

To what extent can elliptic curves be used to establish a shared secret over an insecure channel?

Outline

1. Describe \mathbb{Z}_p^\times as a group
 - a. Group properties: closure, invertibility, existence of identity, associativity
2. Describe the discrete log problem
 - a. Go over an example
3. Describe how the discrete log problem is used for diffie-hellman key exchange
4. A sketch/example on index calculus with finite field diffie-hellman
 - a. Then explain general number field sieve and how that as a special form of index calculus can speed things up.
5. Describe how elliptic curves form a group
 - a. Then, how elliptic curves can also be used for diffie-hellman key exchange.
6. Formalize pollard's ρ algorithm, and how it can attack discrete logs for groups in general, in $O(\sqrt{n})$ time.
7. Describe the use of Diffie-Hellman in the real world with the TLS 1.3 algorithm
8. Compare the performance of a single group operation in both finite field diffie-hellman and elliptic curve diffie-hellman and form conclusions
9. Comparison for space efficiency for elliptic curves, size of group elements for elliptic curves compared to finite fields.

Tentative Table of Contents

1. Group Theory
 - a. Multiplicative group modulo a prime
 - b. The discrete log problem
2. Finite Field Cryptography and Attacks
 - a. Diffie-hellman key exchange
 - b. Index Calculus
 - c. General Number Field Sieve
3. Elliptic Curve Cryptography
 - a. Elliptic curve groups
 - b. Elliptic curve diffie-hellman
 - c. Pollard's ρ algorithm
4. Evaluation
 - a. Diffie-Hellman used in the real world: TLS 1.3
 - b. Performance of group operations
 - c. Comparison of space efficiencies