

## SECTION 11

Quantum Hall effect in  
Graphene

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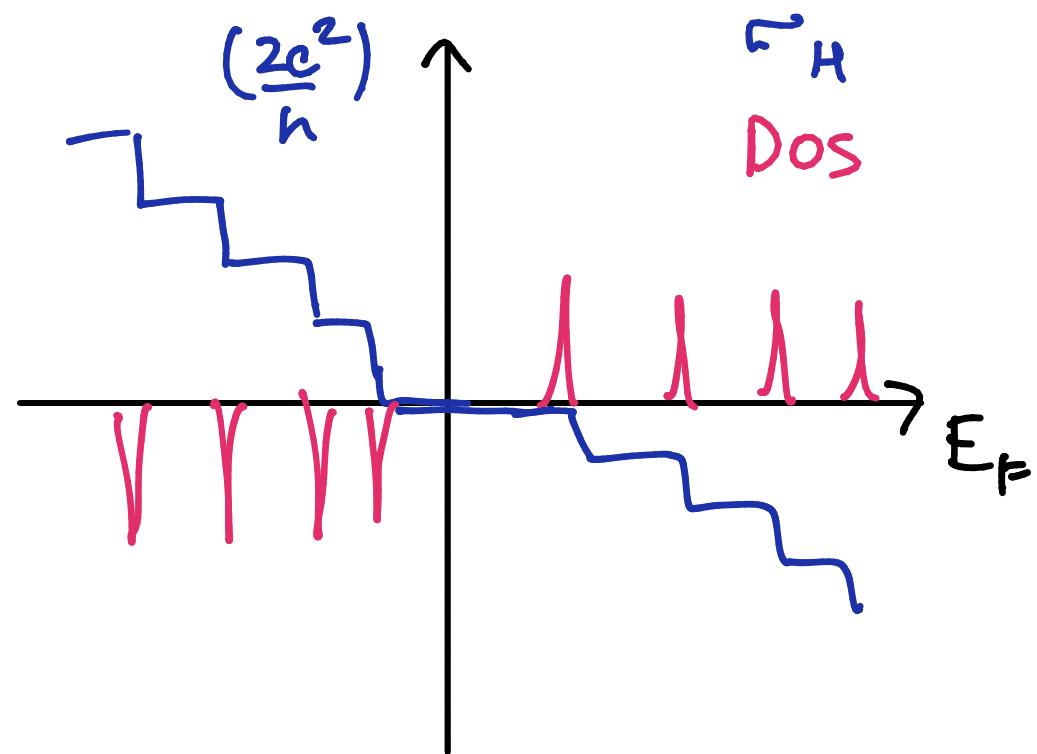
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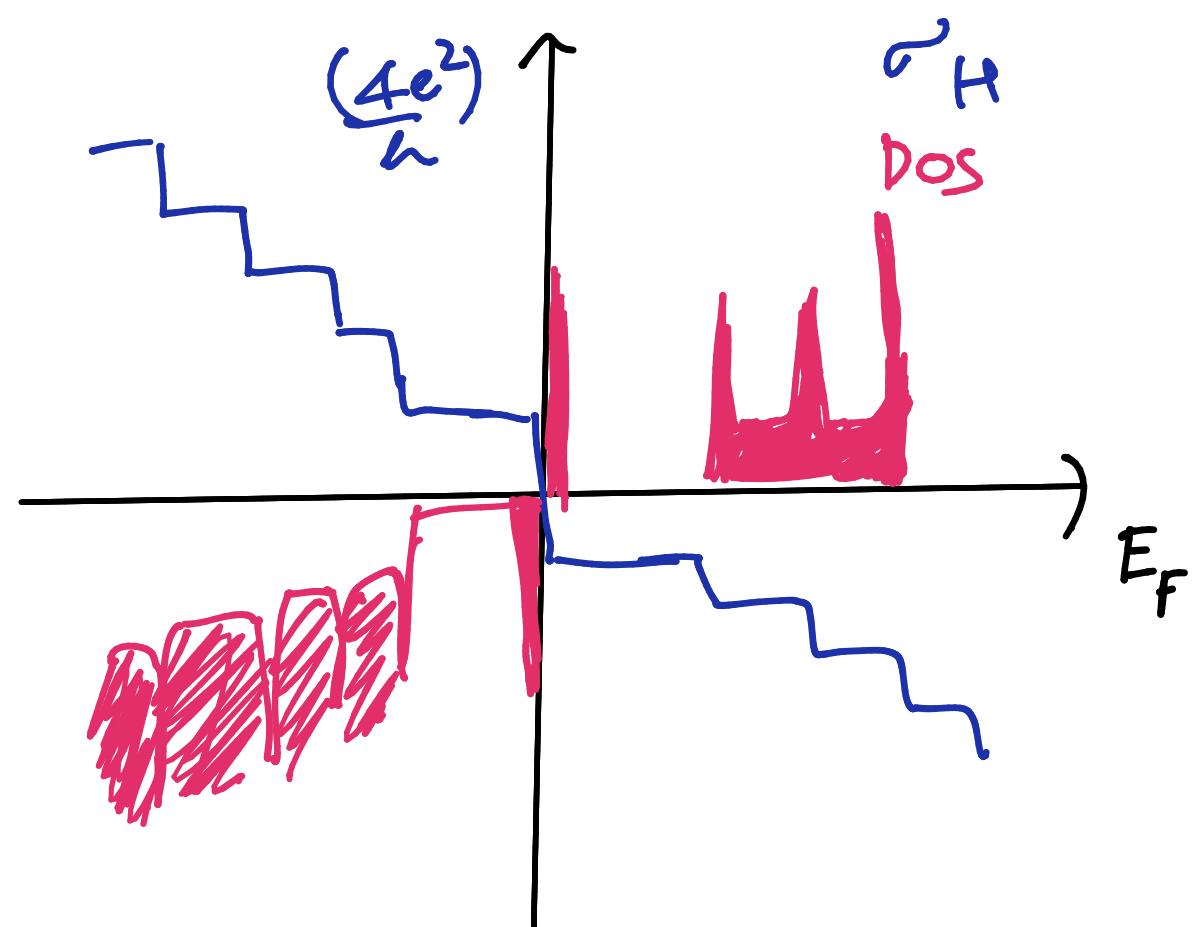
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2D E G



