Department of Mathematics, IIT Madras

MA1102 Series & Matrices

Assignment-2 (Series Representation of Functions)

1. Determine the interval of convergence for each of the following power series:
   (a) $\sum_{n=1}^{\infty} \frac{x^n}{n}$ (b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ (c) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$.
2. Determine the interval of convergence of the series $\frac{2x}{1} - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \cdots$.
3. Determine power series expansion of the functions (a) $\ln(1+x)$ (b) $\frac{\ln(1+x)}{1-x}$.
4. The function $\frac{1}{1-x}$ has interval of convergence $(-1, 1)$. However, prove that it has power series representation around any $c \neq 1$.
5. Find the sum of the alternating harmonic series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$.
6. Give an approximation scheme for $\int_0^a \frac{\sin x}{x}\, dx$ where $a > 0$.
7. Show that $1 + \frac{1}{2}\frac{1}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \cdots = \frac{\pi}{2}$.
8. Find the Fourier series of $f(x)$ given by: $f(x) = 0$ for $-\pi \le x < 0$; and $f(x) = 1$ for $0 \le x \le \pi$. Say also how the Fourier series represents $f(x)$. Hence give a series expansion of $\pi/4$.
9. Considering the fourier series for $|x|$, deduce that $\frac{\pi^2}{8} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$.
10. Considering the fourier series for x, deduce that $\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.
11. Considering the fourier series for $f(x)$ given by: $f(x) = -1$, for $-\pi \le x < 0$ and $f(x) = 1$ for $0 \le x \le \pi$. Deduce that $\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$.
12. Considering $f(x) = x^2$, show that for each $x \in [0, \pi]$,
$$\frac{\pi^2}{6} + 2\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2} = \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^{n+1} + 2(-1)^n - 2}{n^3\pi} \sin nx.$$
13. Represent the function $f(x) = 1 - |x|$ for $-1 \le x \le 1$ as a cosine series.