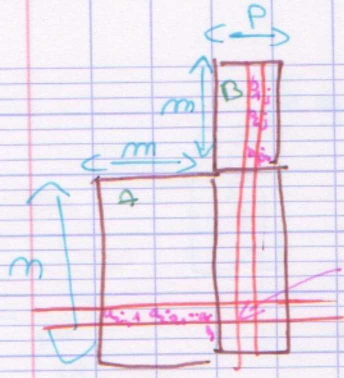
Maths Neem

1



$$(AB)_{i,j} = a_{i,1} \times b_{1,j} + a_{i,2} \times b_{2,j} \dots$$

$$\sum_{k=1}^{m} a_{i,k} \times b_{k,j}$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \; ; \; y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$(x|y) = x_1 y_1 + \dots + x_n y_n$$

$$x^T y = \begin{bmatrix} x_1 & x_2 & \dots \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\begin{bmatrix} y_1 & \dots & y_n \end{bmatrix}$$

$$\underset{(n,1)}{x}\,\underset{(1,n)}{y^T} = \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{C}$$

(n,n)

$|c|$

$n = 2$

$a_{1,1} x_1 + a_{1,2} x_2 = b_1$ ← un hyperplan dans $\mathbb{R}^2$ c-a-d une droite

$a_{2,1} x_1 + a_{2,2} x_2 = b_2$ → une deuxième droite

$n = 3$ un hyperplan dans $\mathbb{R}^3$ c'est à dire un plan

$a_{1,1} x_1 + a_{1,2} x_2 + a_{1,3} x_3 = b_1 \to \Pi_1$

$a_{2,1} x_1 + a_{2,2} x_2 + a_{2,3} x_3 = b_2 \to \Pi_2$

$a_{3,1} x_1 + a_{3,2} x_2 + a_{3,3} = b_3 \to \Pi_3$

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Exercice du cours chap 2

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & & & \\ a_{m,1} & & \cdots & a_{m,m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \quad \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \to b$$

$Ax = b$

On trouve le point x^* vérifiant $(*)$

$$x^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ b \end{bmatrix} \leftarrow i^*$$

$u \perp v$

$$\Leftrightarrow (u|v) = u^T v$$
$$= \langle u, v \rangle = 0$$

$$u_1 v_1 + u_2 v_2 + \cdots + u_m v_m = 0$$

$$x \in H \Leftrightarrow (x - x^* | \alpha) = 0$$
$$\Leftrightarrow (x|\alpha) - (x^*|\alpha) = 0$$
$$= x\alpha_1 + x_2\alpha_2 + \cdots + x_m\alpha_m$$

Maths Num

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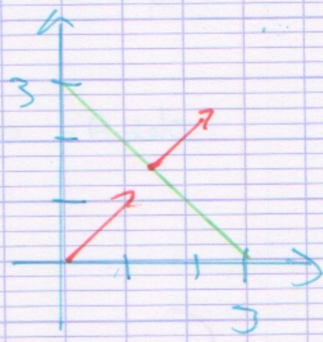
$x_1 + x_2 = 3$

$\boxed{\begin{array}{l} x_1 = 3 \\ x_2 = 0 \end{array}}$ $\boxed{\begin{array}{l} x_1 = 0 \\ x_2 = 3 \end{array}}$

$a = 1$
$b = 1$



Données :

$A \in \mathbb{R}^{n \times m} = \mathcal{M}_{n,m}(\mathbb{R})$ — nb lignes — nb colonnes

$b \in \mathbb{R}^m$

$\underline{Ax = b}$

Exercice diapo 20 ch 2

1) $A = [a]$ $\quad a \in \mathbb{R}$

$\det(A) = \det(1 \times a)$

$= a \det(1)$ par linéarité

$\Rightarrow \det(1) = 1$ par définition

2) $j = 1 \quad n = 2$

$$\sum_{i=1}^{2} (-1)^{i+2} a_{i,2} \left(\det [A]_{(-i,-2)}\right)$$

$$= (-1)^3 a_{1,2} \left(\det[A]_{(-1,-2)}\right) + (-1)^4 a_{2,2} \left(\det[A]_{(-1,-2)}\right)$$

$$= -a_{1,2} \left(\det[A]_{(-1,2)}\right) + a_{2,2} \left(\det[A]_{(-1,2)}\right)$$

