IIT M-CS1200 : Discrete Math (Mar - Jul 2023) Release Date: 22-03-2023

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1. Let $X = \{3, 4, 7, 8\}$

   $Y = \{5, 6, 7, 8\}$

   $Z = \{1, 2, 3, 4\}$. Find

   (a) $(X - Y) \cup (Y - X)$

   (b) $(Y - Z) \cap (Z - Y)$

   (c) $(X \oplus Y) \oplus Z$

   (d) $X \oplus (Y \oplus Z)$

2. You have seen different relations in class. Now, find the relation between the following sets.

   (a) $A = \{-2, -3\}$
   $B = \{x : x$ is a solution of $x^2 + 5x + 6 = 0\}$

   (b) $X = \{9^n - 8n - 1 : n \in \mathbb{N}\}$
   $Y = \{64(n - 1) : n \in \mathbb{N}\}$

3. Construct examples for the following.

   (a) There exists infinite sets A, B and C such that $A \cap (B \cup C)$ is finite.

   (b) Let $S = \{1\}$. All possible subsets of S are $\{\}, \{1\}$. The set of all subsets of S is called the *Power set* of S denoted by $P(S)$. i.e. $P(S) = \{\{\}, \{1\}\}$.

   Now, consider $S_1 = \{a, b\}$.

   1. Find $P(S_1)$.
   2. Write the subset ($\subseteq$) relation between each of the elements of $P(S_1)$. Can you think of a way to visualise this relation? (Hint: Using digraph)
   3. Find $P(P(S_1))$.
   4. Using parts (1) and (3), find $P(S_1) \cap P(P(S_1))$.

4. Let S be a set and $P = P(S)$. Let $D \subseteq P$ such that for any two elements of D (subsets of S), say S_i, S_j, either $S_i \subset S_j$ or $S_j \subset S_i$.

   Let us see an example. Let S = {1,2} The following are some of the valid possibilities for D.

   D = { {}, {1} }. Here, observe that the first element is a proper subset of the second element of D. i.e. $\{\} \subset \{1\}$.

   D = { {1,2}, {1} }. Here, $\{1\} \subset \{1, 2\}$.

   Now consider $A = \{a, b, c, d\}$. Let D_1 be a collection of distinct subsets of A such that for any two subsets S_i, S_j (of A) in D_1, either $S_i \subset S_j$ or $S_j \subset S_i$. What is the maximum size of D_1?

5. Construct two relations R_1, R_2 on the set $S = \{a, b, c\}$ such that R_1 and R_2 are equivalence relations but $R_1 \cup R_2$ is not an equivalence relation.

   Is it possible to construct such relations R_1, R_2 if $S = \{a, b\}$?

6. (a) Construct a relation R on $\mathbb{N}$ such that R is reflexive and symmetric but R is not an equivalence relation.

   (b) Find a set $S \subset \mathbb{N}$ such that the size of S is at least two and every relation R on S has the following property.
   If R is reflexive and symmetric then R is an equivalence relation.

7. (a) Let R be a relation on set S such that, if aRb and bRc, then cRa, for all $a, b, c \in S$. Is the relation R transitive?

   (b) A relation R on a set S has the following properties.

   - Reflexive i.e. aRa, for all $a \in S$
   - If aRb and bRc, then cRa, for all $a, b, c \in S$

   1. Is the relation R transitive?
   2. Is the relation R symmetric?

8. A relation R on set A is called $CS1200$ relation if R is reflexive and xRy, yRx for all $x, y \in A$. The number of different $CS1200$ relations possible on A, if A contains 10 elements is x and if A contains 23 elements is y. Find the value of $x + y + (x * y)$.

9. Let $S = \{a, b, c, d\}$.

   (a) Construct a relation R on the set S such that it is reflexive, symmetric, antisymmetric and transitive. Is this R unique?

   (b) Construct a relation R on the set S such that it is reflexive, symmetric, and transitive. Find all the equivalence classes for R.

   (c) Try to construct a relation R on the set S such that it is reflexive, symmetric, antisymmetric but not transitive. Is such a relation R possible? If not, why?

   (d) Construct a relation R on the set S such that it is total order. Find the size of R. Let R_1 be another total order on the set S. Find the size of R_1. Comment on the size of R and R_1.

10. Let $S = \{a, b, c\}$.

    (a) Find the smallest and largest sized symmetric relations on S.

    (b) Construct a graph G_1 with vertex set $V(G_1)$ and edge set $E(G_1)$ for the largest sized symmetric relation.

    (c) Construct a graph G_2 with vertex set $V(G_2)$ and edge set $E(G_2)$ for any of the non-empty transitive relations on the set S. (Hint: Using digraph)