Assignment - 6

Quantum Hall effect (QHE)

Ex 1.4 (S. Dutta)

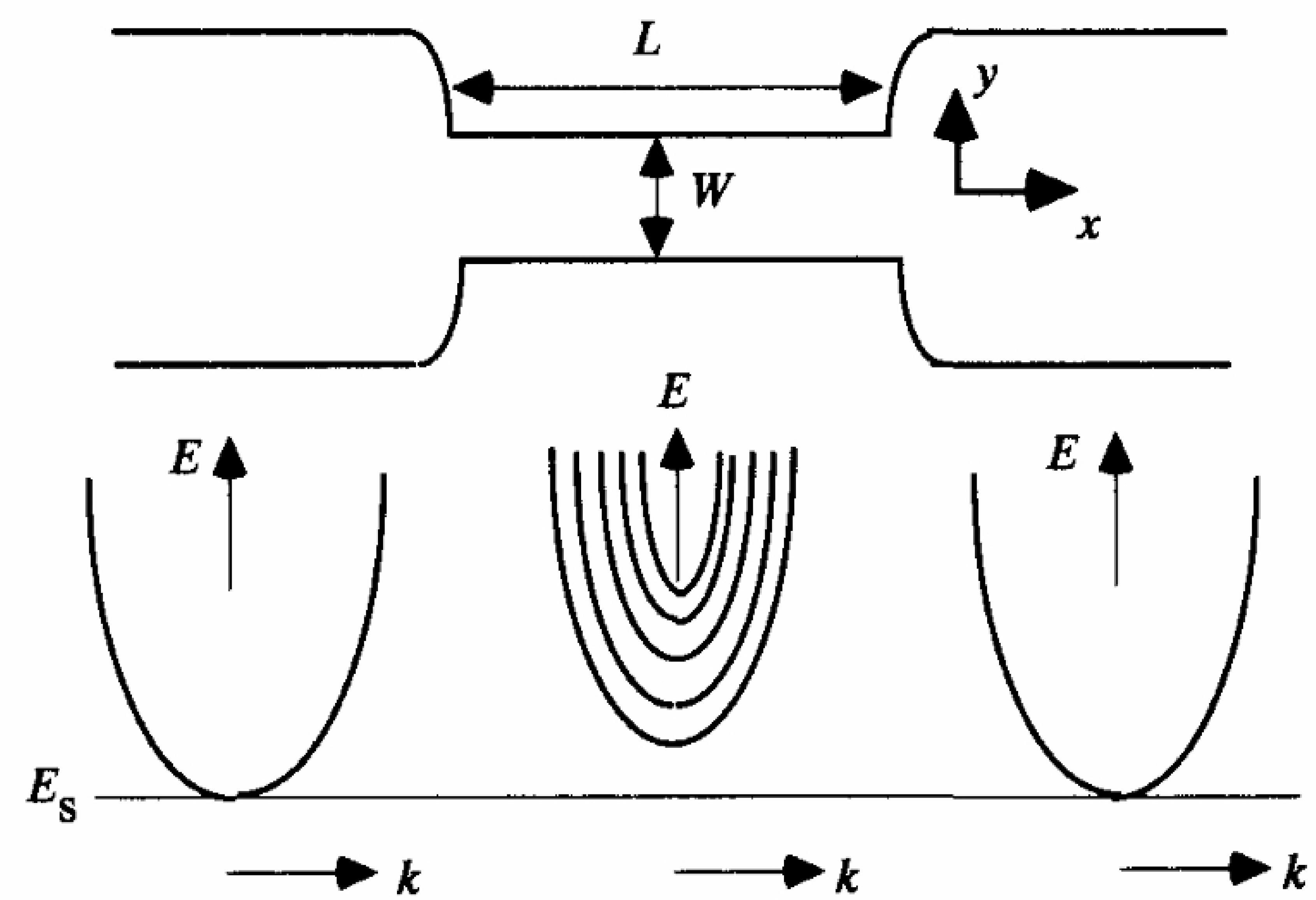


Fig. E.1.3. Narrow conductor etched out of a wide conductor. In the wide regions the transverse modes are essentially continuous, but in the narrow region the modes are well-separated in energy.

