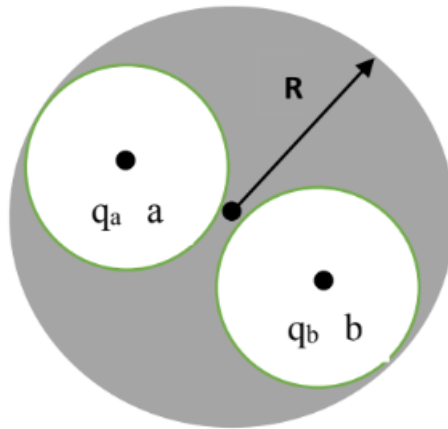


PH-1020
Problem Set - 2
Department of Physics, IIT Madras
Conductors, Multipole expansion
March-June 2023 Semester

Notation:

- Notation throughout follows that of Griffiths, Electrodynamics.
- Bold face characters, such as \mathbf{v} , represent three-vectors.

1. Two spherical cavities, of radii a and b , are hollowed out from the interior of a (neutral) conducting sphere of radius R as shown in the figure below. At the center of each cavity respective point charge q_a and q_b are placed.
 - (a) Find the surface charge densities σ_a , σ_b and σ_R .
 - (b) What is the field outside the conductor?
 - (c) What is the field within each cavity?
 - (d) What is the force on q_a and q_b ?
 - (e) Which of these answers would change if a third charge q_c , were brought near the conductor?



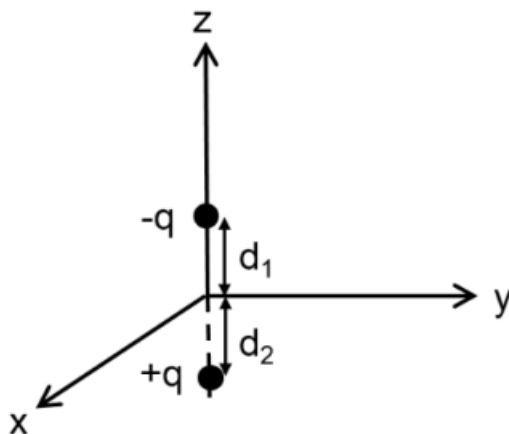
2. Two large metal plates, each of area A , are separated by a distance d . If we put charge Q on each plate, what will be the electrostatic pressure on the plates?
3. A “pure” dipole \mathbf{p} is situated at the origin, pointing in the z -direction. Find the electric potential to be:

$$V_{dipole}(r, \theta) = \frac{\hat{\mathbf{r}} \cdot \mathbf{p}}{4\pi\epsilon_0 r^2} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

Using $V_{dipole}(r, \theta)$, calculate the electric field in the following coordinate-free form:

$$E_{dipole}(r) = \frac{1}{4\pi\epsilon_0 r^3} [3(\mathbf{p} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}]$$

4. Two charges are located as shown in the figure below. Find the quadrupole moment when (i) $|\mathbf{d}_1| = |\mathbf{d}_2|$, and (ii) $|\mathbf{d}_1| = 1/2$ and $|\mathbf{d}_2| = 3/2$.



5. (a) Show that, for a charge distribution, if the total charge (the monopole moment) is zero, the dipole moment is independent of the origin.
 (b) If a distribution has non-zero monopole moment, show that you can always find a new origin about which the dipole moment vanishes.
 (c) Show that, for a spherically symmetric distribution, all moments higher than the monopole vanish about the centre of symmetry.
6. A metal sphere of radius R carries a total charge Q . What is the force of repulsion between the northern and southern hemisphere?
7. Two positive charges occupy diagonally opposite corners and two equal, negative charges occupy the remaining corner of a square as shown in the figure below. Find the expression for the quadrupole moment for such a charge distribution.