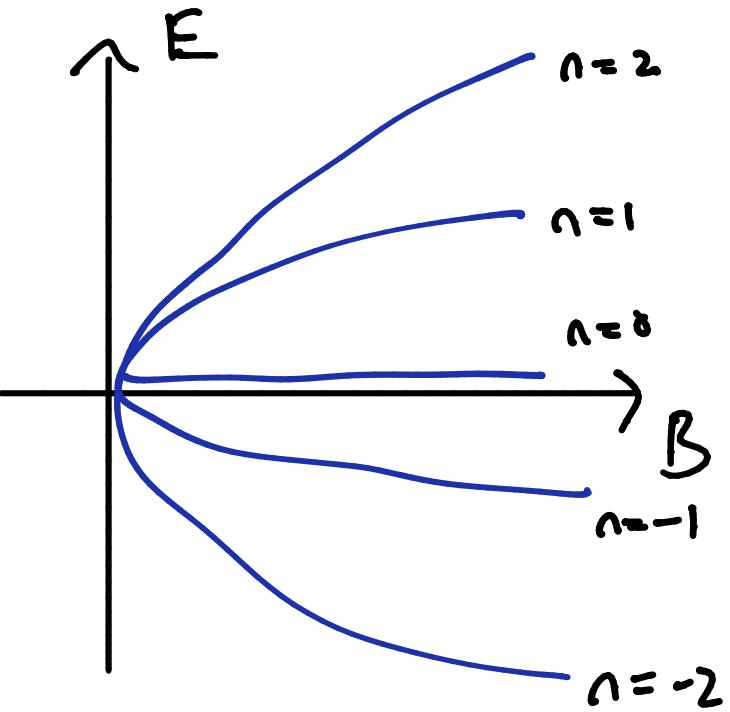
GRAPHENE



In 2DEG,  $R_H = \frac{h}{2e^2 M}$ , M GEN

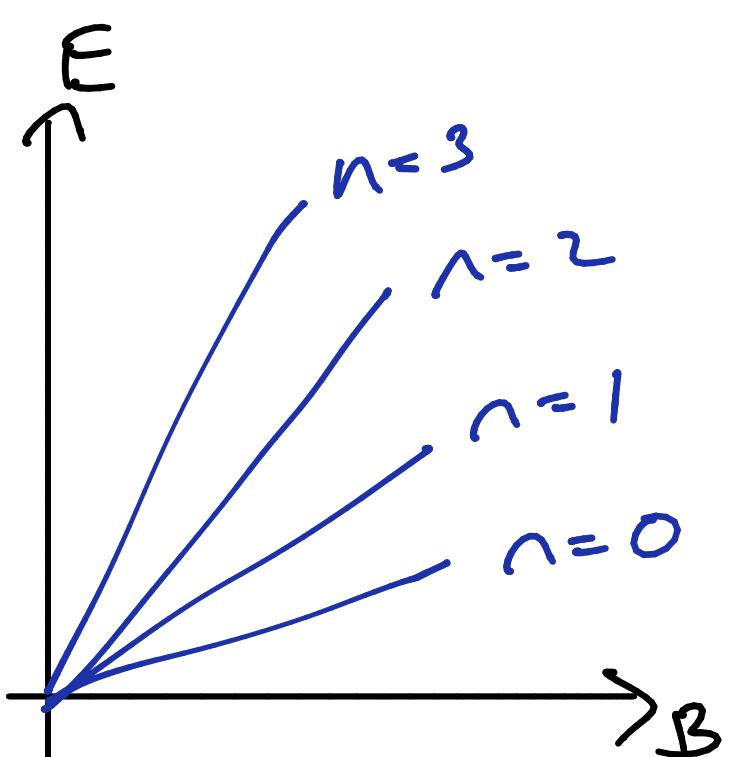
But  $R_H$  in graphene is :

$$R_{H_G} = \frac{h}{4e^2(M + \frac{1}{2})} \quad M \in \mathbb{W}$$



$E$  vs  $B$  for graphene

$$E_n = v_F \operatorname{sgn}(n) \sqrt{2e\hbar B} |n|$$



$E$  vs  $B$  for 2DEG

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega_c$$

1. Energy spacing of LL in graphene  $\Delta E_{LL} \propto \sqrt{B}$ . While energy spacing of LL in 2DEG,  $\Delta E \propto B$ .
2. Separation between  $n=0$  and  $n=1$ , LL in graphene is much larger than 2DEG.  
 $LL \propto \sqrt{n}$  (in graphene)  
 $LL \propto n$  (in 2DEG)

