Multivariate Statistics

HOME WORK # 1

Deadline: 12:00, November 2, 2019

Problem 1. Given the data matrix

$$\mathbf{X} = \begin{bmatrix} 3 & 4 \\ 6 & -2 \\ 3 & 1 \end{bmatrix}$$

(a) Graph the scatter plot in $p = 2$ dimensions, and locate the sample mean on your diagram.

(b) Sketch the $n = 3$-space representation of the data, and plot the deviation vectors $y_1 - \overline{x}_1 1$ and $y_2 - \overline{x}_2 1$.

(c) Sketch the deviation vectors in (b) emanating from the origin. Calculate their lengths and the cosine of the angle between them. Relate these quantities to $\mathbf{S}_n$ and $\mathbf{R}$.

Problem 2. Calculate the generalized sample variance $|\mathbf{S}|$ for the data matrix $\mathbf{X}$ in Problem 1.

Problem 3. Let $\mathbf{X}$ be $N_3(\mu, \Sigma)$ with $\mu' = [2, -3, 1]$ and

$$\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

Find the distribution of $3X_1 - 2X_2 + X_3$.

Problem 4. Let $\mathbf{X}$ be distributed as $N_3(\mu, \Sigma)$, where $\mu' = [1, -1, 2]$ and

$$\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Which of the following variables are independent? Explain.

(a) X_1 and X_2

(b) X_1 and X_3

(c) X_2 and X_3

(d)(X_1, X_3) and X_2

(e) X_1 and $X_1 + 3X_2 - 2X_3$

Problem 5. Let X_1, X_2, X_3 and X_4 be independent $N_p(\mu, \Sigma)$ random vectors.

(a) Find the marginal distributions for each of the random vectors

$$\mathbf{V}_1 = \frac{1}{4}\mathbf{X}_1 - \frac{1}{4}\mathbf{X}_2 + \frac{1}{4}\mathbf{X}_3 - \frac{1}{4}\mathbf{X}_4$$

and

$$\mathbf{V}_2 = \frac{1}{4}\mathbf{X}_1 + \frac{1}{4}\mathbf{X}_2 - \frac{1}{4}\mathbf{X}_3 - \frac{1}{4}\mathbf{X}_4$$

(b) Find the joint density of the random vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ defined in (a).