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9-04-2023 (Lecture-12)

$$\frac{dx}{dt} + ax + b = 0$$

$$x(t_0) = c$$

a, b, c are real constants. What is $x(t)$ for $t > t_0$?

$$x e^{\int a\,dt} = \int -b\, e^{\int a\,dt}\, dt$$

$$x e^{at} = -b \int e^{at}\, dt$$

$$\boxed{x e^{at} = -b\,\frac{e^{at}}{a} + K}$$

at $t = t_0$

$x = c$

$$\Rightarrow \quad c\, e^{at_0} = \left(\frac{-b}{a}\right) e^{at_0} + K$$

$$\Rightarrow K = e^{at_0}\left(c + \frac{b}{a}\right)$$

So $\Rightarrow x = \left(\frac{-b}{a}\right) + e^{-at}\left[e^{at_0}\left(c + \frac{b}{a}\right)\right]$

$$\Rightarrow \boxed{x = \frac{-b}{a} + \left(\frac{b}{a} + c\right) e^{-a(t-t_0)}}$$

Second order differential eq^n.

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx + c = 0$$

$$x(t_0) = d, \quad \left.\frac{dx}{dt}\right|_{t=t_0} = f$$

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a, b, c, d, f

sol $a^2 - 4b = 0$

$x(t) =$

Now Case

for

$a^2 - 4b$

$x(t)$

$a^2 -$

$x(t$

11-04

NOTE:

$e^{\alpha t}$

So

(51) a, b, c, d, f are real constants. What is $x(t)$ for $t > t_0$?

Sol: $\boxed{a^2 - 4b = 0}$

$$\boxed{x(t) = e^{-at/2}\left[c_1 + c_2 t\right] - \frac{c}{b}}$$

Now Cases to get c_1, c_2

for $t = t_0$

$$\left.\begin{array}{l} x(t_0) = d \\ \left.\frac{dx}{dt}\right|_{t_0} = f \end{array}\right\} \text{Solve for } c_1, c_2$$

$\boxed{a^2 - 4b < 0}$

$$\boxed{x(t) = -\frac{c}{b} + e^{-at/2}\left[c_1 \cos\left(\frac{\sqrt{4b - a^2}}{2} t\right) + c_2 \sin\left(\frac{\sqrt{4b - a^2}}{2} t\right)\right]}$$

get c_1, c_2 by given conditions

$\boxed{a^2 - 4b > 0}$

$$\boxed{x(t) = e^{-at/2}\left[c_1 e^{\sqrt{a^2 - 4b}\, t/2} + c_2 e^{-\sqrt{a^2 - 4b}\, t/2}\right] - \frac{c}{b}}$$

get c_1, c_2 by given conditions.

11-04-23

* Note:

$$\frac{dy}{dt} + fy = 0 \quad (\text{'}f\text{' is a real number}) \longrightarrow (1)$$

$e^{\alpha t}$ is one of sol^n.

$$\frac{d(e^{\alpha t})}{dt} + f e^{\alpha t} = \alpha e^{\alpha t} + f e^{\alpha t} = 0$$

$$\Rightarrow \alpha = -f$$

So we got one thing that e^{-ft} satisfies (1)

52 $\boxed{Ke^{-ft}}$ (case-2)

LHS of ① $= Ke^{-ft}(-f) + Kf(e^{-ft}) = 0$

hence Ke^{-ft} satisfies ①

Now,

$$\frac{dx}{dt} + ax + b = 0 \rightarrow ②$$

define $y = x + \frac{b}{a} \Rightarrow x = y - \frac{b}{a}$

$$\frac{d(y-\frac{b}{a})}{dt} + a\left(y - \frac{b}{a}\right) + b = 0 \Rightarrow \frac{dy}{dt} + ay = 0 \rightarrow ③$$

So Ke^{-at} satisfies ③

$$\boxed{y(t) = Ke^{-at}}$$

$$\boxed{x(t) = Ke^{-at} - \frac{b}{a}}$$

12-04-2022

$$\frac{d^2y}{dt^2} + g\frac{dy}{dt} + hy = 0 \longrightarrow ①$$

where 'g' & 'h' are real constants.

Lets see $Ke^{\alpha t}$ is a possible solⁿ or not, (allowing K, α ~~can~~ to be non real also)

$$\frac{d^2(Ke^{\alpha t})}{dt^2} + g\frac{d(Ke^{\alpha t})}{dt} + hKe^{\alpha t} = 0$$

$$\Rightarrow K\alpha^2 e^{\alpha t} + gK\alpha e^{\alpha t} + hKe^{\alpha t} = 0$$

$$\Rightarrow Ke^{\alpha t}(\alpha^2 + g\alpha + h) = 0$$

If ~~provided~~ $\alpha^2 + g\alpha + h = 0$,

let α_1 & α_2 be roots of $\alpha^2 + g\alpha + h = 0$

(53) Suppose $\alpha_1 \neq \alpha_2$, $K_1 e^{\alpha_1 t}$ & $K_2 e^{\alpha_2 t}$ satisfy (1) & $K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t}$ satisfies (1)

Now,

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx + c = 0 \longrightarrow (2)$$

a, b, c are real constants

Define $y = x + \frac{c}{b} \Rightarrow x = y - \frac{c}{b}$

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + by = 0 \text{ --- } (3)$$

So, from our previous arguments, $K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t}$ satisfies eq (3)

and where α_1, α_2 are roots of $\alpha^2 + a\alpha + b = 0$

So,

$$\boxed{x(t) = K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t} - \frac{c}{b}} \longrightarrow \text{satisfies } (2)$$

Initial conditions:-

$$x(t_0) = d, \quad \left.\frac{dx}{dt}\right|_{t=t_0} = f \rightarrow (4)$$

Initial value problem:- (2) & (4)

$$K_1 e^{\alpha_1 t_0} + K_2 e^{\alpha_2 t_0} - \frac{c}{b} = d \longrightarrow (5)$$

$$K_1 \alpha_1 e^{\alpha_1 t_0} + K_2 \alpha_2 e^{\alpha_2 t_0} = f \longrightarrow (6)$$

Suppose $\alpha_1 \neq \alpha_2$

If $a^2 - 4b > 0$ then α_1, α_2 are real

If $a^2 - 4b < 0$ then α_1 & α_2 are complex conjugate

Let, $\alpha_1 = u + jv$, where u, v are real

$\alpha_2 = u - jv$

$(j = +\sqrt{-1}$

5 & 6

$$K_1 e^{(u+jv)t_0} + K_2 e^{(u-jv)t_0} - \frac{c}{b} = d$$

17-04-23

Now,

for

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx + c = 0$$

$$\begin{cases} a^2 - 4b < 0 \\ K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t} - \frac{c}{b} \end{cases}$$ satisfies the equation. Here α_1, α_2 → (1)

are complex conjugate.

Is there a real $x(t)$ that satisfies the above equation?

Let $K_1 = l_1 + jl_2$, $K_2 = l_3 + jl_4$ where l_1, l_2, l_3, l_4 are real.

Now, ~~Substitute~~ Substitute in eqn (1)

and imaginary part after substitution is

$$(l_1 - l_3) e^{ut} \sin(vt) + (l_2 + l_4) e^{ut} \cos(vt)$$

Make $l_1 = l_3$ and $l_2 = -l_4$

i.e K_1 & K_3 are complex conjuage of each other.

Then solution is $Re\left[K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t} - \frac{c}{b}\right]$

$$= l_1 e^{ut} \cos(vt) - l_2 \sin(vt) + l_1 e^{ut} \cos(vt) - l_2 e^{ut} \sin(vt) - \frac{c}{b}$$

$$= 2l_1 e^{ut} \cos(vt) - 2l_2 e^{ut} \sin(vt) - \frac{c}{b}$$

$$= c_1 e^{ut} \cos(vt) + c_2 e^{ut} \sin(vt) - \frac{c}{b}$$

Now,

(55) if $a^2 - 4b = 0$

$$\frac{d^2y}{dt^2} + g\frac{dy}{dt} + hy = 0$$

$$\alpha^2 + g\alpha + h = 0$$

Roots are $\dfrac{-g \pm \sqrt{g^2-4h}}{2}$

$$g^2 - 4h = 0$$

Roots are $\dfrac{-g}{2}$

$$\alpha = -g/2$$

$k_1 e^{\alpha t}$ satisfies (1).

and $k_2 t e^{\alpha t} \longleftarrow$ does this satisfy (1) ?

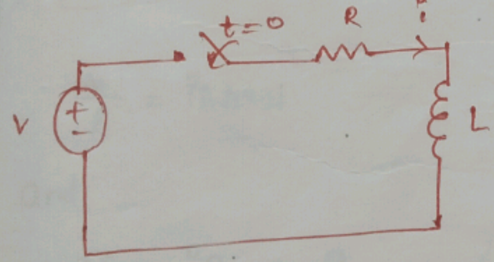
$\boxed{\text{Yes}}$.

So $k_1 e^{\alpha t} + k_2 t e^{\alpha t}$ is also solution.

So for $\dfrac{d^2x}{dt^2} + a\dfrac{dx}{dt} + bx + c = 0 \longrightarrow$ (2)

$k_1 e^{\alpha t} + k_2 t e^{\alpha t} - \frac{c}{b}$ satisfies (2).

Q.



