ME1480 Engineering Drawing

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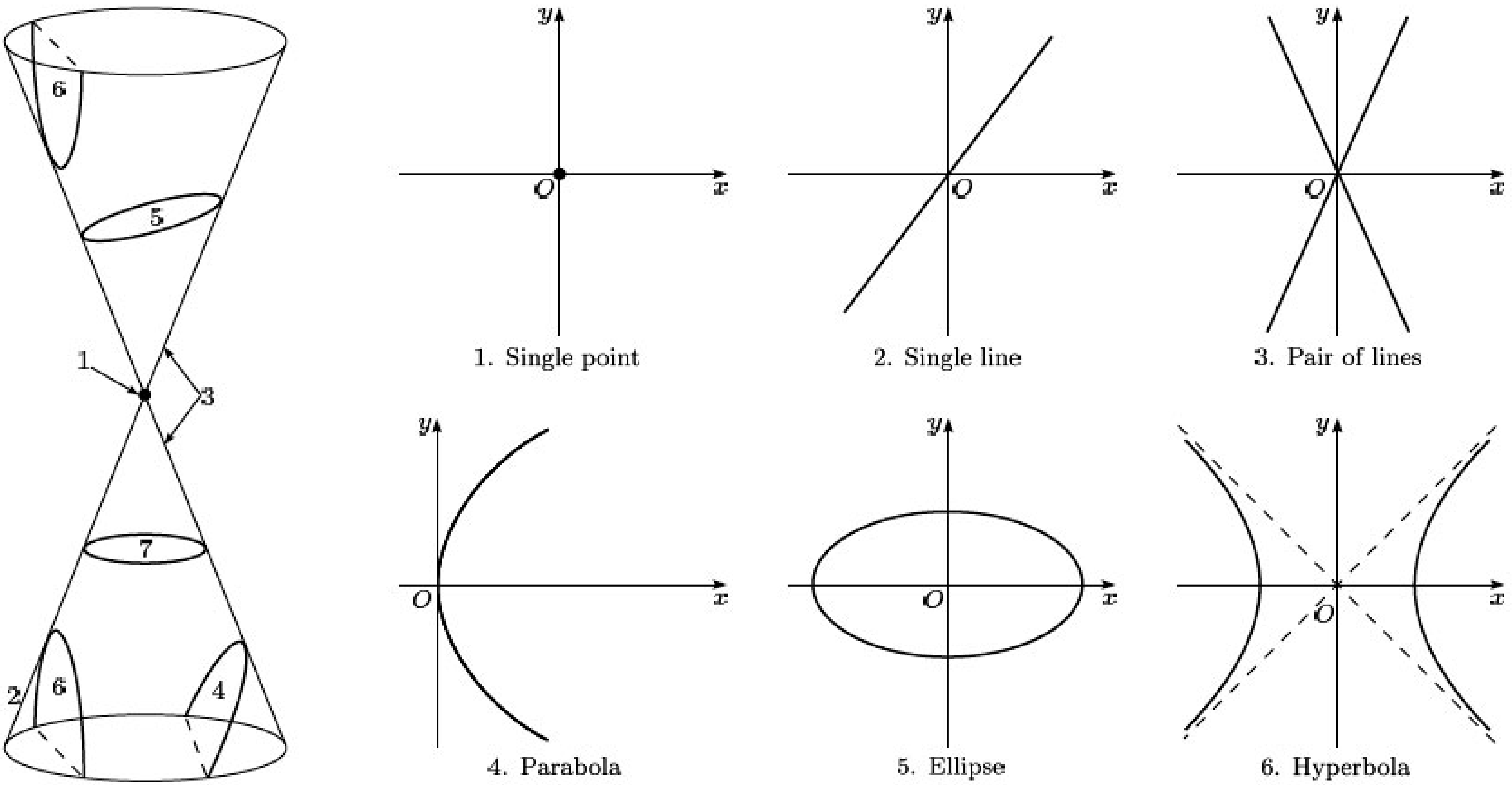
Conic Sections

Conic Sections are the curves that appear when a plane intersects a double right circular cone at different angles.



Common Definition of Conic Sections

Conic section curve is the locus of a point moving in a plane such that the ratio of its distance from a fixed point (focus) and from a fixed line (directrix) remains constant. The ratio is called eccentricity.

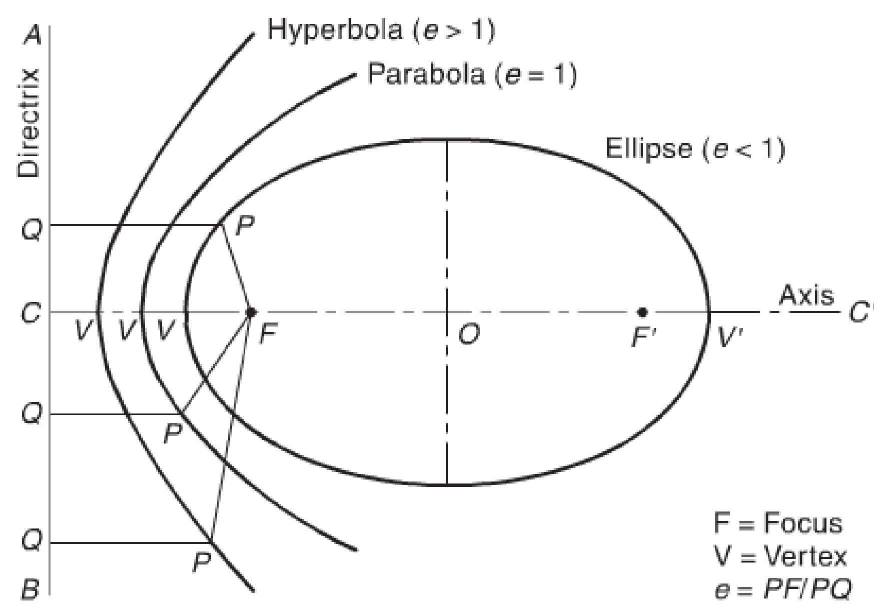
$$Eccentricity(e) = \frac{Distance\ of\ the\ point\ from\ the\ focus}{Distance\ of\ the\ point\ from\ the\ directrix}$$

The portion of a tangent to a conic section curve cut off between the directrix and the curve subtends a right angle at the focus.

