PH-1020
Problem Set - 4
Department of Physics, IIT Madras
Magnetostatics
March-June 2023 Semester

Notation:

- Notation throughout follows that of Griffiths, Electrodynamics.
- Bold face characters, such as $\boldsymbol{v}$, represent three-vectors.

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1. Suppose that a hollow cylinder of length L and radius R has uniform surface current $K_0\hat{e}_\phi$. Calculate the axial magnetization. Discuss your results when L tends to infinity. Compare your results to that of a very long solenoid, consisting of n closely wound turns per unit length on a cylinder of radius R and carrying a steady current I.

2. A large parallel plate capacitor, aligned along the xy-plane, has uniform surface charge densities σ and $-\sigma$ on the upper and lower plates respectively. The capacitor is moving with a constant velocity $V_0\hat{e}_x$.

   (a) Find the magnetic field everywhere.

   (b) Find the magnetic force per unit area both on the upper and lower plates.

   (c) For what value of V_0, magnetic force balances the electric force.

3. A long cylindrical conductor with radius R has a cylindrical cavity of radius b ($b < R$). The axes of the conductor and cavity are parallel and are separated by distance d. The conductor carries a uniform current density $\boldsymbol{J}$ parallel to its axis. Show that the magnetic field in the cavity is constant.

4. Find the vector potential above and below the current sheet, lies in the xy-plane, with uniform current density $\boldsymbol{K} = K\boldsymbol{x}$ (See example 5.8 of Griffith 3^{rd} edition). Also, verify the magnetostatic boundary condition for the vector potential.

5. A circular loop of wire, with radius R, lies in the xy-plane, centered at the origin, and carries a current I running counterclockwise as viewed from the positive z-axis. Calculate the magnetic field of this loop assuming it to be a dipole.

Suggested Question

6. Just as $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ allows us to express $\boldsymbol{B}$ as the curl of a vector potential ($\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$), so $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$ permits us to write $\boldsymbol{A}$ itself as the curl of a "higher" potential: $\boldsymbol{A} = \boldsymbol{\nabla} \times \boldsymbol{W}$. (And this hierarchy can be extended ad infinitum.)

   (a) Find the general formula for $\boldsymbol{W}$ (as an integral over $\boldsymbol{B}$), which holds when $\boldsymbol{B} \to \boldsymbol{0}$ at ∞.

   (b) Determine $\boldsymbol{W}$ for the case of a *uniform* magnetic field $\boldsymbol{B}$.

   (c) Find $\boldsymbol{W}$ inside and outside an infinite solenoid.