SECTION 3

Buttiker's conductance

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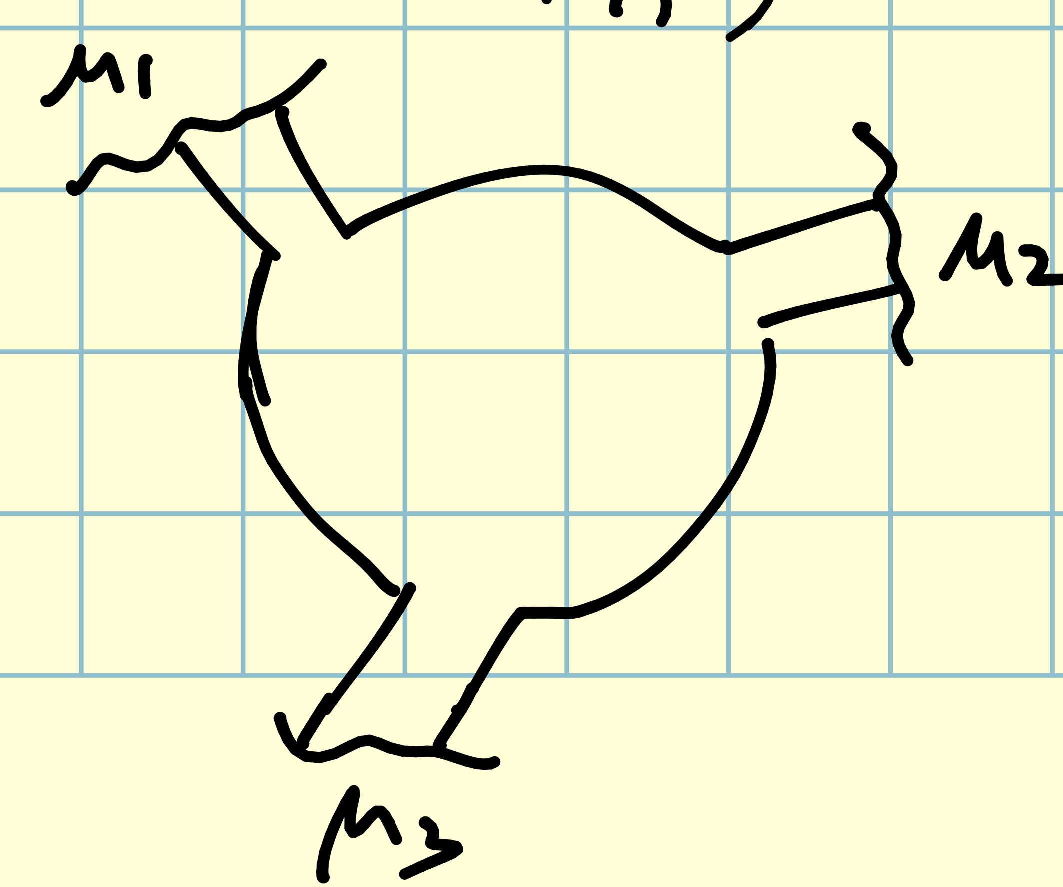
Büttiker's approach to calculating the conductance of multiprobe conductors:

1. Treat all probes (regardless of whether they are current or voltage) on an equal footing.

2. Extend the Landauer 2T formula: $I = \frac{2q}{h} \bar{T}(\mu_1 - \mu_2)$ to all probes or contacts.

$$\bar{T} = NT = \sum_{n=1}^{N} T_n \quad (\text{where, } T_1 = T_2 = \cdots = T_n)$$



$$I_p = \frac{2Q}{h} \sum_{q \neq p} \left(\bar{T}_{q \leftarrow p} \mu_p - \bar{T}_{p \leftarrow q} \mu_q \right)$$

where p and q are probe indices

Q is the charge of particle or hole

$$I_p = \sum_{q \neq p} \left(G_{q \leftarrow p} V_p - G_{p \leftarrow q} V_q \right)$$

where

$$G_{q \leftarrow p} = \frac{2Q^2}{h} T_{q \leftarrow p} \;, \qquad V_p = \frac{\mu_p}{Q}$$

Sum rule:

When voltages / potentials are equal, then the current is 0.

$$\Rightarrow \sum_q G_{qp} = \sum_q G_{pq}$$

Proof:

Suppose we have a 3 terminal system:

$$G_{21} + G_{31} = G_{12} + G_{13}$$

$$I_1 = \sum_q \left(G_{q1} V_1 - G_{1q} V_q\right)$$

If $V_1 = V_2 = V_3 \Rightarrow I_1 = 0$

$$\Rightarrow (G_{21} + G_{31} - G_{12} - G_{13}) V = 0$$

$$\Rightarrow \sum_q G_{pq} = \sum_q G_{qp}$$

$$I_p = \sum_{q \neq p} G_{pq} \left(V_p - V_q\right) = \frac{2e^2}{h} \sum_{q \neq p} T_{pq} \left(V_p - V_q\right)$$

