Quick Recap.

An n-ary relation on sets $A_1, A_2, \ldots, A_n$ is any subset of the Cartesian Product $A_1 \times A_2 \times \ldots \times A_n$.

n is called the arity of the relation.

Binary relation : relation of arity 2 (aka 2-ary relation).

(subset of $A_1 \times A_2$)

- homogeneous: $A_1 = A_2$
- heterogeneous: $A_1 \neq A_2$.

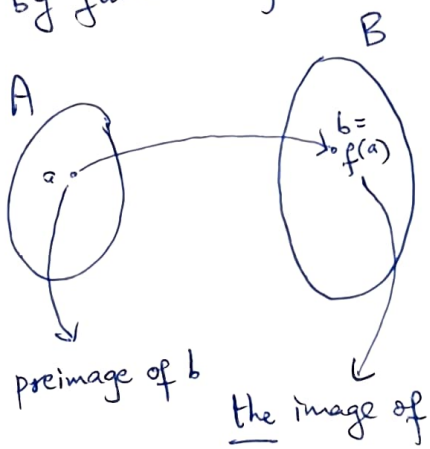
Viewing functions as relations :

A relation from A to B that contains exactly one ordered pair (a,b) for every element $a \in A$ is just a function from A to B.

A, B : nonempty sets

A function f from A to B is an assignment of exactly one element of B to each element of A.

For any element $a \in A$, we write $f(a) = b$ to indicate that b is the element of B assigned to a by function f.

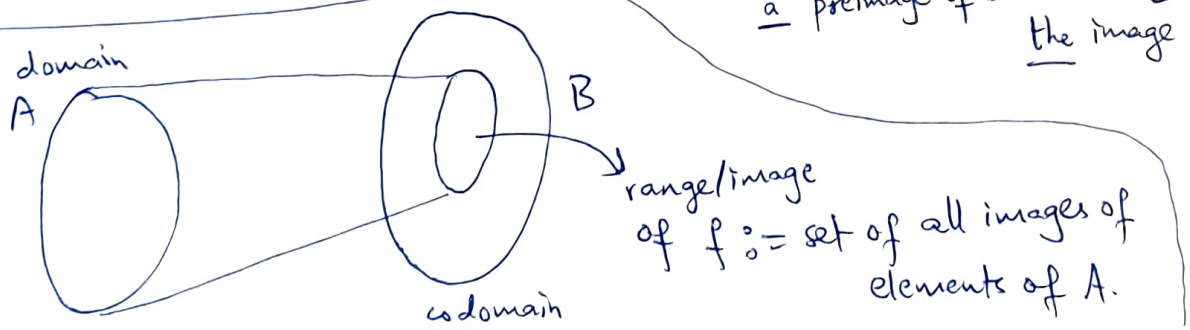


a preimage of b

the image of a

Notation : $f : A \to B$

- A : domain of f
- B : codomain of f



range/image of f := set of all images of elements of A.

Examples of functions:

① $f: \mathbb{Z} \to \mathbb{Z}$ defined as $f(n) = n^2$

② $f: \mathcal{D} \to \mathcal{G}$

↓ set of all digraphs ↘ set of all graphs

$f(D)$: underlying undirected graph obtained by forgetting all directions

③ $f: \mathcal{G} \to \mathcal{G}$

$f(G)$: underlying simple graph obtained by:

(i) throwing away all loops

(ii) for all distinct $u, v \in V(G)$, replacing all edges joining u & v (if any exist) by a single edge

↓

DIY: What is the image/range of each of these functions?

Graphs & digraphs – More Terminology:

Adjacency & Incidence:

For a graph $G := (V, E)$:

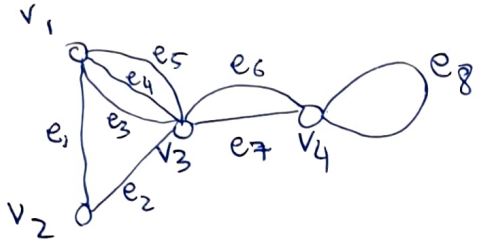
u & w are ends of e.
e joins u & w.

1) For two distinct vertices $u, w \in V$, we say that u & w are **adjacent** if there is an edge $e \in E$ joining u & w.

2) For two distinct edges $e, f \in E$, we say that e & f are **adjacent** if e & f have a common end.

3) For a vertex $v \in V$ and an edge $e \in E$, we say that e is **incident** at v (or that e & v are incident with each other) if v is an end of e.

Example:



v_1 & v_2 are adjacent

v_1 & v_4 are NOT adjacent

e_1 & e_4 are adjacent

e_3 & e_4 are adjacent

e_6 & e_8 are adjacent

e_1 & e_7 are NOT adjacent

e_1 is incident with v_1 (& with v_2)

e_6 is incident with v_3 (& with v_4)

e_8 is incident with v_4

e_7 is NOT incident with v_2

a few adjacencies & incidences are listed

same as graphs: replace "edge" by "arc".

Rule of thumb: adjacency is used for "things" of same type

example: vertex-vertex, edge-edge

whereas incidence is used for "things" of different types

example: vertex-edge, edge-vertex

ends of an arc: head of arc & tail of arc

DIY: Write all definitions for vertex-vertex adjacency, arc-arc adjacency & vertex-arc adjacency for digraphs.