Capacitance

- Capacitor is an electric device which used for saving energy (electric charge) and enable us to use it later.

Capacitance

$q = CV$ → Voltage

Charge

q
C | V

→ C, F ← C, V → V

* SI Unit of Capacitane is → farad "F"

Example 1:

1.25×10^{-12}

A capacitor with capacitance of 1.25 pF is charged by applying a voltage of 12 V across its ends. The total charge of the capacitor is:

$q = CV$

Solution:

(D)

(A) 12 pC
(B) 13 pC
(C) 14 pC
(D) 15 pC ✓

$q = 1.25 \times 10^{-12} \times 12 = 1.5 \times 10^{-11}$

$q = 15\,pC$

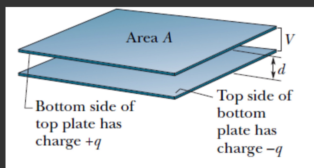
εA
C | d

Calculating the Capacitance

A Parallel-Plate Capacitor

$$C = \frac{\varepsilon_0 A}{d}$$

A → Area, d → distance



Example 2:

$(25 \times 10^{-2})^2 = 25 \times 10^{-4}$

A parallel-plate capacitor with plate's area 25 m^2 and separation of 17.7 mm is charged by applying a voltage of 12 V across its ends. The capacitance of the capacitor is:

Solution:

$C = \frac{\varepsilon_0 \times 25 \times 10^{-4}}{17.7 \times 10^{-3}}$

$C = 1.25 \times 10^{-12}$

(A) 0.83 pF
(B) 1.25 pF ✓
(C) 2.73 pF
(D) 3.09 pF

Example 3:

A parallel-plate capacitor has a capacitance of 8 μF. Its capacitance if the plate separation is doubled is:

كل ما زادت المسافة نقص الـ Capacitance

Solution:

(C)

8 → Normal
4 → doubled

$1d = 8\mu F$

$\frac{2d}{2} = \frac{8\mu F}{2}$ $\therefore d = 4$

(A) 2 μF
(B) 3 μF
(C) 4 μF ✓
(D) 5 μF

Example 4:

Referring to Example 3, if the plate area of the capacitor is doubled. The capacitance will be:

كلما زادت مساحة الصفيحة زادت الـ Capacitance

Solution:

(D)

$1 = 8\mu F$
$2 = X$ $= 16\mu F$

(A) 10 μF
(B) 12 μF
(C) 14 μF
(D) 16 μF ✓

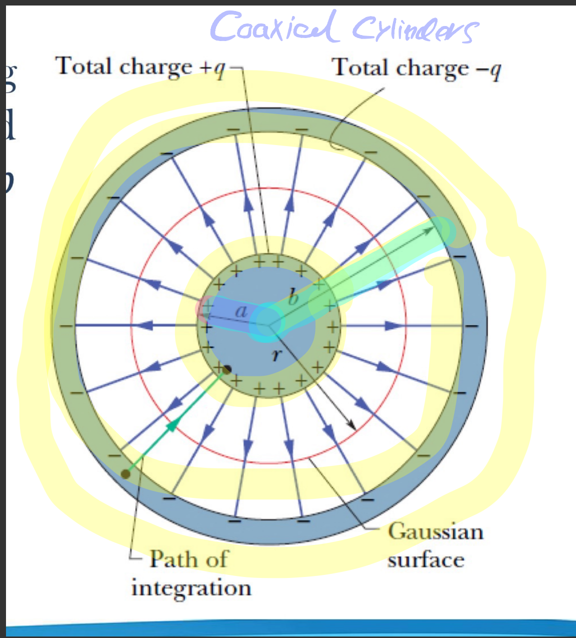
A Cylindrical Capacitor

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

length

outer radius

inner radius



Example 5:

Two radiuses

A coaxial cable of radii 5 mm and 3 mm is connected by a battery of 12 V. If the charge on each cable is 6 nC, the length of the capacitor is:

$r_{inner} = 3\times10^{-3}$

$r_{outer} = 5\times10^{-3}$

$V = 12V$

$q = 6\times10^{-9}$

$L = ?$

$C = \frac{q}{V} = 5\times10^{-10}$

$C = 2\pi\varepsilon_0 \frac{L}{\ln\frac{out}{in}}$

Solution:

$L = 4.59m$

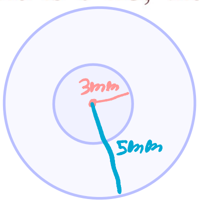
$L \simeq 4.6m$

(A) 5.4 m

✓(B) 4.6 m

(C) 2.9 m

(D) 1.8 m



Always $\ln\frac{out}{in}$

A Spherical Capacitor

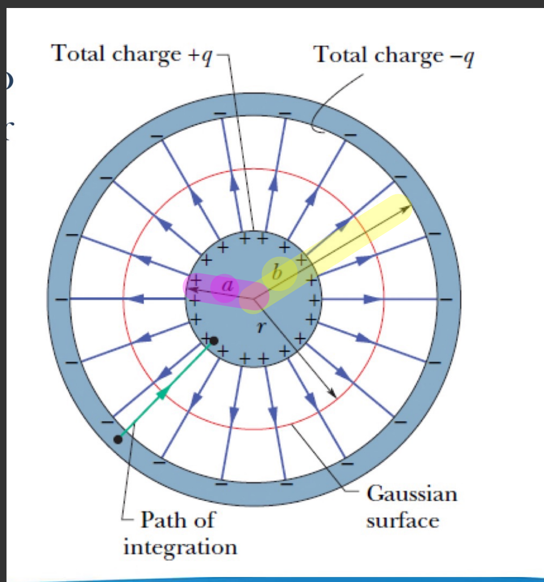
$$C = 4\pi\varepsilon_0 \frac{ab}{b - a}$$

inner

outer

inner

outer



Example 6:

Two concentric spherical shells of radii 4 cm and 3 cm has a charge of 5 nC. The potential difference across the capacitor is:

$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$

Solution:

$C = 1.33\times10^{-11}$

$\frac{q}{C} = \frac{5\times10^{-9}}{1.33\times10^{-11}}$

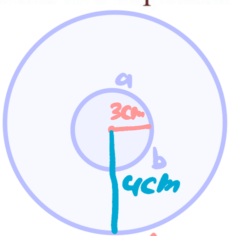
$V = \frac{375V}{1000} = \boxed{0.375KV}$

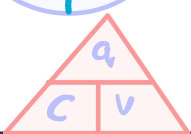
(A) 0.083 KV

✓(B) 0.375 KV

(C) 1.124 KV

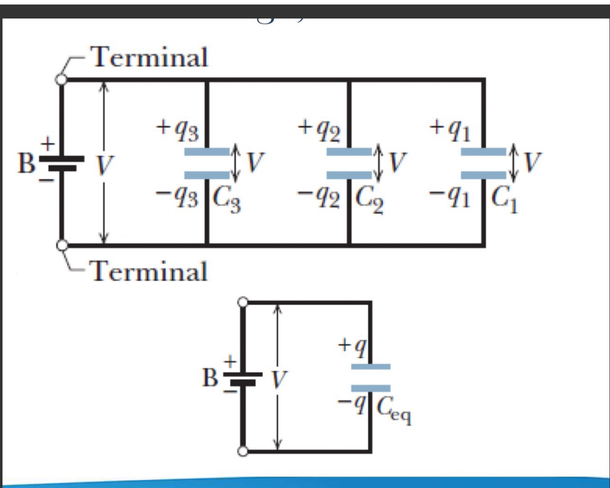
(D) 2.361 KV





Capacitors in Parallel and in Series

Capacitors in Parallel

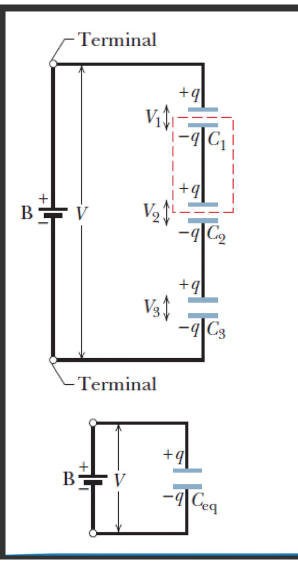


$$C_{eq} = \sum_{j=1}^{n} C_j$$

مجموع المكثفات "التوازي"

