Myhill-Nerode relations

An equivalence relation $\equiv$ over a language $L \subseteq \Sigma^*$ is said to be a Myhill-Nerode relation if

1. $\equiv$ is a right congruence

   if $x \equiv y$, then $x\sigma \equiv y\sigma \quad \forall \sigma \in \Sigma$

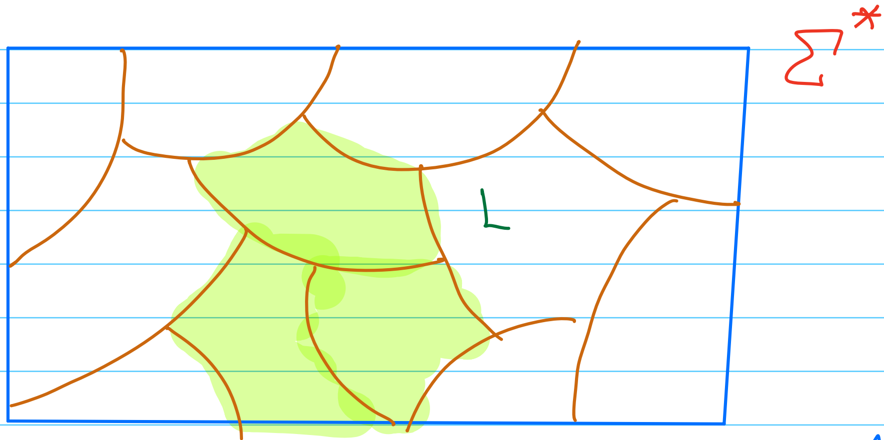
2. $\equiv$ is a refinement of L

   if $x \equiv y$ then $x \in L \iff y \in L$

3. $\equiv$ is of finite index

* If L is regular, & $M = (Q, \Sigma, \delta, q_0, F)$ accepts M,

- Then $\equiv_M$ is a Myhill-Nerode relation



- right congruence: $\quad x \equiv_M y$ if $\hat{\delta}(q_0, x) = \hat{\delta}(q_0, y)$

  Then $\forall \sigma \in \Sigma$, $\hat{\delta}(q_0, x\sigma) = \delta(\hat{\delta}(q_0, x), \sigma)$

  $= \delta(\hat{\delta}(q_0, y), \sigma)$

  $= \hat{\delta}(q_0, y\sigma)$

$$\implies x\sigma \equiv_M y\sigma$$

- $\equiv_M$ is a refinement of L

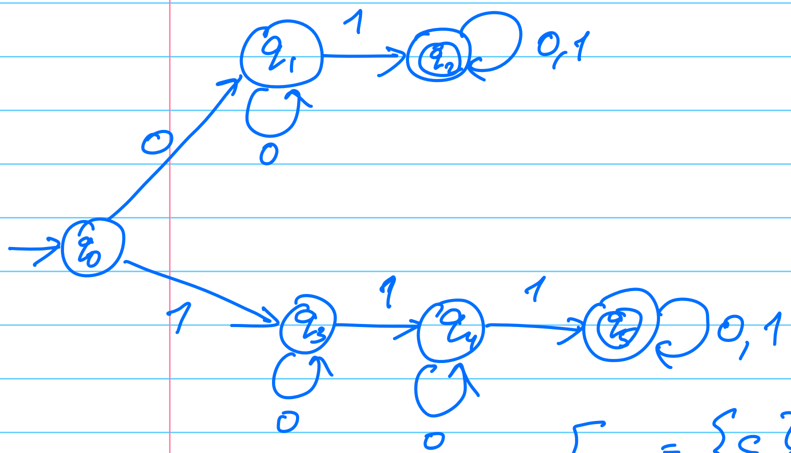
  if $x \equiv_M y$ then $\hat{\delta}(q_0, x) = \hat{\delta}(q_0, y)$

  $$\implies x \in L \iff y \in L$$

- $\equiv_M$ is of finite index

  # of equivalence classes $= |Q|$

$L = \{ w \mid \exists x, y \in \{0,1\}^* - \varepsilon,\ w = xy \ \&\ \#1(y) > \#1(x) \}$



$E_0 = \{\varepsilon\}$

$\equiv_M =$

$E_1 = \{0, 00, 000, \ldots\}$

$E_2 = \{ w \mid w$ starts with 0 & has at least one 1 $\}$

$E_3 = \{ w \mid w$ starts with 1 & no 1s after that $\}$

$E_4 = \{ w \mid w$ starts with 1 & has exactly one more 1

$E_5 = \{ w \mid w$ starts with 1 & $w \in L \}$

Obs: (i) $\forall x, y \in E_2 \cup E_5$, $\forall z$ $xz \in L \Leftrightarrow yz \in L$

(ii) $\forall x, y \in E_1 \cup E_4$, $\forall z$, $xz \in L \Leftrightarrow yz \in L$

How does $\equiv_L$ look like?

E_0, $E_1 \cup E_4$, E_3, $E_2 \cup E_5$