Example uses and abuses

① $L = \{\langle M \rangle \mid M \text{ has 2024 states}\}$

- decidable / recursive
- can create TM with and without 2024 states that accept Σ^*

② $L = \{\langle M \rangle \mid L(M) \text{ is accepted by a TM with 2200 states}\}$

- trivial property

③ $L = \{\langle M \rangle \mid L(M) \text{ is accepted by a TM with } < 2200 \text{ states}\}$

- Rice's theorem is applicable

Rice's theorem: Let P be a non-trivial property of r.e languages.

Let $L = \{ \langle M \rangle \mid L(M) \in P \}$

Then L is undecidable.

Proof: Suppose that $P(\phi) = \underline{\perp}$

(The empty language does not have the property)

[Analogous argument can be done when $P(\phi) = T$]

$\exists\, L'$ s.t $P(L') = T$. Let N be the TM s.t $L' = L(N)$

We will show that $HP \leq_m L$

$A = HP \qquad B = L$

$f(\langle M \rangle, x) \longmapsto \langle M' \rangle$ s.t

- if M halts on x, then $\langle M' \rangle \in L$
- if M does not halt on x, then $\langle M' \rangle \notin L$

$M'(y)$: Simulate M on x
if M halts on x, then
simulate N on y
else reject

* if M halts on x, then $L(M') = L'$
Since $P(L') = T$, we have $\langle M' \rangle \in L$
* if M does not halt on x, then $L(M') = \phi$
Since $P(\phi) = \perp$, we have $\langle M' \rangle \notin L$