Find $i(t)$ at $t>0$.

$$V = iR + L\frac{di}{dt}$$

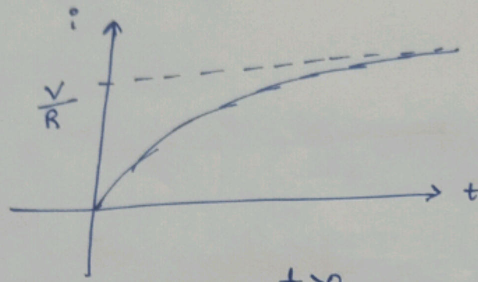
$$V - iR = L\frac{di}{dt} \Rightarrow \frac{-R\,dt}{L} = \frac{-di\,R}{V-iR} \Rightarrow \frac{-Rt}{L} = \ln|V-iR|$$

$$\cancel{k}e^{-Rt/L} = 1 - \frac{iR}{V} \Rightarrow i = (1 - \cancel{k}e^{-Rt/L})\frac{V}{R}$$

So $i(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$

Note: $\frac{(L)}{R}$ has dimension of time.

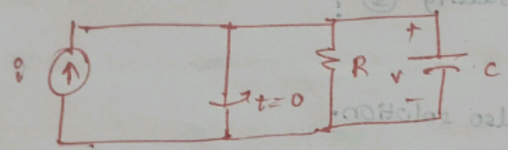
$\frac{(L)}{R}$ is time constant of given circuit.



$\frac{V}{R}$ = steady state value of (i)

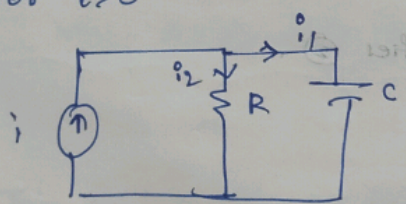
$t \geq 0$

Q.)



Find $v(t)$ at $t > 0$.

For $t > 0$



$\Rightarrow i_2 R = \frac{\cancel{\int i_1 dt}\ Q}{C}$

$\cancel{\frac{i}{\int t}}$. $\Rightarrow \boxed{\left(\frac{di_2}{dt}\right) R = \frac{i_1}{C}}$

$\Rightarrow \boxed{i_2 + RC\frac{di_2}{dt} = i}$

$$RC\frac{di_2}{dt} = i - i_2$$

$$-\frac{di_2}{i-i_2} = -\frac{dt}{RC}$$

$$\left(\ln(i-i_2)\right)_{i_0}^{i_f} = -\frac{t}{RC}$$

and

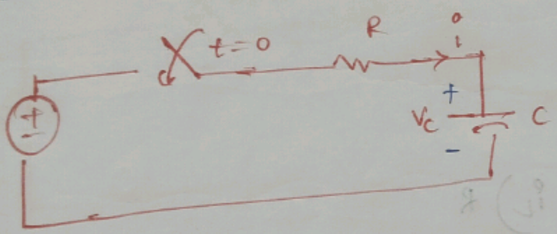
$$i = \frac{V}{R} + C\frac{dv}{dt}$$

$$\frac{dv}{dt} + \frac{V}{RC} - \frac{i}{C} = 0$$

and $v(0) = 0$

$$\boxed{v(t) = iR - iR\,e^{-t/RC}}$$

here RC is the time constant of the circuit

Q) 

Find $i(t)$ for $t>0$.

$$\frac{V}{R} = i_{initial}$$

$$\boxed{\frac{V}{R}e^{-\frac{t}{RC}} = i_f}$$

time const of circuit

and

$$V = iR + \frac{q}{C}$$

~~$V = \frac{dq}{dt}R$~~

$$V - iR = \frac{q}{C}$$

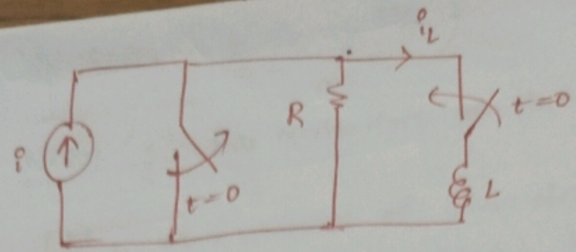
$$\Rightarrow 0 - R\frac{di}{dt} = \frac{i}{C}$$

$$\Rightarrow -RC\frac{di}{dt} = i$$

$$\Rightarrow -\frac{t}{RC} = \ln\left(\frac{i_f}{\frac{V}{R}}\right)$$

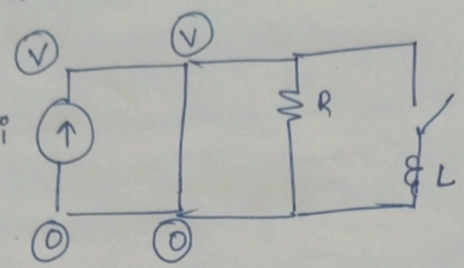
$$-\frac{di}{i} = \frac{dt}{RC}$$

(58) Q)



Find i_L as $f(t)$ for $t>0$.

for $t<0$



& hence

$$\boxed{i_L(0^-)=0}$$

for $t>0$

$$L\frac{di_L}{dt} = (i - i_L)R$$

$$L\frac{di_L}{dt} = R(i - i_L)$$

$$-\frac{d(i_L)}{i - i_L} = \frac{-R\,dt}{L}$$

$$\ln\left(|i - i_L|\right)\Big|_{0}^{i_L} = -\frac{Rt}{L}$$

$$e^{-\frac{Rt}{L}} = \frac{|i - i_L|}{|i|}$$

(59) $|i - i_L| = |i| e^{-Rt/L}$

$i_f(1 - e^{-Rt/L}) = i(t)$

* Solution of the differential equation:-

$x(t) \leftarrow$ solution

Steady state value of $x(t) = \lim_{t\to\infty} x(t)$

$x(t)$ = transient expression + steady state expression.

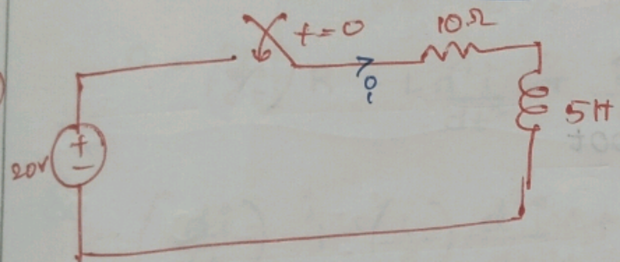
In steady state, all derivatives are zero.

$\frac{dx}{dt} + ax + b = 0$. In steady state $\boxed{x = \frac{-b}{a}}$.

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx + c = 0$$

In steady state $\boxed{x = \frac{-c}{b}}$

(6)



Find $i(t)$ for $t>0$.

Sol.

$$V_A - 10i - L\frac{di}{dt} = V_B$$

$$20 = 10i + 5\frac{di}{dt}$$

$$4 = 2i + \frac{di}{dt}$$

and $i(0^+) = 0$

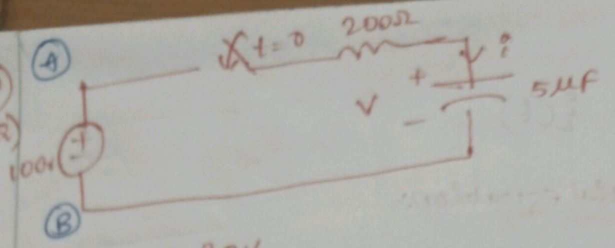
$$\frac{di}{dt} = 4 - 2i \Rightarrow \frac{-2\,di}{4-2i} = -2dt$$

$$\ln\left|\frac{4-2i}{4}\right| = -2t$$

$\Rightarrow$ $\boxed{2 - 2e^{-2t} = i}$

$\boxed{2(1-e^{-2t}) = i}$

60 Q)



$V(0) = 30V$

$i(t) = ?$ for $t>0$; also find $V(t)$ for $t>0$.

$$V_A - 200i - V = V_B$$

$$100 = 200i + V$$

$$0 = 200\frac{di}{dt} + \frac{i}{5\times10^{-6}}$$

& $\left(100 = 200 i_{0i} + 30 \quad \frac{7}{20} = i_0\right)$

$$0 = 2\frac{di}{dt} + \frac{i}{5}\times10^{4}$$

$$0 = \frac{2di}{dt} + i(2\times10^{3})$$

$$\frac{di}{dt} = -1000i$$

$$\Big[\ln(i)\Big]_{\frac{7}{20}}^{i(t)} = -1000t$$

$$\Rightarrow \boxed{\frac{7}{20}e^{-1000t}\ \text{Amp} = i(t)}$$

$$\frac{dv}{dt} = \frac{i}{C}$$

$$\frac{dv}{dt} = \frac{7}{20(5\times10^{-6})}e^{-1000t}$$

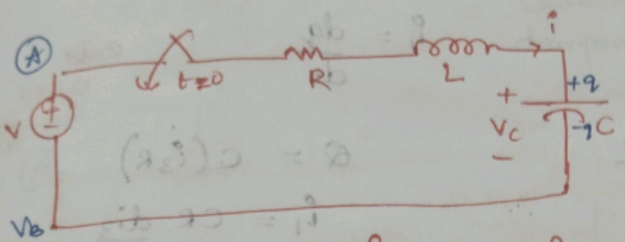
$$V - 30 = \frac{7}{10^{-4}}\left(\frac{e^{-1000t}}{-1000}\right)_0^t$$

(61)

$$V - 30 = (-7\times 10^4)\left(\frac{e^{-1000t}}{1000} - \frac{1}{1000}\right)$$

$$V - 30 = +70\left(1 - e^{-1000t}\right)$$

$$\boxed{V_{(t)} = 100 - 70e^{-1000t}} \text{ volts.}$$

Q.) 

