* $L = \{ a^n b^n c^n \mid n \geq 0 \}$

variable-count
\# of a's, b's & c's

$\vdash \; \cancel{a} \; \cancel{a} \; \cancel{a} \; \cancel{b} \; \cancel{b} \; \cancel{b} \; \cancel{c} \; \cancel{c} \; \cancel{c} \; \dashv \; \sqcup$

Configuration of a TM: $Q \times \Gamma^* \times \mathbb{N}$

- Q: current state
- Γ^*: contents of the tape
- $\mathbb{N}$: position of the tape head

Initial configuration: $(s, \underbrace{\vdash w}, 0)$

after $|w|$, there are only $\sqcup$ symbols.

* A TM M accepts a string $x \in \Sigma^*$ if

$$(s, \vdash x, 0) \xrightarrow{*} (t, y, n)$$

y: contents of the tape
n: head position

* A TM M rejects a string $x \in \Sigma^*$ if

$$(s, \vdash x, 0) \xrightarrow{*} (r, y, n)$$

- A language L is accepted by a TM M
  if $L = \{ x \in \Sigma^* \mid M \text{ accepts } x \}$

A language L is recursively enumerable (r.e)
if $\exists$ TM M s.t $L = L(M)$

- A TM M is total if $\forall x \in \Sigma^*$
  M either accepts x or rejects x

A language L is said to be recursive if
$\exists$ total TM M s.t $L(M) =$

Multi-tape Turing Machines

- TMs that have multiple (but fixed #) tapes
- Each tape has an independent tape head

$M = (Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r)$ k-tape machine

$$\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k$$

2-tape TM

$\vdash$	w_1	w_2	w_3	w_4^{0}	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$

$\vdash$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup^{1}$	$\sqcup$	$\sqcup$	$\sqcup$

✓

$\forall \gamma \in \Gamma$
$\hat{\gamma} \in \hat{\Gamma}$

$\hat{\vdash}$	w_1	w_2	w_3	w_4	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$
$\hat{\vdash}$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$

one tape

$$\boxed{\Gamma' = (\Gamma \cup \hat{\Gamma})^2}$$

$(\hat{\vdash}, \hat{\vdash}), (w_1, \sqcup), (w_2, \sqcup) \ldots$
$(\vdash, \vdash) \rightarrow$ left endmarker

q $\rightarrow$ $\downarrow$ $\delta(q, (w_2, x_6)) \rightarrow q', (w_2', x_6'), (L, R))$

$\vdash$	$\hat{w}_1$	w_2'	w_3	w_4	w_5	$\sqcup$	$\sqcup$	$\sqcup$
$\vdash$	x_1	x_2	x_3	x_4	x_5	x_6'	$\hat{x}_7$	$\sqcup$

Configuration of the 1-tape TM

$\Downarrow$

Tape 1 has w_2 under the tape head & Tape 2 has x_6 under the tape head

Universal Turing Machine

- Every TM can be encoded as a binary string

$(Q, \Sigma, \delta, \vdash, \sqcup, \delta, s, t, r)$

Interpreter for programs: $\mathcal{U} \rightarrow$ TM that takes <M> - encoding of M and an i/p x & Simulates M on x

$\mathcal{U}$ has 3 tapes

T1: Store the encoding of M

T2: Store x & simulate the tape of M

T3: Store the state & position of tape head of M while executing x

Theorem: $\exists$ a universal TM