Deep Learning

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Outline

Dilated/Atrous Convolution

Definition 1

Let $F : \mathbb{Z}^2 \to \mathbb{R}$ *be a discrete function. Let* $\Omega_r : [-r, r] \cap \mathbb{Z}^2$ *and let* $k : \Omega_r \to \mathbb{R}$ *be a discrete filter of size* $(2r + 1)^2$*. The discrete convolution operator* $*$ *can be defined as*

$$(F * k)(p) = \sum_{s+t=p} F(s)\, k(t)$$

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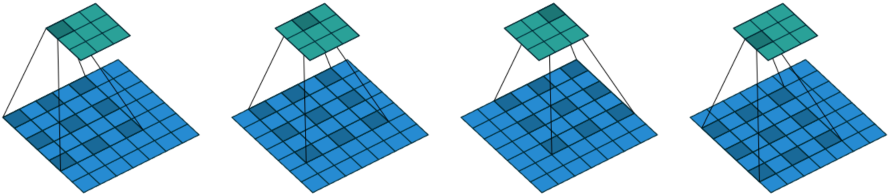
Let $F : \mathbb{Z}^2 \to \mathbb{R}$ *be a discrete function. Let* $\Omega_r : [-r, r] \cap \mathbb{Z}^2$ *and let* $k : \Omega_r \to \mathbb{R}$ *be a discrete filter of size* $(2r+1)^2$*. The discrete l-dilated convolution operator* $*_l$ *can be defined as*

$$(F *_l k)(p) = \sum_{s+lt=p} F(s)\, k(t)$$

Dilated/Atrous Convolution



1D Dilated Convolution

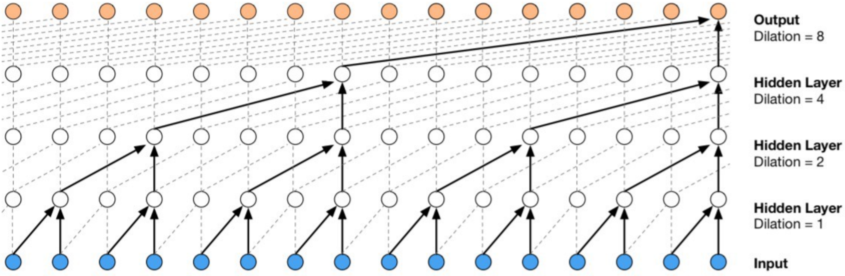
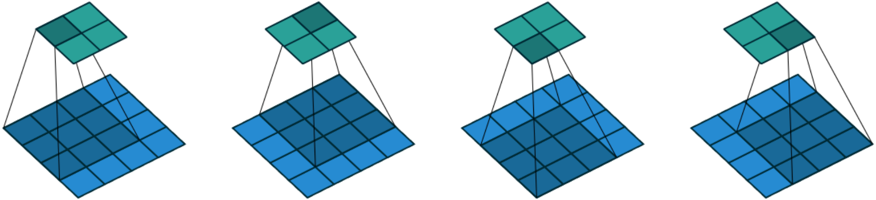


Figure 3: Visualization of a stack of *dilated* causal convolutional layers.

Convolution as a Matrix Operation



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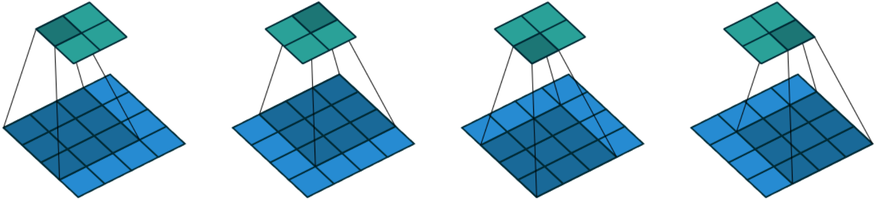
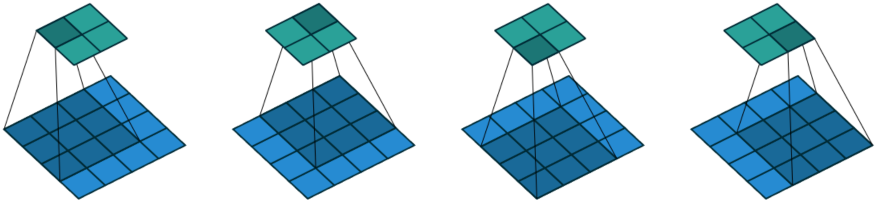


