

Portfolio Optimization via Pair Copula-GARCH-EVT-CVaR Model

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Abstract

This paper uses CVaR as the risk measure and applies EVT to model the tails of the return series so as to estimate risk of assets more accurately. This paper also applies pair Copula to capture the inter-dependence structure between assets and constructs pair Copula-GARCH-EVT model; then, we combine it with Monte Carlo Simulation and Mean-CVaR model to optimize portfolio. Finally, an empirical study of four Indexes from Chinese Stock Market is performed and the result suggests that pair Copula can better characterize the inter-dependence structure between assets and the performance of pair Copula-GARCH-EVT-CVaR model is better than that of multivariate t Copula-GARCH-EVT-CVaR model in portfolio optimization.

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1. Introduction

Markowitz [1] pioneers the construction of the optimal portfolio taking the combination of risk and return into account and proposes the Mean-Variance model, characterizing portfolio return with expected return and estimating portfolio risk with variance, which laid the cornerstone of the development of modern portfolio theory. Variance has no direction, but most investors only regard the adverse deviation between return and expected return as risk, thus subsequent researchers introduce other risk measures such as semi-variance and semi-deviation. Baumol [2] first proposes Value-at-Risk (VaR), which summarizes the worst loss over a target horizon at a given confidence level. Because VaR is easy to calculate and agrees instincts of investors, it has been widely accepted as the risk measure by many financial institutions and financial regulators. However, when the asset return is non-normal distribution, VaR is not sub-additive and convex, so it is not the coherent measure of risk proposed by Artzner et al. [3]. Taking the non-normal features of asset returns into account, in order to overcome the shortcomings of VaR, Uryasev and Rockafellar [4] suggests an alternative risk measure: Conditional Value at Risk (CVaR). CVaR has close relationship with VaR, inherits most advantages of VaR and satisfies sub-additivity and convexity, so it is a coherent measure of risk. This paper uses CVaR as the risk measure and applies Mean-CVaR model for portfolio optimization.

Since VaR and CVaR are greatly affected by the tail distribution of risk factors, it is able to characterize tail distributions and estimate risk of assets more accurately applying extreme value theory (EVT) to research the tails of the return series. However, the return series are not always independently and identically distributed but

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leptokurtosis, fat tails and volatility clustering, it is inappropriate to apply EVT to the return series directly. Following the approach of Frey and McNeil [5], we will first use GARCH model to fit the return series and then apply EVT to the innovations.

Additionally, in order to accurately estimate CVaR of the portfolio, we need to capture the non-linear inter-dependence between tails of asset returns effectively. This paper will apply Copula to model the inter-dependence between innovations of the return series, which can characterize various dependence structures comprehensively and effectively. Sklar [6] first proposes Copula to measure the non-linear inter-dependence between variables. After that, Schweizer and Wolff, Genest and Mackay, Joe, H., Nelsen and so on further develop and improve Copula and it has become an important approach to construct multivariable joint distribution and capture dependence structure between variables. Embrechts et al. [7] first introduce Copula to financial research; Cherubini [8] systematically summarizes the applications of Copula in finance; Jondeau and Rockinger [9] propose Copula-GARCH model and apply it to extract the dependence structure between stock markets. In China, Zhang [10] first introduces Copula; Wei and Zhang [11] apply Copula-GARCH model to extract the dependence structure between Shanghai Stock Market and Shenzhen Stock Market; Wu and Chen [12] apply Copula-GARCH model to analyze portfolio risk in Chinese Stock Market; Wen and Yang propose to use a copula-based correlation measure to test the interdependence among stochastic variables and find that the new correlation coefficient C_0 is more suitable to describe the interdependence among stochastic variables than the Gini correlation coefficient [13].

However, most empirical researches about Copula have focused on capturing bivariate inter-dependence structure. The reason is that the complexity of multivariate Copula increases sharply along with its dimensions and it is inclined to ignore the influence of its dimensions and the differences of tail dependence between variables. Berdford and Cooke [14] introduce probability structure model of multivariate distribution via simple building block—pair Copula based on vine structure, which decomposes multivariate density into a series of pair Copula times marginal density.