3. Each LL in graphene has twice as many electrons & holes as in 2DEG. There is 4 fold degeneracy in graphene due to both spin up & spin down along with valleys in  $K$  &  $K'$  contributing to conductance
4. The  $0^{\text{th}}$  LL in graphene @  $E=0$  is shared equally by both electrons
5. Electrons in graphene have very high mobility and as a consequence quantum hall effect in graphene can be seen at room temperatures.

$$\mu_{\text{Graph.}} @ 300K = 10^5 \text{ cm}^2 / (\text{V} \cdot \text{s})$$

$$\mu_{\text{2DEG}} @ 300K = 10^3 \text{ cm}^2 / \text{V} \cdot \text{s}$$

Condition to see QHE :  $\tau_c \ll \text{Jm}$

Mobility in graphene @  $T=300K$ ,

$$\mu_{\text{Graph}} = 10^5 \text{ cm}^2 / \text{V} \cdot \text{s} \quad ; \quad \mu_{\text{2DEG}} = 10^3 \text{ cm}^2 / \text{V} \cdot \text{s}$$

$$\frac{2\pi}{\omega_c} \ll \gamma_m \Rightarrow \omega_c \gg \frac{2\pi}{\gamma_m} \Rightarrow \frac{1}{2\pi} \omega_c \gg \frac{1}{\gamma_m}$$

$$\Rightarrow \hbar \omega_c \gg \frac{\hbar}{\gamma_m}$$

$$m = \frac{c \gamma_m}{m^*} \Rightarrow \gamma \omega_c \gg \frac{hc}{m^* \mu}$$

$$\Rightarrow \frac{eB}{m^*} \gg \frac{hc}{\mu} \Rightarrow B \gg \frac{2\pi}{\mu}$$

