PH-1020
Problem Set - 6
Department of Physics, IIT Madras
Electrodynamics I
March-June 2023 Semester

Notation:

- Notation throughout follows that of Griffiths, Electrodynamics.
- Bold face characters, such as $\mathbf{v}$, represent three-vectors.

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1. A square loop of wire with side length a lies at a distance s from a very long straight wire carrying current I. Assume the loop and the wire on the same plane.
   (a) Find the magnetic flux through the loop.
   (b) When the loop moves directly away from the wire, at a speed v, calculate the emf generated. Also indicate the direction of the current flow in the loop.
   (c) What will happen when the loop moves with speed v in the direction of the current flow?
   (d) Evaluate the mutual inductance between the wire and the loop.

2. Consider a square loop of wire with side length a lying in a region with uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. If the square loop is co-planar to the $xy-$ plane and rotates with and angular speed ω, plot the induced emf as a function of time.

3. (a) Determine the self-inductance per unit length of a long coaxial cable of inner radius a and outer radius b. The region between the inner and the outer conductor is filled with a linear magnetic material with relative permeability μ_r.
   (b) Calculate the energy stored (per unit length) in this inductor if a current I flows along the inner conductor and returns along the outer conductor.
   (c) Calculate the power transported down the cables when both the conductors are held at a potential difference V.

4. The electric and magnetic fields are given by

$$\mathbf{E}(r,t) = -\frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\theta(vt-r)\hat{\mathbf{r}}, \qquad \mathbf{B}(r,t) = 0,$$

   Where

$$\theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

   Show that these fields satisfy the four Maxwell's equation. Find the charge density ρ and the current density $\mathbf{J}$. What kind of physical situation gives rise these fields?

5. A transformer has two coils wrapped around an iron core in such a way that the same flux passes through every turn of both coils. The primary coil has n_1 turnsand the secondary coil has n_2 turns.

   (a) If the current I_1 through the primary coil is changing, what is the ratio of the emfs in the primary and secondary coils.

   (b) Show that, $M^2 = L_1 L_2$ where M is the mutual inductance and L_1,L_2 are individual self inductances.

6. Consider a long solenoid whose axis of symmetry is along z axis and a constant current I runs through it. A metal coin is allowed to fall under gravity along the axis of the solenoid. Find the induced emf and hence, the force on the coin due to the magnetic interaction when the coin is at a distance b from the top of the solenoid. (Neglect the mutual inductance for simplicity.)

   (Useful parameters: radius of the coin is a, thickness of the coin is τ, conductivity of the coin is σ, cross-sectional area of the solenoid is A. You can neglect the mutual inductance)