Suppose with the same 3 terminal system,

Probe 3 @ μ_3 is voltage probe (doesn't draw any current)

$$G_{2t} = G_{12,12}$$

current voltage

Bais is : $V_1 - V_2 > 0$

$$I_1 = -I_2 = I \quad \& \quad I_3 = 0$$

$$I_1 = G_{12}(V_1 - V_2) + G_{13}(V_1 - V_3)$$

$$I_2 = G_{21}(V_2 - V_1) + G_{23}(V_2 - V_3)$$

$$I_3 = 0$$

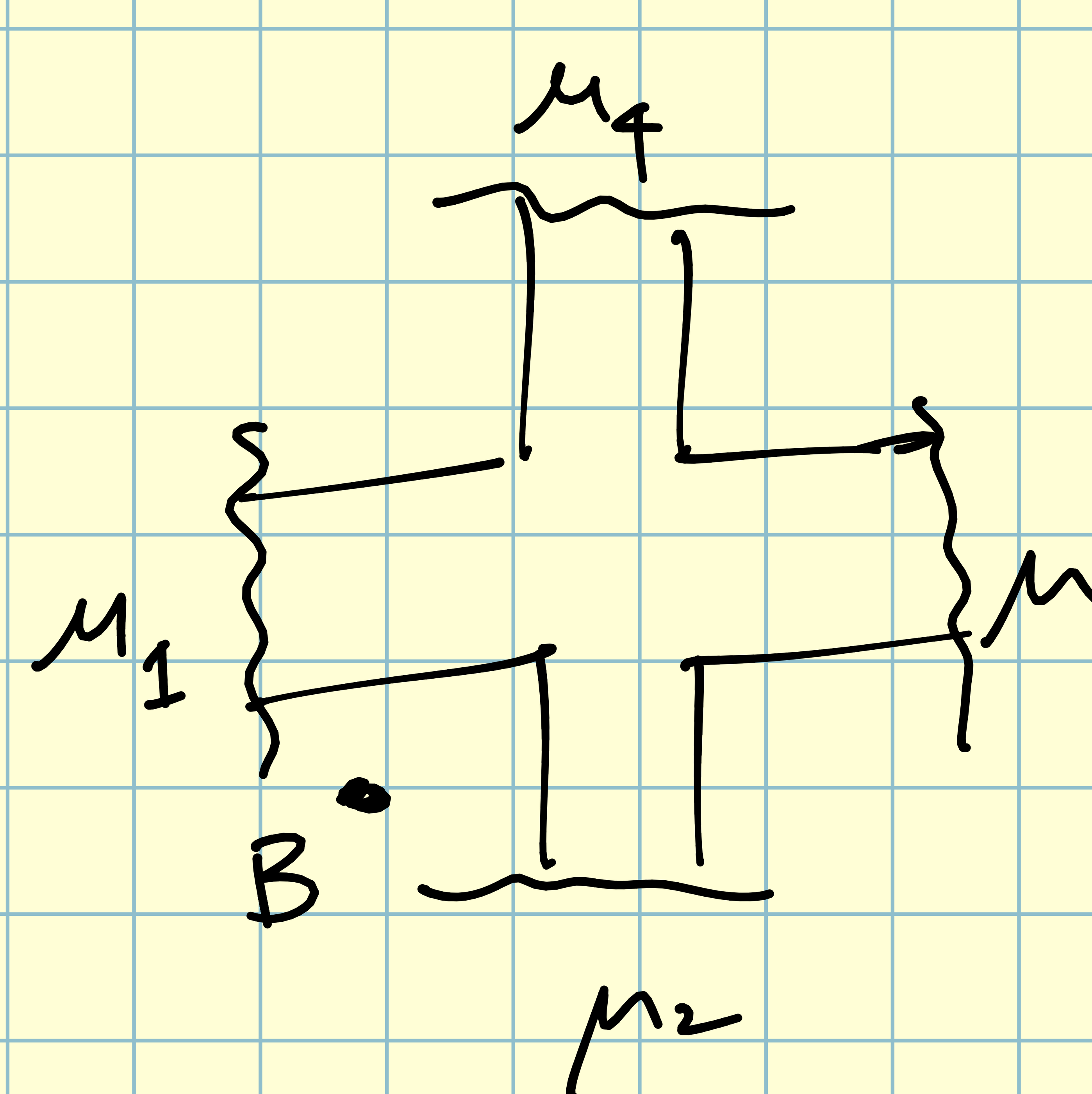
$$\Rightarrow \quad V_3 = \frac{G_{31} V_1 + G_{32} V_2}{G_{31} + G_{32}}$$

$$I_1 = G_{12}(V_1 - V_2) + G_{13}\left[V_1 - \frac{G_{31} V_1 + G_{32} V_2}{G_{31} + G_{32}}\right]$$

$$= \left[G_{12} + \frac{G_{13} G_{32}}{G_{31} + G_{32}}\right](V_1 - V_2)$$

$$G_{2T} = G_{12,12} = \frac{I_1}{V_1 - V_2} = \underset{\substack{\uparrow \\ \text{coherent} \\ \text{transport}}}{G_{12}} + \underset{\substack{\uparrow \\ \text{incoherent transport}}}{\frac{G_{13} G_{32}}{G_{31} + G_{32}}}$$

