BAYESIAN STATISTICS

HOME WORK # 2

Deadline: 15:00, Saturday, March 21, 2020

Problem 1. Using a random sample of size 2, estimate the proportion p of defectives produced by a machine when we assume our prior density of p is

$$p(\theta) = \begin{cases} k, & \text{if} \quad 1/2 < \theta < 1 \\ 0, & \text{otherwise} \end{cases}$$

Problem 2. The time of burn for the first stage of a rocket is a normal random variable with a standard deviation of 0.8 minute. Assume a normal prior distribution for μ with a mean of 8 minutes and a standard deviation of 0.2 minute. If 10 of these rockets are fired and the first stage has an average burning time of 9 minutes, find a 95% Bayes interval for μ.

Problem 3. An electrical firm manufactures light bulbs, that have a length of life that is approximately normally distributed with standard deviation of 100 hours and mean μ. The firm belives, that μ is surely between 770 and 830 hours and it is felt that a more realistic Bayesian approach would be to assume the prior distribution

$$f(\mu) = \begin{cases} \frac{1}{60}, & \text{if} \quad 770 < \mu < 830 \\ 0, & \text{otherwise.} \end{cases}$$

If a random sample of 25 bulbs gives an average life of 780 hours, find the posterior distribution

$$p(\mu|x_1, x_2, ..., x_{25}).$$

Problem 4. Suppose that the time to failure T of a certain hinge is an exponential random variable with probability density

$$f(t) = \begin{cases} \theta\, e^{-\theta\, t}, & \text{if} \quad t > 0 \\ 0, & \text{otherwise.} \end{cases}$$

From prior experience we are led to believe that θ is a value of an exponential random variable with probability density

$$f(\theta) = \begin{cases} 2\,e^{-2\,\theta}, & \text{if} \quad \theta > 0 \\ 0, & \text{otherwise.} \end{cases}$$

If we have a sample of n observations on T, find the posterior distribution of θ.