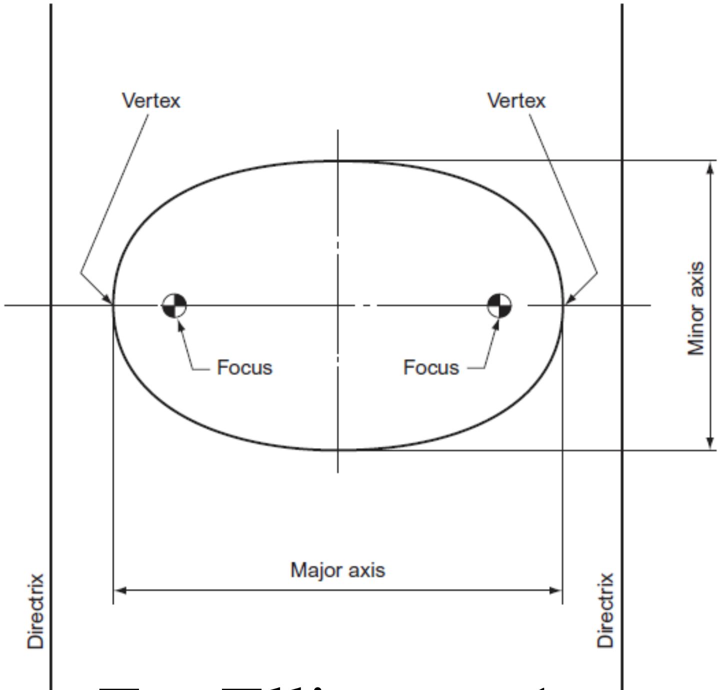
For Ellipse $e<1$

For Parabola $e=1$

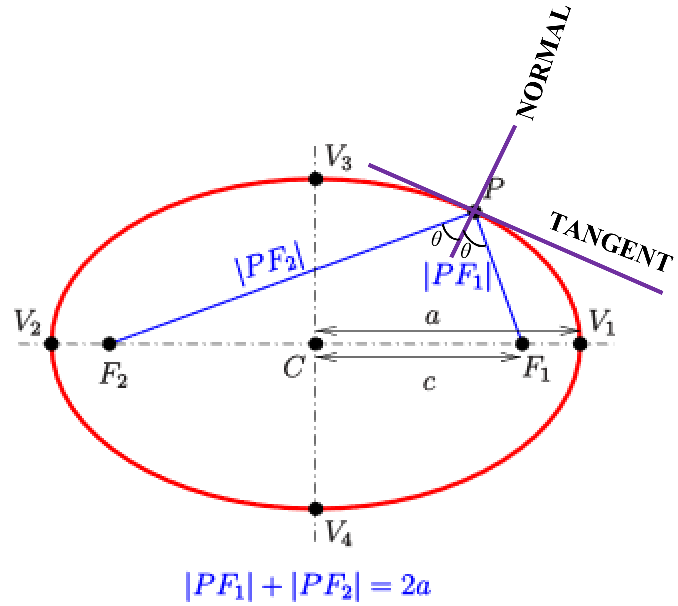
For Hyperbola $e>1$



Ellipse



For Ellipse $e<1$

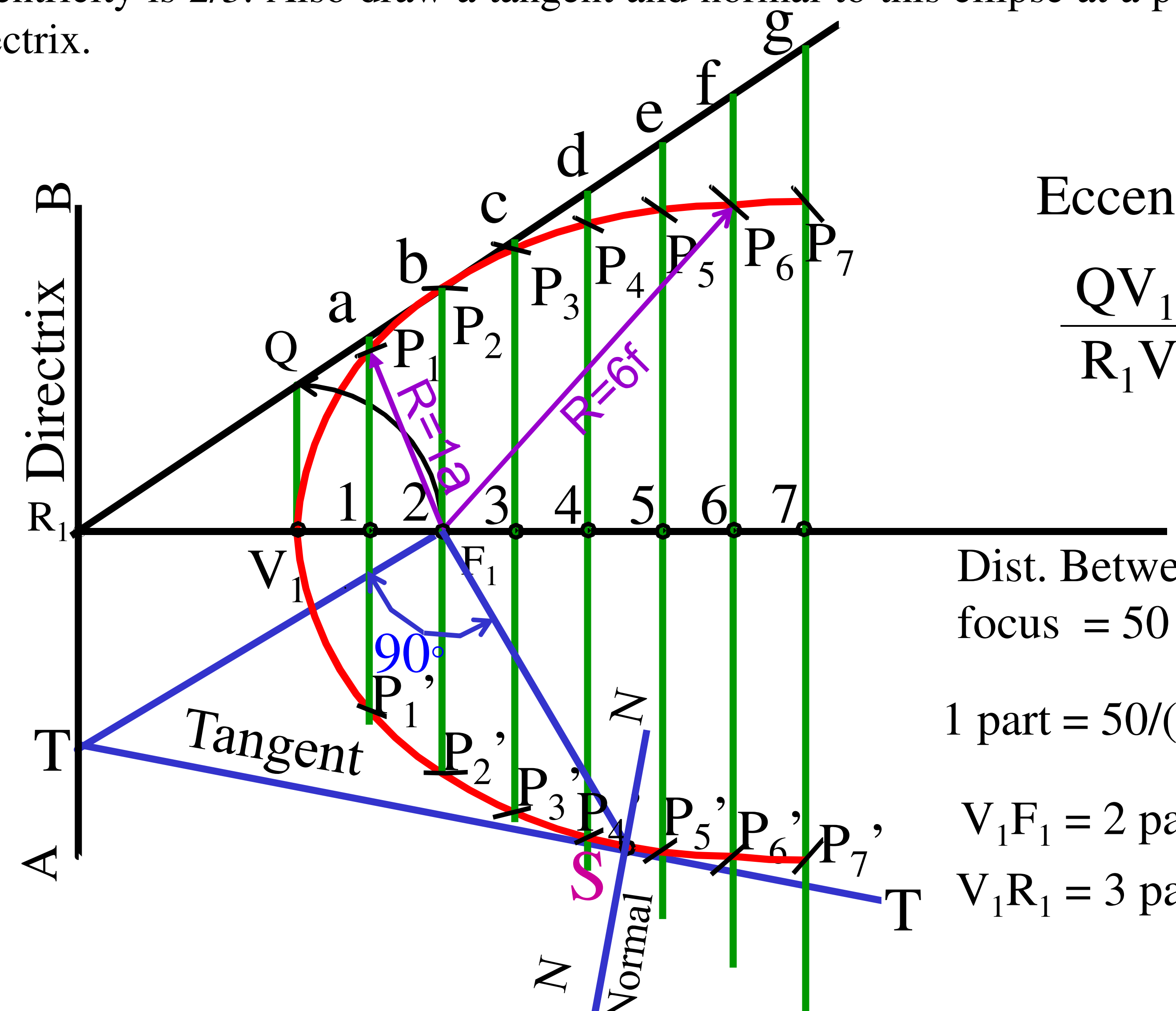


An ellipse is the locus of a point moving in a plane such that the sum of its distance from two fixed points (foci) is constant.

The normal at any point on the ellipse bisects the angle subtended by the foci on that point.

Ellipse (Focus-Directrix method)

Draw an ellipse when the distance of its focus from its directrix is equal to 50 mm and the eccentricity is 2/3. Also draw a tangent and normal to this ellipse at a point 75 mm from the directrix.



Eccentricity = 2/3

$$\frac{QV_1}{R_1V_1} = \frac{V_1F_1}{R_1V_1} = \frac{2}{3}$$

Dist. Between directrix & focus = 50 mm

1 part = 50/(2+3)=10 mm

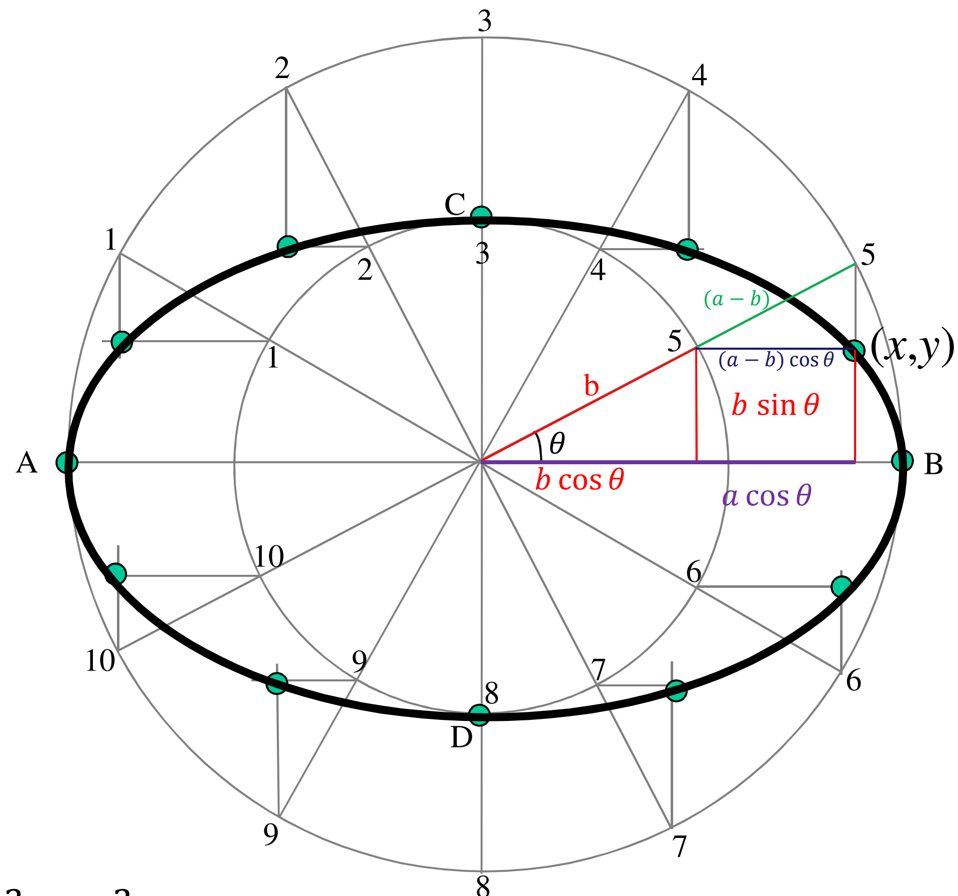
V_1F_1 = 2 parts = 20 mm

V_1R_1 = 3 parts = 30 mm

Ellipse (Concentric Circle Method)

Draw an ellipse by **concentric circle method** Take major axis 120 mm and minor axis 90 mm long.

1. Draw both axes as perpendicular bisectors of each other & name their ends as shown.
2. Taking their intersecting point as a center, draw two concentric circles considering both as respective diameters.
3. Divide both circles in 12 equal parts & name as shown.
4. From all points of outer circle draw vertical lines downwards and upwards respectively.
5. From all points of inner circle draw horizontal lines to intersect those vertical lines.
6. Mark all intersecting points properly as those are the points on ellipse.
7. Join all these points along with the ends of both axes in smooth possible curve. It is required ellipse.



