② $L = \{ 0^n 1^m \mid n \geq m \}$

- Prover chooses $k > 0$

Bad choice of strings

- Suppose spoiler chooses $x = \varepsilon, y = 0^n, z = 1^m$ $\quad n \gg m$

  Prover can choose u, v, w in any way & $xuv^iwz \in L$ $\forall i$ $\longrightarrow$ Spoiler does not win

- Spoiler chooses $x = 0^{k+1}, y = 1^{k+1}, z = \varepsilon$
- Prover chooses u, v, w as

  $u = 1^l, \; v = 1^m, \; w = 1^n$ s.t $l + m + n = k+1$
- Spoiler chooses $i = 0$: $0^{k+1} 1^l 1^n \notin L$

③ $L = \{ 0^{n!} \mid n \geq 0 \}$

- Prover chooses $k > 0$
- Spoiler chooses $x = z = \varepsilon, \; y = 0^{k!}$
- Prover chooses l, m, n s.t $l + m + n = k!$
- Spoiler - find i s.t

  $|xuv^iwz| = \underbrace{l + im + n}_{k! + (i-1)m} \neq j!$ for some j

  $i - 1 = (k+1)!$: $\quad k! + (i-1)m = k!(1 + m(k+1))$

if $k!(1 + m(k+1)) = j!$ then $(k+1)(k+2) \ldots j = 1 + m(k+1)$

④ $L = \{0^p \mid p \text{ is a prime number}\}$

- Prover chooses $k > 0$
- Spoiler chooses $p >> k$ &

  $x = \varepsilon,\ z = \varepsilon,\ y = 0^p$
- Prover chooses l, m, n s.t

  $l + m + n = p$
- Spoiler chooses $i \geq 0$ s.t

  $l + im + n \neq$ prime number

  $\downarrow$

  $p + (i-1)m \longmapsto$ choose $i = p+1$

  $p + (i-1)m = p(1+m)$

⑤ $L = \{ww \mid w \in \{0,1\}^*\}$

- Prover chooses $k > 0$ (even)
- What about 0^{2k} ?

  $x = \varepsilon,\ y = 0^{2k},\ z = \varepsilon$

  Prover chooses $u = \varepsilon,\ v = 0^{2k},\ w = \varepsilon$

  Spoiler can never win

what about $0^k 1^k 0^k 1^k$?

$x = 0^k \quad y = 1^k 0^k \quad z = 1^k$

↳ what are possibilites for the Prover's play ?

v can be $1^i 0^j$ or 1^i or 0^j } show that there is a winning play for the spoiler in each of these case

A different choice for Spoiler

$x = 0^k 1^k \quad y = 0^k \quad z = 1^k$