We continue our deep dive into the theory of posets:

$(S, \preceq)$: a poset

$\underline{a} \in S$

We say that:

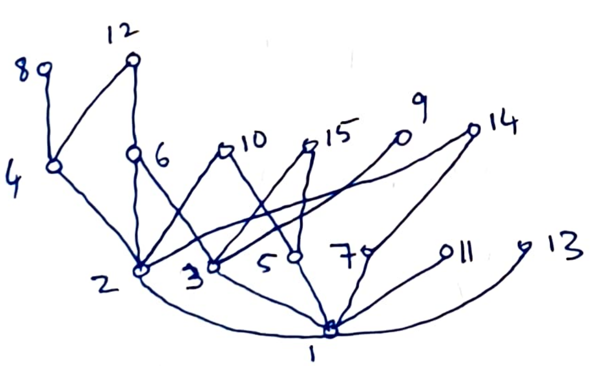
① $\underline{a}$ is $\boxed{\text{minimal}}$ if $\nexists\, b \in S$ such that $b \prec \underline{a}$.

② $\underline{a}$ is $\boxed{\text{minimum}}$ if $\forall\, b \in S$, $\underline{a} \preceq b$.

③ $\underline{a}$ is $\boxed{\text{maximal}}$ if $\nexists\, b \in S$ such that $\underline{a} \prec b$.

④ $\underline{a}$ is $\boxed{\text{maximum}}$ if $\forall\, b \in S$, $b \preceq a$.

Example 1 $(\{1, 2, \ldots, 15\}, \mid)$



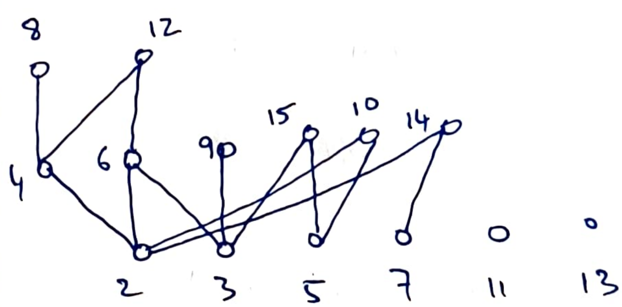
minimal elements: $\{1\}$

maximal elements: $\{8, 12, 10, 15, 9, 14, 11, 13\}$

minimum elements: $\{1\}$

maximum elements: $\emptyset$

Example 2 $(\{2, 3, \ldots, 15\}, \mid)$



minimal elements: $\{2, 3, 5, 7, 11, 13\}$

maximal elements: $\{8, 12, 9, 15, 10, 14, 11, 13\}$

minimum elements: $\emptyset$

maximum elements: $\emptyset$.

Throughout your CSE curriculum, you will encounter the terms minimal & maximal many times, and it is important to distinguish them from minimum & maximum, respectively.

TIY: Prove that every finite poset has a minimal element & a maximal element.

means (at ~~least~~ least one)

(on Assignment-2)

A poset $(S, \preceq)$ is $\boxed{\text{finite}}$ if S is finite.

Let us now revisit our proof of the following theorem:

Theorem: Let G be a graph. If each vertex (of G) has degree two or more then G has a cycle.

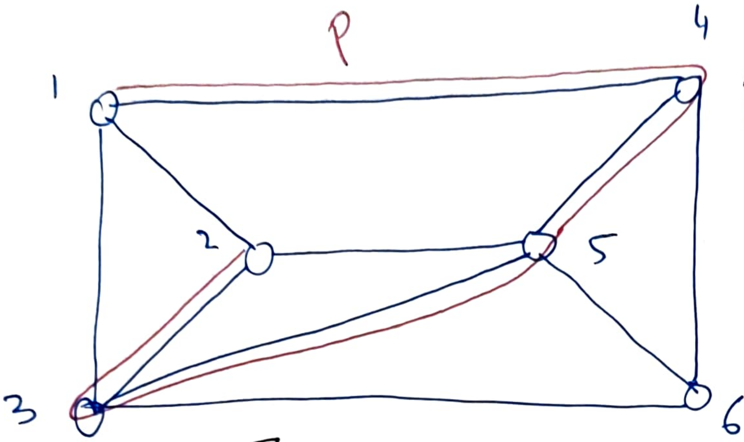
A subgraph H (of a graph G) is a proper subgraph of G if $H \neq G$.

Recall the proof: we considered a longest path (in our proof).

What if — instead of longest path — we consider a maximal path?

A path P (of a graph G) is $\boxed{\text{maximal}}$ if P is NOT a proper subgraph of any other path of G.

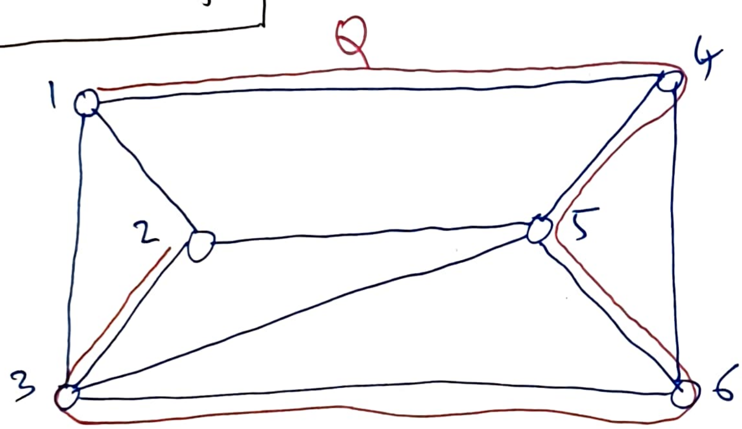
For example: (maximal vs maximum/longest paths)



P := 14532 is a **maximal** path and its length is 4.

because there is NO proper supergraph of P (in given graph) that is also a path.

G is a supergraph of H if H is a subgraph of G.



However, P is NOT a longest ("maximum") path since Q := 145632 is a longest path.

Observe that [redacted] P is NOT a subgraph (subpath) of Q.

Let G be a graph.
DIY: Come up with a poset such that maximal elements of this poset are precisely the maximal paths of G.

Many times, the words maximal/minimal will be used. In most such cases, there is an underlying poset.