

$$\textcircled{a} \quad \frac{100(10)}{9} = (R + jX)^2 + (jX_C)^2 \rightarrow \textcircled{1}$$

$$\Rightarrow \frac{100}{9} = (R^2 + X^2) + X_C^2 \rightarrow \textcircled{2}$$

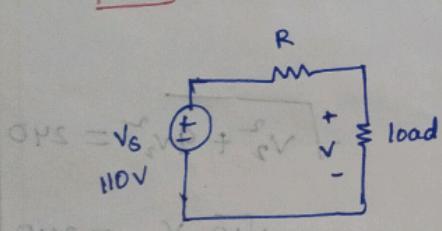
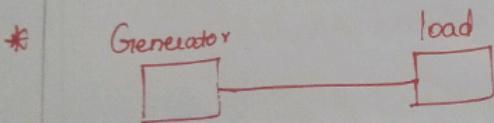
$$\Rightarrow \frac{100}{9} = 16^2 + 16X_C^2 + 12R$$

$$\frac{36}{16} = X_C^2 = \frac{9}{4} \leftarrow \textcircled{2} \text{ & } \textcircled{1}$$

$$\Rightarrow 36 = 16X_C^2 + 12R$$

$$\Rightarrow \boxed{9 = 4X_C^2 + 3R} \quad \text{and use this in eqn } \textcircled{2} \text{ we get}$$

$X_C$  & R



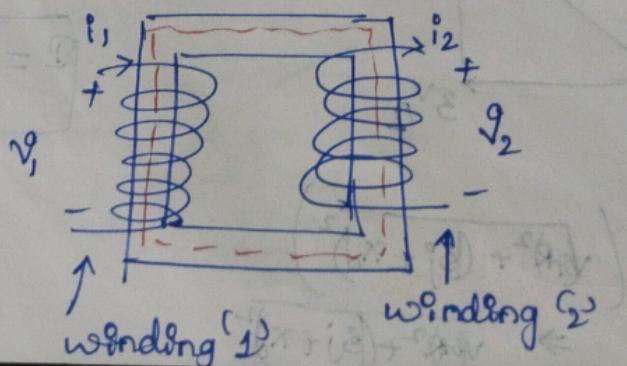
$$-S_1 + S_2 \sqrt{\Delta} = M$$

$$-S_2 + S_1 \sqrt{\Delta} = N$$

$$S_2 X + S_1 R \sqrt{\Delta} = G$$

### \* Transformers:-

### \* Ideal transformer:-



$V_1$  and  $V_2$  are sinusoidal voltages.

$$\textcircled{1} = \text{OP.E.V.}$$

$$\textcircled{2} = \text{OP.E.V.}$$

$N_1$ : No. of turns

$N_2$ : No. of turns

Assumptions:-

(1) Winding

(2) Reciprocal

Magnetic field

Let  $\phi$  be

Ampere's

$\oint H \cdot dL$

No.

$R_1$

$R_2$

$P_1$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$U$

$\Rightarrow$

(q1)  $N_1$ : No. of turns in winding 1

$N_2$ : No. of turns in winding 2

Assumptions:-

① Winding resistances are zero

② Reciprocal of permeability of the core = 0

Magnetic flux is restricted to the core.

Let  $\phi$  be the magnetic flux.

$$v_1 = N_1 \frac{d\phi}{dt}, v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

Ampere's law:

$$\oint \overline{H} \cdot d\overline{L} = \text{current enclosed (in red path)}$$

$\overline{H}$  = magnetic field intensity.

$$\text{for main flux: } 0 = N_1 i_1 - N_2 i_2 \Rightarrow \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

$$\text{for漏 flux: } 0 = M + \frac{i_1}{R_1} L = 0$$

Now deviating from ideality.

$R_1$ : Resistance of winding 1

$R_2$ : Resistance of winding 2

⇒ Permeability of core is finite.

⇒ This results in leakage flux.