The paper is organized as follows. In section 2, we introduce the pair Copula decomposition of a general multivariate distribution and its parameters estimation. In section 3, combining GARCH model and EVT we construct pair Copula-GARCH-EVT model. In section 4, we construct Mean-CVaR model and combine pair Copula-GARCH-EVT model to optimize portfolio. In sections 5 we carry out empirical analysis. Section 6 concludes.

2. Pair Copula decomposition of multivariate distribution and parameter estimation

According to Sklar theory, multivariate distribution F with marginals $F_1(x_1), F_2(x_2), \dots, F_n(x_n)$ can be written as follows with some appropriate n -dimensional Copula C .

$$F(x_1, x_2, \dots, x_n) = C\{F_1(x_1), F_2(x_2), \dots, F_n(x_n)\} \quad (1)$$

If F is absolutely continuous and strictly increasing, its density can be factorized as

$$f(x_1, x_2, \dots, x_n) = c_{1 \dots n}\{F_1(x_1), F_2(x_2), \dots, F_n(x_n)\} \cdot f_1(x_1) \cdots f_n(x_n) \quad (2)$$

In formula (2), $f_i(x_i)$ is the probability density of $F_i(x_i)$, $c_{1 \dots n}(\cdot)$ is n -dimensional Copula density.

We can know from formula (2) that multivariate density contains two parts: inter-dependence structure among variables and marginal density. Marginal density is easy to get but dependence structure is very complex. Pair Copula decomposition provides an approach to decompose multivariate inter-dependence structure, which decomposes multivariate density into a series of pair Copula times marginal density according to certain logical structure [15].

Multivariate distribution has a number of possible pair Copula constructions. For example, a five-dimensional density has 240 possible different constructions. Berdford and Cooke [16] introduce the regular vine to describe pair Copula decomposition. Kurowicka and Cooke [17] introduce two special cases of regular vines: the canonical vine and the D-vine. Figure 1 and figure 2 show the specifications corresponding to a four-dimensional canonical vine and a four-dimensional D vine respectively.

Canonical vine and D vine are suitable for different data sets. When a particular variable acts as the key variable that governs interactions in the data set, it is advantageous to fit a canonical vine and locate the variable at the root of the canonical vine. However, the character limits the freedom of pair Copula decomposition and its use. When there is no pilot variable, the data set is suitable for D vine.

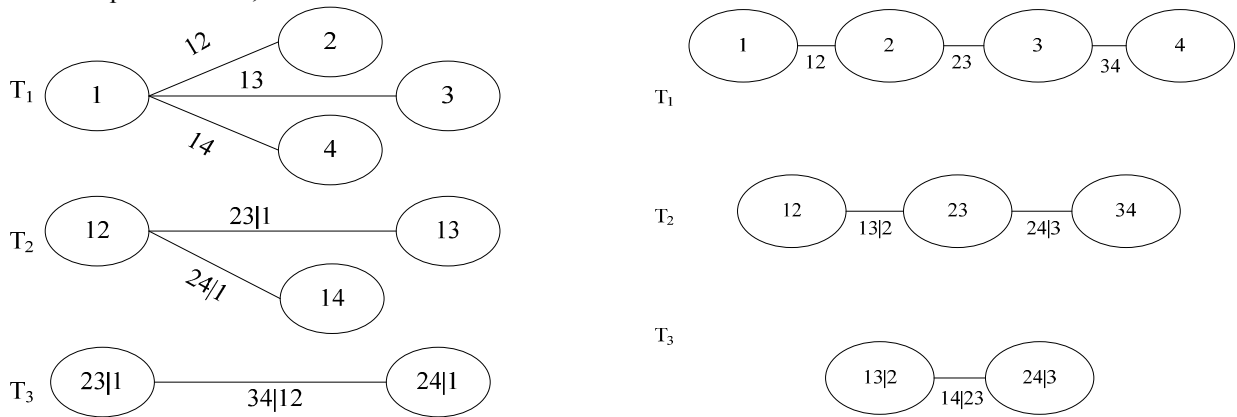


Fig. 1. (a) A canonical vine with 4 variables; (b) A D-vine with 4 variables

According to Bedford and Cooke [14], the n -dimensional density corresponding to a canonical vine can be written as

