YSU ASDS, Statistics, Fall 2019

Lecture 10-11

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25 Sep 2019

Descriptive Statistics / Probability Reminder

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- Give the definition of the Sample Covariance

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Example:

Example: Construct the Ranks Dataset for

$$x: \ 4, -3, 1, 55, 6, 2$$

Solution: OTB

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Spearman's ρ

Now assume we have two numerical 1D Datasets of the same size:

$$x: \; x_1, x_2, ..., x_n, \qquad \text{and} \qquad y: \; y_1, y_2, ..., y_n.$$

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$$x: \; x_1, x_2, ..., x_n, \qquad \text{and} \qquad y: \; y_1, y_2, ..., y_n.$$

First we calculate the ranks:

$$Rank(x): \; r_1, r_2, ..., r_n \qquad \text{and} \qquad Rank(y): \; i_1, ..., i_n.$$

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Definition: The **Spearman's rank correlation coefficient** ρ is defined as:

$$\rho = cor(Rank(x), Rank(y)).$$

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Definition: The **Spearman's rank correlation coefficient** ρ is defined as:

$$\rho = cor(Rank(x), Rank(y)).$$

So, basically, Spearman's ρ between x and y is the Correlation between the ranks of x and y.

Spearman's ρ

There is another formula to calculate the Spearman's *rho*: if we will denote $d_k = rank(x_k) - rank(y_k)$, then

$$\rho = 1 - \frac{6 \cdot \sum_{k=1}^{n} d_k^2}{n(n^2-1)}.$$

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Idea: ρ is measuring monotonic relationship between the Datasets. $\rho = \pm 1$ means (in case we do not have repeatitionsin the Datasets) there exists a monotonic relationship between the Datasets.

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Idea: ρ is measuring monotonic relationship between the Datasets. $\rho = \pm 1$ means (in case we do not have repeatitionsin the Datasets) there exists a monotonic relationship between the Datasets.
Note: Pearson's Correlation Coefficient is sensitive to outliers, but Spearman's ρ is robust wrt outliers.

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$$x: \ 0, 3, 1 \qquad \text{and} \qquad y: \ -3, -2, -1$$

Solution: OTB

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$$x:\ 0, 3, 1 \qquad \text{and} \qquad y:\ -3, -2, -1$$

Solution: OTB

```
x <- c(0,3,1); y <- -3:-1
cor(x,y)
```

```
## [1] 0.3273268
```

```
cor(x,y,method = "spearman")
```

```
## [1] 0.5
```

Examples:

```r
x <- runif(100,-1,1)
y <- x^2 + 0.3* rnorm(100)
cor(x,y)
```

```
## [1] -0.05774362
```

```r
cor(x,y,method = "spearman")
```

```
## [1] -0.005640564
```

Examples:

```r
x <- runif(100,-1,1)
y <- x^3 + 0.3* rnorm(100)
cor(x,y)
```

```
## [1] 0.7267431
```

```r
cor(x,y,method = "spearman")
```

```
## [1] 0.7118032
```

Examples:

```r
x <- runif(100,0,1)
y <- x^4 + rnorm(100)
z <- y; z[1] = 100 ## Introducing an outlier
cor(x,y)
```

```
## [1] 0.1829122
```

```r
cor(x,y,method = "spearman")
```

```
## [1] 0.1264086
```

```r
cor(x,z)
```

```
## [1] -0.164603
```

```r
cor(x,z,method = "spearman")
```

```
## [1] 0.1224002
```

Kendall's τ

Another measure of the Rank Correlation is the **Kendal's** τ: again assume we have two numerical 1D Datasets of the same size:

$$x: \; x_1, x_2, ..., x_n, \qquad \text{and} \qquad y: \; y_1, y_2, ..., y_n.$$

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$$x: \ x_1, x_2, ..., x_n, \qquad \text{and} \qquad y: \ y_1, y_2, ..., y_n.$$

We say that the pair (x_i, y_i) and (x_j, y_j) $(i \neq j)$ are *concordant*, if either

$$x_i < x_j \qquad \text{and} \qquad y_i < y_j$$

or

$$x_i > x_j \qquad \text{and} \qquad y_i > y_j.$$

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We say that the pair (x_i, y_i) and (x_j, y_j) $(i \neq j)$ are *discordant*, if either

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or

$$x_i > x_j \qquad \text{and} \qquad y_i < y_j.$$

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Now, let $con(x, y)$ be the number of concordant pairs in x, y, and $dis(x, y)$ be the number of discordant pairs.