(4)

$$\det A = \sum_{i=1}^{?} (-1)^{i+1} a_{i,j} \det [A]_{(i,j)}$$

$$\det A = a_{1,1} a_{2,2} - a_{2,1} a_{1,2}$$

$$\begin{Bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{Bmatrix}$$

$$a_{1,1} \begin{vmatrix} a_{2,2} & a_{2,3} \\ a_{3,2} & a_{3,3} \end{vmatrix} - a_{2,1} \begin{vmatrix} a_{1,2} & a_{1,3} \\ a_{3,2} & a_{3,3} \end{vmatrix} + a_{3,1} \begin{vmatrix} a_{1,2} & a_{1,3} \\ a_{2,2} & a_{2,3} \end{vmatrix}$$

Système triangulaire

$$\begin{matrix} t_{1,1} & t_{1,2} & \cdots & t_{1,n} \\ 0 & t_{2,2} & \cdots & t_{2,n} \\ 0 & 0 & t_{3,3} \cdots & t_{3,n} \\ & & & \vdots \\ & & & \vdots \end{matrix}$$

Maths Num (5)

$a_{1,1} \neq 0$

$$coef_i \cdot \left(a_{1,1} x_1 + a_{1,2} x_2 + \dots + a_{1,m} x_m = b_1 \right)$$

$$a_{i,1} x_1 + a_{i,1} x_2 + \dots + a_{i,m} x_m = b_i$$

$i \in [\![2, m]\!]$

$coef_i = \alpha_i$

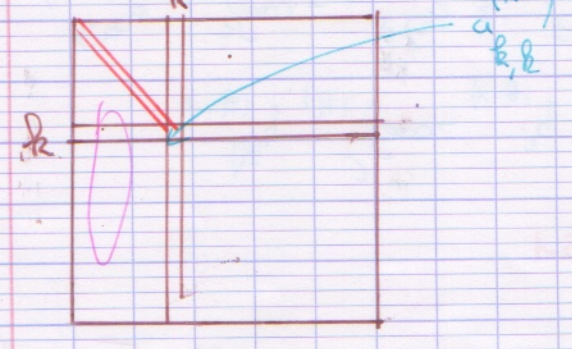
$$\overbrace{(a_{i,1} - coef_i \cdot a_{1,1})}^{a_{i,1}^{(1)}} x_1 + \overbrace{(a_{i,2} - coef_i \cdot a_{1,2})}^{a_{i,2}^{(1)}} x_2 + \dots + \overbrace{(a_{i,m} - coef_i \cdot a_{1,m})}^{a_{i,m}^{(1)}} x_m$$

$$= \underbrace{b_i - coef_i \cdot b_1}_{b_i^{(1)}}$$

Remarque $a_{i,1} = 0$ - Il faut et il suffit

$$\boxed{coef_i = \frac{a_{i,1}}{a_{1,1}}}$$

Diapo 49 : Exo



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$\text{équ}_i : \quad i > k$

$$a_{i,k}^{(k-1)} x_k + a_{i,k+1}^{(k-1)} x_{k+1} + \dots + a_{i,j}^{(k-1)} x_j + \dots + a_{i,n}^{(k-1)} x_n = b_i^{(k-1)}$$

$$0 \neq a_{k,k}^{(k-1)} x_k + a_{k,k+1}^{(k-1)} x_{k+1} + \dots + a_{k,j}^{(k-1)} x_j + \dots + a_{k,n}^{(k-1)} x_n \neq b_k^{(k-1)}$$

