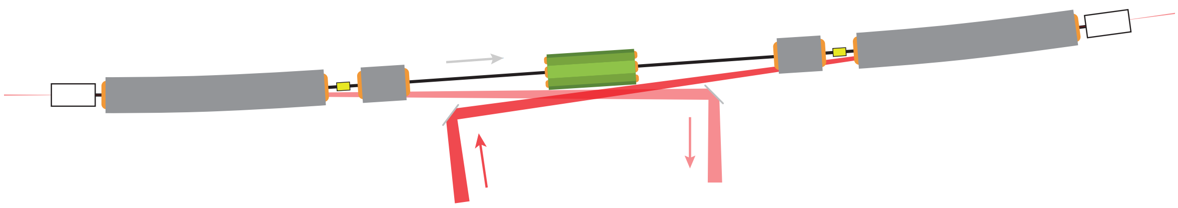
ERC Starting Grant 2023
Research Proposal [Part B1]

Staging of **P**lasma **A**ccelerators for **R**ealizing **T**imely **A**pplications

SPARTA



Cover page:

– *Principal Investigator (PI):* Carl Andreas Lindstrøm
– *Host Institution:* University of Oslo
– *Proposal Duration:* 60 months

High-energy physics is headed for an impasse: the next particle collider will cost several billion euros, and while designs have been ready for a decade, they are so expensive that no host country has come forward—a problem that will soon impact progress in the field.

Plasma acceleration is a novel technology promising to fix this issue—with accelerating fields 1000 times larger than in conventional machines, the size and cost of future accelerators can be drastically reduced. However, there is a gap between what current plasma accelerators can do and what the next collider requires. Therefore, a recent R&D roadmap (European Strategy for Particle Physics) calls for intensified plasma-accelerator research, as well as an intermediate demonstrator facility.

SPARTA tackles two basic problems in plasma acceleration: to reach high energy by connecting multiple accelerator stages without degrading the accelerated beam, and to do so in a stable manner. Access to stable, high-energy electron beams at a fraction of today's cost will enable ground-breaking advances in strong-field quantum electrodynamics (SFQED), an important near-term experiment that doubles as a demo facility.

I have proposed two concepts for overcoming these problems: nonlinear plasma lenses for transport between stages, and a new mechanism for self-stabilization. Can these concepts be realized in practice?

Making use of numerical simulations and beam-based experiments at international accelerator labs, this project has 3 objectives:

1. Develop nonlinear plasma lenses experimentally;
2. Investigate self-stabilization, theoretically and experimentally;
3. Design a plasma-accelerator facility for SFQED.

Reaching this goal will not only impact high-energy physics, producing advances in SFQED and as a major step toward realizing a collider, but also society at large: applications of high-energy electrons, from bright x-ray beams to advanced cancer treatments, will all become significantly more affordable.

Key words: Plasma accelerators, plasma lenses, particle-beam optics, strong-field quantum electrodynamics

Section A: Extended Synopsis of the scientific proposal

Particle accelerators and high-energy physics: it's too expensive

Over the past century, particle accelerators have grown exponentially in their ability to deliver higher particle energy and flux, but also greatly increasing in size and cost. The increase in capability has led to a rich variety of applications well beyond the original particle physics, which are now indispensable for other fields of science and society in general: examples include synchrotron light sources[1] and free-electron lasers[2] (FELs), used to observe atomic structures, as well as radio- and proton therapy for treating cancer[3].

At the forefront of particle physics, new particle colliders are being proposed to collide electrons and positrons at TeV-scale energies for precision tests of the Standard Model. In this case, a *linear collider* is needed to avoid synchrotron radiation[4]. While these machines can be made with conventional accelerator technology, such as the linear-collider proposals ILC[5] and CLIC[6], they will be tens of kilometres long and extremely expensive to build. So far, no one has been willing[7] to foot the multi-billion-dollar[8] bill.

To make progress, the high-energy-physics community is pushing the development of novel technologies for smaller and cheaper accelerators: as stated in the Accelerator R&D Roadmap[9] of the 2020 European Strategy for Particle Physics, *"Development and exploitation of laser/plasma acceleration techniques"* is one of *"five key areas where an intensification of R&D is required."* In addition, there is also consensus, across both European and US roadmaps[10], that in order for this technology to mature and meet the demands of a collider, a near-term application should be sought as an intermediate step, ideally related to high-energy physics.