$$f(x_1, x_2, \dots, x_n) = \prod_{k=1}^n f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1, \dots, j-1} \{F(x_j | x_1, \dots, x_{j-1}), F(x_{j+i} | x_1, \dots, x_{j-1})\} \quad (3)$$

the n -dimensional density corresponding to a D vine can be written as

$$f(x_1, x_2, \dots, x_n) = \prod_{k=1}^n f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|1, \dots, i+j-1} \{F(x_i | x_{i+1}, \dots, x_{i+j-1}), F(x_{i+j} | x_{i+1}, \dots, x_{i+j-1})\} \quad (4)$$

There are a couple of marginal conditional distributions $F(x|v)$ in the pair Copula construction $c_{xv_j|v_{-j}}(\cdot)$ from formula (3) and (4). According to Joe [18], for every j ,

$$F(x|v) = \frac{\partial C_{x,v_j|v_{-j}} \{F(x|v_{-j}), F(v_j|v_{-j})\}}{\partial F(v_j|v_{-j})} \quad (5)$$

where v is a d -dimensional vector, v_j is an arbitrarily chosen component of v , v_{-j} denotes the v -vector excluding the component v_j .

Parameters of the n -dimensional density corresponding to a canonical vine and D vine can be estimated through maximum likelihood approach. The log-likelihood for a canonical vine is given by

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^T \log \left[c_{j,j+i|1, \dots, j-1} \{F(x_{j,t} | x_{1,t}, \dots, x_{j-1,t}), F(x_{j+i,t} | x_{1,t}, \dots, x_{j-1,t})\} \right] \quad (6)$$

the log-likelihood for a D-vine is given by

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^T \log \left[c_{i,i+j|1, \dots, i+j-1} \{F(x_{i,t} | x_{i+1,t}, \dots, x_{i+j-1,t}), F(x_{i+j,t} | x_{i+1,t}, \dots, x_{i+j-1,t})\} \right] \quad (7)$$

Estimating the parameters of formula (6) and (7), we should first get values of the parameters from each tree in the vine, take them as the initial values of parameters and maximize the log-likelihood to get the final values of parameters[19].

3. Construction of pair Copula-GARCH-EVT model

EVT is an effective approach to estimate the extreme case of market risk. It only focuses on the distribution of extreme values and it can describe the tail quantiles of a distribution accurately with the overall distribution unknown. EVT mainly includes Block Maxima model and Peaks over Threshold model. We have to set window parameters artificially and can not import other explanatory variables when using BMM, so it lacks flexibility in practical application. While POT is built on the hypothesis that the distribution of returns over thresholds follows the generalized Pareto distribution (GPD), which only model returns data over some high enough threshold. POT focuses on the approximate description to the tails, not the overall distribution, so it overcomes the shortcomings of other approaches to fit fat tail distribution [20]. Therefore POT based on GPD is widely used in actual research.

The generalized Pareto distribution can be written as

$$G_{\xi, \beta}(x) = \begin{cases} 1 - (1 + \frac{\xi x}{\beta})^{-1/\xi}, & \xi \neq 0, \\ 1 - \exp(-\frac{x}{\beta}), & \xi = 0, \end{cases} \quad (8)$$

where β, ξ are the scale and shape parameters respectively. When $\xi \geq 0, x \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$.