$$\text{equ}_i^{(k)} := \text{equ}_i^{(k-1)} - \text{coef}_i^{(k)} \, \text{equ}_k^{(k-1)}$$

$$x_k \underbrace{\left(a_{i,k}^{(k-1)} - \text{coef}_i^{(k)} a_{k,k}\right)}_{=0} + \dots + \overbrace{\left(a_{i,j} - \text{coef}_i^{(k)} a_{k,j}\right)} x_j + \dots$$

$$= b_i^{(k-1)} - \text{coef}_i^{(k)} b_k^{(k-1)}$$

On choisi $\text{coef}_1^{(k)} := a_{i,k}^{(k-1)} / a_{k,k}^{(k-1)}$

pour i de k + 1 à n

$\text{coef}_i^{(k)} := a_{i,k}^{(k-1)} / a_{k,k}^{(k-1)}$

$a_{i,k}^{(k)} := 0$

Pour j de k+1 à n :

$a_{i,j}^{(k)} := a_{i,j}^{(k-1)} - \text{coef}_i^{(k)} \cdot a_{k,j}^{(k-1)}$

$b_i^{(k)} = b_i^{(k-1)} - \text{coef}_i^{(k)} b_k^{(k-1)}$

Exercice diapo 51

$B(n,m)$ $\qquad\qquad M = I - y\,(e^k)^T$

Maths Num

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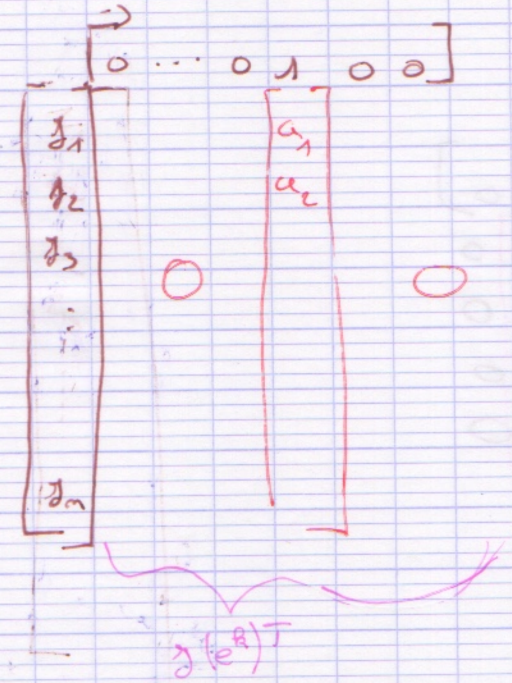
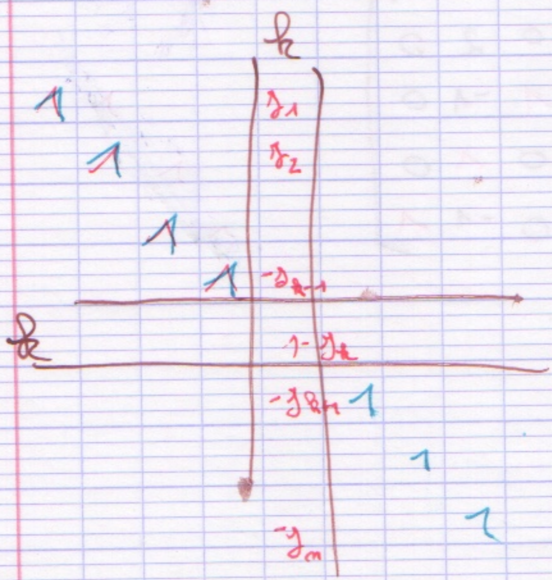
Matrice identité