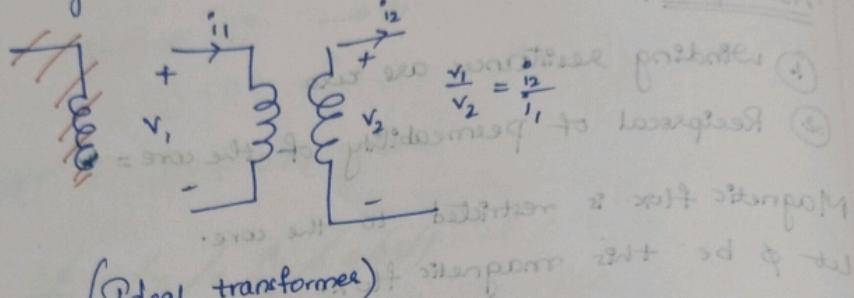
⇒ Leakage flux links few or all turns of only one winding.

⇒ Represented by leakage inductance

(a)

$L_{1e}$ : leakage inductance of winding ① flow of current to side 1,  $i_1$

$L_{2e}$ : leakage inductance of winding ② flow of current to side 2,  $i_2$



(b)

$$L = L_{1e} + N_1 \phi_1$$

$$L_2 = L_{2e} + N_2 \frac{\phi_1}{i_2}$$

$$M_{12} = N_1 \phi_2$$

$$M_{21} = \frac{N_2 \phi_1}{i_1}$$

Assumptions:

① Area of cro

② Magnetic

③ Let 'A' be  
Magnetic

Magnetic

Let 'A'

 $\phi_1 =$  $\frac{ib}{ab}$  $\frac{ib}{ab}$  $M_{12}$ let  $M$ 

$\Phi_1$ : flux linkage of winding ①

$\Phi_2$ : flux linkage of winding ②

$L_1$ : self inductance of winding ①

$L_2$ : self inductance of winding ②

$\Phi_1 =$  flux linkage of winding ①

$\Phi_2 =$  flux linkage of winding ②

$$\Phi_1 = L_1 i_1 + M_{12} i_2$$

$$\Phi_2 = L_2 i_2 + M_{21} i_1$$

$L_1$ : leakage inductance of winding ①

$L_2$ : leakage inductance of

winding ② with coil

$L_1$ : self inductance of winding ① to neutral

$L_2$ : self inductance of winding ② to neutral

$M_{12}$  &  $M_{21}$  are mutual inductances to primary

$\phi_1$ : flux in the core due to  $i_1$ .

$\phi_2$ : flux in the core due to  $i_2$ .

unlinked field between

$$l = l_0 + \frac{N_1 \phi_1}{i_1}$$

$$l_2 = l_0 + \frac{N_2 \phi_2}{i_2}$$

$$M_{12} = \frac{N_1 \phi_2}{i_2} = \frac{ib}{l} M + \frac{ib}{l} \left( M \left( \frac{i_2}{i_1} + 1 \right) + i_2 \right) = \frac{ib}{l} M + \frac{ib}{l} i_2 + \frac{ib}{l} M \left( \frac{i_2}{i_1} + 1 \right) = \frac{ib}{l} M + \frac{ib}{l} i_2 + \frac{ib}{l} M \left( \frac{i_2}{i_1} + 1 \right)$$

$$M_{21} = \frac{N_2 \phi_1}{i_1} = \frac{ib}{l} M + \frac{ib}{l} i_1 + \frac{ib}{l} M \left( \frac{i_1}{i_2} + 1 \right) = \frac{ib}{l} M + \frac{ib}{l} i_1 + \frac{ib}{l} M \left( \frac{i_1}{i_2} + 1 \right)$$

Assumptions:

(1) Area of cross section of core is small.

(2) Magnetic field intensity in the core is uniform.

(3) Let 'l' be the length of the magnetic path in the core.

