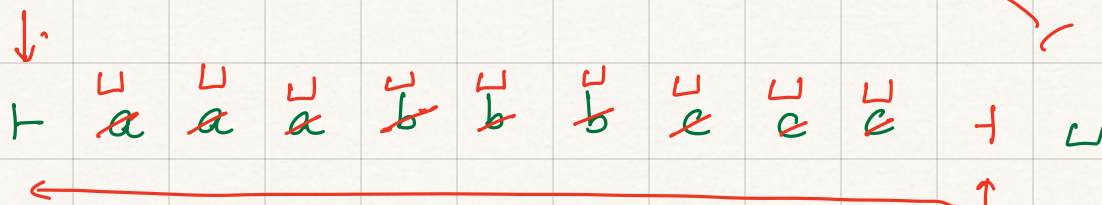


$$* L = \{ a^n b^n c^n \mid n \geq 0 \}$$

variable - count # of a's b's c's



Configuration of a TM:  $Q \times \Gamma^* \times \mathbb{N}$

current state  $\downarrow$  contents of the tape  $\hookrightarrow$  position of the tape head

Initial configuration:  $(s, \sqcup^w, 0)$

after  $|w|$ , there are only  $\sqcup$  symbols.

\* A TM  $M$  accepts a string  $x \in \Sigma^*$  if

$$(s, \sqcup^x, 0) \xrightarrow{*} (t, y, n)$$

$\downarrow$  contents of the tape  $\hookrightarrow$  head position

\* A TM  $M$  rejects a string  $x \in \Sigma^*$  if

$$(s, \sqcup^x, 0) \xrightarrow{*} (r, y, n)$$

- A language  $L$  is accepted by a TM  $M$

$$\text{if } L = \{ x \in \Sigma^* \mid M \text{ accepts } x \}$$

A language  $L$  is recursively enumerable (r.e)

if  $\exists$  TM  $M$  s.t.  $L = L(M)$

- A TM  $M$  is **total** if  $\forall x \in \Sigma^*$   
 $M$  either accepts  $x$  or rejects  $x$

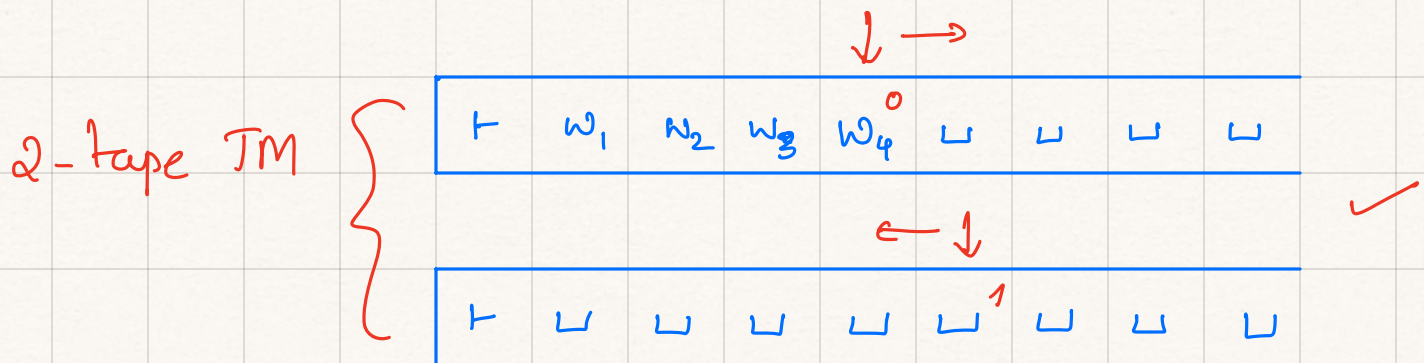
A language  $L$  is said to be **recursive** if  
 $\exists$  total TM  $M$  s.t.  $L(M) = L$

## Multi-tape Turing Machines

- TMs that have multiple (but fixed #) tapes
- Each tape has an independent tape head

$M = (Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r)$   $k$ -tape machine

$$\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R\}^k$$



$\forall \gamma \in \hat{\Gamma}$   
 $\hat{\gamma} \in \hat{\Gamma}$



$$\Gamma' = (\Gamma \cup \hat{\Gamma})^2$$

$(\hat{\vdash}, \hat{\vdash}), (w_1, \sqcup), (w_2, \sqcup) \dots$   
 $(\vdash, \vdash) \rightarrow$  left end marker



Configuration  
of the 1-tape TM  
↓

Tape 1 has  $w_2$  under the  
tape head & Tape 2 has  $x_6$  under the tape head

$q \rightarrow \downarrow$

$\delta(q, (w_2, x_6)) \rightarrow q', (w'_2, x'_6), (L, R)$

$\vdash$	$\hat{w}_1$	$w'_2$	$w_3$	$w_4$	$w_5$	$\sqcup$	$\sqcup$	$\sqcup$
$\vdash$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x'_6$	$\hat{x}_7$	$\sqcup$

## Universal Turing Machine

- Every TM can be encoded as  
a binary string

$(Q, \Sigma, \delta, \vdash, \sqcup, \bar{\delta}, s, t, r)$

Interpreter  
for programs  $\left\{ \begin{array}{l} \mathcal{U} \rightarrow \text{TM that takes } \langle M \rangle - \text{encoding of } M \\ \text{and on ip } x \text{ \& } \\ \text{simulates } M \text{ on } x \end{array} \right.$

$\mathcal{U}$  has 3 tapes

T1: store the encoding of  $M$

T2: store  $x$  & simulate the tape of  $M$

T3: store the state & position of tape head  
of  $M$  while executing  $x$

$\therefore$  Theorem:  $\exists$  a universal TM