Find the expression for $i(t)$ for $t>0$.

Sol.

$$V_A - iR - L\frac{di}{dt} - \frac{q}{C} = V_B$$

$$V = iR + L\frac{di}{dt} + \frac{q}{C}$$

$$0 = \left(\frac{di}{dt}\right)R + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

So $$\left(\frac{di}{dt}\right) + \left(\frac{L}{R}\right)\frac{d^2i}{dt^2} + \frac{i}{RC} = 0$$

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0} \rightarrow (1)$$

$$i(0^+) = 0$$

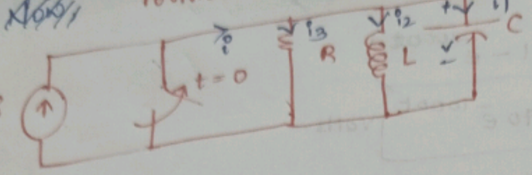
and $$V_A - L\left(\frac{di}{dt}\right)_{at\ t=0} - V_C(0) = V_B$$

$$V = V_C(0) + L\left(\frac{di}{dt}\right)_{at\ t=0}$$

$$\boxed{\frac{V - V_C(0)}{L} = i'(0) \text{ and } i(0) = 0}$$

(62) Ans//

Q) Formulate an initial value problem for this circuit $i_L(0)=0$



Sol: At $t>0$

$$i_1 = \frac{dq}{dt}$$

$$i = i_1 + i_2 + i_3$$

$$i_3 R = L\frac{di_2}{dt} = \frac{q}{C}$$

$$Q = C(i_3 R)$$

$$i_1 = CR\frac{di_3}{dt}$$

$$i = RC\frac{di_3}{dt} + i_2 + i_3$$

$$\left(\frac{di_2}{dt}\right)_{t=0} = \frac{V(0)}{L}$$

$$i = (RC)\left(\frac{L}{R}\right)\frac{d^2 i_2}{dt^2} + i_2 + \frac{L}{R}\frac{di_2}{dt}$$

$$i = LC\frac{d^2 i_2}{dt^2} + \frac{L}{R}\frac{di_2}{dt} + i_2$$

So

$$\boxed{\frac{d^2 i_2}{dt^2} + \frac{1}{RLC}\frac{di_2}{dt} + \frac{i_2 - i}{LC} = 0}$$

Now, $i_2(0^+) = 0$

$$i_2'(0^+) = \frac{V_{at\,t=0}}{L} = 0 \quad (\text{as } i_L(0)=0 \ \forall t<0)$$

and other form is,

$$\frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

$$v(0) = 0, \quad \left.\frac{dv}{dt}\right|_{t=0^+} = \frac{i}{C}$$

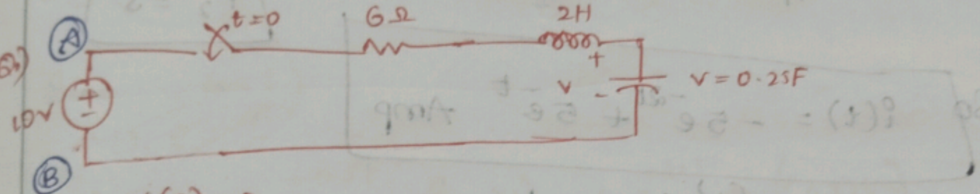
(63) NOTE:

Roots are real & distinct ⟶ Overdamped response / Overdamped circuit

Roots are complex conjugate ⟶ Underdamped circuit & underdamped response

Roots are equal & real. ⟶ Critically damped & Critically damped response.

(63)



$v(0) = 0$

Find $i(t)$ for $t>0$

$Q = 0.25\,V$

$i = \frac{dV}{dt}(4)$;

$V_A - 6i - 2\frac{di}{dt} - v = V_B$

$10 = 6i + 2\frac{di}{dt} + v$

$0 = 6\frac{di}{dt} + 2\frac{d^2i}{dt^2} + 4i$

So $\frac{d^2i}{dt^2} + 3\frac{di}{dt} + 2i = 0$

$y^2 + 3y + 2 = 0$

$\frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} = -2, -1$

So it is overdamped circuit,

$i(t) = K_1 e^{-2t} + K_2 e^{-t}$

and $i(0) = 0 \Rightarrow \boxed{K_1 + K_2 = 0}$

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$10 = 6i + 2\frac{di}{dt}$

$5 = \left(\frac{di}{dt}\right)_{t=0^+}$

$\cancel{-2k_1 + k_2 = 0}$

$\cancel{k_2 = -2k_1}$

and

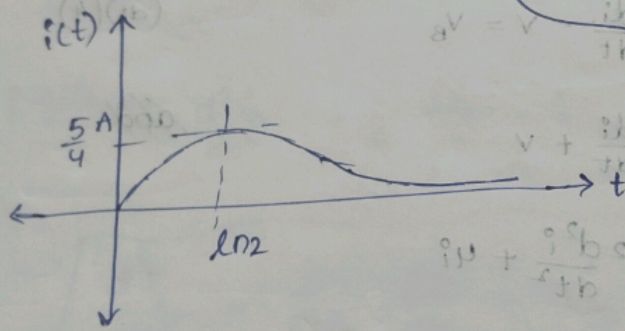
$k_1(-2) + k_2(-1) = 5 \Rightarrow -2k_1 + k_1 = 5$

$\Rightarrow \boxed{k_1 = -5}$

$k_1 = -k_2 \Longrightarrow \boxed{k_2 = 5}$

So $\boxed{i(t) = -5e^{-2t} + 5e^{-t} \text{ Amp}}$

(t) in seconds.



$5(-1)e^{-t} + 5(2)e^{-2t} = 0$

$5e^{-t} = 10e^{-2t}$

$5e^{t} = 10$

$e^{t} = 2$

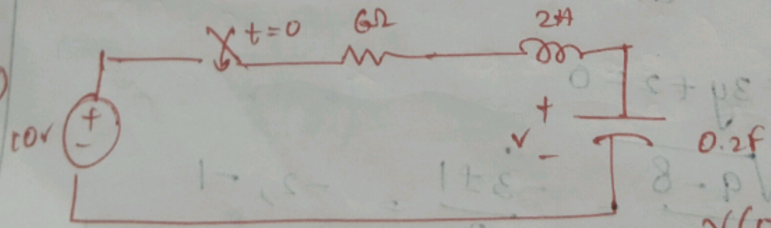
$-5e^{-2\ln 2}$

$+ 5e^{-\ln 2}$

$-5e^{\ln(\frac{1}{4})} + 5e^{\ln(\frac{1}{2})}$

$-\frac{5}{4} + \frac{5}{2}$

Q)



$v(0) = 0$

Sol.

$10 = 6i + 2\frac{di}{dt} + v \qquad i = (0.2)\frac{dv}{dt}$

$0 = 6\frac{di}{dt} + 2\frac{d^2i}{dt^2} + 5i$

68

$\frac{d^2 i}{dt^2} + 3$

$y^2 + 3$

$-3 \pm$

So

i(t)

i(0)

& $\frac{d}{d}$

So

(68)

$$\frac{d^2i}{dt^2} + \frac{3\,di}{dt} + \frac{5}{2}i = 0$$

$$y^2 + 3y + \frac{5}{2} = 0$$

$$\frac{-3 \pm \sqrt{9-10}}{2}$$

$$\frac{-3 \pm i}{2}$$ it is underdamped situation.

So $\cancel{B_1 e^{-\left(\frac{3+i}{2}\right)}}$

$$i(t) = C_1 e^{-1.5t}\cos((0.5)t) + C_2 e^{-1.5t}\sin((0.5)t)$$

$$i(0) = 0$$

& $\frac{di}{dt}\Big|_{t=0^+} = 5$

So $C_1(-1.5) + \cancel{C_2(-1.5)}\ C_2(0.5) = 5$

$\cancel{C_1 + C_2}$ $\boxed{C_1 = 0}$

$$\boxed{C_2 = 10}$$

So, $$\boxed{10e^{-1.5t}\sin(0.5t)A = i(t)}$$

$\cancel{10}\, e^{-1.5t}(0.5)\cos(0.5t)$

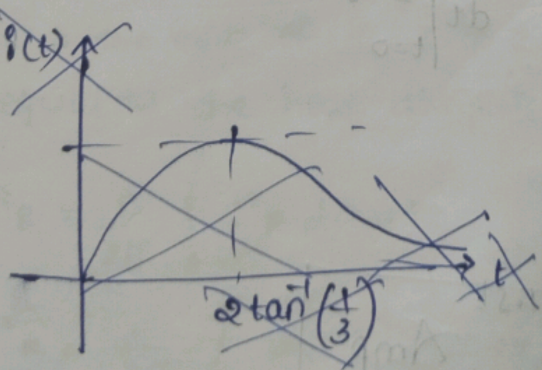
$\cancel{10(0.5)}$

$+ \cancel{10}\sin(0.5t)(-1.5)e^{-1.5t}$

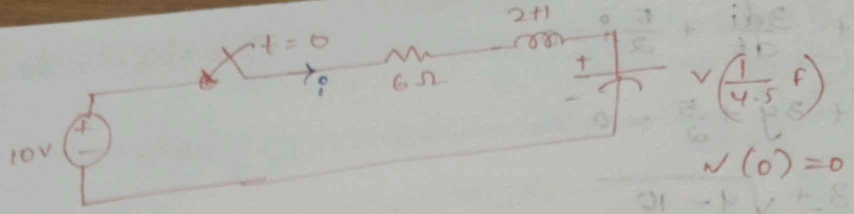
$\cancel{0.5}\cos(0.5t)\ 3$

$= \cancel{1.5}\sin(0.5t)$

$\tan\left(\frac{t}{2}\right) = \frac{1}{3}$



(66) Q)



