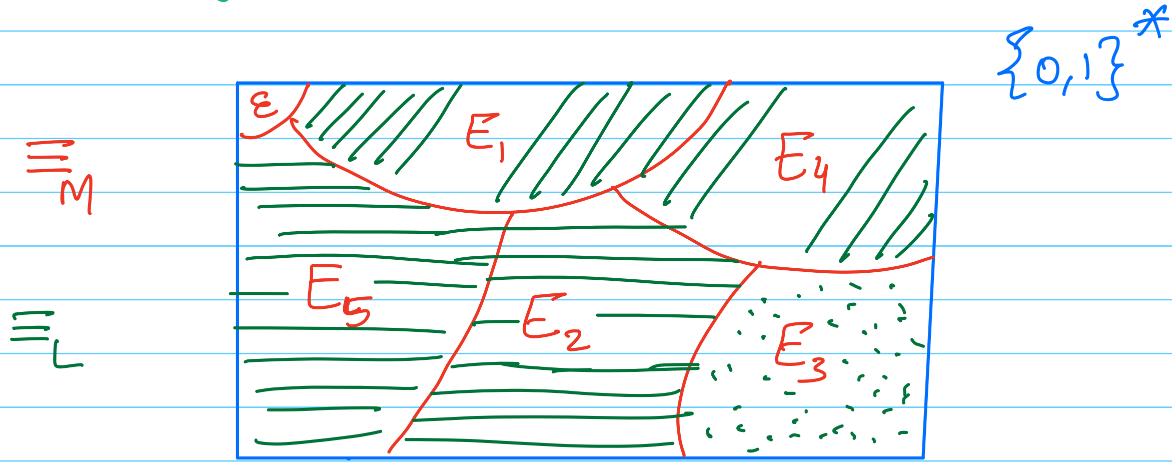
Property of $\equiv_M$ & $\equiv_L$



Fact: $\equiv_M$ is a refinement of $\equiv_L$

Defn: An equivalence relation $\equiv_1$ refines $\equiv_2$ (denoted as $\equiv_1 \subseteq \equiv_2$) if

$$x \equiv_1 y \implies x \equiv_2 y$$

* The equivalence classes of $\equiv_2$ are unions of equivalence classes of $\equiv_1$
* $\equiv_2$ is a refinement of $\equiv_1$
* $\equiv_1$ is a coarsening of $\equiv_2$

Coarsest Myhill-Nerode relation

Theorem: Let L be a regular language and let $\equiv$ be a Myhill-Nerode relation

Then $\equiv \subseteq \equiv_L$

Proof: Let $\equiv$ be any Myhill Nerode relation

if $x \equiv y$, then $x\sigma \equiv y\sigma \quad \forall \sigma \in \Sigma$

inductively $\forall z \in \Sigma^* \quad xz \equiv yz$

Since $\equiv$ refines L

$$xz \equiv yz \Rightarrow xz \in L \Leftrightarrow yz \in L$$

$\therefore$ if $x \equiv y$ then $\forall z \; xz \in L \Leftrightarrow yz \in L$

$$\Rightarrow x \equiv_L y$$

Theorem: Let $\equiv$ be a Myhill Nerode reln. of index k over Σ^* w.r.t a language L.
Then $\exists$ DFA M with k states accepting L

Proof: $Q = \{ [a] \mid a \in \Sigma^* \}$

↳ equivalence classes of $\equiv$

$q_0 = [\varepsilon]$

$F = \{ [w] \mid w \in L \}$

$\delta([w], \sigma) = [w\sigma]$

(Is δ well-defined? If $w_1 \neq w_2$ & $[w_1] = [w_2]$ then is $\delta([w_1], \sigma) = \delta([w_2], \sigma)$?)

Right congruence: if $[w_1] = [w_2]$, then $w_1 \equiv w_2$
$\Rightarrow w_1\sigma \equiv w_2\sigma \Rightarrow [w_1\sigma] = [w_2\sigma]$

* $w \in L \Leftrightarrow [w] \in F$

Claim: $L(M) = L$

Proof: $w \in L(M) \Leftrightarrow \hat{\delta}([\varepsilon], w) \in F$

$\Leftrightarrow [w] \in F$

$\Leftrightarrow w \in L$

Eg: $L = \{ w \mid \exists x, y \in \{0,1\}^* - \varepsilon,\ w = xy,\ \#1(x) < \#1(y) \}$

$S_0 = \{\varepsilon\}$

$S_1 = \{0\}$

$S_2 = L$

$S_3 = \{ 10^i \mid i \geq 0 \}$

$S_4 = \{ 10^i 1 0^j \mid i, j \geq 0 \} \cup \{ 0^i \mid i \geq 1 \}$

- Verify that this is a Myhill-Nerode relation

① $S_0 \cup S_1 \cup S_2 \cup S_3 \cup S_4 = \Sigma^*$

② Right congruence

③ Refines L