Ex. 2.3



2 & 4 are voltage probes

$\Rightarrow I_2 = I_4 = 0$

$$T_F = \bar{T}_{13} = \bar{T}_{31} = \bar{T}_{42} = \bar{T}_{24}$$

$$T_R = \bar{T}_{21} = \bar{T}_{32} = \bar{T}_{43} = \bar{T}_{14}$$

$$T_L = \bar{T}_{41} = \bar{T}_{12} = \bar{T}_{23} = \bar{T}_{34}$$

$$R_{2T} = R_{13,13} = \frac{V_1 - V_3}{I_1}$$ with $I_2 = I_4 = 0$

(local resistace)

$$R_H = R_{13,24} = \frac{V_2 - V_4}{I_1}$$ with $I_2 = I_4 = 0$

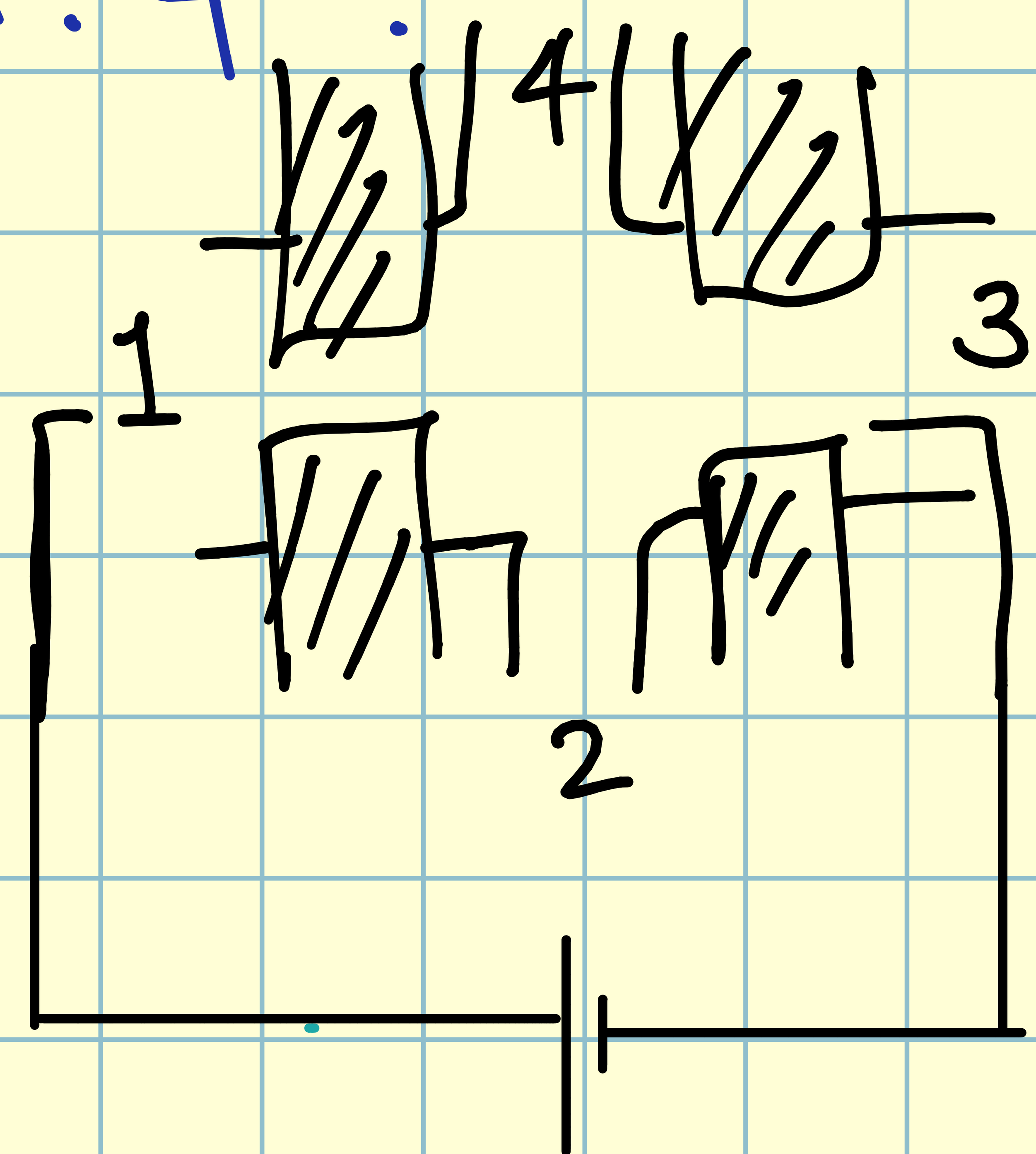
(Hall resistance)

$$R_B = R_{12,34} = \frac{V_3 - V_4}{I_1}$$ with $I_2 = I_4 = 0$

(non-local resistance.

