YSU ASDS, Statistics, Fall 2019

Lectures 02 - 03

Michael Poghosyan

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Descriptive Statistics

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- What is a **Population**?

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- What is a **Representative Sample**?

Stages of Doing a Statistical Analysis

Important Stages of the Statistical Analysis are:

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First, for doing Statistics, Statisticians are modelling the process of Data Collection, they are *Designing the Experiment and the Sampling Methodology*. Correct design is very important for doing a correct analysis.

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Definition: We say that our Sample is *Representative* (obtained by a Simple Random Sampling), if it is obtained in the process where all Samples of size k have the same probability of being chosen.

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Which one gives a Simple Random Sample?

Few Sampling Methods

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Simple Random Sampling is not so easy to perform, so people are using different simpler Sampling Strategies (although they are not always giving exactly Representative Samples):

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- *Systematic (Interval) Sampling*, we fit the population into a list, enumerate it; choose n; choose at random a starting element from the first n elements in the list; and from that element on, every n-th member of the population is selected;

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- *Cluster Sampling*, where the total population is divided into subgroups (clusters), then some clusters are randomly chosen. Then we include all elements of chosen clusters into our Sample.

Classification of Data wrt its Dimension

Data can be

- **Univariate** (1D) - here the observations are on a single Variable
- **Bivariate** (2D) - here the observations are on two Variables
- **Multivariate** (n-D, $n \geq 2$) - when the observations are on more than a one Variable (usually, more than two)

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Examples:

- *No. of Children*, *No. of Customers* ,... are Discrete
- *Height*, *Weight*, *Age*, ... are Continuous

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But even if we are enumerating the Sex or the Color, the average Sex or the average Color is not meaningful, we cannot deal with the assigned numbers as above!

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Yeah, there is an **order** in the second Variables, *Stat Final Letter Grade* and *Year of University Study*.

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Strongly Disagree | Disagree | Neither | Agree | Strongly Agree

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Let me recall that we use the Descriptive Statistics at the beginning of our Statistical Study to examine, explore the dataset.

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And we start by describing some of the *Graphical Summaries*.

Here, for the beginning, we will assume that we have a univariate (mostly numerical) data (dataset), $x_1, x_2, ..., x_n$. In this case we will say that we are given a (univariate, 1D) dataset x.

Frequency Tables

Here we assume that we have a univariate *discrete* numerical or categorical data $x_1, x_2, ..., x_n$.

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Definition: The **frequency** of a value t in observations $x_1, x_2, ..., x_n$ is the number of times t occurs in observations:

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Definition: The **relative frequency** (or percentage) of a value t in observations $x_1, x_2, ..., x_n$ is the ratio of frequency of t divided by the total number of observations, n:

$$\text{Relative Frequency of } t = \frac{\text{Frequency of } t}{\text{Total Number of Observations}} =$$

$$= \frac{\text{Frequency of } t}{n}.$$

Frequency Tables, Example

Example: Given the following Dataset:

$$1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1$$

obtain the Frequency and Relative Frequency Tables.

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Example: Let's construct the Frequency Table of the above Dataset using **R**:

```
x <- c(1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1)
table(x)
```

```
## x
## -1  1  2  3  4  7
##  1  4  4  1  2  1
```

Frequency Tables, Example

Now, consider the *iris* dataset in **R**:

```r
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
## 6          5.4         3.9          1.7         0.4  setosa
```

Frequency Tables, Example

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```

Frequency Tables, Example, Cont'd

To get the *Species* Variable of the iris Dataset, we use

```
iris$Species
```

Frequency Tables, Example, Cont'd

To get the *Species* Variable of the iris Dataset, we use

```
iris$Species
```

And to calculate the Frequency of each of the Species, we use

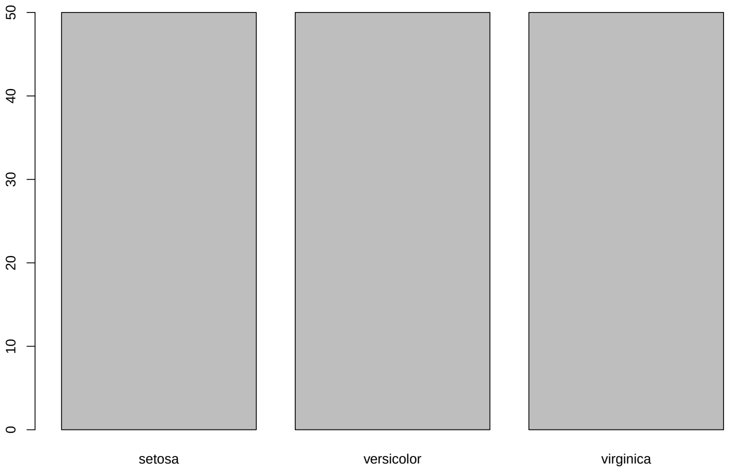
```
table(iris$Species)
```

```
##
##     setosa versicolor  virginica
##         50         50         50
```