This proposal aims to bridge the gap between current *plasma accelerators* and the next linear collider: first by tackling the most crucial problems, and then by providing the blueprints for a medium-scale demonstrator facility that enables ground-breaking experiments in *strong-field QED* at a reasonable cost.

Plasma acceleration: more affordable accelerators

The size and cost of linear colliders and other high-energy linear accelerators, such as FELs, is a result of the limited accelerating field[11] (~100 MV/m) achievable in conventional radio-frequency accelerators. Beyond this, breakdowns occur in the walls of the metallic cavities, rapidly causing irreversible damage. Plasma accelerators, on the other hand, use the result of the breakdown itself (a plasma) as the accelerating medium, instead of metallic cavities. Here, the achievable accelerating field is several orders of magnitude larger than in conventional accelerators: 100 GV/m fields[12] are all but routine in many plasma accelerators.

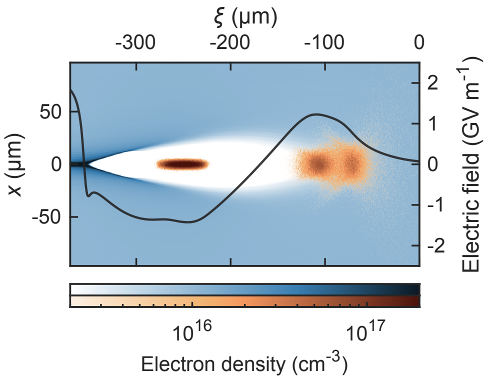


Fig. 1: Simulation of a beam-driven plasma accelerator: an intense particle bunch (orange colour bar), travelling to the right, drives a strong accelerating field (black line) in a plasma (blue colour bar), in which a trailing particle bunch is both accelerated and strongly focused (by the plasma ions). *From: Lindstrøm et al. (submitted).*

Plasma acceleration[13] was introduced in 1979 by Tajima and Dawson[14], who found that if a sufficiently intense laser or (as later suggested[15]) charged-particle beam traverses a plasma, it will drive a charge-density wave behind it—a *plasma wake*. Within this wake, plasma electrons are separated from plasma ions, resulting in strong electromagnetic fields, called *plasma wakefields*, able to both accelerate bunches of particles as well as keep them focused over long distances. Increasing the plasma density means higher accelerating fields, but also a smaller size plasma wake—at GV/m, the accelerator cavity is at the 100-µm scale (Fig. 1), which is significantly smaller than in a conventional accelerator.

Since their inception, plasma accelerators have matured significantly: following the first proof-of-principle acceleration[16] in the 1980's, almost all aspects of a working accelerator have now been demonstrated in experiment. This includes large accelerating fields[17], large energy gain[18], positron acceleration[19], high energy efficiency[20], high repetition rate[21], and preservation of low energy spreads[22]. Over the past 8 years, I have been involved with several of these experiments, including energy-spread preservation, for which I was the PI—an experiment which was part of a wider push to improve the *beam quality* (or density in phase space) of the accelerated bunches. Recently, this culminated in two demonstrations of free-electron lasing by a plasma accelerator[23]—an important milestone in using plasma accelerators for real-life applications.

Building a plasma-based linear collider, however, remains challenging. For this, all the above aspects must be demonstrated simultaneously, in many cases with much stricter requirements, while adding further demands such as beam polarization[24]. However, before any such demonstration can be attempted, two crucial remaining problems in plasma acceleration must first be overcome:

- ***Problem #1:*** **Higher energy by using multiple accelerator stages,** without loss of beam quality.
- ***Problem #2:*** **Stabilization of the acceleration process**, to be resistant to imperfections and instabilities.

Improvements to beam power, energy efficiency, repetition rate, etc. are only meaningful after these basic problems have been solved. If we could reach this intermediate step and provide stable, high-energy electrons in an affordable way, new experiments would become (economically) feasible for the first time.