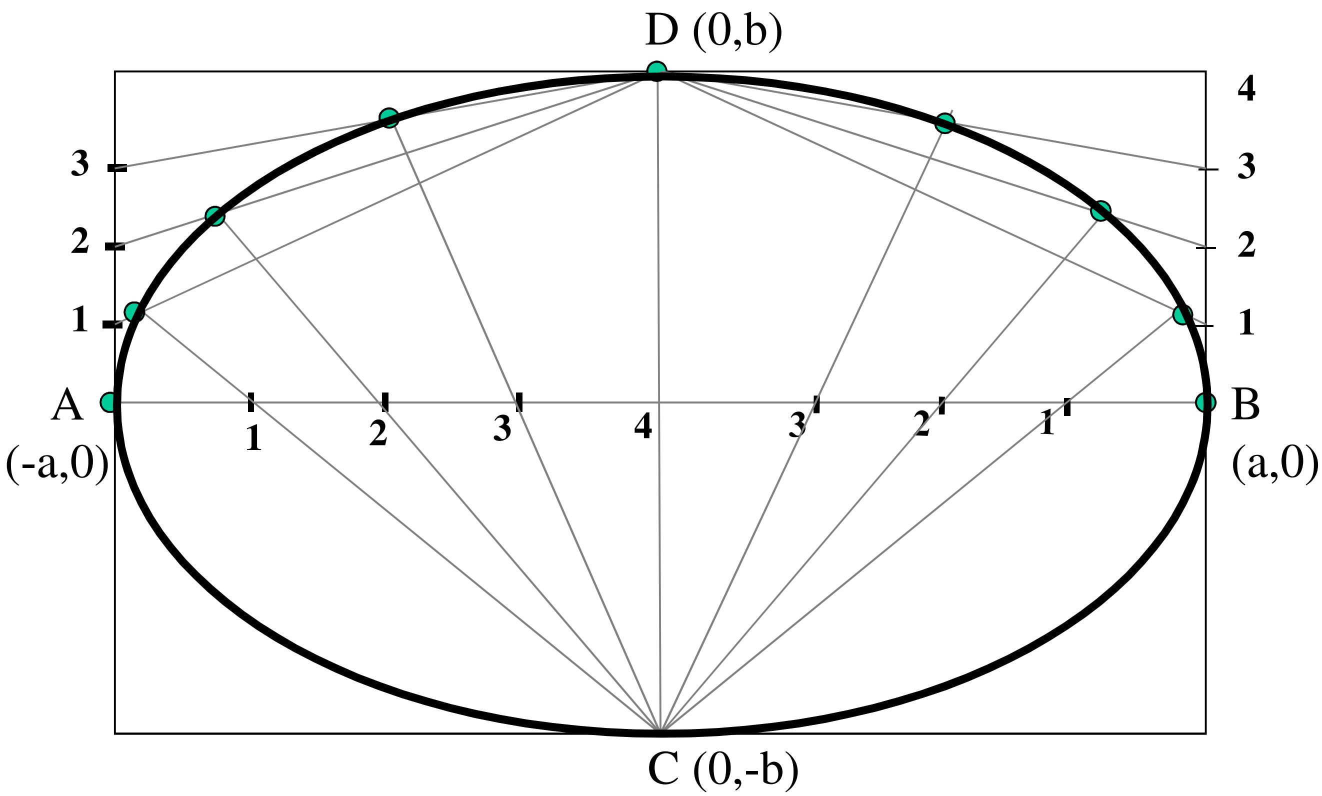
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$x = a\cos\theta; \quad y = b\sin\theta;$$

Ellipse (Rectangle method)

Draw an ellipse by **Rectangle method** Take major axis 120 mm and minor axis 90 mm long.

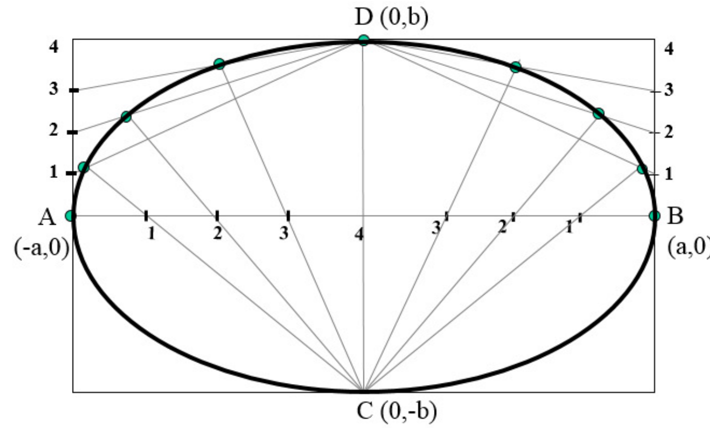
1. Draw a rectangle taking major and minor axes as sides.
2. In this rectangle draw both axes as perpendicular bisectors of each other.
3. For construction, select upper left part of rectangle. Divide vertical small side and horizontal long side into same number of equal parts. (here divided in four parts)
4. Name those as shown.



5. Now join all vertical points 1,2,3,4, to the upper end of minor axis and all horizontal points i.e.1,2,3,4 to the lower end of minor axis.
6. Then extend C-1 line up to D-1 and mark that point. Similarly, extend C-2, C-3, C-4 lines up to D-2, D-3, & D-4 lines.
7. Mark all these points properly and join all along with ends A and D in smooth possible curve. Do similar construction in right side part along with lower half of the rectangle. Join all points in smooth curve.

Reason (Ellipse Rectangle method)

Assume that the half of the major and minor axis is divided in n parts.



Coordinate of i^{th} point on the vertical line passing through B (V_i) in the first quadrant = $\left(a, \frac{ib}{n}\right)$

Coordinate of i^{th} point on the horizontal line passing through B (H_i) in the first quadrant = $\left(\frac{(n-i)a}{n}, 0\right)$

Eq. of line passing through D $(0,b)$ and V_i $\left(a, \frac{ib}{n}\right)$

$$y - Y_1 = \frac{Y_2 - Y_1}{X_2 - X_1}(x - X_1)$$

$$y - b = \frac{\frac{ib}{n} - b}{a}(x) \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Eq. of line passing through C $(0,-b)$ and H_i $\left(\frac{(n-i)a}{n}, 0\right)$

$$y - Y_1 = \frac{Y_2 - Y_1}{X_2 - X_1}(x - X_1)$$

$$y + b = \frac{b}{\frac{(n-i)\,a}{n}}(x) \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

$$y = \frac{\frac{ib}{n} - b}{a}(x) + b = \frac{b}{\frac{(n-i)\,a}{n}}(x) - b$$

Reason (Ellipse Rectangle method)

$$y = \frac{\frac{ib}{n} - b}{a}(x) + b = \frac{b}{\frac{(n-i)\,a}{n}}(x) \text{ -b}$$

$$\frac{b}{a}x\left[\frac{n}{n-i} - \frac{i-n}{n}\right] = 2b$$

$$x\left[\frac{n}{n-i} + \frac{n-i}{n}\right] = 2a$$

$$x\left[\frac{n^2+(n-i)^2}{n(n-i)}\right] = 2a$$

$$x = \frac{2an(n-i)}{n^2+(n-i)^2} \qquad \left(\frac{x}{a}\right)^2 = \frac{4n^2(n-i)^2}{[n^2+(n-i)^2]^2}$$

From eq.(1), $y = \frac{bn}{(n-i)a}\,x - b$

Simplifying we get, $y = \left(\frac{bn}{(n-i)}\right)\left[\frac{2n(n-i)}{n^2+(n-i)^2}\right] - b$

$$y = \frac{2bn^2}{[n^2+(n-i)^2]} - b$$

$$y = \frac{b(n^2-(n-i)^2)}{[n^2+(n-i)^2]}$$

$$\left(\frac{y}{b}\right)^2 = \frac{[n^2-(n-i)^2]^2}{[n^2+(n-i)^2]^2}$$

Let us assume $n^2 = p$ and $(n-i)^2 = q$

$$\left(\frac{x}{a}\right)^2 = \frac{4pq}{(p+q)^2} \quad \text{and} \quad \left(\frac{y}{b}\right)^2 = \frac{(p-q)^2}{(p+q)^2}$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \frac{4pq}{(p+q)^2} + \frac{(p-q)^2}{(p+q)^2} = \frac{4pq+(p-q)^2}{(p+q)^2}$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Ellipse (Parallelogram method)

The sides of a parallelogram are 120 mm and 80 mm. The included angle between them is 75°. Inscribe an ellipse in the given parallelogram.

CONSTRUCTION Figure 5.17

1. Draw a parallelogram *KLMN* with sides KL = 80 mm, LM = 120 mm and $\angle KLM = 75°$.
2. Mark *A*, *B*, *C*, *D* as mid-points of *KL*, *MN*, *NK*, *LM* respectively.
3. Mark *O* as the perpendicular bisectors of *AB* and *CD*.
4. Divide lines *OA* and *KA* into same number of equal parts, say 4. Mark 1, 2, 3 on *OA* and 1′, 2′, 3′ on *KA*.
5. Join point *C* with the points 1′, 2′, 3′.

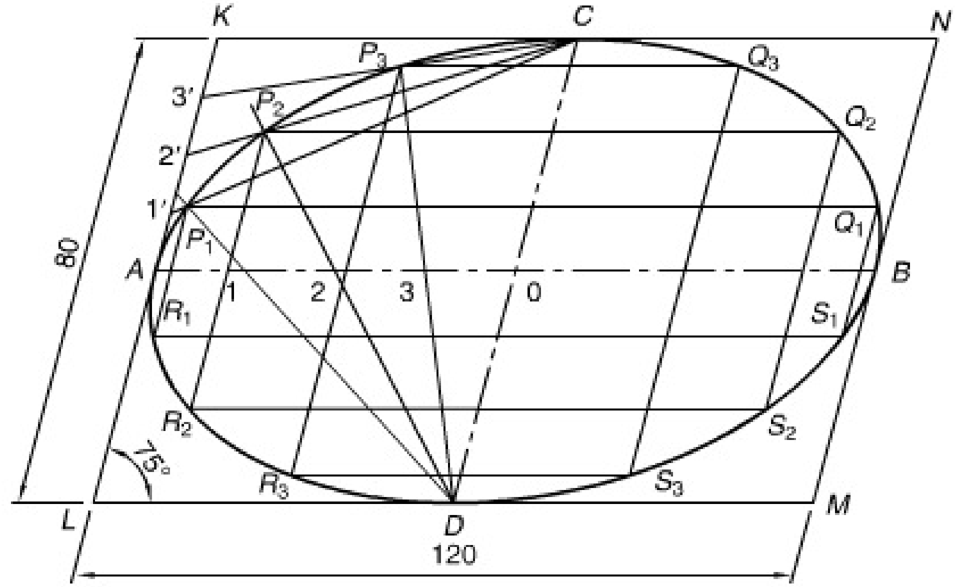


Fig. 5.17

6. Draw lines from point *D*, to join points 1, 2 and 3 and produce to intersect lines *C*1′, *C*2′ *C*3′ at points P_1, P_2, P_3 respectively.
7. Draw a smooth curve through *A*, P_1, P_2, P_3, *C*. This is one-quarter of an ellipse.
8. Draw lines parallel to *AB* through points P_1, P_2, P_3 and make each of them equal on either sides of *CD* and obtain Q_1, Q_2, Q_3.
9. Similarly, draw lines parallel to line *CD* passing through points P_1, P_2, P_3 and points Q_1, Q_2, Q_3. Make each of them equal on either sides of *AB* and obtain points R_1, R_2, R_3, S_1, S_2, S_3.
10. Join the points obtained in steps 8 and 9 with a smooth curve.

Ellipse (Arcs of circle method)

Two fixed points M and N are 100 mm apart. Trace the complete path of point P moving in such a way that the sum of its distance from M and N is always the same and equal to 130 mm. Name the traced curve.

CONSTRUCTION Figure 5.14

1. Draw a major axis AB 130 mm long and locate its mid-point O.
2. Mark foci M and N on AB with symmetry about O such that $MN = 100$ mm.
3. Mark points 1, 2, 3, etc., on OM at any convenient distances, which need not be equal.
4. With foci M and N as the centres and lengths equal to $A1$ and $B1$ as the radii respectively, draw arcs to intersect each other at P_1 and P_1'.
5. With foci M and N as the centres and radii $B1$ and $A1$ respectively, draw arcs to intersect each other at Q_1 and Q_1'.
6. Repeat step 4 and step 5 with the remaining points 2, 3 and 4 to obtain additional points P_2, P_2', Q_2, Q_2', P_3, … etc.
7. Draw a smooth curve passing through all these points. The curve obtained is an ellipse.

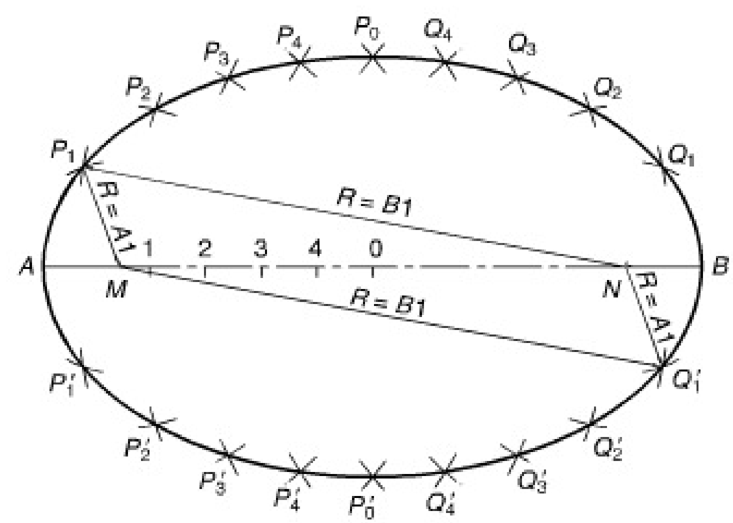


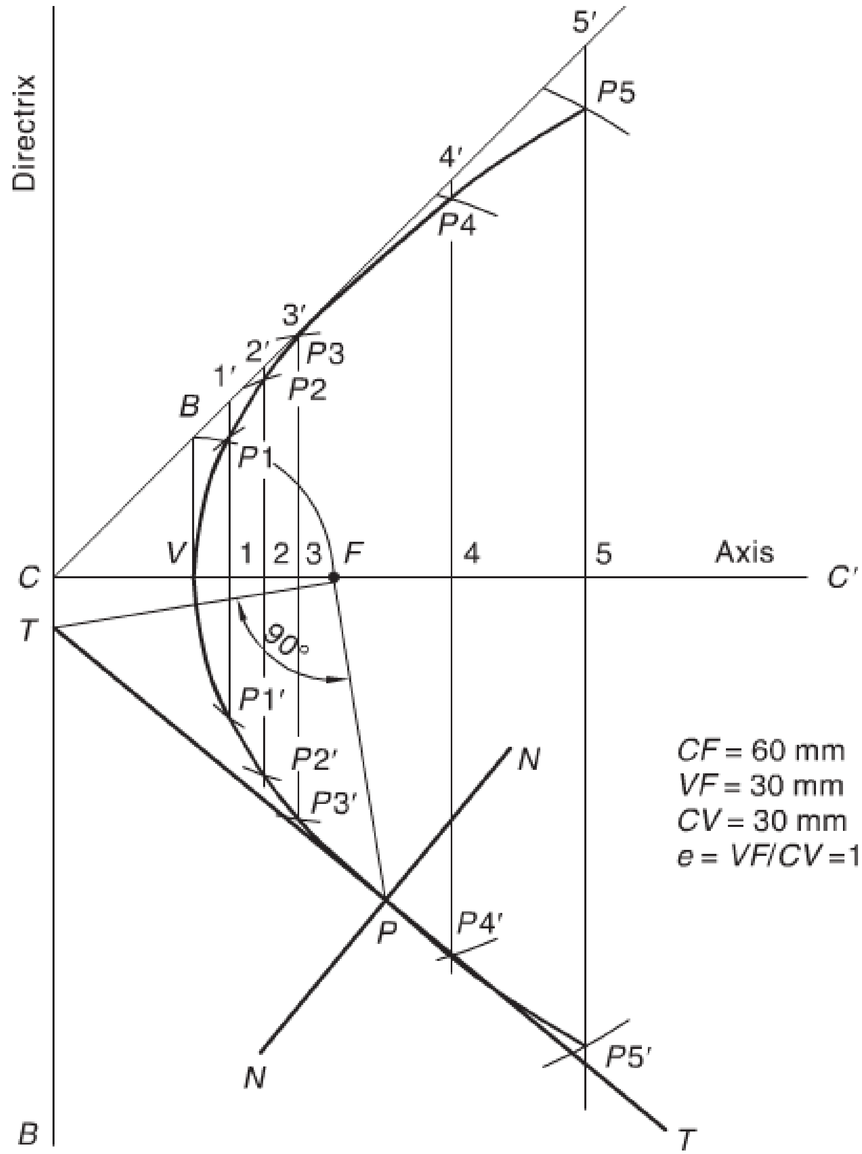
Fig. 5.14

Parabola (Focus-Directrix method)

Draw a Parabola for distance of the focus from the directrix being 60 mm

For Parabola eccentricity is 1.

1. Draw directrix *AB* and axis *CC′* as shown.
2. Mark *F* on *CC′* such that *CF* = 60 mm.
3. Mark *V* at the midpoint of *CF*. Therefore, *e* = *VF*/*VC* = 1.
4. At *V*, erect a perpendicular *VB* = *VF*. Join *CB*.
5. Mark a few points, say, 1, 2, 3, … on *VC′* and erect perpendiculars through them meeting *CB* produced at 1′, 2′, 3′, …
6. With *F* as a centre and radius = 1–1′, cut two arcs on the perpendicular through 1 to locate *P*1 and *P*1′. Similarly, with *F* as a centre and radii = 2–2′, 3–3′, etc., cut arcs on the corresponding perpendiculars to locate *P*2 and *P*2′, *P*3 and *P*3′, etc.
7. Draw a smooth curve passing through *V*, *P*1, *P*2, *P*3 … *P*3′, *P*2′, *P*1′.