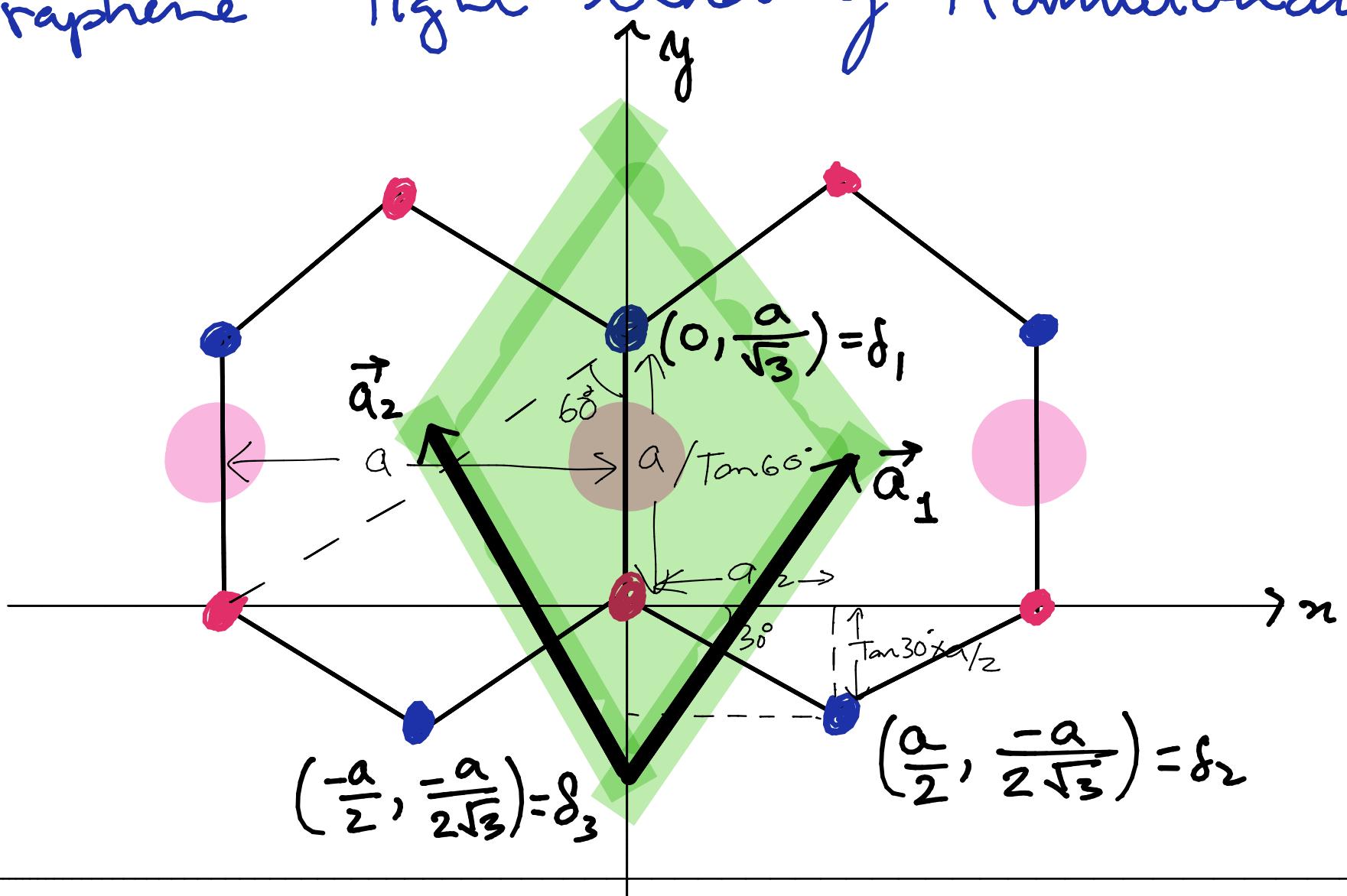
Graphene @ 300K ,

$$B \gg \frac{2\pi}{10^4 \text{ cm}^2/\text{V.s}} = \frac{2\pi}{10 \text{ m}^2/\text{V.s}} = 0.628 \text{ T}$$

2DEG @ 300K ,

$$B \gg 62.8 \text{ T}$$

Graphene tight-binding Hamiltonian:



KEY:- 1) There is a two atom basis that decorates each lattice point.  
2) The magnitude of lattice vector,  $\vec{R}$  is  $|\vec{R}| = a$

- $\rightarrow$  A sublattice
- $\rightarrow$  B sublattice
- $\rightarrow$  Lattice point

$\rightarrow$  Primitive unit cell (PUC);  $\vec{a}_1$  &  $\vec{a}_2$   $\rightarrow$  primitive unit vectors.

A and B atoms form a basis for honeycomb lattice, graphene.

## Step 1: Hamiltonian from tight-binding model

The graphene lattice hamiltonian can be modelled by a tight-binding hamiltonian with hopping between nearest neighbours A & B.

$$\hat{H} = -t \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{a}_i)$$

where  $a_i^\dagger$  creates an electron on an A site at position  $i$  &  $b_j^\dagger$  creates an electron on a B site at  $j$ . The sum is over nearest neighbour pairs  $\langle i,j \rangle$ .

## Step 2:- Fourier transform to momentum space:

Let  $N$  be the number of unit cells.

To write the annihilation operators at site A & B as a Fourier transform of momentum basis, we can consider the Wannier functions:

$$w(\vec{r}-\vec{R}) = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{R}} \quad \Psi_{\vec{R}}(\vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{i\vec{k}(\vec{r}-\vec{R})} u_{\vec{k}}(\vec{r})$$

where  $\vec{r}$  is the position vector  $\vec{R}$  is any lattice vector (linear combination of primitive lattice vectors,  $\vec{R} \in \{n\vec{a}_1 + m\vec{a}_2 : n, m \in \mathbb{Z}\}$ )

$\Psi_{\vec{R}}(\vec{r})$  is the Bloch wavefunction which can be written as  $\Psi_{\vec{R}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}(\vec{r})$  where  $u_{\vec{k}}(\vec{r} + \vec{R}) = u_{\vec{k}}(\vec{r})$ .

The corresponding operator in Wannier basis ( $\omega_{X,\vec{R}}$ ) is (annihilates a particle at  $\vec{R}$ )

$\hat{a}_i := \hat{\omega}_{A,\vec{R}} = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{i(\vec{k} \cdot \vec{R}_i)} \hat{a}_{\vec{k}}$ , where  $\hat{a}_{\vec{k}}$  is the momentum space annihilation operator for A sub lattice. Similarly,

$$\hat{b}_j := \hat{\omega}_{B,\vec{R}} = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{i(\vec{k} \cdot \vec{R}_j)} \hat{b}_{\vec{k}}$$

we have a  $e^{i\vec{k} \cdot \vec{R}_i}$  as we are doing an inverse Fourier transform from momentum space to real space. The  $i \neq j$  symbolize the different site index.

The wave function for the  $\vec{k}$ -subspace can be written as:

$$\Psi_{\vec{k}} = \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \end{pmatrix},$$

where  $a_{\vec{k}}$  &  $b_{\vec{k}}$  are the complex amplitudes for the A & B sub-lattice components.

Step 3:- Hamiltonian in the  $\vec{k}$ -momentum subspace

Since  $\Psi_{\vec{k}}$  has two components describing the different sub-lattices, the tight-binding hamiltonian can be written in  $\vec{k}$  basis:

$$H(\vec{k}) = \begin{bmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{bmatrix}, \text{ where } f(\vec{k}) = -t \sum_{j=1}^3 e^{i\vec{k} \cdot \vec{\delta}_j}$$

**Proof:**  
We found this form of the hamiltonian by substituting  $\vec{A}$ ,  $\vec{B}$  in  $\vec{R}$ . Let me [3] symbolize the 3 nearest neighbours.

$$\begin{aligned}\hat{a}_i^+ \hat{b}_{i+\delta_m} &= \frac{1}{N} \sum_{\vec{k}'} \sum_{\vec{k}} e^{-i\vec{k}' \cdot \vec{R}_i} e^{i\vec{k} \cdot \vec{R}_i + i\delta_m} \hat{a}_{\vec{k}'}^+ \hat{b}_{\vec{k}} \\ &= \frac{1}{N} \sum_{\vec{R}'} \sum_{\vec{R}} e^{i(\vec{R}-\vec{R}') \cdot \vec{R}_i} e^{i\delta_m} \hat{a}_{\vec{R}'}^+ \hat{b}_{\vec{R}}\end{aligned}$$