Sol: $6i + 2\frac{di}{dt} + q(4.5) = 10$

$$6\frac{di}{dt} + 2\frac{d^2i}{dt^2} + i(4.5) = 0$$

$$\frac{d^2i}{dt^2} + 3\frac{di}{dt} + \frac{9}{4}i = 0$$

$$y^2 + 3y + \frac{9}{4} = 0$$

$$\frac{-3 \pm \sqrt{9-9}}{2}$$

$$\boxed{\frac{-3}{2} = y}$$

and hence

$$i(t) = (A + Bt)\, e^{-\frac{3}{2}t}$$

$i(0) = 0$ & $10 = 2\left(\frac{di}{dt}\right)_{at\ t=0}$

$A = 0$

$$\left.\frac{di}{dt}\right|_{t=0} = 5$$

$$B e^{-\frac{3}{2}(0)} = 5$$

$$\boxed{B = 5}$$

hence $\boxed{i(t) = 5t\, e^{-1.5t} \text{ Amp}}$

(67) A.C. circuits :- All voltages & all current vary sinusoidally with time.

$I_m \sin(\omega t + \phi) = i$

here I_m, ω, ϕ are real constants

I_m : max or peakvalue of 'i'

ω : angular frequency in rad/s

$\omega = 2\pi f$ where 'f' is frequency in Hz

ϕ = phase angle.

$\Rightarrow i_D$ (DC current)

i_D : rated current

R : resistance

power $= i_D^2 R$

$\Rightarrow$ A conductor is designed to carry i_D based on amount of heat dissipated.

Now,

$i = I_m \sin(\omega t + \phi)$

Average heat dissipated,

Q) If this conductor is required to carry AC current $i = I_m \sin(\omega t + \phi)$ what is the value of I_m such that the average heat dissipated is equal to the heat dissipated due to flow of current i_D?

$$i_D^2 R = \frac{1}{2\pi} \int_0^{2\pi} i^2 R \, d(\omega t)$$

$$i_D^2 R = \frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2(\omega t + \phi) R \, d(\omega t)$$

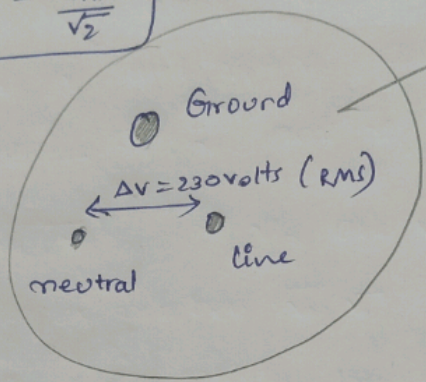
$I_m = \sqrt{2}\, i_0$

$\frac{I_m}{\sqrt{2}}$ ← RMS value (root mean square value)

denoted by I

$$\frac{I_m}{\sqrt{2}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \, d(\omega t)}$$

$$\boxed{I = \frac{I_m}{\sqrt{2}}}$$



socket in houses.

$$\boxed{v = \sqrt{2} V \sin(\omega t + \phi)}$$

RMS value of the instantaneous voltage.

⇒ A sinusoidally varying quantity is called phasor.

★ Phasor representation using a complex number:

phasor $v \longrightarrow V\angle\phi = Ve^{j\phi} = \bar{V}$

Polar form.

$V\angle\phi = V\cos\phi + jV\sin\phi$ (rectangular form)

$\Rightarrow v = \mathrm{Im}\left(\sqrt{2}\, V(\angle\phi \times \angle\omega t)\right)$

(69) $v_1 = \sqrt{2}\, V_1 \sin(\omega t + \phi_1)$

$v_2 = \sqrt{2}\, V_2 \sin(\omega t + \phi_2)$

let $\bar{V_1} = V_1 \angle \phi_1$, $\bar{V_2} = V_2 \angle \phi_2$

let $v = v_1 + v_2$

Q. Is v is a phasor? If so, what is the RMS value of v and what is the phase angle of v?

$$\sqrt{2}\left(\sqrt{V_1^2+V_2^2}\right)\left[\frac{\sqrt{2}\, V_1 \sin(\omega t+\phi_1)}{\sqrt{2}\sqrt{V_1^2+V_2^2}} + \frac{\sqrt{2}\, V_2 \sin(\omega t+\phi_2)}{\sqrt{2}\sqrt{V_1^2+V_2^2}}\right] = v$$

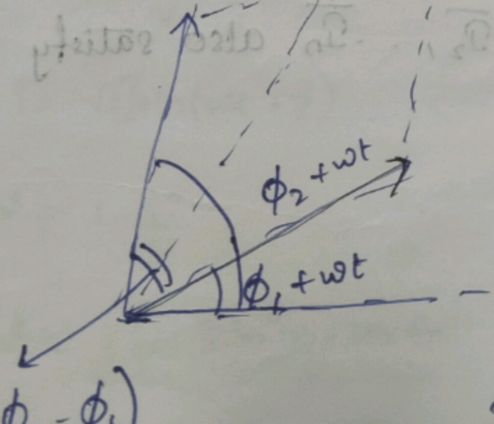
$$\Rightarrow \cancel{\sqrt{2}\left(\sqrt{V_1^2+V_2^2}\right)\left[\sin(\omega t+\phi_1+\phi)\right] = v}$$

$$\Rightarrow \cancel{\tan\phi = \frac{V_1}{V_2}}$$

$$\Rightarrow v = \sqrt{2}\, V_1 \sin(\omega t+\phi_1) + \sqrt{2}\, V_2 \sin(\omega t+\phi_2)$$

$$v = \sqrt{2}\left[V_1 \sin\omega t \cos\phi_1 + V_1 \cos\omega t \sin\phi_1 + V_2 \sin\omega t \cos\phi_2 + V_2 \cos\omega t \sin\phi_2\right)$$

(or)



~~magnitude~~ $= \cancel{\sqrt{2}\sqrt{V_1^2+V_2^2+\sqrt{2}V_1V_2\cos(\phi_2-\phi_1)}}$

~~So~~

So,

$$\underset{\text{instantaneous}}{\overset{\text{phasor}}{V}} = \sqrt{2V_1^2 + 2V_2^2 + 4V_1V_2\cos(\phi_2-\phi_1)}\ \sin(\omega t + \phi_1 + \omega t + \cancel{\omega t}\ \phi)$$

$$\underset{\text{instantaneous}}{\overset{\text{phasor}}{V}} = \sqrt{2}\sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos(\phi_2-\phi_1)}\ \sin(\omega t + \text{phase angle})$$

$$\text{phase angle} = \tan^{-1}\left(V_1\cos(\cancel{\omega t}+\phi_1) + V_2\cos(\cancel{\omega t}+\phi_2),\ V_1\sin(\cancel{\omega t}+\phi_1) + V_2\sin(\cancel{\omega t}+\phi_2)\right)$$

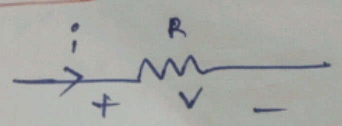
* If v_1 & v_2 are phasors then $v_1 + v_2$ is a phasor.

* If v_1, v_2 and $v_1 + v_2$ are represented by complex numbers $\bar{v}_1, \bar{v}_2$ and $\bar{v}$ respectively then $\bar{V} = \bar{V}_1 + \bar{V}_2$

* If $v_1, v_2, \ldots, v_n$ are voltage phasors represented by complex numbers $\bar{V}_1, \bar{V}_2, \ldots \bar{V}_n$ respectively and if $v_1, v_2, \ldots v_n$ satisfy KVL then $\bar{V}_1, \bar{V}_2, \ldots, \bar{V}_n$ also satisfy KVL.

If $i_1, i_2, \ldots, i_n$ are current phasors represented by complex numbers $\bar{I}_1, \bar{I}_2, \ldots \bar{I}_n$ respectively and $i_1, i_2, \ldots i_n$ satisfy KCL then $\bar{I}_1, \bar{I}_2, \ldots \bar{I}_n$ also satisfy KCL.