Financial time series have typical non-normal characters, such as leptokurtosis, fat tails, volatility clustering and leverage effect, it is able to describe the time series effectively using GJR-GARCH (1, 1) and so this paper applies GJR-GARCH (1, 1) to fit each asset return series. Assuming that a portfolio consists of n assets, the return series for asset i is $\{y_{i,t}, t=1,2,\dots\}$. Then GJR-GARCH (1, 1) can be written as

$$\begin{aligned} y_{i,t} &= \mu_i + \varepsilon_{i,t} \\ \varepsilon_{i,t} &= \sigma_{i,t} z_{i,t} \quad z_{i,t} \sim i.i.d \\ \sigma_{i,t}^2 &= w_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \psi[\varepsilon_{i,t-1} < 0] \varepsilon_{i,t-1}^2 \end{aligned} \quad (9)$$

where $\mu_i, w_i, \alpha_i, \beta_i, \gamma_i$ are parameters, $y_{i,t}$ is the return series of asset i , γ_i is the leverage coefficient and $z_{i,t}$ is the innovation process, which is independently and identically distributed. Typically, $z_{i,t}$ follows a fat-tailed distribution. In the empirical studies, we assume $z_{i,t}$ follows Student t distribution.

Then, we use GPD to model the innovation z_i in the lower and upper tails and the empirical distribution in the remaining part. Then the marginal distribution of each innovation is given by

$$F_i(z_i) = \begin{cases} \frac{T_{u_i^L}}{T} \left(1 + \xi_i^L \frac{u_i^L - z_i}{\beta_i^L} \right)^{-1/\xi_i^L} & z_i < u_i^L \\ \varphi(z_i) & u_i^L < z_i < u_i^R \\ 1 - \frac{T_{u_i^R}}{T} \left(1 + \xi_i^R \frac{z_i - u_i^R}{\beta_i^R} \right)^{-1/\xi_i^R} & z_i > u_i^R \end{cases} \quad (10)$$

where u_i^L, u_i^R are the lower and upper threshold respectively, $\varphi(z_i)$ is the empirical distribution on the interval $[u_i^L, u_i^R]$, T is the number of z_i and $T_{u_i^L}$ is the number of innovation whose value is less than u_i^L and $T_{u_i^R}$ is the

number of innovation whose value is greater than u_i^R . In the empirical studies, we choose the exceedances to be 10 percent of the innovation for lower and upper tails respectively.

Finally, we apply pair Copula to extract the dependence structure between the innovations of each asset return, which needs to select canonical vine or D vine to describe the logical structure of the decomposition firstly. According to the character of canonical vine and D vine, we first fit a bivariate Student Copula to each pair of innovations and obtain the estimated degrees of freedom for each pair; a small number of degrees of freedom indicates strong dependence. Then we can select an appropriate vine for the pair Copula decomposition [19]. Theoretically, we can choose the best bivariate Copula for each Copula, but this will make the decomposition so complicated and the result may not be the best, such as Aas and Czado [19]. Because Student Copula is both lower and upper tail dependent, we choose Student Copula for all pairs of the decomposition.

4. Mean-CVaR model

CVaR is the expected losses that exceed the VaR at some confidence level, which can be written as [21]:

$$CVaR_{\beta}(y) = E[-y | -y \geq VaR_{\beta}(y)] \quad (11)$$

where y is the returns of a portfolio, β is the confidence level, $VaR_{\beta}(y)$ is the VaR at the β confidence level and $CVaR_{\beta}(y)$ is the expected losses of the portfolio at the β confidence level, which reflects the number of the potential losses when the losses exceed the threshold $VaR_{\beta}(y)$. So $CVaR_{\beta}(y) \geq VaR_{\beta}(y)$. In risk management, if we can control CVaR, then we can also control VaR at the same time [22].

Assuming that there are n assets in a portfolio, $X=(x_1, \dots, x_n)^T$ is the position for each asset, $x_i \geq 0 (i=1, \dots, n)$ and $\sum_{i=1}^n x_i = 1$, corresponding asset return is $Y=(y_1, \dots, y_n)^T$, so the expected return of the portfolio is $\sum_{i=1}^n x_i y_i$, the expected loss is $-\sum_{i=1}^n x_i y_i$. The loss function of the portfolio is $f(X, Y) = -\sum_{i=1}^n x_i y_i = -X^T Y$, assuming Y has density $P(Y)$.

