BAYESIAN STATISTICS

HOME WORK # 3

Saturday, April 18, 2020

Problem 1. Suppose that the prior distribution of θ is uniform on some interval. Show that the posterior distribution $p(\theta/x)$ is exactly proportional to the likelihood function $f(x/\theta)$.

Problem 2. Suppose that X is a geometric random variable:

$$f(x/p) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, ...$$

Let p have a prior distribution that is uniform on $[0, 1]$.

a) What is the posterior distribution of p?

b) What is the Bayes estimate of p under squared error loss?

Problem 3. Suppose that X has binomial distribution with parameters $n = 2$ and p. Compare the risk functions for the following estimates of p using square error loss:

$$p_1 = \frac{X}{2},$$

$$p_2 = \frac{X+1}{3},$$

and

$$p_3 = \frac{X+1}{4}.$$

Problem 4. Suppose that a parameter Θ, takes on values $\theta_1 = 1$, $\theta_2 = 10$, and $\theta_3 = 20$. The distribution of X is discrete and depends on Θ as shown in the following table:

$$\begin{array}{cc} & \begin{matrix} \theta_1 & \theta_2 & \theta_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} & \begin{pmatrix} 0.1 & 0.2 & 0.4 \\ 0.1 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ 0.6 & 0.4 & 0.2 \end{pmatrix} \end{array}$$

Assume a prior distribution of Θ is

$$p(\theta_1) = 0.5, \qquad p(\theta_2) = 0.25, \qquad p(\theta_3) = 0.25$$

a) Suppose that x_2 is observed. What is the posterior distribution of Θ?

b) What is the Bayes estimate under squared error loss in this case?

c) What is the Bayes estimate for the loss function $L(\theta, \widehat{\theta}) = |\theta - \widehat{\theta}|$?