Strong-field QED: an ideal near-term application

Quantum electrodynamics (QED), an extremely precise and well-tested theory, becomes non-perturbative and ultimately breaks down when electromagnetic fields approach and surpass the so-called Schwinger limit[25]. This regime, known as *strong-field QED*, is not even reachable with the world's most powerful lasers—they fall short by at least a factor 1000. However, a trick can be played: in the rest frame of an ultra-relativistic electron, a laser pulse travelling in the opposite direction will be relativistically contracted and thus its field amplified by the Lorentz factor. By colliding a laser of intensity greater than 10^{21} W/cm^2 with an electron bunch of energy higher than 10 GeV, fields beyond the Schwinger limit can be probed. Clearly, this requires a very big laser and a very large particle accelerator, and is therefore expensive.

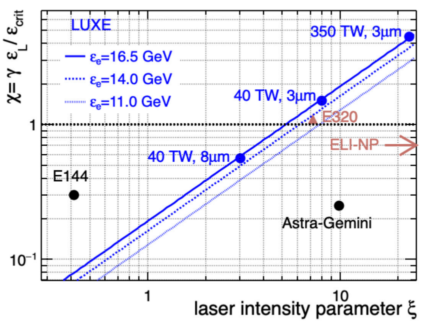


Fig. 2: Plot of the fraction χ of the Schwinger field achieved, as a function of laser intensity ξ, across various planned and performed experiments that collide intense lasers with high-energy electrons. *From: LUXE Collaboration, Eur. Phys. J. Spec. Top. 230, 2445 (2021).*

While some early experiments have been performed, most notably the E144 experiment[26] at SLAC and another using the Astra-Gemini[27] laser at RAL, none have so far reached higher than a fraction $\chi \approx 0.3$ of the Schwinger limit (Fig. 2). Proposals exist for going well beyond it, including LUXE[28]: an experiment that would collide 16.5 GeV electrons from the EuXFEL (a €1.2-billion accelerator) with a 350 TW laser, reaching $\chi \approx 1$–5. However, this experiment is not yet approved, and if it were, would only run for a limited time (ending 2030).

Clearly, progress in strong-field QED would benefit greatly from a dedicated and affordable source of high-energy electrons, built for a fraction of the cost of a conventional machine. Since the required energy is beyond what is attainable in a single-stage plasma accelerator, but the requirements for beam quality and repetition rate are modest, this makes a strong-field QED experiment the ideal application for an intermediate demonstration facility for high-energy plasma accelerators.

This project aims to solve the two major basic problems in plasma acceleration (staging and stability) by exploiting *two new concepts that I have recently developed*, as described below. If verified experimentally, these concepts enable the design and realization of a near-term high-energy-physics facility.

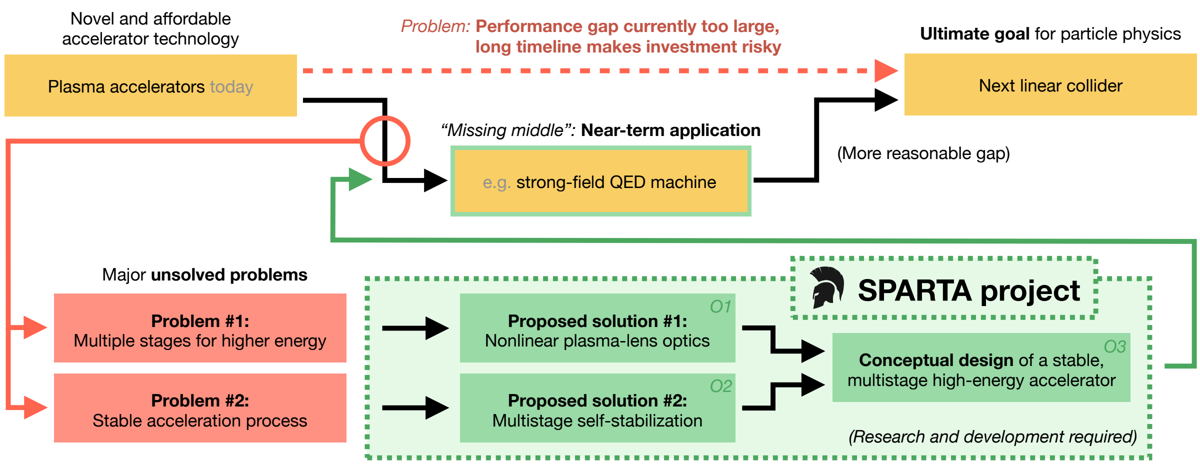


Fig. 3: Flow chart outlining the SPARTA project, including the overall motivation, problems addressed, and solutions proposed.

