, we can conclude that:
$f(0,0)$ is a local minimum at the critical point (0,0).

The function $f(x,y)$ = 1 + x^2 + y^2 has a local minimum at the point (0,0).

2)

$$f(x, y) = x^4 + y^4 - 16xy$$

The partial derivative with respect to x is:

$\partial f/\partial x$ = 4x^3 − 16y

The partial derivative with respect to y is:

$\partial f/\partial y$ = 4y^3 − 16x

Setting both partial derivatives to zero, we get:

4x^3 − 16y = 0

4y^3 − 16x = 0

Solving for x and y, we get:

$x = y$

x^3 = 4

$x = \sqrt[3]{4}$

The critical point is $(x,y) = (\sqrt[3]{4},\sqrt[3]{4})$.

Critical point is a maximum, minimum, or saddle point test

The second partial derivative with respect to x is:

$\partial^2 f/\partial x^2 = 12x^2$

The second partial derivative with respect to y is:

$\partial^2 f/\partial y^2 = 12y^2$

The mixed partial derivative is:

$\partial^2 f/\partial x\partial y = -16$

At the critical point $(\sqrt[3]{4},\sqrt[3]{4})$, we have:

$f(\sqrt[3]{4},\sqrt[3]{4}) = 0$

$\partial^2 f/\partial x^2(\sqrt[3]{4},\sqrt[3]{4}) = 12(\sqrt[3]{4})^2 = 48 > 0$

$\partial^2 f/\partial y^2(\sqrt[3]{4},\sqrt[3]{4}) = 12(\sqrt[3]{4})^2 = 48 > 0$

$\partial^2 f/\partial x\partial y(\sqrt[3]{4},\sqrt[3]{4}) = -16$

The determinant of the Hessian matrix is:

$\partial^2 f/\partial x^2(\sqrt[3]{4},\sqrt[3]{4})\ \partial^2 f/\partial x\partial y(\sqrt[3]{4},\sqrt[3]{4})$

$\partial^2 f/\partial x\partial y(\sqrt[3]{4},\sqrt[3]{4})\ \partial^2 f/\partial y^2(\sqrt[3]{4},\sqrt[3]{4})$

$= (48)(-16) - (-16)(48)$

$= 0$

Since the determinant is zero and the second partial derivatives have different signs, we can conclude that the critical point $(x,y) = (\sqrt[3]{4},\sqrt[3]{4})$ is a saddle point.

The function $f(x,y) = x^4 + y^4 - 16xy$ has a saddle point at the point $(\sqrt[3]{4},\sqrt[3]{4})$.

3)

$$f(x, y) = 15x^3 - 3xy + 15y^3$$

The partial derivative with respect to x is:

$\partial f/\partial x = 45x^2 - 3y$

The partial derivative with respect to y is:

$\partial f/\partial y = 45y^2 - 3x$

Setting both partial derivatives to zero, we get:

$45x^2 - 3y = 0$

$45y^2 - 3x = 0$

Solving for x and y, we get:

$x^2 = 1/15$

$x = \pm(1/\sqrt{15})$

$y = \pm(\sqrt{5}x)$

The critical points are (x,y) =

- $(1/\sqrt{15}, \sqrt{5}/\sqrt{15})$,
- $(-1/\sqrt{15}, -\sqrt{5}/\sqrt{15})$,
- $(1/\sqrt{15}, -\sqrt{5}/\sqrt{15})$,
- $(-1/\sqrt{15}, \sqrt{5}/\sqrt{15})$.

Critical points are a maximum, minimum, or saddle point test:

The second partial derivative with respect to x is:

$\partial^2 f/\partial x^2 = 90x$

The second partial derivative with respect to y is:

$\partial^2 f/\partial y^2 = 90y$

The mixed partial derivative is:

$\partial^2 f/\partial x\partial y = -3$
At the critical point $(1/\sqrt{15},\sqrt{5}/\sqrt{15})$, we have:

$f(1/\sqrt{15},\sqrt{5}/\sqrt{15}) = 10(1/\sqrt{15})^3 + 15(\sqrt{5}/\sqrt{15})^3 = 10/3\sqrt{15}$

$\partial^2 f/\partial x^2(1/\sqrt{15},\sqrt{5}/\sqrt{15}) = 90(1/\sqrt{15}) = 6\sqrt{15} > 0$

$\partial^2 f/\partial y^2(1/\sqrt{15},\sqrt{5}/\sqrt{15}) = 90(\sqrt{5}/\sqrt{15}) = 30\sqrt{5} > 0$

$\partial^2 f/\partial x\partial y(1/\sqrt{15},\sqrt{5}/\sqrt{15}) = -3 < 0$

The critical point $(1/\sqrt{15},\sqrt{5}/\sqrt{15})$ is a saddle point.

the critical point $(-1/\sqrt{15},-\sqrt{5}/\sqrt{15})$ is also a saddle point

while the critical points $(1/\sqrt{15},-\sqrt{5}/\sqrt{15})$ and $(-1/\sqrt{15}, \sqrt{5}/\sqrt{15})$ are local minima.

Student name: Khalid Nimri
Student ID: 2140145