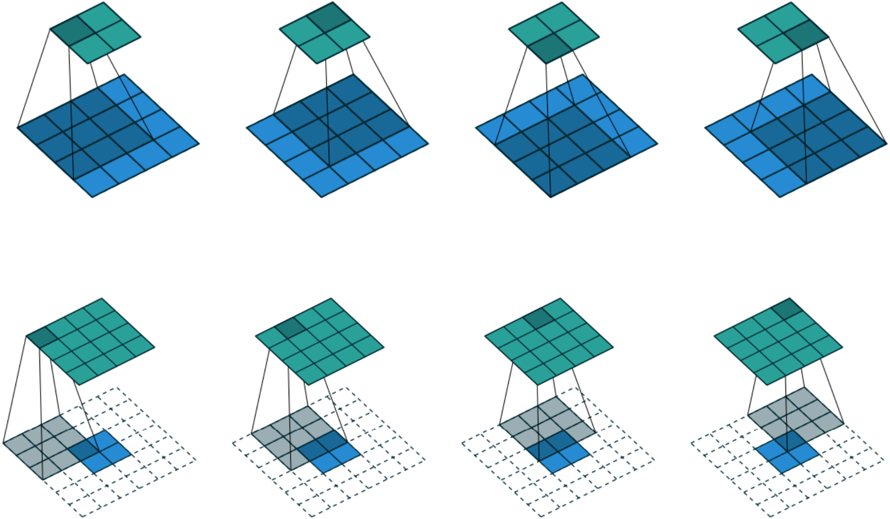
$$\begin{pmatrix} w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & w_{0,0} & w_{0,1} & w_{0,2} & 0 & w_{1,0} & w_{1,1} & w_{1,2} & 0 & w_{2,0} & w_{2,1} & w_{2,2} \end{pmatrix}$$

This linear operation takes the input matrix flattened as a 16-dimensional vector and produces a 4-dimensional vector that is later reshaped as the 2×2 output matrix.

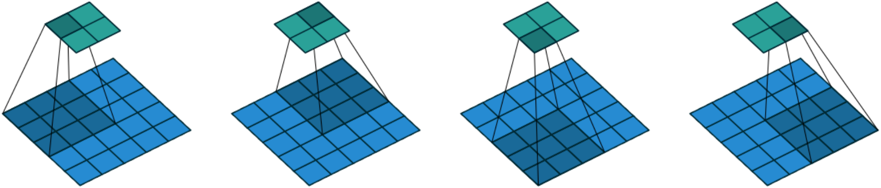
Transposed Convolution (stride=0)



Transposed Convolution (stride=0)



Transposed Convolution (stride=1)



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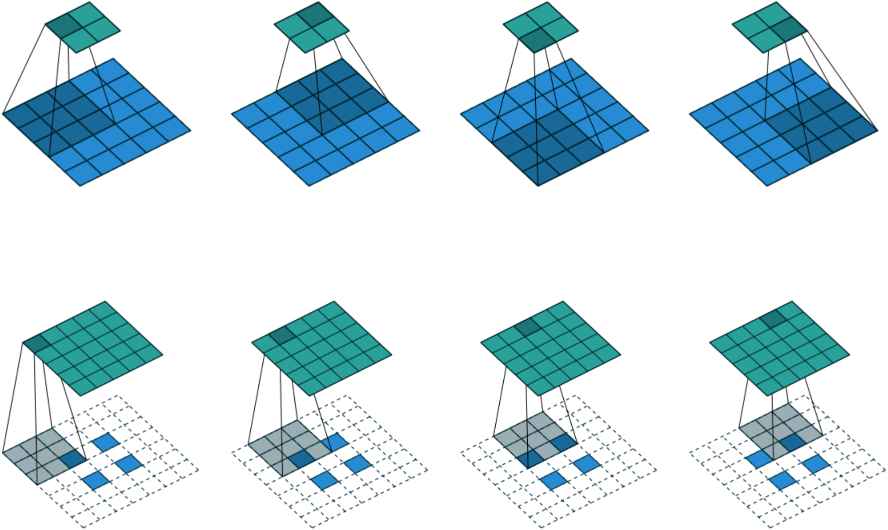
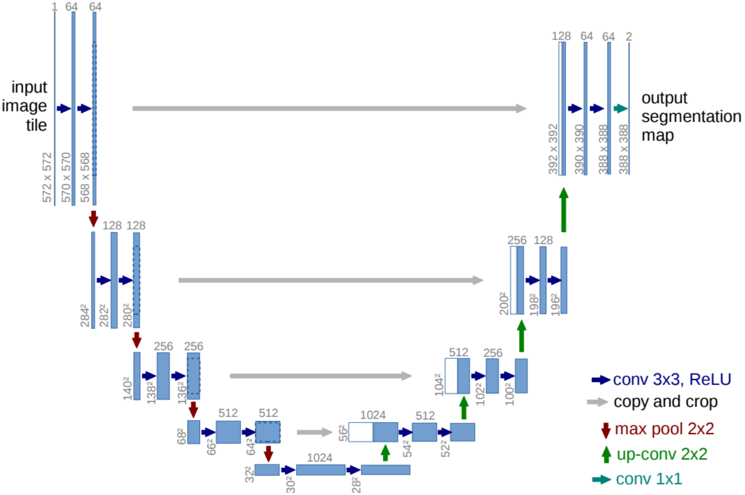