Problem #1: Staging

The energy used to drive a plasma accelerator is delivered through either a beam driver or a laser driver, which typically only has enough energy to accelerate particles by 10 GeV or less[29]. Reaching higher energies requires use of multiple plasma-accelerator stages, known as *staging*. In a conventional accelerator, this is easy: just line up one accelerator module after the other. In a plasma accelerator, however, this is highly nontrivial[30]: the laser or beam driver must be extracted and swapped for a fresh one after each stage; the resulting separation between stages causes the tightly-focused bunches to rapidly diverge, such that they must be refocused into the next stage—this introduces *chromaticity*, or energy-dependent focusing, which is highly detrimental to the beam quality; to correct for this, complex beam optics must be used between stages, decreasing the average accelerating gradient of the machine. Proposals exist to avoid staging altogether, for instance by using a single plasma stage driven by a very-high-energy proton bunch[31], but unfortunately none of these schemes are currently compatible with the high energy efficiency and particle flux required for most applications (including a collider). For these, staging is the only option.

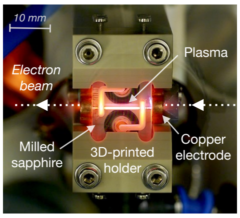


Fig. 4: An active plasma lens experiment at CERN.

One promising technology for compact staging is plasma lensing, which enables strong focusing of particle beams: either an *active plasma lens*[32] (APL), using the strong magnetic field formed in a plasma when passing a current through it; or a *passive plasma lens*[33], which is effectively a short plasma accelerator used only for focusing. Plasma lenses are ideal for staging[34] as they have short focal lengths and focus in both transverse planes (as opposed to quadrupole magnets, which focus in one plane and defocus in the other). In a recent experiment at CERN (Fig. 4), my colleagues and I showed[35] that APLs can in principle preserve the beam quality (*emittance*). However, while more compact, these lenses still introduce chromaticity which must be corrected.

New Concept #1: Nonlinear plasma lenses

Inspired by advanced beam optics used for final focusing[36] in colliders, a setup devised to solve a similar problem, I have recently proposed[37] a concept to correct chromaticity in a compact fashion, by introducing a completely new kind of plasma lens—a *nonlinear plasma lens*—which simultaneously replaces the quadrupoles and sextupoles used in the conventional setup. Nonlinear plasma lenses, which *do not yet exist*, would focus particles more strongly on one side than the other, much like sextupoles. Combined with an energy–transverse correlation (i.e., *dispersion*) from a magnetic dipole, and arranged in a mirror-symmetric beamline to cancel nonlinear effects (Fig. 5), the chromaticity is cancelled and the beam quality preserved.

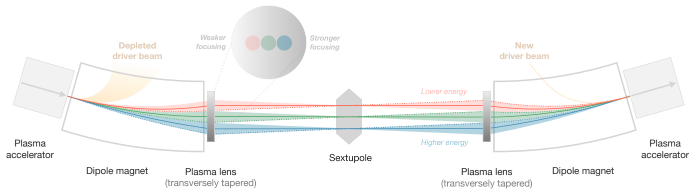


Fig. 5: Schematic layout of an achromatic beamline. The beam is dispersed (not to scale) using dipoles and focused by two nonlinear plasma lenses into the next stage, with no chromaticity. A sextupole cancels the second-order dispersion.