The Hamiltonian is:

$$\begin{aligned}H &= -t \sum_{i=1}^N \sum_{m=1}^3 [\hat{a}_i^+ \hat{b}_{i+\delta_m} + \hat{b}_{i+\delta_m}^+ \hat{a}_i] \\ &= -t \sum_{m=1}^3 \left[ \sum_{i=1}^N \sum_{\vec{k}'} \sum_{\vec{k}} e^{i(\vec{R}-\vec{R}') \cdot \vec{R}_i} e^{i\vec{R} \cdot \vec{\delta}_m} \hat{a}_{\vec{k}'}^+ \hat{b}_{\vec{k}} \right. \\ &\quad \left. + \sum_{i=1}^N \sum_{\vec{R}'} \sum_{\vec{R}} e^{i(\vec{R}'-\vec{R}) \cdot \vec{R}_i} e^{-i\vec{R}' \cdot \vec{\delta}_m} \hat{a}_{\vec{R}'}^+ \hat{b}_{\vec{R}} \right] \times \frac{1}{N} \\ &= -t \sum_{\vec{R}} \sum_{m=1}^3 \left[ e^{i(\vec{R} \cdot \vec{\delta}_m)} \hat{a}_{\vec{R}}^+ \hat{b}_{\vec{R}} + e^{-i(\vec{R} \cdot \vec{\delta}_m)} \hat{b}_{\vec{R}}^+ \hat{a}_{\vec{R}} \right] \\ &= -t \sum_{\vec{R}} (f(\vec{R}) \hat{a}_{\vec{R}}^+ \hat{b}_{\vec{R}} + f^*(\vec{R}) \hat{b}_{\vec{R}}^+ \hat{a}_{\vec{R}})\end{aligned}$$

$$\Rightarrow H = \sum_{\vec{R}} [\hat{a}_{\vec{R}}^+ \hat{b}_{\vec{R}}^+] \begin{bmatrix} 0 & f(\vec{R}) \\ f^*(\vec{R}) & 0 \end{bmatrix} \begin{bmatrix} \hat{a}_{\vec{R}} \\ \hat{b}_{\vec{R}} \end{bmatrix}$$

where, (after substituting  $\vec{\delta}_m$  in  $f(\vec{R}) = \sum_{m=1}^3 e^{i(\vec{R} \cdot \vec{\delta}_m)}$ ):

$$f(\vec{R}) = -t \left[ e^{ik_y a/\sqrt{3}} e^{i[k_x a/2 - k_y a/(\sqrt{3})]} + e^{-i(k_x a/2 + k_y a/(\sqrt{3}))} \right]$$

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$$\Rightarrow H(\vec{K}) = \begin{bmatrix} 0 & f(\vec{K}) \\ f^*(\vec{K}) & 0 \end{bmatrix}$$

To find eigen energies,

$$\Rightarrow |H(\vec{K}) - \lambda I| = 0$$

$$\Rightarrow \lambda^2 - |f(\vec{K})|^2 = 0 \Rightarrow E(\vec{\varepsilon}) = \pm |f(\vec{K})|$$


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Step 4:- Finding the Dirac points ( $E(\vec{\varepsilon})=0$ )

$$f\left[\vec{K} = \left(\frac{4\pi}{3a}, 0\right)\right] = -t \left[ e^{i0} + 2 \cos\left(\frac{2\pi}{3}\right) \right] = 0$$

The other point is  $K' = \left(-\frac{4\pi}{3a}, 0\right)$

where  $f(K') = 0$

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Step 5:- Linearizing near the K valley:

$$\vec{k} = \vec{K} + \vec{q} ; \vec{q} = (q_x, q_y) \text{ (small)}$$

$$f(\vec{K} + \vec{q}) \approx f(\vec{K}) + \left. \nabla_{\vec{K}} f \right|_{\vec{K}=\vec{K}} \cdot \vec{q}$$

$$\partial_{K_x} f = - \left[ 0 + \frac{i\alpha}{2} e^{i\left(k_x \frac{a}{2} - k_y \frac{a}{2\sqrt{3}}\right)} - \frac{i\alpha}{2} e^{-i\left(k_x \frac{a}{2} + k_y \frac{a}{2\sqrt{3}}\right)} \right]$$

$$\Rightarrow \partial_{k_x} f = -t a \frac{\sqrt{3}}{2}$$

$$\partial_{k_y} f = -t \left[ \frac{ia}{\sqrt{3}} e^{ik_y \frac{a}{\sqrt{3}}} + \frac{-ia}{2\sqrt{3}} e^{i(k_x \frac{a}{2} - k_y \frac{a}{2\sqrt{3}})} \right. \\ \left. - \frac{ia}{2\sqrt{3}} e^{-i(k_x \frac{a}{2} + k_y \frac{a}{2\sqrt{3}})} \right]$$

$$\Rightarrow \partial_{k_y} f = -t \frac{a\sqrt{3}}{2} i$$

$$\Rightarrow f(\vec{k} + \vec{a}) \approx -t \left[ a \frac{\sqrt{3}}{2} q_x - a \frac{\sqrt{3}}{2} q_y i \right]$$

