

# Effective Computability

- what can be solved algorithmically?

## Hilbert's Formalist Program

- Axiomatize mathematics

- Entscheidungsproblem - Given a mathematical statement, check if it is true

(Algorithm for this)

Completeness theorem: FOL is axiomatizable (Gödel)

$$\forall x \forall y \exists z \quad x < z < y \quad \dots$$

or  $y < z < x$   
or  $x = y = z$

not valid since the truth value depends on the interpretation of  $x, y, z$

Valid formula:  $\forall x \forall y (x < y \Rightarrow x < y)$

Multiple notions of effective computability

- \*  $\lambda$ -Calculus (Alonzo Church)  $\rightarrow$  Functional Program
- \*  $\mu$ -recursive functions (Kurt Gödel)
- \* Turing machines - intuitively very clear as to why this is the right defn.

All the various definitions compute the same class of functions

Church-Turing thesis: Any physically realizable computationally device can be simulated by a Turing machine

: Turing Machines

$M = (Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r)$

Annotations for  $M$ :

- $Q$ : input alphabet
- $\Sigma$ : tape alphabet
- $\Gamma$ : left end of the tape
- $\vdash$ : blank symbol
- $\sqcup$ : start state
- $\delta$ : transition function
- $s$ : symbol to be written in the current cell
- $t$ : accept state
- $r$ : reject state

$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

Annotations for  $\delta$ :

- $Q$ : current state
- $\Gamma$ : symbol under the tape head
- $Q$ : new state
- $\Gamma$ : direction in which the tape head should move

Language of  $M$   $L(M) \subseteq \Sigma^*$

- one-way infinite tape

$\vdash w_1 w_2 w_3 w_4 \sqcup \sqcup \sqcup \sqcup \sqcup \dots$

Diagram showing a tape head moving over the tape symbols.

Constraints:  $\forall p \in Q, \exists q \in Q$  s.t.

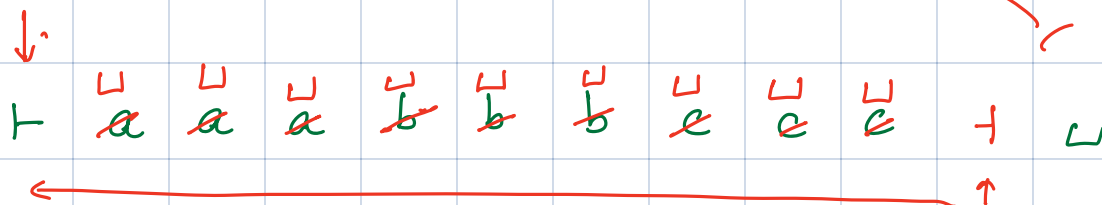
$$\delta(p, \vdash) = (q, \vdash, R)$$

$$\delta(t, b) = (t, b', L/R)$$

$$\delta(r, b) = (r, b', L/R)$$

$$* L = \{ a^n b^n c^n \mid n \geq 0 \}$$

variable - count # of a's b's c's



Configuration of a TM:  $Q \times \Gamma^* \times \mathbb{N}$

current state  $\downarrow$  contents of the tape  $\hookrightarrow$  position of the tape head

Initial configuration:  $(s, \uparrow w, 0)$

after  $|w|$ , there are only  $\sqcup$  symbols.

\* A TM  $M$  accepts a string  $x \in \Sigma^*$  if

$$(s, \uparrow x, 0) \xrightarrow{*} (t, y, n)$$

$\downarrow$  contents of the tape  $\hookrightarrow$  head position

\* A TM  $M$  rejects a string  $x \in \Sigma^*$  if

$$(s, \uparrow x, 0) \xrightarrow{*} (r, y, n)$$