UNet



1
64
64
input
image
tile
572 x 572
570 x 570
568 x 568
128
128
284²
282²
280²
256
256
140²
138²
136²
512
512
68²
66²
64²
1024
32²
30²
28²
1024
56²
54²
512
52²
512
104²
102²
256
100²
256
200²
128
198²
196²
128
64
64
2
392 x 392
390 x 390
388 x 388
388 x 388
output
segmentation
map
conv 3x3, ReLU
copy and crop
max pool 2x2
up-conv 2x2
conv 1x1

Outline

KL Divergence

The KL divergence (also called relative entropy) is a measure of how one probability distribution is different from a second, reference probability distribution:

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but there is no symmetry, i.e. $K\left(P||Q\right) \neq K\left(Q||P\right)$.

Jensen-Shannon Divergence

JS Divergence is the following

$$JS\left(P||Q\right) = \frac{1}{2}K\left(P||M\right) + \frac{1}{2}K\left(M||Q\right),$$

where $M = \dfrac{P+Q}{2}$.

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Recall that probability density function (if it exists) of multivariate normal distribution with mean μ and with (non-singular, symmetric, positive definite) covariance matrix Σ is the following function:

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Suppose that we have two multivariate normal distributions: $\mathcal{N}_1(\mu_1, \Sigma_1), \mathcal{N}_2(\mu_2, \Sigma_2)$. Then

$$KL(\mathcal{N}_1, \mathcal{N}_2) = \frac{1}{2}\left(\operatorname{tr}\left(\Sigma_2^{-1}\Sigma_1\right) + (\mu_2 - \mu_1)^{\mathsf{T}}\Sigma_2^{-1}(\mu_2 - \mu_1) - k + \ln\frac{|\Sigma_2|}{|\Sigma_1|}\right).$$

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In one dimensional case we will have

$$KL(\mathcal{N}_1, \mathcal{N}_2) = \log\frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1-\mu_2)^2}{2\sigma_2^2} - \frac{1}{2}.$$

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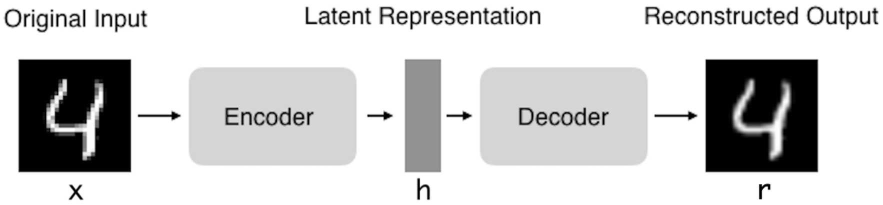
Autoencoders



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- Data denoising.
- In a lot of different tasks.

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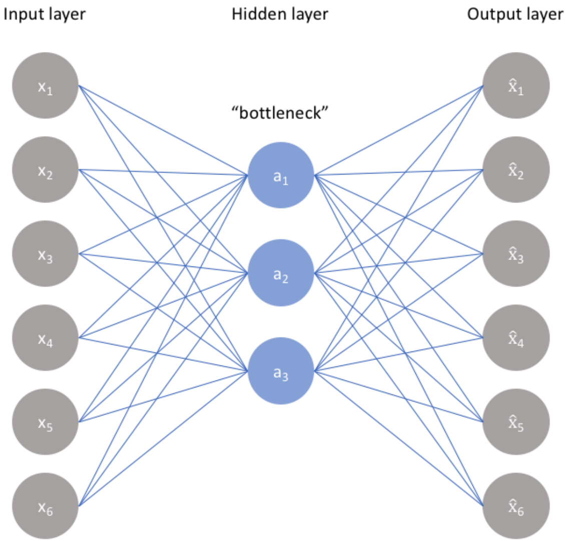
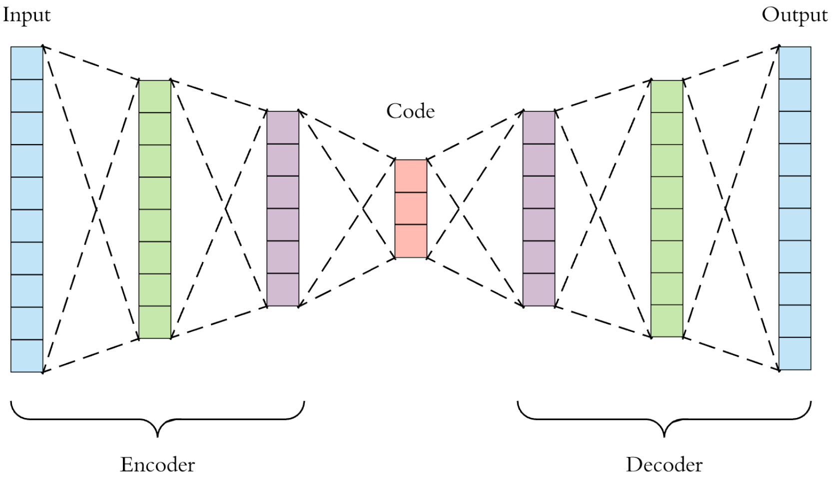
    - Denoising Autoencoders
- Variational Autoencoders

