Complexes

Forme algébrique

$z = a + ib$

$\mathrm{Re}(z) = a$

$\mathrm{Im}(z) = b$

$\bar{z}$: conjugué de z

$\bar{z} = a - ib$

avec $z \in \mathbb{C}$

$$\frac{1}{z} = \frac{1}{a+ib} = \frac{a-ib}{(a+ib)(a-ib)}$$

$$= \frac{a-ib}{a^2+b^2}$$

$$= \frac{a-ib}{|z|}$$

$z \neq 0$

Forme exponentielle

$z = r e^{i\theta}$

Module : $|z| = r$

$\mathrm{Arg}(z) = \theta$

$\bar{z} = r e^{-i\theta}$

avec $z \in \mathbb{C}$

$\cancel{\sqrt{-1}}$

$\cancel{\sqrt{i}}$

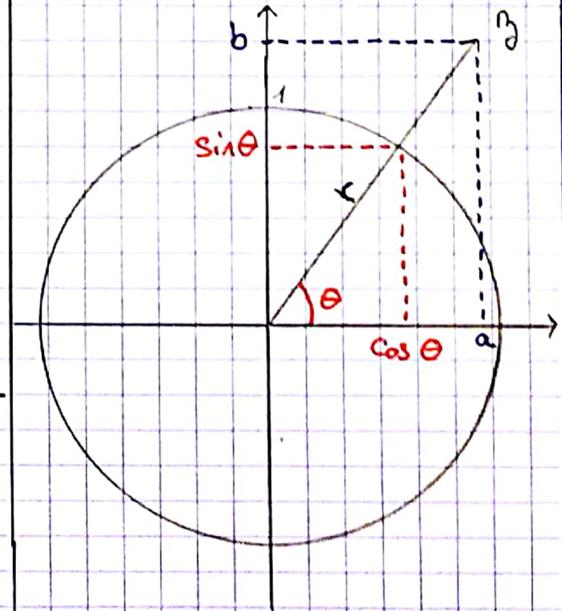
$z^2 = i$

$z^2 = -1$

Forme trigonométrique

$z = r(\cos\theta + i\sin\theta)$

$\bar{z} = r(\cos\theta - i\sin\theta)$



avec $z \in \mathbb{C}$

$i^2 = -1$

$$\left.\begin{aligned} \cos\theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin\theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \end{aligned}\right\}\text{formule Euler}$$

$r = \sqrt{a^2 + b^2}$

$$\left.\begin{aligned} a &= r\cos\theta \\ b &= r\sin\theta \end{aligned}\right] \leftarrow e^{i\theta} = \cos\theta + i\sin\theta$$

$$\begin{aligned} zz' &= (a+ib)(a'+ib') \\ &= aa' + ia'b + iab' - bb' \\ &= (aa' - bb') - i(a'b + ab') \\ &= z'' \begin{pmatrix} aa' - bb' \\ a'b + ab' \end{pmatrix} \end{aligned}$$

$$\begin{aligned} zz' &= re^{i\theta} + r'e^{i\theta'} \\ &= rr'e^{i(\theta+\theta')} \end{aligned}$$