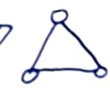
Friends & Strangers at a Party (continued in language of Graph Theory)

Consider a complete graph K_n (where $n \geq 1$), and color the edges pink & blue.

Will there always be either a pink K_3 

or a blue K_3? 

Most popular version.
wikipedia: Theorem on friends & strangers.

Partial answer: Yes — once n is "big enough".

How big? (TIY) Try to formulate a conjecture (& prove it).

Why do we care about K_3? (nontrivial)

It is the smallest case that is interesting.

Ramsey number $R_{p,p}$: smallest integer n such that any blue-pink-K_n contains either pink K_p or blue K_p.

This simple-to-state problem leads to the entire field of Ramsey Theory (a branch of combinatorics) (a subset of discrete math)

$R_{3,3}$ known
$R_{4,4}$ known
$R_{5,5}$ NOT known! (between 43 & 48)

Frank Ramsey: British philosopher, mathematician & economist
contributed to all fields
died at 26 :-(

Clearly, every symmetric relation can be represented/modeled using a finite/infinite graph.

Example: $U = \{1,2,3,4,5,6,7,8,9,10\}$

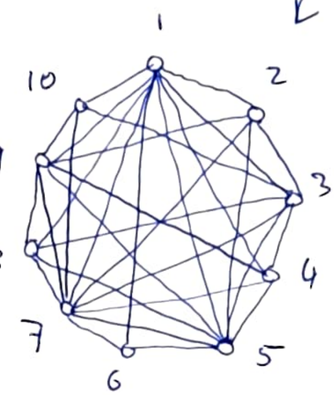
Consider the relation R "are coprime".

Two integers a & b are $\boxed{\text{coprime}}$ if the ONLY positive integer that divides both a & b is 1.

→ We construct a graph G:
$V(G) = U$
Two distinct vertices $a, b \in V(G)$ are adjacent in G if aRb.

Corresponding Graph:

G:



Question: Can every finite/infinite graph be thought of as just a symmetric relation?

YES (& NO – later.)

↓

first we will define relations more generally.

↓

Any relation R (defined on a set U) can be thought of as a subset of $U \times U$.

Set operation:

Cartesian Product

A, B: sets

$\boxed{\text{Cartesian Product of } A \,\&\, B}$, denoted by $A \times B$, is the set of all ordered pairs (a,b) where $a \in A$ & $b \in B$.

→ For example:

The relation "divides" (on $\mathbb{N}$) can be viewed as a subset of $\mathbb{N} \times \mathbb{N}$. Some elements of this relation:

$(3,6), (3,9), (4,12), (6,18), (10,100), \ldots\ldots$

<u>Definition</u>: A $\boxed{\text{relation } R \text{ on a set } U}$ is any subset of $U \times U$.

<u>DIY</u>: Define all the special properties of relations discussed earlier (reflexivity, symmetry, transitivity, antisymmetry) in this language (or, in other words, using this viewpoint).

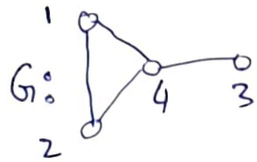
<u>Example</u>: A relation R (on a set U) is $\boxed{\text{symmetric}}$ if whenever $(a,b) \in R$ then $(b,a) \in R$ (for all distinct $a, b \in U$).

<u>Question</u>: Can every finite/infinite graph be thought of as just a symmetric relation?

YES

<u>Example</u>:

G:



$U = \{1,2,3,4\}$

$R = \{(1,2),(2,1),(1,4),(4,1),$
$(2,4),(4,2),(4,3),(3,4)\}$

NO

Graphs are more general than symmetric relations.

<u>Recall</u>: A graph $G := (V, E)$ has:

(i) $V = V(G)$: a set of $\boxed{\text{vertices/nodes}}$

(ii) $E = E(G)$: a set of $\boxed{\text{edges}}$

each edge is an unordered pair of vertices

these are called $\boxed{\text{ends}}$ of the edge

<u>Example</u>: