Quick Recap:

We have seen how to combine two propositions using

AND ($\wedge$) & OR ($\vee$).

AND ($\wedge$) $\updownarrow$ similar to intersection ($\cap$) in set theory

OR ($\vee$) $\updownarrow$ similar to union ($\cup$) in set theory

Let's think about other set theoretic operations and see what notions in logic we can come up with.

(similar to complement of a set)

NOT of a proposition:

Examples:

17 is a prime (T) | NOT (17 is a prime) (F)

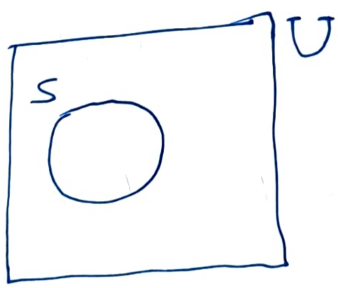
8 is an odd number (F) | NOT (8 is an odd number) (T)

Notation: $\neg P$ (NOT P)

As one would expect, if P is TRUE then $\neg P$ is FALSE.

And, if P is FALSE then $\neg P$ is TRUE.

P	$\neg P$
T	F
F	T



U: universe

S: set (subset of U)

$\bar{S} := \{x \in U : x \notin S\}$

$= \{x \in U : \neg(x \in S)\}$

$\bar{S}$ contains all elements of U that do NOT belong to S ← How to read?

Exclusive OR (XOR) of two propositions

(similar to symmetric difference in set theory)

P, Q : propositions

P	Q	P XOR Q ($P \oplus Q$)
T	T	F
T	F	T
F	T	T
F	F	F

Examples:

(17 is^T prime) $\oplus$ (8 is^T even) = F

(17 is^T prime) $\oplus$ (8 is^F odd) = T

(17 is^F NOT prime) $\oplus$ (8 is^F odd) = F

DIY: ① Define symmetric difference of two sets using XOR

② Define total order using XOR

Note: We are using same symbol for XOR & symmetric difference → The meaning should be clear from context.

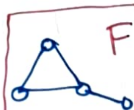
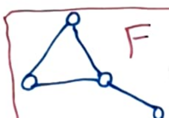
Combining propositions using IF....THEN...:

P, Q : propositions

$\boxed{\text{If P then Q}}$ [aka P implies Q; notation: $P \Rightarrow Q$] is TRUE always except when P is TRUE & Q is FALSE.

P	Q	If P then Q ($P \Rightarrow Q$)
T	T	T
T	F	F
F	T	T
F	F	T

Examples

1. if $\boxed{\text{8 is}^{T}\text{ even}}$ then $\boxed{\text{17 is}^{T}\text{ prime}}$ — TRUE
2. if $\boxed{\text{8 is}^{T}\text{ even}}$ then $\boxed{\text{17 is}^{F}\text{ even}}$ — FALSE
3. if  $\boxed{\text{}^{F}\text{ is 2-regular}}$ then $\boxed{\text{10 is}^{F}\text{ prime}}$ — TRUE
4. if  $\boxed{\text{}^{F}\text{ is 2-regular}}$ then $\boxed{\text{17 is}^{T}\text{ prime}}$ — TRUE

pay close attention: $P \Rightarrow Q$ is always TRUE when P is FALSE

(when P is FALSE, Q does NOT matter)

↓

This may NOT agree with how you (or most people) use natural language.

The good news is that generally mathematicians & computer scientists don't use such strange examples unless their goal is to mess with the other person.

Let us recall the following theorem:

Theorem: (Let G be a graph.)

$\boxed{\text{If}}$ each vertex of G has degree 2 or more $\boxed{\text{then}}$ G has a cycle.

↓

Let us apply this statement to different graphs to create ~~[illegible]~~ propositions: