(Nondeterministic) Push-Down Automata (PDA)

Finite-State machine
- input read-once tape
- unbounded stack

- at every step, one of two things
  - read a symbol of the input, & modify stack
  - modify stack without reading anything from the input tape

$M = (Q, \Sigma, \Gamma, \delta, s, \perp, F)$

Σ ↓ input alphabet

Γ ↳ tape alphabet

$\perp$ initial stack symbol

$\delta \subseteq (Q \times \Sigma \cup \{\varepsilon\} \times \Gamma) \times (Q \times \Gamma^*)$

* $(q, \sigma, A), (q', B_1 B_2 \ldots B_k) \in \delta$
  - read σ, pop A, push $B_1 \ldots B_k$
* $(q, \varepsilon, A), (q', B_1 B_2 \ldots B_k) \in \delta$
  - pop A & push $B_1 \ldots B_k$ onto stack without reading any input symbol.

Configurations $\subseteq Q \times \Sigma^* \times \Gamma^*$

current state

input yet to be read

Current state of the stack

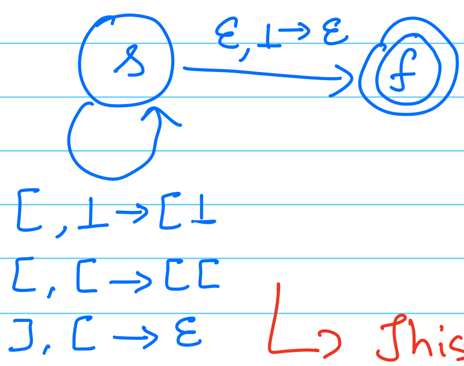
- if $((q, \sigma, A), (q', \beta)) \in \delta$, then
  $\forall\, w \in \Sigma^*,\ \gamma \in \Gamma^*$
  $$(q, \sigma w, A\gamma) \xrightarrow{1} (q', w, \beta\gamma)$$

- if $((q, \varepsilon, A), (q', \beta)) \in \delta$, then
  $\forall\, w \in \Sigma^*,\ \gamma \in \Gamma^*$
  $$(q, w, A) \xrightarrow{1} (q', w, \beta\gamma)$$

Acceptance condition: M accepts x

if $(s, x, \perp) \xrightarrow{*} (q, \varepsilon, \gamma)$ $\quad q \in F$

Dyck language



Non-deterministic transition from s to f

This is also accepting by empty stack.

Alternate accepting condition (by empty stack)

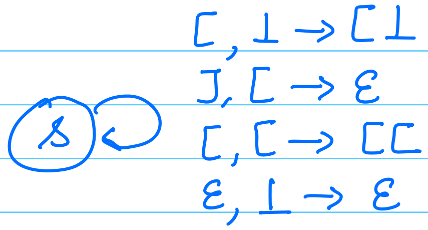
M accepts x if

$$(s, x, \perp) \xrightarrow{*} (q, \varepsilon, \varepsilon)$$

for some state $q \in Q$ not necessarily final

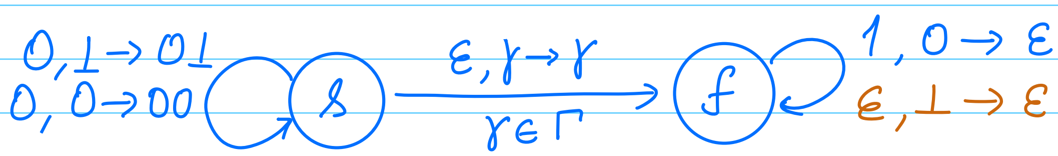
Equivalence: If a string x is accepted by M via the final state, then $\exists\, M'$ that accepts x via the empty stack, and vice-versa.

Dyck language (accepting via the empty stack)



$\exists$ one state PDA that accepts the Dyck language via the empty stack

* $L = \{0^n 1^n \mid n \geq 0\}$



* $L = \{ ww^R \mid w \in \{0,1\}^* \}$