Defining  $v_F = \frac{\sqrt{3}ta}{2\pi}$ ,

$$f(\vec{k} + \vec{a}) \approx \hbar v_F (q_x - iq_y)$$

The Hamiltonian of Graphene in 'K' valley:

$$1. H = \begin{bmatrix} 0 & \hbar v_F (k_x - ik_y) \\ \hbar v_F (k_x + ik_y) & 0 \end{bmatrix} = \hbar v_F \vec{\sigma} \cdot \vec{k}$$

2. Eigen energies:

$$E_{\pm} = \pm \hbar v_F \sqrt{k_x^2 + k_y^2}$$

3. Eigen vectors:

$$\Psi_{\pm} = \begin{pmatrix} 1 \\ \pm e^{i\theta} \end{pmatrix} e^{i(k_x x + k_y y)}$$

$$\theta = \tan^{-1} \left( \frac{k_y}{k_x} \right)$$

In presence of magnetic field:

$$H = \begin{bmatrix} 0 & \hbar v_F \Pi_- \\ \hbar v_F \Pi_+ & 0 \end{bmatrix}$$

$$\vec{B} = -\vec{B}_y$$

$$\Pi_+ = k_+ - \frac{q A_+}{\hbar}, \quad \Pi_- = k_- - \frac{q A_-}{\hbar}$$

$$k_+ = k_x + i k_y, \quad k_- = k_x - i k_y$$

$$A_+ = A_x + i A_y, \quad A_- = A_x - i A_y$$

Landau gauge:

$$\vec{A} = B_y \hat{x} \Rightarrow A_{\pm} = A_x = B_y$$

The spinors:

$$\psi = \begin{pmatrix} u_1(y) \\ u_2(y) \end{pmatrix} e^{ik_x x}$$

$$H \psi = E \psi$$

$$\Rightarrow \begin{bmatrix} 0 & \hbar v_F \Pi_- \\ \hbar v_F \Pi_+ & 0 \end{bmatrix} \begin{bmatrix} u_1(y) \\ u_2(y) \end{bmatrix} e^{ik_x x} = E \begin{bmatrix} u_1(y) \\ u_2(y) \end{bmatrix} e^{ik_x x}$$

$$\Rightarrow \hbar v_F \Pi_- u_2(y) e^{ik_x x} = E u_1(y) e^{ik_x x}$$

$$\hbar v_F \Pi_+ u_1(y) e^{ik_x x} = E u_2(y) e^{ik_x x}$$

$$\text{where } k_y = -i \frac{\partial}{\partial y}$$

$$\hbar v_F \left( k_x - \frac{\partial}{\partial y} - \frac{q}{\hbar} B_y \right) u_2(y) = E u_1(y) \quad -①$$

$$\hbar v_F \left( k_x + \frac{\partial}{\partial y} - \frac{q}{\hbar} B_y \right) u_1(y) = E u_2(y) \quad -②$$

$$\textcircled{1} \times \hbar v_F \left( k_x + \frac{\partial}{\partial y} - \frac{qB_y}{\hbar} \right)$$

$$E \hbar v_F \left( k_x + \frac{\partial}{\partial y} - \frac{qB_y}{\hbar} \right) u_1(y) = \hbar^2 v_F^2 \left( k_x + \frac{\partial}{\partial y} - \frac{qB_y}{\hbar} \right) \times \\ \left( k_x - \frac{\partial}{\partial y} - \frac{qB_y}{\hbar} \right) u_2(y)$$

$$E^2 u_2(y) = \hbar^2 v_F^2 \left[ k_x^2 u_2 - k_x \frac{\partial u_2}{\partial y} - k_x \frac{qB_y}{\hbar} u_2 + \frac{\partial}{\partial y} k_x u_2 \right. \\ \left. - \frac{\partial^2}{\partial y^2} u_2 - \frac{\partial}{\partial y} \frac{qB_y}{\hbar} u_2 - \frac{qB_y}{\hbar} k_x u_2 + \frac{qB_y}{\hbar} \frac{\partial u_2}{\partial y} \right. \\ \left. + \left( \frac{qB_y}{\hbar} \right)^2 u_2 \right] \quad (\text{using } \textcircled{2})$$

$$E^2 u_2(y) = \hbar^2 v_F^2 \left[ k_x^2 u_2 - \frac{2k_x qB_y}{\hbar} u_2 - \frac{\partial^2 u_2}{\partial y^2} - \cancel{\frac{qB}{\hbar} \frac{\partial u_2}{\partial y}} \right. \\ \left. + \left( \frac{qB_y}{\hbar} \right)^2 u_2 + \cancel{\frac{qB_y}{\hbar} \frac{\partial u_2}{\partial y}} - \frac{qB}{\hbar} u_2 \right]$$

$$E^2 u_2(y) = \hbar^2 v_F^2 \left[ k_x^2 - \frac{2k_x qB_y}{\hbar} - \frac{\partial^2}{\partial y^2} - \frac{qB}{\hbar} + \left( \frac{qB_y}{\hbar} \right)^2 \right] u_2$$

$$E^2 u_2(y) = \hbar^2 v_F^2 \left[ -\frac{\partial^2}{\partial y^2} + \left( k_x - \frac{qB_y}{\hbar} \right)^2 - \frac{qB}{\hbar} \right] u_2$$