Magnetic field intensity in the core due to  $i_1 = \frac{N_1 i_1}{l}$

Magnetic field intensity in the core due to  $i_2 = \frac{N_2 i_2}{l}$

Let 'A' be the area of cross section of core,

$$\phi_1 = \frac{ib}{l} M + \frac{ib}{l} \left( \frac{i_2}{i_1} + 1 \right) = \frac{\mu A N_1 i_1}{l}, \quad \phi_2 = \frac{\mu A N_2 i_2}{l}$$

$$l_0 = \frac{ib}{l} + \frac{\mu A N_1^2}{l} + \frac{ib}{l} M + \frac{ib}{l} i_2 =$$

$$l_2 = l_0 + \frac{\mu A N_2^2}{l} + \frac{ib}{l} \left( \frac{i_2}{i_1} + 1 \right) =$$

$$M_{12} = \frac{\mu A N_1 N_2}{l} = M_{21} \quad M \frac{i_2}{i_1} + i_2 =$$

Let  $M = M_{12} = M_{21}$ ,

$$l = l_0 + \frac{N_1}{N_2} M$$

$$l_2 = l_0 + \frac{N_2}{N_1} M$$

(a)

$$N_1 = R_1 i_1 + \frac{d \varphi_1}{dt}$$

$$\varphi_1 = R_1 i_1 + \frac{d}{dt} (L_1 i_1 + M i_2')$$

$$v_1 = R_1 i_1 + \left( L_1 + \left( \frac{N_1}{N_2} \right) M \right) \frac{di_1}{dt} + \frac{M d \varphi_1}{dt}$$

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{N_1}{N_2} M \frac{di_1}{dt} - \frac{M di_2}{dt} = M$$

let  $i_1' = \frac{N_2}{N_1} i_2$

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{N_1}{N_2} M \left( \frac{di_1}{dt} - \frac{di_2'}{dt} \right) \quad \text{①}$$

$$v_2 = R_2 i_2 + \frac{d \varphi_2}{dt}$$

$$v_2 = R_2 i_2' + \frac{d}{dt} (L_2 i_2' + M_{21} i_1)$$

$$= R_2 i_2' + \frac{d}{dt} (M_{21} i_1) - L_2 \frac{di_2}{dt}$$

$$= -R_2 i_2 + M \frac{d}{dt} i_1 - L_2 \frac{N_1}{N_2} \frac{di_1'}{dt}$$

~~$$v_2 = -R_2 \left( \frac{N_1}{N_2} \right) i_1' + M \frac{di_1'}{dt} - \frac{N_1 L_2}{N_2} \frac{di_1'}{dt}$$~~

and  $L_2 = L_{21} + \frac{N_2}{N_1} M$

$$v_2 = -R_2 \left( \frac{N_1}{N_2} \right) i_1' + M \frac{di_1'}{dt} - \frac{N_1}{N_2} \left( L_{21} + \frac{N_2 M}{N_1} \right) \frac{di_1'}{dt}$$

(b)

$$v_2 = -R_2 i_2' - L_2 i_2$$

let  $v_1' = \frac{N_1}{N_2}$

$$v_2' = 1$$

①  $\rightarrow v_1 =$

②  $\rightarrow v_2$

so,

$$i_1 \xrightarrow{R_1}$$

of electy?

$$v_1$$

$$i_2 \xrightarrow{R_2}$$

+  $v_2$

and,  $\frac{v_1'}{v_2'}$

Now

$$v_1'$$

$$i_1$$

$$v_2'$$

$$i_2$$

$$v_2 = -R_2 i_2 - L_2 \frac{di_2}{dt} + M \left( \frac{di_1}{dt} - \frac{di_1'}{dt} \right) \quad (2)$$

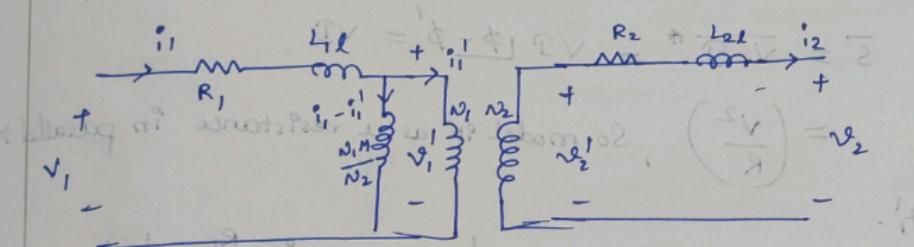
Let  $v_1' = \frac{N_1}{N_2} M \left( \frac{di_1}{dt} - \frac{di_1'}{dt} \right)$

$$v_2' = M \left( \frac{di_1}{dt} - \frac{di_1'}{dt} \right)$$

$$(1) \Rightarrow v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + v_1' \quad (1)$$

$$(2) \Rightarrow v_2 = -R_2 i_2 - L_2 \frac{di_2}{dt} + v_2' \quad (2)$$

So,

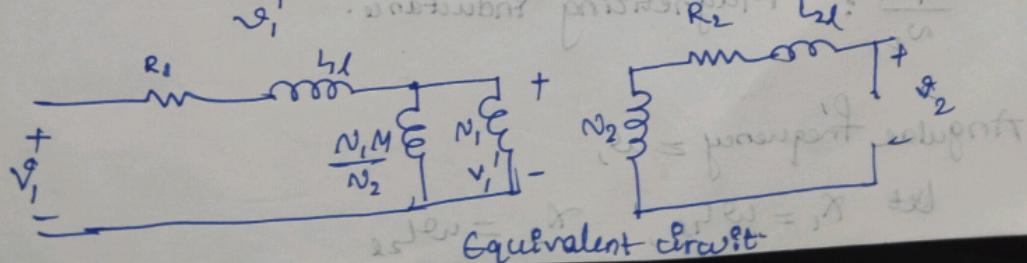


and,  $\frac{v_1'}{v_2'} = \frac{N_1}{N_2} \frac{i_1}{i_2}$

Now

$$v_1' = \frac{N_1}{N_2} M \left( \frac{d}{dt} (i_1 - i_1') \right)$$

$i_1 - i_1'$  → Model it as a self-inductance.  
 $+ \left( \frac{N_1}{N_2} M \right)$

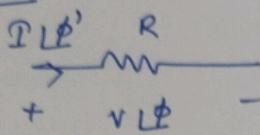


Q6 Core loss:

- 1) Eddy current loss
- 2) Hysteresis loss

Avg core loss is approximately proportional to the RMS value  $V_1'$  or  $V_2'$

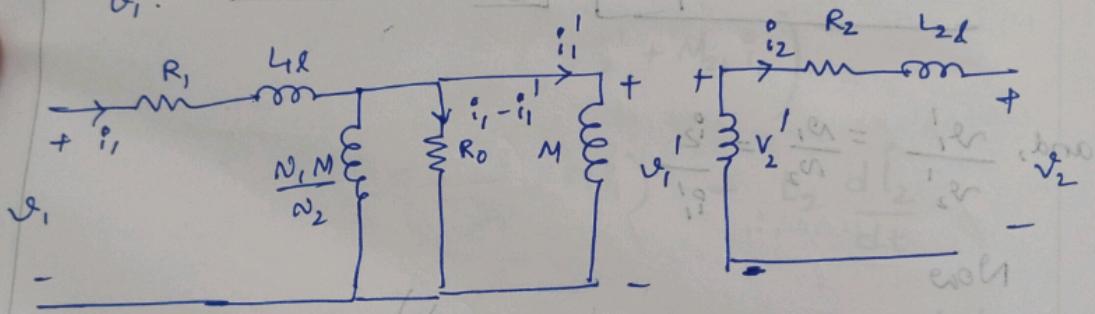
NOTE:



$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = R = \frac{V_L\phi}{I L\phi'} \Rightarrow \boxed{\phi = \phi'}$$

$$\bar{s} = \sqrt{\bar{I}^*} = \sqrt{\bar{I}(\phi - \phi')} = \sqrt{I}$$

$s = \left( \frac{V^2}{R} \right)$ , so model it as a resistance in parallel to  $\bar{V}_1$ .



