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Tutorial-5

1. Construct CFG and prove it is correct.

a) $L = \{w \in \{0,1\}^* \mid \#0(w) = \#1(w)\}$

$S \to SS \mid 0S1 \mid 1S0 \mid \epsilon$. Induction proof.

b) $L = \{w\#x \mid w \neq x,\ w, x \in \{0,1\}^*\}$ L is language s.t. $L \subseteq \{0,1,\#\}^*$.

$L_1 = \{w\#x \mid |w| > |x|,\ w, x \in \{0,1\}^*\}$

$L_2 = \{w\#x \mid |w| < |x|,\ w, x \in \{0,1\}^*\}$

CFG for L_1

$S \to 0S0 \mid 0S1 \mid 1S0 \mid \cancel{1S0}\ 1S1 \mid \cancel{1A\#}\ 0A\# \mid 1A\#$

$A \to 0A0 \mid 1A0 \mid 0A1 \mid 1A1 \mid \epsilon$

Observe here we always create a larger string on left side of # than rightside.

→ Similarly we can create CFG for L_2.

→ Note $L_1 \cup L_2 = \{w\#x \mid |w| \neq |x|,\ w, x \in \{0,1\}^*\}$.

~~$L_3 = \{w\#x \mid |w| = |x|,\ w \neq x,\ w, x \in \{0,1\}^*\}$~~

~~$S \to$~~

~~$L_3 = \{w\#x \mid \exists i$ s.t $w[i] \neq x[i],\ w, x \in \{0,1\}^*\}$~~

~~$S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid$~~

$L_3 = \{w\#x \mid |w| = |x|$ s.t $w \neq x\}$.

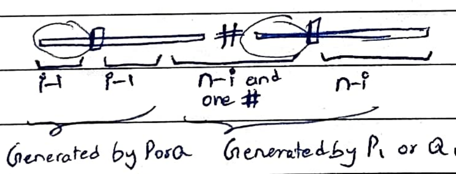
$S \to PQ_1 \mid P_1Q \mid Q_1P \mid QP_1$

$P \to 0P0 \mid 0P1 \mid 1P0 \mid 1P1 \mid 0$

$P_1 \to \cancel{0P\# \mid \#P0 \mid 1P\# \mid \#P1}\ \#\sigma P \sigma \mid \sigma P \sigma \#$ $\sigma \to 0 \mid 1$

$Q \to 0Q0 \mid 0Q1 \mid 1Q0 \mid 1Q1 \mid 1$

$Q_1 \to \cancel{0Q\# \mid \#Q0 \mid 1Q\# \mid \#Q1}\ \#\sigma Q \sigma \mid \sigma Q \sigma \#$



c) $L = \{0,1\}^* - \{0^n 1^n \mid n \geq 0\}$

$S \to 0A \mid 0S1 \mid 1B$

$A \to \cancel{0 \mid 0A}\ 0A \mid \epsilon$

$B \to 1B \mid \epsilon$

d) $L = \{a^i b^j c^k \mid i \neq j$ or $j \neq k\}$

$L_1 = \{a^i b^j \mid i \neq j\}$ ← Can be created similar to part c)

$L_2 = \{c^k\}$

$L_3 = \{b^j c^k \mid j \neq k\}$

$L_4 = \{a^i\}$

Observe $L = L_1 \cdot L_2 + L_4 \cdot L_3$ // Hence CFG construction follows from closure property

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Index of comments

1.1 This is not correct. L inclues strings of the form 10*, and these are not generated. Please argue why the construction is correct.