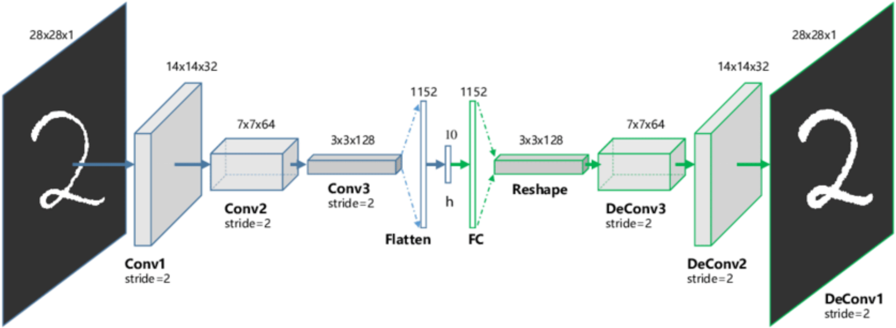
Vanilla Autoencoders



Multilayer/Deep Autoencoders



Convolutional Autoencoders



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- The end result is to reduce the learned representation's sensitivity towards the training input.

Contractive Autoencoders

Let f is our encoder, g is the decoder and D is our training dataset. In the previous cases we minimize this kind of loss function:

$$\sum_{x \in D} L\left(x, g\left(f\left(x\right)\right)\right).$$

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In the case of contractive autoencoders we will minimize this one

$$\sum_{x \in D} \left(L\left(x, g\left(f\left(x\right)\right)\right) + \lambda \left\| J_f\left(x\right) \right\|_F^2 \right),$$

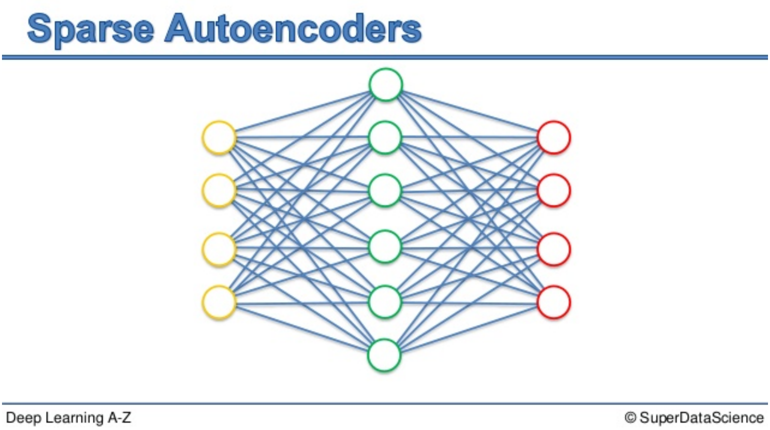
where the added summand is the square of Frobenius norm of the following Jacobian matrix:

$$\left[J_f\left(x\right)\right]_{i,j} = \frac{\partial f_j\left(x\right)}{\partial x_i}$$

i.e.

$$\left\| J_f\left(x\right) \right\|_F^2 = \sum_{i,j} \left(\frac{\partial f_j\left(x\right)}{\partial x_i} \right)^2.$$

Sparse Autoencoders



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- Sparsity penalty is introduced on the hidden layer. This is to prevent output layer copy input data. This prevents overfitting.

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Let f is our encoder, g is the decoder and D is our training dataset, which has n samples. Denote

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where

$$KL(\rho || \rho_j) = -\rho \log \frac{\rho_j}{\rho} - (1-\rho) \log \frac{1-\rho_j}{1-\rho}.$$