Problem #2: Stability

There are two reasons why plasma accelerators are more unstable than conventional accelerators: the dimensions of the accelerator cavity are much smaller, meaning tolerances on alignment and synchronization are made correspondingly stricter (often on the sub-µm/fs scale); and that plasma accelerators are prone to instabilities, partly due to the nonlinear nature of plasma as a medium, and partly due to the strong resonance that can occur between properties of the driver or accelerating bunch and those of the plasma wake. This affects both the longitudinal and transverse phase space of the accelerated bunches: in the longitudinal, any error in the synchronization, even at the fs level, can affect the observed accelerating field (see Fig. 1), resulting in excess or insufficient energy gain; in the transverse, instabilities with exponentially growing amplitude, such as the *beam-breakup instability*[38], can quickly destroy the emittance of the accelerated beam. Active-feedback techniques are now starting to be employed in plasma accelerators[39], which have improved the energy stability. However, passive or *self-stabilization* techniques are needed to make plasma accelerators truly stable. Several such techniques have been proposed[40] to mitigate the beam-breakup instability.

New Concept #2: Self-stabilization in the longitudinal phase space

Self-stabilization requires a mechanism for feedback within the system: if the bunch is offset, it must be corrected. This happens readily in the transverse phase space, but not in the longitudinal: the reason being that inside a plasma accelerator, the bunch is relativistically locked in phase. The solution is to use multiple stages and introduce *magnetic chicanes*[41] between them to produce a non-zero longitudinal dispersion (R_{56}): a correlation between longitudinal position and energy that

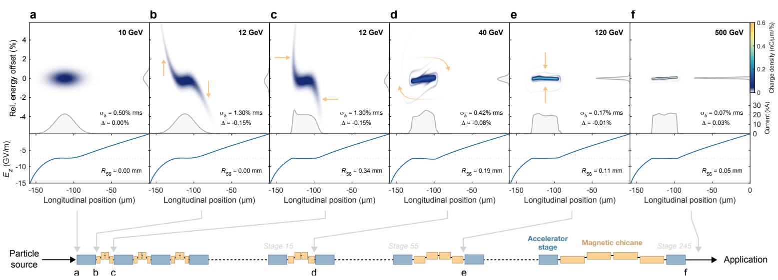


Fig. 6: Self-correction occurs in longitudinal phase space when a multistage plasma accelerator is separated by magnetic chicanes, resulting in higher-quality beams and significantly higher tolerance in the input parameters. *From: Lindstrøm (arXiv: 2104.14460).*

enables feedback. The achromatic beamline proposed above (Fig. 5) provides the required R_{56}. Preliminary simulations that I have performed (Fig. 6) using a large number of stages show that this leads to a remarkable self-correction mechanism[42]: not only are particles *phase focused*[43] (i.e., oscillate around the optimal point) in longitudinal phase space, but the particles *automatically redistribute* in a way that "flattens" the plasma wakefield, in a process known as *beam loading*[44]. This leads to low energy spreads and high energy stability.

Objective #1: Developing nonlinear plasma lenses

The most pressing question that will be answered by this project is: can plasma lenses be made suitably nonlinear in practice? For APLs, which promise the most compact overall beamline, this corresponds to whether the correct magnetic-field structure can be made (Fig. 7). To ascertain this, the first task is to determine the requirements on these lenses and the surrounding beamline (strengths, lengths, apertures, plasma densities, etc.), which will be investigated in numerical simulations using particle tracking and particle-in-cell (PIC) codes like ELEGANT[45] and HiPACE++[46], respectively. Next, 3D magnetohydrodynamics simulations of an APL must be performed (e.g., using COMSOL[47]), to identify a mechanism for creating and tuning the nonlinearity. One possibility is that an external magnetic field could deform the current density inside the APL to produce a transverse gradient. Lastly, following the construction of suitable hardware in collaboration with colleagues in the ADVANCE lab at DESY, the magnetic-field structure of the new lens will be measured experimentally in an accelerator test facility: e.g., the CLEAR facility[48] at CERN, where I have previously conducted similar beam-based plasma-lens experiments[49].

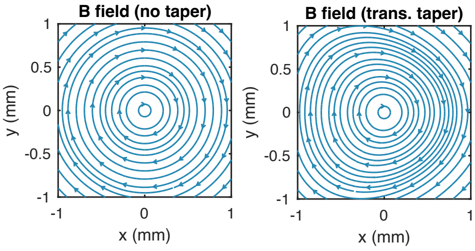


Fig. 7: Transverse magnetic-field structure in a linear (left) vs. a nonlinear APL (right). The linear APL has already been achieved experimentally; Objective #1 is to also achieve the nonlinear APL.

