PH-1020
Problem Set - 8
Department of Physics, IIT Madras
Introduction to Quantum Mechanics
March-June 2023 Semester

Notation:

- Notation follows that of **Concepts Of Modern Physics By Arthur Beiser**.

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1. What is the classical assumption adapted by Rayleigh and Jeans to derive the formula of the spectral energy density of blackbody radiation? How is it modified by Planck to make the theoretically calculated spectral density agreeable with the experimental results?

2. Typical energy of a neutron released from nuclear fission is $3.2 \times 10^{-13} J$. Ignoring the relativistic effect, find out the associated de Broglie wavelength. These neutrons are used to determine the crystal structure where the typical interatomic spacing is $2 \times 10^{-10} m$. How much should you slow down the neutron to make the crystal structure detection possible?

3. The size of a given nucleus is $2 \times 10^{-15} m$. How much kinetic energy should an electron possess to become a part of the nucleus?

4. Establish the uncertainty principle for a normalized state given by,
$$\psi(x) = \frac{1}{(\pi\Delta^2)^{1/4}} \exp\left(-\frac{(x-a)^2}{2\Delta^2}\right)$$
This is a Gaussian wave function with Δ quantifying the spread of it.

5. A particle is moving in one dimension, has ground state wavefunction given by $\psi_0 = Ae^{-\frac{\alpha}{2\hbar}x^2}$ (where α is some real constant and A is the normalization constant) belonging to the energy eigenvalue $E_0 = \frac{\hbar\alpha}{2m}$ (where m is the mass). Determine the potential in which the particle moves.

6. An electron having de Broglie wavelength $2 \times 10^{-12} m$. Calculate its
   (a) Kinetic Energy.
   (b) Phase velocity.
   (c) Group Velocity.

7. Do you think $\frac{1}{3}\cos\left(\frac{\pi}{6}x\right)$ is a valid normalized eigenfunction for an infinite potential well? Appropriately prove your answer. If yes, sketch the potential well and the eigenfunction.

Suggested questions:

1. By considering a cubical cavity of volume L^3, show that the density of standing waves in the frequency range ν and $\nu + d\nu$ is
$$G(\nu)d\nu = \frac{8\pi\nu^2}{c^2}d\nu$$