Ingeniørhøjskolen Århus

Discrete Mathmatics

# Case 3

*Written by:*
Nicolai Glud *Studienummer:* 11102
Johnny Kristensen *Studienummer:* 10734

September 11, 2013

# Chapter 1

# Problems

## 1.1 Disprove the statement: $n \in 0, 1, 2, 3, 4, then 2^n + 3^n + n(n-1)(n-2)$ is prime.

We assume $2^n + 3^n + n(n-1)(n-2)$ produces a prime for n = 4.
Therefore:

$$2^4 + 3^4 + 4(4-1)(4-2) = 16 + 81 + 24 = 121 = 11^2 \tag{1.1}$$

$11^2$ is not a prime. This disproves the original statement with a counterexample.

## 1.2 Let $a, b \in \mathbb{Z}$ Disprove the statement: if ab and $(a+b)^2$ are of opposite parity, then $a^2b^2$ and a+ab+b are of opposite parity.

Let $a = 2k+1$ and $b = 2k+1$ We insert into the equations:

$$(2k+1)*(2k+1) = 4k^2 + 4k + 1 = 2(2k^2 + k) + 1 \tag{1.2}$$

$$((2k+1) + (2k+1))^2 = (4k+2)^2 = 16k^2 + 16k + 4 = 2(8k^2 + 8k + 2) \tag{1.3}$$

Then

$$(2k+1)^2*(2k+1)^2 = (4k^2+4k+1)*(4k^2+4k+1) = 16k^4+32k^3+24k^2+8k+1 = 2(8k^4+16k^3+12k^2+4k)+1 \tag{1.4}$$

$$(2k+1)+(2k+1)*(2k+1)+(2k+1) = (2k+1)+4k^2+4k+1+(2k+1) = 4k^2+8k+3 = 2(2k^2+4k+1)+1 \tag{1.5}$$

## 1.3 Let $a, b \in \mathbb{R}^+$. Use a proof by contradiction to prove that x<y, then $\sqrt{x} < \sqrt{y}$

Hest

## 1.4 Prove that there is no largest negative rational number.

(Note: -1 is larger than -2.)

## 1.5 Prove that there exists no pisitive integer x such that 2x $< x^2 <$ 3x.

hest.jpg

## 1.6 Prove that if n is an odd integer, then 7n-5 is even by

a) direct proof,
b) proof by contrapositive,
c) proof by contradition.

hest.png

## 1.7 Show that there exist two distinct irrational numbers a and b such that $a^b$ is rational.

hest.m

## 1.8 Disprove the statement: There is an integer n such that $n^4 + n^3 + n^2 + n$ is odd.

hest.exe