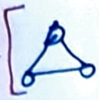
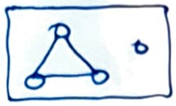
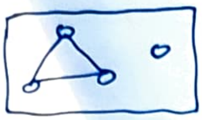
1. If [each vertex of  has degree 2 or more] (T)
then [ has a cycle.] (T) TRUE

2. If [each vertex of  has degree 2 or more] (F)
then [ has a cycle.] (T) DOES NOT MATTER TRUE

The statement

"if each vertex of G has degree 2 or more
then G has a cycle"

is $\boxed{\text{NOT}}$ a proposition — because we do NOT know what G is.

Likewise,
"x equals 2" is $\boxed{\text{NOT}}$ a proposition — because we do NOT know what x is.

They "become" propositions
when we specify what $\underbrace{G/x}_{G \text{ OR } x}$ are.

---

for example, $7-5=2$ (T) & $7-4=2$ (F)
are propositions

Let us go back to the theorem:

Theorem: Let G be a graph. → THIS IS IMPORTANT

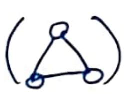
If degree of each vertex is at least two
then G has a cycle. → By itself, NOT a proposition.

We have already proved this theorem.
Let us make sure that we all understand
its meaning (AND the SAME meaning).
(for all of us)

$\underbrace{P(G)}_{\text{propositional function}}$:= if each vertex of G has degree 2 or more then G has a cycle

→ NOT a proposition
"becomes" a proposition when we "plug in" some graph in G

↓

for example: P(), P(), P()
are propositions.

Again:

What is the meaning of the theorem?

Theorem: Let G be a graph. If degree of each vertex is two or more then G has a cycle.

means → for each (every) graph G, P(G) (is TRUE).

definition of P(G)

P(G) := if each vtx (of G) has degree 2 or more than G has a cycle.

another way to write this

$\forall G \in \mathcal{G}$

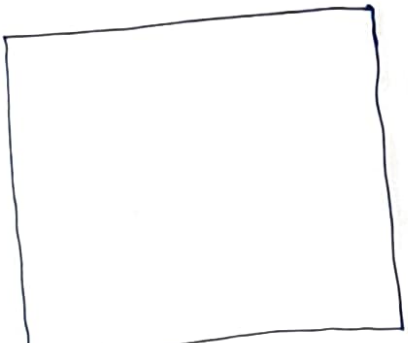
(read: for each/every/all)

universal quantifier

(same) Theorem: $\forall G \in \mathcal{G} : P(G)$ (is TRUE).

$\mathcal{G}$: set of all graphs

U



The [quantified proposition]

$\boxed{\forall x \in U : P(x)}$

is TRUE IF

the proposition P(x) is TRUE for each x in the universe U; othewise it is FALSE.

(Same) Theorem : $\forall\, G \in \mathcal{G}$:

$\underbrace{d(v) \geq 2 \quad \forall\, v \in V(G)}$ $\Longrightarrow$ G has a cycle.

read: degree of each vtx (of G) is at least two

read: implies (that)

it appears here

Yes, we will discuss one more.

We have discussed the universal quantifier ($\forall$) .... are there any other quantifiers?

G has a cycle — what does this mean?

family of all cycle graphs

There exists a cycle graph $C \in \mathcal{C}$ such that C is a subgraph of G.

(Same) Theorem : $\forall\, G \in \mathcal{G}$:

$d(v) \geq 2 \;\forall\, v \in V(G) \Longrightarrow \exists\; C \in \mathcal{C}$ such that C is a subgraph of G.

U

The quantified proposition $\boxed{\exists\, x \in U : P(x)}$ is TRUE IF the proposition P(x) is TRUE for at least one element x in universe U; otherwise it is FALSE.