Ingeniørhøjskolen Århus

Discrete Mathmatics

Hand in 3

Written by:
Nicolai Glud *Studienummer:* 11102
Johnny Kristensen *Studienummer:* 10734

September 17, 2013

Problems

1 Which of the following sets are well-ordered? (Why/why not?)

a. $S = x \in \mathbb{Q} \ : \ x \geq -10$
b. $S = -2, -1, 0, 1, 2$
c. $S = x \in \mathbb{Q} \ : \ -1 \leq x \leq 1$
d. $S = p \ : \ pisprime = 2, 3, 5, 7, 9, 11, 13, ...$

First we look at the definition:
A set is well-ordered if every nonempty subset has a least element.
Then we look at the sets:

a. S is not well-ordered set because we can make a subset that doesn't contain a least element. e.g. x > -10.
b. S is a well-ordered set because we can always produce a least element from the subsets of S.
c. S is not a well-ordered set because we can make a subset that doesn't contain a least element. e.g. 0 < x ≤ 1.
d. S is a well-ordered set because we can always produce a least element from the subsets of S.

2 Use mathematical induction to prove that $1 + 5 + 9 + ... + (4n - 3) = 2n^2 - n$ for every positive integer n.

We have $n \in \mathbb{Z}^+$.
We start by establishing the base case:

$$(4n - 3) = 2n^2 - n \Rightarrow (4 * 1 - 3) = 2 * 1^2 - 1 \tag{1}$$

3 Prove that $2^n > n^3$ for every integer $n \geq 10$

Note: you will need to really work with inequalities.

4 Use the method for minimum counterexample to prove that $3|(2^{2n} - 1)$ for every positive integer n.

5 Use the Strong Principle of Mathmatical Induction to prove the following:

Let $S = i \in \mathbb{Z} \ : \ i \geq 2$ and let P be a subset of S with the properties that $2, 3 \in P$ and if $n \in S$, then either $n \in P$ or $n = ab$, where $a, b \in S$. Then every element of S either belongs to P or it can be expressed as a product of elements of P.
Note: read Theorem 11.17, though the proof of 11.17 is not the proof of this question.

6 Use the Strong Principle of Mathematical Induction to prove that for each integer $n \geq 12$, there are non-negative integers a and b such that $n = 3a + 7b$.

Note: this uses generalized strong induction and minimum counterexamples.