$Z \rightarrow$ ligne

$(e^k) \rightarrow$ colonne





$(e^i)^T A = A_j$

$A e^j = A^j$

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exemple:

$$z = \begin{bmatrix} -2 \\ 1 \\ 0 \\ -1 \end{bmatrix} \qquad e^3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \begin{matrix} n=4 \\ k=3 \end{matrix}$$

$$z(e^3)^T \quad \begin{array}{c|cccc} & 0 & 0 & 1 & 0 \\ \hline -2 & 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 \end{array}$$

z

$-2 + 1 + 0 - 1$

Matrice Identité I_n

⚠

$$I - z(e^3)^T = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 1 \\ 0 & 1 \\ 3 & -2 \\ -1 & 3{,}5 \end{bmatrix}$$

Meth Num

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$$MB = \begin{bmatrix} [5,1] + 2[3,-2] \\ [0,1] - 1[3,-2] \\ [3,-2] - 0[3,-2] \\ [-1;3,5] + 1[3;-2] \end{bmatrix} = \begin{bmatrix} 11 & -3 \\ -3 & 3 \\ 3 & -2 \\ 2 & 1,5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 0 & 1 \\ 3 & -2 \\ -1 & 3,5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 11 & \\ & \\ & \\ & \end{bmatrix}$$

$$(MB)_{i,:} = (e^i)^T (MB) = (e)^T \underbrace{\left(\left(I - y(e^k)^T \right) B \right)}$$

$$M = I - y(e^k)^T$$

$$I \times B - \cancel{k}y(e^k)^T B$$

$$= (e^i)^T (I \times B - y(e^k)^T B)$$

$$= (e^i)^T (B - B - y B_k$$

$$\begin{bmatrix} 1 & 0 & \cdots & y_1 & & & \\ 0 & 1 & & \vdots & & 0 & \\ & & \ddots & y_{k-1} & & & \\ & & & 1-y_k & & & \\ & 0 & & \vdots & \ddots & & \\ & & & y_m & & & 1 \end{bmatrix}$$

Solution

② $MB = (I - y(e^k)^T)B = B - y(e^k)^T B$
$= B - y B_k$

d'où ... Voir diapo $\boxed{52}$

Exercice n°53 (Diapo 53)

$$LM \overset{?}{=} I$$

$LM = (I + y(e^k)^T)(I - y(e^k)^T)$

$= I \times I - I \times y(e^k)^T + y(e^k)^T \times I + y(e^k)^T \times y(e^k)^T)$

$u^T v = (u \mid v) = u_1 v_1 + u_2 v_2 + \cdots + u_m v_m$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_m \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & u_3 & \cdots & u_m \end{bmatrix}$$

Maths Num

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$= I -$

$$\begin{bmatrix} 1 & 0 & \cdots & y_1 & \\ 0 & & & y_2 & \\ & & & \vdots & \\ & & & 1-y_k & \\ & & & \vdots & \\ & & & y_m & \end{bmatrix}$$

$$LM = I + y(e^k)^T - y(e^k)^T - y(e^k)^T y (e^k)^T$$

$$= I - y((e^k)^T y)(e^k)^T$$

$$= I - y(y(e^k))(e^k)^T$$

$$= I$$

Dispo 61

$$U = M^{(n-1)} M^{(n-2)} \dots M^{(2)} M^{(1)} A$$

$$L^{(n-1)} U = \underbrace{L^{(n-1)} M^{(n-1)}}_{I} M^{(n-2)} \dots M^{(2)} M^{(1)} A$$