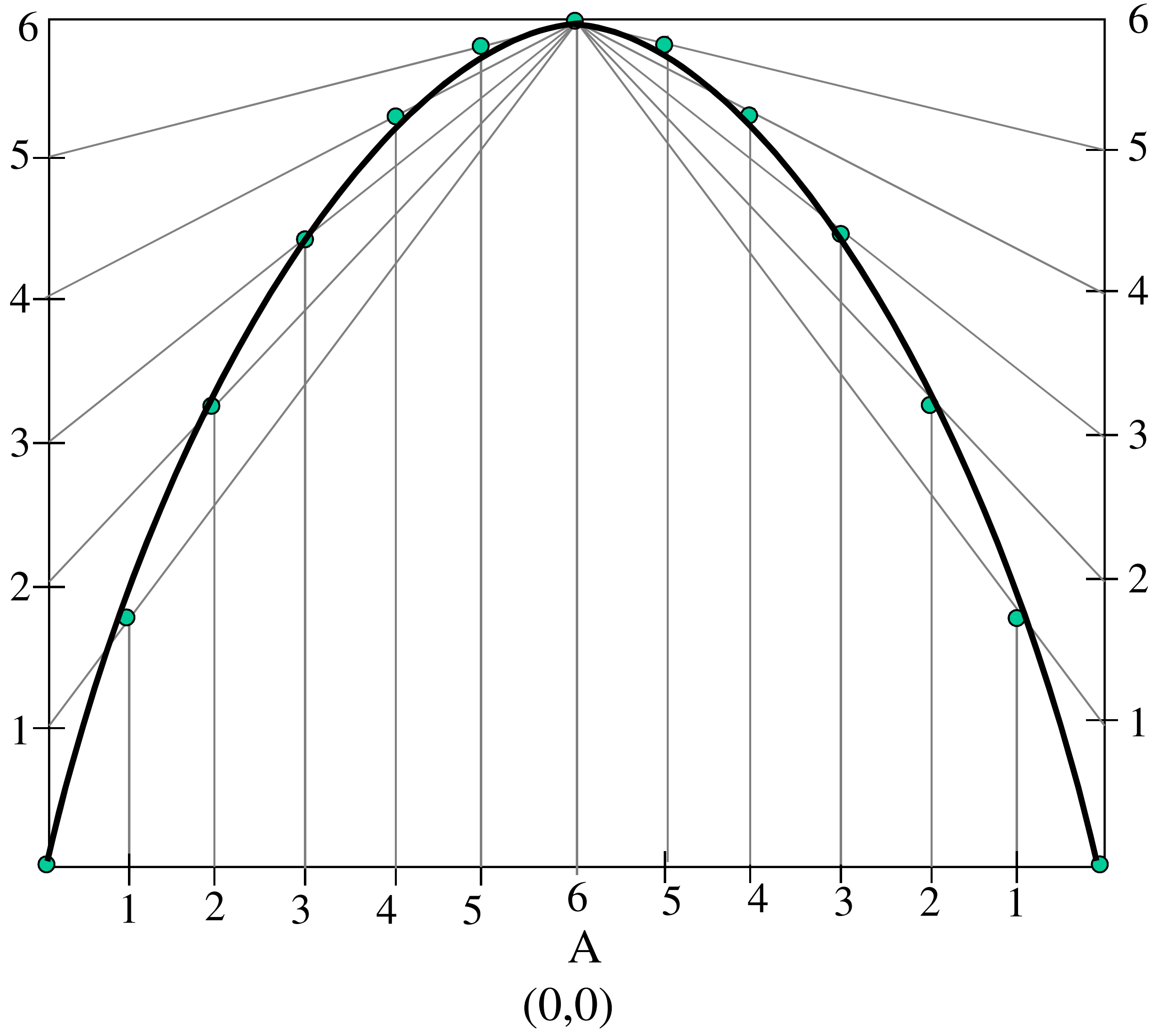


Parabola (Rectangle method)

A ball thrown in the air attains 100cm height and covers a horizontal distance of 150cm on the ground. Draw the path of the ball (projectile).

1. Draw rectangle of above size and divide it in two equal vertical parts.
2. Consider left part for construction. Divide height and length in equal number of parts and name those 1,2,3,4,5& 6.
3. Join vertical 1,2,3,4,5 & 6 to the top center of rectangle.
4. Similarly draw upward vertical lines from horizontal1,2,3,4,5 and wherever these lines intersect previously drawn inclined lines in sequence. Mark those points and further join in smooth possible curve.
5. Repeat the construction on right side rectangle also join all in sequence.

This locus is Parabola.



Reason (Parabola Rectangle method)

Consider five points $A(0,0), B(0,a), C\left(-\frac{b}{2},0\right), D\left(\frac{b}{2},0\right)$, and E $\left(\frac{b}{2},a\right)$.

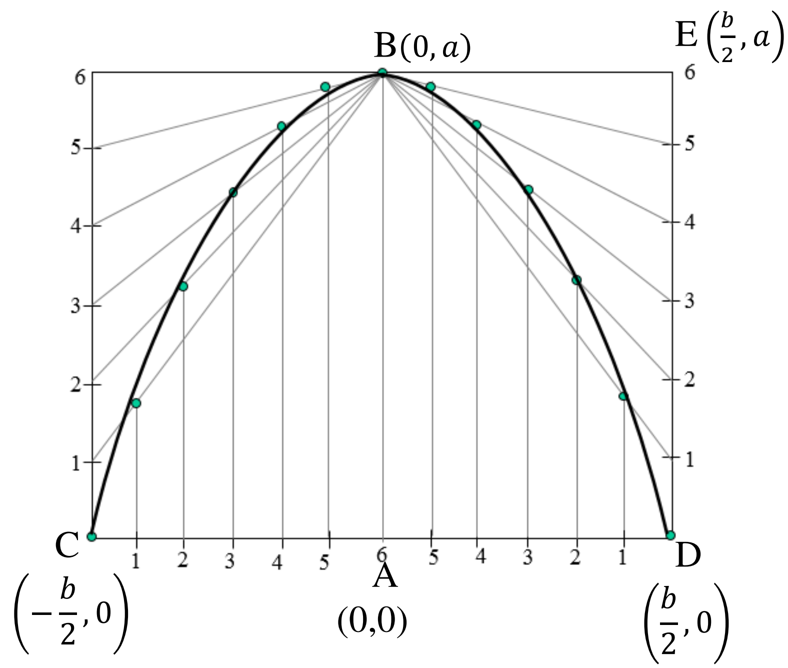
Divide the *DE* in *n*-parts and consider the i^{th} point on the vertical line passing through *D*, $V_i\left(\frac{b}{2}, \frac{ia}{n}\right)$

Divide *AD* in *n*-parts and consider the i^{th} point on the horizontal line passing through D, $H_i\left(\left(\frac{n-i}{n}\right)\frac{b}{2},0\right)$.

Equation of the vertical line through $H_i\left(\left(\frac{n-i}{n}\right)\frac{b}{2},0\right)$,

$$x = \frac{n-i}{n}.\frac{b}{2}$$

$$\frac{2x}{b} = \frac{n-i}{n}$$



Reason (Parabola Rectangle method)

Equation of the vertical line through $H_i\left(\left(\frac{n-i}{n}\right)\frac{b}{2}, 0\right)$,

$$\frac{2x}{b} = \frac{n-i}{n}$$

Equation of line passing through $V_i\left(\frac{b}{2}, \frac{ia}{n}\right)$ and $B(0, a)$,

$$y - Y_1 = \frac{Y_2 - Y_1}{X_2 - X_1}(x - X_1)$$

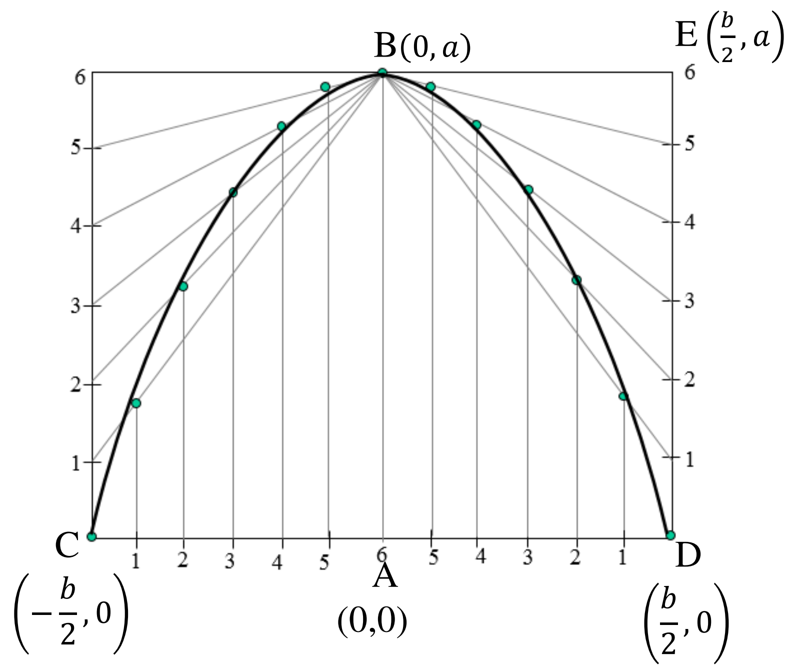
$$y - a = \frac{\frac{ia}{n} - a}{\frac{b}{2}}(x)$$

$$y - a = a\left(\frac{i}{n} - 1\right)\left(\frac{2x}{b}\right)$$

$$y - a = -a\left(\frac{n-i}{n}\right)\left(\frac{2x}{b}\right)$$

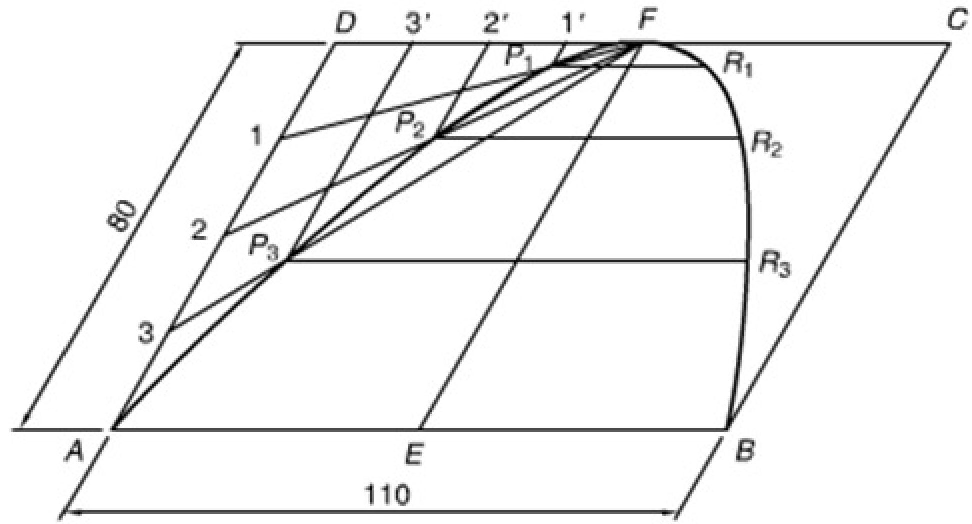
$$y - a = -a\left(\frac{2x}{b}\right)^2$$

$$y - a = -\frac{4ax^2}{b^2}$$



Parabola (Parallelogram method)

Inscribe a parabola in a parallelogram of 110 mm × 80 mm sides, the included angle being 60°. Consider the longer side of the parallelogram as the base of the parabola.