Rockafellar and Uryasev [23] combine CVaR and VaR through a special function $F_{\beta}(X, \alpha)$ and transform the problem of minimizing CVaR to the problem of minimizing a continuously differentiable and convex function.

$$F_{\beta}(X, \alpha) = \alpha + \frac{1}{1-\beta} \int_{y \in R^m} [f(X, Y) - \alpha]^+ P(Y) dY \quad (12)$$

$$VaR_{\beta}(X) \in \arg \min_{\alpha \in R} F_{\beta}(X, \alpha) \quad (13)$$

$$CVaR_{\beta}(X) = \min_{\alpha \in R} F_{\beta}(X, \alpha) = F_{\beta}(X, VaR_{\beta}(X)) \quad (14)$$

where $[U]^+ = \max(U, 0)$, $F_{\beta}(X, \alpha)$ is convex and continuously differentiable with respect to α .

In practice, the density of Y is usually unknown, we can generate a collection of Y under q different situations by Monte Carlo simulations, then the corresponding approximation to $F_{\beta}(X, \alpha)$ is

$$\tilde{F}_{\beta}(X, \alpha) = \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^q [-X^T Y^k - \alpha]^+ \quad (15)$$

Then we construct Mean-CVaR model using CVaR to take the place of variance in Mean-Variance model:

$$\begin{aligned}
 \min \quad & \alpha + \frac{1}{q(1-\beta)} \sum_{k=1}^q u^k \\
 \text{s.t.} \quad & \begin{cases} X^T Y^k + \alpha + u^k \geq 0 \\ u^k \geq 0 \\ \frac{1}{q} X^T \sum_{k=1}^q Y^k \geq \rho \\ \sum_{i=1}^n x_i = 1 \\ x \geq 0 \end{cases}
 \end{aligned} \tag{16}$$

where ρ is the investors' expected return and Y^k can be generated through Monte Carlo simulations combining pair Copula-GARCH-EVT model. In section 3, we use pair Copula to capture the dependence structure between innovations, so we have to simulate the innovations first and then simulate the returns of the portfolio using GJR-GARCH (1, 1). The progress to simulate a return sample corresponding to a canonical vine is as follows:

Step 1. Sample w_1, w_2, \dots, w_n independent uniform on $[0, 1]$, assuming $z_1 = w_1$;

Step 2. According to formula (5), we can get $F(z_2|z_1)$, assuming $w_2 = F(z_2|z_1)$;

Step 3. According to formula (5), we can get $F(z_3|z_1)$, then $F(z_3|z_1, z_2) = \frac{\partial C_{3,2|1} \{F(z_3|z_1), F(z_2|z_1)\}}{\partial F(z_2|z_1)}$, assuming

$w_3 = F(z_3|z_1, z_2)$;

Step 4. Through the same way, we can get $w_4 = F(z_4|z_1, z_2, z_3), \dots, w_n = F(z_n|z_1, z_2, \dots, z_{n-1})$;

Step 5. According to the marginal distribution of z_i , we can get $z_i = F^{-1}(w_i|z_1, z_2, \dots, z_{i-1})$, then we can get a simulation of the innovation $\{z_1, z_2, \dots, z_n\}$;

Step 6. Simulate the returns of the portfolio using GJR-GARCH (1, 1);

Step 7. Repeat the above steps and simulate q times, we can get the simulated return series of the portfolio $\{y_{1,t}, \dots, y_{n,t}, t=1, \dots, q\}$.

