YSU ASDS, Statistics, Fall 2019

Lecture 25

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Last Lecture ReCap

- Give the Hypo Testing Framework.

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- Give the Hypo Testing Framework.
- Give the usual types of Hypotheses

Last Lecture ReCap

- Give the Hypo Testing Framework.
- Give the usual types of Hypotheses
- How to choose the Null Hypothesis?

From the last lecture: Type I and II errors

Assume we are Testing the Hypothesis

$$\mathcal{H}_0 \qquad vs \qquad \mathcal{H}_1.$$

Then the following cases can happen:

Test Decision \ Reality	$\mathcal{H}_0$ is True	$\mathcal{H}_0$ is False (i.e., $\mathcal{H}_1$ is True)
Reject $\mathcal{H}_0$	**Type I Error (False Positive)**	Correct Decision (True Negative)
Do Not Reject $\mathcal{H}_0$	Correct Decision (True Positive)	**Type II Error (False Negative)**

Significance and Power, From the Last Lecture

Here are the Probabilities of correct/incorrect decisions for a Hypothesis testing:

Probabilities of Correct/InCorrect Decisions:

Test Decision \ Reality	$\mathcal{H}_0$ is True	$\mathcal{H}_0$ is False (i.e., $\mathcal{H}_1$ is True)
Reject $\mathcal{H}_0$	$\alpha =$**Significance**	$1-\beta =$**Power**
Do Not Reject $\mathcal{H}_0$	$1-\alpha$	β

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot\smile$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

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Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

Step 2: We State the Hypotheses: we take Θ_0 and Θ_1 such that $\Theta = \Theta_0 \cup \Theta_1$ and $\Theta_0 \cap \Theta_1 = \varnothing$, and state the Hypothesis:

$$\mathcal{H}_0: \ \theta \in \Theta_0 \qquad \textit{vs} \qquad \mathcal{H}_1: \ \theta \in \Theta_1$$

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Step 3: We choose the desirable Significance Level: we choose α (small enough, say, $\alpha = 0.05$).

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[1] If *n* is free to choose, then we can find it by taking the desirable Power of the Test.

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Step 4: Assume we know how many Observations we will have[1], say, n. Then we take a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta.$$

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$$T = T(X_1, X_2, ..., X_n)$$

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$$RR = \{|T| > c\} \qquad or \qquad RR = \{T > c\} \qquad or \qquad RR = \{T < c\}.$$

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Here c is called **the Critical Value** of the Test. We will choose it (soon). We Reject $\mathcal{H}_0$ if and only if $T \in RR$.

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- Reject $\mathcal{H}_0$, if $T(x_1, ..., x_n) \in RR$;

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- Reject $\mathcal{H}_0$, if $T(x_1, ..., x_n) \in RR$;
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$$\overline{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right)$$

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Great!

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Rejection Region: Now we choose the **RR**. The idea is:

If $\mathcal{H}_0$ is True, then Z is close to

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We consider our 3 cases:

Case 1: for Testing $\mathcal{H}_0: \ \mu = \mu_0$ vs $\mathcal{H}_1: \ \mu \neq \mu_0$

In this case we will Reject $\mathcal{H}_0$, if

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Case 2: for Testing $\mathcal{H}_0: \ \mu = \mu_0 \quad$ vs $\quad \mathcal{H}_1: \ \mu > \mu_0$

In this case we will not believe in $\mathcal{H}_0$, if

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In this case we will Reject $\mathcal{H}_0$, if Z will be far from 0, i.e., we choose $RR = \{|Z| > c\}$. The Critical Value c is yet to be determined.

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Rejection Region: Now we choose the **RR**. The idea is:

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Example

Example: I have generated in **R** a Sample of Size 50 from $\mathcal{N}(3, 2^2)$ and made some rounding:

```
set.seed(20112019)
s.size <-50; sigma <- 2
obs <-  rnorm(s.size, mean = 3, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.68  5.48  0.98  3.08  4.79  5.03  1.64  2.35  0.
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I will assume I do not know μ (which is 3, of course), and will just assume my Observation is coming from $\mathcal{N}(\mu, 2^2)$, with some μ. And I will test, at the 5% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 4 \qquad vs \qquad \mathcal{H}_1: \ \mu \neq 4.$$

Example, Cont'd

First, I calculate Z-statistic:

```
mu0 <- 4
Z <- (mean(obs) - mu0)/(sigma/sqrt(s.size)); Z
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## [1] -3.63665
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abs(Z) > z
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```
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So the decision is: **Reject** $\mathcal{H}_0$. In this case we say that the result was **Statistically Significant**.

Example

Example: Now, with the same Observations from the last example, let us test, at the 5% level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3.3 \qquad \textit{vs} \qquad \mathcal{H}_0: \ \mu \neq 3.3.$$

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$$\mathcal{H}_0: \ \mu = 3.3 \qquad vs \qquad \mathcal{H}_0: \ \mu \neq 3.3.$$

```
mu0 <- 3.3; a <- 0.05
Z <- (mean(obs) - mu0)/(sigma/sqrt(s.size))
cat("Z-statistics = ", Z)
```

```
## Z-statistics =  -1.161776
```

```
z <- qnorm(1-a/2)
cat("critical value = ", z)
```

```
## critical value =  1.959964
```

```
if (abs(Z) > z) cat("Reject") else cat("Do Not Reject")
```

```
## Do Not Reject
```

Z-Test, relation to the Normal μ CI

Generally, it can be proven that there is a duality/association between Hypothesis Testing and a CI construction. Let me give the relation for our case, for Normal $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is known.

