## Friends & Strangers at a Party (a "real world" discrete math problem)

$n$ people attend a party. ($n \in \mathbb{N} - \{0\}$)
Any two distinct persons are either friends/acquaintances or strangers (but NOT both).

Will there always be 3 people $a, b$ & $c$ such that either $a, b, c$ are pairwise-friends or $a, b, c$ are pairwise-strangers. ?

(Answer: later) (YES/NO/depends on $n$?)

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Let's rephrase in the language of sets & relations:

Consider $U := \{1, 2, \ldots, n\}$ where $n \in \mathbb{N} - \{0\}$.

Consider a symmetric relation $F$ (are friends):

For distinct $a, b \in U$, we write $aFb$ if $a$ & $b$ are friends.

(Note: if $aFb$ holds then $bFa$ also holds.)

Will there always be 3 elements — say $x, y, z \in U$ — such that either $xFy$, $yFz$ and $xFz$ or $x \not F y$, $y \not F z$ and $x \not F z$?

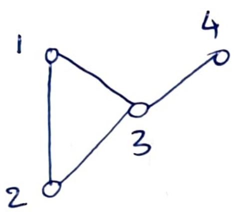
(Answer: later)

## Friends & Strangers at a Party (continued)

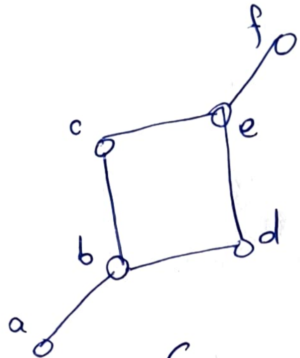
Is there a more visual description of the "problem"?

Yes, through the lens of Graph Theory:

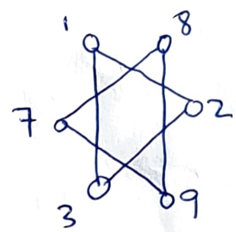
### Examples of (drawings of) Graphs:



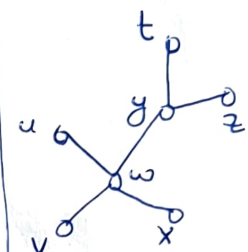

$G_1$



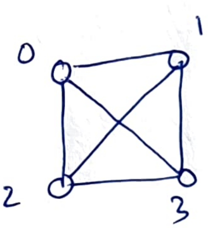

$G_2$



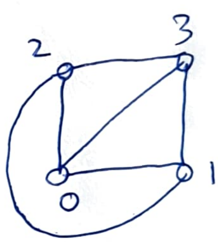

$G_3$




$G_4$




$G_5$




$G_6$

### What is a graph?

singular: vertex/node (vtx)

A $\boxed{\text{graph}}$ $G := (V, E)$ has:

(i) $V$: a set of $\boxed{\text{vertices/nodes}}$

(ii) $E$: a set of $\boxed{\text{edges}}$

The set of vertices may be finite/infinite. A graph with infinite vtx set is called an $\boxed{\text{infinite graph}}$.

each edge is an unordered pair of vertices ↓ these vertices are called $\boxed{\text{ends}}$ of that edge

Drawing a graph is one of the most popular methods of representing a graph. In such a drawing, each vertex is shown as a point/disc, and each edge is shown as a line/curve joining its vertices/ends.

## Friends & Strangers at a Party (Continued in language of Graph Theory)

Consider a graph $G := (V, E)$ on $n$ vertices (that is, $|V| = n$).

cardinality / size of set $V$

cardinal: 1, 2, 3, ....
ordinal: 1st, 2nd, 3rd, ....

Will there always be 3 vertices $a$, $b$ & $c$ such that either $a, b, c$ are pairwise-adjacent or $a, b, c$ are pairwise-nonadjacent?

$G := (V, E)$ : graph
Two distinct vertices $u$ & $v$ are $\boxed{\text{adjacent}}$ if there is an edge joining them.

Otherwise, $u$ & $v$ are $\boxed{\text{nonadjacent}}$.

Still NOT too visual. Especially this.

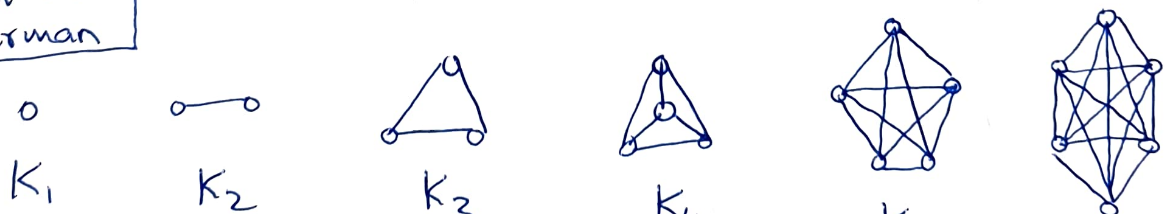
Can we make it more colorful?

Literally!

## Special Graph Classes / Families:

Why K?
Why NOT C?
Komplett in German

### Complete Graphs



$K_1$ $K_2$ $K_3$ $K_4$ $K_5$ $K_6$

Complete graph: A graph with $n$ vertices and ~~one~~ an edge joining each pair of vertices. Denoted by $K_n$.

Question: How many edges are there in $K_n$?