5. Empirical studies and analysis

5.1. Data source and statistics

In the empirical studies, we choose four indexes from Chinese Stock Market: Shanghai Composite Index (H), Shenzhen Component Index (Z), SME Composite Index (M) and Shanghai Fund Index (F). The data employed is from December 1, 2008 to May 31, 2011 and the data source is from CSMAR solution. We employ the daily log returns defined as $R_t = \ln p_t - \ln p_{t-1}$. The sample consists of 1334 observations of log returns in total. All the numerical computations are run in MATLAB 7.10. The preliminary descriptive statistics of the sample data are presented in Table 1.

Table 1. The preliminary descriptive statistics of the sample data

	Shanghai Composite Index	Shenzhen Component Index	SME Composite Index	Shanghai Fund Index
Mean	0.0007	0.0011	0.0011	0.0013
Maximum	0.0903	0.0916	0.0935	0.0941
Minimum	-0.0926	-0.0975	-0.0970	-0.0934
Std.Dev	0.0198	0.0221	0.0223	0.0195
Skewness	-0.4449	-0.4491	-0.6229	-0.0205
Kurtosis	5.5127	4.7954	4.8295	5.9315

Jarque-bera	394.9360 (0.0010)	224.0177 (0.0010)	272.3162 (0.0010)	477.7518 (0.0010)
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As shown in Table 1, the kurtosis of each index is larger than 3 and the skewness is less than 0, which indicates that all series have fat tails and leptokurtosis. From the JB statistics we know that they do not follow normal distribution. So it is effective to fit the series using GJR-GARCH (1, 1) model.

5.2. GARCH-EVT application

We use GARCH-EVT Model to fit the marginal distribution of each return series, estimations of the parameters are presented in Table 2.

Table 2. Estimated parameters for GARCH-EVT model

	Shanghai Composite Index	Shenzhen Component Index	SME Composite Index	Shanghai Fund Index
θ	0.0022	0.0342	0.0580	0.0188
μ	0.0020	0.0020	0.0023	0.0013
w	3.5105E-06	6.3091E-06	1.6844E-05	3.2126E-06
α	0.0798	0.0643	0.1206	0.0923
β	0.9237	0.9222	0.8561	0.9146
γ	-0.0094	0.0083	-0.0055	-0.0137
ν	4.8045	5.7745	6.0627	4.6589
u^L	-1.2875800	-1.2914500	-1.4047100	-1.1608800
ζ^L	-0.0022584	-0.0489275	0.0393374	0.0430824
β^L	0.6993850	0.7434620	0.6515340	0.6126730
u^R	1.0935300	1.1466600	1.0894700	1.2057800
ζ^R	-0.0190134	-0.0956770	-0.0377330	-0.0386688
β^R	0.4493150	0.5009950	0.4155000	0.6049040

5.3. Pair Copula decomposition and parameter estimation

We use Student Copula to obtain the estimated degrees of freedom for each couple of innovations, which is given in Table 3, and then we can determine the decomposition structure of pair Copula.

Table 3. Estimate degrees of freedom for each couple of innovations

	Shenzhen Component Index	SME Composite Index	Shanghai Fund Index
Shanghai Composite Index	4.0749	4.0297	2.3169
Shenzhen Component Index	0.0000	4.0830	3.7251
Shanghai Fund Index	0.0000	0.0000	3.8022

As shown in Table 3, the degrees of freedom between Shanghai Fund Index and other three indexes are smaller, so it is advantageous to select the canonical vine to describe the pair Copula decomposition, as shown in figure 3.

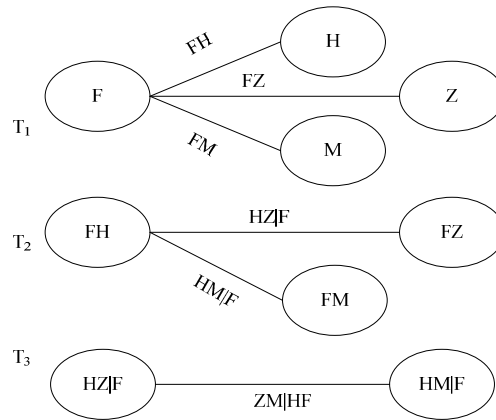


Fig.2. Canonical vine structure for the sample data

Having chosen the decomposition structure for pair Copula, we estimate the parameters. The result is given in Table 4.