$$L^{(n-1)} U = M^{(n-2)} M^{(\ )} \dots M^{(1)} A$$

$$L^{(n-2)} L^{(n-1)} U = L^{(n-2)} M^{(n-2)} M^{(n-3)} \dots M^{(1)} A$$

Dispo Interpolation

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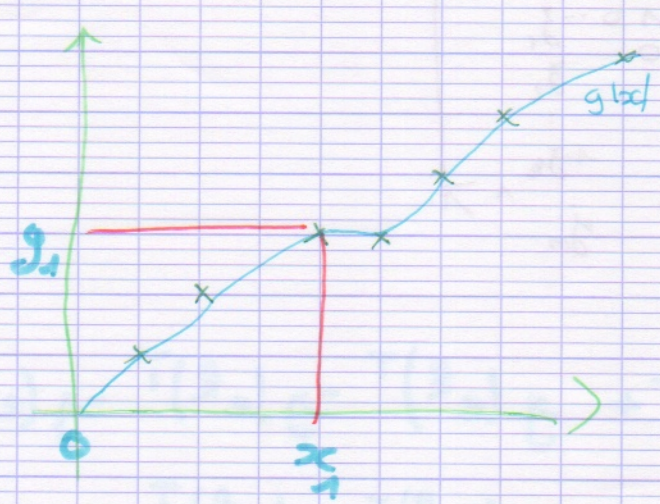
~~Exp~~ Diapo



g(x) une fonction qui passe par toutes les points

spline : morceau de polynome

φ_j -> facilemt calculable

Retenir Base de Lagrange:

$$L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \left(\frac{x - x_j}{x_i - x_j}\right), i \in [0, n]$$

Il a la propriété suivante: $L_i(x_j) = \delta_{i,j} = \begin{cases} 1 \text{ pour } i=j \\ 0 \text{ sinon} \end{cases}$

Maths Num

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$(\mathcal{L}_0, \mathcal{L}_1, \dots, \mathcal{L}_n)$ est une base

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

$$f(x) = \sum_{i=0}^{n} y_i \mathcal{L}_i(x)$$

$$p(x_k) = y_k \quad , \forall k \in [\![0, n]\!]$$

$$\Rightarrow \mathcal{L}_i(x_k) = \delta_{i,k}$$

Mq (Montrons) que

$$\mathcal{L}_i(x_k) = \delta_{i,k}$$

$$\mathcal{L}_i(x_k) = \prod_{j=0}^{n} \left(\frac{x_k - x_j}{x_i - x_j}\right)$$

$k \neq i$

$k = i$

Dans le cas où $k \neq i$

$$\mathcal{L}_i(x_k) = \prod_{\substack{j=0 \\ j \neq i \\ j \neq k}}^{n} \left(\frac{x_k - x_j}{x_i - x_j}\right) \times \left(\frac{x_k - x_k}{x_i - x_k}\right)$$

$= 0$

(14)

Pour $k = i$ $\prod_{\substack{j=0 \\ j\neq i \\ k=i}} \left(\frac{x_i - x_j}{x_i - x_j}\right) = 1$

$x_0 = -1$; $x_1 = 2$; $x_2 = 3$

calculez $\mathcal{L}_0$, $\mathcal{L}_1$, $\mathcal{L}_2$

$\mathcal{L}_0(-1) = 1$

$\mathcal{L}_0(2) = 0$

$\mathcal{L}_0(3) = 0$

$\mathcal{L}_0(x) = \prod_{\substack{j=0 \\ j\neq 1 \\ \to j=1}}^{2} \left(\frac{x - x_j}{x_i - x_j}\right) = \prod_{j=1}^{2} \left(\frac{x - x_j}{-1 - x_j}\right) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$

$\mathcal{L}_0(x) = \left(\frac{x-2}{-3}\right)\left(\frac{x-3}{-4}\right)$

Prenons $x = -1 \to \left(\frac{-1-2}{-3}\right)\left(\frac{-1-3}{-4}\right) = 1 \times 1 = 1$

$\mathcal{L}_0(2) = \left(\frac{2-2}{-3}\right)\left(\frac{2-3}{-4}\right) = 0 \times \left(\frac{2-3}{-3}\right) = 0$

$\mathcal{L}_0(3) = \left(\frac{3-2}{-3}\right)\left(\frac{3-3}{-4}\right) = 0$

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$$\prod_{\substack{j=0\\ j\neq i}}^{n} \left(\frac{x-x_j}{x_i-x_j}\right) = \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)$$

$$= \left(\frac{x+1}{3}\right)\left(\frac{x-3}{-1}\right)$$

On est bon !

