Chomsky Normal Form

A CFG G is in CNF if every production is of the form

$A \rightarrow BC$ or $A \rightarrow \sigma$ $\sigma \in \Sigma$

CNF grammars do not generate ε

- Remove $A \rightarrow \varepsilon$
- For every production $B \rightarrow \alpha A \beta$

  Add $B \rightarrow \alpha\beta$

  $B = \alpha_1 A \alpha_2 A \alpha_3$

  $B = \alpha_1 A \alpha_2 A \alpha_3 \mid \alpha_1 \alpha_2 A \alpha_3 \mid \alpha_1 A \alpha_2 \alpha_3 \mid \alpha_1 \alpha_2 \alpha_3$

- Remove $A \rightarrow B$

  Replace $B \rightarrow \beta$ with $A \rightarrow \beta$

Eg:

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \varepsilon$

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB$

$A \rightarrow B \mid S$

$B \rightarrow b \mid \varepsilon$

Removing ε-productions

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a$

$A \rightarrow B \mid S \mid \varepsilon$

$B \rightarrow b$

$S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a \mid SA \mid AS \mid S$

$A \rightarrow B \mid S$

$B \rightarrow b$

Removing unit productions

* $S_0 \rightarrow S$

$S \rightarrow ASA \mid aB \mid a \mid SA \mid AS$

$A \rightarrow B \mid S$

$B \rightarrow b$

* $S_0 \rightarrow ASA \mid aB \mid a \mid SA \mid AS$

$S \rightarrow ASA \mid aB \mid a \mid SA \mid AS$

$A \rightarrow B \mid S$

$B \rightarrow b$

* $S_0 \to ASA \mid aB \mid a \mid SA \mid AS$
  $S \to ASA \mid aB \mid a \mid SA \mid AS$
  $A \to b \mid S$
  $B \to b$

* $S_0 \to ASA \mid aB \mid a \mid SA \mid AS$
  $S \to ASA \mid aB \mid a \mid SA \mid AS$
  $A \to b \mid ASA \mid aB \mid a \mid SA \mid AS$
  $B \to b$

* $S_0 \to AA_1 \mid UB \mid a \mid SA \mid AS$
  $S \to AA_1 \mid UB \mid a \mid SA \mid AS$
  $A \to b \mid AA_1 \mid UB \mid a \mid SA \mid AS$
  $B \to b$
  $U \to a$
  $A_1 \to SA$

$\hat{P}$ - smallest set of productions containing P s.t

(i) if $A \to \alpha B \gamma$ & $B \to \varepsilon \in \hat{P}$, then
$A \to \alpha\gamma \in \hat{P}$

(ii) if $A \to B$ & $B \to \beta \in \hat{P}$, then
$A \to \beta \in \hat{P}$