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Definition: Kendall's Rank Correlation Coefficient τ is defined by:

$$\tau = \frac{con(x, y) - dis(x, y)}{con(x, y) + dis(x, y)} = \frac{con(x, y) - dis(x, y)}{\binom{n}{2}}.$$

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Another, explicit way, of writing this is:

$$\tau = \frac{\sum_{i<j} sign(x_i - x_j) \cdot sign(y_i - y_j)}{\frac{n(n-1)}{2}}.$$

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Facts:

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- If the rankings of x and y are the same (so x and y are in increasing relationship), then $\tau = 1$
- If the rankings of x and y are inverse of each other (so x and y are in decreasing relationship), then $\tau = -1$

Example:

```
x <- c(1,3,4); y <- c(-5, 10, 1000)
cor(x,y)
```

```
## [1] 0.7643896
```

```
cor(x,y,method = "spearman")
```

```
## [1] 1
```

```
cor(x,y,method = "kendall")
```

```
## [1] 1
```

Notes and Additions: Rank Correlations

- There can be ties (when $x_i = x_j$ or $y_i = y_j$), and there are different methods to deal with ties. See Wiki.

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- There can be ties (when $x_i = x_j$ or $y_i = y_j$), and there are different methods to deal with ties. See Wiki.
- Rank correlations are more robust than the Pearson's Correlation Coefficient;
- Rank correlations can be calculated even for Ordinal Variable Datasets
- There are other Rank Correlation measures, see Wiki

Correlation Coefficient as a Cosine

Recall from Algebra/Calculus, that if

$$\mathbf{a} = [a_1, ..., a_n]^T \qquad \text{and} \qquad \mathbf{b} = [b_1, ..., b_n]^T,$$

then we define the angle α between **a** and **b** in the following way:

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where the numerator is the dot product of $\mathbf{a}$ and $\mathbf{b}$ and

$$\|\mathbf{a}\| = \sqrt{(a_1)^2 + (a_2)^2 + ... + (a_n)^2}$$

is the length of the vector $\mathbf{a}$.

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Now, assume we have two Datasets

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We center our Datasets: calculate

$$x - \bar{x}: \; x_1 - \bar{x}, ..., x_n - \bar{x}, \qquad \text{and} \qquad y - \bar{y}: \; y_1 - \bar{y}, ..., y_n - \bar{y}.$$

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then it is easy to see that the Correlation Coefficient $cor(x, y)$ is the Cosine of the angle between **a** and **b**.

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then it is easy to see that the Correlation Coefficient $cor(x, y)$ is the Cosine of the angle between **a** and **b**.

See also Wiki

Correlation is not Causation

- Some Examples: Spurious Correlations

Anscombe Quartet

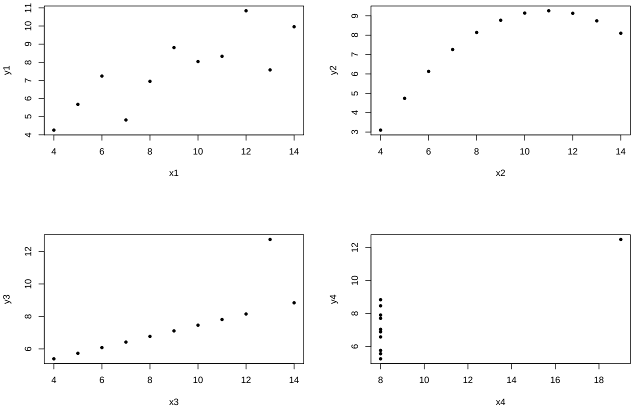
See Wiki

```
anscombe
```

```
##    x1 x2 x3 x4    y1   y2    y3    y4
## 1  10 10 10  8  8.04 9.14  7.46  6.58
## 2   8  8  8  8  6.95 8.14  6.77  5.76
## 3  13 13 13  8  7.58 8.74 12.74  7.71
## 4   9  9  9  8  8.81 8.77  7.11  8.84
## 5  11 11 11  8  8.33 9.26  7.81  8.47
## 6  14 14 14  8  9.96 8.10  8.84  7.04
## 7   6  6  6  8  7.24 6.13  6.08  5.25
## 8   4  4  4 19  4.26 3.10  5.39 12.50
## 9  12 12 12  8 10.84 9.13  8.15  5.56
## 10  7  7  7  8  4.82 7.26  6.42  7.91
## 11  5  5  5  8  5.68 4.74  5.73  6.89
```

Anscombe Quartet

```
attach(anscombe)
par(mfrow=c(2,2))
plot(y1~x1, pch = 20); plot(y2~x2, pch = 20);
plot(y3~x3, pch = 20); plot(y4~x4, pch = 20);
```



