PageWork

Tutorial - 6

1. a) $L = \{a^i b^j c^m d^n \mid i = m \text{ and } j = n\}$

Take $z = 0^k 1^k 0^k 1^k$. Done, in class.

b) $L = \{0^i 1^j \mid j \text{ divides } i\}$

Spoiler chooses $z = 0^{k^3} 1^{k^2}$

Case: $z = uvwxy$. $|vwx| \le k$ $vx \ne \varepsilon$

Case 1: vwx entirely zeroes. Say v and x have c zeroes.

$uv^0wx^0y = 0^{k^3 - c} 1^{k^2}$.

* As $c \le k \Rightarrow \frac{k^3 - c}{k^2} = k - \frac{c}{k^2} \ne$ integer

Case 2: vwx entirely ones. Say v and x have c ones.

$uv^0wx^0y = 0^{k^4} 1^{k^3 - c}$.

* As $c \le k \Rightarrow$ ~~$\frac{k^3}{k^2 - c}$~~. Observe $\frac{k^4}{k^3 - c} > k$.

~~Observe, $c \le k \Rightarrow k^2 - k \le k^2 - c$~~

~~$\Rightarrow (k+1)(k^2 - k) \le (k+1)(k^2 - c)$~~

~~$\Rightarrow k^3 - k \le (k+1)(k^2 - c)$~~

~~$\Rightarrow$~~

~~Observe, $c > k \Rightarrow k^3 - k < k^2 - c$~~

~~$\Rightarrow (k+1)(k^2$~~

Observe, $c \le k$. So $\frac{k^4}{k^3 - c} \le \frac{k^4}{k^3 - k}$

~~$\Rightarrow \frac{k^4}{k^3 - c} \le k + \frac{k}{k^2 - 1} \le k + 1$ for $k \ge 2$~~

$\Rightarrow \frac{k^4}{k^3 - c} < k + 1$

* Hence k^4 not divided by $k^3 - c \Rightarrow uv^0wx^0y \notin L$.

Case 3: vwx has some of both.

If $v = \varepsilon, x \ne \varepsilon \Rightarrow$ Just pump until number of ones more

If $v \ne \varepsilon, x = \varepsilon \Rightarrow$ just do $uv^0wx^0y = 0^{k^4 - c} 1^{k^3}$ $c \le k$ $\notin L$

If $v \ne \varepsilon, x \ne \varepsilon \Rightarrow$ If $\#1(vx) > \#0(vx)$

Just pump until number ones more than zeroes $\notin L$

$\Rightarrow$ Else say vx has c_1 zeroes, c_2 ones $c_2 \le c_1$ and $c_2 + c_1 \le k$

uv^0wx^0y has $k^4 - c_1$ zeroes, $k^3 - c_2$ ones

(1) Observe if $\frac{k^4 - c_1}{k^3 - c_2} = k \Rightarrow c_1 = c_2 k$ Not possible $\Rightarrow \frac{k^4 - c_1}{k^3 - c_2} \ne k$

(2) Also observe ~~$\frac{k^4 - c_1}{k^3 - c_2} < k + 1 \Leftrightarrow (k+1)c_2 - c_1 < k^3$ True~~

$\frac{k^4 - c_1}{k^3 - c_2} > k - 1 \Leftrightarrow (k-1)c_2 - c_1 > -k^3$ True

Hence $\frac{k^4 - c_1}{k^3 - c_2}$ not integer $\Rightarrow uv^0wx^0y \notin L$.

c) $L = \{w_1 \# w_2 \# w_3 \dots w_k \mid k \geq 2,\ w_i \in \{0,1\}^*$ and $w_i = w_j$ for some $i \neq j\}$

Spoiler chooses $0^k 1^k \# 0^k 1^k$

Case 1: vwx totally inside left $0^k 1^k$. Take $uv^0wx^0y \notin L$. ($uv^0wx^0y = \dots$)

Case 2: Similar when vwx totally inside right $0^k 1^k$

Case 3: vwx crosses #

Subcase 1: If v or x contains #. Take power 0. uv^0wx^0y does not have # $\Rightarrow uv^0wx^0y \notin L$.

Subcase 2: v and x do not contain #

~~Sub Subcase 1~~

$v = 1^l$, $x = 0^r$ l and r not zero together

$uv^0wx^0y \notin L$ clearly.

2. One of below is context free, other is not.

a) $L_1 = \{w \# x \mid w, x \in \{0,1\}^*,\ w$ is substring of $x\}$ Not context free.

