

Equations used in Optical levitation of high purity nanodiamonds without heating

DIPOLE TRAP EQUATIONS

In the limit of small oscillations and the Rayleigh approximation, the bead is a point dipole trapped in a harmonic potential. Using the paraxial approximation (this is said to be valid since the aperture of the objective is large [1]) the spring constant of the potential is given by

$$k_{trap} = 4\pi^3 \frac{\alpha P}{c\varepsilon_0} \frac{(\text{N.A.})^4}{\lambda^4}, \quad (1)$$

where α is the polarisability of the bead, P is the optical power, N.A. is the numerical aperture of the trapping lens, c is the speed of light, ε_0 is the vacuum permittivity, and λ is the trapping laser wavelength [2]. Whilst the dipole trap forms a harmonic potential, collisions with the surrounding gas induce Brownian dynamics [3] and the motion of the bead is therefore governed by

$$m\ddot{x}(t) + m\Gamma_0\dot{x}(t) + m\omega_0^2x(t) = f_B(t), \quad (2)$$

where $x(t)$ is the time dependent position along the x axis (transverse to the optical axis), m is the mass, Γ_0 is the damping rate, $\omega_0 = \sqrt{k_{trap}/m}$, and $f_B(t)$ is a Gaussian random force with $\langle f_B(t) \rangle = 0$ and $\langle f_B(t)f_B(t-t') \rangle = 2m\Gamma_0 k_B T_{cm} \delta(t-t')$, where T_{cm} is the centre-of-mass temperature. Similar equations may be written for the y (transverse) and z (along the optical axis) directions. It can then be shown that the power spectral density is

$$S_x(\omega) = \frac{2k_B T_{cm}}{m} \frac{\Gamma_0}{(\omega^2 - \omega_0^2)^2 + \omega^2\Gamma_0^2}. \quad (3)$$

We can express equation 3 as

$$S_x(\omega) = A \frac{\Gamma_0}{(\omega^2 - \omega_0^2)^2 + \omega^2\Gamma_0^2}, \quad (4)$$

for fitting experimental data, where

$$A = \frac{2C^2 k_B T_{cm}}{m} \quad (5)$$

is a fitting parameter and C is a calibration factor in units of V/m. The other fitting parameters are Γ_0 and ω_0 . There are a few equations of Γ_0 available in the literature. For example, in the paper by Ranjit *et. al.* [4], the damping coefficient is given by

$$\Gamma_0 = \frac{16p}{\pi\rho\bar{v}r}, \quad (6)$$

where p is the gas pressure, ρ is the density, and $\bar{v} = \sqrt{8k_B T_0 / \pi m}$ is the gas mean speed, and r is the nanoparticle radius (equivalent sphere). Vovrosh *et. al.* [5] on the other hand use

$$\Gamma_0 = 0.619 \frac{9\pi\xi d_m^2 p}{\sqrt{2}\rho k_B T_0 r}, \quad (7)$$

where ξ is the viscosity of air, and d_m is the diameter of air molecules. Equation 7 seems to give slightly smaller radii than equation 6, e.g. for the nanodiamond discussed in the main text, the radius is found to be either 18 nm or 12 nm using equations 6 and 7 respectively.

We can use the radius to find the mass of the equivalent sphere, and subsequently calibrate the detector signal into nanometers by substituting this mass into equation 5 if we know the centre-of-mass temperature of the trapped nanoparticle, leaving C as the only remaining unknown.

HEAT ABSORPTION AND DISSIPATION IN AN OPTICALLY TRAPPED NANOPARTICLE

The rate of absorption can be written as

$$\frac{dE_{abs}}{dt} = C_x I, \quad (8)$$

where $C_x = 3kV\text{Im}\frac{\epsilon-1}{\epsilon+2}$ is the absorption cross-section of a dielectric nanoparticle satisfying the Rayleigh scattering regime ($r \ll \lambda$), and I is the optical intensity of the trapping light at the focus [6].

For heat dissipation due to gas, we consider gas molecule collisions where the degree of thermalisation is determined by an accommodation coefficient α_g , where $0 \leq \alpha_g \leq 1$. For nanodiamond, we take the accommodation coefficient to be 1 [7]. The number of gas molecules per cubic meter at a given pressure is

$$N = N_0 \frac{p}{p_0}, \quad (9)$$

where N_0 is the number of gas molecules per cubic meter at atmospheric pressure, p is the pressure, and p_0 is the atmospheric pressure. The rate of collisions the levitated bead will receive is then

$$N_c = \frac{1}{2} \bar{v} N A, \quad (10)$$

where $\bar{v} = \sqrt{8k_B T_0/m}$ is the mean speed of the gas molecules, and A is the surface area of the nanoparticle. The heat dissipated is then $-3\alpha_g N_c k_B(T - T_0)$, where k_B is Boltzmann's constant, T is the nanoparticles internal temperature, and T_0 is the gas temperature. This can be recast into

$$\frac{dE_{gas}}{dt} = -6\alpha_g \pi r^2 \bar{v} N_0 \frac{p}{p_0} k_B (T - T_0), \quad (11)$$

where r is the nanoparticle radius.

The nanoparticle absorbs blackbody radiation at a rate [8]

$$\frac{dE_{bb}}{dt} = \frac{72\zeta(5)V}{\pi^2 c^3 \hbar^4} \left(\text{Im} \frac{\epsilon_{bb} - 1}{\epsilon_{bb} + 2} \right) k_B^5 T_0^5, \quad (12)$$

and dissipates black-body radiation at a rate equal to the negative of equation 12 with the substitution of T for T_0 , where all other variables have the same definitions as stated in the main text. Therefore the heat dissipated from raising the temperature of the bead from T_0 to T is

$$\frac{dE_{bb}}{dt} = \frac{72\zeta(5)V}{\pi^2 c^3 \hbar^4} \left(\text{Im} \frac{\epsilon_{bb} - 1}{\epsilon_{bb} + 2} \right) k_B^5 (T^5 - T_0^5). \quad (13)$$

Subtracting this from equation 12 leaves us with

$$\frac{72\zeta(5)V}{\pi^2 c^3 \hbar^4} \left(\text{Im} \frac{\epsilon_{bb} - 1}{\epsilon_{bb} + 2} \right) k_B^5 T^5. \quad (14)$$

In the steady-state, if we subtract the total heat dissipated from the total heat generated from absorption, we are left with the excess heat remaining in the bead

$$VC_V(T - T_0) = 3IkV\text{Im}\frac{\epsilon-1}{\epsilon+2} - 6\alpha_g \pi r^2 \bar{v} N_0 \frac{p}{p_0} k_B (T - T_0) - \frac{72\zeta(5)V}{\pi^2 c^3 \hbar^4} \left(\text{Im} \frac{\epsilon_{bb} - 1}{\epsilon_{bb} + 2} \right) k_B^5 T^5 \quad (15)$$

where C_V is the volumetric heat capacity of the bead. We then solve this equation numerically for T , as no analytic solutions exist for equations of the form $y' = ax^5$.

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