Ex 2.4 :

$\cdot B$



$$G_{2T} = G_{13,13} = \frac{I}{V_1 - V_3} \quad \text{with } I_2 = I_4 = 0$$

$$I_p = \sum_{q \neq p} G_{pq} (V_p - V_q)$$

$$I_1 = G_{12}(V_1 - V_2) + G_{13}(V_1 - V_3) + G_{14}(V_1 - V_4)$$

$$I_2 = G_{21}(V_2 - V_1) + G_{23}(V_2 - V_3) + G_{24}(V_2 - V_4)$$

$$I_3 = G_{31}(V_3 - V_1) + G_{32}(V_3 - V_2) + G_{34}(V_3 - V_4)$$

$$I_4 = G_{41}(V_4 - V_1) + G_{42}(V_4 - V_2) + G_{43}(V_4 - V_3)$$

Put $V_3 = 0$, $I_2 = I_4 = 0$

$$G_{12} = M \cdot \frac{2e^2}{h} T_{12} = T_{12} = T_L = G_{23} = G_{41}$$

$$G_{13} = T_F = G_{31} \quad ; \quad G_{42} = T_F' = G_{24}$$

$$G_{43} = T_R = G_{14} = G_{21} = G_{32}$$

$$G_{13,13} = \frac{2e^2}{h}\left[M - \left(\frac{T_F'(T_L + T_R) + 2T_L T_R}{T_R + T_L + 2T_F'} \right) \right]$$

where $M = T_L + T_F + T_R$

Summary:-

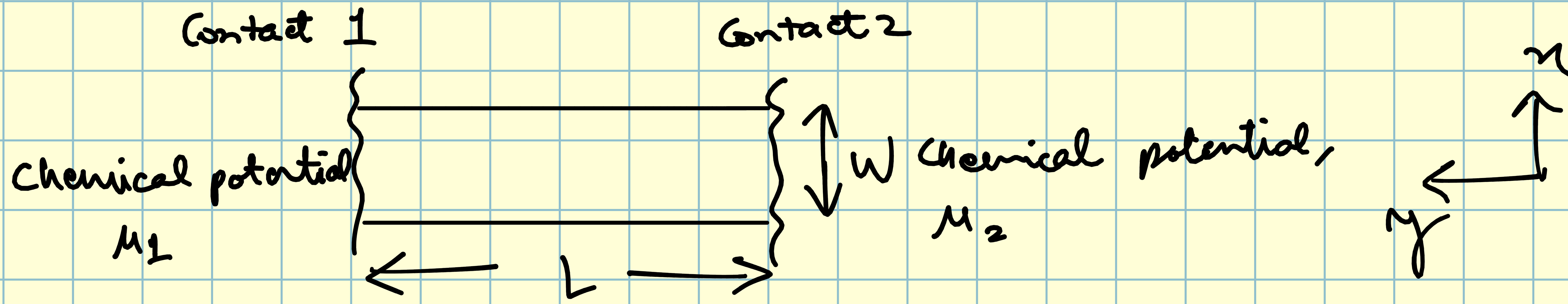
(1) $$I_p = \sum_{q \neq p} G_{p \leftarrow q}\,(V_p - V_q)$$

