1. **[2 marks]** Consider the following statement about the language $L = \{0^n 1^n \mid n \geq 0\}$.

   Let $L' \subseteq L$ be any subset of L. If L' is infinite, then L' is not regular.

   Is the statement correct? Justify your answer.

   Solution: The statement is true.

   For $L' \subseteq L$, define the set $S = \{0^i \mid 0^i 1^i \in L'\}$. This is a distinguishing set for L' since for $x = 0^i$ and $y = 0^j$, we can choose $z = 0^i$, and we will have $0^i 1^i \in L'$ and $0^j 1^i \notin L'$.

   You can also use the pumping lemma.

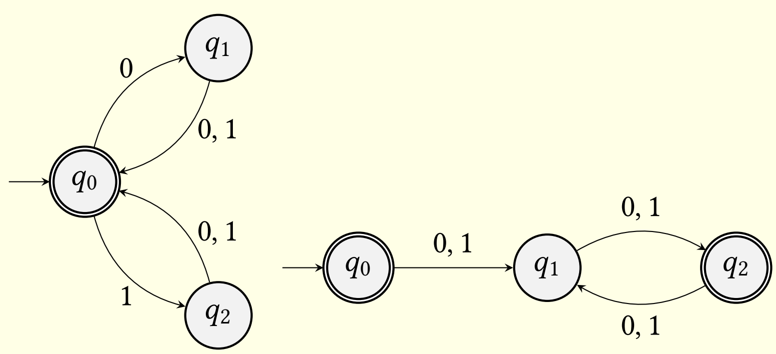
   - The prover chooses $k > 0$
   - Since L' is infinite, there is a string $0^{k'} 1^{k'} \in L'$ where $k' > k$. The spoiler chooses $x = \varepsilon$, $y = 0^{k'}$, $z = 1^{k'}$.
   - The prover chooses $u = 1^l$, $v = 1^m$, and $w = 1^n$, where $l + m + n = k'$.
   - Spoiler chooses $i = 0$, and we have $xuwz = 0^{k'-m} 1^{k'} \notin L$, and therefore $xuwz \notin L'$.

2. **[2 marks]** Consider the language

   $$L = \{w \in \{0,1\}^* \mid |w| \text{ is even}\}.$$

   Draw two non-isomorphic DFAs with the same number of states accepting L. Explain why the DFAs you constructed are non-isomorphic.

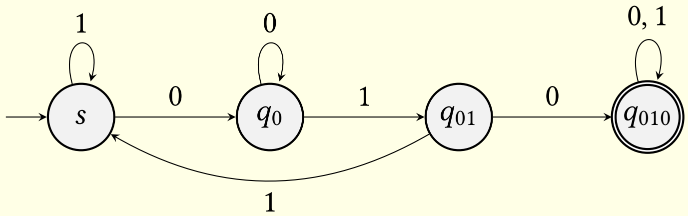
   Solution: Multiple solutions possible



   Both the DFAs have 3 states. They are non-isomorphic because they have different number of final states.

3. **[3 marks]** Let $L = \{w \mid w \text{ contains } 010 \text{ as a substring}\}$. Write down the equivalence classes corresponding to the indistinguishability relation $\equiv_L$ for this language. **Give a clear explicit description of each of the equivalence classes**. Justify your answer.

Solution: First, construct the minimal DFA



This is the minimal DFA since q_{010} is not equivalent with any of the other states. The states q_0 and q_{01} are not equivalent via the string 0. States s and q_{01} are not equivalent via the string 0. Finally, s and q_0 are not equivalent via the string 10.

Since this is the minimal DFA, the Myhill-Nerode relation corresponding to this DFA is $\equiv_L$. The equivalence classes are as follows:

- $[s] = \{w \mid w$ does not contain 010 and ends in $11\} \cup \{\epsilon, 1\}$.
- $[q_0] = \{w \mid w$ does not contain 010 and ends in $0\}$.
- $[q_{01}] = \{w \mid w$ does not contain 010 and ends in $01\}$.
- $[q_{010}] = L$.

4. **[3 marks]** Consider the language

$$L = \{w \mid w \text{ has exactly one occurrence of the substring } 11\}.$$

Consider the following subsets of Σ^*.

$$\begin{aligned}
S_1 &= \{w|w \text{ ends with 1 \& does not contain 11 as a substring}\},\\
S_2 &= \{w|w \text{ ends with 0 \& does not contain 11 as a substring}\},\\
S_3 &= \{w|w \text{ has exactly one occurrence of 11 \& ends with 11}\},\\
S_4 &= \{w|w \text{ has exactly one occurrence of 11 \& ends with 0}\},\\
S_5 &= \{w|w \text{ has exactly one occurrence of 11 \& ends with 01}\},\\
S_6 &= \{w|w \text{ contains} > 1 \text{ occurrences of 11 as a substring}\},\\
S_7 &= \{\varepsilon\}.
\end{aligned}$$

Prove that these subsets correspond to a Myhill-Nerode relation w.r.t L. Construct the equivalent DFA corresponding to this Myhill-Nerode relation.

Solution: Let's divide Σ^* into three parts P_0 is set of strings that do not contain 11, $P_1 = L$, and $P_{>1}$ is the set of strings that contain more than one occurrence of 11. Clearly, $P_0 = S_1 \cup S_2 \cup S_7$ since every string ends with either 0 or 1, or is ε. Similarly, $P_1 = S_3 \cup S_4 \cup S_5$, and

$P_{>1} = S_6$. Firstly, from the definition it clear that P_0, P_1, and $P_{>1}$ are all mutually disjoint. The set S_1, S_2, and S_7 are also mutually disjoint since S_7 contains ε alone, and S_1 and S_2 contain strings that end with 1 and 0, respectively. Similarly, S_3, S_4 and S_5 are mutually disjoint since the last two bits of the strings in these sets are all different. Thus these sets for a partition of Σ^*, and hence define an equivalence relation.

Since $L = S_3 \cup S_4 \cup S_5$, these sets refine L. We need to show that the equivalence relation is right congruent.

- If $x, y \in S_1$, then $x0, y0 \in S_2$ and $x1, y1 \in S_3$.
- If $x, y \in S_2$, then $x0, y0 \in S_2$ and $x1, y1 \in S_1$.
- If $x, y \in S_3$, then $x0, y0 \in S_4$ and $x1, y1 \in S_6$.
- If $x, y \in S_4$, then $x0, y0 \in S_4$ and $x1, y1 \in S_5$.
- If $x, y \in S_5$, then $x0, y0 \in S_4$ and $x1, y1 \in S_6$.
- If $x, y \in S_6$, then $x0, y0 \in S_6$ and $x1, y1 \in S_6$.

Thus, the equivalence relation is a Myhill-Nerode relation. The DFA is as follows.