We have two wide 2D conductors on the left and right side of the etched narrow conductor. The parabolic confinement in y dir^n is given by :

$$U(y) = \frac{1}{2} m \omega_0^2 y^2 \quad , \text{ where } \quad \omega_0 = \left(\sqrt{\frac{8 E_{fs}}{m}}\right) \times \frac{1}{W} \quad ,$$

$$E_{n,k} = E_s + \left(n + \frac{1}{2}\right) \hbar \Omega + \frac{\hbar^2 k_x^2}{2m^*} \frac{\omega_0^2}{\Omega^2} \qquad - E1.4$$

$$\Omega^2 = \omega_0^2 + \omega_c^2 \quad , \qquad \omega_c^2 = \left(\frac{eB}{m^*}\right)^2$$

$B \to$ magnetic field in z-dirⁿ

Find no: of transverse modes as a functioi of magnetic field, B, assuming:

(a) constant Fermi surface

(b) constant electron density

Solutioi :

The number states in the etched conductor per unit length, n_L:

$$n_L = \frac{1}{L} \times 2 \times \frac{\text{Total volume occupied by modes in k space}}{\text{Volume occupied by 1 state in k space}}$$

$$= \frac{1}{\cancel{L}} \times 2 \times \sum_{i=1}^{i_{max}} \frac{k_{xi}}{2\pi/\cancel{L}} \qquad -(1)$$

where i_{max} denotes maximum no: of modes that can be occupied. From Eq. E1.4 :

$$k_{xi} = \frac{1}{\hbar} \frac{\Omega}{\omega_0} \sqrt{2m^*\left(E_f - E_s - \left(i + \frac{1}{2}\right)\hbar\Omega\right)} \qquad -(2)$$

Substituting (2) in (1) :

$$n_L = \frac{1}{\pi\hbar}\sum_{i=1}^{i_{max}} \frac{\Omega}{\omega_0}\left[\sqrt{2m^*\left(E_f - E_s - \left(i+\frac{1}{2}\right)\hbar\Omega\right)}\right]$$

Taking a factor of $\frac{\Omega}{\omega_0}$ outside the root:

$$n_L = \frac{1}{\pi\hbar}\left(\frac{\Omega}{\omega_0}\right)^{3/2}\sum_{i=1}^{i_{max}}\sqrt{2m^*\left(\frac{E_f - E_s}{\Omega/\omega_0} - \left(i+\frac{1}{2}\right)\hbar\omega_0\right)}$$

Let $E_1 = \frac{\hbar^2\pi^2}{2mW^2}$. Multiplying & dividing by $\sqrt{E_1}$,

$$n_L = \frac{1}{\pi\hbar}\left(\frac{\Omega}{\omega_0}\right)^{3/2}\cdot\sqrt{E_1}\sum_{i=1}^{i_{max}}\sqrt{\frac{E_f - E_s}{\Omega/\omega_0 E_1} - \left(i+\frac{1}{2}\right)\frac{\hbar\omega_0}{E_1}}$$

$$= \frac{1}{\cancel{\pi\hbar}}\left(\frac{\Omega}{\omega_0}\right)^{3/2}\cdot\frac{\cancel{\hbar}\cancel{\pi}}{\cancel{\sqrt{2m}}\,W}\cancel{\sqrt{2m}}\sum_{i=1}^{i_{max}}\sqrt{\frac{E_f - E_s}{\Omega/\omega_0 E_1} - \left(i+\frac{1}{2}\right)\frac{\hbar\omega_0}{E_1}}$$

$$\Rightarrow \frac{n_L \cdot W}{(\Omega/\omega_0)^{3/2}} = \sum_{i=1}^{i_{max}} \sqrt{\frac{E_f - E_s}{\Omega/\omega_0\, E_1} - \left(i+\frac{1}{2}\right)\frac{\hbar\omega_0}{E_1}}$$

(a) $E_f = \text{const}$, we need to find greatest integer, i_{max} s.t:

$$\left(i_{max} + \frac{1}{2}\right)\hbar\Omega \leq E_f - E_s$$

$$\Rightarrow i_{max} \leq \frac{E_f - E_s}{\hbar\Omega} - \frac{1}{2}$$

No: of modes, M is $i_{max} + 1$ as i_{max} ranges from 0 to n $n \in \mathbb{N}$.

$$\Rightarrow M \leq \frac{E_f - E_s}{\hbar\Omega} + \frac{1}{2} \quad \Rightarrow M = \left[\frac{E_f - E_s}{\hbar\Omega} + \frac{1}{2}\right]$$

where $[\cdot]$ is the greatest integer function.

$\hbar\omega_0 = 3.9\,meV$, $m^* = 0.067\,m_0$

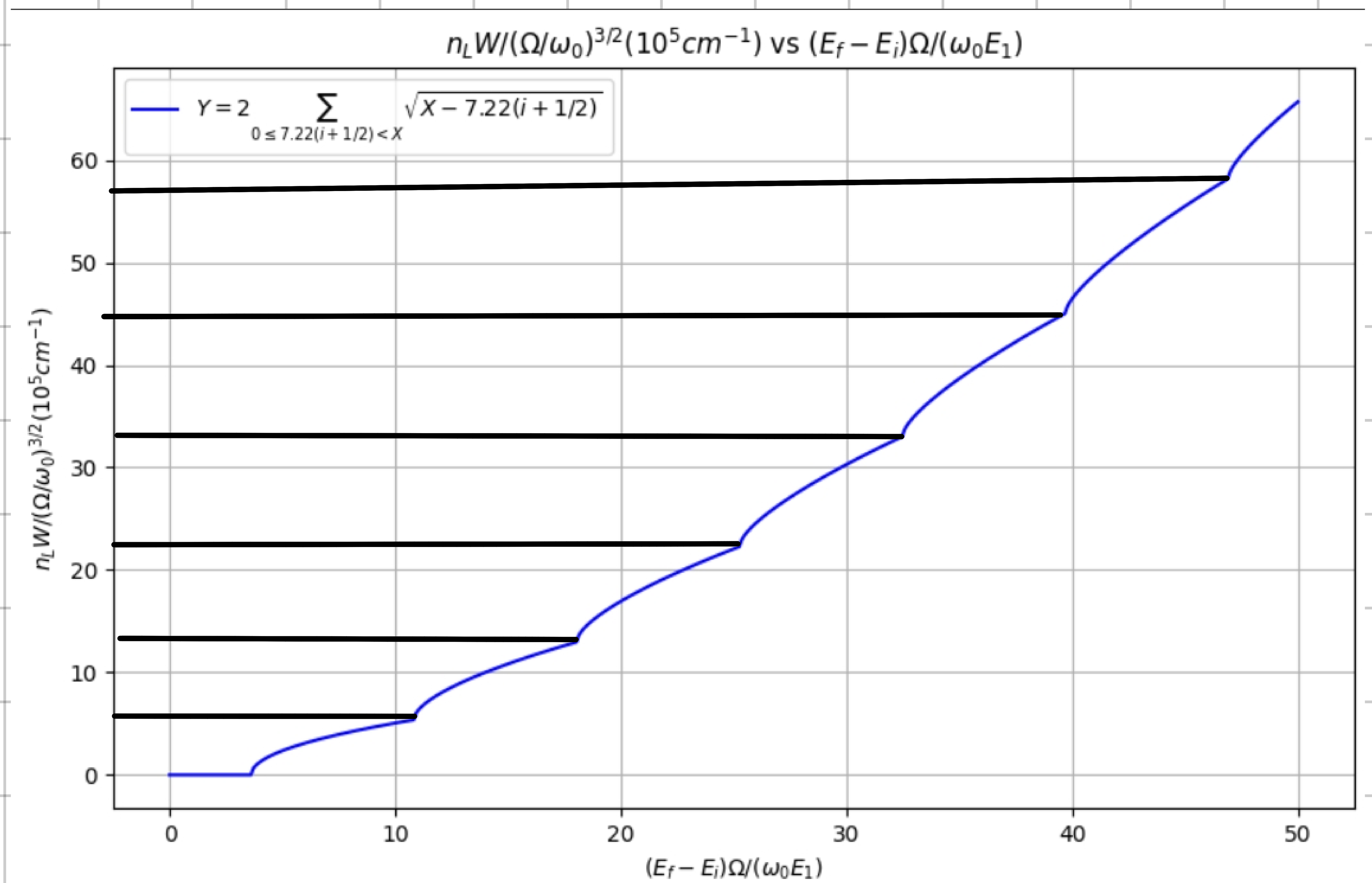
M	$\hbar\Omega$ bounds	B(T)
4	$4.9\,meV > \hbar\Omega > 3.9\,meV$	$1.7T > B$
3	$6.9\,meV > \hbar\Omega > 4.9\,meV$	$3.3T > B > 1.7T$
2	$11.5\,meV > \hbar\Omega > 6.9\,meV$	$6.2T > B > 3.3T$
1	$34.4\,meV > \hbar\Omega > 11.5\,meV$	$19.7T > B > 6.2T$
0	$\hbar\Omega > 34.4\,meV$	$> 19.7T$