Table 4. Estimated parameters for the pair Copula decomposition

Parameter	Initial values	Final values
ρ_{FH}	0.8891	0.8891
ρ_{FZ}	0.8618	0.8616
ρ_{FM}	0.7265	0.7265
$\rho_{HZ F}$	0.5864	0.5887
$\rho_{HM F}$	0.3919	0.3934
$\rho_{ZM HF}$	0.3831	0.3841
v_{FH}	2.2533	2.4980
v_{FZ}	3.6697	4.3998
v_{FM}	3.6379	4.3993
$v_{HZ F}$	4.3066	4.5715
$v_{HM F}$	5.3025	5.5195
$v_{ZM HF}$	6.5760	6.5766
log-likelih.	3.4085E+03	3.4111E+03

5.4. Portfolio optimization

We generate a set of 10,000 samples using the simulation procedure described in section 4. Then at the confidence level of $\beta=95\%$, we apply the Mean-CVaR model to optimize the portfolio and obtain the CVR efficient frontier of the portfolio under different expected returns, as shown in figure 4.

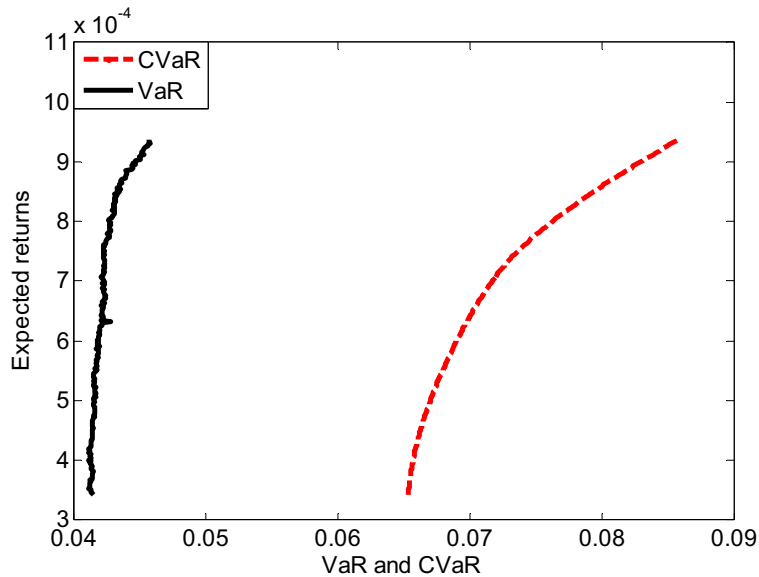


Fig.4. The efficient frontiers of VaR and CVaR under Mean-CVaR model

Figure 4 shows that CVaR is always larger than VaR under Mean-CVaR model and VaR is not convex.

At the same time, we can obtain the optimal portfolios using Mean-Variance model and calculate the CVaR efficient frontier, as shown in figure 5.

