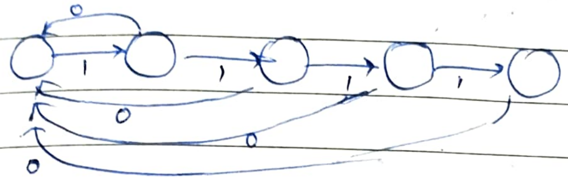
Problem Set #3

1. (a) $L = \{ w \mid w \text{ ends with } 1111 \}$

$\{\epsilon, 1, 11, 111, 1111\}$ is ~~too~~ a distinguishable set.

ϵ distinguishes 1111 with others
1 distinguishes 111 with previous
11 distinguishes 11 with previous.

Note that we can create a DFA with 5 states!



We can prove this DFA is correct easily.

Here $|X| = 5 = |Y|$ so X is largest

→ Size of largest distinguishable set = Size of minimal DFA

For any dist. set X, DFA with states Y. We can say $|X| \le |Y|$.

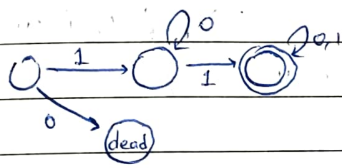
(b) $L = \{1^k x \mid x \in \{0,1\}^* \text{ and } \#1(x) \geq k,\ k \geq 1\}$

$\{\epsilon, 0, 10, 11\}$

→ 1 distinguishes ~~$(\epsilon, 0)$~~ $(\epsilon, 10)$ $(\epsilon, 11)$ → 0 distinguishes $(\epsilon, 0)$

→ 0 distinguishes $(0, 10)$ → ϵ distinguishes $(0, 11)$

→ 0 distinguishes $(10, 11)$



* Observe the above language is equivalent to saying string s is ~~accepte~~ in L iff $|s| \geq 2$, s starts with 1 and has atleast 2 ones.

* The DFA on left accepts these strings precisely

Hence as $|X| \le |Y|$ we say it is largest dist. set

(c) $L = \{1^k x \mid x \in \{0,1\}^* \text{ and } \#1(x) \le k,\ k \geq 1\}$

$X = \{1, 1^2, 1^3, 1^4, \ldots 1^i \ldots\}$

→ Observe 1^i distinguishable from 1^j where $j < i$ with string 01^i as $1^i 0 1^i \in L$ but $1^j 0 1^i \notin L$.

X is infinite. Because largest distinguishable set cardinality = States in smallest DFA ⇒ No DFA possible.

Irregular language.

(g) $L = \{cabc \mid$ ~~abc are~~ abc are nonempty strings$\}$

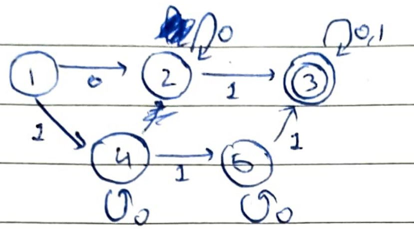
$X = \{1, 1^2 \ldots 1^i \ldots\}$

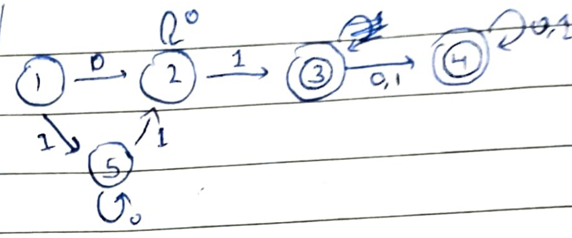
→ Observe 1^i is distinguisable from 1^j where $j < i$ with string 0001^i0. As $1^i 0001^j 0 \notin L$ (can easily be shown), $1^j 0001^j 0 \in L$

→ ~~Hence~~ We can say that $|X| = \infty$. Largest distinguishable set.

2. Consider $L=\{w \mid \exists x,y\in\{0,1\}^* \text{ s.t } w=xy,\ \#1(y)>\#1(x)\}$

Draw 2 non-isomorphic DFAs with the same number of states





$E_1=\{\epsilon\}$

$E_2=\{0^i \mid i\geq 1\}$

$E_3=\{w \mid w\in L\}$

$E_4=\{10^i \mid i\geq 0\}$

$E_5=\{10^i10^j \mid i,j\geq 0\}$

$E_1=\{\epsilon\}$

$E_2=\{0^i \mid i\geq 1\}$

$E_3=\{w \mid w\in L$ and no proper prefix of w in $L\}$

$E_4=\{w \mid w\in L$ and atleast one proper prefix of w in $L\}$

$E_5=\{10^i \mid i\geq 0\}$

3.

a) Show $L(M/\approx)=L(M)$

b) Show $\equiv_L \leq \equiv_{M/\approx}$, hence conclude $M/\approx$ is minimal Automata

Observe if $x\equiv_L y$ iff $xw\in L \Leftrightarrow yw\in L$

For any x,y s.t $x\equiv_L y \Leftrightarrow \left(\hat\delta(\hat\delta(q_0,x),w)\in F \Leftrightarrow \hat\delta(\hat\delta(q_0,y),w)\in F \quad \forall w\in\Sigma^*\right)$

$$\Leftrightarrow \hat\delta(q_0,x)\approx\hat\delta(q_0,y)$$ } Definition

$$\Leftrightarrow [\hat\delta(q_0,x)]=[\hat\delta(q_0,y)]$$

$$\Leftrightarrow \hat\delta'(q_0,x)=\hat\delta'(q_0,y)$$

$$\Leftrightarrow x\equiv_{M/\approx} y$$

Therefore $\equiv_L$ is refinement of $\equiv_{M/\approx}$