Many-one reduction

Let $A, B \subseteq \Sigma^*$ be any two languages
Then $A \leq_m B$ (A many-one reduces to B)
if $\exists$ an $f: \Sigma^* \rightarrow \Sigma^*$ that is
1. total
2. Effectively computable such that

$$x \in A \text{ iff } f(x) \in B$$

Observations:

- If B is recursive and $A \leq_m B$ then A is recursive
- If B is r.e and $A \leq_m B$, then A is r.e

Contrapositive form

- if $A \leq_m B$ and A is not recursive then B is not recursive
- if $A \leq_m B$ and A is not r.e, then B is not r.e

Observation:
- $\leq_m$ is transitive: if $A \leq_m B$ & $B \leq_m C$ then $A \leq_m C$
- If $A \leq_m B$, then $\overline{A} \leq_m \overline{B}$

What we have seen so far

1. $HP \leq_m MP$
2. $MP \leq_m HP$
3. $L = \{ \langle M \rangle \mid L(M) \text{ is regular} \}$

   $\overline{HP} \leq_m L$

   Corollary: L is not r.e

Observation: If L is r.e, then $L \leq_m MP$ & consequently $L \leq_m HP$

More examples

1. $MP_\varepsilon = \{ \langle M \rangle \mid \varepsilon \in L(M) \}$

   Try to reduce from problems of a similar flavor

   Observation: MP_ε is r.e

   Attempt $MP \leq_m MP_\varepsilon$

   $A = MP \qquad B = MP_\varepsilon$

   $f(\langle M \rangle, x) \rightarrow \langle M' \rangle$

   $M'(y) =$ if M accepts x, accept y

Observation: $x \in L(M)$ iff $\varepsilon \in L(M)$

Corollary: $FULL = \{ \langle M \rangle \mid L(M) = \Sigma^* \}$ is not recursive

$INFIN = \{ \langle M \rangle \mid L(M) \text{ is infinite} \}$ is not recursive