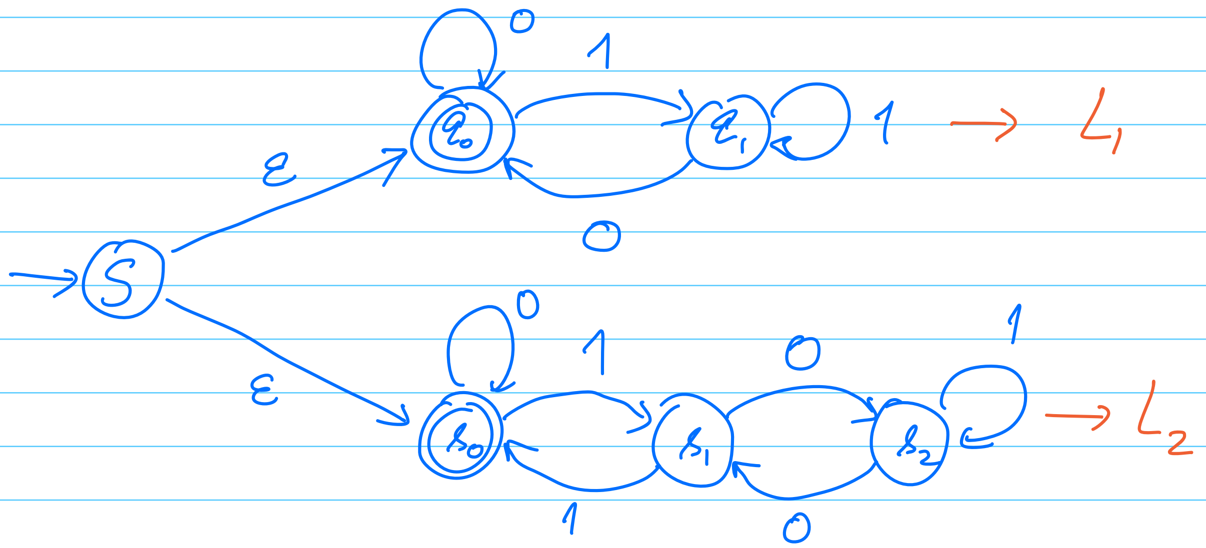
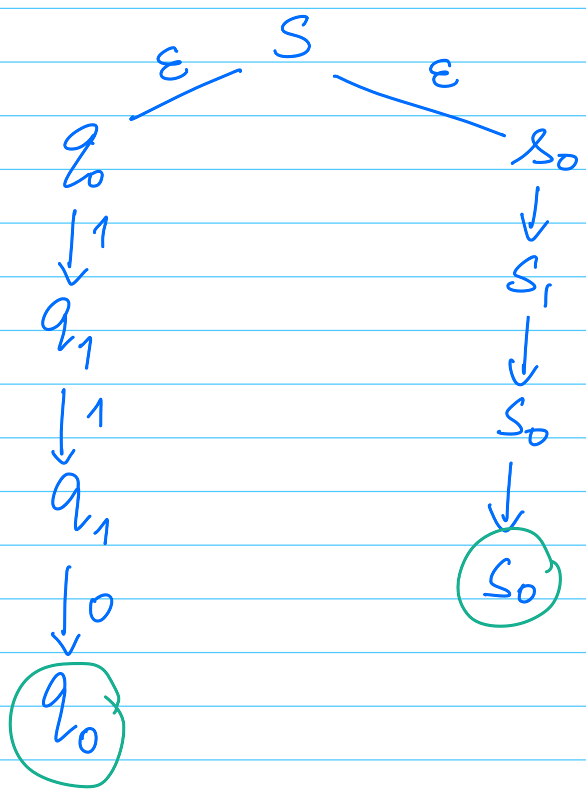
Example using ε-transition

$L_1 \cup L_2$: $L_1 = \{ w \mid n(w) \equiv 0 \pmod 2 \}$

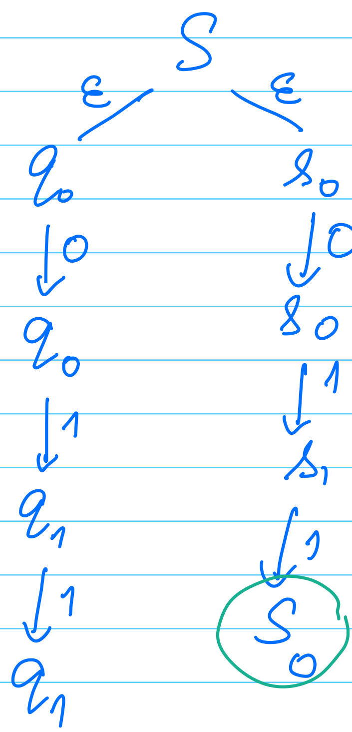
$L_2 = \{ w \mid n(w) \equiv 0 \pmod 3 \}$