$$= \frac{2e^2}{h} \sum T_{p \leftarrow q}\,(V_p - V_q)$$

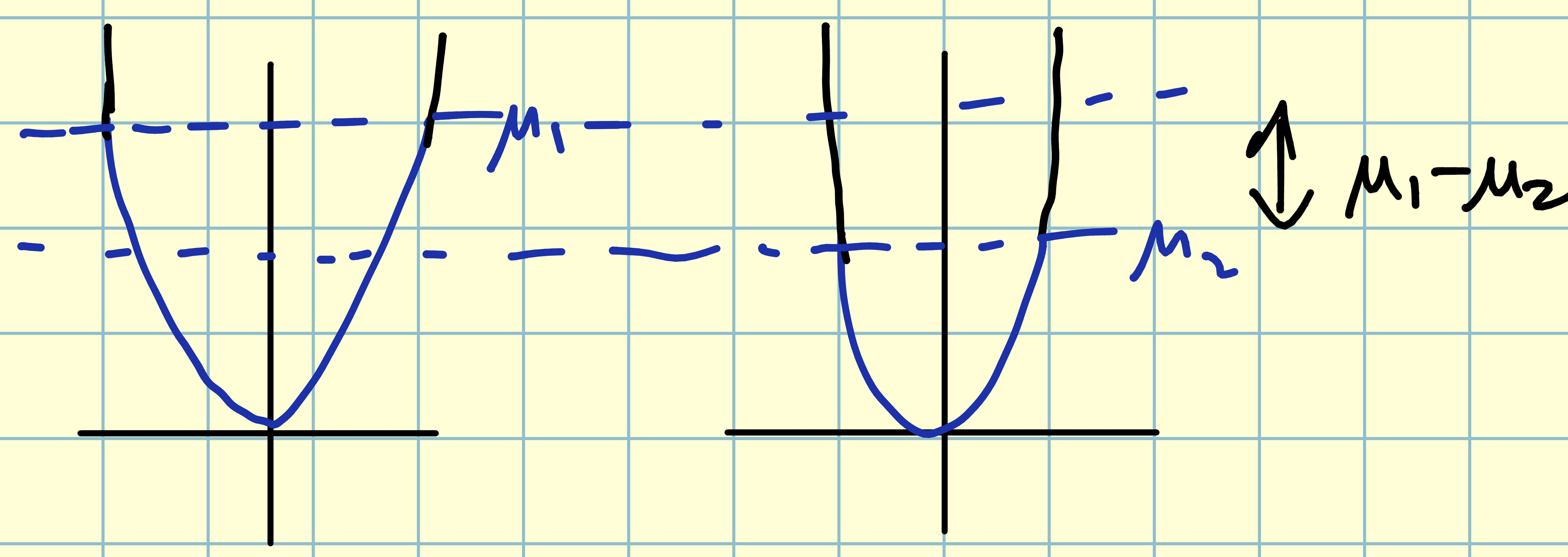
SECTION 1

Landauer conductance

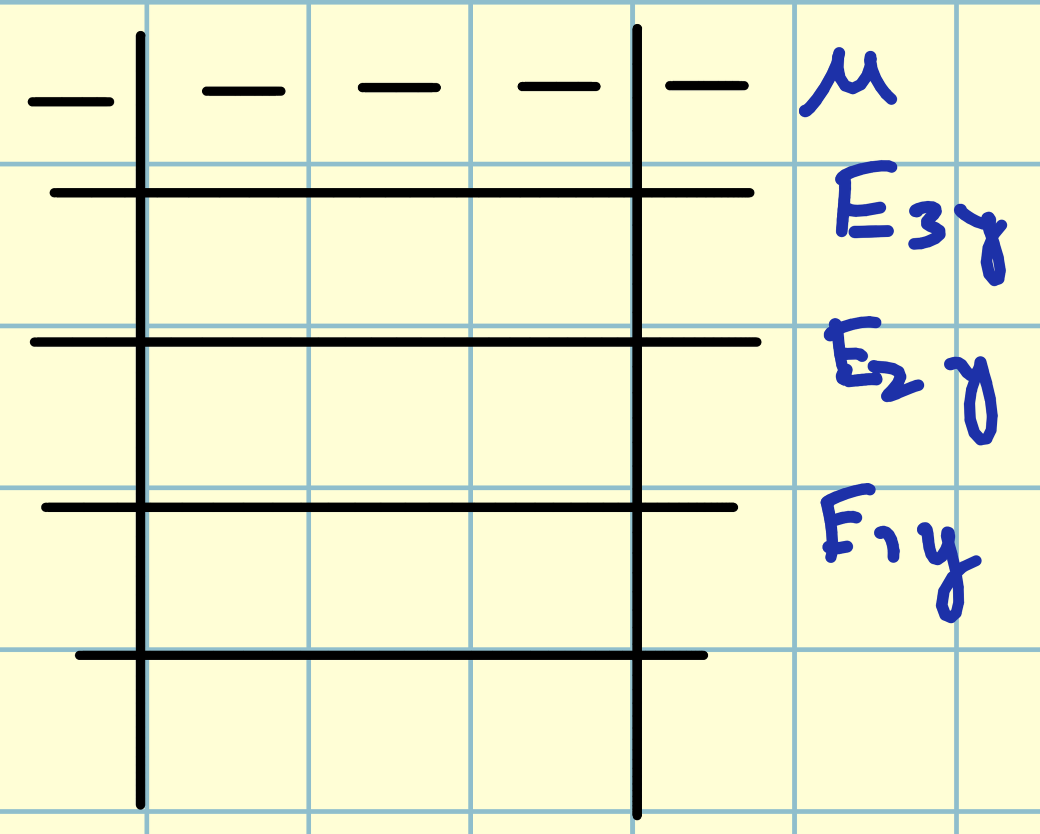
Consider the sample with two reservoirs:



The length scale is: $\lambda_F, L < l_e < l_i$



Right moving states are filled upto a higher chemical potential ($\mu_1 > \mu_2$).



$N_{max} = 3$

$$E_n = E_{ny} + \frac{\hbar^2 k_{nx}^2}{2m^*}$$

$$k_{ny} = \frac{2\pi}{L}$$

$$E_{ny} = \frac{\hbar^2 k_{ny}^2}{2m^*}$$

$$k_{ny}^2 \gg k_{nx}^2$$

$$k_{nx}^2 + k_{ny}^2 = k_F^2$$

Landauer 2 Terminal conductance:

Particle current in a particular mode/subband/channel:

$$J_n = q \int_0^{E_F} v_n(E)\, \rho_{1D,n}(E)\, dE$$

where $v_n(E)$ is the particle velocity and

$\rho_{1D\,n}$ = 1D density of states.

The net current is given by (from a given contact):

$$J_p(\mu) = \sum_{n=1}^{N_{max}} J_{p,n}(E_F) = q \sum_n^{N_{max}} \int_0^{\mu} v_n(E)\, \rho_{1D,n}(E)\, dE$$

$$v_n = \frac{1}{\hbar}\frac{dE_n}{dk} \quad , \quad \rho_{1D,n} = \frac{1}{L}\frac{dN}{dE_n} = \frac{1}{L}\frac{\frac{dN}{dk}}{\frac{dE_n}{dk}}$$

$$v_n \cdot \rho_{1D,n} = \frac{1}{\hbar}\frac{\cancel{dE_n}}{\cancel{dk}} \times \frac{1}{L} \times \frac{\frac{dN}{dk}}{\cancel{\frac{dE_n}{dk}}} = \frac{1}{L}\frac{dN}{dk}\cdot\frac{1}{\hbar} = \frac{1}{L}\frac{1}{\frac{2\pi}{L}}\times\frac{1}{\hbar} = \frac{1}{h}$$

For a single channel,

$$J_{p,net} = J_{p,n,right}(\mu_1) - J_{p,n,left}(\mu_2)$$

$$J_{p,net} = g_s \cdot q \cdot \frac{\mu_1 - \mu_2}{h}$$

Net electronic current:

$$J_{e,n,net} = \frac{-2e\Delta\mu}{h}$$

$$\Delta\mu = q V_{12} = -e V_{12}$$

$$\Rightarrow J_{e,n,net} = \frac{2e^2 V_{12}}{h}$$

Therefore total net electronic current: [Harmans & Datta]

$$J = \sum_{n=1}^{N} J_{e,n,net} = \frac{N \cdot 2e^2}{h} V_{12}$$

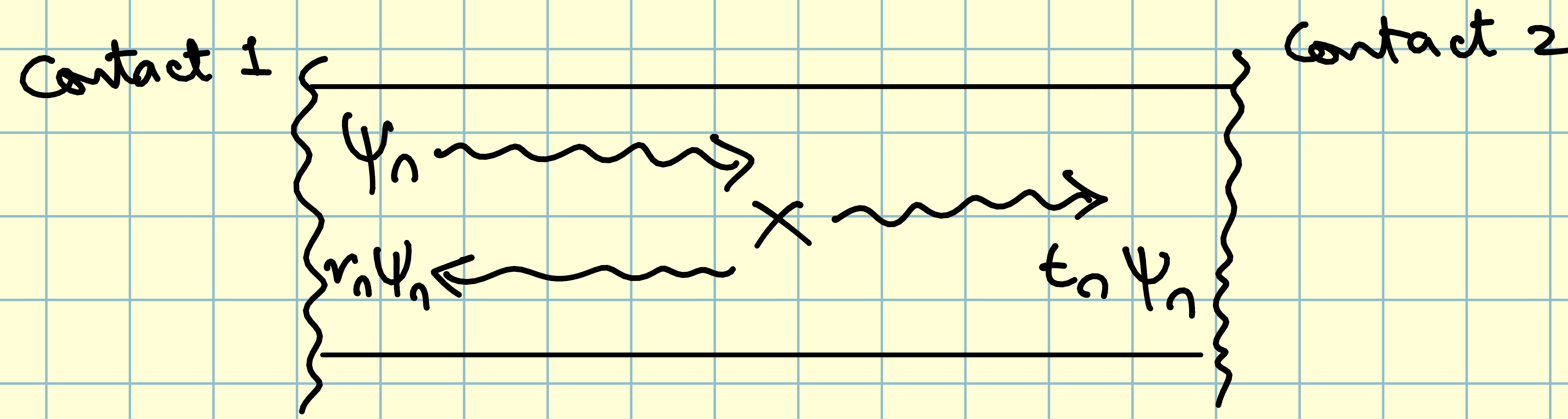
Landauer 2 terminal conductance:

$$G_{2T} = G_{21} = \frac{J}{V_{12}} = \frac{2e^2}{h} \cdot N$$

$$k_{x,n} = \frac{n\pi}{W} \qquad k_{x,n}^2 + k_{y,n}^2 = k_F^2 \qquad \Rightarrow \qquad k_{x,N} \leq k_F \leq k_{x,N+1}$$

$$k_{y,N} \ll k_F \Rightarrow k_{x,N} \simeq k_F \Rightarrow W = N \cdot \frac{\pi}{k_F} = \frac{N \pi}{2\pi/\lambda_F} = \frac{N \lambda_F}{2}$$

What happens if electrons are partially transmitted?



The transmitted wave : $t_n \, \psi_{n,k_n}$

reflected wave : $r_n \, \psi_{n,k_n}$

$$\sum_n \left(t_n^* t_n + r_n^* r_n \right) = \sum_n \left(T_n + R_n \right) = 1$$

Landauer formula:

$$J_p = \sum_{n}^{N} \frac{J_{e,n}}{V_{12}} = \frac{2e^2}{h} \sum_{n}^{N} T_n$$

where T_n is the transmission probability for mode 'i'.