$R_1, R_2$  represent loss in the conductors.

$L_1L, L_2L$ : leakage inductances.

$R_0$ : Represents core loss

$\frac{N_1 M}{N_2}$ : Magnetizing inductance.

Angular frequency  $\omega$ ,

Let  $X_1 = \omega L_{1e}$ ,  $X_2 = \omega L_{2e}$

$$f_m = \frac{w}{2\pi}$$

$A_1, X_2 :$   
 $X_m :$

$$\frac{I_1}{I_2}$$

$$\frac{V_1}{V_2}$$

Suppos

for

other base

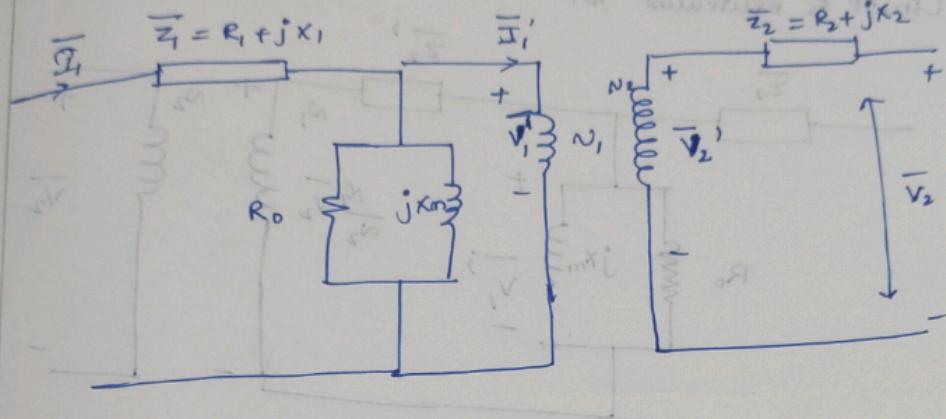
By

$\Rightarrow$

$$f_m = \frac{\omega N_1}{N_2} M$$

$\chi_1, \chi_2$ : leakage reactance

$\chi_m$ : Magnetizing reactance



$$\frac{\bar{V}_1'}{\bar{V}_2'} = \frac{N_1}{N_2} = \frac{R_2 + jX_2}{R_1 + jX_1} = \frac{R_2 + jX_2}{R_1 + jX_1} \left( \frac{1}{N_1} \right) = \frac{R_2 + jX_2}{R_1 + jX_1}$$

$$\text{Suppose } V_1' = \sqrt{2} V_1 \sin(\omega t + \phi_1)$$

$$V_2' = \sqrt{2} V_2 \sin(\omega t + \phi_2)$$

$$\text{for } \frac{V_1'}{V_2'} = \frac{V_1}{V_2} \frac{\sin(\omega t + \phi_1)}{\sin(\omega t + \phi_2)} = \frac{N_1}{N_2}$$

$$\text{Hence } (\phi_1 = \phi_2) \text{ & } \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\boxed{V_1 = \frac{N_1}{N_2} V_2}$$