* Resistance:



$$v = \sqrt{2}\, V \sin(\omega t + \phi)$$

$$i = \frac{v}{R} = \sqrt{2}\frac{V}{R}\sin(\omega t + \phi)$$

$$= \sqrt{2}\, I \sin(\omega t + \phi) \quad \text{where } I = \frac{V}{R}$$

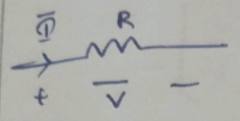
$v \longrightarrow V \angle \phi = \overline{V}$

$i \longrightarrow I \angle \phi = \overline{I}$

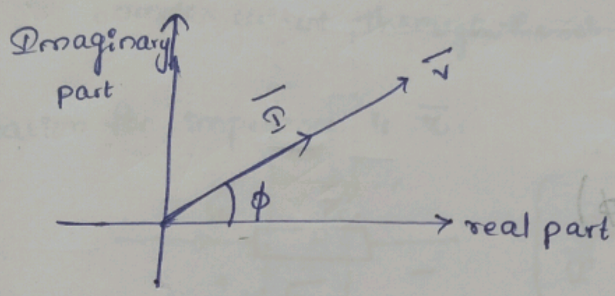
$$\frac{\overline{V}}{\overline{I}} = \frac{V}{I} = R$$

~~and~~ because $R_1 \angle \theta_1 \; R_2 \angle \theta_2 = R_1 R_2 \angle \theta_1 + \theta_2$

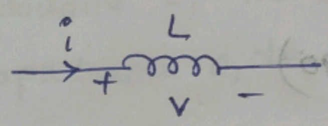
$$\frac{R_1 \angle \theta_1}{R_2 \angle \theta_2} = \frac{R_1}{R_2} \angle \theta_1 - \theta_2$$



phasor diagram:-



* Inductance:-



let $i = \sqrt{2} I \sin(\omega t + \phi)$

$$v = L \frac{di}{dt}$$

$$v = \underbrace{L(\sqrt{2} I)(\omega)}\cos(\omega t + \phi) = \sqrt{2}(I \omega L) \sin\left(\omega t + \frac{\pi}{2} + \phi\right)$$

So $\boxed{V = I \omega L}$ and hence, $V = I X_L$

and here $X_L = \omega L$ (inductive reactance)

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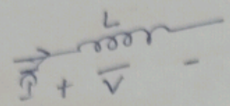
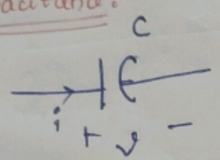
$$\frac{\bar{V}}{\bar{I}} = \frac{V}{I} \angle 90^\circ = X_L \angle 90^\circ = jX_L$$



Phasor diagram

* Capacitance:-



let $v = \sqrt{2}\, V \sin(\omega t + \phi)$

then $i = C\dfrac{dv}{dt}$

$$i = C\sqrt{2}\, V \omega \cos(\omega t + \phi)$$

$$i = \frac{\sqrt{2}\, V}{X_C} \sin(\omega t + \phi + 90^\circ)$$

where $X_C = \dfrac{1}{\omega C}$

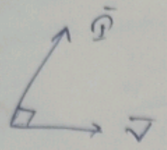
X_C is capacitive reactance

$$i = \sqrt{2}\, I \sin(\omega t + \phi + 90^\circ)$$

where $I = \dfrac{V}{X_C}$

so $\dfrac{V}{I} = X_C$ and $\dfrac{\bar{V}}{\bar{I}} = X_C \angle -90^\circ \Rightarrow -jX_C$

(a) phasor diagram



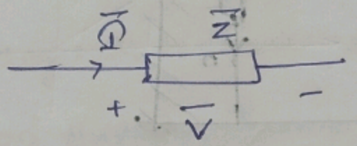
Impedence:-

Impedence of a circuit element (R, L, C)

$$= \left[\frac{\bar{V}}{\bar{I}}\right]$$

= ratio of complex voltage across the circuit element to the complex current through the circuit element.

Notation for impedence is $\bar{z}$.



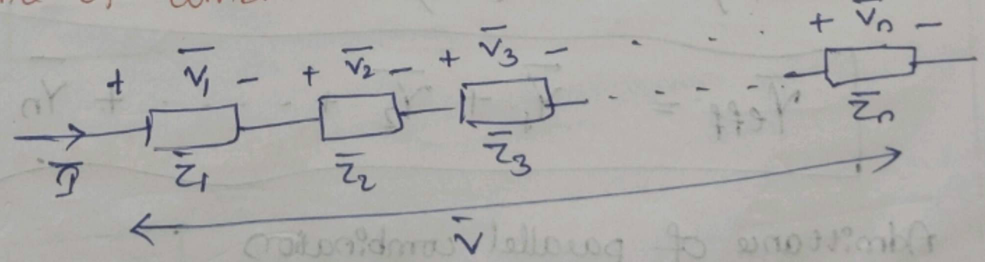
$$\boxed{\frac{\bar{V}}{\bar{I}} = \bar{Z}}$$

For resistance R, the impedance $\bar{z} = R$

for inductance L, the impedance $\bar{z} = j\omega L = jX_L$

for capacitance C, the impedance $\bar{z} = \frac{-j}{\omega C} = -jX_C$

Impedance of combination of circuit elements:-



$$\bar{V} = \bar{V}_1 + \bar{V}_2 + \cdots + \bar{V}_n$$

$$\bar{I}\,\bar{z}_{eff} = \bar{I}\bar{z}_1 + \bar{I}\bar{z}_2 + \cdots + \bar{I}\bar{z}_n \Rightarrow \bar{z}_{eff} = \bar{z}_1 + \bar{z}_2 + \cdots + \bar{z}_n$$

So $\bar{z} = \overset{\nearrow \text{Resistance}}{\text{Real part}} + j\,(\overset{\nearrow \text{Reactance}}{\text{Imaginary part}})$

$$\boxed{\bar{z} = R + jX}$$ (notation)

* Admittance

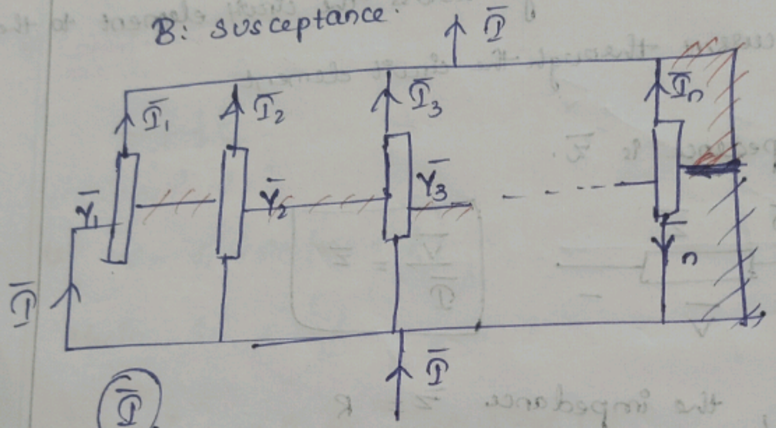
Admittance = $\dfrac{1}{\text{Impedance}}$

Notation is $\bar{y}$

$\bar{y} = G + jB$

G : conductance

B: susceptance



$\bar{I}$.

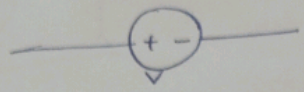
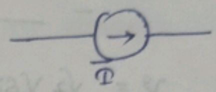
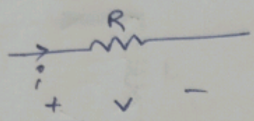
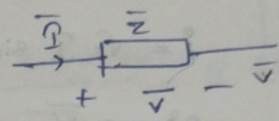
$\bar{I} = \bar{I}_1 + \bar{I}_2 + \dots + \bar{I}_n$

$$\bar{V}(\bar{Y}_{eff}) = \bar{V}(\bar{Y}_1) + \bar{V}(\bar{Y}_2) \dots + \bar{V}(\bar{Y}_n)$$

$$\boxed{\bar{Y}_{eff} = \bar{Y}_1 + \bar{Y}_2 + \dots + \bar{Y}_n}$$

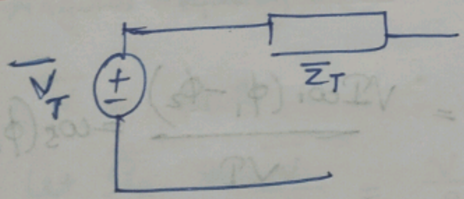
Admittance of parallel combination

$$= \bar{Y}_1 + \bar{Y}_2 + \dots + \bar{Y}_n$$

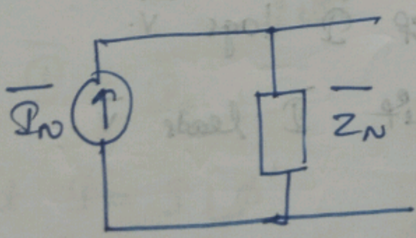
DC circuit	AC circuit
(All voltages and all currents are constants)	(All voltages and all currents are phasors)
1. 	1.) 
2. 	2.) 
3.  $\boxed{v = iR}$	3.)  $\bar{V} = \bar{I}\bar{Z}$
4. v	4.) $\bar{V}$
5. i	5.) $\bar{I}$

* complex voltages & currents satisfy KCL, KVL.
* Superposition theorem, Thevenin thm & Norton thm are valid for AC circuits.

* <u>Thevenin equivalent circuit:-</u>



* <u>Norton equivalent circuit:-</u>



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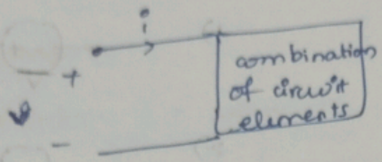
Let (v) be the phasor voltage across a combination of circuit element, and let (i) be the current phasor through this combination of circuit elements.



