Deep Learning

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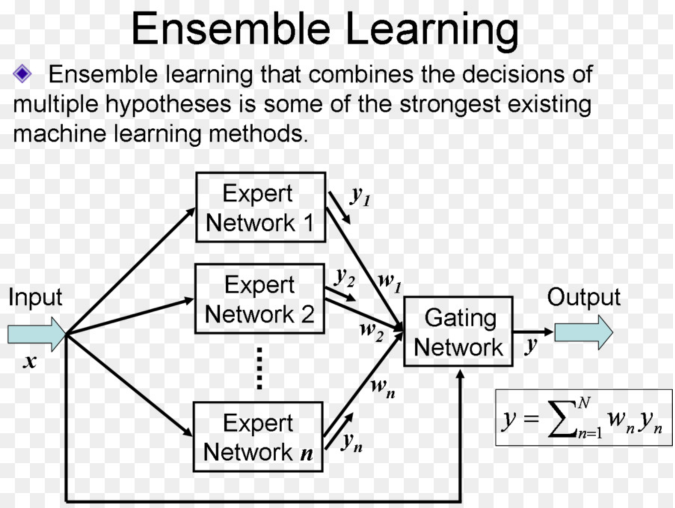
Outline

Ensemble of Neural Networks



Ensemble of Neural Networks

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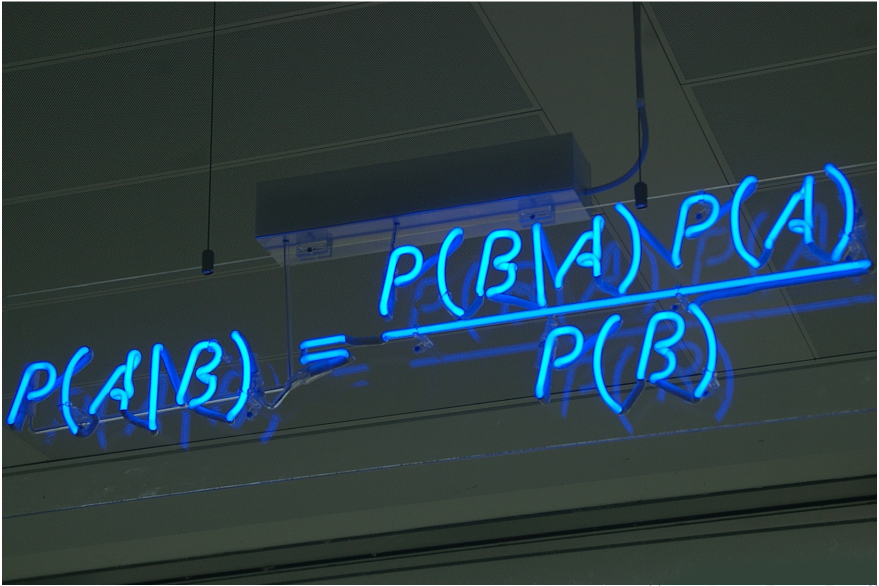
Can we do ensemble learning with infinite number of neural networks?

Outline

BNNs



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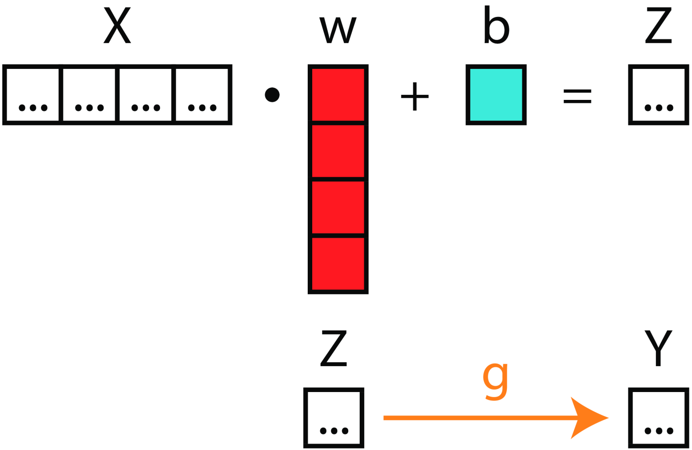

