Quick Recap: We have seen our first nontrivial proof:

Theorem: Let G be a graph. If each vertex (of G) has degree at least two then G has a cycle.

$\downarrow$ means

G contains a cycle (graph) as a subgraph

$\updownarrow$ same as

G is NOT a forest.

We wrote a proof with lots of details & explanations. Here is the same proof with very few details/explanations:

Proof: (NOT acceptable for CS1200)
Consider a longest path P, and let v denote an end of P.
Observe that there IS an edge $f \notin E(P)$ incident at vertex v.
Now we have a cycle.

why can you consider this?
why?
where?

NO explanations provided

NOT acceptable in CS1200

Rule of Thumb (for CS1200):
When in doubt (to explain or NOT to explain), explain!

Same theorem stated differently:

Theorem: Every forest has a vertex of degree at most one.

$\downarrow$ means zero or one, right?

DIY: A vertex of degree one (in a tree/forest) is called a LEAF.

Theorem: Every forest with at least one edge has a leaf.

Corollary: Every tree except K_1 has a leaf.

Corollary: Every tree except K_1 has a leaf.

meaning: a direct or natural consequence or result.

DIY:

① prove this corollary directly using same/similar ideas

② is it possible to prove something stronger?

We will find this fact extremely useful in a few weeks.

Next Goal: Generalize "everything" to digraphs.

Directed walk: defined on Assignment-1

Directed trail: a directed walk but NO repetition of arcs allowed

Directed path: a directed walk but NO repetition of vertices allowed

(aka dipath)

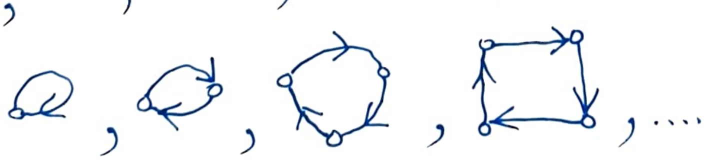
(So, no repetition of arcs allowed, right?)

Directed cycle: same as directed path EXCEPT first vertex = last vertex

(aka dicycle)

same as

Directed paths: 

Directed cycles: 

DIY: write down formal definitions of ALL.

<u>Subdigraph</u>: A labeled digraph F is a <u>subdigraph</u> of a labeled digraph D if F can be obtained from D by deleting vertices and/or arcs.

<u>DIY</u>: Define isomorphism for digraphs.

<u>Subdigraph</u> (unlabeled version): A digraph F is a <u>subdigraph</u> of a digraph D if some digraph isomorphic to F can be obtained from D by deleting vertices and/or arcs.

<u>Recall</u>:

A graph G is a **forest** if it is acyclic.

means

[does NOT contain any cycle (graph) as a subgraph]

So, forest are also called **acyclic graphs**.

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let us do the same for digraphs

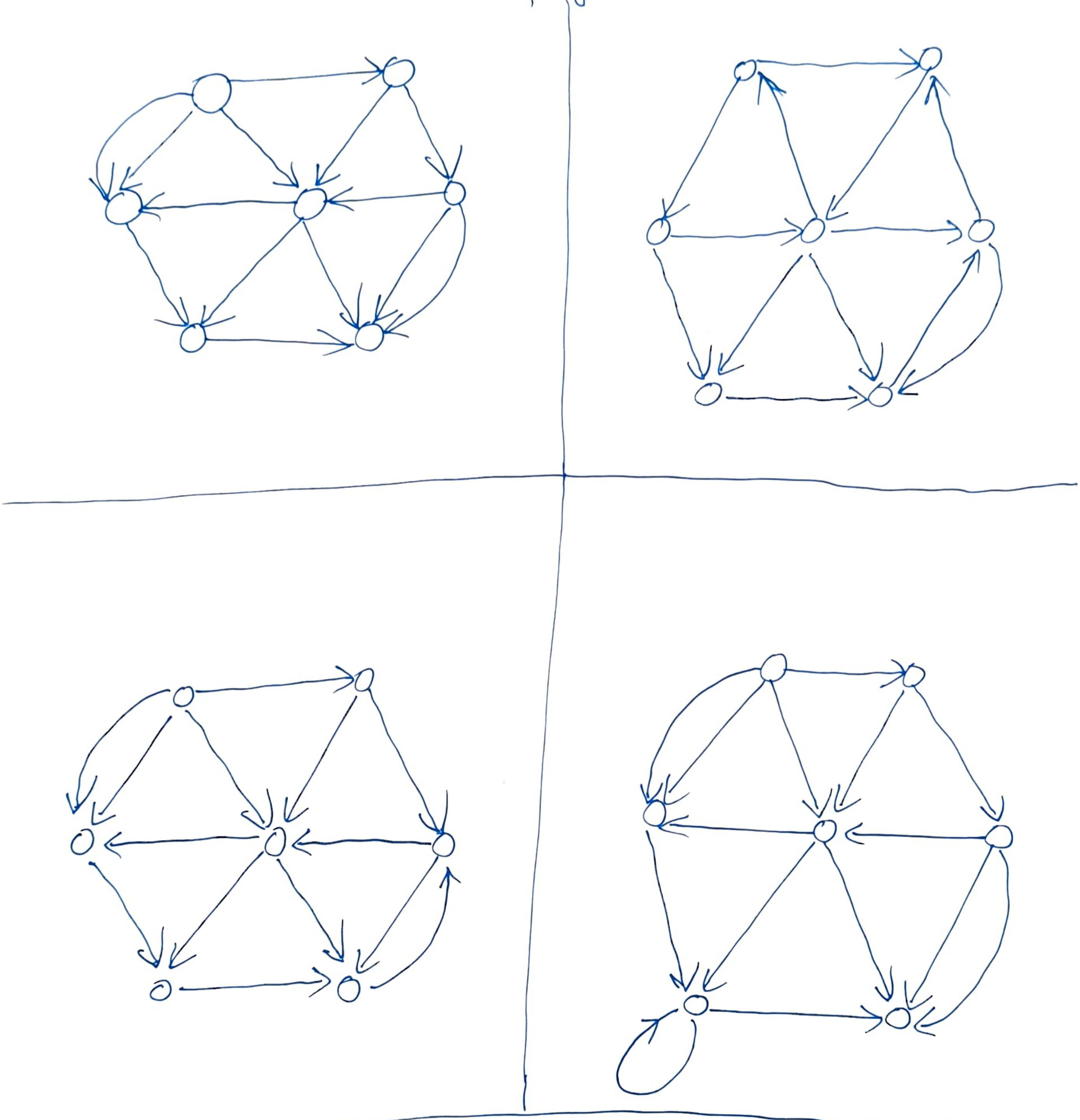
<u>DAGs (Directed Acyclic Graphs)</u>:

A directed graph D is a DAG if it is acyclic.

means

[does NOT contain any dicycle as a subdigraph]

DIY: Which of the following is a DAG? (if any)



We also discussed adjacency & incidence matrices for digraphs but these appear earlier in the notes: see adjacency & incidence matrices for undirected graphs.