This works only for small conductors

(b) Electron density n_L is constant. Hence LHS is constant.

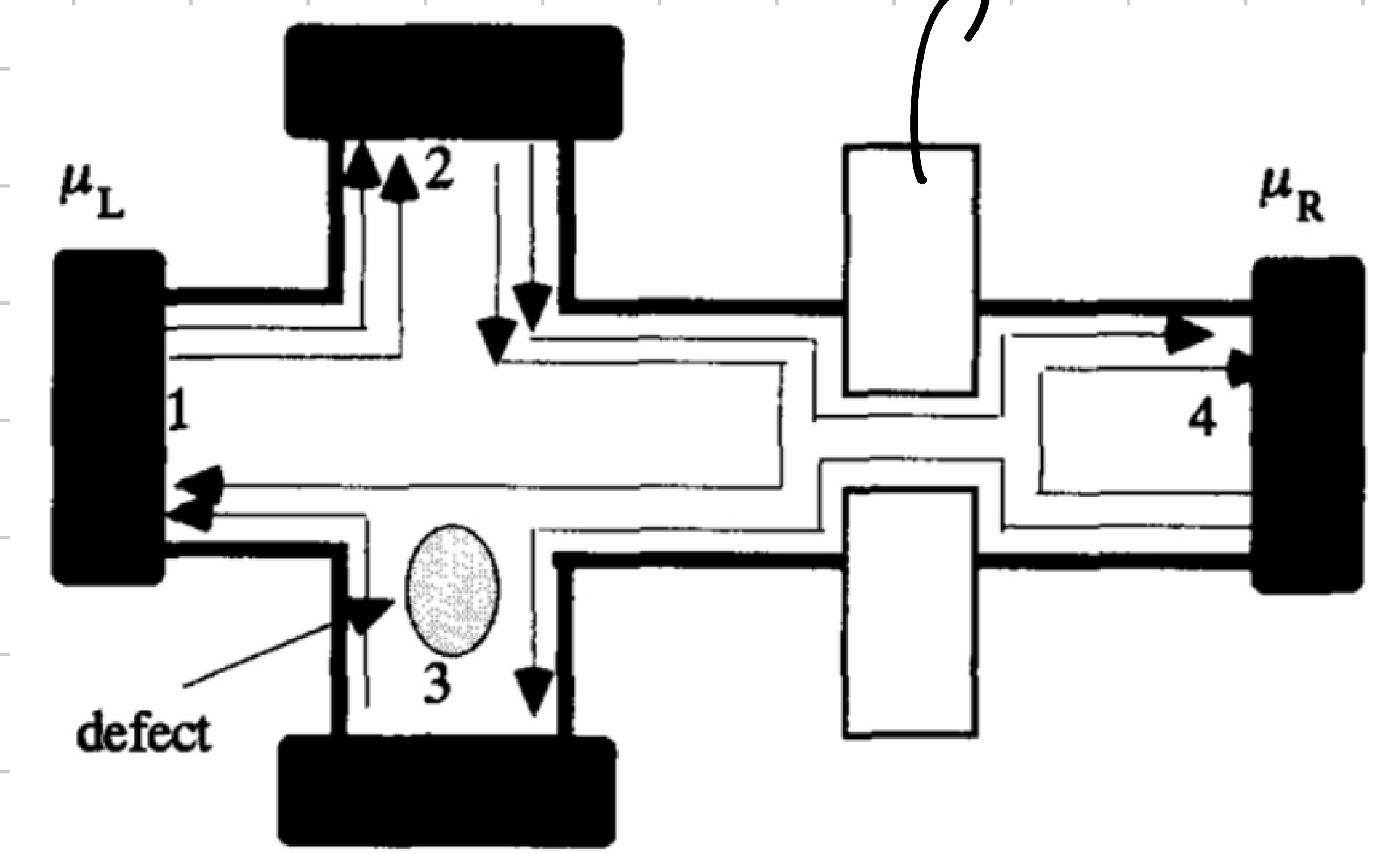
$$\frac{n_L \cdot W}{(\Omega/\omega_0)^{3/2}} = \sum_{i=1}^{i_{max}} \sqrt{\frac{E_f - E_s}{\Omega/\omega_0 \, E_1} - \left(i+\frac{1}{2}\right)\frac{\hbar\omega_0}{E_1}}$$

