Undecidability & Diagonalization

Qn: Given M and i/p x, check if M halts on x

↳ either accepts x or rejects x

$HP = \{(\langle M \rangle, x) \mid M \text{ halts on i/p } x\}$

↳ encoding of the TM M

Fact: HP is r.e

Use the Universal TM to simulate M on x

Theorem (Turing): HP is not recursive

Proof: Assume that ∃TM K that decides HP

K $(\langle M \rangle, x)$ { Halts and accepts if M halts on x

Halts and rejects if M does not halt on x

for an $x \in \{0,1\}^*$ Let M_x denote the TM encoded by x

(If x does not encode any TM, then let M_x be a trivial TM that accepts all strings)

$\forall x, y \in \{0,1\}^*$ $K(x,y)$ halts and accepts if M_x halts on y

$K(x,y)$ halts and rejects if M_x does not halt on y

$\langle M_x \rangle = x$

x is the encoding of M

Consider the following TM N

This TM N can be constructed from the description on K which exists by our assumption

```
N(x)
    Run K(x,x)
    if K(x,x) accept
        enter into an infinite
        loop
    else
        accept and halt
```

The TM cannot be equal to any TM M

$N(x)$ halts and accepts iff M_x does not halt on x

Thus $N(\langle M \rangle) \neq M(\langle M \rangle)$