Formal defn: A CFG is a 4-tuple (N, Σ, P, S), where

(i) N - set of non-terminals
↳ symbols on the lhs of productions

(ii) Σ - set of terminals
↳ alphabet of the language generated by the grammar

(iii) P - set of production rules
↳ rewrite rules to generate sentences of the language

$P \subseteq N \times (N \cup \Sigma)^*$

(iv) S - start symbol
↳ non-terminal from which production starts

$S \in N$

Eg: $L = \{0^n 1^n \mid n \geq 0\}$

$S \to 0S1 \mid \varepsilon$

* A string $\beta \in (N \cup \Sigma^*)$ is derivable from $\alpha \in (N \cup \Sigma)^*$ (denoted $\alpha \to \beta$) if there is a production rule to replace a non-terminal in α and obtain β.

$\alpha = 0^i S 1^i$
$\beta = 0^{i+1} S 1^{i+1}$
$\beta' = 0^i 1^i$

* A string in $(N \cup \Sigma)^*$ is known as a sentential form if it is derivable from S

$0^i S 1^i$ is a sentential form

* A string in Σ^* is a sentence if it is derivable from S.

$\alpha \xrightarrow{0} \alpha \quad \forall \alpha$

$\alpha \rightarrow \beta$ if $\exists$ production rule where a non-terminal can be replaced

$\alpha \xrightarrow{n} \beta$ if $\exists \gamma$ s.t $\alpha \xrightarrow{n-1} \gamma$ & $\gamma \rightarrow \beta$

$\alpha \xrightarrow{*} \beta$ if $\exists n$ s.t $\alpha \xrightarrow{n} \beta$

$L(G) = \{ w \in \Sigma^* \mid S \xrightarrow{*} w \}$

② $L = \{ w \mid w = w^R, \ w \in \{0,1\}^* \}$

$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$

③ Dyck language (Balanced paranthesis)

if-else statements

$[\,][[\,][\,]] \in L \qquad [\,]] \notin L$

$S \rightarrow [S] \mid SS \mid \varepsilon$

Proving correctness of constructions

L = set of balanced paranthesis

$S \to [S] \mid SS \mid \varepsilon \quad : \quad G$

(i) $l(w) = r(w)$

(ii) $\forall$ prefix y, $l(y) \geq r(y)$

Thm: $L(G) = L$

Proof: (i) $L(G) \subseteq L$

for any $\beta \in (S \cup \{[,]\})^*$ s.t $S \xrightarrow{*} \beta$

β satisfies (i) & (ii)

Base case: $S \xrightarrow{0} \beta$. Then $w = S$

and the condition is satisfied

Induction step: $S \xrightarrow{n+1} \beta$

$\exists\, \alpha$ s.t $\underbrace{S \xrightarrow{n} \alpha}_{\text{Induction Hypothesis}} \to \beta$

α satisfies (i)&(ii)

$\alpha = \alpha_1 S \alpha_2 \longrightarrow \beta$

(i) $\alpha_1 S \alpha_2 \to \alpha_1 \alpha_2$ (deleted S - does not contain [or])

(ii) $\alpha_1 S \alpha_2 \to \alpha_1 SS \alpha_2$ (added SS - does not contain [or])

(iii) $\alpha_1 S \alpha_2 \to \alpha_1 [S] \alpha_2$ (try out all prefixes)