PH-1020
Problem Set - 1
Department of Physics, IIT Madras
Electrostatic Potential, Work and Energy
March-June 2023 Semester

Notation:

- Notation throughout follows that of Griffiths, Electrodynamics.
- Bold face characters, such as $\boldsymbol{v}$, represent three-vectors.

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1. Consider a square sheet with each side of length L, and carrying a uniform charge density σ.

   (a) Calculate the electric field '$\mathbf{E}$' at a distance 'h' above the centre of the plate.

   (b) Find the limits of the aboove electric field for L $\to \infty$.

   (c) Verify the results of (b) using Gauss's law.

   (d) Check whether the boundary condition given below is satisfied.

$$\boldsymbol{E}_{above} - \boldsymbol{E}_{below} = \frac{\sigma}{\epsilon_o}\hat{n}$$

2. Use Gauss's law to find the field inside and outside a long hollow cylindrical tube, which carries a uniform surface charge density σ. Check that your result is consistent with the equation above.

3. Consider a uniformly charged (and non-conducting) disk of radius R and charge density σ_0. Find the electric potential at a point of distance z from the centre of the disk, and lying along the central axis.

   (a) Calculate the limit $z \to 0$ of the potential, and provide an interpretation of the result.

   (b) Calculate the electric field $\mathbf{E}$ corresponding to the above potential, and its $z \to 0$ limit. (*Be careful about the* $|z|$*!*)

4. Calculate the electric potential of a uniformly charged sphere of radius R and charge density ρ_0. Find the work W necessary to create such a sphere.

5. In the 1930's, to explain the forces amongst the nucleons (protons and neutrons), the following model of the potential was proposed by Hideki Yukawa:

$$V(r) = \frac{A exp[-kmr]}{r}$$

   where A, k are (non-zero) scaling constants, and m is the mass of the pion, a particle that was proposed as a mediator of the nuclear force.

   (a) Find the charge density $\rho(r)$ corresponding to the above potential, *assuming standard equations of electrostatics that we have learnt in the course*

(b) Calculate the total charge Q corresponding to the above charge density. Does your result make intuitive sense? (Making a rough sketch of $\rho(r)$ might be helpful).

(c) For what values of A and m does the above potential reduce to the standard Coulomb potential? (What do you make up for the result for m)?

Note:

As we learnt in the first tutorial, a point charge is described by a charge density $\rho(r)= q\delta^3(r - r_o)$. As the above problem shows, such a charge density will not yield the Yukawa potential in standard electrostatics. In fact, the Yukawa potential arises most naturally in a theory in which the potential satisfies an equation different from the Possion equation.

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6. At the beginning of the 20th century, the mass-energy relation, $U = mc^2$ (m=mass, U= energy), derived by Einstein from his theory of Special Relativity, led many people to speculate that the *entire* mass of the electron might be purely electrical in origin. Imagine the electron as a sphere of radius r_0 with constant volume charge density, and total charge e.

   (a) Using the result of Problem 3 above (or otherwise), calculate the potential energy corresponding to this system.

   (b) Equate the above potential energy to mc^2 and find r_o. Estimate its numerical value.

Note:

You can find a very illuminating account of the history and technical issues associated with the idea of electromagnetic mass in the *The Feynman lectures on Physics*, Vol. 2, Chapter 28.