Objective #1: Developing nonlinear beam optics	Year 1		Year 2		Year 3		Year 4		Year 5	
↳ Simulating achromatic beamlines based on nonlinear plasma lenses										
↳ Identifying a mechanism for tuning the nonlinearity in a plasma lens										
↳ Experimental demonstration of a nonlinear plasma lens										

Table 1: Timeline and sub-objectives for Objective #1.

Objective #2: Investigating self-stabilization in plasma accelerators

Two studies will be conducted, investigating self-stabilization in the transverse and longitudinal phase space, respectively—both will be required for overall stability. The first is an experimental study of the transverse instability, first demonstrating it and then mitigating it using one of several proposed methods, performed in a plasma-accelerator test facility: either FACET-II[50] at SLAC or FLASHForward[51] at DESY, in both of which I have long experience with performing experiments. The second study is purely simulation-based, investigating self-stabilization in multiple stages, in order to understand the complexities of the mechanism in full (*6D*) phase space. This will be performed within a start-to-end framework, using both particle-tracking and PIC codes, that will be developed and run on a high-performance computing cluster (Norway's Sigma2).

Objective #2: Investigating self-stabilization in plasma accelerators	Year 1		Year 2		Year 3		Year 4		Year 5	
↳ Experimental study of transverse self-stabilization in a plasma-accelerator stage										
↳ Simulation study of longitudinal self-stabilization across multiple accelerator stages										

Table 2: Timeline and sub-objectives for Objective #2.

Objective #3: Conceptual designs of multistage plasma-accelerator facilities

The end goal of the project is to provide blueprints for a multistage plasma accelerator with utility to high-energy applications, and in particular strong-field QED. This work will proceed in two parts: the first is to design a two-stage test facility (Fig. 8), which can be either beam- or laser-driven, in collaboration with an accelerator lab: both DESY and SLAC have expressed an interest in implementing such a facility. Here, quality-preserving staging can be demonstrated using the setup developed in Objective #1. Secondly, by combining all the knowledge gained in the above objectives, a concept for a large-scale,

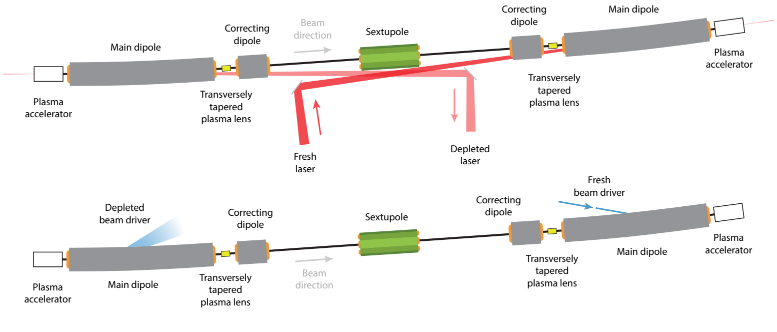


Fig. 8: Layouts for a laser-driven (top) or beam-driven (bottom) staging test facility.