$w = 110$



$w = 011$



- Removing ε-transitions

ε-closure(q) = set of states reachable from q using only ε-transitions

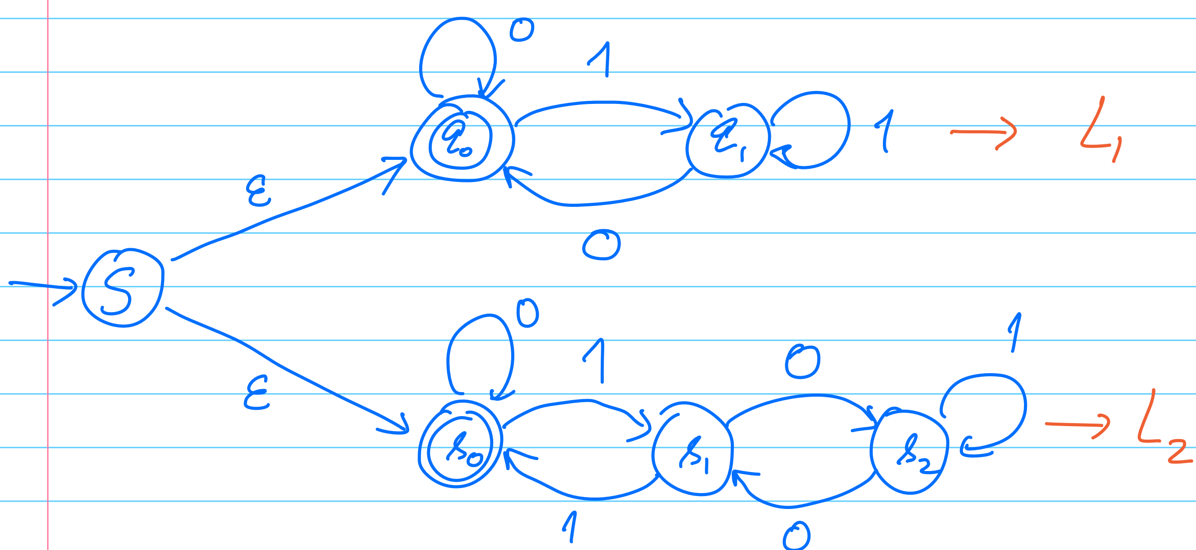
$$N_1 = (Q, \Sigma \cup \{\varepsilon\}, \Delta, q_0, F)$$

$$\downarrow$$

$$N_2 = (Q, \Sigma, \Delta', q_0, F')$$

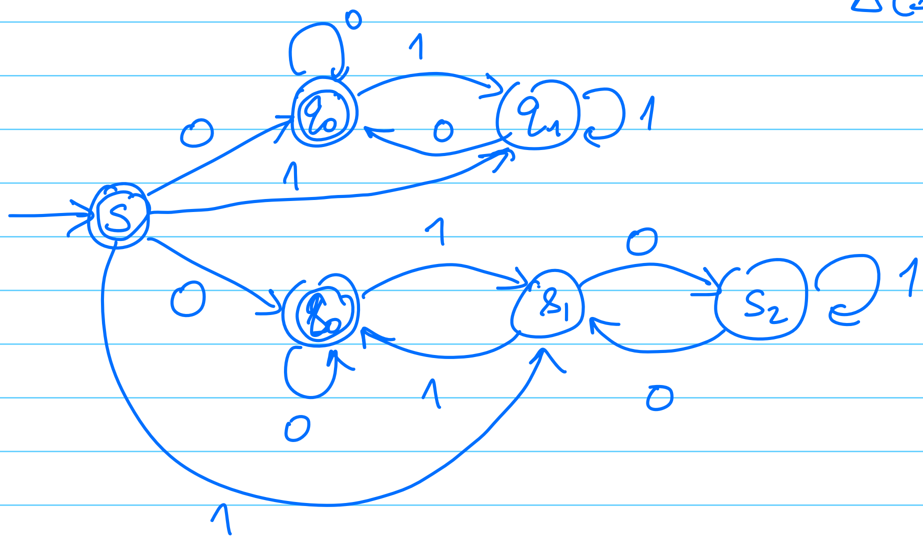
$$\Delta'(q, a) = \bigcup_{q' \in \varepsilon\text{-closure}(q)} \Delta(q', a)$$

$$F' = \{ q \mid \varepsilon\text{-closure}(q) \cap F \neq \emptyset \}$$



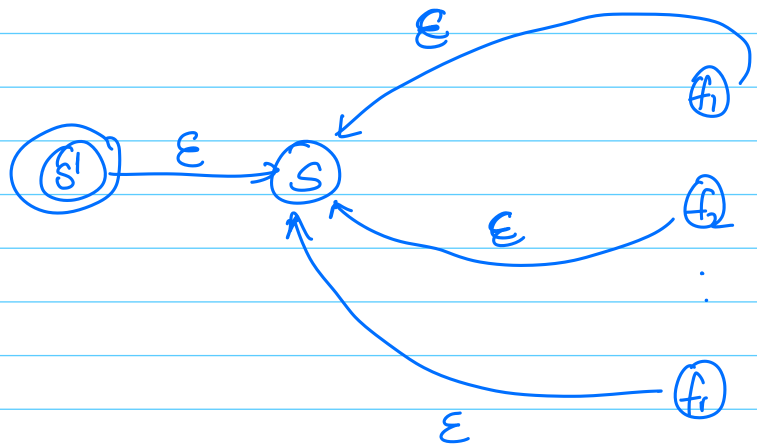
ε-closure$(s) = \{s, q_0, s_0\}$

$\Delta'(s,0) = \Delta(s,0) \cup \Delta(q_0,0) \cup \Delta(q_1,0)$



- Concatenation: $L_1 = L(M_1)$ & $L_2 = L(M_2)$
  add ε-transitions from final states of M_1 to the start state of M_2

- Kleene star: $L = L(M)$: obtain M^* for L^*



Equivalence of NFAs and DFAs

$N = (Q, \Sigma, \Delta, q_0, F)$

$\downarrow$ simulate the NFA

$M = (\mathcal{P}(Q), \Sigma, \delta, \{q_0\}, F')$

$$\delta(Q', \sigma) = \bigcup_{q \in Q'} \Delta(q, \sigma) = \hat{\Delta}(A, \sigma)$$

$Q' \subseteq Q$

$F' = \{ Q' \mid F \cap Q' \neq \emptyset \}$