YSU ASDS, Statistics, Fall 2019

Lecture 24

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- Give the definition of AsympTotic CI.

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- Were Etruscans Italians?

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- Y_k-s are also IID,
- X_k-s and Y_j-s are Independent.

Our Problem was, given α, to construct a $(1-\alpha)$-level CI for the difference

$$\mu_X - \mu_Y.$$

Here μ_X is the Mean of the Distribution behind X_k, and μ_Y is the Mean of the Distribution of Y_k.

CI for the Difference between the Means of Two Samples

We have already considered the first case:

Case 1: X_k, Y_k are Normal, with known Variances

Our Pivot was

$$Z = \frac{(\overline{X} - \overline{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \sim$$

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and a $(1-\alpha)$-level CI for $\mu_X - \mu_Y$ was:

$$(\overline{X} - \overline{Y}) \pm z_{1-\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}.$$

Case 2: X_k, Y_k are Normal, with unknown, but equal Variances

In this case we assume again that

$$X_1, X_2, ..., X_n \overset{IID}{\sim} \mathcal{N}(\mu_X, \sigma_X^2) \qquad \textit{and} \qquad Y_1, Y_2, ..., Y_m \overset{IID}{\sim} \mathcal{N}(\mu_Y, \sigma_Y^2),$$

X_k-s and Y_j-s are Independent, but now we assume that σ_X^2 and σ_Y^2 are **unknown** and **equal**, $\sigma_X^2 = \sigma_Y^2 =: \sigma^2$.

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In this case we consider the following as a Pivot:

$$t = \frac{(\overline{X} - \overline{Y}) - (\mu_X - \mu_Y)}{S_p \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}},$$

where S_p is the **Pooled Sample Deviation**:

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where S_p is the **Pooled Sample Deviation**:

$$S_P^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2} = \frac{\sum_{k=1}^n (X_k - \overline{X})^2 + \sum_{k=1}^m (Y_k - \overline{Y})^2}{n+m-2}.$$

Case 2: X_k, Y_k are Normal, with unknown, but equal Variances

It can be proved that

$$t \sim t(n+m-2),$$

hence, the $(1-\alpha)$-level CI for $\mu_X - \mu_Y$ will be

$$(\overline{X} - \overline{Y}) \pm t_{n+m-2,1-\alpha/2} \cdot S_P \cdot \sqrt{\frac{1}{n} + \frac{1}{m}}$$

Case 3: X_k, Y_k are Normal, with unknown, unequal Variances

In this, maybe more realistic case, we assume again that

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We consider the following as a Pivot:

$$t = \frac{(\overline{X} - \overline{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}}.$$

The exact Distribution is not so easy to find, but it is proven that the approximate distribution of t is $t(\nu)$, where ν is calculated by some complicated formula that I am lazy to bring here.

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So we can construct a $(1-\alpha)$-level approximate CI for $\mu_X - \mu_Y$:

$$(\overline{X} - \overline{Y}) \pm t_{\nu,1-\alpha/2} \cdot \sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}$$

CI for the Difference in Proportions

Assume we have two Bernoulli Random Samples:

$$X_1, X_2, ..., X_n \sim Bernoulli(p_X)$$

and

$$Y_1, Y_2, ..., Y_m \sim Bernoulli(p_Y),$$

and assume X_k-s and Y_j-s are independent.

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It can be shown, that the following will serve as an Asymptotic CI of level $1-\alpha$ for $p_X - p_Y$:

$$\left(\overline{X} - \overline{Y}\right) \pm z_{1-\alpha/2} \cdot \sqrt{\frac{\overline{X}(1-\overline{X})}{n} + \frac{\overline{Y}(1-\overline{Y})}{m}}$$

Hypothesis Testing

RF

Sorry, no translation:

Экзамен, студентка валится безвозвратно. За дверью стоит толпа и думает, как ее выручить. Наконец в аудиторию врывается парень и кричит: — Иванова, у тебя сын родился! Препод ее, естественно, поздравляет, ставит оценку, расписывается.

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As always, we assume we have a Dataset coming as a realization of a Random Sample from some unknown Parametric Distribution $\mathcal{F}_\theta$:

$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta.$$

In this case we want to Test a Hypothesis about θ: say, see whether $\theta = \theta_0$, a given number, or not.

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The Idea The idea is the following: even if our coin was fair, the Probability of Heads $p = 0.5$, it is possible to have some deviation from the expected number of Heads, 50 (in 100 tosses).

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Hypothesis Testing: Problem Setting and Formalization

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Next, we have a Random Sample from $\mathcal{F}_\theta$:

$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta, \qquad \theta \in \Theta,$$

and using this Sample, we want to Test if we can **Reject $\mathcal{H}_0$ in favor of $\mathcal{H}_1$ or not**, i.e., we want to see if **we have enough evidence in our Data to Reject $\mathcal{H}_0$**.

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Example: In the above example about the coin fairness, if p is the Probability of a Head, then our Hypotheses are:

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And here $\Theta = [0, 1]$, $\Theta_0 = \{0.5\}$ and $\Theta_1 = \Theta \setminus \Theta_0$.

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So, when we will have our Data, we will see if we can **Reject** $\mathcal{H}_0$. Btw, hopefully, we will Reject, and not because of my new methodology, but **because of you**.

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So the conclusion of the Hypothesis Testing need to be either:

Reject $\mathcal{H}_0$ or **Fail to Reject** $\mathcal{H}_0$.

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Moral, and Choosing Null Hypotheses

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One is using this general idea to choose the Null and Alternative Hypotheses:

Moral, and Choosing Null Hypotheses

Moral: In Hypothesis tesing, if we have enough evidence from Data against $\mathcal{H}_0$, we Reject it, otherwise, we say that we do not have enough evidence to Reject $\mathcal{H}_0$, so we Fail to Reject it, and keep believing in it.

One is using this general idea to choose the Null and Alternative Hypotheses: **we will keep believing in Null, if the Data will not show strong evidence against**.

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This is an example of **A/B Testing**.

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$$\mathcal{H}_0 \qquad vs \qquad \mathcal{H}_1.$$

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Can you guess the Null Hypo?

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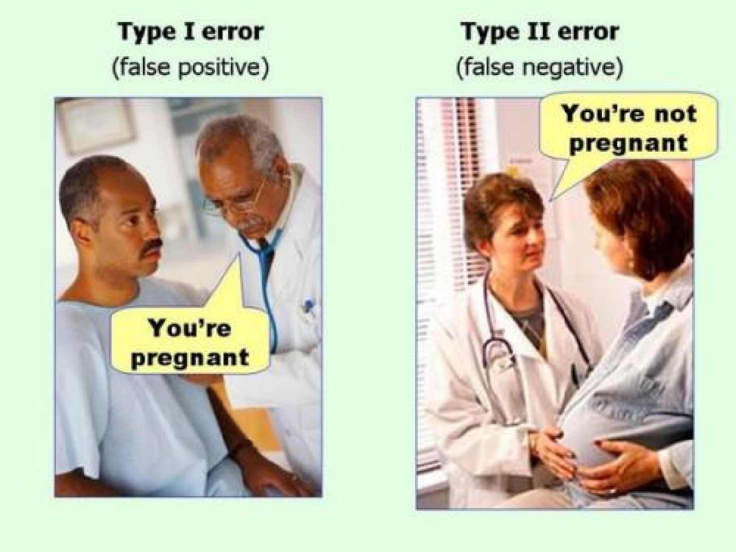

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It is easy to see that

$$\textit{Power} = 1 - \beta = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

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Probabilities of Correct/InCorrect Decisions:

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Reject $\mathcal{H}_0$	$\alpha =$ **Significance**	$1-\beta =$ **Power**
Do Not Reject $\mathcal{H}_0$	$1-\alpha$	β

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Common values of α are $0.05, 0.01$ and 0.1 (corresponding to $95\%, 99\%$ and 90% Confidence Level!).

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- What is means that the Significance level of our Test, α, is small ?
- What it means that the Power of our Test, $1 - \beta$, is high ?