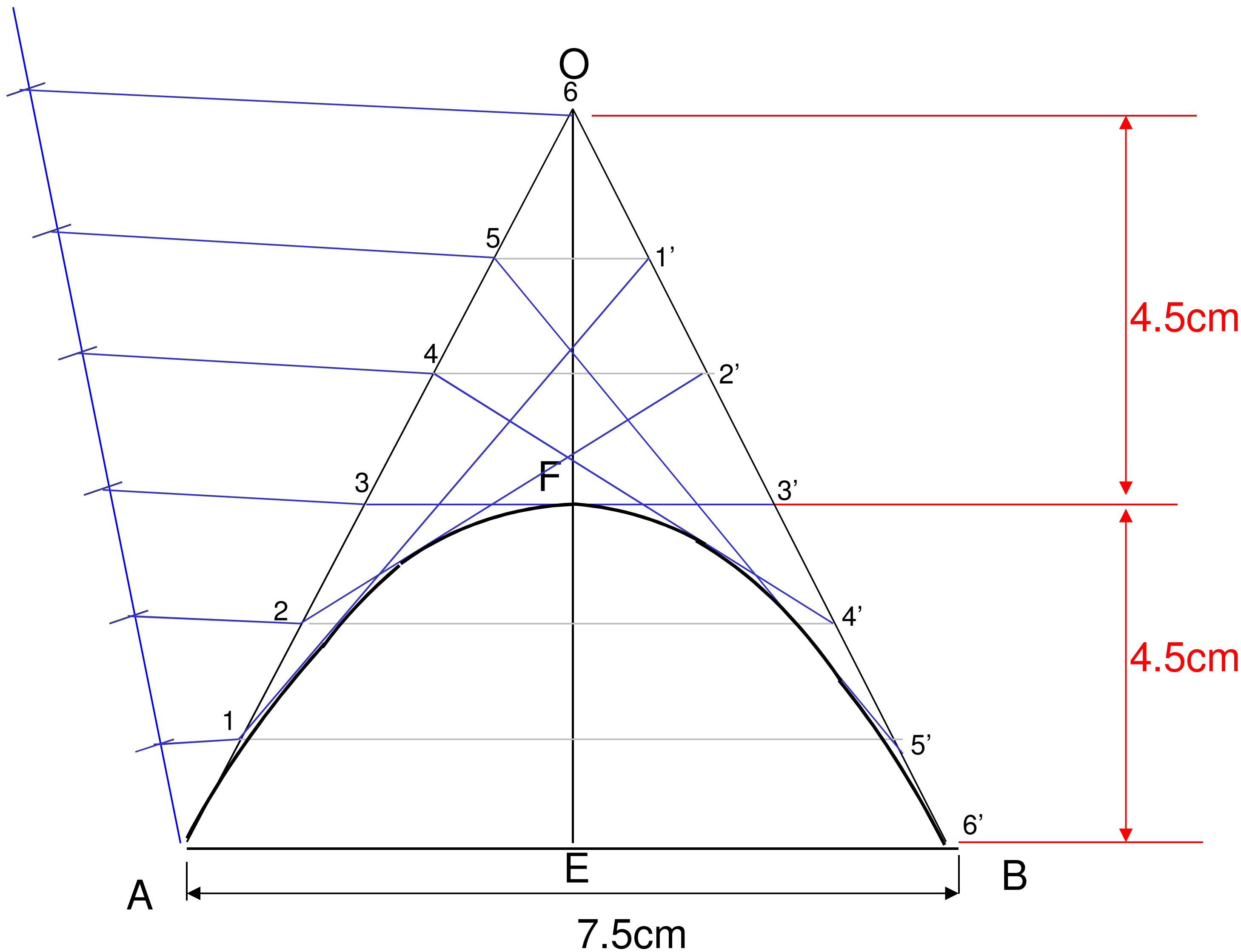


CONSTRUCTION Figure 5.32

1. Draw a parallelogram *ABCD*, *AB* = *110* mm and *AD* = 80 mm. Let ∠*DAB* = 60°.
2. Mark *E* and *F* as the mid-points of *AB* and *CD* respectively.
3. Divide lines, *FD* and *DA*, into same number of equal parts, say 4. Mark divisions of *DA* as 1, 2, 3 and divisions of *FD* as 1′, 2′, 3′.
4. Connect point *F* with points 1, 2, 3.
5. Through 1′, 2′, 3′ draw lines parallel to axis *EF* to meet lines *F*1, *F*2, *F*3 at points P_1, P_2, P_3 respectively.
6. Draw a curve through *F*, P_1, P_2, P_3, *A*. This is one-half of the parabola.
7. Draw horizontal lines through points P_1, P_2, P_3. Make their distances equal on either side of *EF* and obtain points R_1, R_2, R_3 of the curve.
8. Draw a curve to pass through points *F*, R_1, R_2, R_3, *B*. This is other half of the parabola.

Parabola (Tangent method)

Draw a parabola by tangent method given base 7.5cm and axis 4.5cm

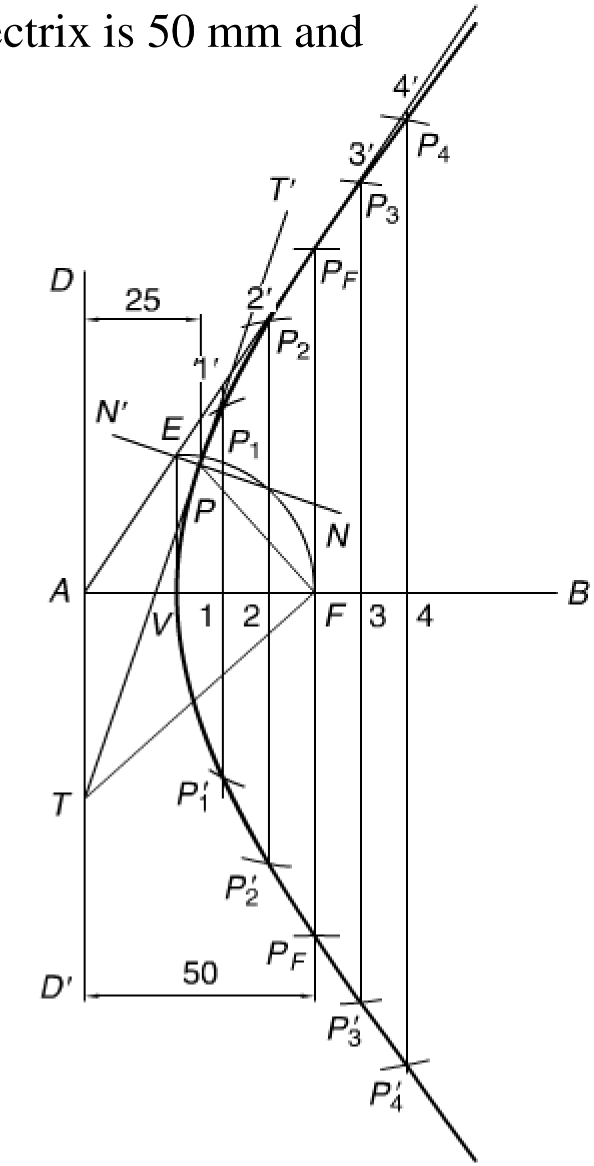


Hyperbola (Focus-Directrix method)

Draw a hyperbola when the distance between its focus and directrix is 50 mm and eccentricity is 1.5.

1. Draw directrix DD' and principal axis AB perpendicular to the DD'.
2. Mark focus F on the principal axis AB at a distance of 50 mm from the directrix DD', i.e., AF = 50 mm.
3. Divide the line AF into five equal parts (as e = 3/2) and mark vertex V on it such that $\frac{VF}{AV} = \frac{2}{3}$. Thus, vertex V satisfies the condition for being a point of the curve.
4. At V, draw a vertical line VE equal to VF. Join AE and extend it to some distance. Thus, in the triangle AVE, $\frac{VE}{AV} = \frac{VF}{AV} = \frac{3}{2}$.
5. Mark any point 1 on the axis and through it, draw a perpendicular line to meet AE produced at 1′. Thus, $\frac{11'}{A1} = \frac{VE}{AV} = \frac{3}{2}$.
6. With centre F and radius equal to $11'$, draw arcs to intersect the perpendicular line $11'$ at point P_1 and P_1'. These are the points of the ellipse because ratio $\frac{11'}{AV} = \frac{3}{2}$.
7. Similarly, mark any number of points 2, 3, 4, … on VB at any convenient distances which need not be equal. Through these points, erect lines $22'$, $33'$, $44'$, …, perpendicular to principal axis AB. With F as the centre and radius equal to $22'$, $33'$, $44'$, …, draw arcs to intersect the perpendicular line $22'$, $33'$, $44'$, … at points P_2 and P_2', P_3 and P_3', … etc., respectively.
8. Join points P_2', P_1', V, P_1, P_2, etc., to form a smooth hyperbolic curve.