Note :-

$$\boxed{\mu_1 = \mu_2 + qV_{12}}$$

Including temperature:

$$I = \frac{2q}{h} \sum_{n=1}^{N} \int dE \left(f_1(E) - f_2(E)\right) T_n(E)$$

at zero temperature, $f_1(E) = \Theta(\mu_1 - E)$

$$I = \frac{2q}{h} \sum_{n=1}^{N} \int_{-\infty}^{\infty} dE \left(\Theta(\mu_1 - E) - \Theta(\mu_2 - E) \right) T_n(E)$$

$$= \frac{2q}{h} \sum_{n=1}^{N} \left[\int_{-\infty}^{\mu_2 + qV} dE\ T_n(E) - \int_{-\infty}^{\mu_2} dE\ T_n(E) \right] = \frac{2q}{h} \sum_{n=1}^{N} \int_{0}^{qV} T_n(E)\, dE$$

What happens if we change N?

$$E_n(k_x) = E_n(k_y) + \frac{\hbar^2 k_{n,x}^2}{2m^*}$$

$$E_n(k_y) = \frac{\hbar^2}{2m^*} k_{n,y}^2 = \frac{\hbar^2}{2m^*} \left(\frac{n\pi}{W} \right)^2 = \frac{\hbar^2}{2m^*} \frac{n^2 \pi^2}{W^2}$$

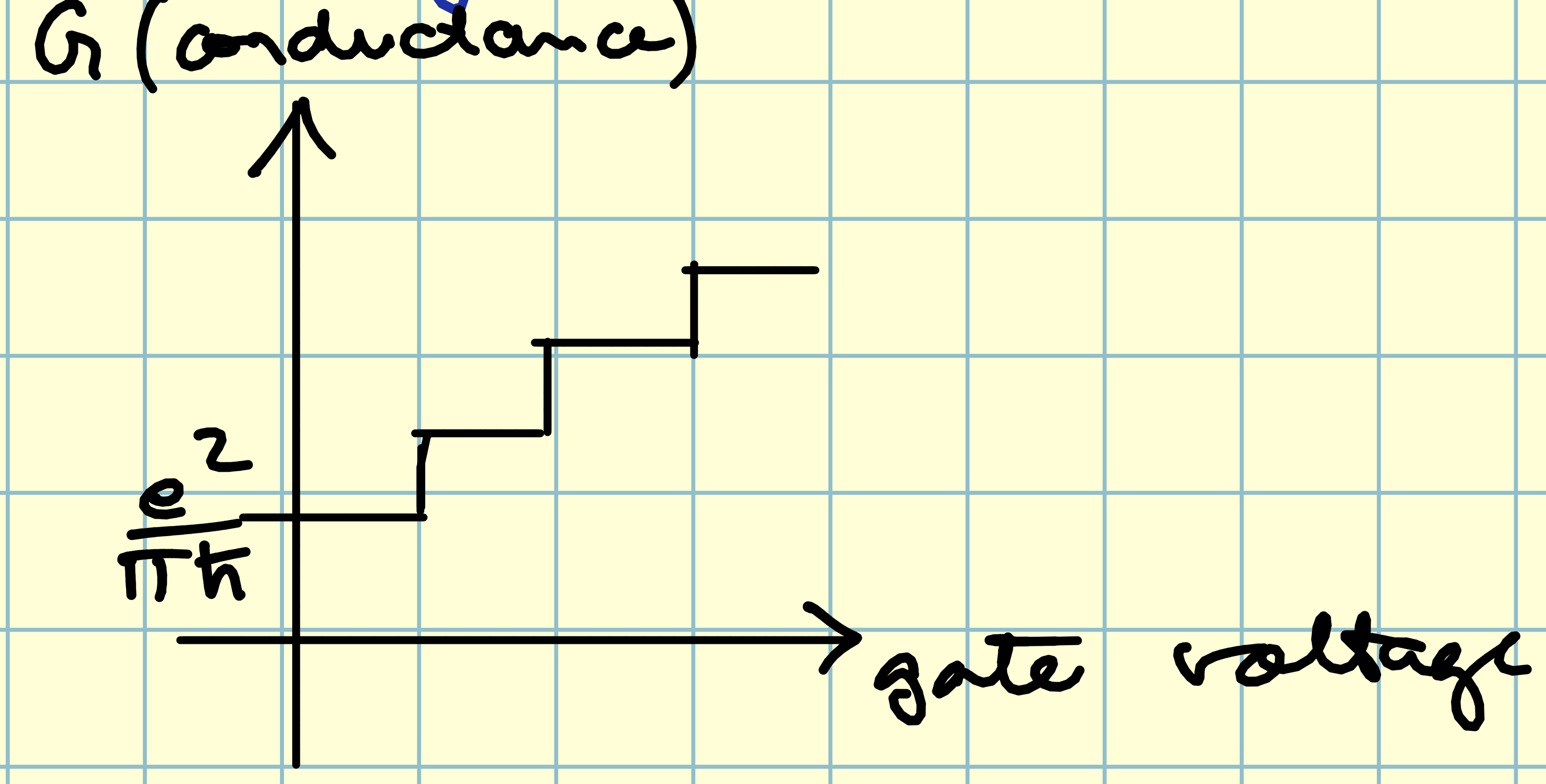
for $n = N$: $E_{n,y} = \frac{\hbar^2}{2m^*} \frac{N^2 \pi^2}{W^2} \simeq E_F$

$$k_F \simeq \frac{N\pi}{W} \quad \text{as} \quad k_{x,N} \ll k_F$$

$$W = \frac{N\pi}{k_F} = N\frac{\pi}{2\pi/\lambda_F} = \frac{N\lambda_F}{2}$$

Note:

A gate voltage can tune the width of a sample.



