INGENIØRHØJSKOLEN ÅRHUS

DISCRETE MATHMATICS

Case 3

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Chapter 1

Problems

1.1 Disprove the statement: $n \in 0, 1, 2, 3, 4, then 2^n + 3^n + n(n-1)(n-2)$ is prime.

We assume $2^n + 3^n + n(n-1)(n-2)$ produces a prime for n $= 4$.
Therefore:

$$2^4 + 3^4 + 4(4-1)(4-2) = 16 + 81 + 24 = 121 = 11^2 \tag{1.1}$$

11^2 is not a prime. This disproves the original statement with a counterexample.

1.2 Let $a, b \in \mathbb{Z}$ Disprove the statement: if ab and $(a+b)^2$ are of opposite parity, then a^2b^2 and a+ab+b are of opposite parity.

For ab to be negative (because something squared can never be negative) either one of them but not both must be negative. We let $a = 1$ and $b = -1$. Substituting this in $a + ab + b$ we get:

$$1 + 1 * (-1) + (-1) = -2 \tag{1.2}$$

1.3 Let $a, b \in \mathbb{R}^+$. Use a proof by contradiction to prove that x<y, then $\sqrt{x} < \sqrt{y}$

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1.4 Prove that there is no largest negative rational number.

(Note: -1 is larger than -2.)

1.5 Prove that there exists no pisitive integer x such that 2x $<$ x^2 $<$ 3x.

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1.6 Prove that if n is an odd integer, then 7n-5 is even by

a) direct proof,
b) proof by contrapositive,
c) proof by contradition.

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1.7 Show that there exist two distinct irrational numbers a and b such that a^b is rational.

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1.8 Disprove the statement: There is an integer n such that $n^4 + n^3 + n^2 + n$ is odd.

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