Spoiler chooses $0^k 1^k \# 0^k 1^k$

Case 1: vwx totally inside left $0^k 1^k$. $uv^2wx^2y \notin L$ Because $uv^2wx^2y = a \# b$ $|a| > |b|$ so a cannot be substring of b.

Case 2. vwx totally inside right $0^k 1^k$. $uv^0wx^0y \notin L$ because $uv^0wx^0y = a \# b$. $|a| > |b|$ so a cannot be substring of b.

Case 3. vwx crosses #.

Subcase 1. v or x contains #. Then $uv^0wx^0y \notin L$ as it does not have #.

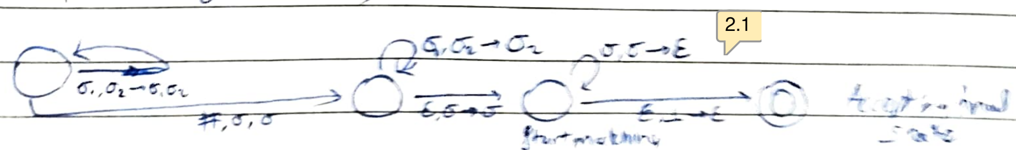
Subcase 2: # in w, $|v| > |x|$ $\Rightarrow uv^2wx^2y = a \# b$ where $|a| > |b|$ so a cannot be substring of b $uv^2wx^2y \notin L$

Subcase 3: # in w, $|v| < |x| \Rightarrow uv^0wx^0y = a \# b$ where $|a| > |b|$ So a cannot be substring of b. $uv^0wx^0y \notin L$

b) $L_2 = \{w \# x \mid w, x \in \{0,1\}^*,\ w^R$ is substring of $x\}$

1. Possible to make ~~PDA~~ non deterministic PDA for this.
(Store w to stack. Then non-deterministically assume where w^R starts in x an pop w by matching)

PDA:



2.1

2. CFG also possible to design
$w \# a w^R b$ where a, b are random strings!

$S \to AB$. $A \to 0A0 \mid 1A1 \mid \#B$. $B \to \varepsilon \mid B0 \mid B1$

3. Suppose G is a grammar in CNF form with n non-terminals. Show that if there is a string w that has a derivation in G of length more than 2^n, then L(G) is infinite.

3.1

→ Take the tree derivation tree. ~~Every non-terminal which does a unit production~~ for ~~remove the~~

→ Remove the terminals from the tree. The length of the string w is still the number of leaves of tree (As leaf terminals were doing $A \to \sigma$ productions only).

→ Property: Binary tree of height h has atmost 2^{h-1} leaves.
Here there are ~~atleast~~ $> 2^n$ leaves.

$2^n <$ number of leaves $\le 2^{h-1}$

$\Rightarrow 2^n < 2^{h-1}$

$\Rightarrow h > n+1$ ⇒ There exists a root leaf path with repeating non-terminal (Suppose A)

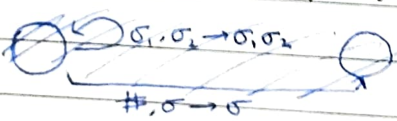
→ We can substitute subtree of upper A to lower A and create larger ~~derivation~~ string.

This can be done infinite times to create infinite strings.

(→ Similar to Pumping Lemma)

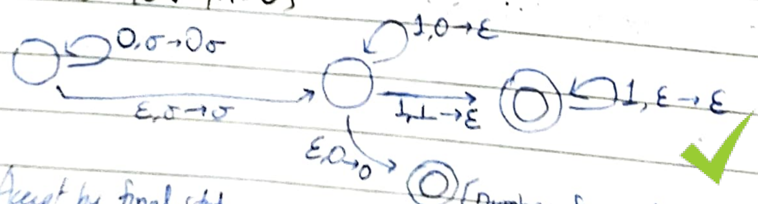
4. Prove below are context free with PDA.

a) $L = \{w\#x \mid w \neq x;\ w, x \in \{0,1\}^*\}$



Non deterministically check if i^{th} character of w, x are different.
Can store i characters ~~in stac~~ in stack as count.

b) $L = \{0,1\}^* - \{0^n 1^n \mid n \geq 0\}$



(~~Number of ones more than zeroes~~)

(~~Number of ones less than zeroes~~)

Accept by final state.

c) $L = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$

→ Similar to above but check for both $i \neq j$ and $j \neq k$

Index of comments

2.1 Idea is clear. The scan resolution of the diagram is not very good.

3.1 Do you mean the parse tree?