LECTURE ONE
ELECTROSTATICS

1.0 Introduction

Electrostatics is the branch of science that deals with the electrical phenomena that arises from stationary electric charges. There are two kinds of electric charge: positive charge (+) and negative charge (-). Like charges repel each other, while unlike charges attract each other. Objects can be charged, either positively or negatively, by the removal or addition of electrons. A hard rubber rod rubbed vigorously with fur and then suspended by a nonmetallic thread will attract a glass rod that has been rubbed with silk. On the other hand, if two charged rubber rods or two charged glass rods) are brought near each other, the two repel each other. This shows that rubber and glass are in two different states of electrification. The electric charge on the rubber rod is called negative and that on the glass rod is called positive.

1.1 Coulomb's Law

Coulomb's law states that in free space, oppositely charged bodies attract each other, while similarly charged bodies repel with a force that varies directly as the product of the magnitude of each charge and inversely as the square of the distance between them, the force being directed along the line joining the charges.

If two particles carrying charges Q_1 and Q_2 are separated by a distance r_{i2} in a vacuum as shown in Figure 1.1, then the electric force exerted by the particle with charge Q_1 on the particle with charge Q_2 is given as

$$F_{12} \propto \frac{Q_1 \, Q_2}{r_{12}^2}$$

$$F_{12} = \frac{kQ_1 \, Q_2}{r_{12}^2}$$

where k is the constant of proportionality and F_{i2} is the force exerted by $Q_2 \, on \, Q_1$.

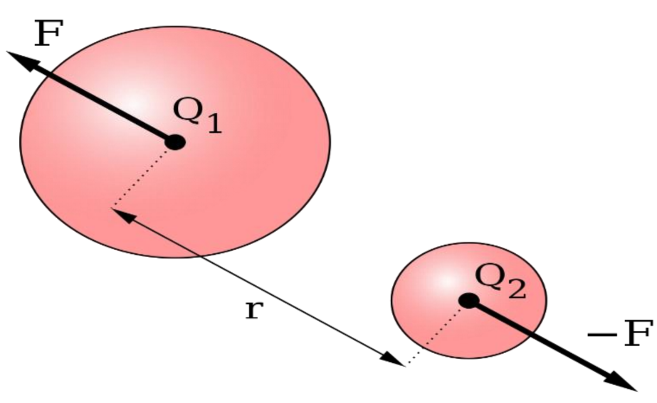


Figure 1.1: Coulomb's law

In vector notation the Coulomb's law can be written as

$$F_{12} = \frac{Q_1\,Q_2}{|r_{12}^2|} r_{12}$$

Where $\hat{r}_{12} = \frac{r_{12}}{|r_{12}|} = \frac{r_1 - r_2}{|r_1 - r_2|}$ is the unit vector along r_{12}

And

$$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^{-12} Nm^2C^{-2}$$

Where$\varepsilon_o = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$

If the charges are similar (i.e., both positive or both negative), **F12** is positive and it is a force of repulsion while if they are unlike charges**, F12** is negative and is a force of attraction.

1.2 Electric Field Strength

An electric field can be defined as a region where a stationary charged particle experiences an electric force. It can be mapped out by lines of electric force. A line of force, also called electric flux, may 1 Refined as a line that the tangent to it at a point P is in the direction of the force on a small position charge placed at P. Figure 1.2 shows electric field lines of force around a positive and a negative charge. The lines originate from the positive charge in Figure 1.2b and terminate in the negative charge Figure 1.2a shows the repulsive lines of force when two positive charges are close to each other.

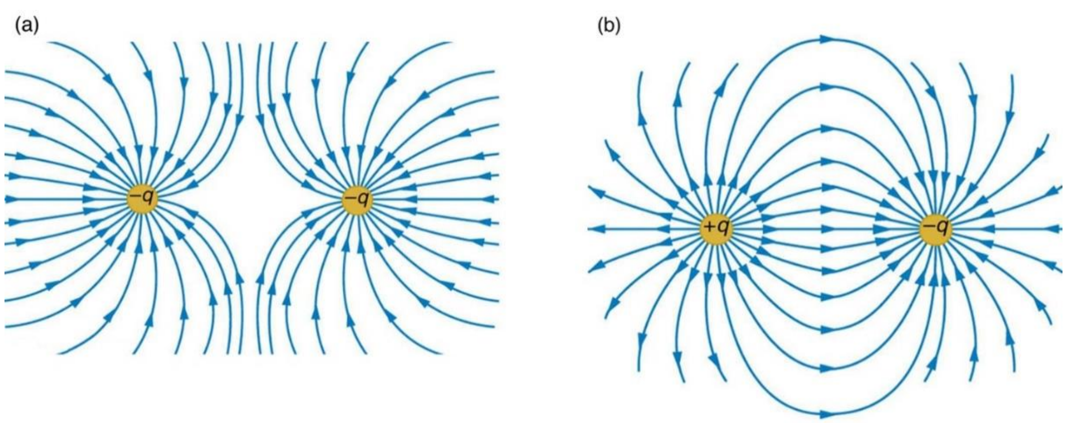


Figure 1.2: Electric field lines

The properties of the electric lines of force include:

- The electric lines of force are drawn such that the magnitude of the electric field is proportional to the number of lines crossing a unit area perpendicular to the lines.
- The tangent to the lines of force at every point gives the direction of the field at that point.
- The lines of force are continuous and they start on positive charges and end only on negative charges.
- Lines of force do not touch or intersect one another.

The electric field strength at any point is defined as the force per unit charge which it exerts at that point,

$$E = \frac{F}{q_o} - - - - - - - - - - - - - 1.3$$

where q_0 is the test charge placed at the point.

If a point charge Q is located at a distance r away from a test charge q_o at point P, then the force exerted on the test charge due to the charge Q, according to Coulomb's law, is

$$F = \frac{kQq_o}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector pointing from Q to q_o Using Equation 1.3, we have?

$$E = \frac{F}{q_o} = \frac{kO}{r^2}\hat{r} - - - - - - - - - - - - - 1.4$$

The unit of the electric field strength is newton per coulomb (**N/C**).

1.3 Electric Field of Continuous Charge Distribution

To evaluate the electric field created by a continuous charge distribution, we first divide the charge distribution into small elements; each of which contains a small charge Δq, as shown in Figure 1.3. We then use Equation 1.4 to calculate the electric field due to one of these elements at a point P. Finally, we evaluate the total field at P due to the charge distribution by summing the contributions of all the charge elements,

$$\Delta E = \frac{k\Delta q}{r^2}\hat{r} - - - - - - - - - - 1.5$$

where r is the distance from the small charge element Δq to point P and $\hat{r}$ is the unit vector directed from the charge element toward P.

The electric field at P to a continuous charge distribution ΔE due to all the sum of the elements Δq of the charge distribution.

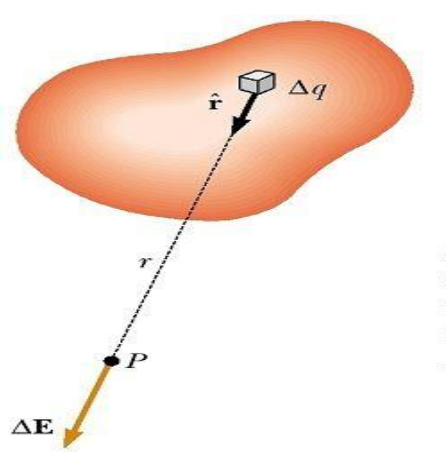


Figure 1.3: Continuous charge distribution

The total electric field at P due to all elements in the charge distribution is

$$E = k \lim_{\Delta q \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{r} = k \int \frac{dq}{r^2} \hat{r} - - - - - - - - -1.6$$

where the integration is over the entire charge distribution.

For example, if a charge Q is uniformly distributed along a line of length l (Figure 1.4), the linear charge density λ is defined by

$$\lambda = \frac{Q}{l}$$

where the units of λ is coulombs per metre (C/m).

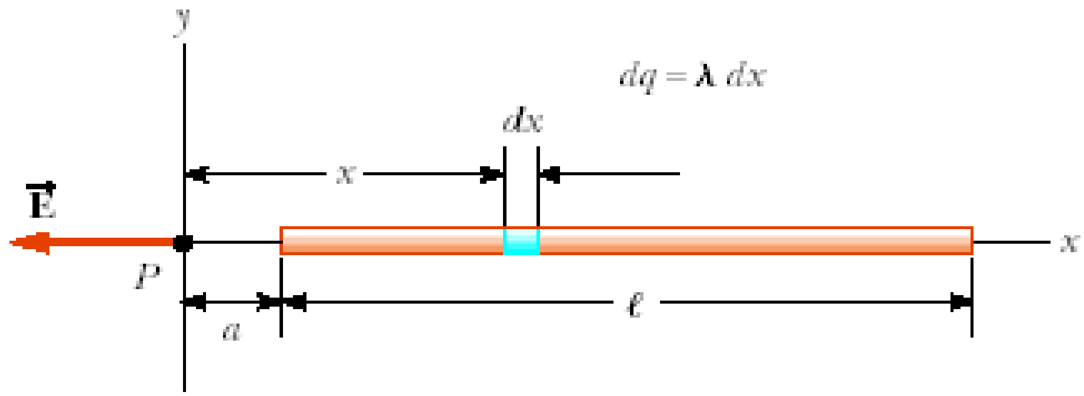


Figure 1.4: Electric field due to a charged rod

Consider a segment of the rod dx having charge $dq = \lambda dx$. The field $\boldsymbol{dE}$ at point **P** due to this length dx of the rod (at x) has magnitude