This means we have to calculate the no: of modes from the plot b/w $\frac{n_L \cdot W}{(\Omega/\omega_0)^{3/2}} = \nu$.



		(in Tesla)
$M=1$	$0 < \nu < 5$	$B > 10.2$
$M=2$	$5 < \nu < 13$	$5 < B < 10.2$
$M=3$	$13 < \nu < 22$	$3.1 < B < 5$
$M=4$	$22 < \nu < 33$	$1.9 < B < 3.1$
$M=5$	$33 < \nu < 45$	$0.9 < B < 1.9$
$M=6$	$45 < \nu < 58$	$B < 0.9$

4.2.1 (Dutta)



scattergates: allow some N modes to transmit out of M total modes

A Quantum Hall setup

M-N m

Let $\mu_L = \mu_4 = 0$. We have $\mu_3 = \mu_2 = 0$ & $\mu_1 = \mu_R$

Use

$$I_p = \sum_{q \neq p} G_{p \leftarrow q} (\mu_p - \mu_q)$$

to show that $R_H = \dfrac{h}{2e^2 M} \dfrac{1}{1-p}$ where $p = \dfrac{M-N}{M}$.

Solution :-

$$I_k = \frac{2e^2 M}{h} \sum_{q \neq k} T_{k \leftarrow q} (V_k - V_q)$$

The defect only sees the outer edge states & the remaining edges bypass it & go directly to 1.

T_{kq}	$q=1$	2	3	4
$k=1$	0	$\left(\frac{M-N}{M}\right)$	$\frac{N}{M}$	0
$k=2$	1	0	0	0
$k=3$	0	0	$\frac{M-N}{M}$	$\frac{N}{M}$
$k=4$	0	$\frac{N}{M}$	0	$\frac{M-N}{M}$

This is done to make sum of rows & column = 1.

$I_2 = 0 = I_3$ as 2 & 3 are voltage probes

$V_2 = V_1$, $V_3 = V_4$ as in Hall setup the edge modes equilibriamize the potential.

$$I_1 = \frac{2e^2}{h} \cdot M \left[T_{12} \cancel{(V_1 - V_2)}^{\,0} + T_{13}(V_1 - V_3) + T_{14}(V_1 - V_4) \right]$$

$$= \frac{2e^2}{h} M \left[\frac{N}{M} (V_2 - V_3) + 0 \right]$$

$$I_1 = \frac{2e^2}{h} M (1-p)(V_2 - V_3)$$

$$\boxed{R_H = \frac{V_2 - V_3}{I_1} = \frac{h}{2e^2 M} \times \frac{1}{1-p}}$$