Theorem: The set of languages $L \subseteq \{0,1\}^*$ is uncountable

The set of all languages $\mathcal{L} = \mathcal{P}(\{0,1\}^*)$ is the power set of $\{0,1\}^*$

By contradiction: Let $f: \{0,1\}^* \to \mathcal{P}(\{0,1\}^*)$ be a bijection

f	ϵ	0	1	00	01	10	11	000	001
ϵ	1	0	0	1	0	0	1	1	1
0	0	1	1	0	0	1	0	1	0
1	0	0	0	1	1	0	0	1	0
00	1	1	1	0	0	0	1	1	1
01	0	1	0	1	0	1	0	1	0
10	1	1	1	1	1	1	1	0	0

1 1 0 0 0 1 . . . .

flip

0 0 1 1 1 0 . . .

↳ This does not exist as any row in the table given

Cantor's theorem: For any set S, there does not exist a bijection $f: S \to \mathcal{P}(S)$

Proof: Suppose that $\exists\, f: S \to \mathcal{P}(S)$ that is a bijection

Consider the set $T = \{x \mid x \notin f(x)\}$

Suppose that $T = f(t)$. Then

$$t \in T \iff t \notin f(t) \iff t \notin T$$

The set of languages is uncountable, whereas the set of C programs is countable

Finite Automata

- Programs that can read the input once
- The values stored in any variable is a constant, independent of the length of the input

Q: Write a program that checks if the length of the input is even

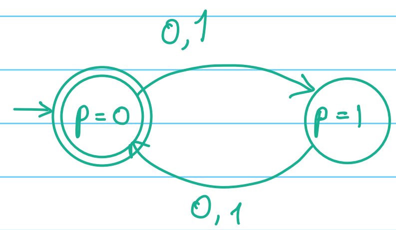
$p = 0$

while $inp[i] \neq \perp$

$\quad p = 1 - p$

return p

$\equiv$



* Captures many simple computational problems

- Lexical analyzer of a compiler