$$dE = k \frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_o} \frac{\lambda dx}{x^2}$$

$$E = \int_a^{l+a} dE = \frac{\lambda}{4\pi\varepsilon_o} \int_a^{l+a} \frac{dx}{x} = \frac{\lambda}{4\pi\varepsilon_o} \left| -\frac{1}{x} \right|_a^{l+a}$$

$$E = \frac{1}{4\pi\varepsilon_o} \frac{Q}{l} \left(\frac{1}{a} - \frac{1}{l+a} \right) = \frac{Q}{4\pi\varepsilon_o a(l+a)}$$

However, if a charge Q is uniformly distributed on a surface of area A, the surface charge density σ is defined by

$$\sigma = \frac{Q}{A}$$

Where the units of σ is coulombs per square metre (C/m^2), and if the charge Q is uniformly distributed throughout a volume V, the volume density p is defined by

$$\rho = \frac{Q}{V}$$

Where the units of ρ is coulombs per cubic metre (C/m^3).

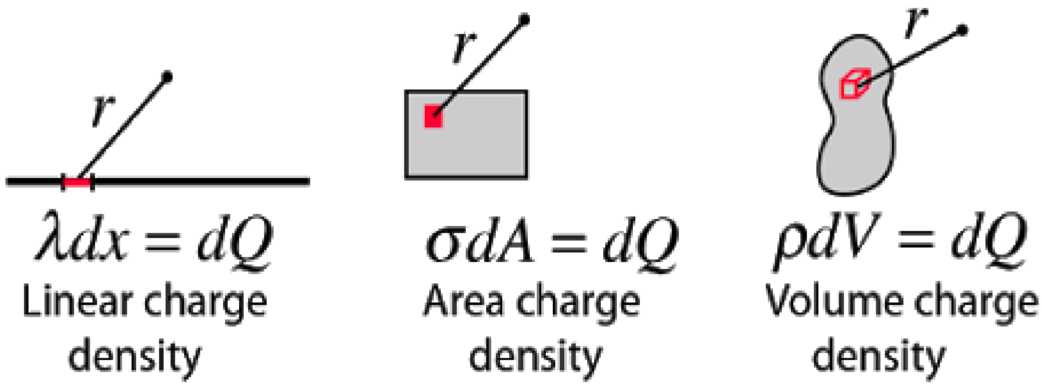


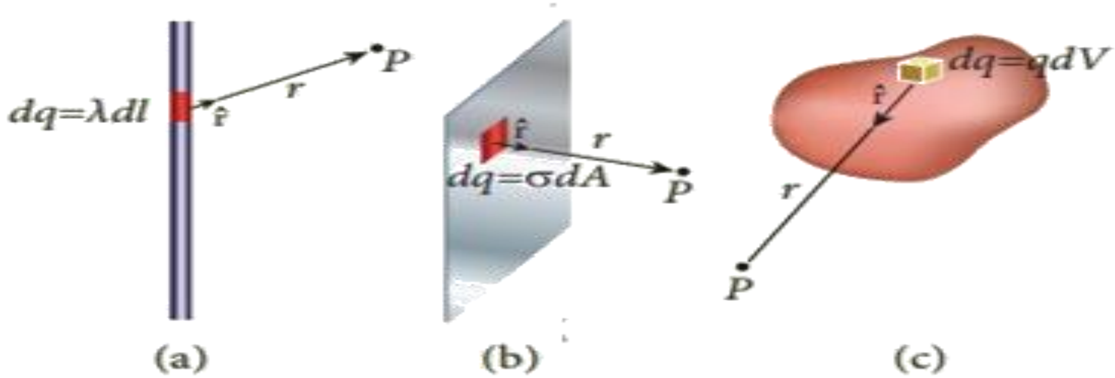
Figure 1.5: Continuous charge distribution of various symmetry.