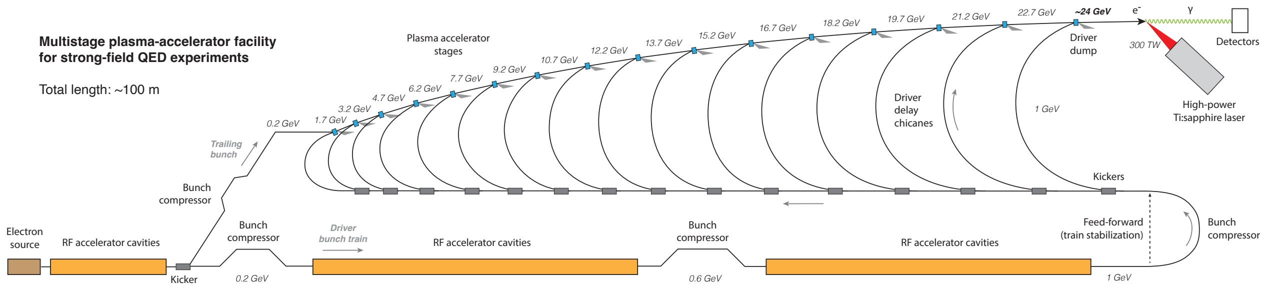


Fig. 9: Preliminary concept for a beam-driven multistage plasma-accelerator facility for strong-field QED. Electron bunches are accelerated to low energies in an RF accelerator, split into a train of 16 drivers and a trailing bunch. The drivers are accelerated to 1 GeV, distributed and synchronised to 16 plasma-accelerator stages. The trailing bunch is matched into the first stage, where it is accelerated before transport to the next stage, etc. The final 24-GeV electrons collide with a high-power laser pulse, producing γ-rays by inverse Compton scattering in order to probe strong-field QED effects.

multistage facility for strong-field QED will be designed (Fig. 9). This starts by determining the required beam parameters and choosing an appropriate driver technology, and proceeds by making use of the start-to-end framework developed in Objective #2 to optimize for stability, beam quality and, ultimately, cost.

Objective #3: Conceptual designs of multistage plasma-accelerator facilities	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**
↳ Technical design of a two-stage test facility					
↳ Conceptual design of a multistage facility for strong-field QED					

Table 3: Timeline and sub-objectives for Objective #3.

Summary and potential impact

SPARTA aims to solve the staging and stability problems of plasma acceleration and apply these techniques to produce low-cost, high-energy electrons with the specific goal of enabling new experiments in strong-field QED—an ideal intermediate step toward the more challenging goal of building an affordable linear collider. This represents a first "return on investment" for the high-energy-physics community. In a broader perspective, any application of high-energy electrons can potentially benefit from these advances, which may provide more affordable access to bright x-ray beams, advanced cancer treatment, etc.

Risks and mitigation strategies

The question of whether plasma lenses can be made suitably nonlinear poses an unavoidable level of risk for this project. To mitigate the effects of this, alternative solutions can be employed: firstly, if active plasma lenses are infeasible, passive plasma lenses will instead be investigated. These are also compact and likely to support the nonlinearity, but are less ideal in terms of the surrounding infrastructure (passive plasma lenses need a laser or beam driver). Secondly, as a backup and to ensure that Objective #3 can be completed regardless of the success of Objective #1, conventional beam optics can be used[52]—certain to work, but non-ideal as the compactness provided by plasma lenses is lost. Risks associated with access to experimental facilities and computing infrastructure are seen as low, as a result of identifying multiple options.

Team structure and resources

Objectives #1 and #2 start in parallel, each lasting 4 years (Tables 1–2), leaving one additional year as buffer to finish experiments and publications. Objective #3, reliant on solutions found in Objectives #1 and #2, starts two years later and lasts until the end of the project (Table 3), with an increasing emphasis over time. The team (Table 4) includes the PI, two postdocs for 3 years each, and two PhD students: one for 3 years, funded by the project, and one for 4 years (with 25% teaching), funded separately by the host institution. This group will be embedded in the high-energy-physics section at the Department of Physics, University of Oslo, which has a vibrant and internationally renowned accelerator-physics group. Access to accelerator test facilities and the high-performance computing cluster is free of charge, and granted based on merit.

SPARTA team contributions	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Objectives #1–3 (all tasks, supervision of PhD students)	**Principal Investigator (Carl A. Lindstrøm) — 60%**				
Objective #1 (mainly simulations, starting experiments)	**Postdoc #1 — 100%**				
Objective #1 (mainly experiments, some simulations)		**PhD #2** (funded by University of Oslo) **— 75%** (25% teaching)			
Objective #2 (mainly experiments, some simulations)	**PhD #1 — 100%**				
Objectives #2–3 (mainly simulations)			**Postdoc #2 — 100%**		

Table 4: Team-structure timeline and contributions. The color coding represent the objectives on which the researcher will be working (see Tables 1–3). The final year of the PhD student period is shown as a fading color, representing their decreasing commitment during the thesis write-up period.

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