CS1200 Module-1 : Discrete Structures

We begin with some examples of theorems & open problems.
↑   ↑
$\begin{pmatrix} \text{statements} \\ \text{whose proofs} \\ \text{are known} \\ \text{to humankind} \end{pmatrix}$ $\begin{pmatrix} \text{unsolved problems} \\ \text{(till today)} \end{pmatrix}$
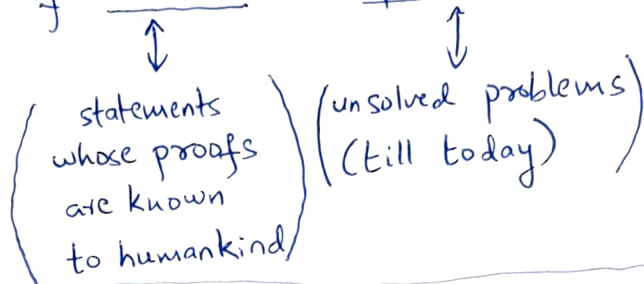
$\mathbb{N}$: set of natural numbers $:= \{0, 1, 2, 3, \ldots \}$
↓
(depends on book/author)

$\mathbb{Z}$: set of integers
$:= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$

Divisibility / Divides :

Examples: ① Does 3 divide 8 ? (Is 8 divisible by 3 ?)

② Does 8 divide 3 ?

③ Does 3 divide 15? ⟶ → YES

④ Does 4 divide 15?   because $15 = 3 \cdot 5$
                                      (q) (p) (s)
⑤ Does 4 divide 17 ?

Definition ~~~~~~ : $p, q \in \mathbb{N}$ and $p \neq 0$.

We say that $\boxed{p \text{ divides } q}$ if there is some ~~~~~ $s \in \mathbb{N}$

such that $q = p \cdot s$ (that is, $q = p * s$).

Many ways to say the same thing:

⓪ q is divisible by p
① p divides q
② p is a factor of q
③ p is a divisor of q
④ q is a multiple of p

Prime numbers :

Definition : A natural number q greater than 1 is called a $\boxed{\text{prime number}}$ if the only factors of q are 1 and q.
↓
$\begin{pmatrix} \text{among natural} \\ \text{numbers} \end{pmatrix}$

Examples: $2, 3, 5, 7, \ldots \ldots, 17, 19, \ldots \ldots 29, 31, \ldots \ldots$

Question: How many prime numbers are there?

Answer: We know the answer! :-)

Theorem: There are infinitely many primes.

↓

requires a proof! (Later.....)

Composite number: positive integer that is NOT prime

Do we know the answers to all mathematical questions?

NO (:-(OR :-) ?)

Definition: Twin primes: a pair of primes (that is, prime numbers) whose difference is 2

Examples: 3 & 5 ; 17 & 19 ; 29 & 31 ; 4967 & 4969

Question: How many twin primes are there?

Answer: We do NOT know! :-(

Really, it is 2023!?!

YES, really! We do NOT know!

Conjecture: (The Twin Prime Conjecture)

There are infinitely many twin primes.

→ Alphonse de Polignac 1849 (French mathematician)

Any other conjectures?

↓

Something that many mathematicians/computer-scientists believe to be true, but NO ONE knows how to prove YET!

Goldbach's Conjecture: (1742)

Every even natural number, greater than 2, is the sum of two primes.