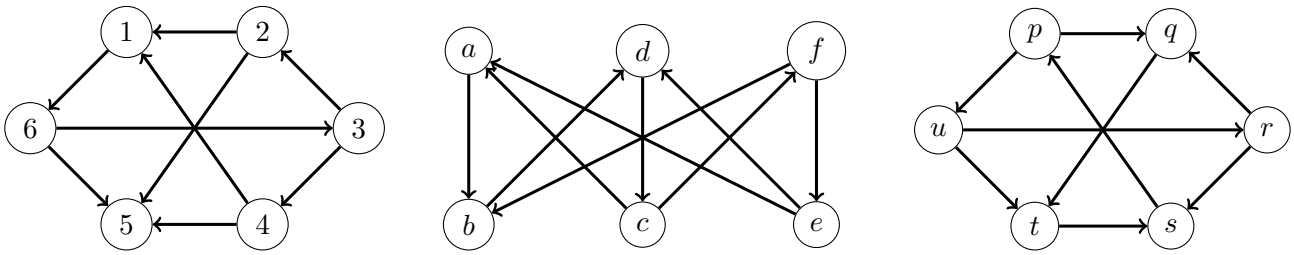
IIT M-CS1200 : Discrete Math (Mar - Jul 2023) Release Date: 27-04-2023

Tutorial No: 3 Tutorial Date: 02-05-2023

1. Let $f : A \to B$ be a function defined as
$$f(x) = \left(\frac{2x+3}{x-3}\right)$$
where, $A = \mathbb{Z} - \{3\}$ and $B = \mathbb{Z} - \{2\}$.
   (a) Is f an injection (aka 1-to-1 function) ?
   (b) Is f a surjection (aka onto function) ?
   (c) Does f have an inverse? If yes, find the inverse of f. If not, justify.

2. Let R_1 and R_2 be two relations on the set $S = \{1, 2, 3, 4, 5\}$ as follows :
$R_1 := \{(a, b) \mid a < b \text{ and } a, b \in S\}$
$R_2 := \{(a, b) \mid a > b \text{ and } a, b \in S\}$
Let G_1 and G_2 be the graphs corresponding to these two relations R_1 and R_2 respectively.
   (a) Draw the graphs G_1 and G_2.
   (b) Are G_1 and G_2 isomorphic to each other ? If yes, define an isomorphism between them. If no, justify.

3. Consider the following digraphs D_1, D_2 and D_3 as shown below.



D_1 D_2 D_3

   (a) Recall that we have seen in class the notion of isomorphism for simple undirected graphs. Define the notion of isomorphism in loopless digraphs.
   (b) Which of the above given digraphs are isomorphic ?
   (c) Define an isomorphism for the isomorphic digraphs among D_1, D_2 and D_3.

4. Let $G := (V, E)$ be a graph and $u, v \in V(G)$. Prove that if there is a walk from u to v in G, then there is a path from u to v in G.

5. Given a non-negative integer n, prove that there is a unique non-negative integer m such that $m^2 \leq n < (m+1)^2$.

6. (a) Prove that *if x and y are integers of opposite parity, then $5x + 5y$ is an odd integer.*
   (b) Use quantifiers and implication (*if-then*) to rewrite the proposition in 6(a). (You may define other propositions as required).
   (c) Write the converse of the above proposition. Is the converse true?

7. (a) Prove that in a digraph $D_1 := (V_1, A_1)$, if every vertex has out-degree at least 1, then there exists a dicycle.
   (b) Let $D_2 := (V_2, A_2)$ be a directed acyclic graph (DAG). Let P be a longest dipath in D_2 where u, v are starting and ending vertices of P, respectively. What can we say (infer) about vertices u and v in D_2?