

$$② L = \{ zz \mid z \in \{0,1\}^* \}$$

choice of the string in L important

- Prover chooses $k > 0$
- Spoiler choose $0^k 1^k 0^k 1^k$
- Prover choose u, v, w, x, y s.t.

$$uvwxy = 0^k 1^k 0^k 1^k$$

$$vx \neq \epsilon, \quad |vwx| \leq k$$

what are the possible choices for the prover?

- $v = 0^l 1^m, w = 1^n, x = 1^r$
 choose $i=0$ $0^{k-l} 1^n 1^{k-m-r} 0^k 1^k$
 $= 0^{k-l} 1^{k-m-r} 0^k 1^k$

- other cases?

$\bar{L} = \{0,1\}^* - \{zz \mid z \in \{0,1\}^*\}$ is context-free.

Two cases: (i) w is of odd length

$$S \rightarrow A|B$$

$$A \rightarrow CAC / 0 \quad C \rightarrow 0/1$$

$$B \rightarrow CBC / 1$$

② Suppose that w is of even length and not of the form zz .
Then $w = u0vz1y$ where

$$|u| = |x| \text{ \& \& } |v| = |y|$$

But then w can be written as
 $u0v'x'1y$ where $|u| = |v'|$
and $|x'| = |y|$

Also every string of the form
 $u0v'x'1y$ where $|u| = |v'|$ & $|x'| = |y|$
will be in L'

$$S \rightarrow AB / BA$$

Final CFG $S \rightarrow AB / BA / A / B$

$$A \rightarrow CAC / 0$$

$$B \rightarrow CBC / 1$$

$$C \rightarrow 0 / 1$$