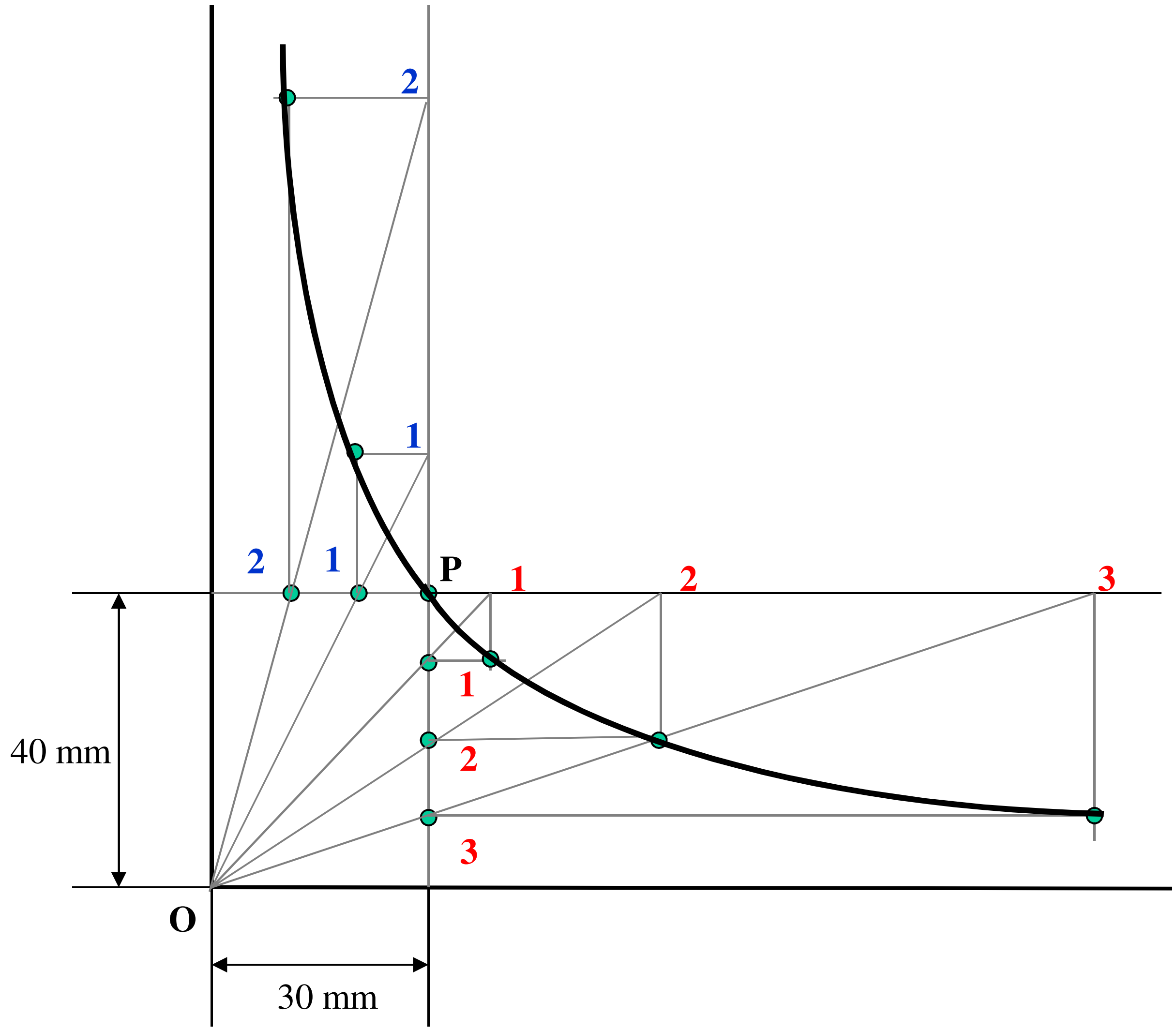
For Hyperbola eccentricity > 1.



Hyperbola (Point method)

Point P is 40 mm and 30 mm from horizontal and vertical axes respectively. Draw Hyperbola through it.

1. Extend horizontal line from P to right side.
2. Extend vertical line from P upward.
3 On horizontal line from P, mark some points taking any distance and name them after P-1, 2,3,4 etc.
4 Join 1-2-3-4 points to pole O. Let them cut part [P-B] also at 1,2,3,4 points.
5. From horizontal 1,2,3,4 draw vertical lines downwards and
6. From vertical 1,2,3,4 points [from P-B] draw horizontal lines.
7. Line from 1 horizontal and line from 1 vertical will meet at P_1.Similarly mark P_2, P_3, P_4 points.
8. Repeat the procedure by marking four points on upward vertical line from P and joining all those to pole O. Name this points P_6, P_7, P_8 etc. and join them by smooth curve.



Reason (Hyperbola Point method)

Consider $O(0,0)$ and $P(an, bn)$.

Divide the PM in n-parts and consider the i^{th} point on the vertical line passing through P, V_i $(an, b(n-i))$

Equation of line passing through O, V_i $(an, b(n-i))$

$$y = \left(\frac{b}{a}\right)\left(\frac{n-i}{n}\right)x$$

Equation of the horizontal line through P

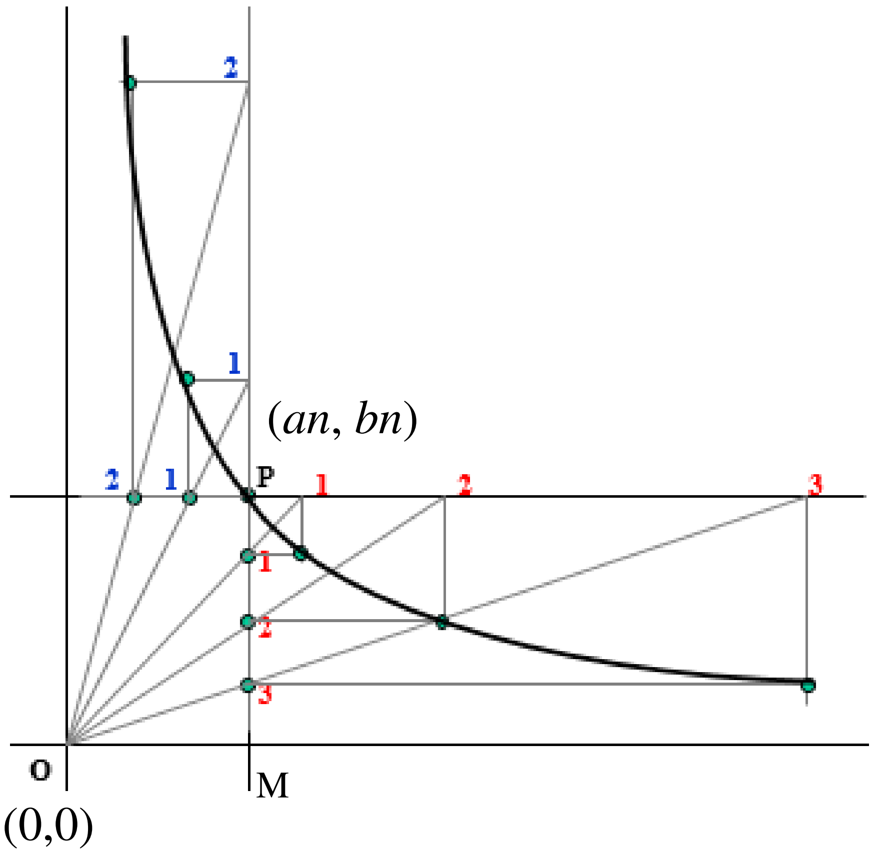
$$y = bn$$

These two lines intersect at H_i

$$bn = \left(\frac{b}{a}\right)\left(\frac{n-i}{n}\right)x$$

Therefore $H_i = \left(\frac{an^2}{n-i}, bn\right)$

and $V_i = (an, b(n-i))$



Reason (Hyperbola Point method)

$H_i = \left(\frac{an^2}{n-i}, bn\right) \qquad V_i = (an, b(n-i))$

The x coordinate of H_i and y coordinate of V_i together form the coordinates of a point on a curve.

