SERIE Entière

$$\sqrt[n]{a_n} \underset{+\infty}{\to} \ell = \frac{1}{R} \begin{cases} \ell = +\infty \to R = 0 \\ \ell = 0 \to R = +\infty \\ \ell \in \mathbb{R}^* \to R = \frac{1}{\ell} \\ \ell = -\infty \to R = 0 \end{cases}$$

Exercice 1:

$\sum \left(\frac{(-3)^n}{n^{\sqrt{n}}} y^{2n} \right)$ on pose $y^2 = z$, $S = \sum \frac{(-3)^n}{n^{\sqrt{n}}} z^n$

on $a_n = \frac{(-3)^n}{n^{\sqrt{n}}}$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(-3)^{n+1}}{n+1^{\sqrt{n+1}}}}{\frac{(-3)^n}{n^{\sqrt{n}}}}$$

$|a_n| = \frac{3^n}{n^{\sqrt{n}}}$

$$= \frac{(-3)^{n+1} n^{\sqrt{n}}}{(n+1)^{\sqrt{n+1}} (-3)^n}$$

$\sqrt[n]{|a_n|} = \left(\frac{3^n}{n^{\sqrt{n}}} \right)^{1/n} = \frac{3}{n^{\frac{\sqrt{n}}{n}}} = \frac{3}{n^{1/\sqrt{n}}}$

$\sqrt[n]{|a_n|} = 3 e^{-\frac{\ln(n)}{\sqrt{n}}}$

$$= -3 \frac{n^{\sqrt{n}}}{(n+1)^{\sqrt{n+1}}}$$

$$= -3 \frac{n \sqrt{n}}{(n+1) \sqrt{n+1}}$$

$\lim\limits_{n \to \infty} \frac{\ln(n)}{\sqrt{n}} = 0$

~~Equivalence~~

$\lim \sqrt[n]{a_n} = 3$

donc $R = \frac{1}{3}$

donc pour $|z| < \frac{1}{3}$, S converge

donc $|y^2| < \frac{1}{3}$, S converge

$|y| < \frac{1}{\sqrt{3}}$, S converge