$v = \sqrt{2} V \sin(\omega t + \phi_1)$

$i = \sqrt{2} I \sin(\omega t + \phi_2)$

$v \longrightarrow \bar{V} = V \angle \phi_1$

$i \longrightarrow \bar{I} = I \angle \phi_2$

* Instantaneous power consumed by combination of circuit elements:-

$\bar{Z} = \dfrac{\bar{V}}{\bar{I}} = \dfrac{V \angle \phi_1}{I \angle \phi_2} = \dfrac{V}{I} \angle \phi_1 - \phi_2$

$$p = vi = \sqrt{2} V \sin(\omega t + \phi_1) \sqrt{2} I \sin(\omega t + \phi_2)$$

$$= VI \cos(\phi_1 - \phi_2) - VI \cos(2\omega t + \phi_1 + \phi_2)$$

Average power, $P = VI \cos(\phi_1 - \phi_2)$

Apparent power, $S = VI$

$$\text{Power factor} = \frac{\text{Average power}}{\text{Apparent power}} = \frac{VI \cos(\phi_1 - \phi_2)}{VI} = \cos(\phi_1 - \phi_2)$$

$$\boxed{0 \le \text{powerfactor} \le 1}$$

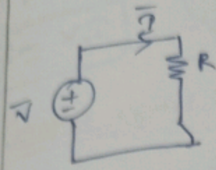
We say "lagging power factor" if $\bar{I}$ lags $\bar{V}$.

We say "leading power factor" if $\bar{I}$ leads $\bar{V}$.

" like Powerfactor is 0.9 lag".

" Power factor is 0.8 lead".

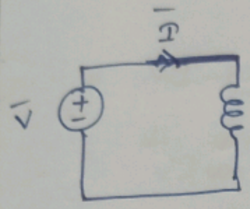
(11) If $\bar{I}$ is in phase with $\bar{V}$ we have "unity power factor".



$\bar{V} = V\angle\phi$. Then $\bar{I} = \frac{\bar{V}}{R} = \frac{V\angle\phi}{R} = I\angle\phi$

where $I = \frac{V}{R}$.

$$\boxed{P = VI = S}$$



$V = V\angle\phi$

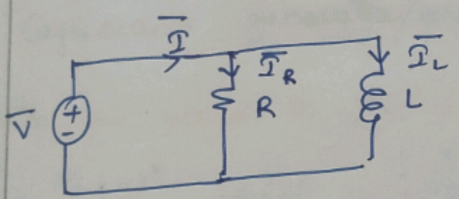
Then $\bar{I} = \frac{\bar{V}}{j\omega L} = \frac{V\angle\phi - 90^\circ}{\omega L} = \bar{I}\angle\phi - 90^\circ$

Angular frequency $= \omega$

So $P = VI\cos(90^\circ)$

$$\boxed{P = 0}$$

$$\boxed{S = VI}$$



Angular frequency $= \omega$

$\bar{V} = V\angle\phi$

Let $\bar{I}_R = \frac{\bar{V}}{R} = \frac{V}{R}\angle\phi = I_R\angle\phi$

$\bar{I}_L = \frac{\bar{V}}{j\omega L} = \frac{V}{\omega L}\angle\phi - 90^\circ = I_L\angle\phi - 90^\circ$

$\bar{I} = \bar{I}_R + \bar{I}_L$

Let $\bar{I} = I\angle\phi'$

So,

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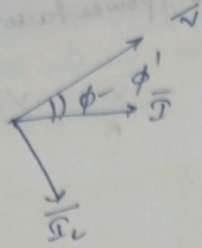
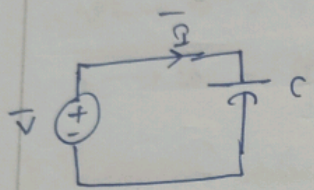


Average power consumed by combination of 'R' & L

$$P = VI\cos(\phi - \phi')$$

$$\boxed{P = VI_R}$$



$\bar{V} = V\angle\phi$

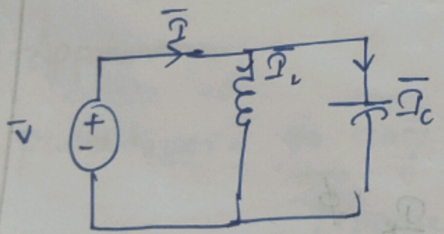
$\bar{I} = V\omega C \angle \phi + 90^\circ$

Angular frequency $= \omega$

$\bar{I} = I \angle \phi + 90^\circ$

$$\boxed{P = VI\cos(-90^\circ) = 0}$$

$$\boxed{S = VI}$$



Angular frequency $= \omega$

$\bar{V} = V\angle\phi$

$\bar{I}_L = I_L \angle \phi - 90^\circ \quad \left(I_L = \dfrac{V}{\omega L}\right)$

$\bar{I}_C = I_C \angle \phi + 90^\circ \quad (I_C = V\omega C)$

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$\boxed{P = 0}$

$\boxed{S = VI}$

Complex p

$\bar{S} =$

=

=

Q'

⇒ Ind

⇒ Ca

Fin

1.

2.

$\boxed{P = 0}$ (L and C have opposite effects on S)
$\boxed{S = VI}$

* Complex power ~~generated~~ consumed by the combination of circuit elements:

$\bar{S} = \bar{V}\,\bar{I}^{*}$ (R, L, C → passive circuit elements)

$= V\angle\phi_1 \cdot I\angle -\phi_2$

$= VI\angle \phi_1 - \phi_2$

$= VI\cos(\phi_1 - \phi_2) + jVI\sin(\phi_1 - \phi_2)$

$= P + jQ$

Q' is called reactive power.

⇒ Inductance consumes/draws reactive power.

⇒ Capacitance generates/supplies reactive power.

* Find the values in the polar form and rectangular form:-

1. $\dfrac{10\angle 45^\circ \quad 20\angle 30^\circ}{35\angle 75^\circ} = \dfrac{\cancel{200}\ 40}{\cancel{35}\ 7}\angle 0^\circ$

$= 5.714\angle 0^\circ$

and in rectangular form 5.714.

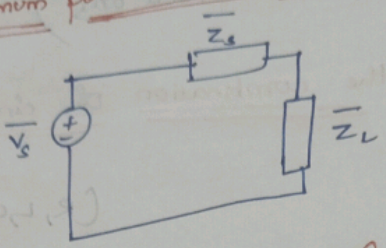
2. $(0.5\angle 30^\circ - 0.8\angle 53.13^\circ)\ 0.707\angle -45^\circ$

$\Rightarrow (0.5)(0.707)\angle -15^\circ \quad - 0.8(0.707)\angle 8.13^\circ$

$\Rightarrow$ ~~0.3535 ∠-15° − 0.5656 ∠8.13°~~

$\Rightarrow$ ~~0.34 + j(-1)(0.0914) − 0.56~~

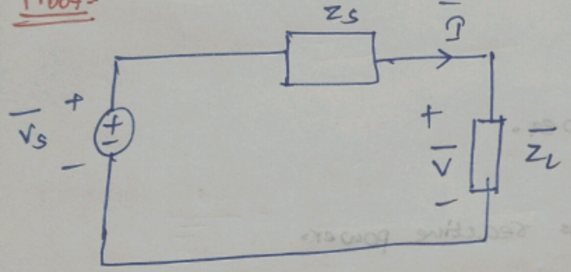
(80) ## Maximum power transfer theorem:-



$\overline{V_s}$ and $\overline{Z_s}$ are fixed. (with $Re(\overline{Z_s}) \neq 0$)

→ The value of $\overline{Z_L}$ which maximises the average power transferred to it is $\overline{Z_s}^*$ (conjugate of $\overline{Z_s}$)

Proof:-



let $\overline{V_s} = V_s \angle 0$ ← without loss of generality

let $\overline{Z_s} = R_s + jX_s$, $\overline{Z_L} = R_L + jX_L$

let $\overline{I} = I \angle \phi$

Average power transferred to $\overline{Z_L}$

$$P = Re(\overline{V}\,\overline{I}^*) = Re(\overline{I}\,\overline{Z_L}\,\overline{I}^*)$$

$$= Re[I \angle\phi (R_L + jX_L) I \angle -\phi]$$

$$= Re[I^2 (R_L + jX_L)] = I^2 R_L$$

$$\overline{I} = \frac{\overline{V_s}}{\overline{Z_s} + \overline{Z_L}} = \frac{V_s}{R_s + jX_s + R_L + jX_L}$$

(8) $I = \dfrac{V_s}{\sqrt{(R_s+R_L)^2 + (X_s+X_L)^2}}$

$$P = \frac{V_s^2 \, R_L}{(R_s+R_L)^2 + (X_s+X_L)^2}$$

maximise $\dfrac{V_s^2 R_L}{(R_s+R_L)^2+(X_s+X_L)^2}$

Here $X_L = -X_s$ as X_L independent on R_L

$\Updownarrow$

minimise $\boxed{\dfrac{(R_s+R_L)^2+(X_s+X_L)^2}{R_L}}$

and hence $\dfrac{(R_s+R_L)^2}{R_L}$ min

$\Updownarrow$

~~minimise~~ $\cancel{\dfrac{R_s^2}{R_L}} + R_L + \cancel{2R_s} + \cancel{\dfrac{X_s^2}{R_L}} + \cancel{\dfrac{X_L^2}{R_L}} + \cancel{\dfrac{2X_s X_L}{R_L}}$

