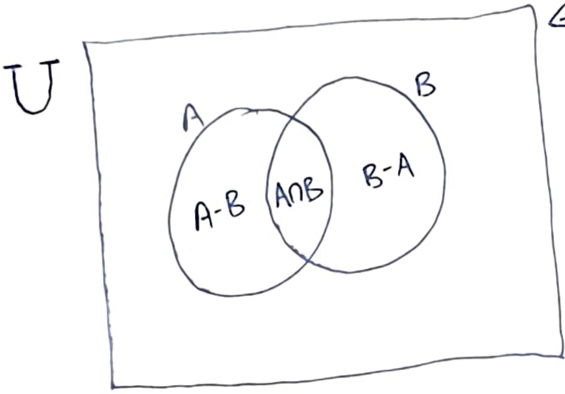
CS1200 Module-1: Discrete Structures

Quick Recap: A, B: sets

We defined $A \cap B$, $A - B$, $B - A$, $A \cup B$ → A union B

$A \cap B$ → A intersection B; $A - B$ → A minus B; $B - A$ → B minus A

Question: What does this box represent?



Question: Any other sets that we can observe?

Answer: ① All elements not in A.
② All elements not in B.

Question: How do we represent these sets?

Answer: ① We subtract A from .....?
② We subtract B from .....?

depends on what you care about

"The Universe" (aka Universal Set) denoted by U

Complement of a set (with respect to a given universe) (w.r.t.)

U: universe

A: some set (subset of U)

Question: What is the complement of U?
Answer: The empty set $\emptyset = \{\}$.

The $\boxed{\text{complement of } A}$ (w.r.t. U), denoted by $\overline{A}$, is the set $U - A$.

complement: (dictionary meaning) one of two mutually completing parts

Note: complement $\neq$ compliment

A, B: sets

Observations: $A \cup B = B \cup A$

$A \cap B = B \cap A$

In general, $A - B \neq B - A$.

DO IT YOURSELF

DIY: U universe; A: set (subset of U)

① What is $A \cap U$?

② What is $A \cup \emptyset$?

③ What is $A \cup U$?

④ What is $A \cap \emptyset$?

⑤ What is $A \cup A$?

⑥ What is $A \cap A$?

⑦ What is $\overline{(\overline{A})}$?

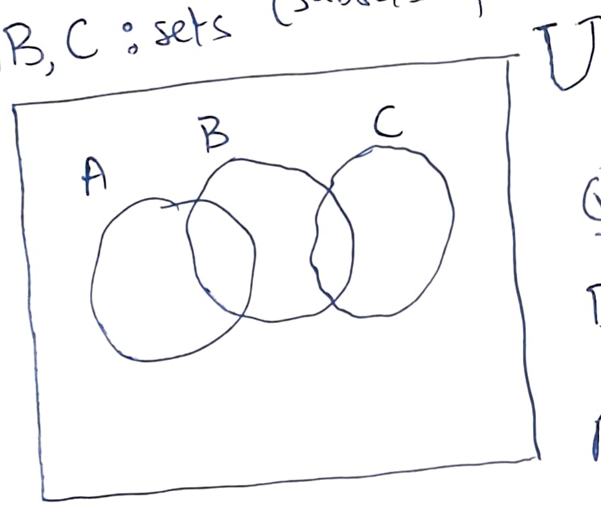
⑧ What is $A \cup \bar{A}$?

⑨ What is $A \cap \bar{A}$?

Question:
Can we generalize any of these operations to more than 2 sets?
What if we have A, B & C?

U: universe

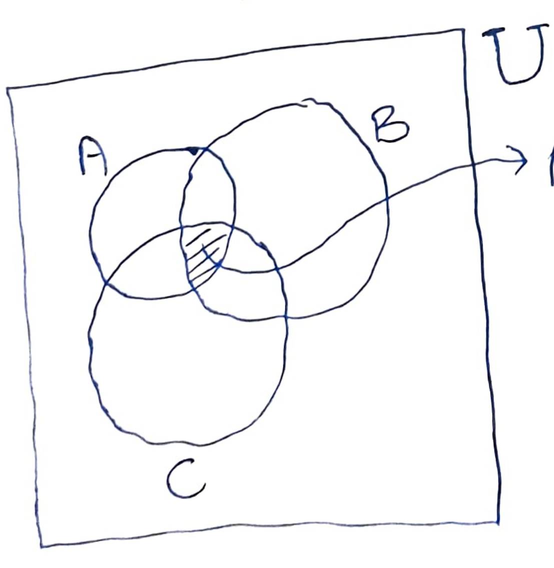
A, B, C: sets (subsets of U)



Question: Is this Venn Diagram general enough?

Answer: NO.

Let's fix this.



$\rightarrow A \cap B \cap C$

Point:

Drawings/figures are great for intuition, but you have to be careful.

Figures can be misleading if one is NOT careful.

Let us generalize intersection to any number of sets:

$A_1, A_2, \ldots, A_n$: sets $(n \geq 1)$

The $\boxed{\text{intersection of } A_1, A_2, \ldots, A_n}$, denoted by $\boxed{A_1 \cap A_2 \cap \ldots \cap A_n}$, is defined as the set that contains those elements that are members of each of the $\underbrace{n \text{ sets}}_{A_1, A_2, \ldots, A_n}$.

Also denoted by $\bigcap_{i=1}^{n} A_i$

Similarly, we can generalize union to any number of sets:

The $\boxed{\text{union of } A_1, A_2, \ldots, A_n}$, denoted by $\boxed{A_1 \cup A_2 \cup \ldots \cup A_n}$, is defined as the set that contains those elements that are members of at least one of the $\underbrace{n \text{ sets}}_{A_1, A_2, \ldots, A_n}$.

Also denoted by $\bigcup_{i=1}^{n} A_i$

Why can't we generalize difference?

$\boxed{A - B \neq B - A}$ (in general) → asymmetric

Whereas $\left.\begin{array}{l} A \cup B = B \cup A \\ A \cap B = B \cap A \end{array}\right\}$ symmetric

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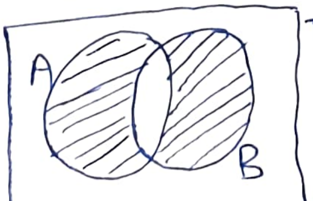
Question: Is it possible to define a "difference operation" that is symmetric?

Answer: YES:



$\dot{\cup}$

The $\boxed{\text{symmetric difference of } A \,\&\, B}$, denoted by $\boxed{A \oplus B}$, is the set containing those elements which belong to exactly one of A & B.

Observe : 1 $A \oplus B = (A-B) \cup (B-A)$ → why? think.

2 $A \oplus B = (A \cup B) - (A \cap B)$ → why? think.

Food for thought : (TIY - TRY IT YOURSELF)

1. Can the symmetric difference operation be generalized to any number of sets?
2. If YES, how? If NO, why NOT?

(Hint: Try examples with 3 & 4 sets.)

(We will answer these questions in Module-2.)

Enough of Set Theory (for now).