LineGraph and BarPlot

Now, let us visualize our Frequency Table:

```
barplot(table(iris$Species))
```



LineGraph and Barplot

Another standard Dataset, *mtcars*, again about cars $\ddot\smile$:

```r
head(mtcars, 3)
```

```
##                mpg cyl disp  hp drat    wt  qsec vs am gear c
## Mazda RX4     21.0   6  160 110 3.90 2.620 16.46  0  1    4
## Mazda RX4 Wag 21.0   6  160 110 3.90 2.875 17.02  0  1    4
## Datsun 710    22.8   4  108  93 3.85 2.320 18.61  1  1    4
```

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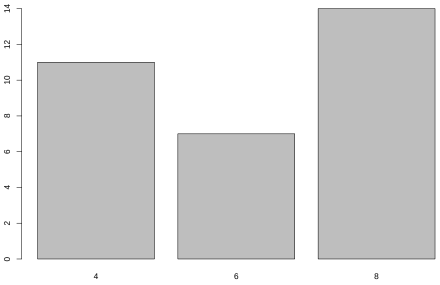
```

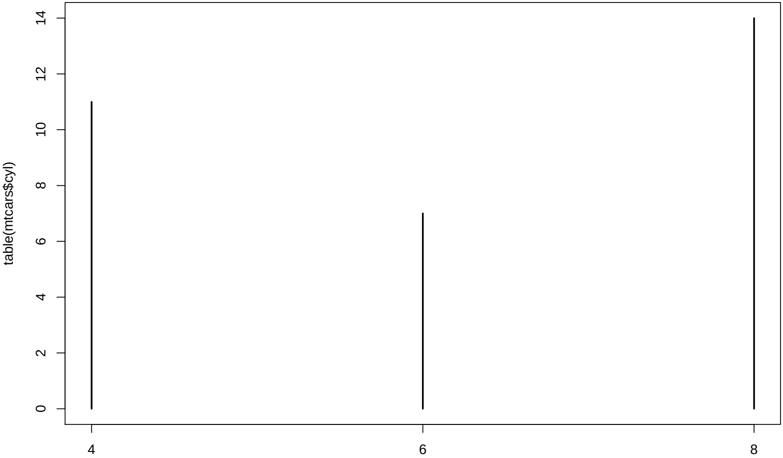
```
barplot(table(mtcars$cyl))
```



LineGraph and Barplot

Now, with the Line Graph:

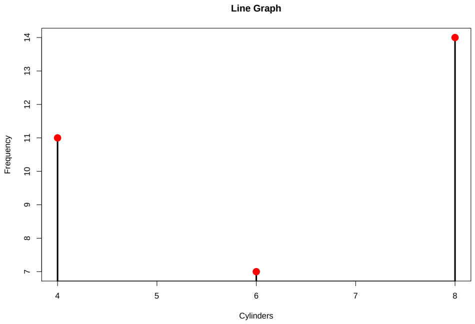
```
plot(table(mtcars$cyl))
```



LineGraph and Barplot

More sophisticated (titiz) version:

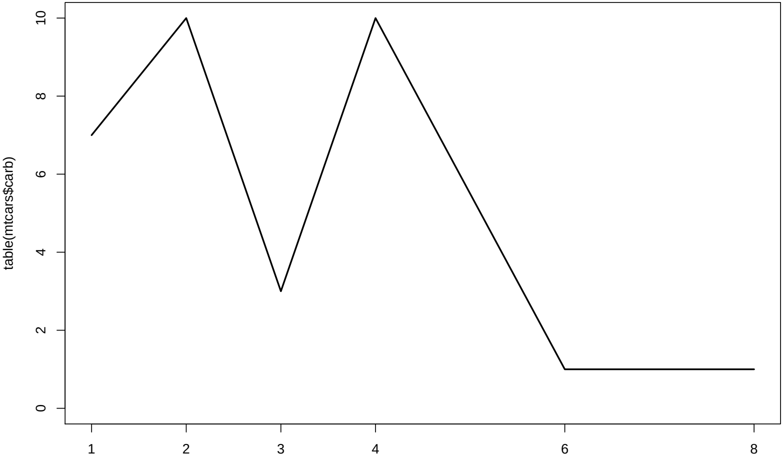
```
x <- mtcars$cyl; y <- as.data.frame(table(x))
a <- as.numeric(as.character(y$x)); b <- y$Freq
plot(a,b,type="h", lwd=3,  xlab = "Cylinders",
     ylab = "Frequency", main = "Line Graph")
points(a,b, pch=16, cex=2, col="red")
```



The Frequency Polygon

Again, same cars, but now the *carb* Variable Frequencies:

```r
plot(table(mtcars$carb), type = "l")
```



Describing the Data Distribution

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Say, when we talk about the height distribution of persons between the ages 20-30, we assume that there is some unknown process that generates that heights.

From our Probability course, we know two complete characteristics of a Random Variable: the **CDF and PD(M)F**. So to describe our Data Distribution, we can try to describe the CDF and/or PD(M)F behind the Data.

Empirical CDF

First let's estimate the CDF. We will estimate CDF by the Empirical CDF:

Definition: The **Empirical Distribution Function, ECDF** or the **Cumulative Histogram** $ecdf(x)$ of our data $x_1, ..., x_n$ is defined by

$$ecdf(x) = \frac{\text{number of elements in our dataset} \le x}{\text{the total number of elements in our dataset}} =$$

$$= \frac{\text{number of elements in our dataset} \le x}{n}, \qquad \forall x \in \mathbb{R}.$$

Example

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$$-1, 4, 7, 5, 4$$

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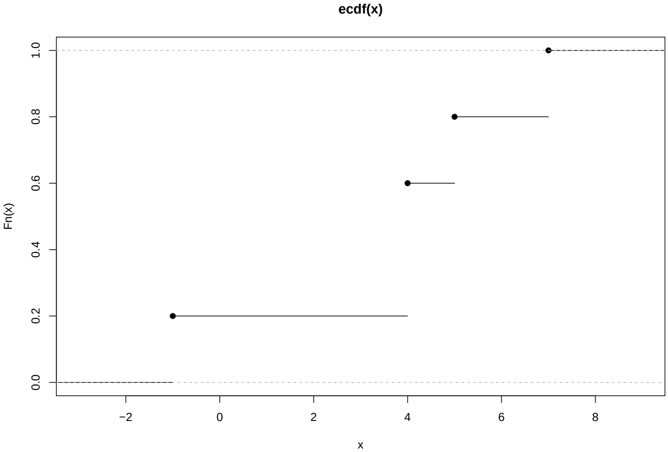
- Sort our Dataset from the lowest to the largest values
- Plot the Data points on the OX axis
- ECDF is 0 for values of x less that the smallest Datapoint, and is 1 for values of x bigger than the largest Datapoint
- For each Data point, calculate the Relative Frequency of that Datapoint (the number of times it occurs in our Dataset over the total number of Datapoints). At that Datapoint, do a Jump of the size of the Relative Frequency, and draw a horizontal line up to the next Datapoint

Example

Now, using **R**:

```
x <-c(-1, 4, 7, 5, 4)
f <- ecdf(x)
plot(f)
```



Note: It is easy to see that the ECDF satisfies all properties of a CDF.

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$$F_n(x) \to F(x) \quad \text{on} \quad \mathbb{R}.$$

Here we need to be more precise about in which sense the convergence holds.

Glivenko-Cantelli Theorem

In fact, the following Theorem Holds:

Theorem (Glivenko, Cantelli): If $X_1, ..., X_n$ are r.v.s from the Distribution with the CDF $F(x)$, and $F_n(x)$ is the ECDF constructed by using $X_1, ..., X_n$, then

$$\sup_x |F_n(x) - F(x)| \to 0 \qquad a.s.$$

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$$\sup_x |F_n(x) - F(x)| \to 0 \qquad a.s.$$

This Theorem says that if you will have enough datapoints from a Distribution, you can approximate the unknown CDF of your Distribution pretty well by using the ECDF.

Estimation of the CDF through ECDF

Let us check this theorem using **R**:

```
plot(pnorm, lwd = 3, col = 'red', xlim = c(-3,3),
     ylim = c(0,1), ylab = "ecdf and CDF")
n <- 30 ; x <- rnorm(n) #Taking a sample of size n from N(0,1)
f <- ecdf(x) #f will be the ECDF of our data x
par(new = TRUE) #this is to keep the previous graph
plot(f, xlim = c(-3,3), ylim = c(0,1), ylab = "ecdf and CDF")
```