PH026

Mars 2026: Simulating future Mars Missions and determining energetically feasible interplanetary transfers

ABSTRACT

The objective of our research is to determine alternative, energetically feasible transfer orbits from Earth to Mars, in order to shorten the time of flight (TOF). Firstly, this work simulates the standard Hohmann transfer model to achieve a theoretical minimum ΔV and TOF, and explains why it is not realistic. To make the model more realistic, this paper uses the FreeFlyer® - a.i. solutions software to find solutions to Lambert's Problem, which determines suitable trajectories for a fixed launch date. This launch date will be set in the future, 26 November 2026, as the potential epoch for an actual Mars mission launch in the future. We then created a plot of total ΔV, and ΔV of first burn against TOF. In order to determine energetically feasible interplanetary transfers, we reference the Centaur III. We will be comparing its fuel consumption with our results obtained, namely the ΔV of the first burn. This paper concludes that for the selected launch date, the lowest total ΔV as well as the corresponding TOF, was a ΔV of 13.3 km s^{-1}, with a TOF of 111 days. Upon further analysis of our data, we have concluded that the TOF can indeed be shortened by 32 days, with extra propellant usage of 5750kg that is small relative to the total mission cost. The result of this research proves that the TOF can be further cut with current technology and presents a method on evaluating the feasibility of reducing TOF.

Section 1 - Introduction

Along with the rising interest in space exploration in recent decades, interplanetary missions are becoming more common. When conducting interplanetary travel, there are two important factors [1] to consider, the time of flight (TOF) as well as mission cost. Mission cost can be reduced by achieving a low total delta-v (ΔV), which is the total change in velocity for the mission, to lower fuel consumption. This is highly favourable, as cost is one of the main factors inhibiting the development of interplanetary space travel. It is important for TOF to be as short as possible too. With lower time in transit, the chances of errors [2] happening will be reduced, and also ensures that scientific research can be conducted as soon as possible. This is crucial for manned missions too, to reduce the time spent in space by the human crew due to factors such as exposure to zero-G and solar radiation [3].

The Hohmann transfer models an ideal case of lowest ΔV [4]. It is an elliptical orbit with the apogee and perigee tangent to the 2 coplanar and circular parking and target orbit as illustrated in Figure 1, with Earth as the departure planet and Mars as the arrival planet. The transfer consists of 2 impulsive burns, one at the perigee and another at the apogee.

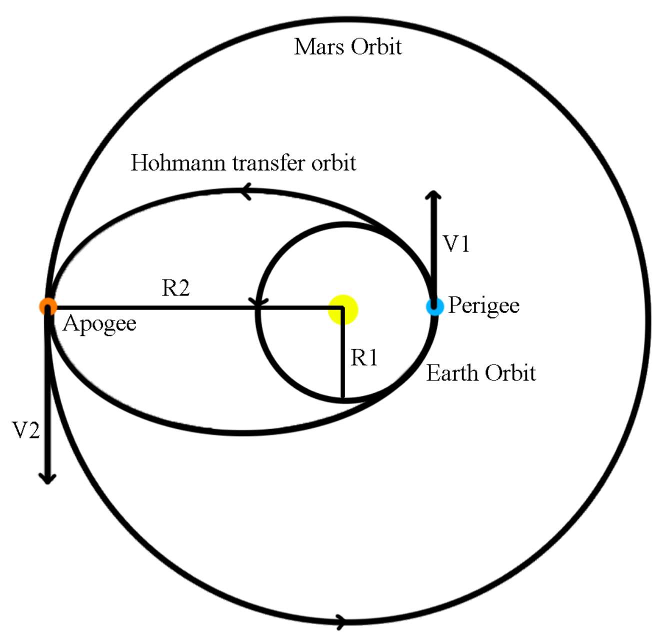

Figure 1: Hohmann transfer orbit from Earth to Mars

Our paper simulates Earth-Mars trajectories as there has been much talk about human colonisation of Mars [5]. While the Hohmann transfer has the lowest ΔV among all the transfers, one problem faced is the long TOF of the transfer. The Hohmann transfer is also unrealistic and limited, merely determining the minimum ΔV required for launch, without variation for differing TOFs. Hence, this paper focuses on reducing the TOF by solving Lambert's Problem while keeping an achievable ΔV with current technology. Lambert's problem is a two-body problem which deals with the determination of an orbit from two position vectors and the TOF. [6] Lambert's theorem states that the transfer time of a body moving between two points on a conic trajectory is a function of the sum of the distances of the two points from the origin of the force ($r_1 + r_2$), the linear distance between the points (d), and the semimajor axis of the conic trajectory (A) [7] as seen in Figure 2 below.

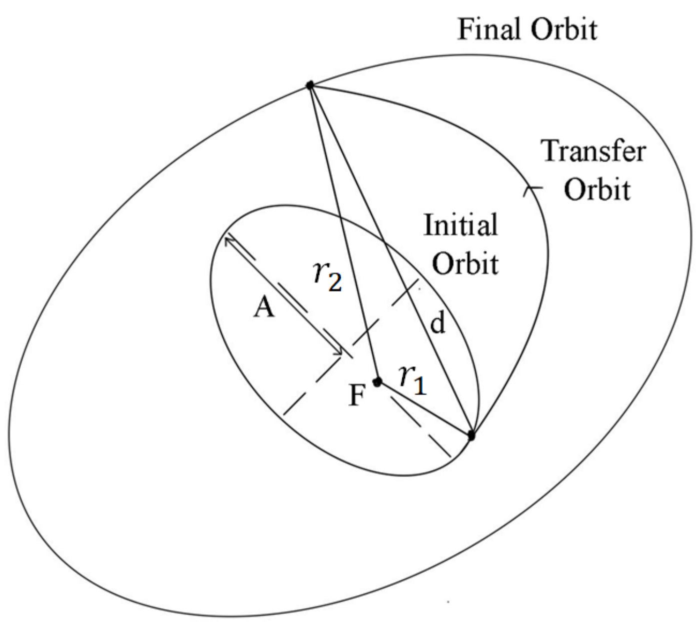

Figure 2: Lambert's problem

Using the Freeflyer software, this paper solves Lambert's Problem to plot a graph of total ΔV against TOF to find the lowest ΔV and its corresponding TOF. It then plots a graph of ΔV of first burn against TOF in comparison with Centaur III and its fuel consumption to determine alternative energetically feasible interplanetary transfers, and determine how much TOF can be shortened. The fundamental assumption made would be that the spacecraft is already orbiting around Earth and the ΔV for the second burn is to get it up to speed with the Mars Orbit.

Section 2 - Research Methodology

2.1 Standard Hohmann transfer model

This paper analyses the standard Hohmann transfer model and derives the ΔV and TOF required for an Earth-Mars trajectory. This model has the following assumptions:

1. The orbits of both Earth and Mars are coplanar and circular
2. All burns used in the transfer orbit are impulsive
3. There is only one body (the Sun) that exerts a gravitational force on the spacecraft

(Refer to Appendix 1 for Nomenclature and Derivation)

Initial velocity V_1 and final velocity V_2 are given by:

$$V_1 = \sqrt{\frac{GM_{sun}}{R_1}} \qquad V_2 = \sqrt{\frac{GM_{sun}}{R_2}}$$

Velocity at perihelion, V_p is found using the vis-viva equation:

$$V_p = \sqrt{2GM_{sun}(\frac{2}{R_1} - \frac{1}{a})}$$

Velocity at aphelion, V_a can be found using:

$$V_a = \frac{R_1}{R_2} V_p$$

Hence, to find the departure ΔV (ΔV_1) and arrival ΔV (ΔV_2),

$$\Delta V_1 = V_p - V_1 \qquad \Delta V_2 = V_2 - V_a$$

Finally, to get the total ΔV required, we take:

$$\Delta V = \Delta V_1 + \Delta V_2$$

Additionally, the time of flight (TOF) in days can be determined with:

$$TOF = (\frac{1}{2}T) \div 86400$$

2.2 Lambert's Problem and the trade-off between transfer time and ΔV

Lambert's problem removes the assumption that the orbits of Earth and Mars are coplanar and circular, providing greater accuracy to the obtained ΔV with Mars having an orbital inclination of 1.85° (relative to Earth's orbit) [8]. Not only that, the orbits are modelled to be elliptical, with an eccentricity of 0.094 [8] and 0.017 [9] for Mars and Earth's heliocentric orbit respectively. In solving Lambert's problem, the TOF from Earth to Mars is an independent variable. In this paper, the Lambert solver in the Freeflyer software, which uses the Newton algorithm [10], is used with several modifications (the full code can be seen in Appendix 2). Hence, the graph of total ΔV, and ΔV of the first burn, against TOF can be plotted, which can help determine alternate trajectories to Mars that reduces the TOF while maintaining a low total ΔV. The departure date epoch is 26 November 2026 [11] to allow our research paper to be of relevance for future space exploration to Mars.

2.3 Propellant mass calculation from total ΔV of trajectories

We plan to find the maximum ΔV viable by plotting a graph of total ΔV, as well as first ΔV against TOF. Then, we will compare it to a reference launch vehicle, the Atlas V. Our interplanetary transfer model requires 2 impulsive burns, hence we will be using the chemical propulsion system [12] for the launch vehicle. This system is used in the Atlas V rocket [13]. Specifically, we are referencing the second stage, Centaur III [14], as it is responsible for orbital insertion. The spacecraft launched will be the Mars Reconnaissance Orbiter (MRO) [2180kg (m_{sat})] [15]. In order to evaluate the energetically feasible interplanetary transfers, we will be using the first burn for the basis of evaluation. The second burn ΔV is not used in our comparison, as in reality, other mechanisms such as aerobraking are in place [16], which affects the ΔV in comparison. The parameters for Centaur III and are as follows [13]:

Spacecraft	**Propellant Mass (m_{fuel})**	**Dry Mass**	**Specific Impulse (I_{sp})**
Centaur III	20,830kg	2,316.0kg	450.50s

Hence, with reference to the data above, to determine the maximum ΔV viable for the first burn, we will be using the Tsiolkovsky rocket equation [17],

$$\Delta V = I_{sp} g \ln\left(1 + \frac{m_{fuel}}{m_{sat}}\right)$$

where I_{sp} refers to the specific impulse dependent on the propellant, m_{sat} refers to the mass of MRO and g refers to the standard acceleration due to gravity on Earth [$9.8067 ms^{-2}$] [18].

Section 3- Results and Discussions

3.1 Standard Hohmann transfer model

Using the standard Hohmann transfer model (Section 2.1), we obtain the following results:

First Burn (ΔV_1)	Second Burn (ΔV_2)	Total ΔV	TOF
2.944km s^{-1}	2.648km s^{-1}	5.592km s^{-1}	259 Days

However, this model is unrealistic due to the various assumptions held. In reality, the orbits of Earth and Mars around the Sun are elliptical and not circular. Furthermore, the orbits are not coplanar and are inclined to one another. This model is also only a special case of Lambert's problem where ΔV is the lowest, and in reality, ΔV and TOF can be varied according to the trajectory chosen. Hence, our paper proceeds to solve Lambert's problem to simulate a more realistic transfer and to find alternative energetically feasible transfers. (Appendix 5)

3.2 Discussions of ΔV against time graphs

These are our findings using the FreeFlyer's Lambert Solver (Appendix 3) for the launch date of 26 November 2026 (Figure 3). For the total ΔV, the graph initially shows a dip in total ΔV as time taken increases until a minimum point is reached at approximately 111 days.

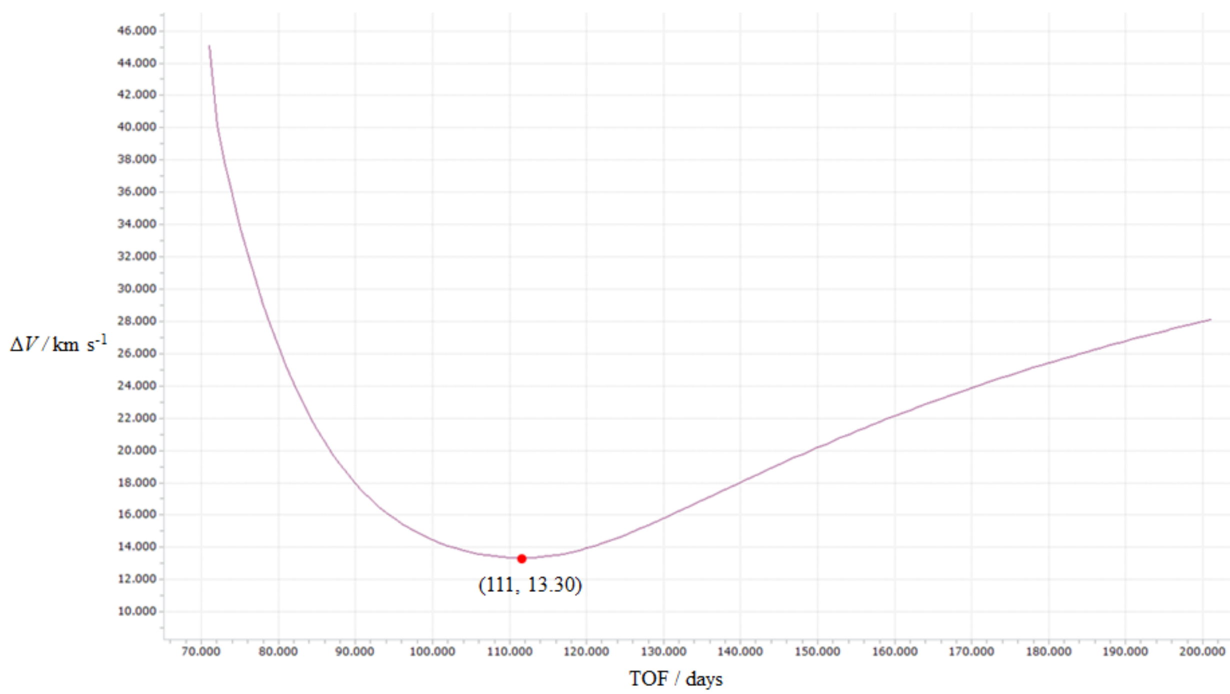

Figure 3: Total ΔV against TOF

There is a minimum point ΔV of 13.30km s^{-1} from the graph of figure 3 is at 111 days. This corresponds to the minimum energy transfer and its TOF from Earth to Mars. The discrepancy between the ΔV and TOF for the Hohmann transfer and the minimum point is due to the breaking of assumptions that the orbits are coplanar and circular. We will use this point as a reference, in order to compare with previous transfers and evaluate the efficiency of our transfer method.

3.3 Lambert's Problem and the trade-off between transfer time and ΔV

To evaluate whether the extra propellant used to shorten TOF is justifiable, we evaluated the maximum mass of fuel that the spacecraft can hold based on the Centaur III's mass of propellant (ΔV = 10.40km s^{-1}) against the graph of ΔV of first burn against the TOF.

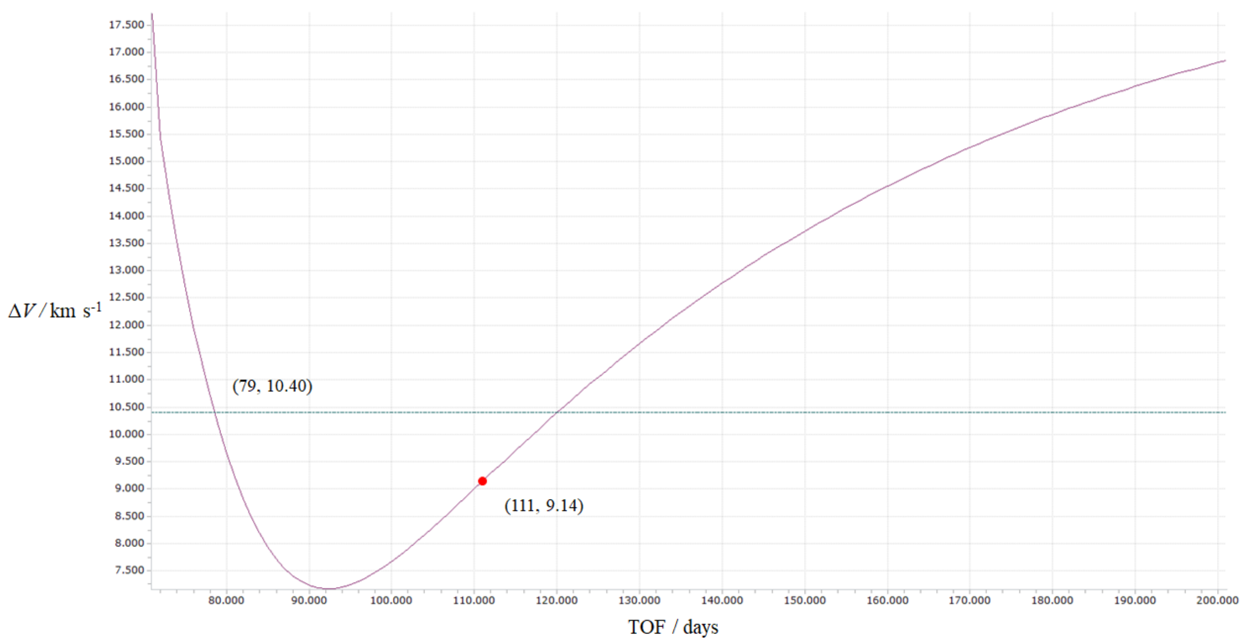

Figure 4: ΔV of the first burn against TOF

Using our data (Appendix 4), we found out that the ΔV produced by the maximum propellant mass of Centaur III for its mission is 10.40km s^{-1}. This would therefore be our assumed maximum ΔV. From the intersection of the line and curve in figure 4, it shows that the shortest TOF possible is 79 days. This is 32 days less than the TOF for the minimum energy transfer burn in figure 3. However, this shorter TOF requires a 1.26km s^{-1} greater ΔV for burn 1 for the manoeuvre, which would result in 5750kg more propellant to be used.

To evaluate whether the extra propellant used is justifiable, we will be looking at 2 factors, namely cost, and purpose of the mission. In terms of cost, we will only be looking at the cost of liquid hydrogen and oxygen and not other costs in relation to the propellant such as storage costs. We would deem the oxidiser to fuel ratio to be 6 [19] (with liquid oxygen as the oxidiser and liquid hydrogen being the fuel). Considering the above, the extra cost required would be $2562 [20][21]. This value is small relative to the total cost of propellants necessary to travel to Mars from the surface of the Earth [22]. The TOF may prove to be crucial for manned missions to reduce the time that astronauts spend in space. This is to prevent time spent by astronauts in Zero-G and the radiation that astronauts are exposed to in space [2]. It is also crucial as it reduces the time in transit during interplanetary travel, reducing the chance for errors [3].

Section 4 - Conclusion

In conclusion, this paper analysed the traditional method of interplanetary travel, the Hohmann transfer, in a trajectory from Earth to Mars, and explains why it is unrealistic and limited. It proceeds to present a method to determine the feasibility of reducing the TOF by solving Lambert's Problem, while considering the limits of current technology. Using the Lambert solver in Freeflyer, we have determined the relationship between the ΔV and the TOF of the manoeuvre from Earth to Mars. Afterwhich, by using Centaur III as reference for maximum fuel capacity, we have determined how much the TOF can be shortened by, with various assumptions and the feasibility of such an increase in propellant to reduce TOF in terms of cost.

For future study, this paper will take into account the gravitational force of the Earth, Mars and other celestial bodies acting on the spacecraft to achieve more accurate ΔV values, and the lift-off from the surface of the Earth. This paper can be adapted to evaluate different kinds of impulsive propulsion systems alongside different chemical propellants, allowing for a deeper exploration and understanding of propulsion systems. Another relevant point of interest will be the adoption of nuclear propulsion systems, as they provide a high I_{sp} and have high thrust, meaning the TOF can be reduced even further for such a propulsion system, bringing us one step closer to the dream of a manned Mars landing.