$$\left[ -\frac{\partial^2}{\partial y^2} + \left( k_x - \frac{qB_y}{\hbar} \right)^2 \right] u_2 = \left( \frac{E}{\hbar v_F} \right)^2 u_2 + \frac{qB}{\hbar} u_2$$

$$\lambda_B^2 = \frac{\hbar}{qB} \quad q_k = \frac{\hbar k_x}{qB}$$

$$\left[ -\frac{\partial^2}{\partial y^2} + \left( \frac{qB}{\hbar} \right)^2 \left( \frac{\hbar k_x}{qB} - y \right)^2 \right] u_2 = \left[ \left( \frac{E}{\hbar v_F} \right)^2 + \frac{qB}{\hbar} \right] u_2$$

$$\Rightarrow \Sigma = \left( n + \frac{1}{2} \right) 2 \sqrt{\frac{1}{\lambda_B^2}} = \left( n + \frac{1}{2} \right) \frac{2}{\lambda_B^2}$$

$$\textcircled{5} \left[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} + \frac{1}{2} m^* \omega_c^2 (y - q_k)^2 \right] \psi = E \psi$$

$$\Rightarrow E_n = \left(n + \frac{1}{2}\right) \hbar \omega_c$$

$$\omega_c = 2 \sqrt{RE \cdot PE}$$

$$\varepsilon_n = (2n+1) \frac{1}{\lambda_B^2}$$

$$\left(\frac{E_n}{\hbar v_F}\right)^2 + \frac{1}{\lambda_B^2} = \frac{2n+1}{\lambda_B^2} = \varepsilon_n$$

$$\Rightarrow E_n^2 = \frac{2n}{\lambda_B^2} \cdot \hbar^2 v_F^2 \Rightarrow E_n = \frac{\hbar v_F \operatorname{sgn}(n)}{\lambda_B} \sqrt{2|n|}$$

$$n \in \mathbb{Z}, \lambda_B = \sqrt{\frac{\hbar}{qB}}$$

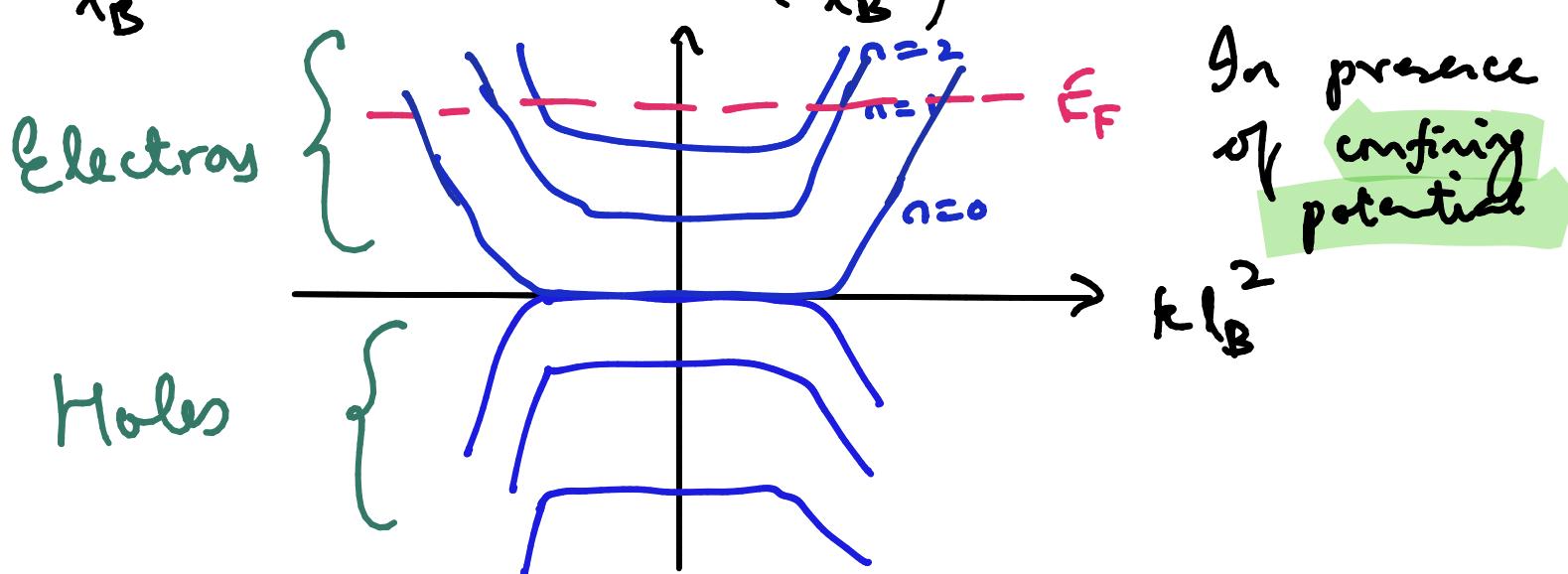
A Landau level @  $n=0$  is 0 energy which did not exist for 2DEG.