If in Equation 1.6 the charge dq is a volume charge distribution, we consider the element $dq = pdV$ that is small enough to be considered as a point charge. The total electric field E at a point P will then be given as

$$E = k\int \frac{\rho dV}{r^2}\hat{r}$$

If it is a surface charge distribution, we have

$$E = k\int \frac{\sigma dA}{r^2}\hat{r}$$



1.4: Motion of Charged Particles in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field E, the charge experiences a force of magnitude qE. This force can cause the body to accelerate according to Newton's second law of motion i.e., $\boldsymbol{F = qE = ma}$ (provided the electric force is the only force exerted on the particle).

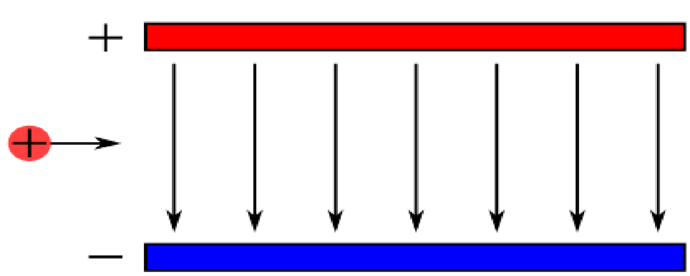


Figure 1.7: Motion of Charged Particles in a Uniform Electric Field

The acceleration of the particle is therefore

$$a = \frac{qE}{m}$$

If **E** is constant in magnitude and direction, that is uniform, then the acceleration **a** is constant. If the particle is positively charged, then its acceleration is in the direction of the electric field. If the particle is negatively charged, then its acceleration is in the direction opposite the electric field.

1.6 Electric Flux

The electric flux is defined as the product of electric field intensity. E and the area A perpendicular to the field. The electric flux is proportional to the number of electric field lines penetrating a surface. For an electric field that is uniform in both magnitude and direction, the electric flux Φ_E (uppercase Greek phi) is defined as

$$\Phi_E = EAcos\,\theta$$

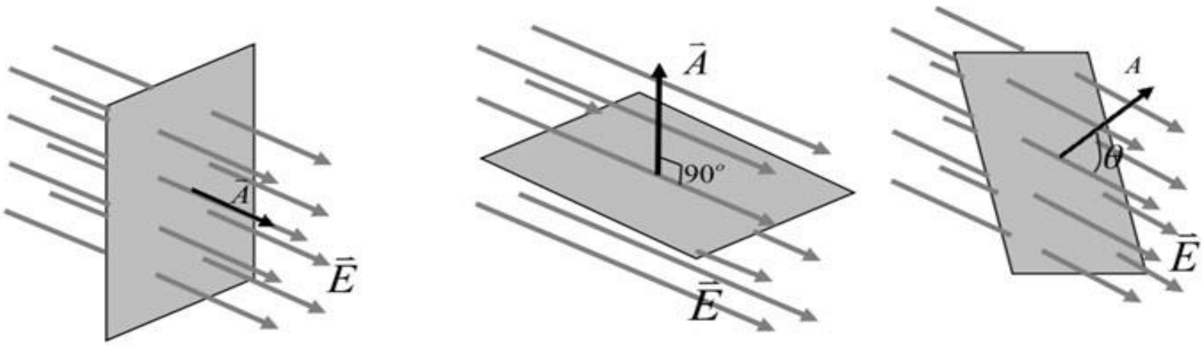


Figure 1.8: Electric flux through a uniform electric field E

Where $Ecos\theta$ is the component of **E** along the perpendicular to the area (Figure 1.8).
The S.I. units of Φ_E is newton-meters squared per coulomb (**Nm²/C**).

In more general situations, the electric field may vary over a surface, Figure 1.6. Imagine that the surface is divided up into a large number of small elements, each of area Φ_E. If the element of area Φ_E is crossed by an electric field E, in the direction which makes an angle θ with the normal to the area, then the electric flux Φ_E crossing the area ΔA is given by

$$\Phi_E = E\Delta Acos\,\theta = E \cdot \Delta A$$

where we have used the definition of a scalar product of two vectors ($A \cdot B\ =\ AB\ cos\theta$).

