YSU ASDS, Statistics, Fall 2019

Lecture 16

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16 Oct 2019

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- Define the MSE.
- How we can define that $\hat{\theta}$ is close to θ?
- Define the Bias of an Estimator.
- What is the definition of the Unbiased/Biased Estimator?

Example

Example: Now, let's do an experiment with Biased and Unbiased Estimators in **R**.

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UnBiased Estimator Case

We consider the Poisson Model:

$$X_1, X_2, ..., X_{10} \sim Pois(\lambda)$$

and we want to estimate λ.

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$$\hat{\lambda} = \frac{X_1 + X_2 + ... + X_{10}}{10}.$$

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Easy to se that $\hat{\lambda}$ is an Unbiased Estimator for λ (OTB!).

Example, cont'd

Now, the code

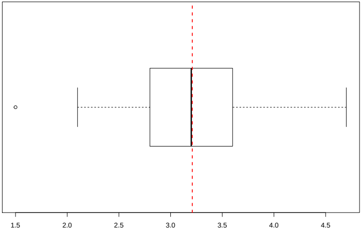
- observing once: ganarating a Sample just once and calculating one Estimate:

```r
lambda <- 3.21
x <- rpois(10, lambda = lambda)
lambda.hat <- mean(x)
lambda.hat
```

```
## [1] 2.4
```

- observing many times: ganarating Samples many times, calculating Estimates, and then averaging:

```
lambda <- 3.21; n <- 10; m <- 200
x <- rpois(n*m, lambda = lambda)
x <- as.data.frame(matrix(x, ncol = m))
lambda.hats <- sapply(x, mean)
boxplot(lambda.hats, horizontal = T);
abline(v = lambda, col="red", lwd = 2, lty = 2)
```

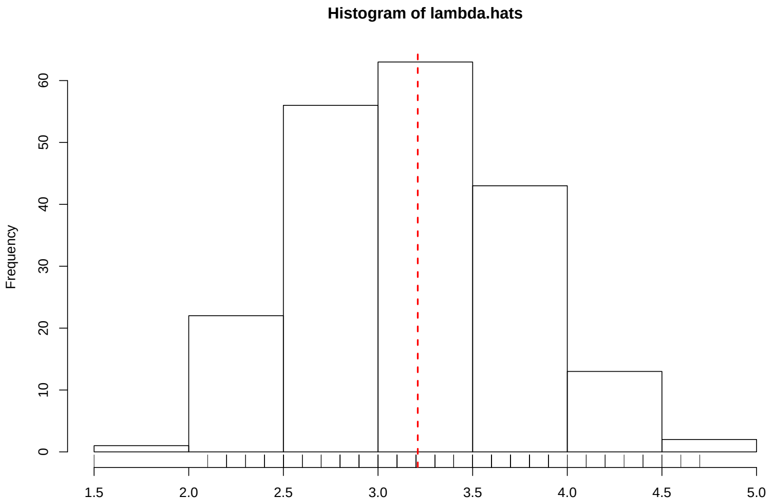



```
mean(lambda.hats)
```

```
## [1] 3.227
```

With a Histogram:

```r
hist(lambda.hats)
rug(lambda.hats)
abline(v = lambda, col="red", lwd = 2, lty = 2)
```



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Biased Estimator Case

Say, let us consider the Exponential Model:

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Easy to se that $\hat{\lambda}$ is an Biased Estimator for λ (OTB!).

Example, cont'd

Now, the code:

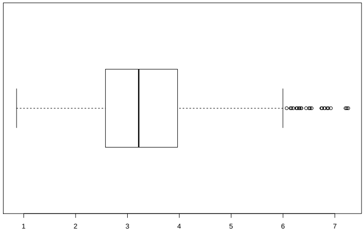
- observing once: ganarating a Sample just once and calculating one Estimate:

```
lambda <- 0.3
x <- rexp(10, rate = lambda)
lambda.hat <- mean(x)
lambda.hat
```

```
## [1] 3.059252
```

- observing many times: ganarating Samples many times, calculating Estimates, and then averaging:

```
lambda <- 0.3; n <- 10; m <- 2000
x <- rexp(n*m, rate = lambda)
x <- as.data.frame(matrix(x, ncol = m))
lambda.hats <- sapply(x, mean)
boxplot(lambda.hats, horizontal = T);
abline(v = lambda, col="red", lwd = 2, lty = 2)
```




```
mean(lambda.hats)
```

```
## [1] 3.319164
```