Total resistance of the system:

Total resistance =

$$R_{2T} = G_{2T}^{-1} = \frac{h}{2e^2} \cdot \frac{1}{\sum_n^N T_n}$$

for a single mode,

$$R_{2T} = \frac{h}{2e^2} \cdot \frac{1}{T}$$

Resistance of sample =

$$R_{2T} - R_{ideal} = \frac{h}{2e^2} \left(\frac{1}{T} - 1 \right)$$

$$R_{sample} = R_{4T} = \frac{h}{2e^2} \left(\frac{1-T}{T} \right)$$

for 'n' modes:

$$R_{2T}^{ideal} = \frac{h}{2e^2} \cdot \frac{1}{N} \quad , \quad R_{2T} = \frac{h}{2e^2} \cdot \frac{1}{\sum_{n}^{N} T_n}$$

$$R_{sample} = \frac{h}{2e^2} \left[\frac{1}{\sum_{n=1}^{N} T_n} - \frac{1}{N} \right]$$

$$= \frac{h}{2e^2} \frac{1}{N \sum_{n}^{N} T_n} \left[\sum_{n=1}^{N} 1 - \sum_{n=1}^{N} T_n \right] = \frac{h}{2e^2} \frac{\sum_{n=1}^{N} (1 - T_n)}{N \sum_{n=1}^{N} T_n}$$

If $T_1 = T_2 \cdots = T_N = T$

$$R_{4T} = R_{sample} = \frac{h}{2e^2} \frac{1}{N} \frac{(1-T)}{T}$$

Contact resistance:

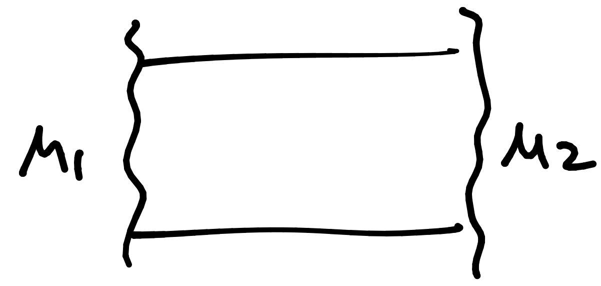
$$R_{2T} - R_{4T} = R_{contact}$$

$$R_{contact} = \frac{h}{2e^2} \cdot \frac{1}{N}$$

Summary

① Landauer formula

Let the sub band index due to confinement be n. The probe current for that band $J_{n,p}$ is given by:



$$J_{n,p} = q\int_{\mu_1}^{\mu_2} v_n(E)\,\rho_n(E)\,dE$$

$\Rightarrow$ Total current $J_p = \sum_{n=1}^{N} J_{n,p}$, where N is decided by the fermi energy, ie. the greatest natural number N s.t $k_{x,N} \le k_F$.

$$v_n(E) = \frac{1}{\hbar}\frac{dE}{dk}$$

$$\rho_n(E) = \frac{1}{L}\cdot\frac{dN}{dk}\times\frac{dk}{dE}$$

$$\Rightarrow v_n(E)\cdot\rho_n(E) = \frac{1}{\hbar}\cancel{\frac{dE}{dk}}\cdot\frac{1}{L}\frac{dN}{dk}\cdot\cancel{\frac{dk}{dE}}$$

In 1 - Dimension:

$$\frac{dN}{dk} = \frac{g_s}{2\pi/L} \Rightarrow v_n(E)\cdot\rho_n(E) = \frac{g_s}{h} = \frac{2}{h}$$

$$\Rightarrow J_{P,\text{Quasi1D}} = \sum_{n=1}^{N}\frac{2q}{h}\int_{\mu_1}^{\mu_2} 1\,dE = \frac{2q}{h}(\mu_2-\mu_1)N$$

$$\Rightarrow R_{\text{ideal, Quasi 1D}} = \frac{V_2-V_1}{J_{P,1D}} = \frac{\cancel{(V_2-V_1)}}{\frac{2(-e)(-e)\cancel{(V_2-V_1)}N}{h}} = \frac{h}{2e^2}\times\frac{1}{N}$$

Introducing transmission probability,

$$J_{P,\text{Quasi1D}} = \frac{2e^2}{h}(V_2-V_1)\sum_{n=1}^{N}T_n$$

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$$R_{2T,\,\text{Quasi 1D}} = \frac{h}{2e^2} \times \frac{1}{\sum_{n=1}^{N} T_n}$$

$$R_{4T} = R_{\text{ideal},\,1D} - R_{2T,\,1D}$$

$$\boxed{R_{4T,} = \frac{h}{2e^2}\,\frac{\sum_{n=1}^{N} (1 - T_n)}{\sum_{n=1}^{N} T_n}}$$

$$R_{\text{contact}} = R_{2T,\,1D} - R_{4T,\,1D}$$