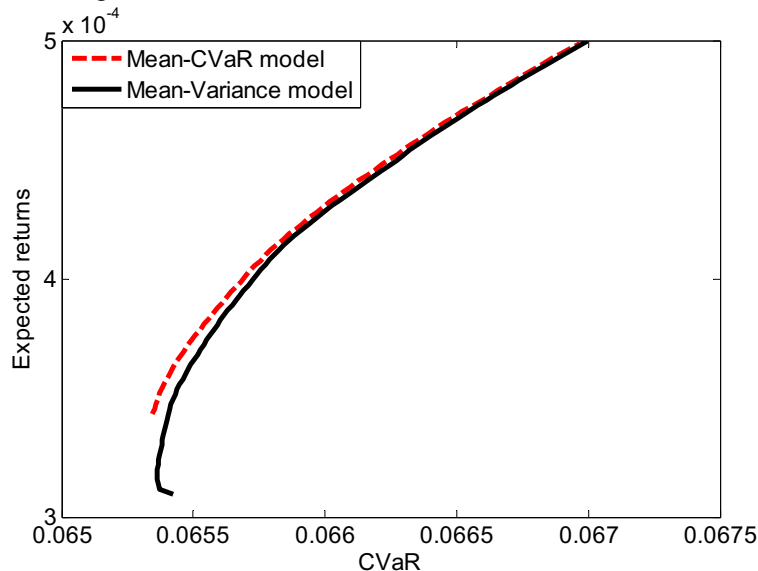


Fig.5. The efficient frontiers of CVaR under Mean-CVaR and Mean-Variance models

From figure 5, we can know that the CVaR under Mean-CVaR model is always less than that under Mean-Variance model when the expected returns are the same, but the difference is very small, which is because the efficient frontier of Mean-CVaR model converges to the efficient frontier of Mean-Variance model when the confidence level approaches 1 [24]. We can also know that the minimum CVaR portfolio is not the minimum variance portfolio.

Finally, we generate a set of 10,000 samples using Student Copula-GARCH-EVT model and Monte Carlo simulation. Then at the confidence level of $\beta=95\%$, we apply the Mean-CVaR model to optimize the portfolio and

obtain the CVR efficient frontier of the portfolio under different expected returns, as shown in figure 6.

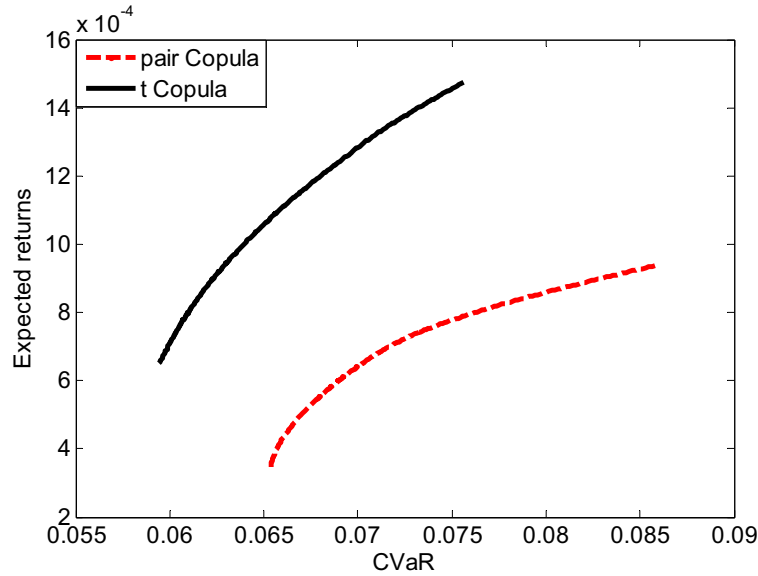


Fig.6. The efficient frontiers of CVaR under pair Copula and Student Copula

From figure 6, we can know that when the expected return is equal, CVaR under pair Copula-GARCH-EVT model is larger than that of Student Copula-GARCH-EVT model. This indicates that it is better to apply pair Copula to capture the dependence structure among assets, so as to estimate CVaR of portfolio more accurately, but CVaR with Student Copula underestimates the risk of portfolio.

6. Conclusion

This paper estimates the risk of portfolio using CVaR and applies Mean-CVaR model to optimize portfolio. In order to estimate the risk of portfolio more accurately, we apply EVT to model the tails of the innovation of each asset return; then, we capture the dependence structure between innovations of asset returns by pair Copula. Pair Copula decomposition model considers both the influence of portfolio dimensions and the differences of tail dependence between assets. Finally, the results of the empirical studies indicate that it is more efficient to optimize portfolio using Mean-CVaR model than Mean-Variance model; the optimal portfolio is better via pair Copula-GARCH-EVT model than that via Student Copula-GARCH-EVT model.

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