Observe: ① $A \oplus B = (A-B) \cup (B-A)$ ⟶ why? think.

② $A \oplus B = (A \cup B) - (A \cap B)$ ⟶ why? think.

Food for thought: (TIY - TRY IT YOURSELF)

① Can the symmetric difference operation be generalized to any number of sets?

② If YES, how? If NO, why NOT?

(Hint: Try examples with 3 & 4 sets.)

(We will answer these questions in Module-2.)

Enough of Set Theory (for now).

Relation (dictionary meaning):

The way in which two (or more) people or $\boxed{\text{things}}$ are connected

two ↓ we will focus on this

things ↑ we care mostly about mathematical creatures

Question: Have we seen any relations so far?

Answer: ① Divisibility / divides

② Subset / is a subset of

Question: Do you know of any other relations in math? Maybe you have seen some in high school?

Answer: $\leq, \geq, =, >, <$

Enough of sets & set operations.... let's move on to relations.

Relation (dictionary meaning): the way in which two (or more) people or things are connected

we care mostly about mathematical things/creatures

Question: Have we seen any relations so far?

Answer: (Lecture-1) divides → aka divisibility

(Lecture-2) is a subset of → aka containment/inclusion

Question: Do you know of any other relations in math?

Answer: Many. Some are: $\leq, \geq, =, >, <,$ (have same) parity

dictionary meaning: the state of being odd or even

Examples: 3 has the same parity as 25

$5 \leq 6$

$7 \geq 4$

4 divides 12

$\mathbb{N} \subseteq \mathbb{Z}$

Notation: $4 \mid 12$

parity relationship.

Notation: $\equiv$

Example: $3 \equiv 25$

Either two elements are related OR they are NOT related

Examples: $3 \not\equiv 26$

$5 \not\leq 4$

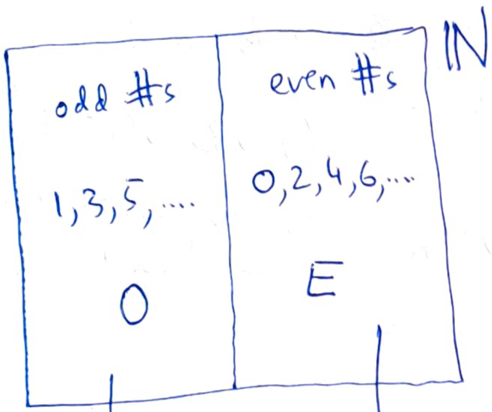
$7 \not\geq 8$

$4 \nmid 13$

$\mathbb{Z} \not\subseteq \mathbb{N}$

Question: There is something special about the parity relationship ($\equiv$) that is NOT true for any ~~other~~ of the other relations we have discussed so far. What is it?

Hint: Consider the relations $\equiv, \leq, \geq,$ and $|$. on the set $\mathbb{N}$.



Observe that all of these are related with each other by $\equiv$.

Observe that all of these are related with each other by $\equiv$.

However, if $x \in O$ & $y \in E$ then $x \not\equiv y$.

So, we are able to nicely $\boxed{\text{break}}\ /\ \boxed{\text{partition}}$ the set $\mathbb{N}$ using the $\equiv$ relation.

Question:

Can we do $\boxed{\text{this}}$ for any other relation ($\leq, \geq, |$) that we have discussed so far?

Answer: NO.

(Why NOT?)

(What is so special about $\equiv$ relation?)

The parity ($\equiv$) relation (on $\mathbb{N}$) satisfies some special properties, and some of these special properties are NOT satisfied by other relations we have discussed (in particular: $\leq, \geq, |$).

Let us discuss & define these special properties:

U: universe

R: a relation (defined on U)

For members a & b of U, we write $\boxed{aRb}$ to indicate that $\boxed{\text{a is related to b through the relation R}}$.

Special Property 1: Reflexivity → dictionary meaning: the fact of someone being able to examine their own feelings, reactions & motives and how these influence what they do or think in a situation (grammar) showing that the person who does the action is also the person who is affected by it

We say that R is $\boxed{\text{reflexive}}$ (or that R $\boxed{\text{satisfies reflexivity}}$) if

aRa for every element a (of U).

For example: $\leq, \geq$ and $\equiv$ (on $\mathbb{N}$) are reflexive, whereas $<$ & $>$ are NOT.

$|$ is NOT reflexive since $0 \nmid 0$.

However, $|$ is reflexive on $\{1,2,3,\ldots\} = \mathbb{N} - \{0\}$.

In the lecture, I made a mistake. As per our defn, 0 does NOT divide 0.

$\boxed{\text{Equivalence relation}}$: a relation that is reflexive, symmetric & transitive.

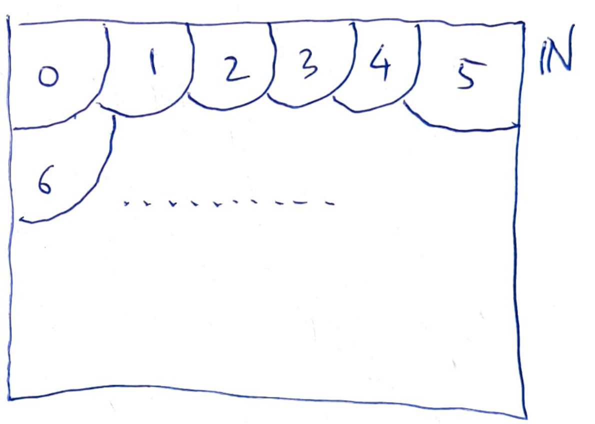
They play an extremely important role in CS & in mathematics.

↓ why?

Any equivalence relation R defined on a set U nicely breaks / partitions the set U into disjoint sets called $\boxed{\text{equivalence classes.}}$

Examples (before formal definitions):

Equivalence classes (of IN) with respect to parity (≡) relation



Equivalence classes (of IN) with respect to equality (=) relation

Special Property 2: Symmetry

We say that R is $\boxed{\text{symmetric}}$
(or that R $\boxed{\text{satisfies symmetry}}$)
if whenever aRb (for any two members ~~a~~ & b of U)
then bRa also holds.

For example: $\equiv$ (on $\mathbb{N}$) is symmetric
whereas $<, >, \leq, \geq$ & $|$ are NOT symmetric.

Special Property 3: Transitivity

We say that R is $\boxed{\text{transitive}}$ (or that R $\boxed{\text{satisfies transitivity}}$)
if whenever aRb & bRc (for any 3 members a, b & c
of U) then aRc also holds.

For example: All of the relations we have discussed
(in particular: $<, >, \leq, \geq, \equiv$ & $|$) on $\mathbb{N}$ are transitive.

TIY: Come up with some "natural" relation (on $\mathbb{N}$)
that is symmetric but is NOT transitive.

A geometric example that is symmetric but NOT
transitive: two lines (in $\mathbb{R}^2$) being perpendicular.