$\Updownarrow$

minimise $\cancel{\dfrac{R_s^2}{R_L}} + \cancel{R_s} + \cancel{\dfrac{X_s^2}{R_L}} + \cancel{\dfrac{X_L^2}{R_L}} + \cancel{\dfrac{2X_s X_L}{R_L}}$

and $\dfrac{R_s^2}{R_L} + R_L + 2R_s$ min

$\Downarrow$

$$\frac{R_s^2}{R_L} + R_L \geq 2\sqrt{R_s^2}$$

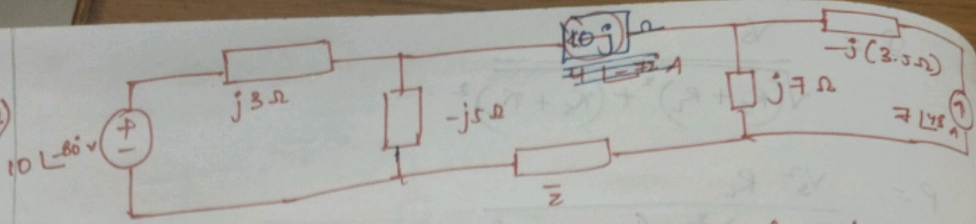
$$\frac{R_s^2}{R_L} + R_L \geq 2R_s \quad (\text{min at } R_s^2 = R_L^2 \Rightarrow R_s = R_L)$$

and hence for whole expression to be min,

$$\left.\begin{aligned} R_s &= R_L \\ X_s &= -X_L \end{aligned}\right\} \Rightarrow \boxed{\bar{Z}_s^{*} = \bar{Z}_L}$$

Hence proved.

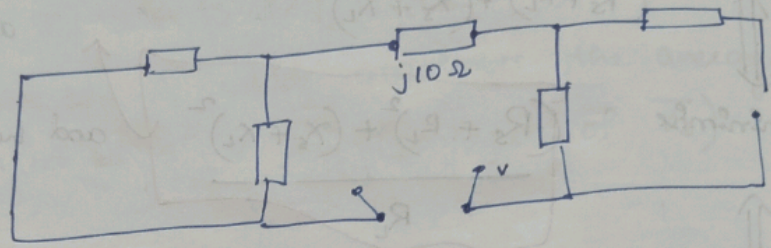
(82) Q)



Find $\bar{Z}$ that maximise average power transferred to it.

Hint: Do you need calculator?

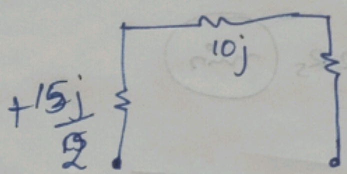
Sol.



$$\frac{-1}{5j} + \frac{1}{3j}$$

$$\frac{1}{Z_1} = \frac{+j}{5} - \frac{j}{3} \Rightarrow -\frac{2j}{15}$$

$$\frac{+15j}{2}$$



So, $(17+7.5)j \Rightarrow \cancel{17j - \frac{2j}{15}}$

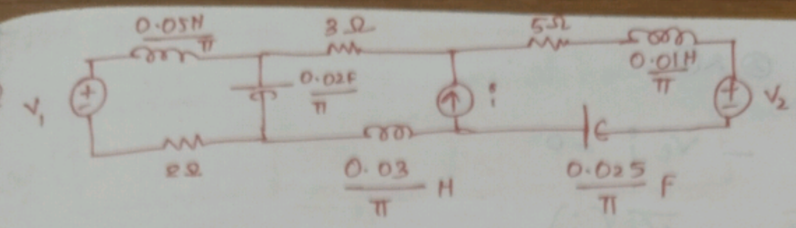
$$\boxed{\bar{Z} = -(24.5)j} \Rightarrow \cancel{17 - 0.13j} \quad \cancel{\boxed{16.87j}}$$

$\cancel{\frac{15}{8}}$ $\cancel{15)20(0.13}$

But it is not correct!!!

(We will come to this later)

Q3



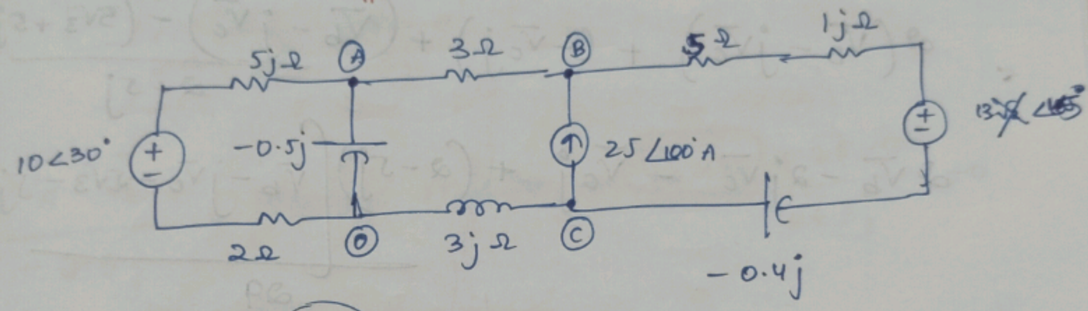
$v_1 = 10\sqrt{2}\sin(100\pi t + 30^\circ)$ V

$v_2 = 13\sqrt{2}\cos(100\pi t + 15^\circ)$ V

$i = -25\sqrt{2}\sin(100\pi t - 80^\circ)$ A

find i_{RMS} through $\frac{0.02F}{\pi}$

Sol:



$$\frac{-j}{\frac{0.02}{\pi}(100\pi)} = \frac{-j}{2}$$

Nodal analysis (at node 'a')

$$\frac{\overline{V_a}}{-0.5j} + \frac{\overline{V_a} - \overline{V_b}}{3} + \frac{\overline{V_a} - 10\angle 30^\circ}{2+5j} = 0 \longrightarrow (1)$$

Nodal analysis (at node 'B')

$$\frac{\overline{V_b} - \overline{V_a}}{3} - 25\angle 100^\circ + \frac{\overline{V_b} - \overline{V_c} - 13\sqrt{2}\angle 15^\circ}{5 + 0.6j} = 0 \longrightarrow (2)$$

Nodal analysis (at node 'c')

$$\frac{\overline{V_c}}{3j} + 25\angle 100^\circ + \frac{\overline{V_c} - \overline{V_b} + 13\sqrt{2}\angle 15^\circ}{5 + 0.6j} = 0 \longrightarrow (3)$$

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Now add ② & ③

$$\bar{V}_b - \bar{V}_a - \bar{V}_c j = 0$$

$$\bar{V}_b - \bar{V}_a = \bar{V}_c (j)$$

$$+2\bar{V}_a j + \bar{V}_a - \bar{V}_b + \frac{\bar{V}_a - 10\angle 30^\circ}{2+5j} = 0 \quad (\text{from eq}^n ①)$$

$$2(\bar{V}_b - j\bar{V}_c) + (-\bar{V}_c j) + \frac{(\bar{V}_b - j\bar{V}_c) - (5\sqrt{3} + 5j)}{2+5j} = 0$$

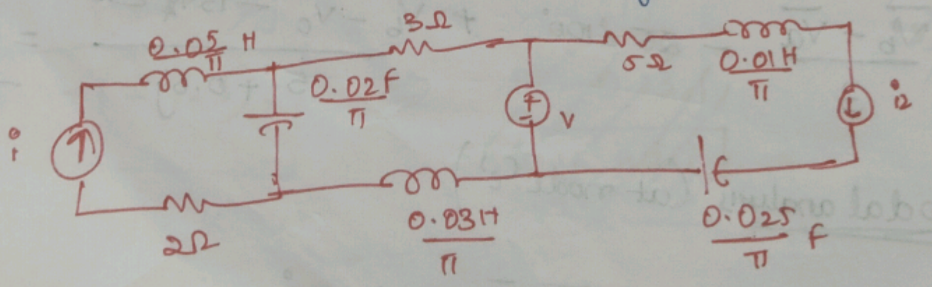
$$2\bar{V}_b - 2j\bar{V}_c - \bar{V}_c j + \frac{(2-5j)\left[\bar{V}_b - j\bar{V}_c - 5\sqrt{3} - 5j\right]}{29} = 0$$

$$\cancel{58\bar{V}_b} - \cancel{58j\bar{V}_c} - \cancel{29\bar{V}_c j} + \cancel{2\bar{V}_b} - \cancel{2j\bar{V}_c} - 10\sqrt{3} - 10j - \cancel{5\bar{V}_b j} - \cancel{5\bar{V}_c} + 25j\sqrt{3} - 25 = 0$$

$$(60 - 5j)\bar{V}_b + (-5 - 89j)\bar{V}_c = (25 + 10\sqrt{3}) + (25\sqrt{3} - 10)j$$

Now find $\bar{V}_a$, $\bar{V}_b$, $\bar{V}_c$ using calculator.

