Department of Mathematics, IIT Madras

MA1102   Series & Matrices

Assignment-3 (Basic Matrix Operations)

1. Show that given any $n \in \mathbb{N}$ there exist matrices $A, B \in \mathbb{R}^{n\times n}$ such that $AB \neq BA$.
2. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. Compute A^n.
3. Let $A \in \mathbb{F}^{m\times n}$; $B \in \mathbb{F}^{n\times k}$. Let $A_1, \ldots, A_m$ be the rows of A and let $B_1, \ldots, B_k$ be the columns of B. Show that
   (a) $A_1B, \ldots, A_MB$ are the rows of AB.   (b) $AB_1, \ldots, AB_k$ are the columns of AB.
4. Let $A \in \mathbb{F}^{n\times n}$; I be the identity matrix of order n.
   Find the inverse of the $2n \times 2n$ matrix $\begin{bmatrix} I & A \\ 0 & I \end{bmatrix}$.
5. If A is a hermitian (symmetric) invertible matrix, then show that A^{-1} is hermitian (symmetric).
6. If A is a lower (upper) triangular invertible matrix, then A^{-1} is lower (upper) triangular.
7. Show that each orthogonal 2×2 matrix is either a reflection or a rotation.
8. Let $u, v, w \in \mathbb{F}^{n\times 1}$. Show that $\{u + v, v + w, w + u\}$ is linearly independent iff $\{u, v, w\}$ is linearly independent.
9. Find linearly independent vectors from $U = \{(a, b, c) : 2a + 3b - 4c = 0\}$ so that the set of linear combinations of which is exactly U.
10. The vectors $u_1 = (1, 2, 2), u_2 = (-1, 0, 2), u_3 = (0, 0, 1)$ are linearly independent in $\mathbb{F}^3$. Apply Gram-Schmidt Orthogonalization.
11. Let $A \in \mathbb{R}^{3\times 3}$ have the first two columns as $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})^T$ and $(1/\sqrt{2}, 0, -1/\sqrt{2})^T$. Determine the third column of A so that A is an orthogonal matrix.
12. Determine linearly independent vectors so that the set of linear combinations of which is $U = \{(a, b, c, d, e) \in \mathbb{R}^5 : a = c = e, b + d = 0\}$.
