Quick Recap: A $\boxed{\text{graph } H \text{ is a subgraph of a graph } G}$ if some graph isomorphic to H can be obtained from G by deleting vertices and/or edges.

we also say: G contains H as a subgraph.

Cycle Graphs: connected 2-regular graphs

recall from Quiz: A graph is $\boxed{k\text{-regular}}$ if each vertex has degree equal to k. $k \in \mathbb{N}$

DIY: what do 1-regular graphs look like? what do 2-regular graphs look like?

FOREST: $\boxed{\text{Acyclic}}$ graph $\xrightarrow{\text{means}}$ does NOT contain any cycle (graph) as a subgraph

TREE: connected forest

Our first nontrivial proof:

NOT trivial $\xrightarrow{\text{means}}$ requires some creativity (or a "leap of imagination")

Theorem: Let G be a graph.
If each vertex of G has degree at least two then G is NOT a forest.

same as $\rightarrow$ G has a cycle (as a subgraph)

GOAL: To prove the following theorem.

Theorem: Let G be a graph.
If each vertex of G has degree at least two then G is NOT a forest.

A (one) Proof:

Let G be a graph whose each vertex has degree at least two.

Observe that G contains a path.
(Why? Each vertex is a path.)

So, G contains a longest path.

means: has maximum length among all paths

(Why? Because G is a finite graph, and so each path has finite length.)

Consider a longest path, say P, in the graph G.

(TIY: Complete the proof yourself.)

How do we prove such a statement?

There are many proofs. We will see one proof (for now).

We will come to this later.

Length of a path / cycle: # of edges