Section 5 - Bibliography of references

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Section 6 - Appendix

Nomenclature:

(Constants to be taken at 5 Significant Figures)

R_1 = Distance of Earth from Sun = 1.4960 x 10^8 km

R_2 = Distance of Mars from Sun = 2.2792 x 10^8 km

GM_{sun} = Standard Gravitational Parameter of Sun = 1.3271 x 10^{11} km^3 s^{-2}

P_1 (Orbital Period of Earth) = 3.1558 x 10^7s

P_2 (Orbital Period of Mars) = 5.9329 x 10^7s

P_3 = Period of transfer orbit

$\triangle V_1$ = Departure Change in Velocity

$\triangle V$ = Total velocity

a = Semi-major axis of transfer orbit

m = Mass of spacecraft

V_1= Initial Velocity in Earth's Orbit

V_2 = Final Velocity in Mars's Orbit

V_p = Velocity at perihelion

V_a = Velocity at aphelion

$\triangle V_2$ = Arrival Change in Velocity

Appendix 1: Derivation for the standard Hohmann transfer model

To find the semi-major axis of the transfer orbit,

$$a = \frac{R_1 + R_2}{2} \quad (1)$$

Using Kepler's Third Law,

$$T^2 = \frac{4\pi^2}{GM_{sun}} a^3$$

Since gravitational pull of Sun will provide the centripetal force on the spacecraft,

$$\frac{GM_{sun} m}{R_1^{\ 2}} = \frac{mV_1^{\ 2}}{R_1}$$

Hence V_1 and V_2 are given by:

$$V_1 = \sqrt{\frac{GM_{sun}}{R_1}} \qquad V_2 = \sqrt{\frac{GM_{sun}}{R_2}}$$

By the Law of Conservation of angular momentum,

$$mV_pR_1 = mV_aR_2$$

$$V_a = \frac{R_1}{R_2}V_p \qquad (2)$$

By the Law of Conservation of energy,

$$-\frac{GM_{sun}m}{R_1} + \frac{1}{2}mV_p^{\ 2} = -\frac{GM_{sun}m}{R_2} + \frac{1}{2}mV_a^{\ 2}$$

$$V_p^{\ 2} - V_a^{\ 2} = -\frac{2GM_{sun}}{R_2} + \frac{2GM_{sun}}{R_1} \qquad (3)$$

Combining both equation (2) and (3),

$$V_p^{\ 2} - (\frac{R_1}{R_2}V_p)^2 = 2GM_{sun}(-\frac{1}{R_2} + \frac{1}{R_1})$$

Hence, V_p is given by:

$$V_p = \sqrt{2GM_{sun}\frac{R_2}{R_1(R_1 + R_2)}}$$

Combining with equation (1), this paper gets the vis-viva equation:

$$V_p = \sqrt{2GM_{sun}(\frac{2}{R_1} - \frac{1}{a})}$$

Hence, to find the departure and arrival velocity,

$$\Delta V_1 = V_p - V_1$$

$$\Delta V_2 = V_2 - V_a$$

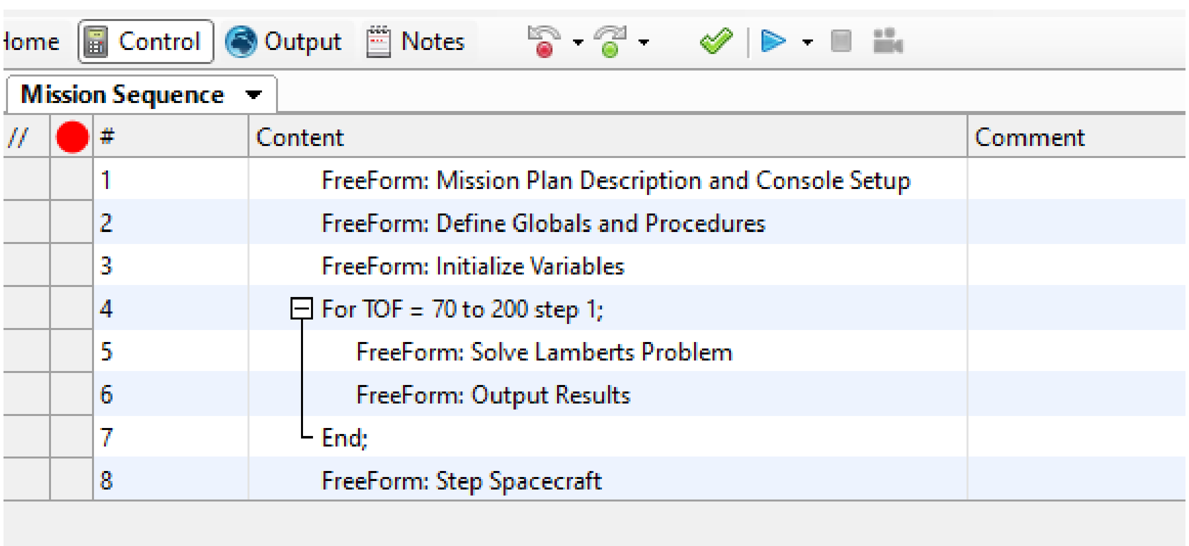
Finally, to get the total ΔV required, we take:

$$\Delta V = \Delta V_1 + \Delta V_2$$

Additionally, the time of flight (TOF) in days can be determined with:

$$TOF = (\frac{1}{2}T) \div 86400$$

Appendix 2: Mission Sequence in FreeFlyer



Appendix 3: Lambert Solver in FreeFlyer with several modifications

ome | Control | Output | Notes

Mission Sequence | 5 - Solve Lamberts Problem

FreeForm Label: Solve Lamberts Problem | Print

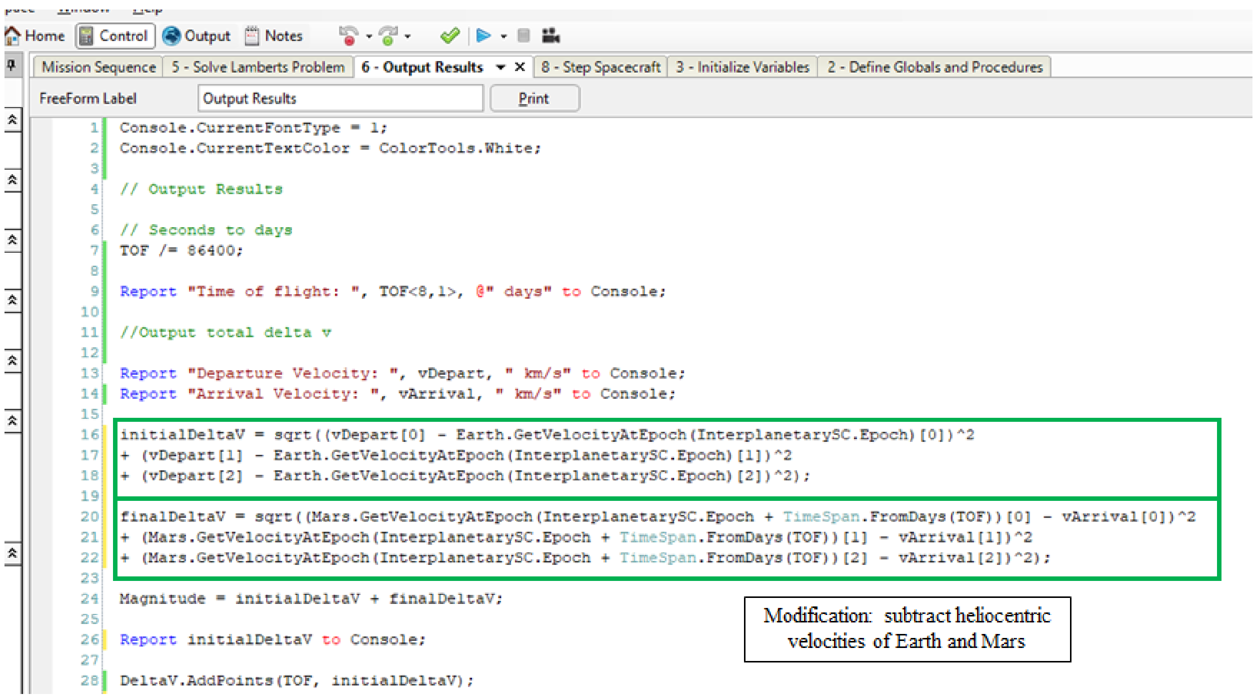
```
// Initialise planets

StartPlanet = 3; // Earth
ArrivalPlanet = 4; // Mars

TOF += 1; // Cycle through TOF

IsPrograde = 1;

// Solve Lambert's Problem

// Convert from days to seconds
TOF *= 86400;
InterplanetarySC.CentralBody = "Sun";
InterplanetarySC.Epoch = DepartTime.ParseCalendarDate();

// Get Planet positions
departurePos = InterplanetarySC.PlanetPosition(StartPlanet);
departurePos[2] = departurePos[2];
InterplanetarySC.Epoch += TimeSpan.FromSeconds(TOF);
arrivalPos = InterplanetarySC.PlanetPosition(ArrivalPlanet);
InterplanetarySC.Epoch -= TimeSpan.FromSeconds(TOF);

// Magnitudes of position vectors
varR1 = departurePos.Norm();
varR2 = arrivalPos.Norm();

// Calculate cross product to determine orbit direction and calculate transfer angle
cross12 = departurePos.CrossProduct(arrivalPos);

theta = departurePos.VertexAngle(arrivalPos);

// Change initial theta to adjust orbit direction
If (IsPrograde == 1);
    If (cross12[2] < 0);
        theta = 360 - theta;
    End;
ElseIf (IsPrograde == 0);
    If (cross12[2] >= 0);
        theta = 360 - theta;
    End;
End;

Const = sin(rad(theta))*sqrt(varR1*varR2/(1 - cos(rad(theta))));
fOut = -1;

// Determine when the F-function crosses 0 to determine an initial z value for easier convergence
While (fOut < 0);

    Call Ffunc(z, TOF, fOut);
```

Mission Sequence | 5 - Solve Lamberts Problem

FreeForm Label: Solve Lamberts Problem | Print

```

// Determine when the F-function crosses 0 to determine an initial z value for easier convergence
While (fOut < 0);

    Call Ffunc(z, TOF, fOut);

    // Errors encountered are due to imaginary numbers in the F-function solution.
    // Increasing the z value at a faster rate if errors.
    If (ErrCount > 0);
        z++;
        ErrCount = 0;
    Else;
        z += 0.1;
    End;
End;

// Iterate the F-function using Newtons algorithm to find a solution for Lamberts problem
While ((abs(ratio) > tol) and (n <= nMax));

    Call Ffunc(z, TOF, fOut);
    Call dFfunc(z, dFOut);
    ratio = fOut/dFOut;
    n++;
    z -= ratio;

End;

If (nMax <= n);
    Console.CurrentTextColor = ColorTools.Red;
    Report "Maximum number of iterations reached, no solution found. Stopping mission plan." to Console;
    Stop;
End;

Call Yfunc(z,yOut);

// Solve the f, g, and gdot functions to determine departure and arrival velocities.
f = 1 - yOut/varR1;
g = Const*sqrt(yOut/Mu);
gdot = 1 - yOut/varR2;

vDepart = 1/g*(arrivalPos - f*departurePos);
vArrival = 1/g*(gdot*arrivalPos - departurePos);

InterplanetarySC.Position = departurePos;
InterplanetarySC.Velocity = vDepart;
```

Appendix 4: Modification to determine ΔV for interplanetary spacecraft in FreeFlyer



Appendix 5: Visualization of Interplanetary Transfer in FreeFlyer of minimum point for total ΔV against TOF graph