Thm: MP is not recursive

Qn: Can you solve MP with subroutine call to HP

Given $\langle M \rangle$ & x, check if $x \in L(M)$

Have an algorithm A s.t $A(\langle M \rangle, x) = \begin{cases} \text{Halts \& accepts} & \text{if } M \text{ halts on } x \\ \text{Halts \& rejects} & \text{o/w} \end{cases}$

Given $\langle M \rangle, x$

$\boxed{M'(y):}$ Simulate M on y

if M accepts y then accept & halt

if M rejects y, then go into an infinite loop

if $A(\langle M' \rangle, x) =$ accept & halt then M accepts x

$\Rightarrow x \in L(M)$

if $A(\langle M' \rangle, x) =$ rejects & halts, then $x \notin L(M)$

* $L = \{ \langle M \rangle \mid L(M) \text{ is regular} \} \rightarrow A'$ is the algorithm for L

Thm: L is not recursive

Idea: Reduction: show that if $\exists$ algorithm for L then $\exists$ algorithm for HP

HP

Input: $\langle M \rangle, x$

Goal: check if M halts on x

An input instance of L and then run the hypothesized algorithm for L

i/p for HP

$\langle M \rangle$ and x : given

$\exists$ a language L' $\to \{0^n 1^n \mid n \geq 0\}$ that is not regular but context-free

$M'(y)$: Simulate M on x

If M enters accept or reject on x

Accept y if $y \in L'$

reject otherwise

What happens if A gets $\langle M' \rangle$ as input

- If $(\langle M \rangle, x) \notin HP$, then $L(M') = \phi$ $\to$ regular
- If $(\langle M \rangle, x) \in HP$, then $L(M') = L'$ $\to$ non-regular

many-one reductions