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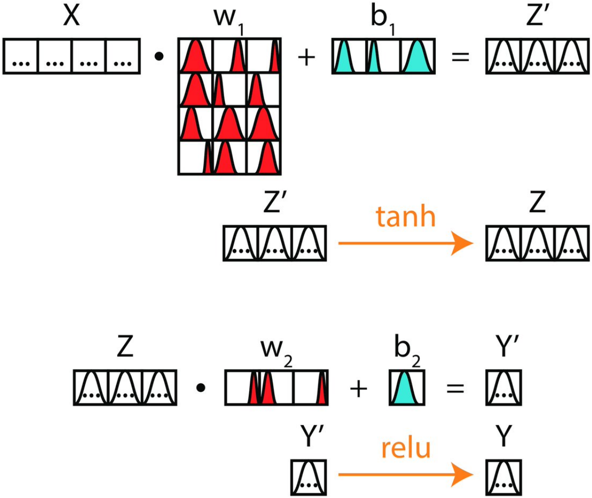

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$$w^{MAP} = \underset{w}{\operatorname{argmax}}\ p\left(w|\mathcal{D}\right) = \underset{w}{\operatorname{argmax}}\ \frac{p\left(\mathcal{D}|w\right) p\left(w\right)}{p\left(\mathcal{D}\right)}$$

$$= \underset{w}{\operatorname{argmax}} \left(\log p\left(\mathcal{D}|w\right) + \log p\left(w\right)\right).$$

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We will assume that prior $p(w)$ is mixture of two Gaussians:

$$p(w) = \prod_j \left(\alpha \mathcal{N}\left(w_j|0, \sigma_1^2\right) + (1-\alpha)\mathcal{N}\left(w_j|0, \sigma_2^2\right)\right)$$

where the first mixture component of the prior is given a larger variance than the second: $\sigma_1 > \sigma_2$.

BNNs

1. Sample $\epsilon \sim \mathcal{N}(0, I)$.
2. Let $\mathbf{w} = \mu + \log(1 + \exp(\rho)) \circ \epsilon$.
3. Let $\theta = (\mu, \rho)$.
4. Let $f(\mathbf{w}, \theta) = \log q(\mathbf{w}|\theta) - \log P(\mathbf{w})P(\mathcal{D}|\mathbf{w})$.
5. Calculate the gradient with respect to the mean

$$\Delta_\mu = \frac{\partial f(\mathbf{w}, \theta)}{\partial \mathbf{w}} + \frac{\partial f(\mathbf{w}, \theta)}{\partial \mu}. \tag{3}$$

6. Calculate the gradient with respect to the standard deviation parameter ρ

$$\Delta_\rho = \frac{\partial f(\mathbf{w}, \theta)}{\partial \mathbf{w}} \frac{\epsilon}{1 + \exp(-\rho)} + \frac{\partial f(\mathbf{w}, \theta)}{\partial \rho}. \tag{4}$$

7. Update the variational parameters:

$$\mu \leftarrow \mu - \alpha \Delta_\mu \tag{5}$$

$$\rho \leftarrow \rho - \alpha \Delta_\rho. \tag{6}$$

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- Can we learn useful representations (e.g., word embeddings) from unlabeled data?

Bigrams from Unlabeled Data

- Given a corpus, extract a training set $(x_i, y_i)_{i=1}^{n}$, where $x_i, y_i \in \mathcal{V}$ and $\mathcal{V}$ is the vocabulary.

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- For example:

Hispaniola quickly became an important base from which Spain expanded its empire into the rest of the Western Hemisphere.

Given a window size of $+/-$ 3, for $x =$ base we get the pairs

(base, became), (base, an), (base, important),
(base, from), (base, which), (base, Spain).

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$$\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t+j} | w_t\right)$$

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  where c is the size of the training context.
- We model $p\left(w_{t+j}|w_t\right)$ using the softmax function:

$$p\left(w_O|w_I\right) = \frac{\exp\left(v'^{T}_{w_O} v_{w_I}\right)}{\sum\limits_{w=1}^{W}\exp\left(v'^{T}_{w} v_{w_I}\right)},$$

  where v_w and v'_w are the “input” and “output” vector representations of w, and W is the number of words in the vocabulary.

Negative Sampling

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- We define Negative sampling by the objective

$$\log \sigma \left(v'^{T}_{w_O} v_{w_I} \right) + \sum_{i=1}^{k} \mathbb{E}_{w_i \sim P_n(w)} \left[\log \sigma \left(-v'^{T}_{w_i} v_{w_I} \right) \right]$$

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- Thus the task is to distinguish the target word w_O from draws from the noise distribution $P_n\left(w\right)$ using logistic regression, where there are k negative samples for each data sample.
- In the original paper authors chose P_n to be the unigram distribution raised to the $3/4$rd power.

Thank you!