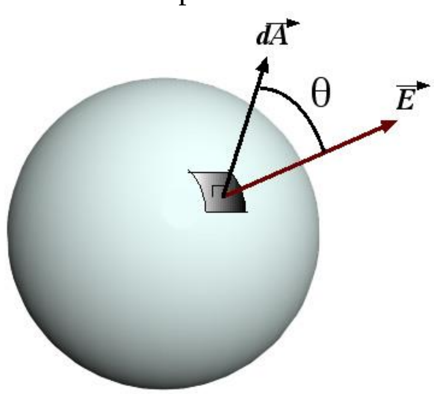


Figure 1.6: Electric flux through a curved surface

The net flux

$$\Phi_E = \sum \Phi_E = \oint E \cdot dA = \oint E\cos\theta \, dA$$

1.7 Gauss's Law

Gauss's law states that the net flux through any closed surface is

$$\Phi_E = \oint E \cdot dA = \frac{Q_{in}}{\varepsilon_o}$$

Where Q_{in} represents the net charge inside the surface, E represents the electric field at any point on the surface and ε_o is the same constant (permittivity of free space) that appears in Coulomb's law.

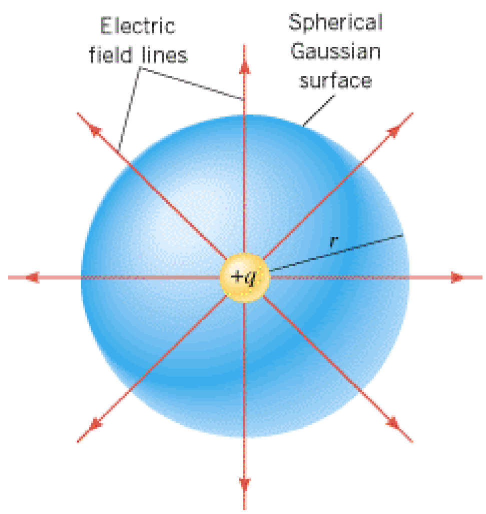


Figure 1.7 Point charge Q at the center

Gauss Law states that the total electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity. The electric flux in an area is defined as the electric field multiplied by the area of the surface projected in a plane and perpendicular to the field.

Let us see how Gauss's law is related to Coulomb's law. Figure 1.7 shows a single isolated charge Q. The Gaussian surface for this point charge is an imaginary sphere of radius r centered on the charge. Since the imaginary sphere is symmetrical about the charge at its centre, we know that E must have the same magnitude at any point on the surface, and that E points radically outward parallel to $d\boldsymbol{A}$, an element of the surface area. We can therefore write Gauss's law as

$$\oint E \cdot dA = E \oint dA = E(4\pi r^2)$$

Note that the surface area of a sphere of radius *r* is $4\pi r^2$, and the magnitude of **E** is the same at all points on the Gaussian spherical surface. Equation 11.32 then becomes

$$\frac{Q}{\varepsilon_0} = \oint E \cdot dA = E = (4\pi\, r^2)$$

Solving for ***E*** we obtain

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

which is the electric field form of Coulomb's law?

We can also do the reverse, that is, we can derive Gauss's law from Coulomb's law for static electric charges. Consider a single point charge Q shown in Figure 11.8 and an imaginary spherical surface A_u which is symmetrical.

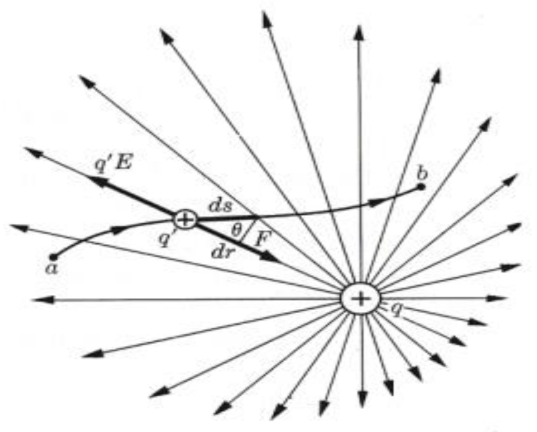


Figure 1.8: A single point charge.

The figure shows an imaginary spherical surface around the charge. From Coulomb's law, we have

$$\oint E \cdot dA = \oint \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}dA = \frac{Q}{4\pi\varepsilon_o r^2}\,(4\pi\, r^2) = \frac{Q}{\varepsilon_0}$$

This is Gauss's law.

We can also have unsymmetrical Gaussian surface marked as A_2 in Figure 1.8. The same number of field lines pass through both surfaces $(A_1 and\ A_2)$, therefore the flux through A_2 is the same as through A_1:

$$\oint_{A_1}^{A_2} E \cdot dA = \frac{Q}{\varepsilon_o}$$

This shows that

$$\oint E \cdot dA = \frac{Q}{\varepsilon_o}$$

would be valid for any surface surrounding a single point charge Q.

Note:

- The net flux through any closed surface is independent of the shape of that surface.
- The net flux through any closed surface surrounding a point charge q is given by q/ε_o.
- The net electric flux through a closed surface that surrounds no charge is zero.

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