Q)



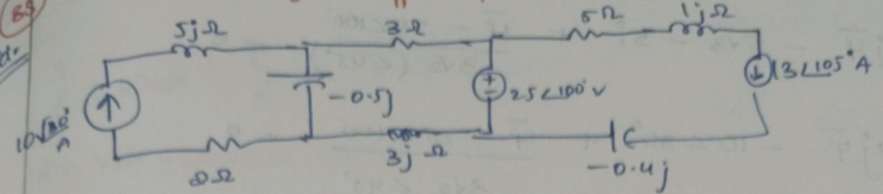
$i_1 = 10\sqrt{2}\sin(100\pi t + 30^\circ)$ A

$i_2 = 13\sqrt{2}\cos(100\pi t + 15^\circ)$ A

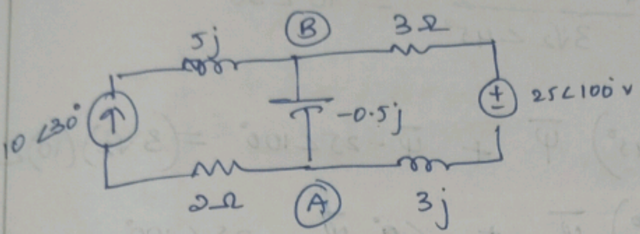
$V = -25\sqrt{2}\sin(100\pi t - 80^\circ)$ V

(85) find i_{RMS} through $\frac{0.02F}{\pi}$ capacitor

Sol.



It can be changed to



$$\frac{\bar{V}_A - \bar{V}_B}{-0.5j} + \cancel{\frac{V_A - V_B}{}}\ 10\angle 30^\circ + \frac{\boxed{\bar{V}_A - \bar{V}_B} + 25\angle 100^\circ}{3+3j} = 0$$

$$2\left(\bar{V}_A - \bar{V}_B\right)j + 10\angle 30^\circ + \frac{\bar{V}_A - \bar{V}_B + 25\angle 100^\circ}{3+3j} = 0$$

and

$$\frac{\bar{V}_B - \bar{V}_A}{-0.5j} + \cancel{\bar{V}_B - \bar{V}}\ \left(-10\angle 30^\circ\right) + \frac{\bar{V}_B - \bar{V}_A - 25\angle 100^\circ}{3+3j} = 0$$

$$+2j\left(\bar{V}_B - \bar{V}_A\right) + \left(-10\angle 30^\circ\right) + \frac{\bar{V}_B - \bar{V}_A - 25\angle 100^\circ}{3+3j} = 0$$

Take $\bar{V}_B - \bar{V}_A = \bar{\varphi}$

$$2j\,\bar{\varphi} - 10\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) + \frac{\bar{\varphi} - 25\angle 100^\circ}{3+3j} = 0$$

find $\bar{\varphi}$ and $\frac{|\bar{\varphi}|}{+0.5} = i_{RMS}$ through asked capacitor

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$$2j\bar{\varphi} - 5\sqrt{3} - 5i + \frac{\bar{\varphi} - 25\angle 100^\circ}{3\sqrt{2}\,(\angle 45^\circ)} = 0$$

$$-2j\bar{\varphi} - 10\angle 30^\circ + \frac{\bar{\varphi} - 25\angle 100^\circ}{3\sqrt{2}\angle 45^\circ} = 0$$

$$(2\angle -90^\circ)\bar{\varphi} + \frac{\bar{\varphi} - 25\angle 100^\circ}{3\sqrt{2}\angle 45^\circ} = 10\angle 30^\circ$$

$$(2\angle 90^\circ)(3\sqrt{2}\angle 45^\circ)\,\bar{\varphi} + \bar{\varphi} - 25\angle 100^\circ = (3\sqrt{2})(10)\angle 75^\circ$$

$$(6\sqrt{2}\angle -45^\circ)\,\bar{\varphi} + \angle 0^\circ\,\bar{\varphi} - 25\angle 100^\circ = 30\sqrt{2}\angle 75^\circ$$

$$\bar{\varphi} = \left[\frac{30\sqrt{2}\angle 75^\circ + 25\angle 100^\circ}{1 + 6\sqrt{2}\angle -45^\circ}\right]$$

$$\bar{\varphi} = -4.083 + 5.871\,i$$

$$|\bar{\varphi}| = 7.151$$

and hence $I_{rms} = (7.151)(2)$

$$\boxed{I_{rms} = 14.302\ \text{Amp}}$$

* NOTE:-

$$\bar{S} = P + jQ$$

(P) : Average power

(or) Active power

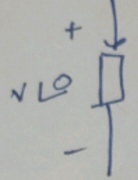
(or) Real power

Q: Reactive power

S
P : watt } x ($\overline{S}$ xs) : VA

Q: Volt - ampere reactive (VAr)

$$\text{power factor} = \frac{\text{Average power}}{\text{Apparent power}}$$



$\overline{Z} = R + jX = Z\angle\phi$

$$= \frac{VI\cos\phi}{VI} = \cos\phi$$

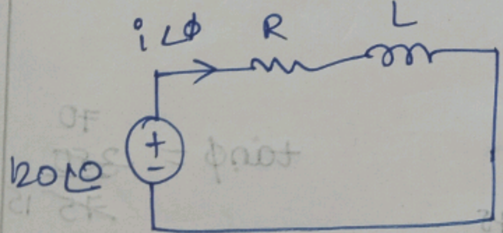
$$= \frac{R}{\sqrt{R^2+X^2}}$$

Q) A series RL circuit consumes 384 W active power at 0.8 powerfactor when connected to 120 V 60 Hz voltage source. find R, L.

V_{RMS}

Sol:

$P = 384W$



and

$P = I^2 R$

$384 = 16 R$

$\boxed{R = 24\Omega}$

$P = 120 I \cos\phi$

$384 = 120 I (0.8) \Rightarrow \boxed{I = 4 \text{ Amp}}$

and $0.8 = \frac{R}{\sqrt{R^2+X^2}}$

we get 'X'

∴ we get 'L'

(88) and $pf = \frac{R}{\sqrt{R^2 + x^2}} = 0.8$ Ω.

Q) The current through the series combination of two passive circuit elements is $(8 - 10j)$ A and the voltage across the series combination is $(50 + 25j)$ V. What are the circuit elements? Find the pf of the series combination. Find active & reactive power consumed by series combination.

$\boxed{R, L, C}$ ★★

Sol:

$$\frac{50 + 25j}{8 - 10j} = \bar{z}$$

$$\frac{25(2 + j)}{2(4 - 5j)} \frac{(4 + 5j)}{(4 + 5j)} = \frac{25(8 + 4j + 10j - 5)}{2(16 + 25)}$$

$$= \frac{25}{2(41)}(3 + 14j)$$

$$= \frac{25}{82}(3 + 14j)$$

$$= 4.38 \angle 77.905°$$

and $R = \frac{75}{82}$ Ω

$\omega L j = \frac{175}{41} j$

and

$\frac{175}{41}$

$\frac{75}{82}$

$$\tan\phi = \frac{\cancel{350}^{70}}{\cancel{75}_{15}}$$

$$= \frac{14}{3}$$

(89) and pf

and $\bar{z}$

Q)

$v_1 =$

Find

Sol.

an

two passive
across the
the circuit
. Find
mbination.

(89) and $pf = \cos(77.905^\circ) \approx 0.21$ (lag)

** (imp to be written)

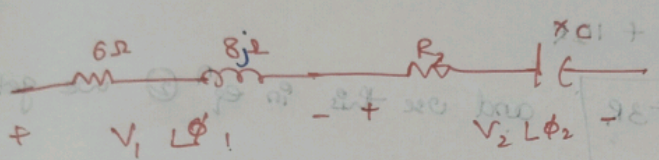
and $\bar{S} = (P + jQ) = \bar{V}\,\bar{I}^* = (50 + 25j)(8 + 10j)$

$= 150 + 700j$

$P = 150W$

$Q = 700$ VAr.

Q)



$V_1 = 3V_2$ & $V_1 \angle \phi_1 + V_2 \angle \phi_2 = 240 \angle \phi$ V $\qquad 8 = \omega L$

$|\phi_1 - \phi_2| = 90^\circ$

Find R, X_C.

Sol.

$V_1 = I\sqrt{6^2 + 8^2}$

$V_2 = I\sqrt{R^2 + X_c^2}$

$10 = 3\sqrt{R^2 + X_c^2}$

$\sqrt{V_2^2 + 9V_2^2} = 240$

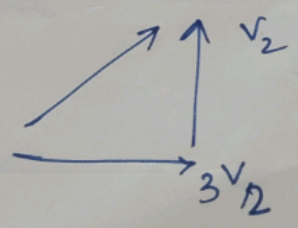
$\sqrt{10}\, V_2 = 240$

$\boxed{V_2 = 24\sqrt{10}}$

$V_1 = \cancel{8\sqrt{10}}$ $72\sqrt{10}$

$\boxed{I = \frac{72}{\sqrt{10}}}$

and



$\cancel{240} = (I)\left(\sqrt{(6+R)^2 + (8j + X_c)^2}\right)$

and $\frac{\cancel{240}^{10}\sqrt{10}}{\cancel{72}_{3}} \Rightarrow \sqrt{(6+R)^2 + (8j + X_c)^2}$

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$\frac{100(10)}{9} = (6+R)^2 + (8+x_c)^2 \longrightarrow (1)$

$\Rightarrow \frac{100}{9} = (R)^2 + x_c^2 \longrightarrow (2)$

$\Rightarrow \cancel{\frac{100}{9}(9)} = 64 + 16x_c + 12R$

$\cancel{\frac{36}{16} = x_c = \frac{9}{4} = 2.25}$

$\Rightarrow 36 = 16x_c + 12R$

$\Rightarrow \boxed{9 = 4x_c + 3R}$ and use this in eq^n (2) we get

x_c & R

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