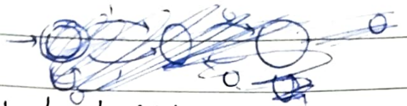
Problem Set #4

1. Write Regular expression for below languages

(a) $L = \{w \in \{0,1\}^* \mid$ w has atmost one pair of consecutive 1's$\}$

$(00^*100^*) + (00^*100^*)11(00^*100^*)$ 1.1

(b) $L = \{w \in \{0,1\}^* \mid$ the number of zeroes in w is divisible by 3$\}$



$(1^*01^*01^*0)^*1^*$

(c) $L = \{w \in \{0,1\}^* \mid$ every odd position of w is a 1$\}$

$(10+11)^*$ 1.2

(d) $L = \{w \in \{0,1\}^* \mid$ every pair of adjacent 0's appear before any pair of adjacent 1's$\}$

~~No 11~~ ~~No 00~~

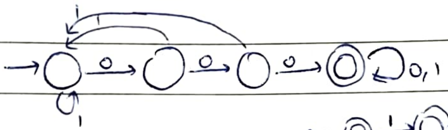
~~$(0^*)(00^*100^*)^*(11^*011^*)^*$~~ $(0+10)^*(1+\epsilon)(1+01)^*(0+\epsilon)$ 1.3

2. Describe the languages below and construct a DFA.

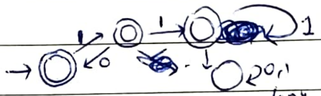
(a) $(0^*1^*)^*000(0+1)^*$

~~Both~~ $\boxed{(0^*1^*)^* = (0+1)^* = \Sigma^*.}$

~~The~~ $L = \{w \mid \exists$ substring 000 in $w\}$



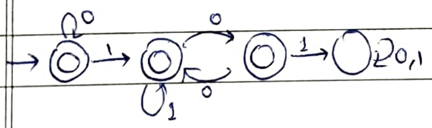
(b) $(0+10)^*1^*$



$L = \{w \mid$ Every 0 appears before every (any) pair of adjacent 1's$\}$

(c) $(0+\epsilon)(00+1(00)^*1)^*(0+\epsilon)$

$L = \{w \mid$ All 0's occur as even groups except at ends of string & where there is no condition$\}$ even



Proof (1) Observe that ~~such~~ any $w \in L$ can be split s.t. w_1 does not have any 11 and w_2 does not have 00. $(w = w_1 w_2)$

[Take w_1 as prefix until last 00 occurence and w_2 as rest]

(2) Observe any string that does not have 11 is in $L((0+10)^*(1+\epsilon))$

As any such string can be split into sequence of 0, and 10.

Similarly any string that does not have 00 is in $L((1+01)^*(0+\epsilon))$

(3) Hence any $w \in L$ is in $L((0+10)^*(1+\epsilon)(1+01)^*(0+\epsilon))$

(4) Any string $w \in L((0+10)^*(1+\epsilon)(1+01)^*(0+\epsilon))$

$(0+10)^*(1+\epsilon)$ will not contain 11

$(1+01)^*(0+\epsilon)$ will not contain 00

Hence in w all 00's will occur before every 11. Hence $w \in L$.

(5) As we showed $w \in L \Leftrightarrow w \in L((0+10)^*(1+01)^*)$ they are equal.

PageWork

3. Denote $\frac{dR}{d\sigma}$ is the language $\{w \mid \sigma w \in L(R)\}$.

a) There exists a DFA M that accepts $L(R)$.

We can construct DFA $N = (Q, \Sigma, \delta, s', F)$ where $s' = \delta(s, \sigma)$. → start of M.

Observe if N accepts w, i.e. $\delta(s', w) \in F$

$\iff \delta(s, \sigma w) \in F \iff M$ accepts σw.

Hence $\frac{dR}{d\sigma}$ is regular. (N is DFA accepting it)

b) $\frac{dR}{d\sigma} = \frac{dR_1}{d\sigma} + \frac{dR_2}{d\sigma}$

c) $\frac{d(R^*)}{d\sigma} = \frac{dR}{d\sigma} R^*$

4. a) $L(R_1(R_2+R_3)) = L(R_1R_2 + R_1R_3)$

b) $L((R_1+R_2)R_3) = L(R_1R_3 + R_2R_3)$

c) $L((R_1+R_2)^*) = L((R_1^* R_2^*)^*)$

A regex R is said to be (+)free if R does not contain + operator.

A regex R is said to be (+) separable if one of the following two conditions:

→ R is (+) free, or

→ $R = R_1 + R_2$ and R_1, R_2 are (+) separable

d) Show that if R is (+) separable then $R = R_1 + R_2 - - R_n$ where R_i's are (+)-free.

Induction on length of R.

Base case: ~~All~~ regex that is (+) free.

I.H. All R upto length k (that are (+) separable) can be represented as

~~R (when (+) free)~~ or $R_1 + R_2 + R_3 - R_n$

Induction Step: Suppose R is length $k+1$ (+) separable regex.

(1) If R is (+) free we are done

(2) If R is (+) separable $R = R_1 + R_2$ where (+) separable

As R_1 and R_2 must have length less than or equal to k

they can be represented as $(R_1 + R_2 .. + R_n)$ form.

Hence R can also be represented as such.

(e) Show that if R_1 and R_2 are (+) separable than there is a (+) separable regex Equivalent to R_1R_2.

$R_1R_2 = (x_1 + x_2 - - x_n)(y_1 + y_2 - - y_\ell)$ in representation (x_i, y_j are (+) free)

Observe $L((x_1 + x_2 - -)(y_1 + y_2 - - y_\ell)) = L(\sum\sum x_i y_j)$

Observe $x_i y_j$ is (+) free.

~~Need~~ to ~~prove~~ ~~any~~ ~~Regex~~

→ Hence $\sum\sum x_i y_j$ regex is (+) separable. Done.

(f) Show that if R is (+) separable, then there exists (+) free regex R_1 such that $L(R^*) = L(R_1^*)$

Base Case: When R is (+) free we are done

Induction Hypothesis: For all (+) separable regex R ~~...~~ upto length k

$L(R^*) = L(R_1^*)$ where R_1 is (+) free.

Induction ~~H~~ransition: Take R that is (+) separable and $k+1$ length

(1) If R is (+) free done

(2) ~~If R is (+) separable~~ Else $R = x_1 + x_2$

By Inductive inductive $L(x_1^*) = L(y_1^*)$, $L(x_2^*) = L(y_2^*)$

where y_1 and y_2 are (+) free.

Observe ~~L(x_1+x_2~~ $L(R^*) = L((x_1+x_2)^*) = L((x_1^* x_2^*)^*) = L((y_1^* y_2^*)^*)$

As y_1 and y_2 are (+) free $\Rightarrow$ $y_1^* y_2^*$ is (+) free.

Hence we are done.

Index of comments

1.1 What about 001001001

1.2 This does not cover odd length strings

1.3 Please give explanations of why these constructions are correct.