Anscombe Quartet

```r
c(mean(x1), mean(x2), mean(x3), mean(x4))
```

```
## [1] 9 9 9 9
```

```r
c(mean(y1), mean(y2), mean(y3), mean(y4))
```

```
## [1] 7.500909 7.500909 7.500000 7.500909
```

```r
c(var(x1), var(x2), var(x3), var(x4))
```

```
## [1] 11 11 11 11
```

```r
c(var(y1), var(y2), var(y3), var(y4))
```

```
## [1] 4.127269 4.127629 4.122620 4.123249
```

```r
c(cor(x1,y1), cor(x2,y2), cor(x3,y3), cor(x4,y4))
```

```
## [1] 0.8164205 0.8162365 0.8162867 0.8165214
```

Anscombe Quartet

Moral: Just calculating numbers (summary statistics) is not enough, visualize your Data if possible.

Reminder on Random Variables and Distributions

Random Variables

Everything starts at the Probability Space (Experiment, Model): we are given

$$(\Omega, \mathcal{F}, \mathbb{P}) \qquad \text{or, we usually use} \qquad (\Omega, \mathbb{P}),$$

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So $X = X(\omega)$, but usually we forget about ω, and use X.

Main Complete Characteristics of a r.v.

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Definition: We say that X is a *Continuous r.v.*, if it has a PDF: a function $f(x)$ such that

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So for a Continuous r.v., another complete characteristic, besides the CDF, is its PDF.

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or, in a table form,

Values of X	x_1	x_2	...
$\mathbb{P}(X = x)$	p_1	p_2	...

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- the Expected Value (Mean):

$$\mathbb{E}(X) = \int_{-\infty}^{+\infty} x \cdot f(x) dx \, (cont.) \quad | \quad \mathbb{E}(X) = \sum_{k} x_k \cdot \mathbb{P}(X = x_k) \, (disc.).$$

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Note:

$$\mathbb{E}(g(X)) = \int_{-\infty}^{+\infty} g(x) \cdot f(x) dx \, (cont.) \quad | \quad \mathbb{E}(g(X)) = \sum_k g(x_k) \cdot \mathbb{P}(X = x_k) \, ($$

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Note:

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- The Variance

$$Var(X) = \mathbb{E}\Big((X - \mathbb{E}(X))^2\Big) = \mathbb{E}(X^2) - \Big[\mathbb{E}(X)\Big]^2.$$

Important Discrete Distributions

Bernoulli Distribution

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- Example:

```
rgeom(10,prob = 0.3)
```

```
##  [1] 11  2  0  0  6  0  2  3  5  0
```

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```
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Important Continuous Distributions

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- Mean and Variance: $\mathbb{E}(X) = \mu$, $Var(X) = \sigma^2$.
- Models: Different things. Important is because of the CLT
- **R** name: `norm` with the parameters `mean = 0`, `sd = 1`

Normal (Gaussian) Distribution

- Parameters: μ (mean) and σ^2 (variance);
- Notation: $X \sim \mathcal{N}(\mu, \sigma^2)$;
- Support: $\mathbb{R}$
- PDF:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Mean and Variance: $\mathbb{E}(X) = \mu$, $Var(X) = \sigma^2$.
- Models: Different things. Important is because of the CLT
- **R** name: `norm` with the parameters `mean = 0`, `sd = 1`
- Example:

```r
rnorm(10, mean = 2, sd = 3)
```

```
##  [1]  1.0604526  2.6384238  1.8464923 -3.6916389  0.627
##  [7]  6.0648668  0.9021995 -1.0345287  3.2685995
```

Normal (Gaussian) Distribution

Note:

- **R** is using $\mathcal{N}(mean, sd)$ format
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So if you want to generate a sample of size 100 from $\mathcal{N}(2, 9)$, use the command `rnorm(100, mean = 2, sd = 3)`.

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So if you want to generate a sample of size 100 from $\mathcal{N}(2, 9)$, use the command `rnorm(100, mean = 2, sd = 3)`.

Additional Properties:

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Z = \dfrac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$ and

$$\mathbb{P}(a < X < b) = \mathbb{P}\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right) =$$

$$= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right).$$

Normal (Gaussian) Distribution

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Normal (Gaussian) Distribution

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- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$\mathbb{P}(-\sigma < X - \mu < \sigma) \approx 0.6827,$$

$$\mathbb{P}(-2\sigma < X - \mu < 2\sigma) \approx 0.9545$$

and

$$\mathbb{P}(-3\sigma < X - \mu < 3\sigma) \approx 0.9973.$$

Additions

- See many other Distributions at Wiki or in different textbooks.

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- Another page for the Relationship: L. Leemis Page