Machine Dynamics - Assignment 4

Student ID: D08522005

Student Name: 張元

Q1

Simulate the dynamics of the engine during $t = 0$ to 10 s.

$$\begin{cases} \theta_2 & = 0 \\ \omega & = 20.94 \text{rad/s} \\ T_2 & = \omega^2 \times 10^{-4} = 0.0438 \text{N} \cdot \text{m} \\ r & = 38 \text{mm} \\ l & = 133 \text{mm} \\ m_2 & = 5 \text{kg} \\ m_3 & = 0.5 \text{kg} \\ m_4 & = 0.3 \text{kg} \\ I_{G2} & = 0.05 \text{kg} \cdot \text{m}^2 \\ I_{G3} & = 0.002 \text{kg} \cdot \text{m}^2 \end{cases}$$

The Lagrange equation:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) - \frac{\partial L}{\partial \theta_2} = T_2 + W_G$$

Total kinetic and potential energy:

$$\begin{cases} T & = \frac{1}{2}(I_{G2}\dot{\theta}_2^2 + I_{G3}\dot{\theta}_3^2 + m_4 v_{G4}^2) \\ V & = g \sin\theta_2 [m_2 r_{G2} + m_3(L + r_{G3})] \\ L & = T - V \end{cases}$$

Applying $\frac{\partial}{\partial \dot{\theta}_2}$ to L:

$$\begin{aligned}\frac{\partial L}{\partial \dot{\theta}_2} =& I_{G2} + I_{G3}\frac{\partial \theta_3^2}{\partial \theta_2} + m_4\frac{\partial v_{G4}^2}{\partial \dot{\theta}_2} \\ &- g\dot{\theta}_2\cos\theta_2[m_2 r_{G2} + m_3(L + r_{G3})]\end{aligned}$$

Applying $\frac{d}{dt}$ to $\frac{\partial L}{\partial \dot{\theta}_2}$:

$$\begin{aligned}\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) =& I_{G2} + 2I_{G3}\frac{\partial \dot{\theta}_3}{\partial \theta_2} + 2m_4\frac{\partial a_{G4}}{\partial \dot{\theta}_2} \\ &- g\ddot{\theta}_2\cos\dot{\theta}_2[m_2 r_{G2} + m_3(L + r_{G3})]\end{aligned}$$

Applying $\frac{\partial}{\partial \theta_2}$ to L:

$$\begin{aligned}\frac{\partial L}{\partial \theta_2} =& I_{G2}\frac{\dot{\theta}_2}{\partial \theta_2} + I_{G3}\frac{\dot{\theta}_2}{\partial \theta_2} + m_4\frac{\partial v}{\partial \theta_2} \\ &- g\cos\theta_2[m_2 r_{G2} + m_3(L + r_{G_3})]\end{aligned}$$

The Lagrange equation of the gas force:

$$\begin{cases}\delta W_G &= F_G\delta r_G \\ \delta r_G &= \delta\theta_2 \cdot \sin\theta_2(-R - \frac{R^2\cos\theta_2}{L\sqrt{1-(\frac{R}{L})^2\sin^2\theta_2}})\end{cases}$$

It can be expressed as:

$$W_G = F\sin\theta_2(-R - \frac{R^2\cos\theta_2}{L\sqrt{1-(\frac{R}{L})^2\sin^2\theta_2}})$$

Vector loop:

$$\begin{cases}l_2\cos\theta_2 + l_3\cos\theta_3 &= x \\ l_2\sin\theta_2 + l_3\sin\theta_3 &= 0\end{cases}$$

Obtain θ_3:

$$\theta_3 = \sin^{-1}(\frac{R}{L}\sin\theta_2)$$

Applying $\frac{d}{dt}$:

$$\begin{cases} l_2\dot{\theta}_2\cos\theta_2 & = -l_3\dot{\theta}_3\cos\theta_3 \\ l_2\dot{\theta}_2\sin\theta_2 & = -l_3\dot{\theta}_3\sin\theta_3 \end{cases}$$

Applying $\frac{\partial}{\partial\dot{\theta}_2}$:

$$\begin{cases} l_2\cos\theta_2 & = -l_3\frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2}\cos\theta_3 \\ l_2\sin\theta_2 & = -l_3\frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2}\sin\theta_3 \end{cases}$$

Obtain $\frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2}$:

$$\frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2} = -\frac{l_2\sin\theta_2}{l_3\cos\theta_3}$$

Q2

In the simulation, find the required time to speed up from 200 **to** 3600 **rpm.**

Q3

According to the result in Assignment 3 to design a flywheel for the crank to achieve $k = 0.05$. Repeat 1 and 2 to find the influence of the flywheel.