With a Histogram:

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hist(lambda.hats)
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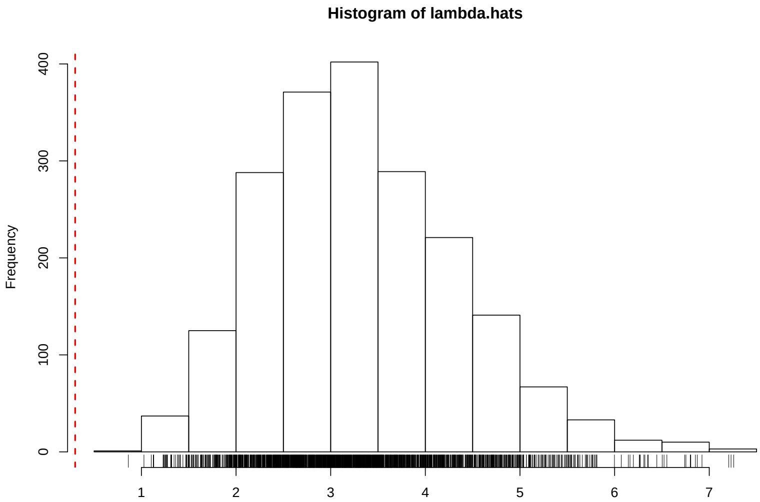

Example

Example: Assume we have a Random Sample for a some Distribution with the Mean μ and Variance σ^2:

$$X_1, X_2, ..., X_n \sim \mathcal{F}_{\mu,\sigma^2},$$

and we want to estimate the Parameters μ and σ^2.

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and we want to estimate the Parameters μ and σ^2.

We consider the following Estimators:

$$\hat{\mu} = \overline{X}_n = \frac{X_1 + X_2 + ... + X_n}{n}$$

and

$$\widehat{\sigma^2} = \frac{\sum\limits_{k=1}^{n} (X_k - \overline{X}_n)^2}{n} \qquad \text{and} \qquad \widehat{\sigma^2} = S^2 = \frac{\sum\limits_{k=1}^{n} (X_k - \overline{X}_n)^2}{n-1}$$

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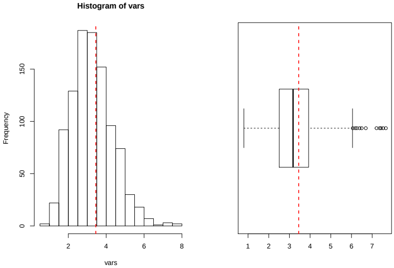
Let us see (OTB) which ones are Biased and which ones are not, and calculate the Biases.

Example

- Biased Case, with n in the Denominator:

```
v <- 3.45; n <- 20; it <- 1000
x <- rnorm(n*it, mean = 2, sd = sqrt(v))
x <- as.data.frame(matrix(x, ncol = it))
my.var <- function(x){return((length(x)-1)*var(x)/length(x))}
vars <- sapply(x, my.var)
par(mfrow = c(1,2))
hist(vars, breaks = 15)
abline(v = v, col = "red", lty = 2, lwd = 2)
boxplot(vars, horizontal = T)
abline(v = v, col = "red", lty = 2, lwd = 2)
```



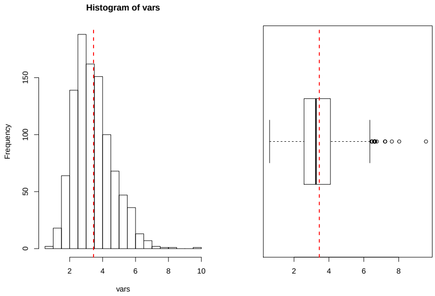

```
par(mfrow = c(1,1))
mean(vars) - v
```

```
## [1] -0.1747914
```

Example

- UnBiased Case, with $n - 1$ in the Denominator:

```
v <- 3.45; n <- 20; it <- 1000
x <- rnorm(n*it, mean = 2, sd = sqrt(v))
x <- as.data.frame(matrix(x, ncol = it))
vars <- sapply(x,var)
par(mfrow = c(1,2))
hist(vars, breaks = 15)
abline(v = v, col = "red", lty = 2, lwd = 2)
boxplot(vars, horizontal = T)
abline(v = v, col = "red", lty = 2, lwd = 2)
```




```
par(mfrow = c(1,1))
mean(vars) - v
```

```
## [1] -0.02689461
```

Asymptotic Unbiasedness

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Idea: If the Sample size is very large, then the behaviour of our Asymptotic Unbiased Estimator is close to an Unbiased one, $Bias(\hat{\theta}_n, \theta) \approx 0$

Example: Say, for the Mean μ of the Population,

$$\hat{\mu} = \frac{X_1 + X_2 + ... + X_n}{n+1}$$

is a Biased, but Asymptotically Unbiased Estimator. OTB, please!