* Votage is the Same in all Capacitor
* the Charge on each Capacitor is different

Capacitors in Series



$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

"التوالي" $\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots\frac{1}{C_n}$

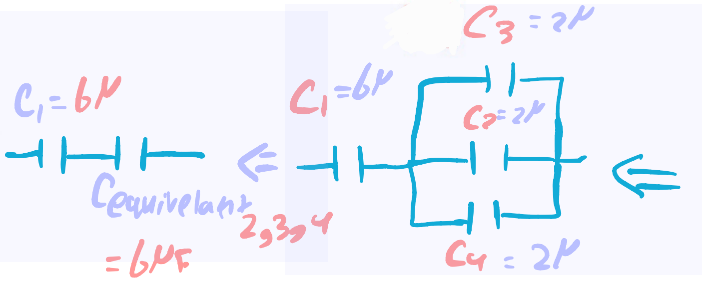
* the Charge is the Same as the total Charge
* the Voltage across each Capacitor is different

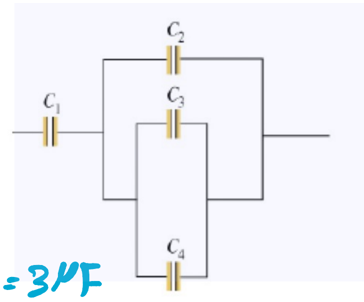
Example 7:

As shown in the figure, C_1= 6μF and C_2= C_3= C_4= 2μF. The equivalent capacitance is: $C_{equivelant_{2,3,4}} = 2\mu + 2\mu + 2\mu = 6\mu F$

Solution:

(A) 4 μF
✓(B) 3 μF
(C) 2 μF
(D) 1 μF





$$C_{equivalent_{1,Ceq_{2,3,4}}} = \frac{1}{6\mu} + \frac{1}{6\mu} = 3\mu F$$

Energy Stored in an Electric Field

$$U = \frac{q^2}{2C}$$

$$U = \tfrac{1}{2} C V^2$$

Energy Stored in an Electric Field

Example 8:

An isolated conducting sphere whose radius R is 6.85 cm has a charge q = 1.25 nC. The potential energy stored in the electric field of this charged conductor is:

$C = 4\pi\varepsilon_0 R \rightarrow$ Special Case

Solution:

$C = 7.62 \times 10^{-12}$

$U = \frac{q^2}{2C} = \frac{(1.25 \times 10^{-9})^2}{2(7.62 \times 10^{-12})} = 1.025 \times 10^{-7}$

$U \approx 1.03 \times 10^{-7}$ J

(A) 9.33×10^{-7} J
(B) 6.48×10^{-7} J
(C) 3.72×10^{-7} J
✓(D) 1.03×10^{-7} J

energy density

$$u = \tfrac{1}{2} \varepsilon_0 E^2$$

where its unit is J/m^3

Energy Stored in an Electric Field

Example 9:

Referring to Example 8, the energy density at the surface of the sphere is:

Solution: $U = \frac{1}{2}\varepsilon_0 E^2$ $E = k\frac{q}{r^2} = 2.39 \times 10^{3}$

$U = 2.54 \times 10^{-5}$ J/m^3

(A) 8.74×10^{-5} J/m^3
(B) 5.89×10^{-5} J/m^3
✓(C) 2.54×10^{-5} J/m^3
(D) 1.58×10^{-5} J/m^3