To what extent can elliptic curves be used to establish a shared secret over an insecure channel?

Outline

1. Describe $\mathbb{Z}_p^\times$ as a group
   a. Group properties: closure, invertibility, existence of identity, associativity
2. Describe the discrete log problem
   a. Go over an example
3. Describe how the discrete log problem is used for diffie-hellman key exchange
4. A sketch/example on index calculus with finite field diffie-hellman
   a. Then explain general number field sieve and how that as a special form of index calculus can speed things up.
5. Describe how elliptic curves form a group
   a. Then, how elliptic curves can also be used for diffie-hellman key exchange.
6. Formalize pollard's ρ algorithm, and how it can attack discrete logs for groups in general, in $O(\sqrt{n})$ time.
7. Describe the use of Diffie-Hellman in the real world with the TLS 1.3 algorithm
8. Compare the performance of a single group operation in both finite field diffie-hellman and elliptic curve diffie-hellman and form conclusions
9. Comparison for space efficiency for elliptic curves, size of group elements for elliptic curves compared to finite fields.

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