# Machine Dynamics - Assignment 4

Student ID: D08522005

Student Name: 張元

## Q1

**Simulate the dynamics of the engine during $t = 0$ to $10$ s.**

$$\begin{cases} \theta_2 & = 0 \\ \omega & = \dot{\theta}_2 = 20.94\text{rad/s} \\ T_2 & = \omega^2 \times 10^{-4} = 0.0438\text{N} \cdot \text{m} \\ R & = l_2 = 38\text{mm} \\ L & = l_3 = 133\text{mm} \\ r_{G2} & = 0.3 l_2 = 11.4\text{mm} \\ r_{G3} & = 0.36 l_3 = 47.88\text{mm} \\ m_2 & = 5\text{kg} \\ m_3 & = 0.5\text{kg} \\ m_4 & = 0.3\text{kg} \\ I_{G2} & = 0.05\text{kg} \cdot \text{m}^2 \\ I_{G3} & = 0.002\text{kg} \cdot \text{m}^2 \end{cases}$$

The Lagrange equation:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) - \frac{\partial L}{\partial \theta_2} = T_2 + W_G$$

Total kinetic and potential energy:

$$\begin{cases} T & = \frac{1}{2}(I_{G2}\dot{\theta}_2^2 + I_{G3}\dot{\theta}_3^2 + m_4 \dot{x}^2) \\ V & = g \sin\theta_2 [m_2 r_{G2} + m_3(L + r_{G3})] \\ L & = T - V \end{cases}$$

Applying $\frac{\partial}{\partial \dot{\theta}_2}$ to $L$:

$$\begin{aligned}\frac{\partial L}{\partial \dot{\theta}_2} =& I_{G2} + I_{G3}\frac{\partial \theta_3}{\partial \dot{\theta}_2} + m_4 \frac{\partial \dot{x}}{\partial \dot{\theta}_2} \\ &- g\dot{\theta}_2 \cos\theta_2 [m_2 r_{G2} + m_3(L + r_{G3})]\end{aligned}$$

Applying $\frac{d}{dt}$ to $\frac{\partial L}{\partial \dot{\theta}_2}$:

$$\begin{aligned}\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) =& I_{G3}\frac{\partial \dot{\theta}_3}{\partial \dot{\theta}_2} + m_4 \frac{\partial \ddot{x}}{\partial \dot{\theta}_2} \\ &- (g\ddot{\theta}_2 \cos\theta_2 + g\dot{\theta}_2 \sin\dot{\theta}_2)[m_2 r_{G2} + m_3(L + r_{G3})]\end{aligned}$$

Applying $\frac{\partial}{\partial \theta_2}$ to $L$:

$$\frac{\partial L}{\partial \theta_2} = - g\cos\theta_2 [m_2 r_{G2} + m_3(L + r_{G_3})]$$

The gas force:

$$W_G = F\sin\theta_2(-R - \frac{\dot{\theta}_3}{\dot{\theta}_2})$$

The result is:

$$\begin{aligned}\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) - \frac{\partial L}{\partial \theta_2} &= I_{G3}\frac{\partial \dot{\theta}_3}{\partial \dot{\theta}_2} + m_4 \frac{\partial \ddot{x}}{\partial \dot{\theta}_2} \\ &- (g\ddot{\theta}_2 \cos\theta_2 + g\dot{\theta}_2 \sin\dot{\theta}_2)[m_2 r_{G2} + m_3(l_3 + r_{G3})] \\ &+ g\cos\theta_2[m_2 r_{G2} + m_3(l_3 + r_{G_3})] \\ &= T_2 + F\sin\theta_2(-l_2 - \frac{l_2^2\cos\theta_2}{l_3\sqrt{1-(\frac{l_2}{l_3})^2\sin^2\theta_2}})\end{aligned}$$

Vector loop:

$$\begin{cases} l_2 \cos\theta_2 + l_3 \cos\theta_3 &= x \\ l_2 \sin\theta_2 + l_3 \sin\theta_3 &= 0 \end{cases}$$