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- calculate $Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$.
- Reject Null if $|Z| > z_{1-\alpha/2}$, otherwise, Do Not Reject Null.

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- Reject Null if $|Z| > z_{1-\alpha/2}$, otherwise, Do Not Reject Null.

This is equilvalent to: Do Not Reject, if

$$-z_{1-\alpha/2} \leq \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \leq z_{1-\alpha/2},$$

Z-Test, relation to the Normal μ CI

i.e., if

$$\mu_0 \in \left[\overline{X} - z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \; ; \; \overline{X} + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right].$$

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Hence, the relation: we Reject $\mathcal{H}_0$, if μ_0 is not in the CI, and otherwise, we Fail to Reject[3].

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Model: $X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu, \sigma^2)$, σ **is known**, the Parameter (our unknown) is μ;

Null Hypothesis: $\mathcal{H}_0: \ \mu = \mu_0$

Significance Level: $\alpha \in (0, 1)$;

Test Statistics: $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

Distrib of the Test-Statistics Under $\mathcal{H}_0$**:** $Z \sim \mathcal{N}(0, 1)$;

$\mathcal{H}_1$ is	RR is
$\mu \neq \mu_0$	$\lvert Z\rvert > z_{1-\frac{\alpha}{2}}$

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t-test Example

Example: Again, I have generated in **R** a Sample of Size 20 from $\mathcal{N}(3.12, 2^2)$ and made some rounding:

```
set.seed(20112019)
s.size <-20; sigma <- 2
obs <-  rnorm(s.size, mean = 3.12, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.80  5.60  1.10  3.20  4.91  5.15  1.76  2.47  0.
## [13]  3.98  4.79  1.98  4.50  3.52  4.13 -0.08  3.87
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Now, let us forget about the fact that the actual value of μ is 3.12 and that $\sigma = 2$, and do some Testing, just assuming that our Observation is coming from a Normal Distribution.

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Now, let us forget about the fact that the actual value of μ is 3.12 and that $\sigma = 2$, and do some Testing, just assuming that our Observation is coming from a Normal Distribution. Say, let us Test, at the 5% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 4 \qquad vs \qquad \mathcal{H}_1: \ \mu \neq 4.$$

Example, Cont'd

First, we calculate *t*-statistic:

```
mu0 <- 4
t <- (mean(obs) - mu0)/(sd(obs)/sqrt(s.size)); t
```

```
## [1] -1.795358
```

Example, Cont'd

First, we calculate t-statistic:

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mu0 <- 4
t <- (mean(obs) - mu0)/(sd(obs)/sqrt(s.size)); t
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```
## [1] -1.795358
```

Now, we calculate the critical value, the quantile $t_{n-1,1-\alpha/2}$:

```
a <- 0.05
c <- qt(1-a/2, df = s.size-1); c
```

```
## [1] 2.093024
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Example, Cont'd

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```

Finally, we check if t is in RR, i.e., if $|t| > t_{n-1,1-\alpha/2}$:

```
abs(t) > c
```

```
## [1] FALSE
```

Example, Cont'd

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## [1] FALSE
```

So the decision is:

Example, Cont'd

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```
abs(t) > c
```

```
## [1] FALSE
```

So the decision is: **Fail to Reject** $\mathcal{H}_0$ at 5% level.

Example, Cont'd

Now, the same, but with an **R** built-in function `t.test`:

```r
t.test(obs, mu = mu0, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  obs
## t = -1.7954, df = 19, p-value = 0.08852
## alternative hypothesis: true mean is not equal to 4
## 95 percent confidence interval:
##  2.524009 4.112991
## sample estimates:
## mean of x
##    3.3185
```

Example, Cont'd

Now, for the same Data, let us Test, at the 10% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3 \qquad \textit{vs} \qquad \mathcal{H}_1: \ \mu > 3.$$

Example, Cont'd

Now, for the same Data, let us Test, at the 10% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3 \qquad vs \qquad \mathcal{H}_1: \ \mu > 3.$$

```
t.test(obs, mu=3,alternative="greater", conf.level=0.9)
```

```
##
##  One Sample t-test
##
## data:  obs
## t = 0.83906, df = 19, p-value = 0.2059
## alternative hypothesis: true mean is greater than 3
## 90 percent confidence interval:
##  2.814508      Inf
## sample estimates:
## mean of x
##    3.3185
```

Note

Note: In **R** `t.test` command, the default values for parameters are:

- `mu = 0`
- `alternative = "two.sided"`
- `conf.level = 0.95`

Note

Note: In many textbooks, you will find the Critical Values and quantiles, calculated using areas of the Right-Tail. So you can meet in textbooks t-Test with the Rejection Region

$$|t| > t_{n-1,\alpha/2}.$$

In fact, here $t_{n-1,\alpha/2}$ is the point such that the area under the PDF of $t(n-1)$ **right to that point** is $\alpha/2$. This coincides with our standard quantile $t_{n-1,1-\alpha/2}$, where we are calculating the area to the **left**.

R can calculate also these type of quantiles:

```
qt(1-0.05, df = 15)
```

```
## [1] 1.75305
```

```
qt(0.05, df = 15, lower.tail = FALSE)
```

```
## [1] 1.75305
```