YSU ASDS, Statistics, Fall 2019

Lecture 06

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Descriptive Statistics

Contents

Last Lecture ReCap

- What is the drawback of the Sample Mean?

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- What is the Sample Trimmed Mean?

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Statistical Measures for the Central Tendency/Location

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Sample Median

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The rigorous definition is: let $x : x_1, x_2, ..., x_n$ be our dataset.

- If n is **odd**, then we define

$$median(x) = x_{\left(\frac{n+1}{2}\right)};$$

- If n is **even**,

$$median(x) = \frac{1}{2} \cdot \left(x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)}\right).$$

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Example: For

$$x : -1, 2, 3, 1, 2, 4, 9,$$

the Median is: OTB

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```
x <- c(1,3,2, 4,2,3,2,2,1)
mean(x)
```

```
## [1] 2.222222
```

```
median(x)
```

```
## [1] 2
```

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median(x)
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```
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```

Now, let's add an outlier:

```
x <- c(x, 1000)
mean(x)
```

```
## [1] 102
```

```
median(x)
```

```
## [1] 2
```

Important Property of the Median

- Half of the Datapoints are to the left of the Median, and half of the Datapoints are to the right

Example: Give OTB

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Remark: Mode can be calculated even for the Nominal Scale Categorical Datasets

Mode Calculation in **R**

We do not have a simple command in basic **R** to calculate all Modes in **R**. Suggesion: write it by yourself!

Other Measures of the Central Tendency

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- Midrange,

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- See others at Wiki

Statistical Measures for the Spread/Variability

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Here we want to answer to the questions: how spread/concentrated are our Datapoints, how much is the variability of our Data?

Deviations from the Mean (or from the Median)

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$$x_k - median(x), \qquad k = 1, ..., n$$

Example

Consider the Dataset `islands` from **R**:

```
head(islands, 3)
```

```
##      Africa Antarctica       Asia
##       11506       5500      16988
```

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```

To calculate Deviations from the Mean for this Dataset, we just use

```
x.bar <- mean(islands)
deviations <- islands - x.bar
head(deviations)
```

```
##        Africa   Antarctica         Asia    Australia Axel
##     10253.271     4247.271    15735.271     1715.271    -
##        Baffin
##     -1068.729
```

Range

The simplest measure of the Spread is the Range:

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Say,

```r
range(islands)
```

```
## [1]    12 16988
```

Example, **R** code to Calculate the Range

We can define our custom function to calculate the Range as the difference:

```
my.range <- function(x){
  return(max(x)-min(x))
}
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We can define our custom function to calculate the Range as the difference:

```
my.range <- function(x){
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```

and run

```
my.range(1:10)
```

```
## [1] 9
```

The Sample Variance

The **Sample Variance** (with the denominator n) of our dataset x is defined by

$$var(x) = s^2 = \frac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n},$$

where $\bar{x}$ is the sample mean of our dataset:

$$\bar{x} = mean(x) = \frac{1}{n} \cdot \sum_{k=1}^{n} x_k.$$

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We will use both, and later we will talk about the difference between these two - there are reasons to prefer one over the other.

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The **Standard Deviation** of x is defined as

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Question: Which measure of the Spread/Variability is better: Variance or SD?

- $sd(x)$ is in the same units as x, but $var(x)$ is in the squared units of x
- $var(x)$ is easy to deal with, has some nice properties, but not $sd(x)$

Example

R is calculating Var and SD by using $n - 1$ in the denominator:

```
x <- 1:5
var(x)
```

```
## [1] 2.5
```

```
sd(x)
```

```
## [1] 1.581139
```

Some Properties of the Variance

The Sample Variance (with the denominator n) can be calculated by the following formula

$$var(x) = \frac{\sum_{k=1}^{n} x_k^2}{n} - \left(\frac{\sum_{k=1}^{n} x_k}{n} \right)^2 = \frac{\sum_{k=1}^{n} x_k^2}{n} - \left(\bar{x} \right)^2.$$

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We can write this, using an analogy with the r.v. Variance,

$$var(x) = mean(x^2) - \Big(mean(x)\Big)^2 = \overline{x^2} - (\bar{x})^2,$$

where x^2 is the dataset $x_1^2, x_2^2, ..., x_n^2$.

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where x^2 is the dataset $x_1^2, x_2^2, ..., x_n^2$. Just remember to use this in the case when the Sample Variance is with the denominator n !

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- $var(x + \beta) = var(x)$.

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By replacing the Mean by the Mode, we will obtain the **Mean Absolute Deviation from the Median**:

$$mad(x) = mad(x, median) = \frac{\sum_{k=1}^{n} |x_k - median(x)|}{n}$$

Quartiles, Quantiles and BoxPlots

Sample Quartiles

- Idea of the Median:

Sample Quartiles

- Idea of the Median:a point on the axis dividing the Dataset into two equal-length portions

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- Idea of Quartiles:3 point on the axis dividing the Dataset into four equal-length portions

[1] See, for example, the Wiki page

Sample Quartiles

- Idea of the Median:a point on the axis dividing the Dataset into two equal-length portions
- Idea of Quartiles:3 point on the axis dividing the Dataset into four equal-length portions

There are different methods to define Quartiles[1], and we will use the following.

Let $x: \ x_1, x_2, ..., x_n$ be our Dataset. First we sort, by using Order Statistics, our Dataset into:

$$x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n-1)} \leq x_{(n)}.$$

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Sample Quartiles and IQR

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- The **second (or middle) Quartile**, Q_2, is the Median of our dataset, $Q_2 = med(x)$;

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- The **third (or upper) Quartile**, Q_3, is the Median of the ordered Dataset of all observations to the right of Q_2 (including Q_2, if it is a Datapoint)

Next, we define the **InterQuartile Range, IQR** to be

$$IQR = Q_3 - Q_1.$$

Example:

Example: Find the Quartiles of

$$x: \ -2, 1, 3, 0, 5, 7, 5, 2, 0$$

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Example: Find the Quartiles of

$$x: \ 1, 1, 2, 3, 1, 1, 3, 4, 5, 2$$

Quartiles and IQR

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Quartiles and IQR

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Note: Recall the idea of Quartiles: the points Q_1, Q_2, Q_3 on the real axis divide our Dataset into (almost) four equal-length portions:

- almost 25% of our Datapoints are to the left to Q_1

Quartiles and IQR

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x <- 1:10
quantile(x,0.25)
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Or, you can use the following commands:

```r
x <- 1:10
fivenum(x)
```

```
## [1]  1.0  3.0  5.5  8.0 10.0
```

```r
summary(x)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    1.00    3.25    5.50    5.50    7.75   10.00
```

Note

Note: Please note that **R** is not using our definition of the Quartiles, so sometimes we will get different results when calculating by a hand or by **R**.

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- the set of all Outliers

$$O = \left\{x_i : x_i \notin \left[Q_1 - \frac{3}{2}IQR, Q_3 + \frac{3}{2}IQR\right]\right\}$$

BoxPlot, Example

Then we draw the points W_1, Q_1, Q_2, Q_3, W_2 on the real line and add all outliers, and make a box over $[Q_1, Q_3]$.

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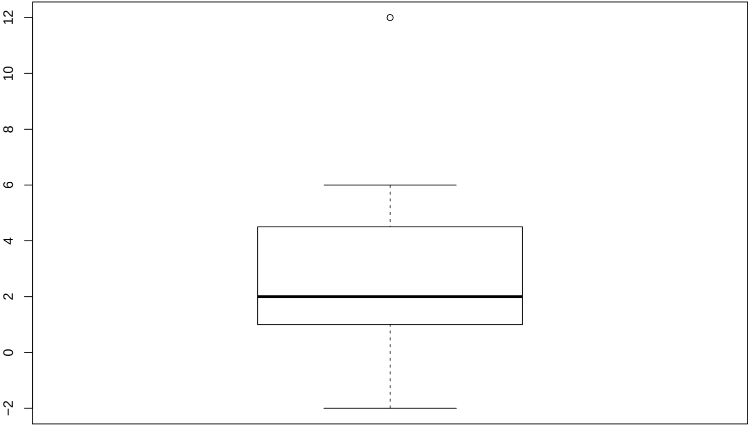
Solution: OTB;

BoxPlot, Example

Now, using **R**:

```
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x)
```



BoxPlot, Example

Another view:

```r
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x, horizontal = T, col = "magenta")
```