Department of Mathematics, IIT Madras

MA1102 Series & Matrices

Assignment-5 (Matrix Eigenvalue Problem)

1. Find the eigenvalues and the associated eigenvectors for the matrices given below.

   (a) $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 0 & 3 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

2. Let A be an $n \times n$ matrix and α be a scalar such that each row (or each column) sums to α. Show that α is an eigenvalue of A.

3. Let $A \in \mathbb{C}^{n\times n}$ be invertible. Show that $\lambda \in \mathbb{C}$ is an eigenvalue of A if and only if λ^{-1} is an eigenvalue of A^{-1}.

4. Show that eigenvectors corresponding to distinct eigenvalues of a unitary (or orthogonal) matrix are orthogonal to each other.

5. Give an example of an $n \times n$ matrix that cannot be diagonalized.

6. Find the matrix $A \in \mathbb{R}^{3\times 3}$ that satisfies the given condition. Diagonalize it if possible.

   (a) $A(a, b, c)^T = (a + b + c,\ a + b - c,\ a - b + c)^T$ for all $a, b, c \in \mathbb{R}$.

   (b) $Ae_1 = 0, \quad Ae_2 = e_1, \quad Ae_3 = e_2$.

   (c) $Ae_1 = e_2, \quad Ae_2 = e_3, \quad Ae_3 = 0$.

   (d) $Ae_1 = e_3, \quad Ae_2 = e_2, \quad Ae_3 = e_1$.

7. Which of the following matrices is/are diagonalizable? If one is diagonalizable, then diagonalize it.

   (a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.