In 2DEG,  $n=0$  level corresponds to energy  $E_0 = \frac{1}{2} \hbar \omega_c$

$$u_2 = H_n \left[ \frac{y - y_R}{\lambda_B} \right] e^{-\frac{(y - y_R)^2}{2\lambda_B^2}}$$

$$u_1 = u_2 \left[ -\sqrt{2n} \left( \frac{H_{n-1}(z)}{H_n(z)} \right) \right]$$

$$z = \frac{y - y_R}{\lambda_B}, \quad \gamma_k = \frac{\hbar k_n}{qB} \varepsilon\left(\frac{\hbar v_F}{\lambda_B}\right)$$



If  $E_F$  comes from ' $M$ ' filled LL,  
we get  $R_M = \frac{h}{4e^2} \frac{1}{M}$

But there is a Landau level at energy which is half occupied by electrons & other half by holes.

Thus,  $R_H$  for  $0^{th}$  LL,

$$R_0 = \frac{h}{2e^2}$$

Total Hall resistance :

$$\frac{1}{R_H} = \frac{1}{R_0} + \frac{1}{R_M} \Rightarrow R_H = \frac{h}{4e^2(M + \frac{1}{2})}$$

Second quantisation method :

$$H = \frac{1}{2m^*} (\nabla_x^2 + \nabla_y^2)$$

$$\nabla_x = p_x + eA_x ; \quad \nabla_y = p_y + eA_y$$

Ladder operators :

$$a = \frac{\hbar_B}{\sqrt{2}\hbar} (\nabla_x - i\nabla_y) ; \quad a^\dagger = \frac{\hbar_B}{\sqrt{2}\hbar} (\nabla_x + i\nabla_y)$$

$$H = \frac{1}{2m^*} \left[ \frac{\hbar^2}{l_B^2} \frac{(a+a^\dagger)^2}{2} - \frac{\hbar^2}{l_B^2} \frac{(a-a^\dagger)^2}{2} \right] ; \quad l_B = \frac{\pi}{eB}$$

$$= \frac{\hbar^2}{4m^* l_B^2} [a a^\dagger + a^\dagger a] ; \quad [a, a^\dagger] = 1$$

$$H = \frac{\hbar^2}{2m^* l_B^2} (2\alpha^\dagger \alpha + 1) = \frac{\hbar^2}{m^* l_B^2} \left( \alpha^\dagger \alpha + \frac{1}{2} \right)$$

$$= \hbar \omega \left( \alpha^\dagger \alpha + \frac{1}{2} \right)$$

$$H\Psi_0 = E_0 \Psi_0$$

$$\alpha \Psi_0 = 0, E_0 = \frac{1}{2} \hbar \omega_c$$

$$\alpha^\dagger \alpha |n\rangle = n|n\rangle$$

$$\alpha^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$

$$\alpha |n\rangle = \sqrt{n} |n-1\rangle$$

$$\alpha |0\rangle = 0, \alpha^\dagger |0\rangle = |1\rangle \Rightarrow (\alpha^\dagger)^2 |0\rangle = \sqrt{2} |2\rangle$$

$$\langle n | H | n \rangle = (n + \frac{1}{2}) \hbar \omega_c = E_n$$

For graphene, Hamiltonian in terms of ladder operators:

$$H = v_F \begin{pmatrix} 0 & \pi n - i\pi y \\ \pi n + i\pi y & 0 \end{pmatrix} = \sqrt{2} \frac{\hbar v_F}{l_B} \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

$$= \hbar \omega \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

$$\Psi_n = \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

$$\Rightarrow \hbar \omega a u_n = E_n u_n$$

$$\hbar \omega' a^\dagger a_n = E_n v_n$$

$$\Rightarrow u_n = \frac{\hbar \omega'}{E_n} a v_n$$

$$\left(\frac{\hbar \omega'}{E_n}\right)^2 a^\dagger a v_n = E_n v_n$$

$$a^\dagger a v_n = \left(\frac{E_n}{\hbar \omega'}\right)^2 v_n$$

$$\text{as } a^\dagger a = n \Rightarrow E_n^2 = (\hbar \omega')^2 \cdot n$$

$$E_{\lambda, n} = \frac{\lambda \pi v_F}{\hbar} \sqrt{2n} \quad \lambda = \pm 1$$

Spinors :

$$\psi_{n=0} = \begin{pmatrix} 0 \\ |n=0\rangle \end{pmatrix} \quad n=0$$

$$\psi_{n \neq 0} = \begin{pmatrix} |n-1\rangle \\ \lambda |n\rangle \end{pmatrix} \quad n \neq 0$$

$$\text{Let } v_n \sim |n\rangle \quad \text{so} \quad a^\dagger a v_n = \left(\frac{E_n}{\hbar \omega'}\right)^2 v_n$$

$$\Rightarrow \hbar \omega' a v_n = E_n u_n$$

$$\Rightarrow u_n \sim a v_n$$

Thus,  $a|n\rangle = |n-1\rangle$  therefore if

$$v_n \propto |n\rangle \Rightarrow u_n \propto |n-1\rangle$$

For the 0th LL,  $a|0\rangle = 0$

Thus,

$$|\Psi_{n=0}\rangle = \begin{pmatrix} u_{n=0} \\ v_{n=0} \end{pmatrix} = \begin{pmatrix} 0 \\ |n=0\rangle \end{pmatrix}$$

in general,

$$|\Psi_{\lambda, n \neq 0}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |n-\lambda\rangle \\ \lambda |n\rangle \end{pmatrix}$$

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