Obtain $\theta_3$:

$$\begin{cases} \theta_3 = \sin^{-1}(\frac{l_2}{l_3}\sin\theta_2) \\ \dot{\theta}_3 = \frac{l_2^2 \cos\theta_2}{l_3\sqrt{1-(\frac{l_2}{l_3})^2 \sin^2\theta_2}}\dot{\theta}_2 \end{cases}$$

Applying $\frac{d}{dt}$:

$$\begin{cases} l_2\dot{\theta}_2 \cos\theta_2 &= \dot{x} - l_3\dot{\theta}_3 \cos\theta_3 \\ l_2\dot{\theta}_2 \sin\theta_2 &= -l_3\dot{\theta}_3 \sin\theta_3 \end{cases}$$

Applying $\frac{\partial}{\partial\theta_2}$ to $\frac{d}{dt}$:

$$\begin{cases} l_2\frac{\partial\dot{\theta}_2}{\partial\theta_2} \sin\theta_2 &= \frac{\partial\dot{x}}{\partial\theta_2} - l_3\frac{\partial\dot{\theta}_3}{\partial\theta_2} \cos\theta_3 \\ l_2\frac{\partial\dot{\theta}_2}{\partial\theta_2} \cos\theta_2 &= -l_3\frac{\partial\dot{\theta}_3}{\partial\theta_2} \sin\theta_3 \end{cases}$$

Applying $\frac{\partial}{\partial\dot{\theta}_2}$ to $\frac{d}{dt}$:

$$\begin{cases} l_2 \cos\theta_2 &= \frac{\partial\dot{x}}{\partial\dot{\theta}_2} - l_3\frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2} \cos\theta_3 \\ l_2 \sin\theta_2 &= -l_3\frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2} \sin\theta_3 \end{cases}$$

Obtain:

$$\begin{cases} \frac{\partial\dot{\theta}_3}{\partial\dot{\theta}_2} &= \frac{\partial\theta_3}{\partial\theta_2} = -\frac{l_2\sin\theta_2}{l_3\sin\theta_3} = -1 \\ \frac{\partial\dot{x}}{\partial\dot{\theta}_2} &= l_2\cos\theta_2 - l_3\cos\theta_3 \\ \frac{\partial\ddot{x}}{\partial\dot{\theta}_2} &= l_2\dot{\theta}_2\sin\theta_2 - l_3\dot{\theta}_3\sin\theta_3 = l_2\sin\theta_2(\dot{\theta}_2 - \dot{\theta}_3) \end{cases}$$

Substitute in to Lagrange equation:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) - \frac{\partial L}{\partial \theta_2} = -I_{G3} + l_2 m_4 \sin\theta_2(\dot{\theta}_2 - \frac{l_2^2\cos\theta_2}{l_3\sqrt{1-(\frac{l_2}{l_3})^2\sin^2\theta_2}}\dot{\theta}_2)$$

$$- (g\ddot{\theta}_2\cos\theta_2 + g\dot{\theta}_2\sin\dot{\theta}_2)[m_2 r_{G2} + m_3(l_3 + r_{G3})]$$

$$+ g\cos\theta_2[m_2 r_{G2} + m_3(l_3 + r_{G_3})]$$

$$= T_2 + F\sin\theta_2(-l_2 - \frac{l_2^2\cos\theta_2}{l_3\sqrt{1-(\frac{l_2}{l_3})^2\sin^2\theta_2}})$$

Among them, the $\ddot{\theta}_2$ ($\alpha_2$) is unknown. Change the items and obtain:

$$\ddot{\theta}_2 = \frac{-I_{G3} + \sin\theta_2[l_2 m_4(\dot{\theta}_2 - \dot{\theta}_3) - F(-l_2 - \frac{\dot{\theta}_3}{\dot{\theta}_2})] + g\cos\theta_2[m_2 r_{G2} + m_3(l_3 + r_{G_3})] - T_2}{g\cos\theta_2[m_2 r_{G2} + m_3(l_3 + r_{G3})]} - \frac{\sin\dot{\theta}_2}{\cos\theta_2}\dot{\theta}_2$$

## Q2

**In the simulation, find the required time to speed up from 200 to 3600 rpm.**

## Q3

**According to the result in Assignment 3 to design a flywheel for the crank to achieve $k = 0.05$. Repeat 1 and 2 to find the influence of the flywheel.**