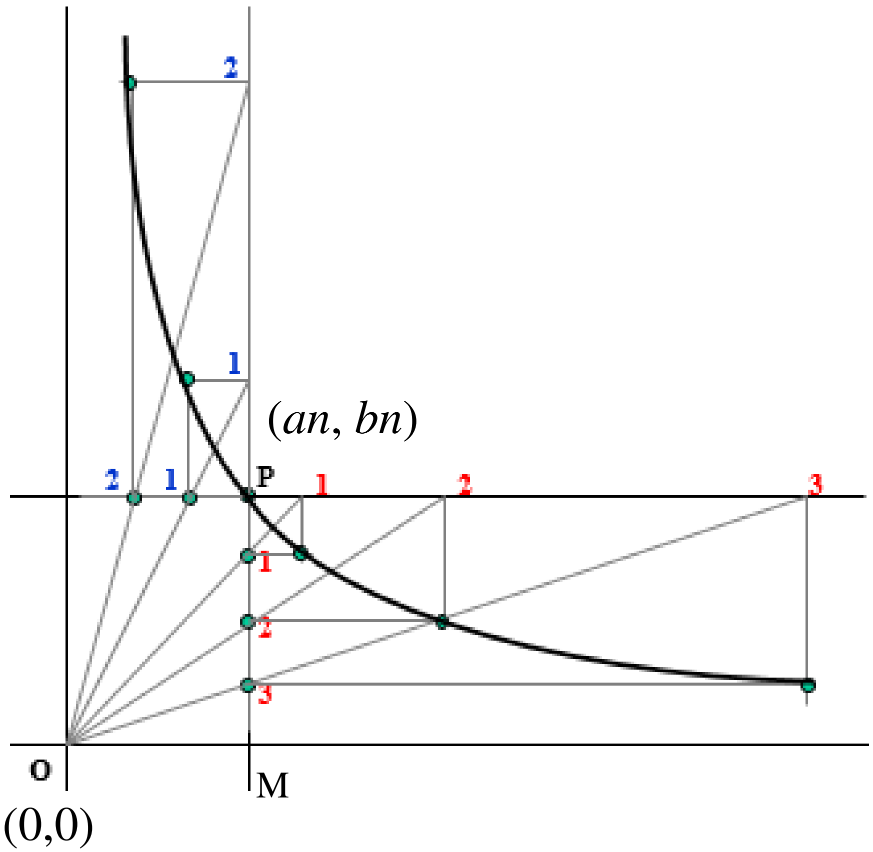
Therefore, the coordinates of a point on the curve $(x, y) = \left(\frac{an^2}{n-i}, b(n-i)\right)$

$$\frac{x}{a} = \frac{an^2}{a(n-i)} = \frac{n^2}{n-i}$$

$$\frac{y}{b} = (n-i)$$

$$\frac{x}{a} = \frac{n^2}{\frac{y}{b}}$$

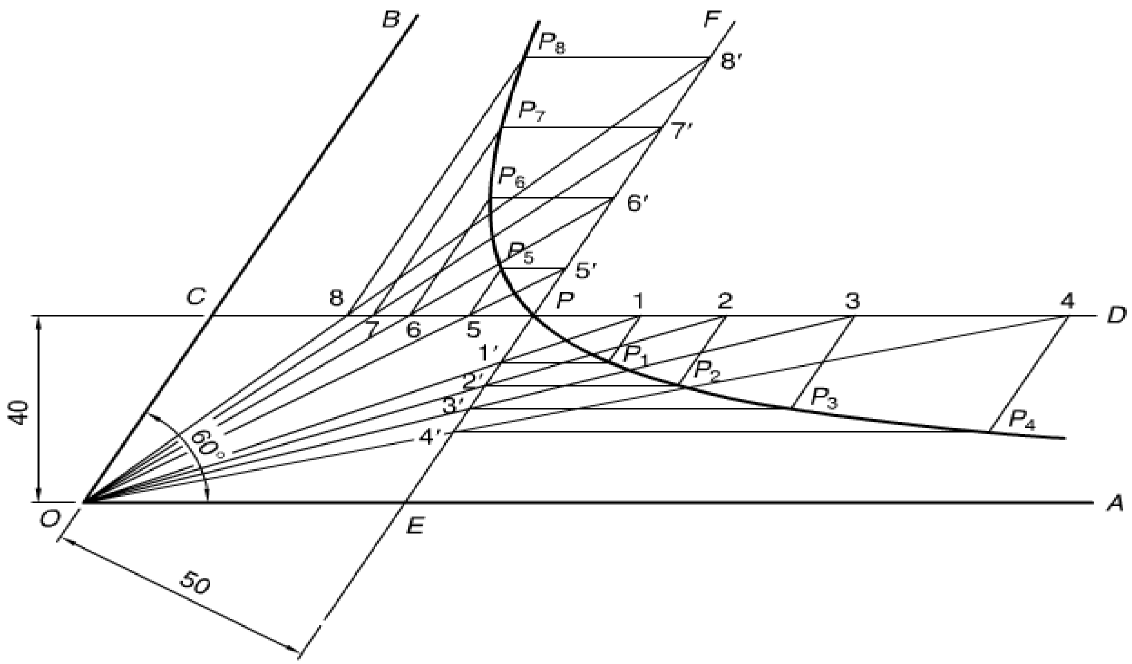
$$xy = abn^2$$



Hyperbola (Oblique Asymptotes Method)

Draw a hyperbola when the asymptotes are inclined at 60° to each other and it passes through a point P at a distance of 40 mm and 50 mm from the asymptotes.

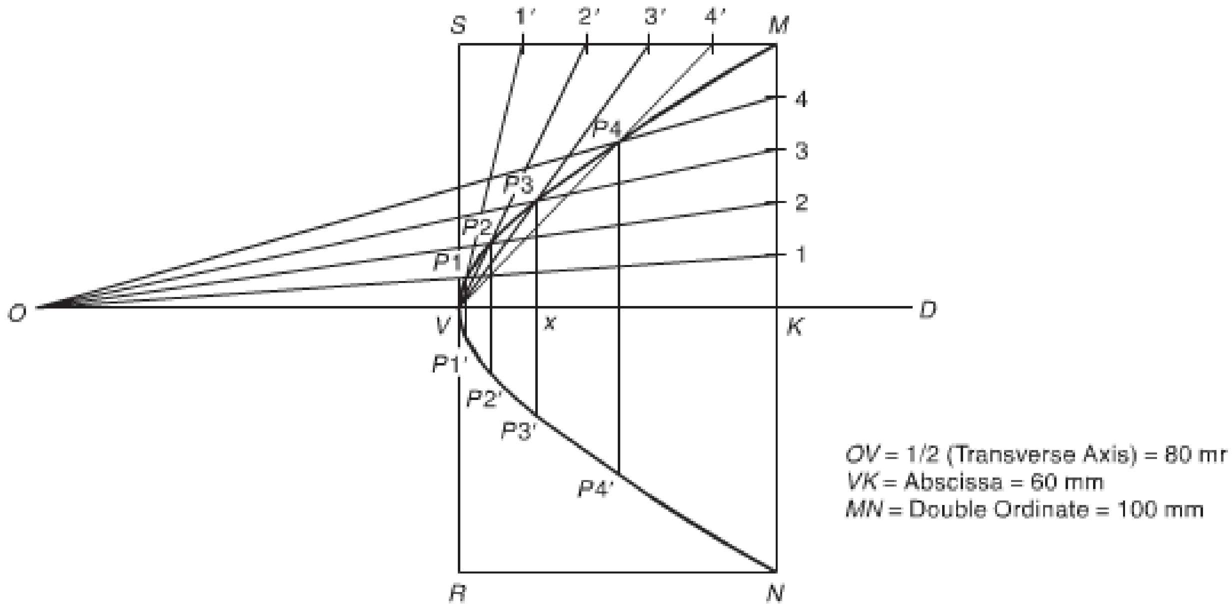
CONSTRUCTION Figure 5.45



1. Draw asymptotes *OA* and *OB* with an included angle of 60°.
2. Mark a point *P* such that its distance from *OA* is 40 mm and from *OB* is 50 mm.
3. Through point *P*, draw lines *CD* and *EF* parallel to asymptotes *OA* and *OB* respectively.
4. Mark points 1, 2, 3, …etc. along *CD* which need not be equidistant and lying on both sides of point *P*. It is advisable to mark points 1, 2, 3, at distances in increasing order.
5. Join *O*1, *O*2, *O*3, …etc. and extend them, if necessary, until they meet the line *EF* at points 1′, 2′, 3′, …etc.
6. Through 1, 2, 3, …etc., draw lines parallel to *OB* and through 1′, 2′, 3′, … etc., draw lines parallel to *OA*. Let them intersect at points P_1, P_2, P_3, …etc. respectively.
7. Draw a smooth curve passing through points P_1, P_2, P_3, …etc. The obtained curve is the required hyperbola.

Hyperbola (Rectangle method)

Draw a hyperbola having the double ordinate of 100 mm, the abscissa of 60 mm and the transverse axis of 160 mm.



1. Draw axis *OD* and mark *V* and *K* on it such that *OV* = 1/2(Transverse Axis) = 80 mm and *VK* = Abscissa = 60 mm.
2. Through *K*, draw double ordinate *MN* = 100 mm.
3. Construct rectangle *MNRS* such that *NR* = *VK*.
4. Divide *MK* and *MS* into the same number of equal parts, say 5. Number the divisions as shown.
5. Join *O*–1, *O*–2, *O*–3, etc. Also join *V*–1′, *V*–2′, *V*–3′, etc. Mark *P*1, *P*2, *P*3, etc., at the intersections of *O*–1 and *V*–1′, *O*–2 and *V*–2′, *O*–3 and *V*–3′, etc., respectively.
6. Obtain *P*1′, *P*2′, *P*3′, etc., in other half in a similar way. *Alternatively*, draw *P*1–*P*1′, *P*2–*P*2′, *P*3–*P*3′, etc., such that *P*3–*x* = *x*–*P*3′ and likewise.

Thank you