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$$\widehat{\sigma^2} = \frac{\sum_{k=1}^{n}(X_k - \overline{X}_n)^2}{n},$$

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$$\widehat{\sigma^2} = \frac{\sum_{k=1}^{n}(X_k - \overline{X}_n)^2}{n},$$

is Biased. But the Bias is

$$Bias(\widehat{\sigma^2}, \sigma^2) = -\frac{\sigma^2}{n} \to 0, \qquad n \to \infty,$$

or, equivalently,

$$\mathbb{E}(\widehat{\sigma^2}) \to \sigma^2,$$

so $\widehat{\sigma^2}$ is an Asymptotically Unbiased Estimator for σ^2.

Bias-Variance Decomposition

This is an important result:

Theorem(Bias-Variance Decomposition of the MSE): If $\hat{\theta}$ is an Estimator for θ, then

$$MSE(\hat{\theta}, \theta) = \left(Bias(\hat{\theta}, \theta)\right)^2 + Var_{\theta}(\hat{\theta}).$$

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Proof: OTB

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Nice Graphical Interpretation: Link

Bias-Variance Decomposition/Tradeoff

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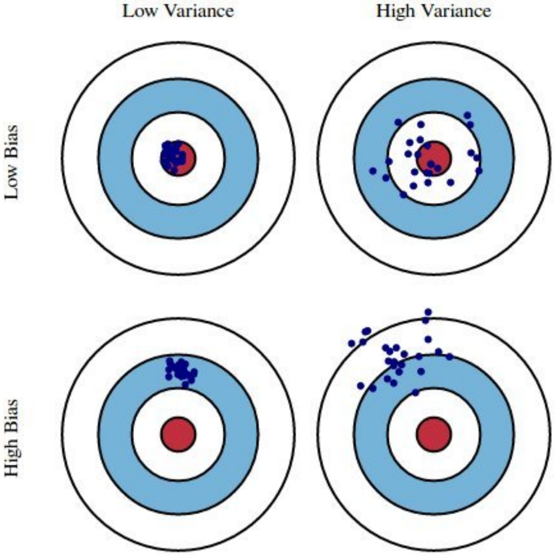

Figure 1: Credits to: http://scott.fortmann-roe.com

Standard Error and Estimated Standard Error

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Usually, the Standard Error will depend on the unknown value of the Parameter θ. If we use the Estimator $\hat{\theta}$, then the **Estimated Standard Error** of $\hat{\theta}$, $\widehat{SE}(\hat{\theta})$ is the Standard Error, where after calculation we plug $\hat{\theta}$ instead of θ.

Example

Example: Assume we are facing an election with Parties A and B, and we want to estimate the percentage of voters for A in advance. So we do a poll, asking 10 persons to give their preferences. Let the result be:

$$A, B, B, B, A, B, B, A, B, B.$$

Problem: Estimate the percentage of voters for the Party A, and give the Estimated Standard Error.

Solution: OTB.

Bias-Variance Decomposition, Corollary

Recall from the last lecture the BVD:

Theorem(Bias-Variance Decomposition of the MSE): If $\hat{\theta}$ is an Estimator for θ, then

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Corollary: If $\hat{\theta}_1$ and $\hat{\theta}_2$ are both *Unbiased* Estimators for the unknown Parameter θ, then $\hat{\theta}_1$ is preferable to $\hat{\theta}_2$ if and only if

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and a strict inequality holds for at least one θ.

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B-V Decomposition, Again

Recall again the B-V D:

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Well, in general, there will be a lot of Unbiased Estimators for the same Parameter.Say, if $\hat{\theta}_0$ and $\hat{\theta}_1$ are Unbiased Estimators of θ, then for any $\alpha \in [0, 1]$, the Estimator

$$\hat{\theta}_\alpha = \alpha \cdot \hat{\theta}_1 + (1 - \alpha) \cdot \hat{\theta}_0$$

will be an Unbiased Estimator too.

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Later we will talk about how to find MVUE for a parameter for some cases.