$$\mathcal{L}_1(-1) = \left(\frac{-1+1}{3}\right)\left(\frac{-1-3}{-1}\right) = 0 \times \left(\frac{-1-3}{-1}\right) = 0$$

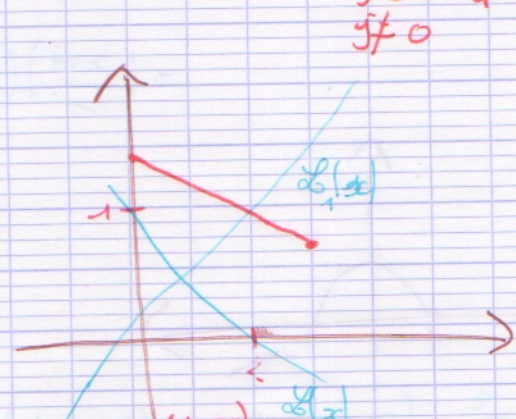
$$\mathcal{L}_1(2) = \left(\frac{2+1}{3}\right)\left(\frac{2-3}{-1}\right) = \frac{3}{3} \times \frac{-1}{-1} = 1$$

$$\mathcal{L}_1(3) = \left(\frac{3+1}{3}\right)\left(\frac{3-3}{-1}\right) = \frac{4}{3} \times \frac{0}{-1} = 0$$

Diapo $\boxed{26 / 84}$

~~Poser~~ $\mathcal{L}_0(x) = \left(\frac{x-x_1}{x_0-x_1}\right) = \frac{x-1}{-1}$

$$\mathcal{L}_1(x) = \prod_{\substack{j=0\\ j\neq 0}}^{1}\left(\frac{x-x_0}{x_1-x_0}\right) = \left(\frac{x-x_0}{x_1-x_0}\right) = x$$



$$p(x) = y_0 \overbrace{\mathcal{L}_0(x)}^{(1-x)} + y_1 \overbrace{\mathcal{L}_1(x)}^{x}$$

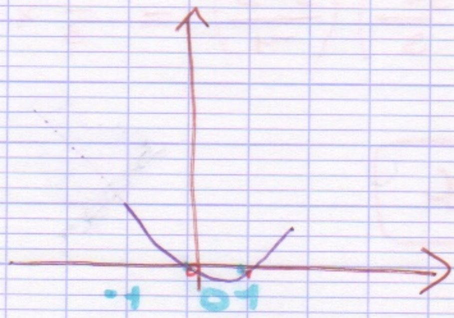
$$p(x) = y_0 + \text{pente}(x-0) \qquad \frac{y_1-y_0}{x_1-x_0} = 1$$

$$= y_0 + (y_1 - y_0)\, x$$

2 $\mathcal{L}_0(x) = \prod_{\substack{j=0 \\ j\neq 0}}^{2} \left(\frac{x - x_j}{x_0 - x_j}\right)$

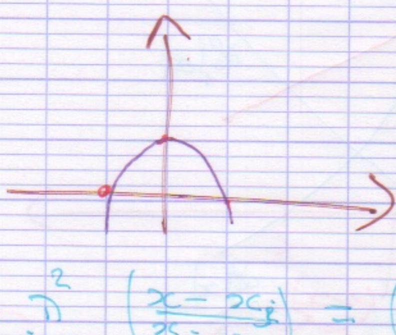
$$= \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$\mathcal{L}_0(x) = \left(\frac{x}{-1}\right)\left(\frac{x+1}{2}\right)$$



$$\mathcal{L}_1(x) = \prod_{\substack{j=0 \\ j\neq i}}^{2} \left(\frac{x - x_j}{x_i - x_j}\right) = \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$$

$$= \left(\frac{x+1}{1}\right)\left(\frac{x-1}{-1}\right)$$



$$\mathcal{L}_2(x) = \prod_{\substack{j=0 \\ j\neq 2}}^{2} \left(\frac{x - x_j}{x_i - x_j}\right) = \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right) = \left(\frac{x+1}{2}\right)\left(\frac{x}{1}\right)$$