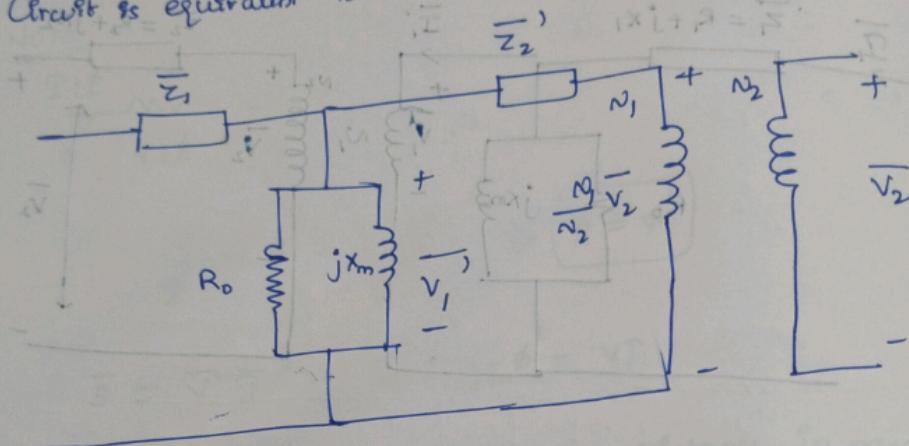
$$\text{By KVL, } \bar{V}_2' = \bar{V}_2 + \bar{I}_2 Z_2$$

$$\Rightarrow \bar{V}_1' = \frac{N_1}{N_2} (\bar{V}_2) + \left( \frac{N_1}{N_2} \right)^2 \bar{R}_1 Z_2$$

$$= \frac{N_1}{N_2} \bar{V}_2 + \bar{I}_1' \bar{z}_2'$$

where  $\bar{z}_2' = \left(\frac{N_1}{N_2}\right)^2 \bar{z}_2$  without magnet M :  $\bar{z}_2 = \bar{R}_2 + j\bar{x}_2$   
another magnet M :  $\bar{z}_2' = \bar{R}_2 + j\bar{x}_2 + \frac{M}{N_2} \bar{V}_2$

Circuit is equivalent to:



$$\bar{z}_2' = \left(\frac{N_1}{N_2}\right)^2 \bar{z}_2, \quad \bar{z}_2 = \bar{R}_2 + j\bar{x}_2$$

$$\Rightarrow \bar{z}_2' = \bar{R}_2 + j\bar{x}_2$$

$$\text{where, } \bar{R}_2' = \bar{R}_2 \left( \frac{N_1^2}{N_2^2} \right) \text{ or } \frac{1}{5} \text{ V } \omega = \frac{1}{5} \text{ V}$$

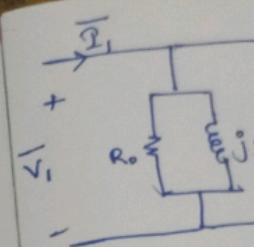
$$\bar{x}_2' = \frac{N_1^2 (\phi + \omega) n_{12}}{N_2^2 (\phi + \omega) n_{12}} \frac{1}{5} \text{ V} = \frac{1}{5} \text{ V}$$

Note:  $R_1, R_1', x_1, x_2'$  are very small when compared with  $R_0$  and  $x_m$ .

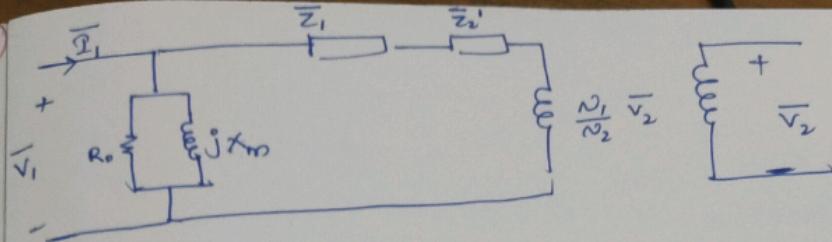
So, shifting  $R_0, x_m$ , we get an approximate equivalent.

$$\sqrt{\frac{R_0}{x_m}} = \sqrt{5}$$

$$\bar{R}' + \bar{V} = \bar{V} \quad \text{But } \bar{R}' \left( \frac{1}{5} \right) + \left( \bar{V} \right) \frac{1}{5} = \bar{V} \Leftarrow$$



(a)



$\bar{V}_1$

$\bar{V}_2$

$\bar{V}_2$

with